Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.7-Trig-functions

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May 23, 2020 Compiled on May 23, 2020 at 8:33am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [950]. This is test number [141].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.37 (944)	% 0.63 (6)
Mathematica	% 98.74 (938)	% 1.26 (12)
Maple	% 95.47 (907)	% 4.53 (43)
Maxima	% 65.47 (622)	% 34.53 (328)
Fricas	% 88.95 (845)	% 11.05 (105)
Sympy	% 41.26 (392)	% 58.74 (558)
Giac	% 73.26 (696)	% 26.74 (254)

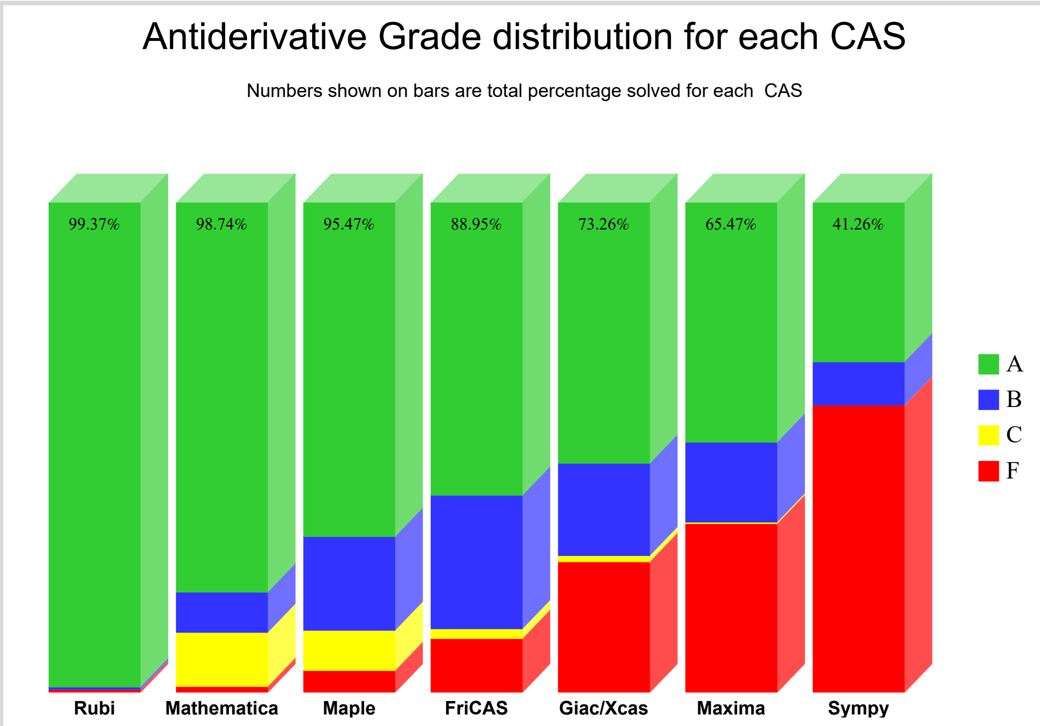
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

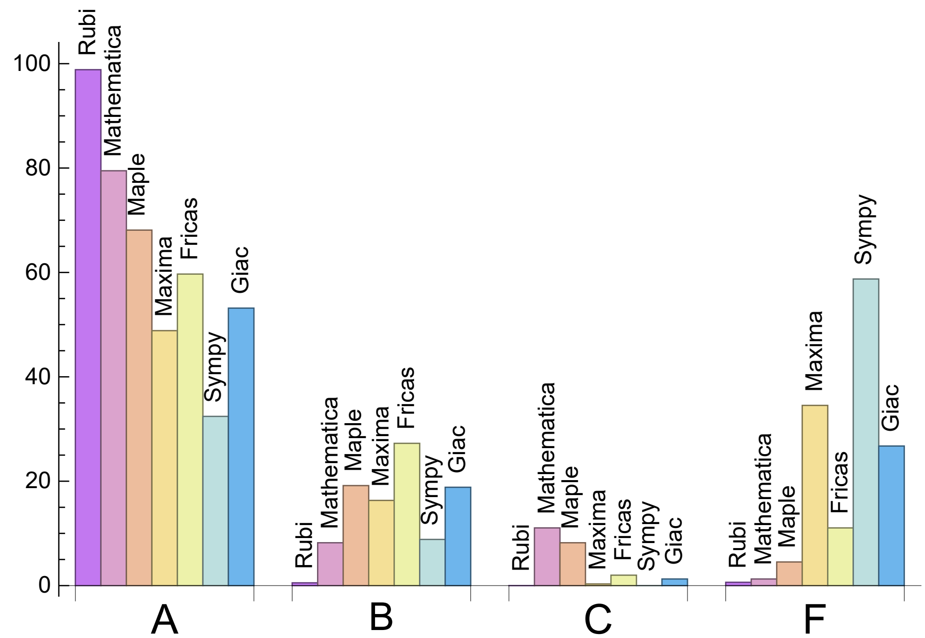
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.84	0.53	0.	0.63
Mathematica	79.47	8.21	11.05	1.26
Maple	68.11	19.16	8.21	4.53
Maxima	48.84	16.32	0.32	34.53
Fricas	59.68	27.26	2.	11.05
Sympy	32.42	8.84	0.	58.74
Giac	53.16	18.84	1.26	26.74

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	76.23	1.01	43.5	1.
Mathematica	1.06	373.47	3.96	40.	1.
Maple	0.51	702.24	4.33	45.	1.16
Maxima	1.19	123.67	2.85	38.	1.34
Fricas	2.49	459.7	5.53	128.	3.89
Sympy	8.33	141.8	5.01	32.	1.34
Giac	1.21	191.4	3.15	52.	1.51

1.4 list of integrals that has no closed form antiderivative

{42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 32, 36, 37, 38, 46, 47, 51, 52, 87, 89, 93, 107, 108, 109, 160, 163, 240, 241, 242, 243, 244, 245, 246, 247, 269, 271, 393, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 497, 556, 557, 558, 559, 560, 561, 566, 588, 597, 630, 859}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

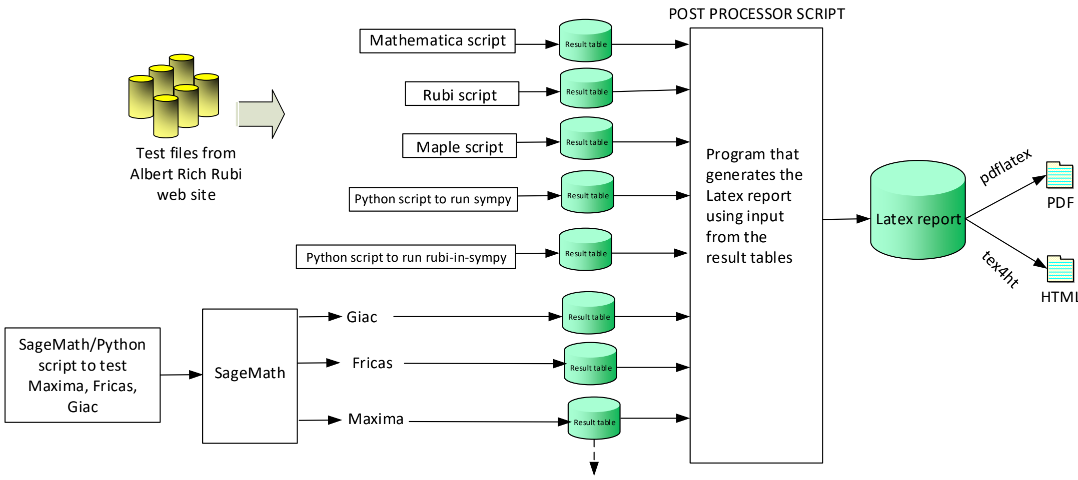
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 913, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 932, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 555, 759, 858, 860, 912 }

C grade: { }

F grade: { 796, 859, 914, 915, 931, 933}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 178, 179, 180, 181, 183, 184, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 268, 270, 272, 273, 275, 276, 279, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 344, 345, 346, 347, 348, 349, 350, 352, 353, 357, 358, 359, 360, 363, 364, 365, 367, 368, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 385, 386, 388, 389, 390, 392, 393, 395, 396, 397, 398, 399, 400, 401, 417, 418, 419, 423, 424, 425, 426, 443, 444, 445, 446, 447, 458, 459, 460, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 637, 639, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 782, 783, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 804, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 827, 828, 829, 830, 831, 832, 833, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 946, 947, 948, 949, 950 }

B grade: { 35, 90, 106, 108, 110, 114, 123, 127, 160, 163, 185, 189, 263, 267, 269, 271, 274, 277, 278, 280, 281, 287, 297, 306, 310, 312, 325, 329, 331, 341, 343, 361, 362, 366, 369, 370, 378, 380, 384, 387, 391, 394, 402, 461, 475, 497, 548, 549, 581, 638, 640, 654, 673, 677, 691, 705, 709, 710, 711, 712, 713, 728, 759, 760, 781, 784, 802, 803, 805, 806, 807, 826, 834, 861, 885, 904, 927, 945 }

C grade: { 31, 34, 36, 37, 38, 46, 51, 52, 62, 85, 87, 89, 105, 107, 109, 112, 125, 174, 175, 176, 182, 228, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 351, 354, 355, 356, 379, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 420, 421, 422, 427, 428, 429, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 503, 510, 511, 512, 513, 514, 517, 556, 557, 558, 559, 560, 561, 588, 597, 634, 635, 636, 657, 910, 912 }

F grade: { 435, 436, 441, 442, 453, 454, 456, 457, 467, 468, 470, 471}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 150, 151, 152, 159, 162, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 201, 202, 203, 204, 206, 207, 208, 209, 211, 212, 213, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 341, 342, 343, 344, 345, 346, 347, 348, 350, 351, 353, 354, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 436, 437, 438, 442, 444, 446, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 490, 491, 492, 496, 499, 500, 504, 505, 506, 510, 511, 514, 516, 518, 519, 520, 522, 523, 524, 526, 527, 528, 530, 531, 533, 534, 567, 568, 569, 570, 571, 576, 582, 583, 584, 590, 591, 593, 599, 600, 602, 603, 604, 605, 606, 611, 612, 613, 614, 637, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 723, 724, 725, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 749, 750, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 835, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 861, 862, 863, 866, 867, 868, 869, 870, 873, 874, 875, 876, 877, 879, 880, 881, 882, 883, 884, 886, 887, 889, 890, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 913, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 934, 935, 936, 937, 938, 939, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 76, 78, 145, 146, 147, 148, 149, 160, 161, 163, 164, 193, 194, 195, 196, 197, 198, 199, 200, 205, 210, 214, 215, 219, 230, 249, 250, 251, 255, 271, 273, 274, 275, 291, 293, 294, 295, 333, 334, 335, 336, 337, 338, 339, 340, 349, 352, 355, 361, 362, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 400, 401, 402, 410, 411, 412, 413, 414, 415, 416, 433, 434, 435, 439, 440, 441, 443, 445, 447, 458, 497, 498, 501, 502, 503, 507, 508, 509, 512, 513, 517, 521, 525, 529, 532, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 572, 573, 574, 575, 577, 578, 579, 580, 581, 589, 598, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 638, 639, 640, 670, 721, 722, 729, 759, 793, 795, 834, 836, 864, 871, 872, 885, 888, 891, 904, 926, 933, 940, 941 }

C grade: { 34, 139, 140, 141, 142, 153, 154, 155, 156, 157, 158, 319, 320, 321, 403, 404, 405, 406, 407, 408, 409, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 487, 488, 489, 493, 494, 495, 515, 588, 597, 633, 634, 635, 636, 709, 710, 711, 712, 713, 741, 742, 743, 744, 745, 746, 747, 748, 751, 776, 796, 809, 842, 859, 860, 878, 912, 914, 927 }

F grade: { 39, 40, 41, 53, 54, 55, 60, 79, 115, 124, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 217, 218, 318, 566, 585, 586, 587, 592, 594, 595, 596, 601, 657, 865, 918 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 44, 45, 48, 49, 50, 56, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 121, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 180, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 207, 208, 212, 213, 220, 221, 222, 223, 224, 225, 227, 229, 231, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 274, 276, 277, 279, 283, 284, 285, 286, 287, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 312, 322, 323, 324, 325, 327, 329, 331, 333, 341, 342, 343, 344, 345, 346, 347, 348, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 504, 505, 510, 511, 512, 514, 515, 516, 517, 518, 519, 522, 523, 528, 567, 568, 569, 582, 583, 584, 590, 599, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 667, 668, 669, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 718, 719, 720, 721, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 745, 746, 749, 753, 754, 755, 756, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 798, 799, 800, 801, 802, 803, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 843, 844, 845, 852, 854, 857, 862, 863, 864, 867, 868, 869, 870, 871, 872, 873, 880, 881, 882, 883, 884, 885, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 908, 909, 910, 911, 913, 914, 916, 917, 920, 923, 924, 925, 926, 929, 932, 934, 935, 936, 937, 940, 941, 943, 945, 946, 948, 950

B grade: { 61, 74, 80, 81, 82, 85, 86, 90, 91, 92, 105, 110, 111, 112, 116, 117, 122, 123, 124, 125, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 189, 190, 219, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 268, 270, 272, 273, 275, 278, 280, 281, 282, 288, 290, 292, 293, 294, 295, 298, 307, 309, 311, 313, 314, 315, 316, 317, 326, 328, 330, 332, 334, 335, 336, 377, 379, 384, 385, 386, 387, 391, 392, 393, 394, 444, 513, 529, 531, 589, 591, 593, 598, 600, 602, 607, 608, 609, 610, 615, 616, 654, 657, 670, 676, 715, 722, 728, 750, 752, 757, 761, 770, 771, 796, 797, 804, 805, 810, 827, 829, 836, 841, 842, 853, 855, 856, 858, 859, 860, 866, 874, 876, 877, 878, 879, 886, 887, 888, 904, 915, 918, 919, 927, 928, 930, 933, 944, 947, 949 }

C grade: { 751, 907, 921 }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 43, 46, 47, 51, 52, 53, 54, 55, 57, 63, 64, 65, 75, 76, 77, 78, 79, 83, 84, 87, 88, 89, 93, 94, 106, 107, 108, 109, 113, 114, 115, 118, 119, 120, 126, 127, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 197, 198, 199, 200, 205, 206, 209, 210, 211, 214, 215, 216, 217, 218, 226, 228, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 269, 271, 289, 291, 318, 319, 320, 321, 337, 338, 339, 340, 349, 350, 351, 352, 353, 354, 360, 361, 362, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 482, 483, 496, 497, 498, 501, 502, 503, 506, 507, 508, 509, 520, 521, 524, 525, 526, 527, 530, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 592, 594, 595, 596, 597, 601, 603, 604, 605, 606, 611, 612, 613, 614, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 649, 666, 693, 717, 743, 744, 747, 748, 762, 846, 847, 848, 849, 850, 851, 861, 865, 875, 906, 912, 922, 931, 938, 939, 942 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 36, 37, 38, 42, 43, 44, 45, 48, 50, 51, 52, 56, 57, 58, 59, 61, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 86, 88, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 111, 113, 117, 121, 124, 126, 129, 130, 132, 134, 136, 138, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 227, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 270, 273, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 288, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 307, 309, 310, 311, 313, 314, 315, 316, 317, 321, 322, 323, 324, 326, 333, 334, 335, 336, 337, 341, 342, 344, 345, 346, 348, 355, 356, 357, 358, 359, 360, 363, 364, 365, 370, 371, 372, 373, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 417, 418, 419, 423, 424, 425, 426, 430, 431, 432, 433, 437, 438, 439, 444, 447, 461, 472, 473, 474, 476, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 501, 504, 505, 506, 507, 508, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 529, 532, 538, 539, 543, 544, 548, 549, 553, 554, 555, 562, 563, 567, 568, 569, 570, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 641, 642, 644, 645, 646, 647, 648, 649, 654, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 696, 700, 705, 706, 708, 715, 717, 723, 724, 725, 726, 727, 729, 730, 731, 734, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 794, 796, 799, 800, 802, 803, 804, 806, 807, 808, 809, 811, 812, 813, 814, 815, 817, 819, 820, 821, 822, 823, 824, 827, 828, 829, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 866, 867, 874, 880, 881, 882, 883, 884, 886, 887, 889, 890, 891, 892, 893, 894, 895, 896, 898, 900, 901, 903, 905, 907, 908, 909, 910, 911, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 929, 930, 933, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 49, 62, 64, 65, 74, 78, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 93, 94, 105, 106, 108, 109, 110, 112, 114, 116, 118, 119, 120, 122, 123, 125, 127, 128, 131, 133, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 154, 156, 159, 160, 162, 163, 195, 196, 197, 198, 209, 214, 215, 216, 219, 226, 228, 229, 230, 231, 267, 269, 271, 272, 274, 281, 283, 287, 289, 291, 304, 306, 308, 312, 318, 319, 320, 325, 327, 328, 329, 330, 331, 332, 338, 339, 340, 343, 347, 349, 350, 351, 352, 353, 354, 361, 362, 366, 367, 368, 369, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 399, 400, 401, 402, 420, 421, 422, 427, 428, 429, 443, 445, 446, 458, 459, 460, 475, 477, 478, 479, 480, 496, 497, 502, 503, 509, 513, 521, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 564, 565, 571, 572, 581, 592, 593, 601, 602, 608, 616, 633, 634, 638, 639, 640, 643, 650, 651, 652, 653, 655, 657, 667, 668, 669, 670, 671, 673, 674, 675, 676, 677, 690, 691, 694, 695, 697, 698, 699, 701, 702, 703, 704, 707, 709, 710, 711, 712, 713, 714, 716, 718, 719, 720, 721, 722, 728, 732, 733, 738, 752, 759, 775, 791, 793, 795, 797, 798, 801, 805, 810, 816, 818, 825, 826, 836, 846, 847, 859, 862, 863, 864, 868, 871, 877, 885, 888, 897, 899, 902, 904, 906, 912, 913, 924 }

C grade: { 31, 32, 46, 47, 161, 164, 184, 498, 579, 580, 751, 869, 870, 872, 873, 875, 876, 878, 879 }

F grade: { 34, 39, 40, 41, 53, 54, 55, 60, 63, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 434, 435, 436, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 865, 931, 932, 938, 939 }

2.1.6 Sympy

A grade: { 1, 8, 15, 17, 18, 19, 22, 29, 30, 33, 35, 42, 44, 45, 48, 49, 50, 56, 58, 59, 62, 66, 67, 68, 69, 70, 71, 72, 73, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 124, 135, 136, 137, 138, 180, 185, 186, 187, 188, 203, 204, 219, 220, 221, 222, 223, 224, 225, 248, 249, 250, 251, 252, 253, 254, 255, 256, 264, 265, 266, 267, 268, 270, 272, 274, 276, 277, 285, 286, 287, 296, 303, 304, 305, 322, 323, 324, 341, 342, 343, 344, 355, 356, 357, 358, 363, 364, 365, 366, 370, 371, 372, 374, 375, 376, 377, 381, 382, 383, 384, 388, 389, 390, 391, 395, 396, 397, 398, 444, 479, 480, 481, 482, 483, 499, 500, 510, 511, 526, 527, 528, 529, 538, 539, 543, 544, 548, 549, 553, 554, 562, 567, 568, 569, 582, 589, 590, 598, 599, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 655, 656, 659, 660, 661, 665, 666, 674, 675, 676, 678, 679, 680, 681, 684, 685, 686, 701, 702, 703, 705, 706, 707, 714, 715, 718, 720, 721, 722, 723, 724, 725, 726, 728, 730, 731, 732, 735, 736, 737, 754, 755, 756, 758, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 799, 800, 801, 806, 807, 808, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 824, 825, 826, 827, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 842, 843, 844, 845, 847, 852, 855, 856, 884, 887, 889, 890, 891, 892, 894, 895, 901, 902, 905, 907, 908, 911, 913, 916, 920, 921, 923, 924, 925, 926, 929, 932, 933, 934, 935, 936, 937, 943, 944, 946 }

B grade: { 3, 4, 5, 80, 90, 110, 121, 122, 123, 128, 129, 130, 131, 132, 133, 134, 139, 140, 201, 202, 207, 208, 210, 212, 213, 263, 273, 275, 278, 280, 282, 295, 297, 306, 325, 472, 473, 474, 475, 476, 477, 478, 591, 600, 653, 654, 672, 677, 694, 695, 719, 727, 738, 751, 753, 759, 760, 775, 793, 798, 802, 803, 804, 805, 809, 821, 822, 823, 828, 836, 841, 846, 848, 849, 854, 862, 880, 881, 882, 883, 885, 888, 909, 945 }

C grade: { }

F grade: { 2, 6, 7, 9, 10, 11, 12, 13, 14, 16, 20, 21, 23, 24, 25, 26, 27, 28, 31, 32, 34, 36, 37, 38, 39, 40, 41, 43, 46, 47, 51, 52, 53, 54, 55, 57, 60, 61, 63, 64, 65, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125,

126, 127, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 205, 206, 209, 211, 214, 215, 216, 217, 218, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 257, 258, 259, 260, 261, 262, 269, 271, 279, 281, 283, 284, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 302, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 360, 361, 362, 367, 368, 369, 373, 378, 379, 380, 385, 386, 387, 392, 393, 394, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 530, 531, 532, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 584, 585, 586, 587, 588, 592, 593, 594, 595, 596, 597, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 650, 657, 658, 662, 663, 664, 667, 668, 669, 670, 671, 673, 682, 683, 687, 688, 689, 690, 691, 692, 693, 696, 697, 698, 699, 700, 704, 708, 709, 710, 711, 712, 713, 716, 717, 729, 733, 734, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 752, 757, 761, 762, 774, 796, 797, 810, 829, 850, 851, 853, 857, 858, 859, 860, 861, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 886, 893, 896, 897, 898, 899, 900, 903, 904, 906, 910, 912, 914, 915, 917, 918, 919, 922, 927, 928, 930, 931, 938, 939, 940, 941, 942, 947, 948, 949, 950
}

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 62, 66, 67, 68, 69, 70, 71, 72, 73, 91, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 117, 119, 120, 121, 123, 124, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 150, 151, 152, 159, 180, 186, 187, 188, 189, 190, 191, 192, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 222, 223, 224, 225, 226, 227, 229, 231, 248, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 272, 274, 276, 279, 281, 283, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 308, 310, 312, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 400, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 476, 477, 479, 480, 481, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 504, 505, 506, 514, 515, 520, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 535, 536, 540, 541, 545, 546, 550, 551, 562, 563, 567, 568, 569, 570, 571, 572, 582, 583, 584, 589, 591, 598, 600, 640, 641, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 661, 665, 666, 668, 669, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 684, 686, 687, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 704, 705, 706, 708, 710, 711, 713, 714, 715, 716, 724, 725, 726, 727, 729, 730, 731, 735, 736, 737, 738, 739, 740, 749, 753, 754, 755, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 798, 799, 800, 802, 803, 804, 806, 807, 808, 810, 811, 812, 813, 814, 817, 818, 819, 820, 821, 822, 823, 824, 826, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 848, 849, 851, 852, 853, 854, 855, 856, 857, 866, 867, 880, 881, 882, 883, 884, 887, 889, 891, 893, 894, 895, 896, 897, 898, 899, 901, 903, 908, 909, 911, 913, 916, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 932, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 64, 65, 75, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 94, 110, 112, 114, 116, 118, 122, 125, 126, 127, 139, 140, 141, 142, 143, 144, 145, 146, 157, 158, 162, 185, 193, 194, 195, 196, 197, 198, 199, 200, 219, 221, 228, 230, 249, 250, 251, 267, 271, 273, 275, 277, 278, 280, 282, 284, 291, 304, 307, 309, 311, 313, 325, 341, 342, 343, 347, 354, 362, 384, 385, 386, 387, 391, 392, 393, 394, 401, 402, 421, 422, 475, 478, 482, 483, 496, 501, 502, 507, 508, 510, 511, 512, 513, 516, 517, 518, 519, 521, 528, 534, 537, 538, 539, 542, 543, 544, 547, 548, 549, 552, 553, 554, 555, 564, 565, 590, 599, 638, 642, 650, 667, 670, 676, 683, 707, 718, 719, 720, 721, 722, 728, 732, 734, 743, 744, 747, 748, 752, 756, 757, 759, 775, 793, 796, 797, 801, 805, 815, 816, 825, 827, 829, 836, 842, 850, 860, 861, 862, 863, 864, 885, 886, 888, 890, 892, 902, 904, 905, 910, 915, 927, 930, 933 }

C grade: { 428, 429, 585, 586, 587, 588, 594, 595, 596, 597, 712, 751 }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 61, 63, 74, 76, 77, 78, 79, 105, 106, 107, 108, 109, 115, 147, 148, 149, 153, 154, 155, 156, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 314, 315, 316, 317, 318, 319, 320, 321, 333, 334, 335, 336, 337, 338, 339, 340, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 423, 424, 425, 426, 427, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 503, 509, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 592, 593, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 643, 660, 662, 663, 664, 685, 688, 689, 699, 709, 717, 723, 733, 741, 742, 745, 746, 750, 809, 858, 859, 865, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 900, 906, 907, 912, 914, 917, 918, 931, 938, 939 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	35	101	32	77
normalized size	1	1.	0.5	0.39	0.8	2.3	0.73	1.75
time (sec)	N/A	0.04	0.036	0.016	1.697	1.426	0.324	1.138

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	35	101	0	77
normalized size	1	1.	0.5	0.39	0.8	2.3	0.	1.75
time (sec)	N/A	0.038	0.028	0.081	1.575	1.454	0.	1.236

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	22	127	343	77
normalized size	1	1.	0.46	0.35	0.46	2.65	7.15	1.6
time (sec)	N/A	0.022	0.046	0.033	1.566	1.47	8.389	1.12

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	22	127	343	77
normalized size	1	1.	0.46	0.35	0.46	2.65	7.15	1.6
time (sec)	N/A	0.02	0.059	0.02	1.602	1.421	10.251	1.104

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	22	127	343	77
normalized size	1	1.	0.46	0.35	0.46	2.65	7.15	1.6
time (sec)	N/A	0.026	0.025	0.04	1.671	1.469	9.454	1.097

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	22	127	0	77
normalized size	1	1.	0.46	0.35	0.46	2.65	0.	1.6
time (sec)	N/A	0.044	0.019	0.061	1.542	1.532	0.	2.459

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	22	127	0	77
normalized size	1	1.	0.46	0.35	0.46	2.65	0.	1.6
time (sec)	N/A	0.041	0.018	0.061	1.526	1.459	0.	1.28

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	73	207	42	53
normalized size	1	1.	0.37	0.28	1.22	3.45	0.7	0.88
time (sec)	N/A	0.026	0.038	0.012	1.628	1.402	0.39	1.21

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	73	207	0	53
normalized size	1	1.	0.37	0.28	1.22	3.45	0.	0.88
time (sec)	N/A	0.046	0.039	0.084	1.665	1.485	0.	1.302

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	46	232	0	53
normalized size	1	1.	0.37	0.28	0.77	3.87	0.	0.88
time (sec)	N/A	0.02	0.058	0.029	1.611	1.405	0.	1.172

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	46	232	0	53
normalized size	1	1.	0.37	0.28	0.77	3.87	0.	0.88
time (sec)	N/A	0.019	0.07	0.017	1.597	1.311	0.	1.186

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	46	232	0	53
normalized size	1	1.	0.37	0.28	0.77	3.87	0.	0.88
time (sec)	N/A	0.03	0.032	0.043	1.6	1.407	0.	1.225

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	46	232	0	53
normalized size	1	1.	0.37	0.28	0.77	3.87	0.	0.88
time (sec)	N/A	0.045	0.024	0.061	1.559	1.432	0.	2.534

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	46	232	0	53
normalized size	1	1.	0.37	0.28	0.77	3.87	0.	0.88
time (sec)	N/A	0.046	0.031	0.055	1.613	1.51	0.	1.374

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	36	101	34	77
normalized size	1	1.	0.52	0.43	0.86	2.4	0.81	1.83
time (sec)	N/A	0.037	0.029	0.011	1.686	1.221	0.307	1.125

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	36	101	0	77
normalized size	1	1.	0.52	0.43	0.86	2.4	0.	1.83
time (sec)	N/A	0.037	0.022	0.073	1.749	1.849	0.	1.188

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	23	124	76	77
normalized size	1	1.	0.46	0.38	0.48	2.58	1.58	1.6
time (sec)	N/A	0.019	0.021	0.024	1.578	1.737	1.204	1.122

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	23	124	76	77
normalized size	1	1.	0.46	0.38	0.48	2.58	1.58	1.6
time (sec)	N/A	0.016	0.041	0.017	1.609	1.783	0.949	1.13

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	23	124	76	77
normalized size	1	1.	0.46	0.38	0.48	2.58	1.58	1.6
time (sec)	N/A	0.027	0.021	0.039	1.572	1.79	1.275	1.138

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	23	124	0	77
normalized size	1	1.	0.46	0.38	0.48	2.58	0.	1.6
time (sec)	N/A	0.042	0.018	0.06	1.607	1.867	0.	1.82

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	23	124	0	77
normalized size	1	1.	0.46	0.38	0.48	2.58	0.	1.6
time (sec)	N/A	0.045	0.019	0.087	1.56	1.843	0.	1.463

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	72	207	39	53
normalized size	1	1.	0.36	0.3	1.18	3.39	0.64	0.87
time (sec)	N/A	0.03	0.027	0.01	1.555	1.825	0.398	1.21

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	72	207	0	53
normalized size	1	1.	0.36	0.3	1.18	3.39	0.	0.87
time (sec)	N/A	0.046	0.035	0.079	1.495	1.755	0.	1.315

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	43	230	0	53
normalized size	1	1.	0.36	0.3	0.7	3.77	0.	0.87
time (sec)	N/A	0.02	0.065	0.03	1.604	1.693	0.	1.222

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	43	230	0	53
normalized size	1	1.	0.36	0.3	0.7	3.77	0.	0.87
time (sec)	N/A	0.019	0.058	0.018	1.663	1.639	0.	1.207

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	43	230	0	53
normalized size	1	1.	0.36	0.3	0.7	3.77	0.	0.87
time (sec)	N/A	0.031	0.031	0.042	1.561	1.625	0.	1.24

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	43	230	0	53
normalized size	1	1.	0.36	0.3	0.7	3.77	0.	0.87
time (sec)	N/A	0.043	0.021	0.059	1.648	1.722	0.	1.873

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	43	230	0	53
normalized size	1	1.	0.36	0.3	0.7	3.77	0.	0.87
time (sec)	N/A	0.047	0.035	0.086	1.667	1.796	0.	1.51

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	76	41	32
normalized size	1	1.	1.	0.83	1.07	2.53	1.37	1.07
time (sec)	N/A	0.035	0.062	0.009	1.11	1.715	0.19	1.145

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	57	65	135	85	62
normalized size	1	1.	0.86	1.02	1.16	2.41	1.52	1.11
time (sec)	N/A	0.067	0.088	0.037	1.116	2.026	0.384	1.189

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	172	229	0	377	0	0
normalized size	1	1.	0.81	1.08	0.	1.77	0.	0.
time (sec)	N/A	0.536	0.322	0.022	0.	2.131	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	238	320	0	950	0	0
normalized size	1	1.	0.88	1.18	0.	3.51	0.	0.
time (sec)	N/A	0.802	0.579	0.015	0.	2.428	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	28	10	11
normalized size	1	1.	1.	0.9	1.1	2.8	1.	1.1
time (sec)	N/A	0.018	0.023	0.011	1.095	1.991	0.334	1.142

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	46	72	0	0	0	0
normalized size	1	1.	1.64	2.57	0.	0.	0.	0.
time (sec)	N/A	0.471	0.709	0.046	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	26	12	23	89	15	15
normalized size	1	1.	2.17	1.	1.92	7.42	1.25	1.25
time (sec)	N/A	0.029	0.02	0.017	1.099	2.021	2.354	1.113

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	272	142	0	323	0	0
normalized size	1	1.	2.72	1.42	0.	3.23	0.	0.
time (sec)	N/A	0.164	5.519	0.013	0.	2.252	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	401	195	0	351	0	0
normalized size	1	1.	3.75	1.82	0.	3.28	0.	0.
time (sec)	N/A	0.192	7.279	0.015	0.	2.251	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	657	295	0	651	0	0
normalized size	1	1.	3.39	1.52	0.	3.36	0.	0.
time (sec)	N/A	0.322	7.721	0.015	0.	2.476	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.096	0.118	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.075	0.07	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.041	0.021	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	5.542	0.04	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	18.71	0.046	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	32	76	41	32
normalized size	1	1.	0.87	0.83	1.07	2.53	1.37	1.07
time (sec)	N/A	0.034	0.069	0.008	1.126	2.299	0.21	1.11

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	57	62	135	85	62
normalized size	1	1.	0.91	1.02	1.11	2.41	1.52	1.11
time (sec)	N/A	0.069	0.104	0.032	1.156	2.323	0.52	1.124

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	172	229	0	398	0	0
normalized size	1	1.	0.81	1.08	0.	1.87	0.	0.
time (sec)	N/A	0.308	0.303	0.016	0.	2.496	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	236	320	0	963	0	0
normalized size	1	1.	0.87	1.18	0.	3.55	0.	0.
time (sec)	N/A	0.562	0.542	0.017	0.	2.643	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	27	8	11
normalized size	1	1.	1.	0.9	1.1	2.7	0.8	1.1
time (sec)	N/A	0.136	0.032	0.011	1.124	2.304	0.902	1.099

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	18	23	112	20	23
normalized size	1	1.	1.	0.82	1.05	5.09	0.91	1.05
time (sec)	N/A	0.189	0.053	0.015	1.59	2.336	1.585	1.099

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	16	22	46	15	26
normalized size	1	1.	0.83	0.67	0.92	1.92	0.62	1.08
time (sec)	N/A	0.493	0.156	0.025	1.139	2.153	7.2	1.098

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	260	142	0	323	0	0
normalized size	1	1.	2.57	1.41	0.	3.2	0.	0.
time (sec)	N/A	0.133	5.135	0.016	0.	2.47	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	400	195	0	338	0	0
normalized size	1	1.	3.74	1.82	0.	3.16	0.	0.
time (sec)	N/A	0.161	6.113	0.02	0.	2.583	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.024	0.122	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.038	0.088	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.006	0.027	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	2.897	0.031	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	10.993	0.05	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	41	10	11
normalized size	1	1.	1.	0.89	1.	4.56	1.11	1.22
time (sec)	N/A	0.01	0.012	0.003	1.014	2.359	0.991	1.103

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	13	16	39	14	16
normalized size	1	1.	1.12	0.81	1.	2.44	0.88	1.
time (sec)	N/A	0.018	0.034	0.007	1.48	2.29	1.02	1.114

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	108	0	0	0
normalized size	1	1.	1.	0.	1.54	0.	0.	0.
time (sec)	N/A	0.091	0.021	0.026	1.521	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	112	59	0	0
normalized size	1	1.	0.95	0.95	5.89	3.11	0.	0.
time (sec)	N/A	0.018	0.676	0.03	1.207	2.375	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	14	19	88	15	30
normalized size	1	1.	1.62	0.88	1.19	5.5	0.94	1.88
time (sec)	N/A	0.018	0.044	0.005	1.498	2.388	0.709	1.121

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	85	150	0	0	0	0
normalized size	1	1.	0.92	1.63	0.	0.	0.	0.
time (sec)	N/A	0.164	1.565	0.305	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	48	0	193	0	1280
normalized size	1	1.	1.	1.37	0.	5.51	0.	36.57
time (sec)	N/A	0.033	0.037	0.045	0.	2.408	0.	2.547

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	48	0	193	0	1280
normalized size	1	1.	1.	1.37	0.	5.51	0.	36.57
time (sec)	N/A	0.033	0.025	0.002	0.	2.486	0.	2.523

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	15	38	20	15
normalized size	1	1.	1.	0.47	1.	2.53	1.33	1.
time (sec)	N/A	0.009	0.005	0.01	0.96	2.304	0.549	1.13

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	39	20	18
normalized size	1	1.	1.	0.82	1.06	2.29	1.18	1.06
time (sec)	N/A	0.008	0.007	0.026	0.971	2.325	0.775	1.109

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	59	20	18
normalized size	1	1.	1.	0.82	1.06	3.47	1.18	1.06
time (sec)	N/A	0.008	0.007	0.029	1.019	2.232	1.776	1.104

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	38	68	78	39
normalized size	1	1.	0.71	0.8	1.09	1.94	2.23	1.11
time (sec)	N/A	0.031	0.043	0.013	0.968	2.273	5.022	1.114

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	32	20	15
normalized size	1	1.	1.	0.8	1.	2.13	1.33	1.
time (sec)	N/A	0.008	0.005	0.013	0.984	2.227	1.864	1.108

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	20	18
normalized size	1	1.	1.	0.82	1.06	2.06	1.18	1.06
time (sec)	N/A	0.008	0.006	0.035	0.985	2.251	0.897	1.148

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	53	20	18
normalized size	1	1.	1.	0.82	1.06	3.12	1.18	1.06
time (sec)	N/A	0.008	0.006	0.035	1.006	2.284	0.957	1.09

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	28	38	66	39	39
normalized size	1	1.	0.69	0.8	1.09	1.89	1.11	1.11
time (sec)	N/A	0.027	0.04	0.011	0.98	2.359	3.709	1.143

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	190	109	0	0
normalized size	1	1.	1.	0.9	9.5	5.45	0.	0.
time (sec)	N/A	0.023	0.013	0.043	1.536	2.364	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	21	38	0	138	0	491
normalized size	1	1.	0.45	0.81	0.	2.94	0.	10.45
time (sec)	N/A	0.052	0.028	0.094	0.	2.541	0.	1.251

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	115	0	329	0	0
normalized size	1	1.	0.97	1.62	0.	4.63	0.	0.
time (sec)	N/A	0.109	0.069	0.166	0.	2.455	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	100	84	0	451	0	0
normalized size	1	1.	0.89	0.75	0.	4.03	0.	0.
time (sec)	N/A	0.17	0.151	0.141	0.	2.736	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	256	0	435	0	0
normalized size	1	1.	0.94	2.88	0.	4.89	0.	0.
time (sec)	N/A	0.271	0.134	0.281	0.	2.78	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	200	0	0	0	0	0
normalized size	1	1.	1.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.165	0.174	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	50	73	19	26
normalized size	1	1.	1.	1.2	5.	7.3	1.9	2.6
time (sec)	N/A	0.022	0.009	0.026	1.496	2.42	1.206	1.13

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	171	109	0	46
normalized size	1	1.	1.	0.85	8.55	5.45	0.	2.3
time (sec)	N/A	0.027	0.017	0.046	1.587	2.326	0.	1.157

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	234	170	0	68
normalized size	1	1.	1.	1.07	8.36	6.07	0.	2.43
time (sec)	N/A	0.051	0.032	0.047	1.579	2.502	0.	1.162

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	70	0	423	0	150
normalized size	1	1.	0.93	0.85	0.	5.16	0.	1.83
time (sec)	N/A	0.196	0.224	0.103	0.	2.702	0.	1.29

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	49	0	243	0	95
normalized size	1	1.	1.	1.29	0.	6.39	0.	2.5
time (sec)	N/A	0.082	0.067	0.069	0.	2.719	0.	1.151

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	174	13	174	97	0	66
normalized size	1	1.	11.6	0.87	11.6	6.47	0.	4.4
time (sec)	N/A	0.015	0.368	0.028	1.712	2.46	0.	1.264

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	109	61	0	68
normalized size	1	1.	0.81	0.86	5.19	2.9	0.	3.24
time (sec)	N/A	0.027	0.008	0.061	1.473	2.536	0.	1.232

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	4845	54	0	394	0	147
normalized size	1	1.	68.24	0.76	0.	5.55	0.	2.07
time (sec)	N/A	0.062	56.622	0.066	0.	2.89	0.	1.298

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	43	0	230	0	159
normalized size	1	1.	0.92	0.69	0.	3.71	0.	2.56
time (sec)	N/A	0.073	0.097	0.079	0.	2.783	0.	1.261

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	678	80	0	498	0	208
normalized size	1	1.	7.98	0.94	0.	5.86	0.	2.45
time (sec)	N/A	0.063	9.28	0.096	0.	2.997	0.	1.319

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	37	9	47	59	15	34
normalized size	1	1.	5.29	1.29	6.71	8.43	2.14	4.86
time (sec)	N/A	0.012	0.006	0.018	1.546	2.347	6.401	1.14

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	15	14	169	177	76	42
normalized size	1	1.	0.33	0.31	3.76	3.93	1.69	0.93
time (sec)	N/A	0.04	0.02	0.064	1.574	2.349	4.899	1.201

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	231	158	0	65
normalized size	1	1.	1.	1.08	8.88	6.08	0.	2.5
time (sec)	N/A	0.025	0.023	0.068	1.56	2.583	0.	1.161

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	84	66	0	759	0	142
normalized size	1	1.	0.51	0.4	0.	4.6	0.	0.86
time (sec)	N/A	0.141	0.11	0.107	0.	2.73	0.	1.356

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	47	0	231	0	92
normalized size	1	1.	0.83	1.31	0.	6.42	0.	2.56
time (sec)	N/A	0.046	0.037	0.093	0.	2.74	0.	1.198

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	6	9	8	28	5	8
normalized size	1	1.	0.75	1.12	1.	3.5	0.62	1.
time (sec)	N/A	0.03	0.012	0.032	1.022	2.424	1.683	1.1

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	19	7	8
normalized size	1	1.	1.	1.12	1.	2.38	0.88	1.
time (sec)	N/A	0.014	0.003	0.012	0.992	2.475	4.921	1.155

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	15	20	22	15
normalized size	1	1.	1.	0.47	1.	1.33	1.47	1.
time (sec)	N/A	0.009	0.005	0.009	0.995	2.298	0.892	1.125

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	22	18
normalized size	1	1.	1.	0.82	1.06	2.06	1.29	1.06
time (sec)	N/A	0.008	0.006	0.034	1.001	2.477	0.618	1.082

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	41	22	18
normalized size	1	1.	1.	0.82	1.06	2.41	1.29	1.06
time (sec)	N/A	0.008	0.006	0.02	1.003	2.5	0.69	1.111

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	28	36	68	44	39
normalized size	1	1.	0.74	0.8	1.03	1.94	1.26	1.11
time (sec)	N/A	0.03	0.049	0.008	0.977	2.523	1.246	1.103

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	39	20	15
normalized size	1	1.	1.	0.8	1.	2.6	1.33	1.
time (sec)	N/A	0.009	0.005	0.018	0.98	2.337	0.786	1.106

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	23	20	18
normalized size	1	1.	1.	0.82	1.06	1.35	1.18	1.06
time (sec)	N/A	0.009	0.006	0.021	0.987	2.361	0.814	1.108

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	59	20	18
normalized size	1	1.	1.	0.82	1.06	3.47	1.18	1.06
time (sec)	N/A	0.008	0.005	0.041	1.008	2.269	0.558	1.155

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	38	66	56	39
normalized size	1	1.	0.71	0.8	1.09	1.89	1.6	1.11
time (sec)	N/A	0.029	0.039	0.012	1.016	2.429	4.911	1.112

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	183	18	180	109	0	0
normalized size	1	1.	9.15	0.9	9.	5.45	0.	0.
time (sec)	N/A	0.025	0.24	0.015	1.577	2.825	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	48	19	0	109	0	0
normalized size	1	1.	2.29	0.9	0.	5.19	0.	0.
time (sec)	N/A	0.024	0.053	0.022	0.	2.856	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	6196	68	0	329	0	0
normalized size	1	1.	87.27	0.96	0.	4.63	0.	0.
time (sec)	N/A	0.083	58.347	0.043	0.	2.789	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	215	72	0	421	0	0
normalized size	1	1.	2.56	0.86	0.	5.01	0.	0.
time (sec)	N/A	0.098	0.598	0.036	0.	2.634	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	679	104	0	435	0	0
normalized size	1	1.	7.63	1.17	0.	4.89	0.	0.
time (sec)	N/A	0.238	8.927	0.049	0.	2.75	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	25	14	50	88	19	26
normalized size	1	1.	2.5	1.4	5.	8.8	1.9	2.6
time (sec)	N/A	0.02	0.015	0.025	0.992	2.517	1.489	1.139

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	36	177	155	0	53
normalized size	1	1.	1.04	0.8	3.93	3.44	0.	1.18
time (sec)	N/A	0.053	0.019	0.101	1.506	2.521	0.	1.142

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	73	30	223	185	0	68
normalized size	1	1.	2.61	1.07	7.96	6.61	0.	2.43
time (sec)	N/A	0.049	0.066	0.134	1.541	2.576	0.	1.14

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	133	82	0	466	0	158
normalized size	1	1.	1.21	0.75	0.	4.24	0.	1.44
time (sec)	N/A	0.155	0.125	0.153	0.	2.605	0.	1.197

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	87	49	0	259	0	95
normalized size	1	1.	2.29	1.29	0.	6.82	0.	2.5
time (sec)	N/A	0.071	0.086	0.2	0.	2.616	0.	1.147

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	179	0	0	0	0	0
normalized size	1	1.	1.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.191	0.223	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	185	97	0	42
normalized size	1	1.	1.	0.87	12.33	6.47	0.	2.8
time (sec)	N/A	0.015	0.007	0.029	1.551	2.29	0.	1.135

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	15	13	103	162	0	42
normalized size	1	1.	0.34	0.3	2.34	3.68	0.	0.95
time (sec)	N/A	0.036	0.017	0.054	1.544	2.488	0.	1.157

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	54	0	393	0	134
normalized size	1	1.	0.94	0.76	0.	5.54	0.	1.89
time (sec)	N/A	0.046	0.107	0.083	0.	2.607	0.	1.31

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	84	68	0	759	0	142
normalized size	1	1.	0.52	0.42	0.	4.66	0.	0.87
time (sec)	N/A	0.129	0.104	0.1	0.	2.81	0.	1.357

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	80	0	500	0	178
normalized size	1	1.	0.95	0.94	0.	5.88	0.	2.09
time (sec)	N/A	0.061	0.087	0.111	0.	2.797	0.	1.335

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	14	26	76	20	28
normalized size	1	1.	1.	1.4	2.6	7.6	2.	2.8
time (sec)	N/A	0.018	0.007	0.026	0.995	2.361	2.574	1.095

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	174	81	427	34
normalized size	1	1.	1.	1.29	12.43	5.79	30.5	2.43
time (sec)	N/A	0.02	0.008	0.034	1.546	2.313	20.535	1.139

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	21	11	47	77	15	11
normalized size	1	1.	3.	1.57	6.71	11.	2.14	1.57
time (sec)	N/A	0.011	0.003	0.017	0.989	2.482	7.943	1.115

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	174	65	17	41
normalized size	1	1.	1.	0.	8.29	3.1	0.81	1.95
time (sec)	N/A	0.026	0.008	180.	1.52	2.538	4.886	1.114

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	66	28	220	174	0	88
normalized size	1	1.	2.54	1.08	8.46	6.69	0.	3.38
time (sec)	N/A	0.026	0.058	0.045	1.553	2.473	0.	1.171

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	80	0	232	0	177
normalized size	1	1.	0.92	1.29	0.	3.74	0.	2.85
time (sec)	N/A	0.07	0.062	0.097	0.	2.567	0.	1.202

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	83	47	0	248	0	136
normalized size	1	1.	2.31	1.31	0.	6.89	0.	3.78
time (sec)	N/A	0.041	0.073	0.059	0.	2.659	0.	1.178

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	234	63	34
normalized size	1	1.	1.	0.79	1.03	7.09	1.91	1.03
time (sec)	N/A	0.031	0.017	0.092	0.994	2.796	11.434	1.12

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	117	71	34
normalized size	1	1.	1.	0.79	1.03	3.55	2.15	1.03
time (sec)	N/A	0.033	0.017	0.038	1.012	2.446	23.391	1.122

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	92	48	26
normalized size	1	1.	1.	0.8	1.04	3.68	1.92	1.04
time (sec)	N/A	0.028	0.016	0.048	0.982	2.368	12.775	1.121

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	154	42	23
normalized size	1	1.	1.	0.78	1.	6.7	1.83	1.
time (sec)	N/A	0.026	0.012	0.051	1.008	2.489	12.85	1.119

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	197	65	34
normalized size	1	1.	1.	0.79	1.03	5.97	1.97	1.03
time (sec)	N/A	0.031	0.015	0.059	1.024	2.762	16.104	1.14

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	31	297	70	31
normalized size	1	1.	1.	0.77	1.	9.58	2.26	1.
time (sec)	N/A	0.03	0.015	0.18	1.007	3.119	11.812	1.104

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	42	80	228	42
normalized size	1	1.	1.	0.78	1.02	1.95	5.56	1.02
time (sec)	N/A	0.043	0.019	0.066	1.012	2.396	52.917	1.161

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	31	127	58	31
normalized size	1	1.	0.96	0.89	1.15	4.7	2.15	1.15
time (sec)	N/A	0.024	0.048	0.011	1.	2.451	1.392	1.131

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	31	124	61	31
normalized size	1	1.	0.96	0.89	1.15	4.59	2.26	1.15
time (sec)	N/A	0.025	0.034	0.01	1.006	2.416	1.216	1.118

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	31	127	58	31
normalized size	1	1.	0.96	0.89	1.15	4.7	2.15	1.15
time (sec)	N/A	0.017	0.025	0.013	0.982	2.296	1.093	1.107

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	31	123	58	31
normalized size	1	1.	0.96	0.89	1.15	4.56	2.15	1.15
time (sec)	N/A	0.019	0.023	0.012	1.017	2.383	3.505	1.11

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	31	173	501	387	7713	109
normalized size	1	1.	0.79	4.44	12.85	9.92	197.77	2.79
time (sec)	N/A	0.066	0.503	0.074	1.131	2.467	10.587	1.163

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	145	392	374	7720	109
normalized size	1	1.	0.82	4.26	11.53	11.	227.06	3.21
time (sec)	N/A	0.068	0.517	0.067	1.085	2.57	10.155	1.166

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	31	177	741	315	0	470
normalized size	1	1.	0.79	4.54	19.	8.08	0.	12.05
time (sec)	N/A	0.032	0.508	0.076	1.166	2.525	0.	1.243

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	149	583	304	0	466
normalized size	1	1.	0.88	4.38	17.15	8.94	0.	13.71
time (sec)	N/A	0.034	0.489	0.08	1.156	2.531	0.	1.203

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	55	471	273	0	231
normalized size	1	1.	0.78	1.53	13.08	7.58	0.	6.42
time (sec)	N/A	0.019	0.224	0.231	1.058	2.603	0.	1.247

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	53	435	263	0	228
normalized size	1	1.	0.79	1.61	13.18	7.97	0.	6.91
time (sec)	N/A	0.018	0.229	0.221	1.091	2.529	0.	1.242

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	169	761	281	0	535
normalized size	1	1.	0.78	4.69	21.14	7.81	0.	14.86
time (sec)	N/A	0.018	0.241	0.234	1.115	2.753	0.	1.256

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	81	724	270	0	536
normalized size	1	1.	0.88	2.45	21.94	8.18	0.	16.24
time (sec)	N/A	0.018	0.219	0.232	1.115	2.67	0.	1.249

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	177	77	63	0	0
normalized size	1	1.	1.	13.62	5.92	4.85	0.	0.
time (sec)	N/A	0.032	0.082	0.217	1.539	2.432	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	587	77	76	0	0
normalized size	1	1.	0.74	18.94	2.48	2.45	0.	0.
time (sec)	N/A	0.053	0.041	0.13	1.555	2.412	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	29	324	111	112	0	0
normalized size	1	1.	0.58	6.48	2.22	2.24	0.	0.
time (sec)	N/A	0.075	0.099	0.167	1.607	2.403	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	254	53	0	16
normalized size	1	1.	1.	1.54	19.54	4.08	0.	1.23
time (sec)	N/A	0.038	0.069	0.132	1.797	2.387	0.	1.146

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	424	66	0	26
normalized size	1	1.	0.68	0.84	13.68	2.13	0.	0.84
time (sec)	N/A	0.068	0.038	0.117	1.855	2.396	0.	1.129

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	576	103	0	36
normalized size	1	1.	0.58	0.68	11.52	2.06	0.	0.72
time (sec)	N/A	0.094	0.1	0.155	1.909	2.543	0.	1.122

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	159	0	531	0	0
normalized size	1	1.	0.97	2.74	0.	9.16	0.	0.
time (sec)	N/A	0.078	0.14	0.247	0.	2.582	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	84	257	0	1013	0	0
normalized size	1	1.	0.99	3.02	0.	11.92	0.	0.
time (sec)	N/A	0.106	0.254	0.412	0.	2.557	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	154	0	516	0	0
normalized size	1	1.	0.98	2.61	0.	8.75	0.	0.
time (sec)	N/A	0.059	0.104	0.125	0.	2.526	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	85	250	0	944	0	0
normalized size	1	1.	0.97	2.84	0.	10.73	0.	0.
time (sec)	N/A	0.103	0.316	0.231	0.	2.687	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	86	338	216	0	435
normalized size	1	1.	0.96	1.72	6.76	4.32	0.	8.7
time (sec)	N/A	0.083	0.176	0.164	1.039	2.576	0.	1.362

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	87	338	216	0	435
normalized size	1	1.	0.96	1.74	6.76	4.32	0.	8.7
time (sec)	N/A	0.082	0.182	0.147	1.051	2.646	0.	1.303

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	487	0	54
normalized size	1	1.	1.	0.75	0.	15.22	0.	1.69
time (sec)	N/A	0.055	0.097	0.065	0.	2.885	0.	1.466

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	512	1003	0	8015	0	0
normalized size	1	1.	2.43	4.75	0.	37.99	0.	0.
time (sec)	N/A	0.528	6.535	0.157	0.	7.232	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	294	1251	0	11173	0	0
normalized size	1	1.	0.87	3.71	0.	33.15	0.	0.
time (sec)	N/A	0.9	1.054	0.144	0.	7.13	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	747	0	103
normalized size	1	1.	1.	0.85	0.	18.68	0.	2.58
time (sec)	N/A	0.605	0.263	0.084	0.	2.938	0.	1.428

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	751	1670	0	10267	0	0
normalized size	1	1.	2.81	6.25	0.	38.45	0.	0.
time (sec)	N/A	0.719	4.344	0.201	0.	7.029	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	499	2061	0	14288	0	0
normalized size	1	1.	1.23	5.06	0.	35.11	0.	0.
time (sec)	N/A	1.075	2.194	0.173	0.	7.743	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	61	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.512	0.17	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	54	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	0.332	0.082	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.206	0.081	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.222	0.083	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.248	0.082	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	87	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.29	0.08	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	113	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.375	1.159	0.079	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	95	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.274	0.823	0.08	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	73	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.62	0.076	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	150	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.662	1.242	0.078	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	231	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.665	1.484	0.079	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	317	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.741	1.817	0.084	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	767	767	247	0	0	0	0	0
normalized size	1	1.	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.351	2.752	0.092	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	555	194	0	0	0	0	0
normalized size	1	1.	0.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.877	1.923	0.092	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	154	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.511	1.241	0.188	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	108	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.646	3.58	0.102	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	193	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	3.55	2.05	0.077	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	154	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	2.155	1.6	0.077	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	150	0	0	180	0	0
normalized size	1	1.	0.88	0.	0.	1.05	0.	0.
time (sec)	N/A	0.973	0.622	0.079	0.	2.357	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	85	154	76	271	0	0
normalized size	1	1.	0.28	0.51	0.25	0.9	0.	0.
time (sec)	N/A	0.438	0.084	0.067	1.561	2.43	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	51	24	31	107	27	69
normalized size	1	1.	2.83	1.33	1.72	5.94	1.5	3.83
time (sec)	N/A	0.098	0.015	0.038	0.975	2.535	3.579	1.171

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	67	52	73	170	61	135
normalized size	1	1.	1.18	0.91	1.28	2.98	1.07	2.37
time (sec)	N/A	0.202	0.082	0.046	1.014	2.573	8.76	1.183

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	80	77	113	213	92	169
normalized size	1	1.	1.07	1.03	1.51	2.84	1.23	2.25
time (sec)	N/A	0.297	0.104	0.054	0.992	2.589	37.164	1.159

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	97	103	159	255	116	201
normalized size	1	1.	0.93	0.99	1.53	2.45	1.12	1.93
time (sec)	N/A	0.403	0.126	0.088	1.008	2.542	174.516	1.169

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	71	46	85	143	0	62
normalized size	1	1.	2.84	1.84	3.4	5.72	0.	2.48
time (sec)	N/A	0.093	0.085	0.03	0.992	2.461	0.	1.179

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	76	71	126	248	0	104
normalized size	1	1.	1.58	1.48	2.62	5.17	0.	2.17
time (sec)	N/A	0.185	0.204	0.032	1.03	2.436	0.	1.191

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	88	95	161	356	0	138
normalized size	1	1.	1.17	1.27	2.15	4.75	0.	1.84
time (sec)	N/A	0.311	0.351	0.035	1.011	2.436	0.	1.165

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	104	119	193	464	0	170
normalized size	1	1.	1.08	1.24	2.01	4.83	0.	1.77
time (sec)	N/A	0.414	0.805	0.037	1.1	2.539	0.	1.156

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	78	228	58	354	0	244
normalized size	1	1.	0.8	2.33	0.59	3.61	0.	2.49
time (sec)	N/A	0.432	0.165	2.795	1.734	2.484	0.	3.309

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	62	199	35	297	0	209
normalized size	1	1.	0.86	2.76	0.49	4.12	0.	2.9
time (sec)	N/A	0.294	0.105	2.882	1.692	2.481	0.	3.235

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	152	18	252	0	155
normalized size	1	1.	1.07	3.45	0.41	5.73	0.	3.52
time (sec)	N/A	0.16	0.037	2.702	1.67	2.431	0.	3.222

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	192	78	354	0	180
normalized size	1	1.	0.76	2.82	1.15	5.21	0.	2.65
time (sec)	N/A	0.195	0.051	3.422	1.709	2.601	0.	2.235

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	73	270	0	559	0	227
normalized size	1	1.	0.79	2.93	0.	6.08	0.	2.47
time (sec)	N/A	0.336	0.372	3.653	0.	2.49	0.	2.291

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	95	322	0	689	0	269
normalized size	1	1.	0.79	2.68	0.	5.74	0.	2.24
time (sec)	N/A	0.482	0.5	3.66	0.	2.615	0.	2.43

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	80	0	93	0	382
normalized size	1	1.	1.	3.2	0.	3.72	0.	15.28
time (sec)	N/A	0.053	0.197	0.337	0.	2.534	0.	1.297

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	91	0	93	0	536
normalized size	1	1.	1.	3.79	0.	3.88	0.	22.33
time (sec)	N/A	0.056	0.125	0.166	0.	2.529	0.	1.328

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	42	41	11
normalized size	1	1.	1.	1.12	1.38	5.25	5.12	1.38
time (sec)	N/A	0.041	0.008	0.022	1.492	2.214	1.503	1.117

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	16	20	107	416	66
normalized size	1	1.	0.67	0.44	0.56	2.97	11.56	1.83
time (sec)	N/A	0.041	0.029	0.022	1.482	2.396	77.885	1.118

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	14	42	12	22
normalized size	1	1.	1.	1.38	1.75	5.25	1.5	2.75
time (sec)	N/A	0.041	0.008	0.037	1.511	2.218	2.488	1.153

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	17	22	104	61	66
normalized size	1	1.	0.62	0.46	0.59	2.81	1.65	1.78
time (sec)	N/A	0.038	0.032	0.03	1.488	2.506	5.674	1.163

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	32	0	30	0	35
normalized size	1	1.	1.	2.29	0.	2.14	0.	2.5
time (sec)	N/A	0.128	0.01	0.029	0.	2.388	0.	1.15

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	73	148	0	558	0	149
normalized size	1	1.	0.7	1.41	0.	5.31	0.	1.42
time (sec)	N/A	0.26	0.154	0.042	0.	2.722	0.	1.14

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	34	44	39	128	143	84
normalized size	1	1.	0.6	0.77	0.68	2.25	2.51	1.47
time (sec)	N/A	0.134	0.089	0.032	1.499	2.569	27.942	1.1

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	23	49	24	39
normalized size	1	1.	1.	1.33	1.53	3.27	1.6	2.6
time (sec)	N/A	0.089	0.013	0.031	1.56	2.34	1.87	1.169

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	78	0	547	0	78
normalized size	1	1.	0.96	1.59	0.	11.16	0.	1.59
time (sec)	N/A	0.15	0.16	0.038	0.	2.879	0.	1.145

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	28	0	30	76	30
normalized size	1	1.	1.	2.15	0.	2.31	5.85	2.31
time (sec)	N/A	0.119	0.01	0.044	0.	2.413	152.837	1.141

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	72	153	0	571	0	126
normalized size	1	1.	0.72	1.53	0.	5.71	0.	1.26
time (sec)	N/A	0.24	0.19	0.033	0.	2.634	0.	1.166

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	31	42	38	131	782	84
normalized size	1	1.	0.55	0.75	0.68	2.34	13.96	1.5
time (sec)	N/A	0.197	0.083	0.038	1.499	2.573	98.621	1.153

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	19	49	51	31
normalized size	1	1.	1.	1.21	1.36	3.5	3.64	2.21
time (sec)	N/A	0.058	0.012	0.029	1.52	2.441	2.204	1.14

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	84	0	617	0	95
normalized size	1	1.	0.96	1.71	0.	12.59	0.	1.94
time (sec)	N/A	0.161	0.149	0.039	0.	2.732	0.	1.171

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	98	135	0	768	0	169
normalized size	1	1.	1.32	1.82	0.	10.38	0.	2.28
time (sec)	N/A	0.246	0.442	0.036	0.	8.934	0.	1.18

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	102	120	0	817	0	149
normalized size	1	1.	1.42	1.67	0.	11.35	0.	2.07
time (sec)	N/A	0.238	0.519	0.038	0.	9.015	0.	1.165

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	94	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.228	0.415	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.177	0.416	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	246	321	347	583	461	427
normalized size	1	1.	1.94	2.53	2.73	4.59	3.63	3.36
time (sec)	N/A	0.078	1.038	0.169	1.009	3.085	10.172	1.132

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	192	285	321	501	821	317
normalized size	1	1.	1.19	1.77	1.99	3.11	5.1	1.97
time (sec)	N/A	0.079	0.707	0.141	1.029	2.915	6.391	1.127

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	156	175	232	354	267	252
normalized size	1	1.	1.66	1.86	2.47	3.77	2.84	2.68
time (sec)	N/A	0.045	0.467	0.105	0.998	2.496	2.971	1.122

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	107	153	184	273	406	165
normalized size	1	1.	0.99	1.42	1.7	2.53	3.76	1.53
time (sec)	N/A	0.044	0.318	0.085	1.012	2.492	1.742	1.131

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	81	75	113	173	117	123
normalized size	1	1.	1.4	1.29	1.95	2.98	2.02	2.12
time (sec)	N/A	0.024	0.337	0.055	0.988	2.633	0.722	1.132

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	92	120	128	68
normalized size	1	1.	0.95	1.27	1.67	2.18	2.33	1.24
time (sec)	N/A	0.019	0.102	0.041	0.97	2.595	0.34	1.11

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	25	32	51	31	32
normalized size	1	1.	1.92	1.04	1.33	2.12	1.29	1.33
time (sec)	N/A	0.014	0.012	0.008	0.981	1.928	0.18	1.136

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	0	312	0	100
normalized size	1	1.	0.96	0.91	0.	6.64	0.	2.13
time (sec)	N/A	0.027	0.06	0.075	0.	2.12	0.	1.282

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	28	132	0	27
normalized size	1	1.	1.	0.66	0.88	4.12	0.	0.84
time (sec)	N/A	0.016	0.04	0.128	0.981	2.159	0.	1.114

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	0	679	0	298
normalized size	1	1.	1.28	1.85	0.	6.59	0.	2.89
time (sec)	N/A	0.057	0.269	0.157	0.	2.262	0.	1.25

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	115	470	0	68
normalized size	1	1.	0.87	0.65	1.17	4.8	0.	0.69
time (sec)	N/A	0.042	0.29	0.181	1.017	2.215	0.	1.173

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	157	514	0	1216	0	794
normalized size	1	1.	1.01	3.29	0.	7.79	0.	5.09
time (sec)	N/A	0.086	1.128	0.222	0.	2.535	0.	1.345

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	182	125	235	984	0	159
normalized size	1	1.	1.21	0.83	1.56	6.52	0.	1.05
time (sec)	N/A	0.069	0.532	0.225	1.05	2.776	0.	1.129

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	205	183	0	0	0	0
normalized size	1	1.	1.1	0.98	0.	0.	0.	0.
time (sec)	N/A	0.096	1.859	1.891	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	256	246	0	0	0	0
normalized size	1	1.	1.95	1.88	0.	0.	0.	0.
time (sec)	N/A	0.058	1.613	1.464	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	143	163	0	0	0	0
normalized size	1	1.	1.09	1.24	0.	0.	0.	0.
time (sec)	N/A	0.058	1.326	1.221	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	268	159	0	0	0	0
normalized size	1	1.	3.57	2.12	0.	0.	0.	0.
time (sec)	N/A	0.03	1.142	1.204	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	92	121	0	0	0	0
normalized size	1	1.	1.23	1.61	0.	0.	0.	0.
time (sec)	N/A	0.03	0.186	1.072	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	219	228	0	0	0	0
normalized size	1	1.	1.59	1.65	0.	0.	0.	0.
time (sec)	N/A	0.058	3.12	1.742	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	145	178	0	0	0	0
normalized size	1	1.	1.02	1.25	0.	0.	0.	0.
time (sec)	N/A	0.055	1.687	1.589	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	277	309	0	0	0	0
normalized size	1	1.	1.41	1.57	0.	0.	0.	0.
time (sec)	N/A	0.086	2.421	2.812	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	153	128	0	0	0	0
normalized size	1	1.	1.27	1.07	0.	0.	0.	0.
time (sec)	N/A	0.07	0.514	1.334	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	199	174	0	0	0	0
normalized size	1	1.	2.65	2.32	0.	0.	0.	0.
time (sec)	N/A	0.045	0.877	1.238	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	133	108	0	0	0	0
normalized size	1	1.	1.77	1.44	0.	0.	0.	0.
time (sec)	N/A	0.043	0.318	1.186	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	27	27	184	112	0	0	0	0
normalized size	1	1.	6.81	4.15	0.	0.	0.	0.
time (sec)	N/A	0.023	0.864	0.935	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	27	27	88	85	0	0	0	0
normalized size	1	1.	3.26	3.15	0.	0.	0.	0.
time (sec)	N/A	0.024	0.109	0.687	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	190	162	0	0	0	0
normalized size	1	1.	2.6	2.22	0.	0.	0.	0.
time (sec)	N/A	0.041	1.112	1.253	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	157	118	0	0	0	0
normalized size	1	1.	2.09	1.57	0.	0.	0.	0.
time (sec)	N/A	0.042	0.742	1.101	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	224	205	0	0	0	0
normalized size	1	1.	1.87	1.71	0.	0.	0.	0.
time (sec)	N/A	0.064	2.102	1.348	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	31	80	43	116	31
normalized size	1	1.	0.97	0.97	2.5	1.34	3.62	0.97
time (sec)	N/A	0.015	0.084	0.029	1.539	2.028	7.149	1.643

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	151	178	46	37	70
normalized size	1	1.	1.	4.87	5.74	1.48	1.19	2.26
time (sec)	N/A	0.018	0.113	0.06	1.003	2.027	0.171	1.154

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	76	112	46	37	70
normalized size	1	1.	1.	2.45	3.61	1.48	1.19	2.26
time (sec)	N/A	0.016	0.082	0.058	1.011	2.037	0.167	1.108

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	73	93	46	37	70
normalized size	1	1.	1.	2.35	3.	1.48	1.19	2.26
time (sec)	N/A	0.014	0.055	0.047	0.999	2.186	0.161	1.164

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	51	26	32	32	26	32
normalized size	1	1.	1.96	1.	1.23	1.23	1.	1.23
time (sec)	N/A	0.014	0.012	0.001	0.987	2.162	0.144	1.129

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	39	35	31	28
normalized size	1	1.	1.	0.79	1.34	1.21	1.07	0.97
time (sec)	N/A	0.015	0.035	0.074	0.991	2.108	0.161	1.125

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	30	49	46	41
normalized size	1	1.	1.	0.74	0.97	1.58	1.48	1.32
time (sec)	N/A	0.016	0.045	0.092	0.983	1.993	0.179	1.116

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	57	39	49	46	49
normalized size	1	1.	1.	1.84	1.26	1.58	1.48	1.58
time (sec)	N/A	0.015	0.047	0.108	1.037	2.	0.182	1.107

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	36	39	49	46	59
normalized size	1	1.	1.	1.16	1.26	1.58	1.48	1.9
time (sec)	N/A	0.015	0.049	0.115	1.031	2.065	0.194	1.141

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	69	57	0	23
normalized size	1	1.	0.97	0.85	2.09	1.73	0.	0.7
time (sec)	N/A	0.016	0.03	0.055	1.573	2.006	0.	2.146

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	69	57	0	34
normalized size	1	1.	0.97	0.85	2.09	1.73	0.	1.03
time (sec)	N/A	0.016	0.029	0.03	1.548	1.917	0.	1.155

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	69	54	0	23
normalized size	1	1.	0.97	0.9	2.23	1.74	0.	0.74
time (sec)	N/A	0.016	0.022	0.033	1.542	2.019	0.	1.681

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	69	57	0	50
normalized size	1	1.	0.97	0.9	2.23	1.84	0.	1.61
time (sec)	N/A	0.017	0.035	0.029	1.542	1.949	0.	1.707

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	69	59	0	50
normalized size	1	1.	0.97	0.85	2.09	1.79	0.	1.52
time (sec)	N/A	0.016	0.033	0.03	1.629	2.081	0.	1.707

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	69	59	0	50
normalized size	1	1.	0.97	0.85	2.09	1.79	0.	1.52
time (sec)	N/A	0.016	0.036	0.029	1.585	1.929	0.	1.795

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	303	199	275	405	308	240
normalized size	1	1.	2.03	1.34	1.85	2.72	2.07	1.61
time (sec)	N/A	0.211	1.227	0.075	1.107	2.397	8.312	1.12

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	96	96	97	196	97	177
normalized size	1	1.	0.96	0.96	0.97	1.96	0.97	1.77
time (sec)	N/A	0.196	0.193	0.06	1.602	2.187	4.699	1.127

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	123	82	128	219	122	116
normalized size	1	1.	1.64	1.09	1.71	2.92	1.63	1.55
time (sec)	N/A	0.136	0.575	0.063	1.1	2.327	5.118	1.149

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	35	72	22	54
normalized size	1	1.	0.93	0.96	1.3	2.67	0.81	2.
time (sec)	N/A	0.054	0.043	0.044	1.566	2.082	1.48	1.146

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	42	16	19	81	24	46
normalized size	1	1.	3.5	1.33	1.58	6.75	2.	3.83
time (sec)	N/A	0.007	0.005	0.002	1.002	2.237	0.108	1.148

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	68	28	32	16
normalized size	1	1.	1.	1.09	6.18	2.55	2.91	1.45
time (sec)	N/A	0.034	0.006	0.048	1.533	2.284	0.6	1.135

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	422	106	0	687	0	127
normalized size	1	1.	6.39	1.61	0.	10.41	0.	1.92
time (sec)	N/A	0.133	3.755	0.079	0.	2.4	0.	1.142

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	57	271	197	508	58
normalized size	1	1.	0.78	1.12	5.31	3.86	9.96	1.14
time (sec)	N/A	0.075	0.166	0.085	1.565	2.442	4.611	1.151

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	2677	967	0	2061	0	498
normalized size	1	1.	17.16	6.2	0.	13.21	0.	3.19
time (sec)	N/A	0.337	6.369	0.113	0.	2.962	0.	1.17

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	86	130	652	509	1727	123
normalized size	1	1.	0.85	1.29	6.46	5.04	17.1	1.22
time (sec)	N/A	0.116	0.332	0.113	1.646	2.561	33.285	1.14

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	54	106	190	119	68	84
normalized size	1	1.	1.8	3.53	6.33	3.97	2.27	2.8
time (sec)	N/A	0.05	0.112	0.064	0.996	2.077	8.265	1.126

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	64	71	38	188	44	27
normalized size	1	1.	2.13	2.37	1.27	6.27	1.47	0.9
time (sec)	N/A	0.102	0.136	0.048	1.486	2.074	4.465	1.157

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	31	45	70	68	44	65
normalized size	1	1.	1.72	2.5	3.89	3.78	2.44	3.61
time (sec)	N/A	0.044	0.026	0.051	0.982	2.03	5.042	1.188

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	19	89	10	19
normalized size	1	1.	0.88	0.94	1.19	5.56	0.62	1.19
time (sec)	N/A	0.07	0.012	0.017	1.458	2.05	1.235	1.131

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	9	38	13	14	26	20	42
normalized size	1	0.69	2.92	1.	1.08	2.	1.54	3.23
time (sec)	N/A	0.006	0.005	0.001	0.967	2.056	0.113	1.178

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	42	23	17	30
normalized size	1	1.	3.2	1.2	8.4	4.6	3.4	6.
time (sec)	N/A	0.024	0.014	0.051	0.979	2.176	0.166	1.123

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	27	15	38	89	0	19
normalized size	1	1.	1.93	1.07	2.71	6.36	0.	1.36
time (sec)	N/A	0.041	0.026	0.059	1.494	2.067	0.	1.098

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	34	17	86	68	306	61
normalized size	1	1.	2.12	1.06	5.38	4.25	19.12	3.81
time (sec)	N/A	0.046	0.021	0.085	1.5	2.078	1.345	1.128

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	62	23	86	188	0	27
normalized size	1	1.	2.38	0.88	3.31	7.23	0.	1.04
time (sec)	N/A	0.069	0.075	0.072	1.513	2.149	0.	1.171

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	39	23	124	116	1064	86
normalized size	1	1.	1.77	1.05	5.64	5.27	48.36	3.91
time (sec)	N/A	0.048	0.047	0.109	1.512	2.096	5.487	1.128

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	143	204	254	703	0	228
normalized size	1	1.	0.94	1.34	1.67	4.62	0.	1.5
time (sec)	N/A	0.22	0.736	0.059	1.	2.178	0.	1.116

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	95	93	108	225	0	290
normalized size	1	1.	0.94	0.92	1.07	2.23	0.	2.87
time (sec)	N/A	0.215	0.257	0.049	1.504	1.927	0.	1.111

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	79	87	117	313	122	116
normalized size	1	1.	1.03	1.13	1.52	4.06	1.58	1.51
time (sec)	N/A	0.136	0.273	0.051	0.988	2.102	46.882	1.183

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	29	39	72	31	70
normalized size	1	1.	0.83	1.	1.34	2.48	1.07	2.41
time (sec)	N/A	0.056	0.13	0.014	1.484	1.95	7.407	1.142

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	25	16	20	97	24	28
normalized size	1	1.	2.08	1.33	1.67	8.08	2.	2.33
time (sec)	N/A	0.007	0.009	0.003	0.988	2.017	0.124	1.147

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	61	30	0	18
normalized size	1	1.	1.	1.08	5.08	2.5	0.	1.5
time (sec)	N/A	0.035	0.016	0.038	1.462	2.062	0.	1.131

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	71	86	0	689	0	144
normalized size	1	1.	1.06	1.28	0.	10.28	0.	2.15
time (sec)	N/A	0.117	0.255	0.051	0.	2.391	0.	1.151

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	77	56	239	181	0	61
normalized size	1	1.	1.54	1.12	4.78	3.62	0.	1.22
time (sec)	N/A	0.079	0.111	0.048	1.517	2.311	0.	1.116

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	150	534	0	1945	0	381
normalized size	1	1.	0.94	3.36	0.	12.23	0.	2.4
time (sec)	N/A	0.338	0.467	0.062	0.	2.937	0.	1.141

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	138	132	671	409	0	126
normalized size	1	1.	1.38	1.32	6.71	4.09	0.	1.26
time (sec)	N/A	0.123	0.339	0.056	1.698	2.538	0.	1.149

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	32	105	169	126	0	30
normalized size	1	1.	1.14	3.75	6.04	4.5	0.	1.07
time (sec)	N/A	0.051	0.073	0.048	1.013	2.034	0.	1.14

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	68	76	109	0	27
normalized size	1	1.	1.	2.27	2.53	3.63	0.	0.9
time (sec)	N/A	0.101	0.045	0.039	1.479	2.094	0.	1.136

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	62	77	44	24
normalized size	1	1.	1.	2.45	3.1	3.85	2.2	1.2
time (sec)	N/A	0.048	0.039	0.037	0.958	2.058	82.194	1.142

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	15	22	47	17	16
normalized size	1	1.	0.75	0.94	1.38	2.94	1.06	1.
time (sec)	N/A	0.069	0.024	0.012	1.494	1.953	8.483	1.153

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	13	16	32	20	24
normalized size	1	1.	2.22	1.44	1.78	3.56	2.22	2.67
time (sec)	N/A	0.005	0.004	0.003	1.011	2.073	0.107	1.175

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	19	32	0	9
normalized size	1	1.	1.29	1.14	2.71	4.57	0.	1.29
time (sec)	N/A	0.027	0.012	0.047	1.456	2.134	0.	1.134

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	31	55	0	14
normalized size	1	1.	0.86	0.79	2.21	3.93	0.	1.
time (sec)	N/A	0.041	0.013	0.044	1.488	2.014	0.	1.153

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	38	74	0	19
normalized size	1	1.	1.29	1.07	2.71	5.29	0.	1.36
time (sec)	N/A	0.047	0.014	0.073	1.512	1.939	0.	1.138

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	17	47	123	0	22
normalized size	1	1.	1.15	0.65	1.81	4.73	0.	0.85
time (sec)	N/A	0.071	0.014	0.073	1.48	1.931	0.	1.215

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	25	53	126	0	30
normalized size	1	1.	1.33	1.04	2.21	5.25	0.	1.25
time (sec)	N/A	0.05	0.013	0.08	1.565	2.192	0.	1.134

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	39	49	157	44	53
normalized size	1	1.	0.86	0.89	1.11	3.57	1.	1.2
time (sec)	N/A	0.034	0.03	0.022	1.006	2.121	18.101	1.165

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	61	32	50	197	42	134
normalized size	1	1.	1.79	0.94	1.47	5.79	1.24	3.94
time (sec)	N/A	0.046	0.02	0.019	0.987	2.122	5.871	1.16

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	22	63	15	31
normalized size	1	1.	0.82	0.68	1.	2.86	0.68	1.41
time (sec)	N/A	0.024	0.016	0.016	0.994	2.116	1.979	1.145

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	15	88	19	12
normalized size	1	1.	2.38	1.5	1.88	11.	2.38	1.5
time (sec)	N/A	0.005	0.004	0.003	0.985	2.15	0.104	1.149

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	5	23	14	0	23
normalized size	1	1.	1.	2.5	11.5	7.	0.	11.5
time (sec)	N/A	0.019	0.004	0.03	0.964	1.976	0.	1.159

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	50	0	8
normalized size	1	1.	1.	0.88	1.	6.25	0.	1.
time (sec)	N/A	0.014	0.003	0.036	1.023	1.978	0.	1.134

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	139	45	0	80
normalized size	1	1.	1.	0.82	8.18	2.65	0.	4.71
time (sec)	N/A	0.038	0.02	0.046	0.999	1.938	0.	1.18

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	18	85	0	18
normalized size	1	1.	2.18	0.82	1.06	5.	0.	1.06
time (sec)	N/A	0.018	0.017	0.043	0.988	2.132	0.	1.15

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	252	66	0	136
normalized size	1	1.	1.	0.8	10.08	2.64	0.	5.44
time (sec)	N/A	0.04	0.016	0.05	1.048	1.994	0.	1.131

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	26	135	0	26
normalized size	1	1.	2.28	0.8	1.04	5.4	0.	1.04
time (sec)	N/A	0.022	0.018	0.053	0.984	2.07	0.	1.157

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	366	96	0	193
normalized size	1	1.	1.	0.79	11.09	2.91	0.	5.85
time (sec)	N/A	0.043	0.017	0.058	1.057	2.218	0.	1.191

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	37	40	780	131	0	0
normalized size	1	1.	0.51	0.55	10.68	1.79	0.	0.
time (sec)	N/A	0.148	0.08	0.157	1.972	2.334	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	576	103	0	0
normalized size	1	1.	0.58	0.68	11.52	2.06	0.	0.
time (sec)	N/A	0.112	0.078	0.118	1.876	2.264	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	424	66	0	0
normalized size	1	1.	0.68	0.84	13.68	2.13	0.	0.
time (sec)	N/A	0.081	0.04	0.088	1.87	2.175	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	254	53	0	0
normalized size	1	1.	1.	1.54	19.54	4.08	0.	0.
time (sec)	N/A	0.049	0.028	0.091	1.801	2.012	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	0	0	393	0	0
normalized size	1	1.	0.73	0.	0.	6.55	0.	0.
time (sec)	N/A	0.091	0.267	0.121	0.	2.463	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	60	450	0	475	0	0
normalized size	1	1.	0.75	5.62	0.	5.94	0.	0.
time (sec)	N/A	0.116	0.155	0.209	0.	2.41	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	69	382	0	512	0	0
normalized size	1	1.	0.7	3.86	0.	5.17	0.	0.
time (sec)	N/A	0.151	0.532	0.191	0.	2.672	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	74	487	0	528	0	0
normalized size	1	1.	0.63	4.13	0.	4.47	0.	0.
time (sec)	N/A	0.18	0.268	0.227	0.	2.604	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	40	35	116	44	47
normalized size	1	1.	0.86	0.91	0.8	2.64	1.	1.07
time (sec)	N/A	0.031	0.032	0.023	0.978	2.102	32.136	1.143

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	30	50	161	42	53
normalized size	1	1.	1.12	0.88	1.47	4.74	1.24	1.56
time (sec)	N/A	0.042	0.011	0.022	0.989	2.276	8.172	1.163

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	16	13	16	68	14	24
normalized size	1	1.	0.73	0.59	0.73	3.09	0.64	1.09
time (sec)	N/A	0.021	0.018	0.016	0.974	2.067	2.594	1.16

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	37	12	15	72	19	39
normalized size	1	1.	4.62	1.5	1.88	9.	2.38	4.88
time (sec)	N/A	0.005	0.004	0.002	0.995	2.118	0.111	1.148

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	7	28	15	0	8
normalized size	1	1.	1.	1.75	7.	3.75	0.	2.
time (sec)	N/A	0.018	0.003	0.03	0.997	1.953	0.	1.105

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	51	0	8
normalized size	1	1.	1.	0.88	1.	6.38	0.	1.
time (sec)	N/A	0.014	0.003	0.03	0.98	2.034	0.	1.147

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	99	82	0	19
normalized size	1	1.	1.	0.82	5.82	4.82	0.	1.12
time (sec)	N/A	0.038	0.009	0.036	0.988	2.095	0.	1.112

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	19	112	0	19
normalized size	1	1.	2.18	0.82	1.12	6.59	0.	1.12
time (sec)	N/A	0.017	0.022	0.037	0.998	2.301	0.	1.17

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	163	139	0	27
normalized size	1	1.	1.	0.8	6.52	5.56	0.	1.08
time (sec)	N/A	0.041	0.012	0.043	1.016	2.457	0.	1.156

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	27	173	0	27
normalized size	1	1.	2.28	0.8	1.08	6.92	0.	1.08
time (sec)	N/A	0.021	0.02	0.039	0.99	2.455	0.	1.147

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	228	207	0	35
normalized size	1	1.	1.	0.79	6.91	6.27	0.	1.06
time (sec)	N/A	0.042	0.013	0.044	1.049	2.302	0.	1.16

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	37	603	111	134	0	0
normalized size	1	1.	0.51	8.26	1.52	1.84	0.	0.
time (sec)	N/A	0.113	0.204	0.235	1.574	2.124	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	29	321	111	112	0	0
normalized size	1	1.	0.58	6.42	2.22	2.24	0.	0.
time (sec)	N/A	0.086	0.076	0.146	1.585	2.245	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	584	77	76	0	0
normalized size	1	1.	0.74	18.84	2.48	2.45	0.	0.
time (sec)	N/A	0.064	0.039	0.104	1.577	2.179	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	174	77	63	0	0
normalized size	1	1.	1.	13.38	5.92	4.85	0.	0.
time (sec)	N/A	0.041	0.026	0.138	1.541	1.985	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	105	0	228	0	0
normalized size	1	1.	0.83	2.02	0.	4.38	0.	0.
time (sec)	N/A	0.078	0.246	0.108	0.	2.325	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	265	0	371	0	0
normalized size	1	1.	0.78	3.68	0.	5.15	0.	0.
time (sec)	N/A	0.094	0.172	0.149	0.	2.479	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	73	454	0	452	0	0
normalized size	1	1.	0.8	4.99	0.	4.97	0.	0.
time (sec)	N/A	0.121	0.647	0.15	0.	2.475	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	74	494	0	529	0	0
normalized size	1	1.	0.67	4.49	0.	4.81	0.	0.
time (sec)	N/A	0.14	0.369	0.167	0.	2.571	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	129	66	92	255	90	1856
normalized size	1	1.	2.35	1.2	1.67	4.64	1.64	33.75
time (sec)	N/A	0.11	0.2	0.022	1.492	2.207	8.14	8.736

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	39	57	151	46	234
normalized size	1	1.	1.05	1.03	1.5	3.97	1.21	6.16
time (sec)	N/A	0.05	0.041	0.02	0.985	2.23	6.665	1.653

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	60	25	38	154	31	239
normalized size	1	1.	2.4	1.	1.52	6.16	1.24	9.56
time (sec)	N/A	0.063	0.096	0.016	1.478	2.114	1.841	1.245

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	11	32	8	15
normalized size	1	1.	1.	1.1	1.1	3.2	0.8	1.5
time (sec)	N/A	0.005	0.003	0.002	0.983	2.267	0.065	1.128

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	34	132	0	38
normalized size	1	1.	1.46	1.	1.42	5.5	0.	1.58
time (sec)	N/A	0.058	0.015	0.037	0.979	2.195	0.	1.124

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	57	32	61	109	0	42
normalized size	1	1.	1.73	0.97	1.85	3.3	0.	1.27
time (sec)	N/A	0.127	0.016	0.041	0.996	2.137	0.	1.154

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	83	56	99	424	0	128
normalized size	1	1.	1.38	0.93	1.65	7.07	0.	2.13
time (sec)	N/A	0.072	0.018	0.052	1.002	2.248	0.	1.139

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	129	64	131	255	0	88
normalized size	1	1.	1.98	0.98	2.02	3.92	0.	1.35
time (sec)	N/A	0.211	0.021	0.054	1.002	2.183	0.	1.148

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	150	0	358	0	177
normalized size	1	1.	0.92	2.03	0.	4.84	0.	2.39
time (sec)	N/A	0.063	0.186	0.059	0.	2.381	0.	1.333

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	82	108	0	487	0	176
normalized size	1	1.	1.09	1.44	0.	6.49	0.	2.35
time (sec)	N/A	0.06	0.297	0.087	0.	2.292	0.	1.251

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	132	177	0	659	0	269
normalized size	1	1.	1.14	1.53	0.	5.68	0.	2.32
time (sec)	N/A	0.11	0.363	0.104	0.	2.494	0.	1.32

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	67	150	0	358	0	177
normalized size	1	1.	0.92	2.05	0.	4.9	0.	2.42
time (sec)	N/A	0.053	0.138	0.055	0.	2.318	0.	1.331

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	82	109	0	489	0	178
normalized size	1	1.	1.08	1.43	0.	6.43	0.	2.34
time (sec)	N/A	0.053	0.226	0.084	0.	2.349	0.	1.33

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	118	204	0	659	0	331
normalized size	1	1.	1.02	1.76	0.	5.68	0.	2.85
time (sec)	N/A	0.108	0.313	0.099	0.	2.434	0.	1.396

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	238	514	478	509	882	387
normalized size	1	1.	0.97	2.09	1.94	2.07	3.59	1.57
time (sec)	N/A	0.169	1.484	0.124	1.015	2.325	4.224	1.244

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	163	250	279	342	415	269
normalized size	1	1.	0.92	1.4	1.57	1.92	2.33	1.51
time (sec)	N/A	0.102	0.661	0.085	1.001	2.255	1.881	1.158

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	111	124	153	196	192	124
normalized size	1	1.	0.96	1.07	1.32	1.69	1.66	1.07
time (sec)	N/A	0.058	0.222	0.061	0.986	2.196	0.618	1.14

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	36	47	80	42	47
normalized size	1	1.	0.97	0.97	1.27	2.16	1.14	1.27
time (sec)	N/A	0.015	0.037	0.005	0.984	2.142	0.164	1.132

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	54	173	0	58
normalized size	1	1.	1.	1.02	1.1	3.53	0.	1.18
time (sec)	N/A	0.036	0.1	0.074	1.	2.156	0.	1.143

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	98	233	0	424	0	216
normalized size	1	1.	0.76	1.81	0.	3.29	0.	1.67
time (sec)	N/A	0.085	0.248	0.13	0.	2.303	0.	1.158

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	420	496	0	1080	0	467
normalized size	1	1.	2.2	2.6	0.	5.65	0.	2.45
time (sec)	N/A	0.133	2.777	0.194	0.	2.985	0.	1.142

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	533	823	0	1643	0	809
normalized size	1	1.	2.06	3.18	0.	6.34	0.	3.12
time (sec)	N/A	0.189	2.114	0.318	0.	5.611	0.	1.171

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	258	308	291	204
normalized size	1	1.	0.86	1.13	1.64	1.96	1.85	1.3
time (sec)	N/A	0.143	0.422	0.075	1.125	2.216	0.884	1.145

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	134	162	170	105
normalized size	1	1.	1.14	1.25	1.65	2.	2.1	1.3
time (sec)	N/A	0.05	0.146	0.054	1.018	2.08	0.377	1.171

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	39	63	39	39
normalized size	1	1.	1.83	1.03	1.34	2.17	1.34	1.34
time (sec)	N/A	0.016	0.016	0.001	0.985	2.145	0.159	1.122

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	57	23	39	157	63	31
normalized size	1	1.	2.28	0.92	1.56	6.28	2.52	1.24
time (sec)	N/A	0.022	0.053	0.087	0.997	2.175	1.204	1.163

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	115	91	122	398	0	116
normalized size	1	1.	1.53	1.21	1.63	5.31	0.	1.55
time (sec)	N/A	0.049	0.538	0.137	1.03	2.215	0.	1.145

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	186	211	257	986	0	231
normalized size	1	1.	1.39	1.57	1.92	7.36	0.	1.72
time (sec)	N/A	0.112	3.08	0.189	1.064	2.353	0.	1.177

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	492	378	414	1782	0	410
normalized size	1	1.	2.38	1.83	2.	8.61	0.	1.98
time (sec)	N/A	0.248	1.627	0.222	1.146	2.7	0.	1.142

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	50	21	38	89	36	28
normalized size	1	1.	2.17	0.91	1.65	3.87	1.57	1.22
time (sec)	N/A	0.021	0.03	0.065	0.987	2.03	0.719	1.132

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	93	60	108	285	168	92
normalized size	1	1.	1.24	0.8	1.44	3.8	2.24	1.23
time (sec)	N/A	0.048	0.184	0.086	1.017	2.122	2.487	1.143

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	135	100	197	405	423	144
normalized size	1	1.	1.1	0.81	1.6	3.29	3.44	1.17
time (sec)	N/A	0.107	0.582	0.099	1.071	2.133	9.654	1.159

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	247	140	281	645	0	188
normalized size	1	1.	1.47	0.83	1.67	3.84	0.	1.12
time (sec)	N/A	0.186	0.974	0.112	1.121	2.143	0.	1.149

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	136	178	254	306	291	204
normalized size	1	1.	0.87	1.13	1.62	1.95	1.85	1.3
time (sec)	N/A	0.134	0.447	0.075	0.997	2.263	0.907	1.138

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	132	161	170	105
normalized size	1	1.	1.14	1.23	1.63	1.99	2.1	1.3
time (sec)	N/A	0.047	0.155	0.054	0.987	2.291	0.382	1.139

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	39	63	39	39
normalized size	1	1.	1.83	1.03	1.34	2.17	1.34	1.34
time (sec)	N/A	0.014	0.015	0.001	0.979	2.007	0.157	1.134

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	50	42	73	159	95	57
normalized size	1	1.	2.	1.68	2.92	6.36	3.8	2.28
time (sec)	N/A	0.021	0.152	0.1	0.987	2.13	1.465	1.147

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	229	110	185	398	0	155
normalized size	1	1.	3.05	1.47	2.47	5.31	0.	2.07
time (sec)	N/A	0.053	0.414	0.151	1.017	2.342	0.	1.156

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	350	272	358	987	0	323
normalized size	1	1.	2.61	2.03	2.67	7.37	0.	2.41
time (sec)	N/A	0.113	0.611	0.196	1.063	2.469	0.	1.298

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	494	416	516	1783	0	490
normalized size	1	1.	2.39	2.01	2.49	8.61	0.	2.37
time (sec)	N/A	0.24	1.089	0.229	1.121	2.642	0.	1.208

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	258	293	291	204
normalized size	1	1.	0.86	1.13	1.64	1.87	1.85	1.3
time (sec)	N/A	0.14	0.464	0.069	0.997	2.308	0.935	1.109

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	134	162	170	107
normalized size	1	1.	1.14	1.25	1.65	2.	2.1	1.32
time (sec)	N/A	0.047	0.151	0.057	0.993	2.067	0.37	1.12

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	39	63	39	39
normalized size	1	1.	1.83	1.03	1.34	2.17	1.34	1.34
time (sec)	N/A	0.016	0.018	0.001	0.969	2.	0.155	1.122

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	93	104	89	136	107	96
normalized size	1	1.	2.82	3.15	2.7	4.12	3.24	2.91
time (sec)	N/A	0.022	0.07	0.093	0.989	2.112	61.581	1.152

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	162	166	250	377	0	250
normalized size	1	1.	1.95	2.	3.01	4.54	0.	3.01
time (sec)	N/A	0.05	0.507	0.162	1.046	2.361	0.	1.202

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	255	639	666	940	0	653
normalized size	1	1.	1.8	4.5	4.69	6.62	0.	4.6
time (sec)	N/A	0.112	2.417	0.178	1.129	2.452	0.	1.192

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	632	1069	1300	1632	0	1368
normalized size	1	1.	2.94	4.97	6.05	7.59	0.	6.36
time (sec)	N/A	0.244	2.917	0.223	1.253	2.91	0.	1.19

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	136	176	254	292	291	204
normalized size	1	1.	0.87	1.12	1.62	1.86	1.85	1.3
time (sec)	N/A	0.136	0.443	0.07	0.991	2.212	0.975	1.11

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	132	161	170	107
normalized size	1	1.	1.14	1.23	1.63	1.99	2.1	1.32
time (sec)	N/A	0.047	0.152	0.055	0.98	2.149	0.368	1.122

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	39	63	39	39
normalized size	1	1.	1.83	1.03	1.34	2.17	1.34	1.34
time (sec)	N/A	0.014	0.012	0.	0.991	2.149	0.153	1.125

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	96	61	84	136	109	101
normalized size	1	1.	2.91	1.85	2.55	4.12	3.3	3.06
time (sec)	N/A	0.022	0.102	0.1	0.996	2.182	61.238	1.195

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	166	178	246	377	0	258
normalized size	1	1.	2.	2.14	2.96	4.54	0.	3.11
time (sec)	N/A	0.053	0.557	0.148	1.006	2.278	0.	1.165

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	261	687	663	942	0	659
normalized size	1	1.	1.84	4.84	4.67	6.63	0.	4.64
time (sec)	N/A	0.108	2.622	0.17	1.1	2.458	0.	1.208

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	636	1149	1295	1635	0	1373
normalized size	1	1.	2.96	5.34	6.02	7.6	0.	6.39
time (sec)	N/A	0.235	1.854	0.224	1.258	2.825	0.	1.182

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	237	335	446	576	707	386
normalized size	1	1.	0.91	1.29	1.72	2.22	2.72	1.48
time (sec)	N/A	0.4	1.089	0.072	1.024	2.259	2.28	1.123

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	144	177	255	338	294	225
normalized size	1	1.	0.85	1.04	1.5	1.99	1.73	1.32
time (sec)	N/A	0.186	0.438	0.06	0.999	2.222	0.903	1.124

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	77	99	135	173	162	109
normalized size	1	1.	0.85	1.09	1.48	1.9	1.78	1.2
time (sec)	N/A	0.046	0.171	0.051	0.981	2.237	0.388	1.129

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	49	28	36	61	34	36
normalized size	1	1.	1.81	1.04	1.33	2.26	1.26	1.33
time (sec)	N/A	0.016	0.014	0.002	0.978	2.02	0.155	1.119

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	0	953	0	123
normalized size	1	1.	0.93	1.	0.	15.62	0.	2.02
time (sec)	N/A	0.084	0.117	0.079	0.	2.339	0.	1.117

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	116	424	0	1796	0	300
normalized size	1	1.	0.96	3.5	0.	14.84	0.	2.48
time (sec)	N/A	0.108	0.341	0.142	0.	2.988	0.	1.142

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	200	3933	0	4077	0	1204
normalized size	1	1.	1.02	19.96	0.	20.7	0.	6.11
time (sec)	N/A	0.198	0.909	0.191	0.	3.457	0.	1.179

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	606	16909	0	8541	0	3606
normalized size	1	1.	2.08	57.91	0.	29.25	0.	12.35
time (sec)	N/A	0.376	2.085	0.291	0.	5.098	0.	1.491

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	399	464	0	0	0	0
normalized size	1	1.	2.16	2.51	0.	0.	0.	0.
time (sec)	N/A	0.268	6.053	2.494	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	349	449	0	0	0	0
normalized size	1	1.	2.51	3.23	0.	0.	0.	0.
time (sec)	N/A	0.139	3.571	2.675	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	326	316	0	0	0	0
normalized size	1	1.	7.24	7.02	0.	0.	0.	0.
time (sec)	N/A	0.031	2.303	2.867	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	128	158	0	0	0	0
normalized size	1	1.	2.84	3.51	0.	0.	0.	0.
time (sec)	N/A	0.037	0.262	2.134	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	528	425	0	0	0	0
normalized size	1	1.	5.62	4.52	0.	0.	0.	0.
time (sec)	N/A	0.054	6.141	3.376	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	430	524	0	0	0	0
normalized size	1	1.	2.3	2.8	0.	0.	0.	0.
time (sec)	N/A	0.2	3.259	6.476	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	436	589	0	0	0	0
normalized size	1	1.	1.87	2.53	0.	0.	0.	0.
time (sec)	N/A	0.26	3.929	5.834	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	347	347	3767	2303	0	0	0	0
normalized size	1	1.	10.86	6.64	0.	0.	0.	0.
time (sec)	N/A	0.532	6.595	10.443	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	2190	1516	0	0	0	0
normalized size	1	1.	7.74	5.36	0.	0.	0.	0.
time (sec)	N/A	0.282	6.286	7.295	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	1408	720	0	0	0	0
normalized size	1	1.	13.04	6.67	0.	0.	0.	0.
time (sec)	N/A	0.071	6.274	3.265	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	285	303	0	0	0	0
normalized size	1	1.	2.64	2.81	0.	0.	0.	0.
time (sec)	N/A	0.07	0.648	2.115	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	1540	2388	0	0	0	0
normalized size	1	1.	8.28	12.84	0.	0.	0.	0.
time (sec)	N/A	0.104	6.374	8.075	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	2408	2967	0	0	0	0
normalized size	1	1.	6.3	7.77	0.	0.	0.	0.
time (sec)	N/A	0.363	6.386	29.057	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	490	490	4116	3876	0	0	0	0
normalized size	1	1.	8.4	7.91	0.	0.	0.	0.
time (sec)	N/A	0.619	6.621	94.	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	130	74	0	293	0	0
normalized size	1	1.	0.94	0.53	0.	2.11	0.	0.
time (sec)	N/A	0.065	0.627	1.355	0.	1.826	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	104	60	0	228	0	0
normalized size	1	1.	1.12	0.65	0.	2.45	0.	0.
time (sec)	N/A	0.04	0.34	1.44	0.	1.754	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	167	0	0
normalized size	1	1.	1.7	1.14	0.	3.8	0.	0.
time (sec)	N/A	0.018	0.038	0.89	0.	1.75	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	101	77	0	428	0	0
normalized size	1	1.	2.1	1.6	0.	8.92	0.	0.
time (sec)	N/A	0.065	0.105	0.89	0.	1.774	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	154	117	0	782	0	383
normalized size	1	1.	1.6	1.22	0.	8.15	0.	3.99
time (sec)	N/A	0.053	0.299	1.286	0.	1.954	0.	1.958

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	180	190	0	998	0	563
normalized size	1	1.	1.27	1.34	0.	7.03	0.	3.96
time (sec)	N/A	0.077	0.403	1.46	0.	1.973	0.	1.972

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	151	86	0	358	0	0
normalized size	1	1.	0.82	0.46	0.	1.94	0.	0.
time (sec)	N/A	0.094	1.922	1.25	0.	1.765	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	127	74	0	293	0	0
normalized size	1	1.	0.91	0.53	0.	2.11	0.	0.
time (sec)	N/A	0.074	0.523	1.248	0.	1.666	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	103	60	0	225	0	0
normalized size	1	1.	1.11	0.65	0.	2.42	0.	0.
time (sec)	N/A	0.038	0.237	1.193	0.	1.71	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	163	0	0
normalized size	1	1.	1.7	1.14	0.	3.7	0.	0.
time (sec)	N/A	0.017	0.042	1.077	0.	1.764	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	99	77	0	269	0	0
normalized size	1	1.	2.02	1.57	0.	5.49	0.	0.
time (sec)	N/A	0.061	0.089	1.171	0.	1.854	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	152	118	0	614	0	336
normalized size	1	1.	1.58	1.23	0.	6.4	0.	3.5
time (sec)	N/A	0.052	0.295	1.116	0.	1.786	0.	1.521

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	178	190	0	830	0	514
normalized size	1	1.	1.25	1.34	0.	5.85	0.	3.62
time (sec)	N/A	0.075	0.394	1.611	0.	1.878	0.	1.577

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	11888	306	0	645	0	0
normalized size	1	1.	46.08	1.19	0.	2.5	0.	0.
time (sec)	N/A	0.179	32.745	1.937	0.	2.007	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	11771	200	0	463	0	0
normalized size	1	1.	61.95	1.05	0.	2.44	0.	0.
time (sec)	N/A	0.122	33.091	1.45	0.	1.957	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	11679	126	0	313	0	0
normalized size	1	1.	92.69	1.	0.	2.48	0.	0.
time (sec)	N/A	0.075	21.959	1.641	0.	1.706	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	11586	113	0	201	0	0
normalized size	1	1.	210.65	2.05	0.	3.65	0.	0.
time (sec)	N/A	0.033	21.979	1.319	0.	1.641	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	63264	172	0	0	0	0
normalized size	1	1.	718.91	1.95	0.	0.	0.	0.
time (sec)	N/A	0.12	33.865	1.432	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	F(-2)	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	0	350	0	0	0	0
normalized size	1	1.	0.	2.19	0.	0.	0.	0.
time (sec)	N/A	0.133	180.001	1.644	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F(-2)	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	0	350	0	0	0	0
normalized size	1	1.	0.	1.55	0.	0.	0.	0.
time (sec)	N/A	0.186	180.002	1.552	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	11602	204	0	463	0	0
normalized size	1	1.	59.19	1.04	0.	2.36	0.	0.
time (sec)	N/A	0.134	34.231	1.753	0.	1.895	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	11512	130	0	313	0	0
normalized size	1	1.	88.55	1.	0.	2.41	0.	0.
time (sec)	N/A	0.081	21.255	1.591	0.	1.883	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	11415	117	0	201	0	0
normalized size	1	1.	200.26	2.05	0.	3.53	0.	0.
time (sec)	N/A	0.038	21.525	1.706	0.	1.861	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	61904	175	0	0	0	0
normalized size	1	1.	680.26	1.92	0.	0.	0.	0.
time (sec)	N/A	0.098	33.882	1.314	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	F(-2)	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	0	363	0	0	0	0
normalized size	1	1.	0.	2.21	0.	0.	0.	0.
time (sec)	N/A	0.126	180.001	1.934	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F(-2)	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	0	363	0	0	0	0
normalized size	1	1.	0.	1.56	0.	0.	0.	0.
time (sec)	N/A	0.17	180.014	2.082	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	80	438	0	1291	0	216
normalized size	1	1.	0.79	4.34	0.	12.78	0.	2.14
time (sec)	N/A	0.096	0.224	0.048	0.	2.148	0.	1.154

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	30	22	25	55	39	22	34
normalized size	1	1.36	1.	1.14	2.5	1.77	1.	1.55
time (sec)	N/A	0.031	0.047	0.043	1.482	1.868	0.312	1.165

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	79	414	0	1277	0	213
normalized size	1	1.	0.81	4.27	0.	13.16	0.	2.2
time (sec)	N/A	0.127	0.193	0.064	0.	2.568	0.	1.174

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	799	0	99
normalized size	1	1.	0.98	1.04	0.	15.67	0.	1.94
time (sec)	N/A	0.076	0.045	0.059	0.	2.14	0.	1.165

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	120	430	0	1539	0	217
normalized size	1	1.	0.85	3.03	0.	10.84	0.	1.53
time (sec)	N/A	0.515	0.289	0.059	0.	21.925	0.	1.236

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	21265	0	0	0	0
normalized size	1	1.	6.71	57.32	0.	0.	0.	0.
time (sec)	N/A	0.447	6.452	2.027	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	12462	0	0	0	0
normalized size	1	1.	13.39	105.61	0.	0.	0.	0.
time (sec)	N/A	0.144	6.254	0.666	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	339	722	0	0	0	0
normalized size	1	1.	2.87	6.12	0.	0.	0.	0.
time (sec)	N/A	0.166	0.919	0.596	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	12572	0	0	0	0
normalized size	1	1.	7.22	52.38	0.	0.	0.	0.
time (sec)	N/A	0.216	6.398	0.589	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	63949	0	0	0	0
normalized size	1	1.	5.5	129.98	0.	0.	0.	0.
time (sec)	N/A	0.519	6.514	1.75	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	0	21015	0	0	0	0
normalized size	1	1.	0.	56.64	0.	0.	0.	0.
time (sec)	N/A	0.39	151.125	0.709	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	0	12460	0	0	0	0
normalized size	1	1.	0.	105.59	0.	0.	0.	0.
time (sec)	N/A	0.147	21.288	0.444	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	506	714	0	0	0	0
normalized size	1	1.	4.29	6.05	0.	0.	0.	0.
time (sec)	N/A	0.152	2.944	0.415	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	0	12562	0	0	0	0
normalized size	1	1.	0.	52.34	0.	0.	0.	0.
time (sec)	N/A	0.211	24.569	0.44	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	492	492	0	63939	0	0	0	0
normalized size	1	1.	0.	129.96	0.	0.	0.	0.
time (sec)	N/A	0.495	28.201	1.151	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	446	0	1277	0	213
normalized size	1	1.	0.82	4.55	0.	13.03	0.	2.17
time (sec)	N/A	0.103	0.224	0.051	0.	2.18	0.	1.162

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	799	0	99
normalized size	1	1.	0.98	1.04	0.	15.67	0.	1.94
time (sec)	N/A	0.072	0.046	0.042	0.	2.167	0.	1.174

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	184	0	1571	0	192
normalized size	1	1.	0.87	1.53	0.	13.09	0.	1.6
time (sec)	N/A	0.533	0.286	0.045	0.	23.526	0.	1.179

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	51	10	19	74	0	30
normalized size	1	1.	2.43	0.48	0.9	3.52	0.	1.43
time (sec)	N/A	0.048	0.025	0.039	1.483	2.193	0.	1.143

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	20627	0	0	0	0
normalized size	1	1.	6.71	55.6	0.	0.	0.	0.
time (sec)	N/A	0.432	6.435	1.618	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	12367	0	0	0	0
normalized size	1	1.	13.39	104.81	0.	0.	0.	0.
time (sec)	N/A	0.144	6.256	0.55	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	339	715	0	0	0	0
normalized size	1	1.	2.87	6.06	0.	0.	0.	0.
time (sec)	N/A	0.166	0.908	0.587	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	12477	0	0	0	0
normalized size	1	1.	7.22	51.99	0.	0.	0.	0.
time (sec)	N/A	0.212	6.415	0.556	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	64199	0	0	0	0
normalized size	1	1.	5.5	130.49	0.	0.	0.	0.
time (sec)	N/A	0.497	6.513	1.804	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	0	20858	0	0	0	0
normalized size	1	1.	0.	56.22	0.	0.	0.	0.
time (sec)	N/A	0.383	53.407	0.677	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	0	12362	0	0	0	0
normalized size	1	1.	0.	104.76	0.	0.	0.	0.
time (sec)	N/A	0.141	12.671	0.435	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	519	705	0	0	0	0
normalized size	1	1.	4.4	5.97	0.	0.	0.	0.
time (sec)	N/A	0.149	2.89	0.403	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	0	12467	0	0	0	0
normalized size	1	1.	0.	51.95	0.	0.	0.	0.
time (sec)	N/A	0.205	21.461	0.433	0.	0.	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	492	492	0	64189	0	0	0	0
normalized size	1	1.	0.	130.47	0.	0.	0.	0.
time (sec)	N/A	0.491	25.974	1.202	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	10	1
normalized size	1	1.	1.	2.	1.	4.	10.	1.
time (sec)	N/A	0.009	0.	0.014	1.473	1.557	0.423	1.147

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	22	1
normalized size	1	1.	1.	2.	1.	4.	22.	1.
time (sec)	N/A	0.009	0.	0.016	1.496	1.875	1.088	1.129

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	34	1
normalized size	1	1.	1.	2.	1.	4.	34.	1.
time (sec)	N/A	0.009	0.	0.016	1.457	1.636	3.321	1.098

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	20	84	36	45
normalized size	1	1.	2.09	0.36	1.82	7.64	3.27	4.09
time (sec)	N/A	0.015	0.005	0.025	1.031	1.857	0.469	1.122

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	8	18	16	43	48	8
normalized size	1	1.	0.62	1.38	1.23	3.31	3.69	0.62
time (sec)	N/A	0.023	0.003	0.031	0.983	1.649	2.396	1.117

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	22	48	51	227	765	50
normalized size	1	1.	0.69	1.5	1.59	7.09	23.91	1.56
time (sec)	N/A	0.027	0.007	0.037	1.005	1.889	9.113	1.153

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	88	2011	27
normalized size	1	1.	1.	1.11	1.33	9.78	223.44	3.
time (sec)	N/A	0.018	0.034	0.038	1.517	1.832	29.425	1.13

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	85	2118	30
normalized size	1	1.	1.	1.09	1.36	7.73	192.55	2.73
time (sec)	N/A	0.02	0.032	0.033	1.463	1.915	32.779	1.111

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	99	2866	35
normalized size	1	1.	1.	1.07	1.33	6.6	191.07	2.33
time (sec)	N/A	0.026	0.04	0.04	1.482	1.811	49.337	1.164

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	25	18	23	128	87	82
normalized size	1	1.	0.47	0.34	0.43	2.42	1.64	1.55
time (sec)	N/A	0.037	0.043	0.055	1.467	1.851	1.291	1.141

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	38	0	433	241	151
normalized size	1	1.	0.84	0.88	0.	10.07	5.6	3.51
time (sec)	N/A	0.149	0.099	0.051	0.	1.992	2.537	1.215

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	36	0	432	267	200
normalized size	1	1.	0.84	0.84	0.	10.05	6.21	4.65
time (sec)	N/A	0.109	0.058	0.044	0.	1.94	2.554	1.125

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	19	16	20	107	0	20
normalized size	1	1.	0.53	0.44	0.56	2.97	0.	0.56
time (sec)	N/A	0.028	0.047	0.046	1.469	1.773	0.	1.113

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	27	36	207	0	36
normalized size	1	1.	0.86	0.55	0.73	4.22	0.	0.73
time (sec)	N/A	0.045	0.137	0.057	1.484	1.821	0.	1.124

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	79	40	61	305	0	53
normalized size	1	1.	1.07	0.54	0.82	4.12	0.	0.72
time (sec)	N/A	0.055	0.185	0.054	1.485	1.843	0.	1.162

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	4	0	1
normalized size	1	1.	1.	4.	1.	4.	0.	1.
time (sec)	N/A	0.012	0.001	0.024	1.487	1.601	0.	1.114

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	4	0	1
normalized size	1	1.	1.	4.	1.	4.	0.	1.
time (sec)	N/A	0.013	0.	0.024	1.49	1.577	0.	1.141

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	4	0	1
normalized size	1	1.	1.	4.	1.	4.	0.	1.
time (sec)	N/A	0.012	0.	0.028	1.508	1.615	0.	1.117

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	19	17	22	104	0	66
normalized size	1	1.	0.51	0.46	0.59	2.81	0.	1.78
time (sec)	N/A	0.031	0.043	0.077	1.482	1.899	0.	1.131

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	64	28	36	201	0	81
normalized size	1	1.	1.36	0.6	0.77	4.28	0.	1.72
time (sec)	N/A	0.04	0.106	0.103	1.495	1.915	0.	1.154

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	39	57	297	0	93
normalized size	1	1.	0.92	0.54	0.79	4.12	0.	1.29
time (sec)	N/A	0.076	0.164	0.121	1.515	1.854	0.	1.131

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	4	5	0	4
normalized size	1	1.	1.	2.	1.33	1.67	0.	1.33
time (sec)	N/A	0.013	0.	0.023	1.485	1.644	0.	1.126

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	4	0	1
normalized size	1	1.	1.	4.	1.	4.	0.	1.
time (sec)	N/A	0.013	0.001	0.027	1.489	1.762	0.	1.142

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	4	5	0	4
normalized size	1	1.	1.	2.	1.33	1.67	0.	1.33
time (sec)	N/A	0.013	0.001	0.029	1.487	1.485	0.	1.122

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	0	660	0	82
normalized size	1	1.	1.	0.82	0.	20.	0.	2.48
time (sec)	N/A	0.05	0.063	0.032	0.	2.147	0.	1.14

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	507	820	0	7385	0	0
normalized size	1	1.	2.12	3.43	0.	30.9	0.	0.
time (sec)	N/A	0.492	3.15	0.105	0.	4.134	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	258	1161	0	11062	0	0
normalized size	1	1.	0.71	3.18	0.	30.31	0.	0.
time (sec)	N/A	0.739	4.032	0.11	0.	4.558	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	149	255	332	348	566	213
normalized size	1	1.	0.76	1.31	1.7	1.78	2.9	1.09
time (sec)	N/A	0.393	0.943	0.035	1.037	1.826	3.519	1.167

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	77	115	151	182	204	107
normalized size	1	1.	0.71	1.06	1.39	1.67	1.87	0.98
time (sec)	N/A	0.099	0.296	0.024	0.991	1.743	0.782	1.139

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	52	0	54	0	70
normalized size	1	1.	1.	2.26	0.	2.35	0.	3.04
time (sec)	N/A	0.089	0.062	0.092	0.	1.677	0.	1.196

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	140	1297	0	1705	0	613
normalized size	1	1.	0.89	8.26	0.	10.86	0.	3.9
time (sec)	N/A	0.415	0.974	0.118	0.	2.165	0.	1.273

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	286	832	0	13609	0	0
normalized size	1	1.	1.18	3.44	0.	56.24	0.	0.
time (sec)	N/A	0.94	0.7	0.095	0.	61.352	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	140	269	751	263	0	323
normalized size	1	1.	0.42	0.81	2.27	0.79	0.	0.98
time (sec)	N/A	0.323	0.894	0.306	1.587	1.836	0.	1.334

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	70	107	252	108	0	132
normalized size	1	1.	0.38	0.58	1.36	0.58	0.	0.71
time (sec)	N/A	0.109	0.193	0.209	1.559	1.817	0.	1.224

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	85	176	0	462	0	281
normalized size	1	1.	0.62	1.28	0.	3.37	0.	2.05
time (sec)	N/A	0.198	0.183	0.164	0.	1.883	0.	1.35

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	144	738	0	1170	0	647
normalized size	1	1.	0.6	3.09	0.	4.9	0.	2.71
time (sec)	N/A	0.272	0.353	0.153	0.	2.108	0.	1.597

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	33	0	31	0	43
normalized size	1	1.	1.	3.	0.	2.82	0.	3.91
time (sec)	N/A	0.081	0.052	0.037	0.	1.7	0.	1.162

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	241	2556	0	13604	0	0
normalized size	1	1.	0.98	10.39	0.	55.3	0.	0.
time (sec)	N/A	0.789	0.579	0.071	0.	65.635	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	153	245	203	342	248	3140
normalized size	1	1.	1.06	1.7	1.41	2.38	1.72	21.81
time (sec)	N/A	0.269	2.32	0.008	1.512	1.857	0.741	7.671

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	88	117	100	174	122	957
normalized size	1	1.	1.22	1.62	1.39	2.42	1.69	13.29
time (sec)	N/A	0.076	0.334	0.005	1.507	1.793	0.326	2.075

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	187	222	217	417	0	275
normalized size	1	1.	1.85	2.2	2.15	4.13	0.	2.72
time (sec)	N/A	0.258	2.041	0.05	1.503	1.74	0.	1.575

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	308	458	566	1266	0	609
normalized size	1	1.	1.56	2.32	2.87	6.43	0.	3.09
time (sec)	N/A	0.535	4.898	0.059	1.592	2.079	0.	1.626

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	147	158	224	236	0	328
normalized size	1	1.	0.52	0.56	0.79	0.83	0.	1.15
time (sec)	N/A	0.226	1.367	0.088	1.525	1.751	0.	1.522

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	58	75	88	95	0	100
normalized size	1	1.	0.48	0.61	0.72	0.78	0.	0.82
time (sec)	N/A	0.101	0.288	0.095	1.558	1.732	0.	1.257

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	88	114	185	163	0	748
normalized size	1	1.	0.64	0.83	1.34	1.18	0.	5.42
time (sec)	N/A	0.188	0.704	0.067	1.519	1.652	0.	2.019

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	268	622	672	764	0	2121
normalized size	1	1.	0.85	1.97	2.13	2.42	0.	6.71
time (sec)	N/A	0.402	3.186	0.094	1.547	1.738	0.	3.006

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	130	246	404	481	0	635
normalized size	1	1.	0.71	1.34	2.2	2.61	0.	3.45
time (sec)	N/A	0.43	0.802	0.073	1.033	1.977	0.	1.28

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	110	170	305	0	258
normalized size	1	1.	0.84	1.45	2.24	4.01	0.	3.39
time (sec)	N/A	0.078	0.265	0.039	1.021	1.748	0.	1.223

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	163	0	670	0	196
normalized size	1	1.	1.05	1.77	0.	7.28	0.	2.13
time (sec)	N/A	0.304	0.385	0.093	0.	2.013	0.	1.238

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	276	1118	0	2871	0	662
normalized size	1	1.	1.2	4.86	0.	12.48	0.	2.88
time (sec)	N/A	0.833	1.576	0.113	0.	3.055	0.	1.332

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	128	387	594	394	0	880
normalized size	1	1.	0.36	1.08	1.65	1.1	0.	2.45
time (sec)	N/A	0.287	0.838	0.264	1.667	2.205	0.	1.667

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	67	208	221	225	0	302
normalized size	1	1.	0.39	1.2	1.28	1.3	0.	1.75
time (sec)	N/A	0.119	0.263	0.238	1.592	2.036	0.	1.36

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	92	157	0	421	0	265
normalized size	1	1.	0.65	1.11	0.	2.96	0.	1.87
time (sec)	N/A	0.213	0.392	0.238	0.	2.174	0.	1.56

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	216	756	0	1739	0	768
normalized size	1	1.	0.65	2.29	0.	5.27	0.	2.33
time (sec)	N/A	0.566	1.011	0.21	0.	2.618	0.	1.886

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	24	8	19
normalized size	1	1.	1.12	0.47	0.	1.41	0.47	1.12
time (sec)	N/A	0.04	0.005	0.065	0.	1.788	0.128	1.137

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	24	10	19
normalized size	1	1.	1.12	0.47	0.	1.41	0.59	1.12
time (sec)	N/A	0.037	0.005	0.057	0.	1.882	0.103	1.144

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	41	7	22
normalized size	1	1.	1.	1.17	1.33	6.83	1.17	3.67
time (sec)	N/A	0.023	0.026	0.022	0.99	1.889	0.138	1.148

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	39	111	244	139	360	104
normalized size	1	1.	0.83	2.36	5.19	2.96	7.66	2.21
time (sec)	N/A	0.041	0.119	0.059	1.511	1.987	2.46	1.161

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	75	113	0	466	0	178
normalized size	1	1.	1.01	1.53	0.	6.3	0.	2.41
time (sec)	N/A	0.068	0.215	0.092	0.	2.039	0.	1.2

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	37	269	333	0	35
normalized size	1	1.	0.97	0.56	4.08	5.05	0.	0.53
time (sec)	N/A	0.057	0.174	0.098	1.078	1.897	0.	1.21

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	222	0	379	0	200
normalized size	1	1.	0.93	2.64	0.	4.51	0.	2.38
time (sec)	N/A	0.058	0.219	0.06	0.	2.134	0.	1.292

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	92	124	0	544	0	203
normalized size	1	1.	1.08	1.46	0.	6.4	0.	2.39
time (sec)	N/A	0.057	0.24	0.09	0.	2.29	0.	1.191

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	122	218	0	728	0	365
normalized size	1	1.	0.95	1.69	0.	5.64	0.	2.83
time (sec)	N/A	0.124	0.585	0.105	0.	2.263	0.	1.296

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	95	544	0	1377	0	240
normalized size	1	1.	0.83	4.73	0.	11.97	0.	2.09
time (sec)	N/A	0.13	0.269	0.053	0.	2.798	0.	1.167

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	254	0	2691	0	282
normalized size	1	1.	1.04	2.25	0.	23.81	0.	2.5
time (sec)	N/A	0.106	0.288	0.094	0.	2.881	0.	1.173

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	326	1109	0	7119	0	1569
normalized size	1	1.	1.63	5.54	0.	35.6	0.	7.84
time (sec)	N/A	0.255	0.804	0.115	0.	4.659	0.	1.317

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	147	153	0	173	58	209
normalized size	1	1.	1.75	1.82	0.	2.06	0.69	2.49
time (sec)	N/A	0.045	0.21	0.08	0.	2.007	1.715	1.133

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	147	284	0	132	51	209
normalized size	1	1.	1.75	3.38	0.	1.57	0.61	2.49
time (sec)	N/A	0.042	0.183	0.08	0.	1.894	0.884	1.156

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	96	542	0	1374	0	239
normalized size	1	1.	0.83	4.67	0.	11.84	0.	2.06
time (sec)	N/A	0.108	0.278	0.06	0.	2.851	0.	1.152

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	123	255	0	2739	0	278
normalized size	1	1.	1.08	2.24	0.	24.03	0.	2.44
time (sec)	N/A	0.101	0.343	0.091	0.	2.94	0.	1.166

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	361	1088	0	7318	0	1423
normalized size	1	1.	1.8	5.44	0.	36.59	0.	7.12
time (sec)	N/A	0.249	0.858	0.125	0.	4.914	0.	1.352

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	152	151	0	167	54	212
normalized size	1	1.	1.79	1.78	0.	1.96	0.64	2.49
time (sec)	N/A	0.046	0.26	0.082	0.	1.928	1.759	1.172

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	152	280	0	127	48	212
normalized size	1	1.	1.79	3.29	0.	1.49	0.56	2.49
time (sec)	N/A	0.046	0.238	0.077	0.	2.09	0.977	1.129

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	98	824	0	1507	0	252
normalized size	1	1.	0.82	6.92	0.	12.66	0.	2.12
time (sec)	N/A	0.113	0.353	0.057	0.	2.667	0.	1.162

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	116	255	0	2799	0	277
normalized size	1	1.	1.05	2.32	0.	25.45	0.	2.52
time (sec)	N/A	0.096	0.357	0.1	0.	2.794	0.	1.177

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	311	881	0	6728	0	1396
normalized size	1	1.	1.58	4.47	0.	34.15	0.	7.09
time (sec)	N/A	0.232	0.821	0.127	0.	4.524	0.	1.321

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	87	195	212	0	178	75	238
normalized size	1	0.95	2.12	2.3	0.	1.93	0.82	2.59
time (sec)	N/A	0.078	0.297	0.079	0.	2.085	1.939	1.165

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	85	195	388	0	153	75	238
normalized size	1	0.94	2.17	4.31	0.	1.7	0.83	2.64
time (sec)	N/A	0.078	0.277	0.085	0.	2.055	1.063	1.169

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	110	954	0	1550	0	269
normalized size	1	1.	0.84	7.28	0.	11.83	0.	2.05
time (sec)	N/A	0.126	0.333	0.056	0.	2.987	0.	1.15

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	137	329	0	3236	0	325
normalized size	1	1.	1.08	2.59	0.	25.48	0.	2.56
time (sec)	N/A	0.123	0.436	0.102	0.	3.057	0.	1.184

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	452	1422	0	8820	0	2033
normalized size	1	1.	1.91	6.	0.	37.22	0.	8.58
time (sec)	N/A	0.277	1.194	0.13	0.	5.721	0.	1.349

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	165	257	0	209	87	278
normalized size	1	1.	1.57	2.45	0.	1.99	0.83	2.65
time (sec)	N/A	0.074	0.413	0.08	0.	2.147	2.331	1.127

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	167	475	0	169	80	278
normalized size	1	1.	1.62	4.61	0.	1.64	0.78	2.7
time (sec)	N/A	0.073	0.421	0.082	0.	2.067	1.38	1.142

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	68	32	70	0	68	0	92
normalized size	1	2.83	1.33	2.92	0.	2.83	0.	3.83
time (sec)	N/A	0.068	0.094	0.102	0.	1.943	0.	1.184

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	7823	3502	0	0	0	0
normalized size	1	1.	20.06	8.98	0.	0.	0.	0.
time (sec)	N/A	0.888	6.902	16.03	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	5218	2238	0	0	0	0
normalized size	1	1.	17.75	7.61	0.	0.	0.	0.
time (sec)	N/A	0.555	6.555	10.746	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	3006	1460	0	0	0	0
normalized size	1	1.	13.13	6.38	0.	0.	0.	0.
time (sec)	N/A	0.332	6.343	8.033	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	1319	777	0	0	0	0
normalized size	1	1.	7.33	4.32	0.	0.	0.	0.
time (sec)	N/A	0.187	6.261	6.956	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	3176	2596	0	0	0	0
normalized size	1	1.	12.7	10.38	0.	0.	0.	0.
time (sec)	N/A	0.323	6.487	9.936	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	5554	3164	0	0	0	0
normalized size	1	1.	14.69	8.37	0.	0.	0.	0.
time (sec)	N/A	0.561	6.847	43.325	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	80	178	0	763	1151	190
normalized size	1	1.	0.95	2.12	0.	9.08	13.7	2.26
time (sec)	N/A	0.152	0.262	0.059	0.	2.638	52.295	1.134

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	114	426	0	995	0	252
normalized size	1	1.	0.97	3.61	0.	8.43	0.	2.14
time (sec)	N/A	0.157	0.46	0.16	0.	2.769	0.	1.146

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	174	1891	0	1891	0	805
normalized size	1	1.	0.94	10.22	0.	10.22	0.	4.35
time (sec)	N/A	0.246	0.957	0.147	0.	3.337	0.	1.222

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	244	5051	0	3085	0	1809
normalized size	1	1.	0.95	19.58	0.	11.96	0.	7.01
time (sec)	N/A	0.405	2.769	0.166	0.	3.838	0.	1.276

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	145	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.61	0.521	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	75	106	108	228	190	101
normalized size	1	1.	0.7	0.99	1.01	2.13	1.78	0.94
time (sec)	N/A	0.082	0.283	0.061	1.006	2.486	4.363	1.131

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	69	65	146	129	62
normalized size	1	1.	0.79	1.13	1.07	2.39	2.11	1.02
time (sec)	N/A	0.034	0.157	0.049	0.999	2.412	1.159	1.134

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	24	49	24	24
normalized size	1	1.	1.9	0.95	1.2	2.45	1.2	1.2
time (sec)	N/A	0.016	0.008	0.001	1.003	2.308	0.199	1.125

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	0	653	0	82
normalized size	1	1.	1.	0.94	0.	13.6	0.	1.71
time (sec)	N/A	0.066	0.076	0.067	0.	2.413	0.	1.135

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	94	139	0	1116	0	157
normalized size	1	1.	0.99	1.46	0.	11.75	0.	1.65
time (sec)	N/A	0.109	0.414	0.113	0.	2.723	0.	1.177

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	640	0	2152	0	340
normalized size	1	1.	0.81	4.3	0.	14.44	0.	2.28
time (sec)	N/A	0.177	0.95	0.148	0.	3.174	0.	1.149

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	202	1138	0	0	0	0
normalized size	1	1.	0.76	4.29	0.	0.	0.	0.
time (sec)	N/A	0.366	1.914	3.159	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	167	844	0	0	0	0
normalized size	1	1.	0.79	3.98	0.	0.	0.	0.
time (sec)	N/A	0.22	1.502	2.825	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	312	0	0	0	0
normalized size	1	1.	0.99	4.11	0.	0.	0.	0.
time (sec)	N/A	0.061	0.114	2.24	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	165	0	0	0	0
normalized size	1	1.	0.92	2.17	0.	0.	0.	0.
time (sec)	N/A	0.067	0.14	2.135	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	101	570	0	0	0	0
normalized size	1	1.	0.71	3.99	0.	0.	0.	0.
time (sec)	N/A	0.094	0.434	3.159	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	201	1554	0	0	0	0
normalized size	1	1.	0.68	5.27	0.	0.	0.	0.
time (sec)	N/A	0.301	1.532	3.382	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	340	2282	0	7900	0	0
normalized size	1	1.	0.74	4.95	0.	17.14	0.	0.
time (sec)	N/A	0.63	0.863	0.148	0.	5.089	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	256	1782	0	5913	0	0
normalized size	1	1.	0.75	5.24	0.	17.39	0.	0.
time (sec)	N/A	0.537	0.746	0.112	0.	4.789	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	788	1284	0	3969	0	0
normalized size	1	1.	3.5	5.71	0.	17.64	0.	0.
time (sec)	N/A	0.319	1.434	0.111	0.	4.885	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	1.628	0.107	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	5.515	0.951	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	4.919	0.837	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	198	0	0	478	0	9918
normalized size	1	1.	1.13	0.	0.	2.73	0.	56.67
time (sec)	N/A	0.297	1.48	180.	0.	2.734	0.	2.587

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	142	0	0	385	0	5636
normalized size	1	1.	1.08	0.	0.	2.94	0.	43.02
time (sec)	N/A	0.227	0.997	180.	0.	2.493	0.	1.856

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	209	0	1395
normalized size	1	1.	0.96	0.	0.	2.61	0.	17.44
time (sec)	N/A	0.131	0.854	180.	0.	2.23	0.	1.502

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	242	108	0	138	0	670
normalized size	1	1.	4.32	1.93	0.	2.46	0.	11.96
time (sec)	N/A	0.102	7.546	0.666	0.	2.084	0.	1.286

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	77	154	54	20	53
normalized size	1	1.	0.69	2.2	4.4	1.54	0.57	1.51
time (sec)	N/A	0.024	0.298	0.454	1.062	2.083	3.598	1.149

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	27	47	19	57
normalized size	1	1.	1.	1.05	1.35	2.35	0.95	2.85
time (sec)	N/A	0.038	0.031	0.035	1.012	1.966	3.463	1.17

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	54	135	80	112	72
normalized size	1	1.	0.91	1.54	3.86	2.29	3.2	2.06
time (sec)	N/A	0.039	0.464	0.316	1.025	2.133	5.418	1.128

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	157	0	0	771	0	0
normalized size	1	1.	1.51	0.	0.	7.41	0.	0.
time (sec)	N/A	0.091	1.019	1.249	0.	2.332	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	102	172	821	1095	0	0
normalized size	1	1.	0.8	1.35	6.46	8.62	0.	0.
time (sec)	N/A	0.181	1.064	0.357	1.27	2.433	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	194	0	0	421	0	9827
normalized size	1	1.	1.1	0.	0.	2.39	0.	55.84
time (sec)	N/A	0.299	1.242	180.	0.	2.951	0.	2.494

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	136	0	0	535	0	4226
normalized size	1	1.	1.03	0.	0.	4.05	0.	32.02
time (sec)	N/A	0.226	0.808	180.	0.	2.594	0.	1.731

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	203	0	1346
normalized size	1	1.	0.89	0.	0.	2.54	0.	16.82
time (sec)	N/A	0.128	0.685	180.	0.	2.292	0.	1.452

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	237	106	0	219	0	494
normalized size	1	1.	4.23	1.89	0.	3.91	0.	8.82
time (sec)	N/A	0.093	7.472	1.171	0.	2.216	0.	1.289

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	22	70	154	54	20	43
normalized size	1	1.	0.65	2.06	4.53	1.59	0.59	1.26
time (sec)	N/A	0.022	0.221	0.663	1.06	2.023	3.096	1.148

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	26	49	20	54
normalized size	1	1.	1.	1.05	1.37	2.58	1.05	2.84
time (sec)	N/A	0.056	0.02	0.043	0.989	1.995	3.695	1.17

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	53	135	81	109	70
normalized size	1	1.	0.94	1.61	4.09	2.45	3.3	2.12
time (sec)	N/A	0.038	0.417	0.372	1.023	1.976	5.109	1.152

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	176	0	0	772	0	0
normalized size	1	1.	1.6	0.	0.	7.02	0.	0.
time (sec)	N/A	0.093	1.14	1.057	0.	2.32	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	130	141	514	1014	0	0
normalized size	1	1.	1.05	1.14	4.15	8.18	0.	0.
time (sec)	N/A	0.183	1.085	0.353	1.653	2.567	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	64	98	0	273	0	0
normalized size	1	1.	0.41	0.62	0.	1.74	0.	0.
time (sec)	N/A	0.445	0.234	0.642	0.	2.057	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	62	88	0	216	0	0
normalized size	1	1.	0.56	0.8	0.	1.96	0.	0.
time (sec)	N/A	0.276	0.181	0.466	0.	2.091	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	44	78	0	159	0	0
normalized size	1	1.	0.61	1.08	0.	2.21	0.	0.
time (sec)	N/A	0.199	0.181	0.445	0.	2.037	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	52	0	96	0	0
normalized size	1	1.	0.91	1.58	0.	2.91	0.	0.
time (sec)	N/A	0.065	0.085	0.357	0.	2.145	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	136	581	525	0	0
normalized size	1	1.	1.62	3.02	12.91	11.67	0.	0.
time (sec)	N/A	0.04	0.129	0.317	1.724	2.382	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	92	387	1416	921	0	0
normalized size	1	1.	1.1	4.61	16.86	10.96	0.	0.
time (sec)	N/A	0.139	0.245	0.424	2.107	2.401	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	105	649	1918	1092	0	0
normalized size	1	1.	0.81	5.03	14.87	8.47	0.	0.
time (sec)	N/A	0.214	0.256	0.451	2.265	2.483	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	116	921	3150	1260	0	0
normalized size	1	1.	0.66	5.23	17.9	7.16	0.	0.
time (sec)	N/A	0.288	0.306	0.413	3.386	2.509	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	85	105	0	348	0	0
normalized size	1	1.	0.41	0.5	0.	1.67	0.	0.
time (sec)	N/A	0.528	0.367	0.519	0.	2.067	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	73	95	0	290	0	0
normalized size	1	1.	0.49	0.64	0.	1.96	0.	0.
time (sec)	N/A	0.348	0.22	0.398	0.	2.057	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	59	85	0	220	0	0
normalized size	1	1.	0.54	0.77	0.	2.	0.	0.
time (sec)	N/A	0.268	0.238	0.34	0.	2.032	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	61	0	165	0	0
normalized size	1	1.	0.68	0.81	0.	2.2	0.	0.
time (sec)	N/A	0.11	0.17	0.429	0.	1.993	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	253	1778	770	0	0
normalized size	1	1.	1.08	3.16	22.22	9.62	0.	0.
time (sec)	N/A	0.06	0.158	0.514	2.112	2.42	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	93	518	1428	941	0	0
normalized size	1	1.	1.08	6.02	16.6	10.94	0.	0.
time (sec)	N/A	0.222	0.249	0.474	2.077	2.541	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	105	792	0	1126	0	0
normalized size	1	1.	0.79	5.95	0.	8.47	0.	0.
time (sec)	N/A	0.256	0.288	0.455	0.	2.376	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	117	1078	0	1310	0	0
normalized size	1	1.	0.64	5.92	0.	7.2	0.	0.
time (sec)	N/A	0.312	0.249	0.457	0.	2.566	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	112	980	0	994	0	0
normalized size	1	1.	0.64	5.6	0.	5.68	0.	0.
time (sec)	N/A	0.6	0.727	0.577	0.	2.434	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	89	673	0	749	0	0
normalized size	1	1.	0.69	5.22	0.	5.81	0.	0.
time (sec)	N/A	0.359	0.392	0.506	0.	2.38	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	67	478	0	626	0	0
normalized size	1	1.	0.76	5.43	0.	7.11	0.	0.
time (sec)	N/A	0.238	0.257	0.467	0.	2.407	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	64	236	0	386	0	0
normalized size	1	1.	1.16	4.29	0.	7.02	0.	0.
time (sec)	N/A	0.076	0.148	0.388	0.	2.271	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	94	301	0	807	0	0
normalized size	1	1.	0.94	3.01	0.	8.07	0.	0.
time (sec)	N/A	0.09	0.332	0.363	0.	2.14	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	166	1030	0	1274	0	0
normalized size	1	1.	1.2	7.46	0.	9.23	0.	0.
time (sec)	N/A	0.279	2.473	0.466	0.	2.292	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	186	1835	0	1503	0	0
normalized size	1	1.	1.02	10.08	0.	8.26	0.	0.
time (sec)	N/A	0.37	3.074	0.462	0.	2.327	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	100	1211	0	886	0	0
normalized size	1	1.	0.56	6.73	0.	4.92	0.	0.
time (sec)	N/A	0.513	1.361	0.454	0.	2.907	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	94	930	0	711	0	0
normalized size	1	1.	0.73	7.27	0.	5.55	0.	0.
time (sec)	N/A	0.306	0.625	0.453	0.	2.857	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	84	433	0	706	0	0
normalized size	1	1.	0.9	4.66	0.	7.59	0.	0.
time (sec)	N/A	0.236	0.61	0.423	0.	2.792	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	83	599	0	699	0	0
normalized size	1	1.	0.89	6.44	0.	7.52	0.	0.
time (sec)	N/A	0.121	0.651	0.406	0.	2.985	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	196	561	0	1148	0	0
normalized size	1	1.	1.42	4.07	0.	8.32	0.	0.
time (sec)	N/A	0.143	3.857	0.388	0.	2.973	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	342	1157	0	1372	0	0
normalized size	1	1.	1.92	6.5	0.	7.71	0.	0.
time (sec)	N/A	0.32	6.198	0.43	0.	2.912	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	356	1787	0	1596	0	0
normalized size	1	1.	1.52	7.64	0.	6.82	0.	0.
time (sec)	N/A	0.497	6.247	0.395	0.	2.914	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	119	0	97	0	0
normalized size	1	1.	1.	7.44	0.	6.06	0.	0.
time (sec)	N/A	0.087	0.033	0.09	0.	2.512	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	119	396	0	431	0	0
normalized size	1	1.	1.72	5.74	0.	6.25	0.	0.
time (sec)	N/A	0.364	5.903	0.141	0.	2.648	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	139	396	0	432	0	0
normalized size	1	1.	1.76	5.01	0.	5.47	0.	0.
time (sec)	N/A	0.567	4.807	0.116	0.	2.657	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	188	761	0	501	0	0
normalized size	1	1.	1.98	8.01	0.	5.27	0.	0.
time (sec)	N/A	0.576	15.322	0.187	0.	2.633	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	51	31	0	150	0	0
normalized size	1	1.	1.7	1.03	0.	5.	0.	0.
time (sec)	N/A	0.059	1.126	0.078	0.	3.092	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	938	137	32	292	129	192
normalized size	1	1.	36.08	5.27	1.23	11.23	4.96	7.38
time (sec)	N/A	0.044	6.559	0.2	0.964	2.621	65.903	1.295

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	31	118	32	217	100	0
normalized size	1	1.	1.19	4.54	1.23	8.35	3.85	0.
time (sec)	N/A	0.043	1.278	0.172	0.963	2.601	18.267	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	67	57	32	150	73	61
normalized size	1	1.	2.58	2.19	1.23	5.77	2.81	2.35
time (sec)	N/A	0.028	0.037	0.119	0.967	2.396	5.013	1.184

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	29	23	30	85	63	57
normalized size	1	1.	1.32	1.05	1.36	3.86	2.86	2.59
time (sec)	N/A	0.048	0.462	0.09	0.966	2.763	7.528	1.307

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	25	32	72	49	146
normalized size	1	1.	1.12	1.04	1.33	3.	2.04	6.08
time (sec)	N/A	0.044	0.31	0.139	0.985	2.355	22.534	1.239

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	25	32	149	80	0
normalized size	1	1.	1.12	0.96	1.23	5.73	3.08	0.
time (sec)	N/A	0.045	0.727	0.171	0.983	2.987	62.24	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.043	0.025	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.035	0.013	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.078	0.026	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.076	0.028	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	31	17	18
normalized size	1	1.	1.	1.08	1.33	2.58	1.42	1.5
time (sec)	N/A	0.022	0.018	0.007	0.969	2.068	0.386	1.078

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	0	59	63	27
normalized size	1	1.	0.95	1.05	0.	2.95	3.15	1.35
time (sec)	N/A	0.025	0.032	0.005	0.	2.08	2.513	1.104

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	112	0	19
normalized size	1	1.	1.	1.2	1.4	22.4	0.	3.8
time (sec)	N/A	0.023	0.021	0.013	1.427	2.182	0.	1.101

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	59	5	7
normalized size	1	1.	1.	1.2	1.4	11.8	1.	1.4
time (sec)	N/A	0.009	2.712	0.008	0.963	2.017	0.534	1.096

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	23	23	219	34	23
normalized size	1	1.	0.75	0.82	0.82	7.82	1.21	0.82
time (sec)	N/A	0.026	1.512	0.009	0.969	2.05	7.895	1.089

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	494	54	27
normalized size	1	1.	1.	0.81	1.04	19.	2.08	1.04
time (sec)	N/A	0.046	4.684	0.067	0.962	2.375	70.251	1.094

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	137	40	139	111	97	53
normalized size	1	1.	3.81	1.11	3.86	3.08	2.69	1.47
time (sec)	N/A	0.087	0.301	0.009	0.981	2.215	16.471	1.088

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	65	7	9
normalized size	1	1.	1.	0.89	1.	7.22	0.78	1.
time (sec)	N/A	0.011	2.754	0.006	0.96	1.995	0.533	1.109

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	26	23	57	26	23
normalized size	1	1.	0.77	0.84	0.74	1.84	0.84	0.74
time (sec)	N/A	0.023	0.121	0.005	0.976	1.903	0.82	1.11

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	162	0	24	82	0	9
normalized size	1	1.	18.	0.	2.67	9.11	0.	1.
time (sec)	N/A	0.072	2.252	0.518	1.444	2.855	0.	1.09

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	49	69	140	0	101
normalized size	1	1.	0.83	0.69	0.97	1.97	0.	1.42
time (sec)	N/A	0.066	0.158	1.081	0.985	2.186	0.	1.11

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	23	36	39	23
normalized size	1	1.	1.	1.	1.28	2.	2.17	1.28
time (sec)	N/A	0.014	0.051	0.005	0.984	2.005	0.512	1.096

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	31	45	54	0
normalized size	1	1.	1.	1.04	1.35	1.96	2.35	0.
time (sec)	N/A	0.015	0.237	0.014	0.983	1.889	9.436	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	24	31	45	51	31
normalized size	1	1.	0.96	1.	1.29	1.88	2.12	1.29
time (sec)	N/A	0.014	0.043	0.01	0.96	2.029	2.769	1.149

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	19	31	0	0
normalized size	1	1.	1.	1.14	1.36	2.21	0.	0.
time (sec)	N/A	0.022	0.035	0.01	1.034	2.089	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	27	42	0	0
normalized size	1	1.	1.	1.16	1.42	2.21	0.	0.
time (sec)	N/A	0.022	0.061	0.02	1.081	2.329	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	22	27	42	0	0
normalized size	1	1.	0.95	1.1	1.35	2.1	0.	0.
time (sec)	N/A	0.023	0.058	0.013	1.07	1.889	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	28	14	16
normalized size	1	1.	1.	1.09	1.36	2.55	1.27	1.45
time (sec)	N/A	0.022	0.006	0.014	0.945	2.072	0.36	1.094

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	0	58	56	26
normalized size	1	1.	0.95	1.05	0.	3.05	2.95	1.37
time (sec)	N/A	0.022	0.02	0.007	0.	2.067	2.57	1.085

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	119	0	22
normalized size	1	1.	1.	1.33	1.33	39.67	0.	7.33
time (sec)	N/A	0.023	0.008	0.016	1.437	2.179	0.	1.08

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	176	0	7
normalized size	1	1.	1.	0.86	1.	25.14	0.	1.
time (sec)	N/A	0.025	0.009	0.026	1.442	2.215	0.	1.107

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	200	0	12
normalized size	1	1.	1.	0.77	0.92	15.38	0.	0.92
time (sec)	N/A	0.026	0.03	0.031	1.437	2.085	0.	1.166

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	48	51	271	0	51
normalized size	1	1.	1.	2.29	2.43	12.9	0.	2.43
time (sec)	N/A	0.024	0.015	0.034	0.949	2.03	0.	1.143

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	30	208	0	30
normalized size	1	1.	1.	0.82	1.07	7.43	0.	1.07
time (sec)	N/A	0.026	0.018	0.023	1.498	2.224	0.	1.093

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	27	14
normalized size	1	1.	1.	0.79	1.	2.57	1.93	1.
time (sec)	N/A	0.034	0.007	0.006	0.964	2.145	0.878	1.116

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	40	17	27	115	0	27
normalized size	1	1.	2.11	0.89	1.42	6.05	0.	1.42
time (sec)	N/A	0.032	0.018	0.036	0.953	3.79	0.	1.097

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	51	3	4
normalized size	1	1.	1.	1.33	1.33	17.	1.	1.33
time (sec)	N/A	0.008	1.483	0.007	0.97	2.15	0.54	1.075

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	116	5	5
normalized size	1	1.	1.	1.25	1.25	29.	1.25	1.25
time (sec)	N/A	0.022	7.687	0.012	0.952	2.151	12.989	1.099

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	11	140	10	39
normalized size	1	1.	1.	2.25	2.75	35.	2.5	9.75
time (sec)	N/A	0.007	0.005	0.009	0.951	2.085	1.723	1.089

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	128	40	53	258	102	53
normalized size	1	1.	3.56	1.11	1.47	7.17	2.83	1.47
time (sec)	N/A	0.077	0.425	0.011	0.983	2.226	16.575	1.14

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	9	13	11	31	12	11
normalized size	1	1.	0.64	0.93	0.79	2.21	0.86	0.79
time (sec)	N/A	0.016	0.01	0.004	0.965	2.166	0.837	1.085

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	20	14	26	88	36	18
normalized size	1	1.	0.8	0.56	1.04	3.52	1.44	0.72
time (sec)	N/A	0.049	0.023	0.577	0.988	2.141	3.247	1.089

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	24	8	9
normalized size	1	1.	1.	0.8	0.9	2.4	0.8	0.9
time (sec)	N/A	0.027	0.013	0.007	0.965	2.034	0.596	1.104

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	22	7	7
normalized size	1	1.	1.	1.	1.17	3.67	1.17	1.17
time (sec)	N/A	0.01	0.011	0.006	0.958	1.954	0.725	1.118

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	9	26	0	16
normalized size	1	1.	0.7	0.7	0.9	2.6	0.	1.6
time (sec)	N/A	0.012	0.021	0.015	3.177	2.157	0.	1.099

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	9	39	0	24
normalized size	1	1.	1.	1.3	0.9	3.9	0.	2.4
time (sec)	N/A	0.011	0.009	0.017	0.996	2.057	0.	1.105

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	35	36	22
normalized size	1	1.	1.	1.	1.29	2.06	2.12	1.29
time (sec)	N/A	0.013	0.017	0.006	0.962	2.027	0.46	1.117

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	23	30	43	51	0
normalized size	1	1.	1.05	1.05	1.36	1.95	2.32	0.
time (sec)	N/A	0.013	0.146	0.015	0.956	2.107	8.674	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	30	43	48	30
normalized size	1	1.	1.	1.	1.3	1.87	2.09	1.3
time (sec)	N/A	0.013	0.046	0.008	1.099	2.058	2.574	1.15

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	18	30	0	18
normalized size	1	1.	1.	1.31	1.38	2.31	0.	1.38
time (sec)	N/A	0.02	0.042	0.011	1.07	2.168	0.	1.089

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	23	26	41	0	0
normalized size	1	1.	1.	1.28	1.44	2.28	0.	0.
time (sec)	N/A	0.021	0.067	0.025	1.062	2.057	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	23	26	41	0	0
normalized size	1	1.	0.95	1.21	1.37	2.16	0.	0.
time (sec)	N/A	0.02	0.06	0.014	1.064	2.095	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	15	107	0	16
normalized size	1	1.	1.82	1.09	1.36	9.73	0.	1.45
time (sec)	N/A	0.034	0.065	0.027	0.953	2.253	0.	1.083

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	20	84	0	23
normalized size	1	1.	2.09	0.36	1.82	7.64	0.	2.09
time (sec)	N/A	0.031	0.006	0.029	0.956	2.059	0.	1.116

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	9	70	0	9
normalized size	1	1.	0.33	0.3	0.33	2.59	0.	0.33
time (sec)	N/A	0.029	0.024	0.031	1.447	2.132	0.	1.087

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	0	101	0	26
normalized size	1	1.	0.95	1.05	0.	5.32	0.	1.37
time (sec)	N/A	0.035	0.178	0.021	0.	2.104	0.	1.141

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	38	27	5
normalized size	1	1.	1.	1.25	1.25	9.5	6.75	1.25
time (sec)	N/A	0.043	0.006	0.046	1.459	2.029	0.93	1.111

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	38	27	5
normalized size	1	1.	1.	1.25	1.25	9.5	6.75	1.25
time (sec)	N/A	0.063	0.005	0.053	1.491	2.12	0.924	1.087

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	6	7	117	0	7
normalized size	1	1.	0.94	0.18	0.21	3.55	0.	0.21
time (sec)	N/A	0.042	0.044	0.06	1.446	2.049	0.	1.093

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	15	16	18	23	124	0	26
normalized size	1	1.5	1.6	1.8	2.3	12.4	0.	2.6
time (sec)	N/A	0.048	0.037	0.055	0.96	2.179	0.	1.134

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	15	16	16	20	126	0	23
normalized size	1	1.5	1.6	1.6	2.	12.6	0.	2.3
time (sec)	N/A	0.053	0.037	0.058	0.963	2.171	0.	1.111

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	74	80	120	1635	0	0
normalized size	1	1.	0.42	0.45	0.68	9.29	0.	0.
time (sec)	N/A	0.14	0.121	0.054	1.455	3.069	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	22	18	23	188	0	23
normalized size	1	1.	0.42	0.34	0.43	3.55	0.	0.43
time (sec)	N/A	0.066	0.055	0.064	1.468	2.103	0.	1.122

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	54	35	38	194	29	39
normalized size	1	1.	1.93	1.25	1.36	6.93	1.04	1.39
time (sec)	N/A	0.087	0.354	0.042	0.959	2.337	3.573	1.111

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	80	85	302	56	86
normalized size	1	1.	1.17	1.51	1.6	5.7	1.06	1.62
time (sec)	N/A	0.138	0.556	0.059	0.991	2.406	4.703	1.097

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	133	143	159	459	95	166
normalized size	1	1.	1.71	1.83	2.04	5.88	1.22	2.13
time (sec)	N/A	0.15	0.897	0.075	0.977	2.989	7.214	1.088

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	109	0	14
normalized size	1	1.	1.	0.92	1.17	9.08	0.	1.17
time (sec)	N/A	0.076	0.036	0.072	0.973	2.022	0.	1.107

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	67	42	34	158	27	34
normalized size	1	1.	2.03	1.27	1.03	4.79	0.82	1.03
time (sec)	N/A	0.092	0.026	0.022	0.96	2.228	92.638	1.102

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	32	24	31	174	41	32
normalized size	1	1.	0.7	0.52	0.67	3.78	0.89	0.7
time (sec)	N/A	0.089	0.227	0.108	1.461	2.403	8.212	1.103

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	38	3	20
normalized size	1	1.	1.	1.25	1.25	9.5	0.75	5.
time (sec)	N/A	0.018	0.002	0.032	1.465	1.954	23.674	1.12

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	18	104	0	20
normalized size	1	1.	1.38	0.67	0.86	4.95	0.	0.95
time (sec)	N/A	0.115	0.031	0.057	0.962	2.08	0.	1.148

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	43	103	11	76	0	0
normalized size	1	1.	4.78	11.44	1.22	8.44	0.	0.
time (sec)	N/A	0.047	0.044	0.164	1.419	2.116	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	47	172	9	135	0	9
normalized size	1	1.	5.22	19.11	1.	15.	0.	1.
time (sec)	N/A	0.047	0.062	0.361	1.48	2.249	0.	1.159

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	46	173	22	224	0	23
normalized size	1	1.	3.29	12.36	1.57	16.	0.	1.64
time (sec)	N/A	0.044	0.052	0.394	0.967	2.155	0.	1.175

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	52	223	41	225	0	192
normalized size	1	1.	2.74	11.74	2.16	11.84	0.	10.11
time (sec)	N/A	0.049	0.529	0.318	1.451	2.263	0.	1.207

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	63	492	27	215	0	27
normalized size	1	1.	2.42	18.92	1.04	8.27	0.	1.04
time (sec)	N/A	0.046	0.113	0.21	1.462	2.349	0.	1.107

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	26	3	4
normalized size	1	1.	1.	1.	1.	6.5	0.75	1.
time (sec)	N/A	0.012	0.06	0.012	0.961	1.984	1.72	1.109

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	25	20	57	61	19	27
normalized size	1	1.	1.47	1.18	3.35	3.59	1.12	1.59
time (sec)	N/A	0.067	0.018	0.013	0.955	2.092	9.181	1.103

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	16	123	0	30
normalized size	1	1.	1.67	1.08	1.33	10.25	0.	2.5
time (sec)	N/A	0.041	0.059	0.025	0.961	2.267	0.	1.129

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	0	103	0	0
normalized size	1	1.	0.95	1.05	0.	5.15	0.	0.
time (sec)	N/A	0.041	0.184	0.023	0.	2.415	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	11	38	3	22
normalized size	1	1.	1.	1.17	1.83	6.33	0.5	3.67
time (sec)	N/A	0.016	0.003	0.017	1.473	2.411	13.162	1.104

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	11	38	27	22
normalized size	1	1.	1.	1.17	1.83	6.33	4.5	3.67
time (sec)	N/A	0.047	0.005	0.025	1.442	2.313	0.939	1.119

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	56	56	62	209	31	92
normalized size	1	1.	2.	2.	2.21	7.46	1.11	3.29
time (sec)	N/A	0.082	0.33	0.046	0.98	2.953	11.597	1.154

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	119	124	417	58	188
normalized size	1	1.	1.17	2.25	2.34	7.87	1.09	3.55
time (sec)	N/A	0.136	0.505	0.065	0.974	3.292	28.694	1.2

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	135	202	217	705	97	313
normalized size	1	1.	1.73	2.59	2.78	9.04	1.24	4.01
time (sec)	N/A	0.139	1.264	0.078	0.991	3.424	75.33	1.173

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	27	5	0
normalized size	1	1.	1.	1.	1.17	4.5	0.83	0.
time (sec)	N/A	0.015	0.076	0.011	0.978	2.239	115.646	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	20	20	12	15	51	14	30
normalized size	1	1.82	1.82	1.09	1.36	4.64	1.27	2.73
time (sec)	N/A	0.046	0.018	0.01	0.96	2.482	0.573	1.109

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	7	23	3	7
normalized size	1	1.	1.	0.8	1.4	4.6	0.6	1.4
time (sec)	N/A	0.033	0.022	0.025	1.455	2.491	0.27	1.089

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	34	8	9
normalized size	1	1.	1.	0.73	0.82	3.09	0.73	0.82
time (sec)	N/A	0.035	0.027	0.019	1.454	2.419	0.308	1.107

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	9	12	9	32	15	9
normalized size	1	1.	1.29	1.71	1.29	4.57	2.14	1.29
time (sec)	N/A	0.032	0.006	0.028	0.955	2.414	0.257	1.09

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	38	6	45	80	5	55
normalized size	1	1.	7.6	1.2	9.	16.	1.	11.
time (sec)	N/A	0.045	0.03	0.032	0.954	2.713	0.95	1.129

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	25	18	51	0	28
normalized size	1	1.	1.	1.92	1.38	3.92	0.	2.15
time (sec)	N/A	0.079	0.017	0.033	1.455	2.411	0.	1.116

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	19	3	7
normalized size	1	1.	1.	1.	1.	4.75	0.75	1.75
time (sec)	N/A	0.022	0.007	0.006	0.956	2.294	0.829	1.12

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	28	7	15
normalized size	1	1.	1.	1.11	1.33	3.11	0.78	1.67
time (sec)	N/A	0.022	0.008	0.006	0.957	2.311	0.927	1.082

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	11	14	76	12	32
normalized size	1	1.	1.67	0.92	1.17	6.33	1.	2.67
time (sec)	N/A	0.055	0.077	0.023	0.975	2.395	1.283	1.467

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	66	47	354	0	0
normalized size	1	1.	1.	1.53	1.09	8.23	0.	0.
time (sec)	N/A	0.095	0.057	0.043	1.464	2.769	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	27	50	0	159
normalized size	1	1.	1.	1.55	1.23	2.27	0.	7.23
time (sec)	N/A	0.091	0.033	0.032	1.433	2.444	0.	1.247

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	20	20	13	16	58	17	30
normalized size	1	1.67	1.67	1.08	1.33	4.83	1.42	2.5
time (sec)	N/A	0.042	0.019	0.017	0.967	2.497	0.482	1.091

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	38	12	19
normalized size	1	1.	1.	0.93	1.14	2.71	0.86	1.36
time (sec)	N/A	0.025	0.031	0.01	0.964	2.347	0.822	1.114

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	4	22	5	4
normalized size	1	1.	1.	2.	1.33	7.33	1.67	1.33
time (sec)	N/A	0.032	0.015	0.015	1.456	2.371	0.232	1.106

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	35	66	200	151	65
normalized size	1	1.	0.95	0.81	1.53	4.65	3.51	1.51
time (sec)	N/A	0.057	0.68	0.033	1.477	2.163	2.602	1.162

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	54	0	28
normalized size	1	1.	1.	1.07	1.36	3.86	0.	2.
time (sec)	N/A	0.077	0.018	0.463	1.445	2.121	0.	1.099

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	38	50	146	0	65
normalized size	1	1.	0.65	0.88	1.16	3.4	0.	1.51
time (sec)	N/A	0.106	0.057	0.973	1.483	2.189	0.	1.182

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	50	69	0	0
normalized size	1	1.	0.65	2.42	1.16	1.6	0.	0.
time (sec)	N/A	0.037	0.062	0.079	1.078	2.143	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	50	69	0	0
normalized size	1	1.	0.65	2.42	1.16	1.6	0.	0.
time (sec)	N/A	0.035	0.03	0.	1.069	2.152	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	90	0	186
normalized size	1	1.	0.56	1.91	0.	1.41	0.	2.91
time (sec)	N/A	0.036	0.068	0.081	0.	2.13	0.	1.244

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	90	0	186
normalized size	1	1.	0.56	1.91	0.	1.41	0.	2.91
time (sec)	N/A	0.038	0.03	0.	0.	2.166	0.	1.265

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	106	50	70	0	0
normalized size	1	1.	0.65	2.47	1.16	1.63	0.	0.
time (sec)	N/A	0.041	0.149	0.07	1.048	2.089	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	106	50	70	0	0
normalized size	1	1.	0.65	2.47	1.16	1.63	0.	0.
time (sec)	N/A	0.035	0.035	0.	1.056	2.039	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	124	0	92	0	263
normalized size	1	1.	0.56	1.94	0.	1.44	0.	4.11
time (sec)	N/A	0.037	0.177	0.072	0.	2.095	0.	1.25

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	124	0	92	0	263
normalized size	1	1.	0.56	1.94	0.	1.44	0.	4.11
time (sec)	N/A	0.037	0.034	0.	0.	2.177	0.	1.253

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	35	0	9
normalized size	1	1.	1.	0.89	1.	3.89	0.	1.
time (sec)	N/A	0.022	0.006	0.017	0.968	1.923	0.	1.08

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	358	26	0	0
normalized size	1	1.	1.	0.89	39.78	2.89	0.	0.
time (sec)	N/A	0.02	0.011	0.016	1.548	2.014	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	5	5	7	14	5
normalized size	1	1.	1.	1.67	1.67	2.33	4.67	1.67
time (sec)	N/A	0.009	0.	0.012	1.457	1.937	0.142	1.091

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	100	55	0	139
normalized size	1	1.	1.	1.12	12.5	6.88	0.	17.38
time (sec)	N/A	0.018	0.005	0.005	1.46	2.144	0.	1.133

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	26	30	73	76	30
normalized size	1	1.	0.82	0.76	0.88	2.15	2.24	0.88
time (sec)	N/A	0.022	0.017	0.01	0.957	2.027	1.208	1.086

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	26	10	8
normalized size	1	1.	1.	0.7	0.8	2.6	1.	0.8
time (sec)	N/A	0.012	0.002	0.003	0.962	2.072	0.284	1.103

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	46	15	12
normalized size	1	1.	1.	0.82	1.	4.18	1.36	1.09
time (sec)	N/A	0.012	0.008	0.012	0.959	2.065	0.372	1.079

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	9	35	8	23
normalized size	1	1.	1.29	1.14	1.29	5.	1.14	3.29
time (sec)	N/A	0.044	0.006	0.033	0.979	2.082	7.664	1.105

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	100	74	0	139
normalized size	1	1.	1.	0.84	5.26	3.89	0.	7.32
time (sec)	N/A	0.019	0.008	0.007	1.466	2.003	0.	1.14

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	26	31	35	82	32	36
normalized size	1	1.	0.7	0.84	0.95	2.22	0.86	0.97
time (sec)	N/A	0.014	0.028	0.01	0.968	1.828	0.486	1.128

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	129	49	176	11	130	236	50
normalized size	1	10.75	4.08	14.67	0.92	10.83	19.67	4.17
time (sec)	N/A	0.324	0.026	0.081	0.983	2.452	0.09	1.117

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	19	8	9	43	19	16
normalized size	1	1.	2.11	0.89	1.	4.78	2.11	1.78
time (sec)	N/A	0.007	0.009	0.002	0.97	2.081	0.152	1.112

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	47	31	0	8
normalized size	1	1.	1.	0.88	5.88	3.88	0.	1.
time (sec)	N/A	0.013	0.016	0.005	0.965	1.975	0.	1.077

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	49	0	20
normalized size	1	1.	1.	0.8	0.	3.27	0.	1.33
time (sec)	N/A	0.029	0.016	0.05	0.	2.145	0.	1.118

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	16	20	57	19	20
normalized size	1	1.	0.87	0.7	0.87	2.48	0.83	0.87
time (sec)	N/A	0.022	0.013	0.013	1.458	2.001	0.319	1.088

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	39	86	42	39
normalized size	1	1.	1.	0.81	1.05	2.32	1.14	1.05
time (sec)	N/A	0.034	0.023	0.024	0.969	2.088	0.609	1.075

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	11	45	8	14
normalized size	1	1.	1.2	0.9	1.1	4.5	0.8	1.4
time (sec)	N/A	0.033	0.017	0.012	0.954	1.928	0.191	1.109

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	5	8
normalized size	1	1.	1.	0.88	1.	2.38	0.62	1.
time (sec)	N/A	0.007	0.001	0.005	0.962	1.894	0.164	1.111

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	23	7	11
normalized size	1	1.	1.	0.9	1.1	2.3	0.7	1.1
time (sec)	N/A	0.012	0.003	0.005	0.967	1.99	0.293	1.105

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	5	8
normalized size	1	1.	1.	0.88	1.	2.38	0.62	1.
time (sec)	N/A	0.009	0.002	0.005	0.982	1.948	0.538	1.084

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	20	7	11
normalized size	1	1.	1.	0.9	1.1	2.	0.7	1.1
time (sec)	N/A	0.008	0.01	0.003	0.957	1.855	0.163	1.087

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	76	19	5	11
normalized size	1	1.	1.	0.88	9.5	2.38	0.62	1.38
time (sec)	N/A	0.063	0.006	0.006	0.973	1.952	0.332	1.082

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	11	90	28	7	14
normalized size	1	1.	1.2	1.1	9.	2.8	0.7	1.4
time (sec)	N/A	0.018	0.023	0.003	0.961	2.039	0.383	1.095

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	46	12	14
normalized size	1	1.	1.	0.91	1.09	4.18	1.09	1.27
time (sec)	N/A	0.01	0.018	0.003	0.964	1.89	0.167	1.113

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	35	12	14
normalized size	1	1.	1.	0.92	1.17	2.92	1.	1.17
time (sec)	N/A	0.005	0.005	0.005	0.951	1.923	1.045	1.084

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	20	24	88	0	24
normalized size	1	1.	1.19	0.95	1.14	4.19	0.	1.14
time (sec)	N/A	0.06	0.018	0.053	0.956	2.016	0.	1.107

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	116	32	53
normalized size	1	1.	1.	0.84	1.05	6.11	1.68	2.79
time (sec)	N/A	0.037	0.009	0.045	0.965	2.142	7.696	1.084

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	35	64	43	88	48	43
normalized size	1	1.	0.74	1.36	0.91	1.87	1.02	0.91
time (sec)	N/A	0.064	0.032	0.023	0.964	2.065	123.675	1.082

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	34	78	32	34
normalized size	1	1.	0.8	1.03	0.97	2.23	0.91	0.97
time (sec)	N/A	0.158	0.047	0.018	0.962	2.031	2.312	1.093

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	14	3	5
normalized size	1	1.	1.	1.25	1.25	3.5	0.75	1.25
time (sec)	N/A	0.007	0.001	0.001	0.961	2.017	0.164	1.099

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	19	5	8
normalized size	1	1.	1.	1.17	1.33	3.17	0.83	1.33
time (sec)	N/A	0.013	0.004	0.005	0.963	2.023	0.302	1.091

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	27	5	9
normalized size	1	1.	1.	1.14	1.29	3.86	0.71	1.29
time (sec)	N/A	0.004	0.008	0.002	1.462	2.168	0.086	1.111

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	11	26	8	11
normalized size	1	1.	2.1	0.9	1.1	2.6	0.8	1.1
time (sec)	N/A	0.009	0.013	0.003	0.961	2.012	0.169	1.072

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	11
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.1
time (sec)	N/A	0.009	0.002	0.005	0.952	2.042	0.17	1.069

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	11
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.1
time (sec)	N/A	0.01	0.003	0.003	0.949	2.059	0.298	1.081

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	11	26	8	11
normalized size	1	1.	2.1	0.9	1.1	2.6	0.8	1.1
time (sec)	N/A	0.012	0.014	0.003	0.968	2.132	0.299	1.078

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	16	5	8
normalized size	1	1.	1.	1.17	1.33	2.67	0.83	1.33
time (sec)	N/A	0.009	0.002	0.001	0.962	2.036	0.296	1.065

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	46	15	19
normalized size	1	1.	1.	1.07	1.36	3.29	1.07	1.36
time (sec)	N/A	0.011	0.03	0.006	0.966	1.955	0.17	1.084

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	45	15	22
normalized size	1	1.	1.	0.85	1.1	2.25	0.75	1.1
time (sec)	N/A	0.017	0.01	0.007	0.961	2.028	1.994	1.093

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	20	62	20	20
normalized size	1	1.	0.7	0.78	0.87	2.7	0.87	0.87
time (sec)	N/A	0.009	0.013	0.004	0.96	2.172	0.471	1.082

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	46	15	22
normalized size	1	1.	1.	0.85	1.1	2.3	0.75	1.1
time (sec)	N/A	0.017	0.002	0.003	0.961	2.008	0.546	1.09

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	45	15	22
normalized size	1	1.	1.	0.85	1.1	2.25	0.75	1.1
time (sec)	N/A	0.017	0.008	0.005	0.975	1.961	0.55	1.095

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	57	7	9
normalized size	1	1.	1.	0.89	1.	6.33	0.78	1.
time (sec)	N/A	0.01	1.371	0.013	0.964	2.031	0.516	1.1

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	41	10	12
normalized size	1	1.	1.	0.91	1.09	3.73	0.91	1.09
time (sec)	N/A	0.032	0.028	0.01	0.948	2.054	0.194	1.074

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	65	11	126	36	82
normalized size	1	1.	1.	6.5	1.1	12.6	3.6	8.2
time (sec)	N/A	0.038	0.018	0.036	0.954	2.036	0.613	1.081

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	12	18	30	8	11
normalized size	1	1.	1.	1.33	2.	3.33	0.89	1.22
time (sec)	N/A	0.029	0.004	0.017	0.968	1.905	3.764	1.092

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	20	9	31	7	9
normalized size	1	1.	1.	4.	1.8	6.2	1.4	1.8
time (sec)	N/A	0.016	0.011	0.008	0.957	2.181	0.077	1.093

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	A	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	13	0	13	30	119	43	0	1072
normalized size	1	0.	1.	2.31	9.15	3.31	0.	82.46
time (sec)	N/A	0.64	0.276	0.13	2.641	2.015	0.	1.162

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	140	61	0	70
normalized size	1	1.	1.	1.11	15.56	6.78	0.	7.78
time (sec)	N/A	0.016	0.017	0.006	0.97	2.052	0.	1.108

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	105	37	19
normalized size	1	1.	1.	0.75	0.95	5.25	1.85	0.95
time (sec)	N/A	0.017	0.012	0.031	0.956	2.086	0.534	1.096

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	20	42	22	20
normalized size	1	1.	1.	0.79	1.05	2.21	1.16	1.05
time (sec)	N/A	0.015	0.012	0.005	0.96	2.052	0.556	1.079

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	39	10	24
normalized size	1	1.	1.	0.93	1.57	2.79	0.71	1.71
time (sec)	N/A	0.015	0.006	0.011	0.953	2.071	0.081	1.097

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	27	116	20	49
normalized size	1	1.	0.91	1.32	1.23	5.27	0.91	2.23
time (sec)	N/A	0.033	0.025	0.015	0.961	2.171	0.096	1.079

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	36	6	7	23	12	7
normalized size	1	1.	7.2	1.2	1.4	4.6	2.4	1.4
time (sec)	N/A	0.015	0.007	0.019	0.955	1.997	2.393	1.093

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	20	6	7	32	12	9
normalized size	1	1.	2.86	0.86	1.	4.57	1.71	1.29
time (sec)	N/A	0.016	0.006	0.013	0.95	1.994	1.985	1.072

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	15	20	14	12
normalized size	1	1.	1.6	1.2	3.	4.	2.8	2.4
time (sec)	N/A	0.022	0.002	0.016	0.975	1.961	5.975	1.086

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	61	20	93	149	32	39
normalized size	1	1.	4.07	1.33	6.2	9.93	2.13	2.6
time (sec)	N/A	0.045	0.009	0.077	1.477	2.178	1.867	1.137

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	26	12	15	55	12	20
normalized size	1	1.	2.36	1.09	1.36	5.	1.09	1.82
time (sec)	N/A	0.046	0.094	0.027	0.951	2.221	0.216	1.089

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	23	61	19	23
normalized size	1	1.	4.1	0.9	1.15	3.05	0.95	1.15
time (sec)	N/A	0.053	0.173	0.015	1.462	2.289	0.576	1.078

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	47	10	16
normalized size	1	1.	1.	1.09	1.36	4.27	0.91	1.45
time (sec)	N/A	0.021	0.008	0.029	0.998	2.212	0.194	1.078

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	1856	46	99	82	0
normalized size	1	1.	1.	71.38	1.77	3.81	3.15	0.
time (sec)	N/A	0.049	0.042	0.566	1.441	2.423	1.261	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	297	132	0	38
normalized size	1	1.	1.62	1.	12.38	5.5	0.	1.58
time (sec)	N/A	0.028	0.032	0.054	0.972	2.317	0.	1.074

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	35	51	76	39	35
normalized size	1	1.	0.72	0.88	1.27	1.9	0.98	0.88
time (sec)	N/A	0.065	0.046	0.007	0.99	2.284	0.323	1.087

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	68	26	26
normalized size	1	1.	0.81	0.81	0.96	2.52	0.96	0.96
time (sec)	N/A	0.01	0.03	0.007	0.947	2.228	0.466	1.072

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	23	47	26	23
normalized size	1	1.	1.35	0.78	1.	2.04	1.13	1.
time (sec)	N/A	0.04	0.024	0.009	0.963	2.234	0.315	1.086

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	41	32	109	27	32
normalized size	1	1.	1.53	1.37	1.07	3.63	0.9	1.07
time (sec)	N/A	0.03	0.018	0.007	0.963	2.339	0.068	1.101

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	16	3	19
normalized size	1	1.	1.	1.17	1.33	2.67	0.5	3.17
time (sec)	N/A	0.011	0.002	0.031	0.953	2.312	1.801	1.112

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	39	8	41
normalized size	1	1.	1.	1.14	1.29	5.57	1.14	5.86
time (sec)	N/A	0.038	0.004	0.031	0.96	2.367	23.318	1.097

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	10	10	18	34	14	18
normalized size	1	1.	0.77	0.77	1.38	2.62	1.08	1.38
time (sec)	N/A	0.014	0.004	0.007	0.958	2.204	0.07	1.085

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	15	68	12	15
normalized size	1	1.	1.	0.91	1.36	6.18	1.09	1.36
time (sec)	N/A	0.008	0.006	0.005	0.967	2.286	0.066	1.091

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	14	66	10	15
normalized size	1	1.	1.	1.7	1.4	6.6	1.	1.5
time (sec)	N/A	0.008	0.01	0.005	0.978	2.466	0.066	1.094

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	23	8	11
normalized size	1	1.	1.	0.9	1.1	2.3	0.8	1.1
time (sec)	N/A	0.008	0.009	0.003	0.953	2.148	0.166	1.073

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	42	32	16
normalized size	1	1.	1.	0.72	0.89	2.33	1.78	0.89
time (sec)	N/A	0.041	0.034	0.015	0.955	2.436	0.857	1.077

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	22	31	11
normalized size	1	1.	1.	0.9	1.1	2.2	3.1	1.1
time (sec)	N/A	0.012	0.005	0.003	0.95	2.191	1.743	1.067

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	25	11	14	66	39	14
normalized size	1	1.	1.56	0.69	0.88	4.12	2.44	0.88
time (sec)	N/A	0.016	0.021	0.006	0.948	2.326	0.573	1.105

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	35	8	12
normalized size	1	1.	1.	1.14	1.29	5.	1.14	1.71
time (sec)	N/A	0.007	0.006	0.003	0.962	2.279	0.127	1.067

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	22	76	15	55
normalized size	1	1.	1.	1.31	1.69	5.85	1.15	4.23
time (sec)	N/A	0.012	0.018	0.002	0.962	2.329	0.979	1.104

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	21	11	14	81	10	14
normalized size	1	1.	4.2	2.2	2.8	16.2	2.	2.8
time (sec)	N/A	0.009	0.015	0.003	0.962	2.296	1.351	1.144

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	9	7	8	20	34	8	26
normalized size	1	1.29	1.	1.14	2.86	4.86	1.14	3.71
time (sec)	N/A	0.045	0.007	0.04	0.966	2.279	6.479	1.082

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	12	20	5
normalized size	1	1.	1.	1.25	1.25	3.	5.	1.25
time (sec)	N/A	0.017	0.001	0.05	0.953	2.223	0.83	1.057

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	21	14	100	69	0	109
normalized size	1	1.	1.62	1.08	7.69	5.31	0.	8.38
time (sec)	N/A	0.02	0.007	0.006	1.47	2.505	0.	1.104

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	28	27	65	20	27
normalized size	1	1.	1.12	1.75	1.69	4.06	1.25	1.69
time (sec)	N/A	0.029	0.025	0.01	1.452	2.393	0.062	1.078

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	34	47	86	36	42
normalized size	1	1.	0.81	1.06	1.47	2.69	1.12	1.31
time (sec)	N/A	0.034	0.022	0.01	1.444	2.47	0.066	1.11

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	19	14	53	12	14
normalized size	1	1.	0.89	1.06	0.78	2.94	0.67	0.78
time (sec)	N/A	0.028	0.008	0.005	0.959	2.235	0.067	1.075

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	29	24	78	32	30
normalized size	1	1.	0.94	0.85	0.71	2.29	0.94	0.88
time (sec)	N/A	0.05	0.01	0.005	0.96	2.398	0.061	1.082

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	33	29	18	66	12	18
normalized size	1	1.	2.54	2.23	1.38	5.08	0.92	1.38
time (sec)	N/A	0.024	0.009	0.005	0.98	2.291	0.062	1.079

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	24	36	22	100	31	22
normalized size	1	1.	0.52	0.78	0.48	2.17	0.67	0.48
time (sec)	N/A	0.056	0.007	0.007	0.96	2.296	0.069	1.078

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	21	27	68	20	26
normalized size	1	1.	1.	2.33	3.	7.56	2.22	2.89
time (sec)	N/A	0.028	0.005	0.02	0.969	2.128	0.36	1.093

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	17	13	18	42	12	18
normalized size	1	1.	1.21	0.93	1.29	3.	0.86	1.29
time (sec)	N/A	0.017	0.005	0.005	0.95	1.992	0.061	1.08

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	0	1
normalized size	1	1.	1.	2.	1.	4.	0.	1.
time (sec)	N/A	0.01	0.	0.003	0.963	1.826	0.06	1.068

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	8	22	7	8
normalized size	1	1.	1.33	1.17	1.33	3.67	1.17	1.33
time (sec)	N/A	0.011	0.002	0.001	0.952	2.091	0.059	1.054

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	23	7	12
normalized size	1	1.	1.	1.11	1.33	2.56	0.78	1.33
time (sec)	N/A	0.009	0.006	0.006	0.96	1.945	0.29	1.069

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	47	19	22	8
normalized size	1	1.	1.	0.88	5.88	2.38	2.75	1.
time (sec)	N/A	0.017	0.005	0.004	0.949	1.925	0.125	1.071

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	20	69	16	5	35
normalized size	1	1.	1.	3.33	11.5	2.67	0.83	5.83
time (sec)	N/A	0.178	0.024	0.035	1.664	1.962	0.553	1.098

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	11	23	8	11
normalized size	1	1.	1.	0.88	1.38	2.88	1.	1.38
time (sec)	N/A	0.197	0.018	0.016	0.953	2.092	0.328	1.062

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	11	23	8	11
normalized size	1	1.	1.5	1.12	1.38	2.88	1.	1.38
time (sec)	N/A	0.011	0.012	0.007	0.958	1.898	0.305	1.067

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	11	22	7	11
normalized size	1	1.	1.	0.88	1.38	2.75	0.88	1.38
time (sec)	N/A	0.184	0.018	0.01	0.963	2.002	0.347	1.068

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	70	55	69	0	767	136	104
normalized size	1	1.27	1.	1.25	0.	13.95	2.47	1.89
time (sec)	N/A	0.17	0.081	0.072	0.	2.476	9.908	1.093

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	70	54	69	0	767	136	104
normalized size	1	1.27	0.98	1.25	0.	13.95	2.47	1.89
time (sec)	N/A	0.133	0.062	0.061	0.	2.41	10.025	1.086

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	80	0	535	432	93
normalized size	1	1.	0.92	1.54	0.	10.29	8.31	1.79
time (sec)	N/A	0.125	0.09	0.045	0.	2.308	63.917	1.089

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	80	0	525	432	96
normalized size	1	1.	0.96	1.54	0.	10.1	8.31	1.85
time (sec)	N/A	0.094	0.056	0.03	0.	2.226	63.931	1.087

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	31	30	0	221	0	82
normalized size	1	1.	1.03	1.	0.	7.37	0.	2.73
time (sec)	N/A	0.035	0.039	0.043	0.	2.323	0.	1.233

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	49	31	0	208	0	42
normalized size	1	1.	1.58	1.	0.	6.71	0.	1.35
time (sec)	N/A	0.032	0.062	0.043	0.	2.051	0.	1.134

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	22	7	8
normalized size	1	1.	1.	0.88	1.	2.75	0.88	1.
time (sec)	N/A	0.013	0.003	0.007	0.956	1.985	0.309	1.074

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	25	55	68	0	19
normalized size	1	1.	0.95	1.32	2.89	3.58	0.	1.
time (sec)	N/A	0.03	0.014	1.99	1.489	1.997	0.	1.092

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	16	24	16	85	476	16
normalized size	1	1.	0.55	0.83	0.55	2.93	16.41	0.55
time (sec)	N/A	0.056	0.016	0.009	0.962	2.134	66.341	1.073

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	37	29	34	139	93	32	46
normalized size	1	1.28	1.	1.17	4.79	3.21	1.1	1.59
time (sec)	N/A	0.133	0.103	0.058	1.477	2.171	0.405	1.096

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	37	29	34	139	93	32	46
normalized size	1	1.28	1.	1.17	4.79	3.21	1.1	1.59
time (sec)	N/A	0.091	0.08	0.061	1.466	2.169	0.424	1.122

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	31	41	171	0	46
normalized size	1	1.	0.86	0.7	0.93	3.89	0.	1.05
time (sec)	N/A	0.056	0.077	0.071	0.962	1.96	0.	1.066

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	88	19	16	196	54	0	0
normalized size	1	4.63	1.	0.84	10.32	2.84	0.	0.
time (sec)	N/A	2.243	0.067	0.26	1.632	2.193	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	B	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	44	0	68	917	699	363	0	0
normalized size	1	0.	1.55	20.84	15.89	8.25	0.	0.
time (sec)	N/A	2.569	0.392	0.435	2.445	2.158	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	72	17	12372	444	5	0	57
normalized size	1	3.79	0.89	651.16	23.37	0.26	0.	3.
time (sec)	N/A	1.707	0.013	0.287	1.662	1.964	0.	1.116

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	37	10	0	72	0	74
normalized size	1	1.	2.85	0.77	0.	5.54	0.	5.69
time (sec)	N/A	0.146	0.045	0.058	0.	2.035	0.	1.292

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	74	29	92
normalized size	1	1.	1.	0.79	1.	5.29	2.07	6.57
time (sec)	N/A	0.044	0.054	0.018	0.955	2.042	0.911	1.097

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	30	34	28	111	0	173
normalized size	1	1.	1.2	1.36	1.12	4.44	0.	6.92
time (sec)	N/A	0.085	0.19	0.067	0.957	2.039	0.	1.092

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	18	38	28	135	0	173
normalized size	1	1.	0.72	1.52	1.12	5.4	0.	6.92
time (sec)	N/A	0.081	0.039	0.084	0.966	2.075	0.	1.096

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.447	0.188	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	84	40	54	184	112	0	131
normalized size	1	1.11	0.53	0.71	2.42	1.47	0.	1.72
time (sec)	N/A	0.162	0.089	0.194	1.479	2.11	0.	1.098

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	55	65	116	150	0	138
normalized size	1	1.	0.68	0.8	1.43	1.85	0.	1.7
time (sec)	N/A	0.159	0.064	0.157	0.986	2.189	0.	1.09

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	98	107	439	0	0
normalized size	1	1.	0.91	1.29	1.41	5.78	0.	0.
time (sec)	N/A	0.535	0.061	0.078	1.528	2.454	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	99	132	144	788	0	0
normalized size	1	1.	0.77	1.03	1.12	6.16	0.	0.
time (sec)	N/A	0.592	0.069	0.066	1.521	2.414	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	147	172	177	1137	0	0
normalized size	1	1.	0.79	0.92	0.95	6.11	0.	0.
time (sec)	N/A	0.57	0.099	0.068	1.561	2.703	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	50	147	112	459	0	0
normalized size	1	1.	0.62	1.81	1.38	5.67	0.	0.
time (sec)	N/A	0.487	0.038	0.077	1.514	3.075	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	75	183	153	821	0	0
normalized size	1	1.	0.69	1.68	1.4	7.53	0.	0.
time (sec)	N/A	0.574	0.06	0.067	1.537	2.688	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	87	221	185	1180	0	0
normalized size	1	1.	0.61	1.55	1.29	8.25	0.	0.
time (sec)	N/A	0.61	0.069	0.069	1.553	2.696	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	108	86	405	500	0	0
normalized size	1	1.	1.03	0.82	3.86	4.76	0.	0.
time (sec)	N/A	0.343	0.079	0.107	1.571	2.412	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	174	200	0	1210	0	0
normalized size	1	1.	0.77	0.89	0.	5.38	0.	0.
time (sec)	N/A	0.531	0.128	0.157	0.	2.676	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	290	250	790	1906	0	0
normalized size	1	1.	0.85	0.73	2.32	5.59	0.	0.
time (sec)	N/A	0.629	0.432	0.226	1.653	3.038	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	85	165	583	910	0	0
normalized size	1	1.	0.6	1.16	4.11	6.41	0.	0.
time (sec)	N/A	0.399	0.237	0.077	1.704	3.121	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	138	254	882	1823	0	0
normalized size	1	1.	0.63	1.15	4.01	8.29	0.	0.
time (sec)	N/A	0.537	0.643	0.098	1.786	3.26	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	191	324	1175	2446	0	0
normalized size	1	1.	0.54	0.91	3.3	6.87	0.	0.
time (sec)	N/A	0.637	1.09	0.09	1.912	4.54	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	54	116	18
normalized size	1	1.	1.	0.8	1.04	2.16	4.64	0.72
time (sec)	N/A	0.031	0.01	0.048	0.946	2.332	14.387	1.065

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	81	114	30
normalized size	1	1.	1.	0.77	1.	2.7	3.8	1.
time (sec)	N/A	0.033	0.01	0.041	0.967	2.348	14.559	1.093

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	82	116	30
normalized size	1	1.	1.	0.77	1.	2.73	3.87	1.
time (sec)	N/A	0.033	0.008	0.036	0.94	2.423	14.256	1.071

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	61	114	26
normalized size	1	1.	1.	0.8	1.04	2.44	4.56	1.04
time (sec)	N/A	0.031	0.009	0.035	0.948	2.386	14.315	1.065

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	7	8
normalized size	1	1.	1.	0.88	1.	2.5	0.88	1.
time (sec)	N/A	0.006	0.006	0.002	0.946	2.355	0.159	1.079

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	97	12	101	54	26
normalized size	1	1.	2.27	8.82	1.09	9.18	4.91	2.36
time (sec)	N/A	0.021	0.078	0.032	0.944	2.4	2.087	1.071

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	18	12	180	42	0	70
normalized size	1	1.	1.64	1.09	16.36	3.82	0.	6.36
time (sec)	N/A	0.019	0.011	0.01	0.954	2.28	0.	1.086

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	24	43	7	11
normalized size	1	1.	1.	0.75	2.	3.58	0.58	0.92
time (sec)	N/A	0.047	0.016	0.047	0.946	2.309	1.683	1.067

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	17	17	27	35	108	29	42
normalized size	1	1.42	1.42	2.25	2.92	9.	2.42	3.5
time (sec)	N/A	0.039	0.015	0.024	0.95	2.311	1.642	1.069

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	27	66	19	27
normalized size	1	1.	1.	0.95	1.42	3.47	1.	1.42
time (sec)	N/A	0.04	0.013	0.017	0.958	2.528	33.438	1.073

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	52	50	66	182	51	657
normalized size	1	1.	1.18	1.14	1.5	4.14	1.16	14.93
time (sec)	N/A	0.038	0.045	0.025	0.943	2.512	0.116	3.069

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	60	45	86	34	45
normalized size	1	1.	0.95	1.62	1.22	2.32	0.92	1.22
time (sec)	N/A	0.033	0.031	0.029	0.957	2.405	0.098	1.274

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	68	58	77	211	61	890
normalized size	1	1.	1.26	1.07	1.43	3.91	1.13	16.48
time (sec)	N/A	0.041	0.092	0.038	0.942	2.552	0.121	2.933

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	42	90	0	45
normalized size	1	1.	1.	0.86	1.14	2.43	0.	1.22
time (sec)	N/A	0.039	0.029	0.031	0.947	2.353	0.	1.07

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	42	46	62	158	49	57
normalized size	1	1.	1.24	1.35	1.82	4.65	1.44	1.68
time (sec)	N/A	0.025	0.01	0.013	0.945	2.599	0.149	1.069

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	68	55	140	42	55
normalized size	1	1.	1.	1.58	1.28	3.26	0.98	1.28
time (sec)	N/A	0.036	0.026	0.054	0.94	2.411	0.099	1.084

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	72	99	275	0	99
normalized size	1	1.	1.	0.83	1.14	3.16	0.	1.14
time (sec)	N/A	0.13	0.06	0.056	0.943	2.708	0.	1.181

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	59	220	0	70
normalized size	1	1.	1.	0.88	1.4	5.24	0.	1.67
time (sec)	N/A	0.118	0.034	0.05	0.939	2.52	0.	1.093

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	77	238	0	77
normalized size	1	1.	1.	0.89	1.22	3.78	0.	1.22
time (sec)	N/A	0.124	0.031	0.051	0.955	2.429	0.	1.098

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	49	70	293	0	81
normalized size	1	1.	0.87	0.82	1.17	4.88	0.	1.35
time (sec)	N/A	0.126	0.124	0.056	0.945	2.286	0.	1.129

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	59	38	55	115	0	0
normalized size	1	1.	1.26	0.81	1.17	2.45	0.	0.
time (sec)	N/A	0.102	0.05	0.044	0.95	2.068	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	71	38	50	140	44	53
normalized size	1	1.	1.65	0.88	1.16	3.26	1.02	1.23
time (sec)	N/A	0.116	0.022	0.024	0.944	2.238	4.251	1.116

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	69	59	220	41	300
normalized size	1	1.	1.	1.64	1.4	5.24	0.98	7.14
time (sec)	N/A	0.041	0.028	0.027	0.952	2.238	0.109	1.145

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	55	123	274	0	97
normalized size	1	1.	0.85	0.74	1.66	3.7	0.	1.31
time (sec)	N/A	0.246	0.078	0.021	1.436	2.453	0.	1.225

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	47	32	238	122	0	1592
normalized size	1	1.	3.36	2.29	17.	8.71	0.	113.71
time (sec)	N/A	0.011	0.04	0.011	1.465	2.053	0.	1.447

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	31	12	136
normalized size	1	1.	1.	0.93	1.14	2.21	0.86	9.71
time (sec)	N/A	0.038	0.015	0.018	0.944	2.131	1.26	1.17

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	72	0	0
normalized size	1	1.	1.	0.91	0.	6.55	0.	0.
time (sec)	N/A	0.054	0.016	0.033	0.	2.652	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	35	26	72	32	0
normalized size	1	1.	0.94	1.03	0.76	2.12	0.94	0.
time (sec)	N/A	0.049	0.025	0.008	1.074	2.023	4.172	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	24	23	50	22	23
normalized size	1	1.	1.19	1.14	1.1	2.38	1.05	1.1
time (sec)	N/A	0.03	0.013	0.013	0.956	1.975	2.073	1.084

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	55	97	69	177	189	78
normalized size	1	1.	0.56	0.98	0.7	1.79	1.91	0.79
time (sec)	N/A	0.06	0.23	0.019	0.984	2.157	1.317	1.076

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	51	80	180	0	95
normalized size	1	1.	1.08	1.38	2.16	4.86	0.	2.57
time (sec)	N/A	0.09	0.06	0.045	1.456	2.204	0.	1.166

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	55	220	90	55
normalized size	1	1.	0.86	0.81	0.96	3.86	1.58	0.96
time (sec)	N/A	0.073	0.091	0.046	0.986	2.255	7.722	1.1

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	243	68	137	0	275	0	0
normalized size	1	4.26	1.19	2.4	0.	4.82	0.	0.
time (sec)	N/A	0.211	0.057	0.157	0.	2.396	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	9	61	7	14
normalized size	1	1.	1.	1.6	1.8	12.2	1.4	2.8
time (sec)	N/A	0.023	0.004	0.018	0.947	2.045	5.5	1.089

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	11	0	11	30	15	30	0	0
normalized size	1	0.	1.	2.73	1.36	2.73	0.	0.
time (sec)	N/A	0.276	0.319	0.109	1.207	2.158	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	24	22	100	85	0	126
normalized size	1	0.	0.89	0.81	3.7	3.15	0.	4.67
time (sec)	N/A	0.063	0.167	0.14	1.443	2.067	0.	1.144

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	62	89	146	100	69
normalized size	1	1.	0.77	0.95	1.37	2.25	1.54	1.06
time (sec)	N/A	0.068	0.087	0.026	0.971	2.06	1.232	1.09

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	14	39	0	0
normalized size	1	1.	1.	1.38	1.75	4.88	0.	0.
time (sec)	N/A	0.023	0.016	0.019	0.946	2.113	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	169	101	0	0
normalized size	1	1.	1.	0.	5.28	3.16	0.	0.
time (sec)	N/A	0.04	0.081	0.139	1.485	2.306	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	61	22	0	4
normalized size	1	1.	1.	1.33	20.33	7.33	0.	1.33
time (sec)	N/A	0.03	0.018	0.043	0.972	2.095	0.	1.097

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	47	73	39	36
normalized size	1	1.	0.91	1.03	1.34	2.09	1.11	1.03
time (sec)	N/A	0.065	0.04	0.002	0.959	1.892	0.337	1.095

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	23	31	5	8
normalized size	1	1.	1.	0.88	2.88	3.88	0.62	1.
time (sec)	N/A	0.007	0.002	0.005	1.085	1.988	0.572	1.103

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	38	0	176	0	39
normalized size	1	1.	0.78	1.03	0.	4.76	0.	1.05
time (sec)	N/A	0.173	0.071	0.01	0.	2.142	0.	1.092

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	30	10	14
normalized size	1	1.	1.	0.79	1.	2.14	0.71	1.
time (sec)	N/A	0.013	0.007	0.004	0.953	2.154	120.493	1.093

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	11	58	7	11
normalized size	1	1.	1.	1.5	1.1	5.8	0.7	1.1
time (sec)	N/A	0.037	0.003	0.022	0.935	2.059	1.731	1.063

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	35	27	20
normalized size	1	1.	1.	0.94	1.18	2.06	1.59	1.18
time (sec)	N/A	0.024	0.021	0.006	0.96	2.221	30.813	1.173

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	120	436	170	213	241	170
normalized size	1	1.	0.93	3.38	1.32	1.65	1.87	1.32
time (sec)	N/A	0.144	0.547	0.248	1.021	2.188	115.585	1.19

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	352	160	5171	294	0	1964
normalized size	1	1.	3.2	1.45	47.01	2.67	0.	17.85
time (sec)	N/A	0.112	0.883	0.148	2.322	2.314	0.	2.433

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	49	27	0	8
normalized size	1	1.	1.	1.17	8.17	4.5	0.	1.33
time (sec)	N/A	0.02	0.018	0.007	0.962	2.112	0.	1.071

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	14	3	5
normalized size	1	1.	1.	1.25	1.25	3.5	0.75	1.25
time (sec)	N/A	0.01	0.002	0.002	0.941	2.017	0.293	1.073

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	24	132	104	0	1273
normalized size	1	1.	1.11	0.89	4.89	3.85	0.	47.15
time (sec)	N/A	0.023	0.015	0.008	1.45	2.024	0.	1.356

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	F	F(-2)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	26	0	26	28	0	0	0	0
normalized size	1	0.	1.	1.08	0.	0.	0.	0.
time (sec)	N/A	0.811	0.431	0.324	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	7.654	0.38	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	A	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	9	0	9	57	111	35	10	112
normalized size	1	0.	1.	6.33	12.33	3.89	1.11	12.44
time (sec)	N/A	0.381	0.123	0.063	1.61	2.12	0.331	1.068

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	60	77	157	150	84
normalized size	1	1.	0.99	0.78	1.	2.04	1.95	1.09
time (sec)	N/A	0.126	0.159	0.012	0.968	2.092	1.1	1.078

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	134	125	177	321	326	193
normalized size	1	1.	0.83	0.78	1.1	1.99	2.02	1.2
time (sec)	N/A	0.27	0.209	0.018	0.971	2.304	8.147	1.123

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	105	84	107	181	201	95
normalized size	1	1.	1.18	0.94	1.2	2.03	2.26	1.07
time (sec)	N/A	0.109	0.155	0.016	0.97	2.095	1.204	1.088

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	167	213	294	420	541	251
normalized size	1	1.	0.58	0.74	1.02	1.46	1.88	0.87
time (sec)	N/A	0.4	0.479	0.023	0.978	2.297	8.988	1.15

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	55	136	0	0	0	0
normalized size	1	1.	0.9	2.23	0.	0.	0.	0.
time (sec)	N/A	0.289	0.181	0.924	0.	0.	0.	0.

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	137	266	0	0	0	0
normalized size	1	1.	0.93	1.8	0.	0.	0.	0.
time (sec)	N/A	0.239	0.276	1.197	0.	0.	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	43	86	68	70	0	42
normalized size	1	1.	1.26	2.53	2.	2.06	0.	1.24
time (sec)	N/A	0.097	0.093	0.125	0.989	2.303	0.	1.823

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	43	86	84	73	0	42
normalized size	1	1.	1.19	2.39	2.33	2.03	0.	1.17
time (sec)	N/A	0.097	0.076	0.122	0.996	2.061	0.	2.133

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	73	184	0	405	0	173
normalized size	1	1.	0.61	1.53	0.	3.38	0.	1.44
time (sec)	N/A	0.701	0.595	0.211	0.	2.411	0.	1.259

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	25	108	78	193	122	65
normalized size	1	1.	0.35	1.5	1.08	2.68	1.69	0.9
time (sec)	N/A	0.151	0.035	0.088	1.434	2.125	28.038	1.233

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	42	56	208	116	76	70
normalized size	1	1.	0.76	1.02	3.78	2.11	1.38	1.27
time (sec)	N/A	0.409	0.204	0.147	1.471	2.238	1.746	1.172

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	33	17	30	39	32	19
normalized size	1	1.	2.06	1.06	1.88	2.44	2.	1.19
time (sec)	N/A	0.054	0.012	0.04	0.952	1.991	0.283	1.087

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	59	31	39
normalized size	1	1.	1.	1.06	1.33	3.28	1.72	2.17
time (sec)	N/A	0.028	0.043	0.026	0.937	2.052	0.484	1.133

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	95	61	0	39
normalized size	1	1.	1.	1.68	5.	3.21	0.	2.05
time (sec)	N/A	0.314	0.058	0.131	1.438	2.121	0.	1.158

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	18	31	41	0	19
normalized size	1	1.	1.94	1.06	1.82	2.41	0.	1.12
time (sec)	N/A	0.175	0.014	0.084	0.946	2.022	0.	1.139

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	56	208	117	0	70
normalized size	1	1.	0.78	1.04	3.85	2.17	0.	1.3
time (sec)	N/A	0.532	0.237	0.26	1.464	2.232	0.	1.228

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	26	108	78	193	0	65
normalized size	1	1.	0.36	1.5	1.08	2.68	0.	0.9
time (sec)	N/A	1.396	0.025	0.156	1.431	2.249	0.	1.225

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [341] had the largest ratio of [1.286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.	14	0.143
2	A	3	3	1.	27	0.111
3	A	2	2	1.	12	0.167
4	A	2	2	1.	14	0.143
5	A	2	1	1.	21	0.048
6	A	2	2	1.	23	0.087
7	A	2	2	1.	21	0.095
8	A	3	3	1.	14	0.214
9	A	4	4	1.	25	0.16
10	A	2	2	1.	14	0.143
11	A	2	2	1.	14	0.143
12	A	2	1	1.	23	0.043
13	A	2	2	1.	23	0.087
14	A	2	2	1.	23	0.087
15	A	2	2	1.	12	0.167
16	A	3	3	1.	25	0.12
17	A	2	2	1.	14	0.143
18	A	2	2	1.	12	0.167
19	A	2	1	1.	21	0.048
20	A	2	2	1.	21	0.095
21	A	2	2	1.	23	0.087
22	A	3	3	1.	14	0.214
23	A	4	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	2	2	1.	14	0.143
25	A	2	2	1.	14	0.143
26	A	2	1	1.	21	0.048
27	A	2	2	1.	21	0.095
28	A	2	2	1.	23	0.087
29	A	6	5	1.	6	0.833
30	A	9	6	1.	6	1.
31	A	8	4	1.	16	0.25
32	A	8	4	1.	19	0.21
33	A	3	3	1.	16	0.188
34	A	3	3	1.	27	0.111
35	A	2	1	1.	11	0.091
36	A	5	5	1.	14	0.357
37	A	6	6	1.	16	0.375
38	A	9	5	1.	16	0.312
39	A	5	3	1.	36	0.083
40	A	4	3	1.	36	0.083
41	A	2	2	1.	34	0.059
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.
44	A	6	5	1.	6	0.833
45	A	9	6	1.	6	1.
46	A	8	4	1.	16	0.25
47	A	8	4	1.	19	0.21
48	A	4	4	1.	21	0.19
49	A	4	4	1.	27	0.148
50	A	5	4	1.	37	0.108
51	A	5	5	1.	14	0.357
52	A	6	6	1.	16	0.375
53	A	5	3	1.	36	0.083
54	A	4	3	1.	36	0.083
55	A	2	2	1.	34	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	0	0	0.	0	0.
57	A	0	0	0.	0	0.
58	A	2	2	1.	12	0.167
59	A	3	3	1.	14	0.214
60	A	6	6	1.	12	0.5
61	A	2	1	1.	33	0.03
62	A	3	3	1.	14	0.214
63	A	2	2	1.	25	0.08
64	A	4	3	1.	16	0.188
65	A	4	3	1.	15	0.2
66	A	1	1	1.	7	0.143
67	A	1	1	1.	7	0.143
68	A	1	1	1.	7	0.143
69	A	4	2	1.	7	0.286
70	A	1	1	1.	7	0.143
71	A	1	1	1.	7	0.143
72	A	1	1	1.	7	0.143
73	A	4	2	1.	7	0.286
74	A	4	3	1.	7	0.429
75	A	9	4	1.	7	0.571
76	A	5	3	1.	7	0.429
77	A	10	4	1.	7	0.571
78	A	10	5	1.	7	0.714
79	A	6	3	1.	7	0.429
80	A	3	2	1.	7	0.286
81	A	3	2	1.	7	0.286
82	A	6	3	1.	7	0.429
83	A	6	3	1.	7	0.429
84	A	7	3	1.	7	0.429
85	A	2	2	1.	7	0.286
86	A	5	5	1.	7	0.714
87	A	4	3	1.	7	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	7	6	1.	7	0.857
89	A	7	4	1.	7	0.571
90	A	2	2	1.	7	0.286
91	A	2	1	1.	7	0.143
92	A	4	2	1.	7	0.286
93	A	4	2	1.	7	0.286
94	A	7	3	1.	7	0.429
95	A	3	2	1.	7	0.286
96	A	2	2	1.	9	0.222
97	A	1	1	1.	7	0.143
98	A	1	1	1.	7	0.143
99	A	1	1	1.	7	0.143
100	A	4	2	1.	7	0.286
101	A	1	1	1.	7	0.143
102	A	1	1	1.	7	0.143
103	A	1	1	1.	7	0.143
104	A	4	2	1.	7	0.286
105	A	4	3	1.	7	0.429
106	A	3	2	1.	7	0.286
107	A	6	4	1.	7	0.571
108	A	6	3	1.	7	0.429
109	A	10	5	1.	7	0.714
110	A	4	3	1.	7	0.429
111	A	9	4	1.	7	0.571
112	A	6	3	1.	7	0.429
113	A	10	4	1.	7	0.571
114	A	7	3	1.	7	0.429
115	A	6	3	1.	7	0.429
116	A	2	2	1.	7	0.286
117	A	2	1	1.	7	0.143
118	A	4	3	1.	7	0.429
119	A	4	2	1.	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	7	4	1.	7	0.571
121	A	3	3	1.	7	0.429
122	A	3	3	1.	9	0.333
123	A	2	2	1.	7	0.286
124	A	5	5	1.	7	0.714
125	A	4	2	1.	7	0.286
126	A	7	6	1.	7	0.857
127	A	7	3	1.	7	0.429
128	A	6	2	1.	9	0.222
129	A	6	2	1.	11	0.182
130	A	5	2	1.	11	0.182
131	A	5	2	1.	9	0.222
132	A	6	2	1.	9	0.222
133	A	6	2	1.	11	0.182
134	A	7	2	1.	13	0.154
135	A	3	2	1.	13	0.154
136	A	3	2	1.	14	0.143
137	A	3	2	1.	13	0.154
138	A	3	2	1.	14	0.143
139	A	4	3	1.	13	0.231
140	A	4	3	1.	14	0.214
141	A	4	3	1.	13	0.231
142	A	4	3	1.	14	0.214
143	A	3	2	1.	13	0.154
144	A	3	2	1.	14	0.143
145	A	3	2	1.	13	0.154
146	A	3	2	1.	14	0.143
147	A	2	2	1.	9	0.222
148	A	3	3	1.	9	0.333
149	A	4	4	1.	9	0.444
150	A	2	2	1.	9	0.222
151	A	3	3	1.	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
152	A	4	4	1.	9	0.444
153	A	4	4	1.	12	0.333
154	A	6	6	1.	12	0.5
155	A	3	3	1.	12	0.25
156	A	5	5	1.	12	0.417
157	A	3	3	1.	14	0.214
158	A	3	3	1.	14	0.214
159	A	2	2	1.	23	0.087
160	A	9	6	1.	24	0.25
161	A	11	7	1.	26	0.269
162	A	2	1	1.	33	0.03
163	A	9	6	1.	34	0.176
164	A	11	7	1.	36	0.194
165	A	5	3	1.	33	0.091
166	A	4	3	1.	33	0.091
167	A	3	3	1.	31	0.097
168	A	4	4	1.	33	0.121
169	A	5	5	1.	33	0.152
170	A	6	5	1.	33	0.152
171	A	11	9	1.	33	0.273
172	A	8	7	1.	33	0.212
173	A	3	3	1.	31	0.097
174	A	11	7	1.	33	0.212
175	A	13	8	1.	33	0.242
176	A	15	8	1.	33	0.242
177	A	20	11	1.	37	0.297
178	A	17	10	1.	37	0.27
179	A	14	9	1.	35	0.257
180	A	0	0	0.	0	0.
181	A	51	17	1.	33	0.515
182	A	34	14	1.	33	0.424
183	A	26	12	1.	31	0.387

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	15	8	1.	22	0.364
185	A	5	5	1.	13	0.385
186	A	6	6	1.	15	0.4
187	A	7	6	1.	15	0.4
188	A	8	6	1.	15	0.4
189	A	4	4	1.	15	0.267
190	A	5	4	1.	15	0.267
191	A	6	4	1.	15	0.267
192	A	7	4	1.	15	0.267
193	A	6	5	1.	17	0.294
194	A	5	5	1.	17	0.294
195	A	4	4	1.	17	0.235
196	A	6	5	1.	17	0.294
197	A	7	6	1.	17	0.353
198	A	8	6	1.	17	0.353
199	A	3	3	1.	16	0.188
200	A	3	3	1.	16	0.188
201	A	4	4	1.	17	0.235
202	A	3	3	1.	17	0.176
203	A	4	4	1.	17	0.235
204	A	3	3	1.	17	0.176
205	A	4	3	1.	22	0.136
206	A	8	5	1.	17	0.294
207	A	5	3	1.	19	0.158
208	A	3	2	1.	20	0.1
209	A	4	3	1.	19	0.158
210	A	4	3	1.	22	0.136
211	A	10	6	1.	17	0.353
212	A	4	2	1.	19	0.105
213	A	3	3	1.	20	0.15
214	A	4	3	1.	19	0.158
215	A	6	6	1.	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
216	A	7	7	1.	17	0.412
217	A	2	2	1.	19	0.105
218	A	2	2	1.	19	0.105
219	A	3	2	1.	19	0.105
220	A	4	2	1.	19	0.105
221	A	3	2	1.	19	0.105
222	A	3	2	1.	19	0.105
223	A	2	1	1.	19	0.053
224	A	2	2	1.	19	0.105
225	A	3	2	1.	17	0.118
226	A	2	2	1.	19	0.105
227	A	1	1	1.	19	0.053
228	A	3	3	1.	19	0.158
229	A	2	2	1.	19	0.105
230	A	4	3	1.	19	0.158
231	A	3	2	1.	19	0.105
232	A	4	3	1.	21	0.143
233	A	3	3	1.	21	0.143
234	A	3	3	1.	21	0.143
235	A	2	2	1.	21	0.095
236	A	2	2	1.	21	0.095
237	A	3	3	1.	21	0.143
238	A	3	3	1.	21	0.143
239	A	4	3	1.	21	0.143
240	A	4	3	1.	21	0.143
241	A	3	3	1.	21	0.143
242	A	3	3	1.	21	0.143
243	A	2	2	1.	21	0.095
244	A	2	2	1.	21	0.095
245	A	3	3	1.	21	0.143
246	A	3	3	1.	21	0.143
247	A	4	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
248	A	1	1	1.	22	0.045
249	A	1	1	1.	22	0.045
250	A	1	1	1.	22	0.045
251	A	1	1	1.	22	0.045
252	A	3	2	1.	20	0.1
253	A	1	1	1.	22	0.045
254	A	1	1	1.	22	0.045
255	A	1	1	1.	22	0.045
256	A	1	1	1.	22	0.045
257	A	1	1	1.	24	0.042
258	A	1	1	1.	24	0.042
259	A	1	1	1.	24	0.042
260	A	1	1	1.	24	0.042
261	A	1	1	1.	24	0.042
262	A	1	1	1.	24	0.042
263	A	8	7	1.	11	0.636
264	A	4	4	1.	11	0.364
265	A	7	6	1.	11	0.546
266	A	4	3	1.	11	0.273
267	A	3	2	1.	9	0.222
268	A	3	3	1.	11	0.273
269	A	6	6	1.	11	0.546
270	A	4	3	1.	11	0.273
271	A	8	8	1.	11	0.727
272	A	4	3	1.	11	0.273
273	A	4	3	1.	7	0.429
274	A	5	4	1.	7	0.571
275	A	4	3	1.	7	0.429
276	A	4	4	1.	7	0.571
277	A	3	2	0.69	5	0.4
278	A	3	3	1.	7	0.429
279	A	3	3	1.	7	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	A	4	3	1.	7	0.429
281	A	4	3	1.	7	0.429
282	A	4	3	1.	7	0.429
283	A	8	7	1.	11	0.636
284	A	4	4	1.	11	0.364
285	A	7	6	1.	11	0.546
286	A	4	3	1.	11	0.273
287	A	3	2	1.	9	0.222
288	A	3	3	1.	11	0.273
289	A	5	5	1.	11	0.454
290	A	4	3	1.	11	0.273
291	A	7	7	1.	11	0.636
292	A	4	3	1.	11	0.273
293	A	4	3	1.	7	0.429
294	A	5	4	1.	7	0.571
295	A	4	3	1.	7	0.429
296	A	4	4	1.	7	0.571
297	A	3	2	1.	5	0.4
298	A	3	3	1.	7	0.429
299	A	3	3	1.	7	0.429
300	A	4	3	1.	7	0.429
301	A	4	3	1.	7	0.429
302	A	4	3	1.	7	0.429
303	A	6	3	1.	9	0.333
304	A	6	5	1.	9	0.556
305	A	4	3	1.	9	0.333
306	A	3	2	1.	7	0.286
307	A	3	3	1.	9	0.333
308	A	2	1	1.	9	0.111
309	A	4	3	1.	9	0.333
310	A	2	0	1.	9	0.
311	A	4	3	1.	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	3	1	1.	9	0.111
313	A	4	3	1.	9	0.333
314	A	6	5	1.	11	0.454
315	A	5	5	1.	11	0.454
316	A	4	4	1.	11	0.364
317	A	3	3	1.	11	0.273
318	A	8	8	1.	11	0.727
319	A	9	9	1.	11	0.818
320	A	10	10	1.	11	0.909
321	A	11	10	1.	11	0.909
322	A	6	3	1.	9	0.333
323	A	6	5	1.	9	0.556
324	A	4	3	1.	9	0.333
325	A	3	2	1.	7	0.286
326	A	3	3	1.	9	0.333
327	A	2	1	1.	9	0.111
328	A	4	3	1.	9	0.333
329	A	2	0	1.	9	0.
330	A	4	3	1.	9	0.333
331	A	3	1	1.	9	0.111
332	A	4	3	1.	9	0.333
333	A	6	5	1.	11	0.454
334	A	5	5	1.	11	0.454
335	A	4	4	1.	11	0.364
336	A	3	3	1.	11	0.273
337	A	8	8	1.	11	0.727
338	A	9	9	1.	11	0.818
339	A	10	10	1.	11	0.909
340	A	11	10	1.	11	0.909
341	A	18	9	1.	7	1.286
342	A	4	3	1.	7	0.429
343	A	9	7	1.	7	1.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
344	A	3	2	1.	5	0.4
345	A	6	6	1.	7	0.857
346	A	11	6	1.	7	0.857
347	A	5	4	1.	7	0.571
348	A	18	6	1.	7	0.857
349	A	3	3	1.	18	0.167
350	A	3	3	1.	18	0.167
351	A	4	4	1.	18	0.222
352	A	3	3	1.	18	0.167
353	A	3	3	1.	18	0.167
354	A	4	4	1.	18	0.222
355	A	6	3	1.	30	0.1
356	A	5	3	1.	30	0.1
357	A	4	3	1.	30	0.1
358	A	3	2	1.	28	0.071
359	A	1	1	1.	30	0.033
360	A	2	2	1.	30	0.067
361	A	3	2	1.	30	0.067
362	A	4	2	1.	30	0.067
363	A	5	4	1.	24	0.167
364	A	4	3	1.	24	0.125
365	A	3	2	1.	22	0.091
366	A	2	2	1.	24	0.083
367	A	4	4	1.	24	0.167
368	A	4	4	1.	24	0.167
369	A	5	5	1.	24	0.208
370	A	2	2	1.	24	0.083
371	A	4	4	1.	24	0.167
372	A	4	4	1.	24	0.167
373	A	5	5	1.	24	0.208
374	A	5	4	1.	24	0.167
375	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	3	2	1.	22	0.091
377	A	2	2	1.	24	0.083
378	A	4	4	1.	24	0.167
379	A	4	4	1.	24	0.167
380	A	5	5	1.	24	0.208
381	A	5	4	1.	24	0.167
382	A	4	3	1.	24	0.125
383	A	3	2	1.	22	0.091
384	A	2	2	1.	24	0.083
385	A	4	4	1.	24	0.167
386	A	4	4	1.	24	0.167
387	A	5	5	1.	24	0.208
388	A	5	4	1.	24	0.167
389	A	4	3	1.	24	0.125
390	A	3	2	1.	22	0.091
391	A	2	2	1.	24	0.083
392	A	4	4	1.	24	0.167
393	A	4	4	1.	24	0.167
394	A	5	5	1.	24	0.208
395	A	6	4	1.	20	0.2
396	A	5	4	1.	20	0.2
397	A	4	3	1.	20	0.15
398	A	3	2	1.	18	0.111
399	A	3	3	1.	20	0.15
400	A	5	5	1.	20	0.25
401	A	5	5	1.	20	0.25
402	A	6	6	1.	20	0.3
403	A	7	7	1.	22	0.318
404	A	6	6	1.	22	0.273
405	A	2	2	1.	22	0.091
406	A	2	2	1.	22	0.091
407	A	3	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
408	A	7	7	1.	22	0.318
409	A	8	7	1.	22	0.318
410	A	7	7	1.	22	0.318
411	A	6	6	1.	22	0.273
412	A	2	2	1.	22	0.091
413	A	2	2	1.	22	0.091
414	A	3	3	1.	22	0.136
415	A	7	7	1.	22	0.318
416	A	8	7	1.	22	0.318
417	A	3	2	1.	22	0.091
418	A	2	2	1.	22	0.091
419	A	1	1	1.	22	0.045
420	A	3	3	1.	22	0.136
421	A	4	4	1.	22	0.182
422	A	5	4	1.	22	0.182
423	A	4	2	1.	22	0.091
424	A	3	2	1.	22	0.091
425	A	2	2	1.	22	0.091
426	A	1	1	1.	22	0.045
427	A	3	3	1.	22	0.136
428	A	4	4	1.	22	0.182
429	A	5	4	1.	22	0.182
430	A	4	2	1.	32	0.062
431	A	3	2	1.	32	0.062
432	A	2	2	1.	32	0.062
433	A	1	1	1.	32	0.031
434	A	3	3	1.	32	0.094
435	A	4	4	1.	32	0.125
436	A	5	4	1.	32	0.125
437	A	3	2	1.	34	0.059
438	A	2	2	1.	34	0.059
439	A	1	1	1.	34	0.029

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	3	3	1.	34	0.088
441	A	4	4	1.	34	0.118
442	A	5	4	1.	34	0.118
443	A	4	4	1.	15	0.267
444	A	3	3	1.36	11	0.273
445	A	5	5	1.	12	0.417
446	A	4	4	1.	15	0.267
447	A	10	8	1.	17	0.471
448	A	7	7	1.	33	0.212
449	A	3	3	1.	33	0.091
450	A	3	3	1.	33	0.091
451	A	4	4	1.	33	0.121
452	A	8	8	1.	33	0.242
453	A	7	7	1.	33	0.212
454	A	3	3	1.	33	0.091
455	A	3	3	1.	33	0.091
456	A	4	4	1.	33	0.121
457	A	8	8	1.	33	0.242
458	A	5	5	1.	12	0.417
459	A	4	4	1.	15	0.267
460	A	9	7	1.	17	0.412
461	A	4	4	1.	15	0.267
462	A	7	7	1.	33	0.212
463	A	3	3	1.	33	0.091
464	A	3	3	1.	33	0.091
465	A	4	4	1.	33	0.121
466	A	8	8	1.	33	0.242
467	A	7	7	1.	33	0.212
468	A	3	3	1.	33	0.091
469	A	3	3	1.	33	0.091
470	A	4	4	1.	33	0.121
471	A	8	8	1.	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	A	2	2	1.	11	0.182
473	A	2	2	1.	11	0.182
474	A	2	2	1.	11	0.182
475	A	2	1	1.	13	0.077
476	A	2	1	1.	13	0.077
477	A	4	3	1.	13	0.231
478	A	2	1	1.	15	0.067
479	A	2	1	1.	15	0.067
480	A	2	1	1.	19	0.053
481	A	2	1	1.	23	0.043
482	A	4	3	1.	20	0.15
483	A	4	3	1.	20	0.15
484	A	4	2	1.	11	0.182
485	A	6	4	1.	11	0.364
486	A	6	4	1.	11	0.364
487	A	2	2	1.	13	0.154
488	A	2	2	1.	13	0.154
489	A	2	2	1.	13	0.154
490	A	4	2	1.	11	0.182
491	A	6	4	1.	11	0.364
492	A	6	4	1.	11	0.364
493	A	2	2	1.	13	0.154
494	A	2	2	1.	13	0.154
495	A	2	2	1.	13	0.154
496	A	2	1	1.	16	0.062
497	A	9	6	1.	18	0.333
498	A	11	7	1.	20	0.35
499	A	5	3	1.	39	0.077
500	A	2	2	1.	37	0.054
501	A	3	3	1.	39	0.077
502	A	9	6	1.	39	0.154
503	A	7	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	4	3	1.	41	0.073
505	A	2	2	1.	41	0.049
506	A	5	5	1.	41	0.122
507	A	8	6	1.	41	0.146
508	A	3	3	1.	27	0.111
509	A	5	3	1.	21	0.143
510	A	7	5	1.	39	0.128
511	A	3	3	1.	37	0.081
512	A	4	4	1.	39	0.103
513	A	6	4	1.	39	0.103
514	A	6	5	1.	41	0.122
515	A	3	3	1.	41	0.073
516	A	3	3	1.	41	0.073
517	A	5	4	1.	41	0.098
518	A	8	7	1.	39	0.18
519	A	5	4	1.	37	0.108
520	A	6	6	1.	39	0.154
521	A	8	7	1.	39	0.18
522	A	7	6	1.	41	0.146
523	A	5	5	1.	41	0.122
524	A	5	5	1.	41	0.122
525	A	7	7	1.	41	0.171
526	A	1	1	1.	21	0.048
527	A	1	1	1.	21	0.048
528	A	1	1	1.	15	0.067
529	A	1	1	1.	21	0.048
530	A	3	3	1.	21	0.143
531	A	3	3	1.	21	0.143
532	A	3	3	1.	22	0.136
533	A	3	3	1.	22	0.136
534	A	4	4	1.	22	0.182
535	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
536	A	4	4	1.	19	0.21
537	A	5	5	1.	19	0.263
538	A	1	1	1.	22	0.045
539	A	1	1	1.	22	0.045
540	A	4	4	1.	19	0.21
541	A	4	4	1.	19	0.21
542	A	5	5	1.	19	0.263
543	A	1	1	1.	22	0.045
544	A	1	1	1.	22	0.045
545	A	4	4	1.	22	0.182
546	A	4	4	1.	22	0.182
547	A	5	5	1.	22	0.227
548	A	1	1	0.95	25	0.04
549	A	1	1	0.94	25	0.04
550	A	4	4	1.	23	0.174
551	A	4	4	1.	23	0.174
552	A	5	5	1.	23	0.217
553	A	1	1	1.	26	0.038
554	A	1	1	1.	26	0.038
555	B	1	1	2.83	30	0.033
556	A	8	6	1.	27	0.222
557	A	7	6	1.	27	0.222
558	A	6	6	1.	27	0.222
559	A	5	5	1.	27	0.185
560	A	6	6	1.	27	0.222
561	A	7	6	1.	27	0.222
562	A	7	7	1.	31	0.226
563	A	8	8	1.	31	0.258
564	A	9	8	1.	31	0.258
565	A	10	8	1.	31	0.258
566	A	4	4	1.	18	0.222
567	A	3	3	1.	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	2	2	1.	18	0.111
569	A	3	2	1.	16	0.125
570	A	4	4	1.	18	0.222
571	A	6	6	1.	18	0.333
572	A	7	7	1.	18	0.389
573	A	8	8	1.	20	0.4
574	A	7	7	1.	20	0.35
575	A	3	3	1.	20	0.15
576	A	3	3	1.	20	0.15
577	A	5	5	1.	20	0.25
578	A	8	8	1.	20	0.4
579	A	13	8	1.	14	0.571
580	A	11	7	1.	14	0.5
581	A	9	6	1.	12	0.5
582	A	0	0	0.	0	0.
583	A	0	0	0.	0	0.
584	A	0	0	0.	0	0.
585	A	15	6	1.	26	0.231
586	A	11	5	1.	26	0.192
587	A	6	6	1.	26	0.231
588	A	4	3	1.	26	0.115
589	A	1	1	1.	23	0.043
590	A	1	1	1.	22	0.045
591	A	3	3	1.	20	0.15
592	A	7	5	1.	24	0.208
593	A	9	9	1.	26	0.346
594	A	15	6	1.	24	0.25
595	A	11	5	1.	24	0.208
596	A	6	6	1.	24	0.25
597	A	4	3	1.	24	0.125
598	A	1	1	1.	21	0.048
599	A	1	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
600	A	3	3	1.	18	0.167
601	A	7	5	1.	22	0.227
602	A	9	9	1.	24	0.375
603	A	5	5	1.	31	0.161
604	A	4	4	1.	31	0.129
605	A	3	3	1.	31	0.097
606	A	2	2	1.	29	0.069
607	A	3	3	1.	20	0.15
608	A	4	4	1.	29	0.138
609	A	5	4	1.	31	0.129
610	A	6	4	1.	31	0.129
611	A	7	7	1.	31	0.226
612	A	5	5	1.	31	0.161
613	A	4	4	1.	31	0.129
614	A	3	3	1.	29	0.103
615	A	5	5	1.	20	0.25
616	A	6	6	1.	29	0.207
617	A	6	6	1.	31	0.194
618	A	7	6	1.	31	0.194
619	A	6	6	1.	31	0.194
620	A	5	5	1.	31	0.161
621	A	4	4	1.	31	0.129
622	A	3	3	1.	29	0.103
623	A	6	5	1.	20	0.25
624	A	7	6	1.	29	0.207
625	A	8	7	1.	31	0.226
626	A	6	6	1.	31	0.194
627	A	5	5	1.	31	0.161
628	A	4	4	1.	31	0.129
629	A	4	4	1.	29	0.138
630	A	7	6	1.	20	0.3
631	A	8	7	1.	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
632	A	9	7	1.	31	0.226
633	A	3	2	1.	13	0.154
634	A	6	4	1.	21	0.19
635	A	6	4	1.	28	0.143
636	A	6	4	1.	30	0.133
637	A	1	1	1.	43	0.023
638	A	1	1	1.	43	0.023
639	A	1	1	1.	43	0.023
640	A	1	1	1.	41	0.024
641	A	1	1	1.	43	0.023
642	A	1	1	1.	43	0.023
643	A	1	1	1.	43	0.023
644	A	0	0	0.	0	0.
645	A	0	0	0.	0	0.
646	A	0	0	0.	0	0.
647	A	0	0	0.	0	0.
648	A	2	2	1.	11	0.182
649	A	2	2	1.	11	0.182
650	A	2	2	1.	13	0.154
651	A	2	2	1.	6	0.333
652	A	4	4	1.	11	0.364
653	A	6	3	1.	13	0.231
654	A	4	3	1.	17	0.176
655	A	2	2	1.	10	0.2
656	A	3	3	1.	21	0.143
657	A	3	3	1.	19	0.158
658	A	3	2	1.	15	0.133
659	A	2	2	1.	17	0.118
660	A	2	2	1.	22	0.091
661	A	2	2	1.	22	0.091
662	A	2	2	1.	17	0.118
663	A	2	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
664	A	2	2	1.	22	0.091
665	A	2	2	1.	11	0.182
666	A	2	2	1.	11	0.182
667	A	2	2	1.	13	0.154
668	A	2	2	1.	15	0.133
669	A	2	2	1.	19	0.105
670	A	4	4	1.	11	0.364
671	A	3	3	1.	15	0.2
672	A	2	2	1.	15	0.133
673	A	3	3	1.	16	0.188
674	A	2	2	1.	6	0.333
675	A	3	2	1.	10	0.2
676	A	2	2	1.	6	0.333
677	A	4	3	1.	17	0.176
678	A	3	3	1.	9	0.333
679	A	4	3	1.	13	0.231
680	A	2	2	1.	17	0.118
681	A	2	2	1.	9	0.222
682	A	2	2	1.	12	0.167
683	A	2	2	1.	18	0.111
684	A	2	2	1.	17	0.118
685	A	2	2	1.	22	0.091
686	A	2	2	1.	22	0.091
687	A	2	2	1.	17	0.118
688	A	2	2	1.	22	0.091
689	A	2	2	1.	22	0.091
690	A	2	2	1.	13	0.154
691	A	2	2	1.	15	0.133
692	A	2	2	1.	13	0.154
693	A	2	2	1.	13	0.154
694	A	3	1	1.	15	0.067
695	A	4	3	1.	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
696	A	3	3	1.	17	0.176
697	A	3	2	1.5	16	0.125
698	A	3	2	1.5	18	0.111
699	A	7	7	1.	15	0.467
700	A	3	3	1.	19	0.158
701	A	3	2	1.	19	0.105
702	A	3	2	1.	21	0.095
703	A	3	2	1.	21	0.095
704	A	2	2	1.	17	0.118
705	A	4	3	1.	17	0.176
706	A	5	5	1.	19	0.263
707	A	2	2	1.	11	0.182
708	A	4	2	1.	17	0.118
709	A	2	2	1.	17	0.118
710	A	2	2	1.	17	0.118
711	A	3	3	1.	15	0.2
712	A	3	3	1.	17	0.176
713	A	3	3	1.	17	0.176
714	A	2	2	1.	9	0.222
715	A	4	3	1.	15	0.2
716	A	2	2	1.	13	0.154
717	A	2	2	1.	13	0.154
718	A	2	2	1.	11	0.182
719	A	4	2	1.	15	0.133
720	A	3	2	1.	19	0.105
721	A	3	2	1.	21	0.095
722	A	3	2	1.	21	0.095
723	A	2	2	1.	11	0.182
724	A	4	4	1.82	13	0.308
725	A	2	2	1.	13	0.154
726	A	2	2	1.	15	0.133
727	A	2	2	1.	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
728	A	3	3	1.	15	0.2
729	A	2	1	1.	15	0.067
730	A	2	2	1.	9	0.222
731	A	2	2	1.	9	0.222
732	A	2	2	1.	19	0.105
733	A	3	2	1.	23	0.087
734	A	2	1	1.	23	0.043
735	A	4	4	1.67	13	0.308
736	A	2	2	1.	15	0.133
737	A	2	2	1.	13	0.154
738	A	3	3	1.	21	0.143
739	A	2	1	1.	15	0.067
740	A	3	2	1.	23	0.087
741	A	4	3	1.	20	0.15
742	A	4	3	1.	19	0.158
743	A	4	3	1.	24	0.125
744	A	4	3	1.	21	0.143
745	A	4	3	1.	20	0.15
746	A	4	3	1.	19	0.158
747	A	4	3	1.	24	0.125
748	A	4	3	1.	21	0.143
749	A	1	3	1.	8	0.375
750	A	1	2	1.	8	0.25
751	A	3	2	1.	11	0.182
752	A	2	2	1.	6	0.333
753	A	4	3	1.	8	0.375
754	A	2	2	1.	9	0.222
755	A	2	2	1.	12	0.167
756	A	3	2	1.	13	0.154
757	A	2	2	1.	8	0.25
758	A	1	1	1.	12	0.083
759	B	25	3	10.75	20	0.15

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
760	A	2	2	1.	6	0.333
761	A	3	3	1.	8	0.375
762	A	4	3	1.	15	0.2
763	A	2	2	1.	15	0.133
764	A	8	4	1.	17	0.235
765	A	3	2	1.	13	0.154
766	A	2	2	1.	6	0.333
767	A	2	2	1.	10	0.2
768	A	2	2	1.	8	0.25
769	A	2	2	1.	10	0.2
770	A	3	3	1.	10	0.3
771	A	3	3	1.	10	0.3
772	A	2	2	1.	8	0.25
773	A	1	1	1.	8	0.125
774	A	3	1	1.	18	0.056
775	A	6	3	1.	16	0.188
776	A	6	3	1.	8	0.375
777	A	2	2	1.	17	0.118
778	A	3	3	1.	7	0.429
779	A	3	3	1.	11	0.273
780	A	3	2	1.	10	0.2
781	A	2	2	1.	8	0.25
782	A	2	2	1.	8	0.25
783	A	3	2	1.	10	0.2
784	A	2	2	1.	10	0.2
785	A	3	3	1.	9	0.333
786	A	2	2	1.	8	0.25
787	A	3	3	1.	8	0.375
788	A	1	1	1.	8	0.125
789	A	3	3	1.	8	0.375
790	A	3	3	1.	8	0.375
791	A	2	2	1.	8	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
792	A	2	2	1.	13	0.154
793	A	1	1	1.	11	0.091
794	A	3	3	1.	9	0.333
795	A	3	2	1.	7	0.286
796	F	0	0	N/A	0	N/A
797	A	2	2	1.	6	0.333
798	A	3	2	1.	11	0.182
799	A	3	2	1.	8	0.25
800	A	3	2	1.	7	0.286
801	A	4	3	1.	9	0.333
802	A	2	2	1.	9	0.222
803	A	2	2	1.	7	0.286
804	A	2	2	1.	13	0.154
805	A	6	4	1.	10	0.4
806	A	2	2	1.	21	0.095
807	A	3	3	1.	15	0.2
808	A	2	2	1.	12	0.167
809	A	5	5	1.	16	0.312
810	A	4	2	1.	11	0.182
811	A	8	4	1.	13	0.308
812	A	1	1	1.	10	0.1
813	A	3	2	1.	13	0.154
814	A	4	2	1.	12	0.167
815	A	3	2	1.	8	0.25
816	A	5	4	1.	10	0.4
817	A	6	4	1.	15	0.267
818	A	4	3	1.	11	0.273
819	A	3	2	1.	13	0.154
820	A	2	2	1.	8	0.25
821	A	2	2	1.	28	0.071
822	A	1	1	1.	12	0.083
823	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
824	A	3	3	1.	7	0.429
825	A	2	2	1.	10	0.2
826	A	2	2	1.	8	0.25
827	A	4	2	1.29	13	0.154
828	A	3	1	1.	20	0.05
829	A	3	3	1.	9	0.333
830	A	5	5	1.	10	0.5
831	A	5	4	1.	9	0.444
832	A	4	4	1.	10	0.4
833	A	5	4	1.	10	0.4
834	A	4	3	1.	10	0.3
835	A	6	4	1.	10	0.4
836	A	5	5	1.	14	0.357
837	A	5	4	1.	13	0.308
838	A	5	2	1.	9	0.222
839	A	5	2	1.	11	0.182
840	A	2	2	1.	7	0.286
841	A	6	2	1.	9	0.222
842	A	13	5	1.	10	0.5
843	A	3	3	1.	18	0.167
844	A	1	1	1.	18	0.056
845	A	3	3	1.	18	0.167
846	A	9	7	1.27	15	0.467
847	A	8	7	1.27	15	0.467
848	A	4	2	1.	15	0.133
849	A	4	2	1.	15	0.133
850	A	3	3	1.	21	0.143
851	A	3	3	1.	21	0.143
852	A	1	1	1.	14	0.071
853	A	4	4	1.	15	0.267
854	A	4	3	1.	14	0.214
855	A	7	5	1.28	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
856	A	7	5	1.28	16	0.312
857	A	4	2	1.	15	0.133
858	B	5	3	4.63	18	0.167
859	F	0	0	N/A	0	N/A
860	B	17	9	3.79	16	0.562
861	A	4	4	1.	12	0.333
862	A	2	2	1.	15	0.133
863	A	6	5	1.	15	0.333
864	A	6	5	1.	15	0.333
865	A	8	5	1.	18	0.278
866	A	10	8	1.11	23	0.348
867	A	7	6	1.	23	0.261
868	A	6	4	1.	16	0.25
869	A	8	5	1.	18	0.278
870	A	10	6	1.	18	0.333
871	A	5	5	1.	16	0.312
872	A	6	6	1.	18	0.333
873	A	7	7	1.	18	0.389
874	A	10	10	1.	16	0.625
875	A	17	14	1.	18	0.778
876	A	21	13	1.	18	0.722
877	A	12	11	1.	16	0.688
878	A	16	13	1.	18	0.722
879	A	21	17	1.	18	0.944
880	A	5	2	1.	11	0.182
881	A	5	2	1.	11	0.182
882	A	5	2	1.	11	0.182
883	A	5	2	1.	11	0.182
884	A	2	2	1.	6	0.333
885	A	1	1	1.	15	0.067
886	A	4	4	1.	9	0.444
887	A	3	2	1.	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
888	A	4	3	1.42	16	0.188
889	A	3	2	1.	15	0.133
890	A	5	4	1.	13	0.308
891	A	3	2	1.	13	0.154
892	A	5	4	1.	13	0.308
893	A	4	3	1.	15	0.2
894	A	5	4	1.	7	0.571
895	A	3	2	1.	13	0.154
896	A	5	4	1.	27	0.148
897	A	5	4	1.	27	0.148
898	A	5	4	1.	27	0.148
899	A	5	4	1.	27	0.148
900	A	5	3	1.	21	0.143
901	A	7	5	1.	23	0.217
902	A	4	3	1.	13	0.231
903	A	7	7	1.	15	0.467
904	A	2	2	1.	10	0.2
905	A	3	2	1.	17	0.118
906	A	5	4	1.	17	0.235
907	A	5	3	1.	8	0.375
908	A	3	2	1.	13	0.154
909	A	4	3	1.	16	0.188
910	A	6	5	1.	13	0.385
911	A	8	3	1.	12	0.25
912	B	22	9	4.26	18	0.5
913	A	3	3	1.	9	0.333
914	F	0	0	N/A	0	N/A
915	F	0	0	N/A	0	N/A
916	A	6	4	1.	12	0.333
917	A	2	2	1.	9	0.222
918	A	1	1	1.	21	0.048
919	A	3	3	1.	10	0.3

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
920	A	8	4	1.	13	0.308
921	A	1	1	1.	8	0.125
922	A	7	5	1.	12	0.417
923	A	1	1	1.	18	0.056
924	A	1	1	1.	14	0.071
925	A	1	1	1.	22	0.045
926	A	7	4	1.	22	0.182
927	A	6	4	1.	22	0.182
928	A	3	3	1.	10	0.3
929	A	3	3	1.	9	0.333
930	A	2	2	1.	14	0.143
931	F	0	0	N/A	0	N/A
932	A	0	0	0.	0	0.
933	F	0	0	N/A	0	N/A
934	A	7	5	1.	28	0.179
935	A	9	6	1.	30	0.2
936	A	7	6	1.	36	0.167
937	A	16	5	1.	38	0.132
938	A	7	6	1.	29	0.207
939	A	11	8	1.	31	0.258
940	A	4	4	1.	27	0.148
941	A	4	4	1.	27	0.148
942	A	7	4	1.	39	0.103
943	A	4	2	1.	39	0.051
944	A	5	3	1.	39	0.077
945	A	6	3	1.	39	0.077
946	A	1	1	1.	31	0.032
947	A	4	2	1.	31	0.065
948	A	2	1	1.	39	0.026
949	A	5	3	1.	39	0.077
950	A	4	2	1.	39	0.051

Chapter 3

Listing of integrals

3.1 $\int \frac{2}{3-\cos(4+6x)}\,dx$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rubi [A] time = 0.0397767, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2657}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(3 - Cos[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{2}{3-\cos(4+6x)}\,dx = 2\int \frac{1}{3-\cos(4+6x)}\,dx$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.036174, size = 22, normalized size = 0.5

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(3 - Cos[4 + 6*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.016, size = 17, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(3-cos(4+6*x)),x)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.69744, size = 35, normalized size = 0.8

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(6\,x+4\right)}{\cos\left(6\,x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A] time = 1.42568, size = 101, normalized size = 2.3

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(6\,x+4\right)-\sqrt{2}}{4\,\sin\left(6\,x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

Sympy [A] time = 0.324355, size = 32, normalized size = 0.73

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(3x+2\right)\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

Giac [A] time = 1.13833, size = 77, normalized size = 1.75

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.2 $\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rubi [A] time = 0.0375237, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {12, 3166, 2657}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
```

```
PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx &= 2\int \frac{\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx\\
&= 2\int \frac{1}{3-\cos(4+6x)}\,dx\\
&= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0277109, size = 22, normalized size = 0.5

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.081, size = 17, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.57537, size = 35, normalized size = 0.8

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A] time = 1.45432, size = 101, normalized size = 2.3

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(6x+4\right)-\sqrt{2}}{4\sin\left(6x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\int\frac{\csc\left(6x+4\right)}{\cot\left(6x+4\right)-3\csc\left(6x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x)
```

```
[Out] -2*Integral(csc(6*x + 4)/(cot(6*x + 4) - 3*csc(6*x + 4)), x)
```

Giac [A] time = 1.23553, size = 77, normalized size = 1.75

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.3 $\int \frac{1}{1+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.0215153, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0464328, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.033, size = 17, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.56648, size = 22, normalized size = 0.46

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.46993, size = 127, normalized size = 2.65

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [B] time = 8.38866, size = 343, normalized size = 7.15

$$\frac{2\sqrt{2}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-2\sqrt{2}}}+\frac{3\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-2\sqrt{2}}}+\frac{2\sqrt{2}\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}\right.}{\sqrt{2\sqrt{2}}}\right.\right.}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)**2),x)
```

```
[Out] 2*sqrt(2)*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 - pi/
2 + 1)/pi))/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2))) + 3*(
atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 - pi/2 + 1)/pi))
/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2))) + 2*sqrt(2)*sqrt
(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3
)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*
sqrt(3 - 2*sqrt(2))) + 3*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(
3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(21*sqrt
(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2)))
```

Giac [A] time = 1.12004, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.4 $\int \frac{1}{2-\cos^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.0198624, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0586315, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.02, size = 17, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-cos(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.60181, size = 22, normalized size = 0.46

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.42053, size = 127, normalized size = 2.65

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [B] time = 10.2511, size = 343, normalized size = 7.15

$$\frac{2\sqrt{2}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-2\sqrt{2}}}+\frac{3\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-2\sqrt{2}}}+\frac{2\sqrt{2}\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}\right.}{\sqrt{2\sqrt{2}}}\right.\right.}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)**2),x)
```

```
[Out] 2*sqrt(2)*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 - pi/
2 + 1)/pi))/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2))) + 3*(
atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 - pi/2 + 1)/pi))
/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2))) + 2*sqrt(2)*sqrt
(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3
)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*
sqrt(3 - 2*sqrt(2))) + 3*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(
3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(21*sqrt
(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2)))
```

Giac [A] time = 1.10448, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.5 $\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.0255043, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.024852, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.04, size = 17, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.67127, size = 22, normalized size = 0.46

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.46917, size = 127, normalized size = 2.65

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [B] time = 9.4536, size = 343, normalized size = 7.15

$$\frac{2\sqrt{2}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-2\sqrt{2}}}+\frac{3\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-2\sqrt{2}}}+\frac{2\sqrt{2}\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}\right.}{\sqrt{2\sqrt{2}}}\right.\right.}{21\sqrt{2}\sqrt{3-2\sqrt{2}}+30\sqrt{3-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

```
[Out] 2*sqrt(2)*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 - pi/
2 + 1)/pi))/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2))) + 3*(
atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 - pi/2 + 1)/pi))
/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2))) + 2*sqrt(2)*sqrt
(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3
)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(21*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 30*
sqrt(3 - 2*sqrt(2))) + 3*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(
3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(21*sqrt
(2)*sqrt(3 - 2*sqrt(2)) + 30*sqrt(3 - 2*sqrt(2)))
```

Giac [A] time = 1.09728, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.6 $\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.0436017, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0185332, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.061, size = 17, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.54248, size = 22, normalized size = 0.46

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.53201, size = 127, normalized size = 2.65

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-2\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(3x+2\right)}{2\tan^{2}\left(3x+2\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(1+2*tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 + 1), x)
```

Giac [A] time = 2.45887, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.7 $\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.041115, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0183495, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.061, size = 17, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(2+cot(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.52606, size = 22, normalized size = 0.46

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.45905, size = 127, normalized size = 2.65

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-2\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{2}(3x+2)}{\cot^{2}(3x+2)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2+cot(2+3*x)**2),x)
```

```
[Out] Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 + 2), x)
```

Giac [A] time = 1.28001, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.8 $\int \frac{2}{1-3\cos(4+6x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0260871, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 2659, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(1 - 3*Cos[4 + 6*x]),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2}{1-3\cos(4+6x)}\,dx &= 2\int \frac{1}{1-3\cos(4+6x)}\,dx\\ &= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-2+4x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0383321, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(1 - 3*Cos[4 + 6*x]),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.012, size = 17, normalized size = 0.3

$$-\frac{\sqrt{2}\text{Artanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(1-3*cos(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.62765, size = 73, normalized size = 1.22

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{2\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{2\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A] time = 1.40245, size = 207, normalized size = 3.45

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(6x+4\right)^{2}-4\left(\sqrt{2}\cos\left(6x+4\right)-3\sqrt{2}\right)\sin\left(6x+4\right)+6\cos\left(6x+4\right)-17}{9\cos\left(6x+4\right)^{2}-6\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*
sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1)
)
```

Sympy [A] time = 0.390018, size = 42, normalized size = 0.7

$$\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)-\frac{\sqrt{2}}{2}\right)}{12}-\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)+\frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x)
```

```
[Out] sqrt(2)*log(tan(3*x + 2) - sqrt(2)/2)/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(
2)/2)/12
```

Giac [A] time = 1.21038, size = 53, normalized size = 0.88

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

3.9 $\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.046477, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {12, 3166, 2659, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```


```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```


Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx &= 2\int \frac{\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx\\ &= 2\int \frac{1}{1-3\cos(4+6x)}\,dx\\ &= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-2+4x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0390506, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.084, size = 17, normalized size = 0.3

$$-\frac{\sqrt{2}\operatorname{Artanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.66518, size = 73, normalized size = 1.22

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{2\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{2\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A] time = 1.48476, size = 207, normalized size = 3.45

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(6x+4)^2-4\left(\sqrt{2}\cos(6x+4)-3\sqrt{2}\right)\sin(6x+4)+6\cos(6x+4)-17}{9\cos(6x+4)^2-6\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*
sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\int\frac{\csc(6x+4)}{3\cot(6x+4)-\csc(6x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) - csc(6*x + 4)), x)
```

Giac [A] time = 1.30167, size = 53, normalized size = 0.88

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

3.10 $\int \frac{1}{-1+3\sin^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0195075, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-1+3\sin^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.0584288, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.029, size = 17, normalized size = 0.3

$$-\frac{\sqrt{2}\text{Artanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+3*sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.61108, size = 46, normalized size = 0.77

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A] time = 1.40464, size = 232, normalized size = 3.87

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4 - 4\cos(3x+2)^2 - 4\left(\sqrt{2}\cos(3x+2)^3 - 2\sqrt{2}\cos(3x+2)\right)\sin(3x+2) - 4}{9\cos(3x+2)^4 - 12\cos(3x+2)^2 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.17198, size = 53, normalized size = 0.88

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\tan(3x+2)\right|}{\left|2\sqrt{2} + 4\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

3.11 $\int \frac{1}{2-3\cos^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0191408, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 206}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{2-3\cos^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.0704723, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.017, size = 17, normalized size = 0.3

$$-\frac{\sqrt{2}\mathrm{Artanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*cos(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.59672, size = 46, normalized size = 0.77

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A] time = 1.31075, size = 232, normalized size = 3.87

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4 - 4\cos(3x+2)^2 - 4\left(\sqrt{2}\cos(3x+2)^3 - 2\sqrt{2}\cos(3x+2)\right)\sin(3x+2) - 4}{9\cos(3x+2)^4 - 12\cos(3x+2)^2 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.18605, size = 53, normalized size = 0.88

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\tan(3x+2)\right|}{\left|2\sqrt{2} + 4\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

3.12 $\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0304304, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0319567, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.043, size = 17, normalized size = 0.3

$$-\frac{\sqrt{2}\text{Artanh}\left(\tan\left(2+3x\right)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.60005, size = 46, normalized size = 0.77

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A] time = 1.40656, size = 232, normalized size = 3.87

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-4\cos\left(3x+2\right)^{2}-4\left(\sqrt{2}\cos\left(3x+2\right)^{3}-2\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-4}{9\cos\left(3x+2\right)^{4}-12\cos\left(3x+2\right)^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.22547, size = 53, normalized size = 0.88

$$\frac{1}{12}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

3.13 $\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0453745, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.0237708, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.061, size = 17, normalized size = 0.3

$$-\frac{\sqrt{2}\operatorname{Artanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.55851, size = 46, normalized size = 0.77

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A] time = 1.43176, size = 232, normalized size = 3.87

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4 - 4\cos(3x+2)^2 - 4\left(\sqrt{2}\cos(3x+2)^3 - 2\sqrt{2}\cos(3x+2)\right)\sin(3x+2) - 4}{9\cos(3x+2)^4 - 12\cos(3x+2)^2 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(3x+2)}{2\tan^2(3x+2) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-1+2*tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 - 1), x)
```

Giac [A] time = 2.53432, size = 53, normalized size = 0.88

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\tan(3x+2)\right|}{\left|2\sqrt{2} + 4\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

3.14 $\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0462344, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.030814, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] time = 0.055, size = 17, normalized size = 0.3

$$-\frac{\sqrt{2}\text{Artanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(2-cot(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.61261, size = 46, normalized size = 0.77

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A] time = 1.50975, size = 232, normalized size = 3.87

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-4\cos\left(3x+2\right)^{2}-4\left(\sqrt{2}\cos\left(3x+2\right)^{3}-2\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-4}{9\cos\left(3x+2\right)^{4}-12\cos\left(3x+2\right)^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\csc^{2}\left(3x+2\right)}{\cot^{2}\left(3x+2\right)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2-cot(2+3*x)**2),x)
```

```
[Out] -Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 - 2), x)
```

Giac [A] time = 1.37396, size = 53, normalized size = 0.88

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

3.15 $\int \frac{2}{3+\cos(4+6x)}\,dx$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rubi [A] time = 0.036651, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 2657}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(3 + Cos[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{2}{3+\cos(4+6x)}\,dx = 2\int \frac{1}{3+\cos(4+6x)}\,dx$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0294782, size = 22, normalized size = 0.52

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(3 + Cos[4 + 6*x]),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.011, size = 18, normalized size = 0.4

$$\frac{\sqrt{2}}{6}\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(3+cos(4+6*x)),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.68609, size = 36, normalized size = 0.86

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{2(\cos(6x+4)+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A] time = 1.22075, size = 101, normalized size = 2.4

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}}{4\sin\left(6x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

Sympy [A] time = 0.306932, size = 34, normalized size = 0.81

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\tan\left(3x+2\right)}{2}\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)/2) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

Giac [A] time = 1.1251, size = 77, normalized size = 1.83

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.16 $\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rubi [A] time = 0.0370735, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {12, 3166, 2657}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
```

```
PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx &= 2\int \frac{\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx\\
&= 2\int \frac{1}{3+\cos(4+6x)}\,dx\\
&= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0220547, size = 22, normalized size = 0.52

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.073, size = 18, normalized size = 0.4

$$\frac{\sqrt{2}}{6}\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.74898, size = 36, normalized size = 0.86

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(6x+4\right)}{2\left(\cos\left(6x+4\right)+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A] time = 1.84853, size = 101, normalized size = 2.4

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}}{4\sin\left(6x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int\frac{\csc\left(6x+4\right)}{\cot\left(6x+4\right)+3\csc\left(6x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x)
```

```
[Out] 2*Integral(csc(6*x + 4)/(cot(6*x + 4) + 3*csc(6*x + 4)), x)
```

Giac [A] time = 1.1875, size = 77, normalized size = 1.83

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.17 $\int \frac{1}{2-\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.0190109, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0210394, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.024, size = 18, normalized size = 0.4

$$\frac{\sqrt{2}}{6}\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-sin(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.57803, size = 23, normalized size = 0.48

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.73682, size = 124, normalized size = 2.58

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [A] time = 1.20373, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

Giac [A] time = 1.1218, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.18 $\int \frac{1}{1+\cos^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.0163445, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+\cos^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0405731, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.017, size = 18, normalized size = 0.4

$$\frac{\sqrt{2}}{6}\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.60938, size = 23, normalized size = 0.48

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.78258, size = 124, normalized size = 2.58

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [A] time = 0.948679, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

Giac [A] time = 1.12962, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.19 $\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.026868, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0205361, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.039, size = 18, normalized size = 0.4

$$\frac{\sqrt{2}}{6}\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.57235, size = 23, normalized size = 0.48

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.79003, size = 124, normalized size = 2.58

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [A] time = 1.2753, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

Giac [A] time = 1.13776, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.20 $\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.0422988, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0184973, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.06, size = 18, normalized size = 0.4

$$\frac{\sqrt{2}}{6}\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(2+tan(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.60711, size = 23, normalized size = 0.48

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.8668, size = 124, normalized size = 2.58

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2(3x+2)}{\tan^2(3x+2)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(2+tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 + 2), x)
```

Giac [A] time = 1.81969, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.21 $\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rubi [A] time = 0.044736, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.0188301, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.087, size = 18, normalized size = 0.4

$$\frac{\sqrt{2}}{6}\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.55973, size = 23, normalized size = 0.48

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A] time = 1.84252, size = 124, normalized size = 2.58

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^2(3x+2)}{2\cot^2(3x+2)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(1+2*cot(2+3*x)**2),x)
```

```
[Out] Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 + 1), x)
```

Giac [A] time = 1.46307, size = 77, normalized size = 1.6

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.22 $\int -\frac{2}{1+3\cos(4+6x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0303837, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 2659, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[-2/(1 + 3*Cos[4 + 6*x]),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int -\frac{2}{1+3\cos(4+6x)}\,dx &= -\left(2\int \frac{1}{1+3\cos(4+6x)}\,dx\right)\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4-2x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right)\\
&= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0266506, size = 22, normalized size = 0.36

$$
-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/(1 + 3*Cos[4 + 6*x]),x]
```

```
[Out] -ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.01, size = 18, normalized size = 0.3

$$
-\frac{\sqrt{2}}{6}\text{Artanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(1+3*cos(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.55537, size = 72, normalized size = 1.18

$$\frac{1}{12}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6\,x+4)}{\cos(6\,x+4)+1}}{\sqrt{2}+\frac{\sin(6\,x+4)}{\cos(6\,x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + si
n(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A] time = 1.82479, size = 207, normalized size = 3.39

$$\frac{1}{24}\,\sqrt{2}\log\left(-\frac{7\,\cos\left(6\,x+4\right)^{2}+4\left(\sqrt{2}\cos\left(6\,x+4\right)+3\,\sqrt{2}\right)\sin\left(6\,x+4\right)-6\,\cos\left(6\,x+4\right)-17}{9\,\cos\left(6\,x+4\right)^{2}+6\,\cos\left(6\,x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*
sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1)
)
```

Sympy [A] time = 0.398481, size = 39, normalized size = 0.64

$$\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)-\sqrt{2}\right)}{12}-\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)+\sqrt{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x)
```

```
[Out] sqrt(2)*log(tan(3*x + 2) - sqrt(2))/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(2)
)/12
```

Giac [A] time = 1.21, size = 53, normalized size = 0.87

$$\frac{1}{12}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

3.23 $\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0457299, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {12, 3166, 2659, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx &= -\left(2\int \frac{\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx\right)\\ &= -\left(2\int \frac{1}{1+3\cos(4+6x)}\,dx\right)\\ &= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{4-2x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0348746, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] -ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.079, size = 18, normalized size = 0.3

$$-\frac{\sqrt{2}}{6}\operatorname{Artanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.49522, size = 72, normalized size = 1.18

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + si
n(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A] time = 1.75495, size = 207, normalized size = 3.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(6x+4)^2+4\left(\sqrt{2}\cos(6x+4)+3\sqrt{2}\right)\sin(6x+4)-6\cos(6x+4)-17}{9\cos(6x+4)^2+6\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*
sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\int \frac{\csc(6x+4)}{3\cot(6x+4)+\csc(6x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) + csc(6*x + 4)), x)
```

Giac [A] time = 1.31549, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

3.24 $\int \frac{1}{-2+3\sin^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0201867, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 207}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.0651653, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.03, size = 18, normalized size = 0.3

$$-\frac{\sqrt{2}}{6}\text{Artanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+3*sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.60356, size = 43, normalized size = 0.7

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A] time = 1.69256, size = 230, normalized size = 3.77

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.22209, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

3.25 $\int \frac{1}{1-3\cos^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0191013, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.0578985, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] -ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.018, size = 18, normalized size = 0.3

$$-\frac{\sqrt{2}}{6}\text{Artanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-3*cos(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.66264, size = 43, normalized size = 0.7

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A] time = 1.63888, size = 230, normalized size = 3.77

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.20706, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

3.26 $\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0307402, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {207}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0313135, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.042, size = 18, normalized size = 0.3

$$-\frac{\sqrt{2}}{6}\text{Artanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.56128, size = 43, normalized size = 0.7

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A] time = 1.62461, size = 230, normalized size = 3.77

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.24016, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

3.27 $\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.0429416, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 207}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.0210346, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.059, size = 18, normalized size = 0.3

$$-\frac{\sqrt{2}}{6}\text{Artanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.64842, size = 43, normalized size = 0.7

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A] time = 1.72201, size = 230, normalized size = 3.77

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4 - 10\cos(3x+2)^2 + 4\left(\sqrt{2}\cos(3x+2)^3 + \sqrt{2}\cos(3x+2)\right)\sin(3x+2) - 1}{9\cos(3x+2)^4 - 6\cos(3x+2)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(3x+2)}{\tan^2(3x+2) - 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-2+tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 - 2), x)
```

Giac [A] time = 1.87334, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 2\tan(3x+2)\right|}{\left|2\sqrt{2} + 2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

3.28 $\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rubi [A] time = 0.046606, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.0349107, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A] time = 0.086, size = 18, normalized size = 0.3

$$-\frac{\sqrt{2}}{6}\text{Artanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A] time = 1.66734, size = 43, normalized size = 0.7

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A] time = 1.79598, size = 230, normalized size = 3.77

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4 - 10\cos(3x+2)^2 + 4\left(\sqrt{2}\cos(3x+2)^3 + \sqrt{2}\cos(3x+2)\right)\sin(3x+2) - 1}{9\cos(3x+2)^4 - 6\cos(3x+2)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\csc^2(3x+2)}{2\cot^2(3x+2)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(1-2*cot(2+3*x)**2),x)
```

```
[Out] -Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 - 1), x)
```

Giac [A] time = 1.51047, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 2\tan(3x+2)\right|}{\left|2\sqrt{2} + 2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

3.29 $\int (x+\sin(x))^2 \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0348256, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2637, 2635, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (x+\sin(x))^2\,dx &= \int \left(x^2+2x\sin(x)+\sin^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\sin(x)\,dx+\int \sin^2(x)\,dx\\ &= \frac{x^3}{3}-2x\cos(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}+2\int \cos(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0619871, size = 30, normalized size = 1.

$$\frac{1}{6}x\left(2x^2+3\right)+2\sin(x)-\frac{1}{4}\sin(2x)-2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^2,x]
```

```
[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

Maple [A] time = 0.009, size = 25, normalized size = 0.8

$$\frac{x}{2}+\frac{x^3}{3}-2\,x\cos(x)+2\,\sin(x)-\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^2,x)
```

```
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.1105, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 - 2x\cos(x) + \frac{1}{2}x - \frac{1}{4}\sin(2x) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

Fricas [A] time = 1.71524, size = 76, normalized size = 2.53

$$\frac{1}{3}x^3 - 2x\cos(x) - \frac{1}{2}(\cos(x) - 4)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

Sympy [A] time = 0.190192, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} - 2x\cos(x) - \frac{\sin(x)\cos(x)}{2} + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**2,x)
```

```
[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*s
in(x)
```

Giac [A] time = 1.14454, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 - 2x\cos(x) + \frac{1}{2}x - \frac{1}{4}\sin(2x) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

3.30 $\int (x+\sin(x))^3\,dx$

Optimal. Leaf size=56

$$\frac{x^4}{4}+\frac{3x^2}{4}-3x^2\cos(x)+\frac{3\sin^2(x)}{4}+6x\sin(x)+\frac{\cos^3(x)}{3}+5\cos(x)-\frac{3}{2}x\sin(x)\cos(x)$$

```
[Out] (3*x^2)/4 + x^4/4 + 5*Cos[x] - 3*x^2*Cos[x] + Cos[x]^3/3 + 6*x*Sin[x] - (3*
x*Cos[x]*Sin[x])/2 + (3*Sin[x]^2)/4
```

Rubi [A] time = 0.0669768, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6742, 3296, 2638, 3310, 30, 2633}

$$\frac{x^4}{4}+\frac{3x^2}{4}-3x^2\cos(x)+\frac{3\sin^2(x)}{4}+6x\sin(x)+\frac{\cos^3(x)}{3}+5\cos(x)-\frac{3}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 + 5*Cos[x] - 3*x^2*Cos[x] + Cos[x]^3/3 + 6*x*Sin[x] - (3*
x*Cos[x]*Sin[x])/2 + (3*Sin[x]^2)/4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int (x+\sin(x))^3\,dx &= \int \left(x^3+3x^2\sin(x)+3x\sin^2(x)+\sin^3(x)\right)\,dx \\
&= \frac{x^4}{4}+3\int x^2\sin(x)\,dx+3\int x\sin^2(x)\,dx+\int \sin^3(x)\,dx \\
&= \frac{x^4}{4}-3x^2\cos(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}+\frac{3\int x\,dx}{2}+6\int x\cos(x)\,dx-\text{Subst}\left(\int \left(1-x^2\right)\,d\right. \\
&= \frac{3x^2}{4}+\frac{x^4}{4}-\cos(x)-3x^2\cos(x)+\frac{\cos^3(x)}{3}+6x\sin(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}-6\int \sin(x)\,d \\
&= \frac{3x^2}{4}+\frac{x^4}{4}+5\cos(x)-3x^2\cos(x)+\frac{\cos^3(x)}{3}+6x\sin(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}
\end{aligned}
$$

Mathematica [A] time = 0.0884516, size = 48, normalized size = 0.86

$$\frac{1}{24}\left(6x\left(x^3+3x+24\sin(x)-3\sin(2x)\right)-18\left(4x^2-7\right)\cos(x)-9\cos(2x)+2\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^3,x]
```

```
[Out] (-18*(-7 + 4*x^2)*Cos[x] - 9*Cos[2*x] + 2*Cos[3*x] + 6*x*(3*x + x^3 + 24*Si
n[x] - 3*Sin[2*x]))/24
```

Maple [A] time = 0.037, size = 57, normalized size = 1.

$$-\frac{\left(2+(\sin(x))^{2}\right)\cos(x)}{3}+3x\left(-1/2\cos(x)\sin(x)+x/2\right)-\frac{3x^{2}}{4}+\frac{3(\sin(x))^{2}}{4}-3x^{2}\cos(x)+6\cos(x)+6x\sin(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)+3*x*(-1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2+3/4*sin(x)^
2-3*x^2*cos(x)+6*cos(x)+6*x*sin(x)+1/4*x^4
```

Maxima [A] time = 1.1164, size = 65, normalized size = 1.16

$$\frac{1}{4}x^{4}+\frac{1}{3}\cos(x)^{3}+\frac{3}{4}x^{2}-3\left(x^{2}-2\right)\cos(x)-\frac{3}{4}x\sin(2x)+6x\sin(x)-\frac{3}{8}\cos(2x)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - 3*(x^2 - 2)*cos(x) - 3/4*x*sin(2*x) + 6*
x*sin(x) - 3/8*cos(2*x) - cos(x)
```

Fricas [A] time = 2.02615, size = 135, normalized size = 2.41

$$\frac{1}{4}x^{4}+\frac{1}{3}\cos(x)^{3}+\frac{3}{4}x^{2}-\left(3x^{2}-5\right)\cos(x)-\frac{3}{4}\cos(x)^{2}-\frac{3}{2}\left(x\cos(x)-4x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - (3*x^2 - 5)*cos(x) - 3/4*cos(x)^2 - 3/2*
(x*cos(x) - 4*x)*sin(x)
```

Sympy [A] time = 0.384255, size = 85, normalized size = 1.52

$$\frac{x^4}{4} + \frac{3x^2\sin^2(x)}{4} + \frac{3x^2\cos^2(x)}{4} - 3x^2\cos(x) - \frac{3x\sin(x)\cos(x)}{2} + 6x\sin(x) - \sin^2(x)\cos(x) - \frac{2\cos^3(x)}{3} - \frac{3\cos^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*cos(x)**2/4 - 3*x**2*cos(x) - 3*x*sin(
x)*cos(x)/2 + 6*x*sin(x) - sin(x)**2*cos(x) - 2*cos(x)**3/3 - 3*cos(x)**2/4
 + 6*cos(x)
```

Giac [A] time = 1.18936, size = 62, normalized size = 1.11

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{3}{4}\left(4x^2 - 7\right)\cos(x) - \frac{3}{4}x\sin(2x) + 6x\sin(x) + \frac{1}{12}\cos(3x) - \frac{3}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 - 3/4*(4*x^2 - 7)*cos(x) - 3/4*x*sin(2*x) + 6*x*sin(x) +
1/12*cos(3*x) - 3/8*cos(2*x)
```

3.31 $\int \frac{\sin(a+bx)}{c+dx^2} dx$

Optimal. Leaf size=213

$$-\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}$$

```
[Out] -(CosIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sin[a - (b*Sqrt[-c])/Sqrt[d]])/(2
*Sqrt[-c]*Sqrt[d]) + (CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sin[a + (b*Sq
rt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*Sin
Integral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sq
rt[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[
d])
```

Rubi [A] time = 0.536306, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3333, 3303, 3299, 3302}

$$-\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x^2),x]
```

```
[Out] -(CosIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sin[a - (b*Sqrt[-c])/Sqrt[d]])/(2
*Sqrt[-c]*Sqrt[d]) + (CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sin[a + (b*Sq
rt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*Sin
Integral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sq
rt[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[
d])
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\sin(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\sin(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$= -\frac{\int \frac{\sin(a+bx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\int \frac{\sin(a+bx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}} + \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}\,d}{2\sqrt{-c}}$$

$$= -\frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} -$$

Mathematica [C] time = 0.321835, size = 172, normalized size = 0.81

$$\frac{i\left(\sin\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{CosIntegral}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)-\sin\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{CosIntegral}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+\cos\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+}{2\sqrt{c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[b*((I*Sqrt[c])/Sqrt[d] + x)]*Sin[a - (I*b*Sqrt[c])/Sqrt
[d]] - CosIntegral[b*(((-I)*Sqrt[c])/Sqrt[d] + x)]*Sin[a + (I*b*Sqrt[c])/Sq
rt[d]] + Cos[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[b*((I*Sqrt[c])/Sqrt[d]
+ x)] + Cos[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(I*b*Sqrt[c])/Sqrt[d] -
b*x]))/(Sqrt[c]*Sqrt[d])
```

Maple [A] time = 0.022, size = 229, normalized size = 1.1

$$b\left(\frac{1}{2d}\left(\mathrm{Si}\left(bx+a-\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)\cos\left(\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)+\mathrm{Ci}\left(bx+a-\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)\sin\left(\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)\right)\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x^2+c),x)
```

```
[Out] b*(1/2/((b*(-c*d)^(1/2)+a*d)/d-a)/d*(Si(b*x+a-(b*(-c*d)^(1/2)+a*d)/d)*cos((
b*(-c*d)^(1/2)+a*d)/d)+Ci(b*x+a-(b*(-c*d)^(1/2)+a*d)/d)*sin((b*(-c*d)^(1/2)
+a*d)/d))+1/2/(-(b*(-c*d)^(1/2)-a*d)/d-a)/d*(Si(b*x+a+(b*(-c*d)^(1/2)-a*d)/
d)*cos((b*(-c*d)^(1/2)-a*d)/d)-Ci(b*x+a+(b*(-c*d)^(1/2)-a*d)/d)*sin((b*(-c*
d)^(1/2)-a*d)/d)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(d*x^2 + c), x)
```

Fricas [C] time = 2.13078, size = 377, normalized size = 1.77

$$\frac{\sqrt{\frac{b^2c}{d}}\mathrm{Ei}\left(i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a+\sqrt{\frac{b^2c}{d}}\right)}-\sqrt{\frac{b^2c}{d}}\mathrm{Ei}\left(i\,bx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a-\sqrt{\frac{b^2c}{d}}\right)}+\sqrt{\frac{b^2c}{d}}\mathrm{Ei}\left(-i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(-i\,a+\sqrt{\frac{b^2c}{d}}\right)}-\sqrt{\frac{b^2c}{d}}\mathrm{Ei}\left(-i\,b\right.}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) - sqrt
(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) + sqrt(b^2*c/d)
*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) - sqrt(b^2*c/d)*Ei(-I*
b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx)}{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx + a)}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x^2 + c), x)
```

3.32 $\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cos\ldots}{}$$

```
[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d - Sqrt[
d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CosIntegral[(b*(d + Sqrt[d^2 -
4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2
- 4*c*e] + (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d -
Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt
[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]
)/Sqrt[d^2 - 4*c*e]
```

Rubi [A] time = 0.801635, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6728, 3303, 3299, 3302}

$$\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cos\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x + e*x^2),x]
```


```
[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d - Sqrt[
d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CosIntegral[(b*(d + Sqrt[d^2 -
4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2
- 4*c*e] + (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d -
Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt
[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]
)/Sqrt[d^2 - 4*c*e]
```


Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$= \frac{(2e)\int \frac{\sin(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\sin(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sin\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sin\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d+\sqrt{d^2-4ce}+2ex}\,d}{\sqrt{d^2-4ce}}$$

$$= \frac{\text{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\text{Ci}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} + \frac{\cos\left(a\right.}{}$$

Mathematica [A] time = 0.578755, size = 238, normalized size = 0.88

$$\frac{\sin\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(-\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right) - \sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right) - \cos\left(a+\right.}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]*Sin[a + (b*(-d + Sq
rt[d^2 - 4*c*e]))/(2*e)] - CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/
(2*e)]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)] - Cos[a + (b*(-d + Sqrt[d
^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x]
- Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)])/Sqrt[d^2 - 4*c*e]
```

Maple [A] time = 0.015, size = 320, normalized size = 1.2

$$b\left(\left(\mathrm{Si}\left(bx+a-\frac{1}{2e}\left(2ae-db+\sqrt{-4b^2ce+b^2d^2}\right)\right)\cos\left(\frac{1}{2e}\left(2ae-db+\sqrt{-4b^2ce+b^2d^2}\right)\right)+\mathrm{Ci}\left(bx+a-\frac{1}{2e}\left(2ae-c\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(b*x+a-1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*
d^2)^(1/2)))*cos(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(b*x+a-1/2
/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))*sin(1/2/e*(2*a*e-d*b+(-4*b^2*c*e
+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(b*x+a+1/2*(-2*a*e+d*b+(
-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2
))/e)-Ci(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin(1/2*(-2*a
*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)
```

Fricas [C] time = 2.42761, size = 950, normalized size = 3.51

$$-\frac{e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\,ae+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)}-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\,ae-e\sqrt{-}}{2\,e}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e*sqr
t(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b^2*d^
2 - 4*b^2*c*e)/e^2))/e) - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b
*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a
*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)
/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^
(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt(-(
b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sqrt(-(b^2*d^2 - 4*
b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/
e^2))/e))/(b*d^2 - 4*b*c*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)
```

3.33 $\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}} \, dx$

Optimal. Leaf size=10

$$-2\cos\left(\sqrt{x-7}\right)$$

```
[Out] -2*Cos[Sqrt[-7 + x]]
```

Rubi [A] time = 0.0179978, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3431, 15, 2638}

$$-2\cos\left(\sqrt{x-7}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]
```

```
[Out] -2*Cos[Sqrt[-7 + x]]
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x\sin(x)}{\sqrt{x^2}}\,dx, x, \sqrt{-7+x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int \sin(x)\,dx, x, \sqrt{-7+x}\right)$$
$$= -2\cos\left(\sqrt{-7+x}\right)$$

Mathematica [A] time = 0.0233957, size = 10, normalized size = 1.

$$-2\cos\left(\sqrt{x-7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]
```

```
[Out] -2*Cos[Sqrt[-7 + x]]
```

Maple [A] time = 0.011, size = 9, normalized size = 0.9

$$-2\cos\left(\sqrt{-7+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-7+x)^(1/2))/(-7+x)^(1/2),x)
```

```
[Out] -2*cos((-7+x)^(1/2))
```

Maxima [A] time = 1.09476, size = 11, normalized size = 1.1

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*cos(sqrt(x - 7))
```

Fricas [A] time = 1.99068, size = 28, normalized size = 2.8

$$-2 \cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*cos(sqrt(x - 7))
```

Sympy [A] time = 0.334359, size = 10, normalized size = 1.

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)**(1/2))/(-7+x)**(1/2),x)
```

```
[Out] -2*cos(sqrt(x - 7))
```

Giac [A] time = 1.14195, size = 11, normalized size = 1.1

$$-2 \cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*cos(sqrt(x - 7))
```

3.34 $\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{x\mathrm{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x*SinIntegral[x])/Sqrt[a - b*x^2]
```

Rubi [A] time = 0.470982, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6721, 23, 3299}

$$\frac{x\mathrm{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x*SinIntegral[x])/Sqrt[a - b*x^2]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}\sin(x)}{x\sqrt{a-bx^2}}\,dx}{\sqrt{1-\frac{bx^2}{a}}}$$

$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int \frac{\sin(x)}{x}\,dx}{\sqrt{a-bx^2}}$$

$$= \frac{\sqrt{b-\frac{a}{x^2}}x\operatorname{Si}(x)}{\sqrt{a-bx^2}}$$

Mathematica [C] time = 0.708935, size = 46, normalized size = 1.64

$$\frac{ix(\text{ExpIntegralEi}(-ix)-\text{ExpIntegralEi}(ix))\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]
```

```
[Out] ((I/2)*Sqrt[b - a/x^2]*x*(ExpIntegralEi[(-I)*x] - ExpIntegralEi[I*x]))/Sqrt
[a - b*x^2]
```

Maple [C] time = 0.046, size = 72, normalized size = 2.6

$$-\left(bx^2-a\right)x\left(-i\operatorname{Si}(x)+\frac{i}{2}\pi\operatorname{csgn}(x)\right)\sqrt{-\frac{-bx^2+a}{x^2}}\sqrt{\frac{-bx^2+a}{bx^2-a}}\left(-bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -(-(-b*x^2+a)/x^2)^(1/2)*(b*x^2-a)/(-b*x^2+a)^(3/2)*x*(1/(b*x^2-a)*(-b*x^2+
a))^(1/2)*(-I*Si(x)+1/2*I*Pi*csgn(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b - \frac{a}{x^2}}\sin(x)}{\sqrt{-bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2 + a}\sqrt{\frac{bx^2-a}{x^2}}\sin(x)}{bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt((b*x^2 - a)/x^2)*sin(x)/(b*x^2 - a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b - \frac{a}{x^2}}\sin(x)}{\sqrt{-bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

3.35 $\int \frac{1}{x(1+\sin(\log(x)))}\,dx$

Optimal. Leaf size=12

$$-\frac{\cos(\log(x))}{\sin(\log(x))+1}$$

```
[Out] -(Cos[Log[x]]/(1 + Sin[Log[x]]))
```

Rubi [A] time = 0.0289448, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2648}

$$-\frac{\cos(\log(x))}{\sin(\log(x))+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Sin[Log[x]])),x]
```

```
[Out] -(Cos[Log[x]]/(1 + Sin[Log[x]]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+\sin(\log(x)))}\,dx &= \text{Subst}\left(\int \frac{1}{1+\sin(x)}\,dx, x, \log(x)\right)\\ &= -\frac{\cos(\log(x))}{1+\sin(\log(x))}\end{aligned}$$

Mathematica [B] time = 0.0196309, size = 26, normalized size = 2.17

$$\frac{2\sin\left(\frac{\log(x)}{2}\right)}{\sin\left(\frac{\log(x)}{2}\right)+\cos\left(\frac{\log(x)}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Sin[Log[x]])),x]
```

```
[Out] (2*Sin[Log[x]/2])/(Cos[Log[x]/2] + Sin[Log[x]/2])
```

Maple [A] time = 0.017, size = 12, normalized size = 1.

$$-2 \left(1+\tan \left(1/2 \ln \left(x\right)\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+sin(ln(x))),x)
```

```
[Out] -2/(1+tan(1/2*ln(x)))
```

Maxima [A] time = 1.09909, size = 23, normalized size = 1.92

$$-\frac{2}{\frac{\sin(\log(x))}{\cos(\log(x))+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="maxima")
```

```
[Out] -2/(sin(log(x))/(cos(log(x)) + 1) + 1)
```

Fricas [A] time = 2.02112, size = 89, normalized size = 7.42

$$-\frac{\cos \left(\log \left(x\right)\right)-\sin \left(\log \left(x\right)\right)+1}{\cos \left(\log \left(x\right)\right)+\sin \left(\log \left(x\right)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="fricas")
```

```
[Out] -(cos(log(x)) - sin(log(x)) + 1)/(cos(log(x)) + sin(log(x)) + 1)
```

Sympy [A] time = 2.35437, size = 15, normalized size = 1.25

$$\frac{2\tan\left(\frac{\log(x)}{2}\right)}{\tan\left(\frac{\log(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(ln(x))),x)
```

```
[Out] 2*tan(log(x)/2)/(tan(log(x)/2) + 1)
```

Giac [A] time = 1.11301, size = 15, normalized size = 1.25

$$-\frac{2}{\tan\left(\frac{1}{2}\log(x)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*log(x)) + 1)
```

3.36 $\int \sin\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=100

$$\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c + d
*x)*Sin[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*Sin[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.163526, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4563, 3297, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c + d
*x)*Sin[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*Sin[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rule 4563

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin\left(\frac{a+bx}{c+dx}\right)dx &= -\frac{\text{Subst}\left(\int \frac{\sin\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\sin\left(\frac{b}{d}\right)\right)\text{S}\cdots}{\cdots}\\
&= \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}
\end{aligned}
$$

Mathematica [C] time = 5.51914, size = 272, normalized size = 2.72

$$
\frac{2\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{ad-bc}{d(c+dx)}\right) + d\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)\left(ic\left(e^{\frac{2ibc}{d(c+dx)}} - e^{2i\left(\frac{a}{c+dx}+\frac{b}{d}\right)}\right) + dx\sin\left(\frac{b}{d}\right)\left(e^{i\left(\frac{2a}{c+dx}+\frac{b}{d}\right)} + e^{\frac{ib\cdots}{d\cdots}}\cdots\right.\right.}{2d^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)],x]
```

```
[Out] (2*(b*c - a*d)*Cos[b/d]*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + (d*(I*c
*(E^(((2*I)*b*c)/(d*(c + d*x))) - E^((2*I)*(b/d + a/(c + d*x)))) + d*(E^((I
*b*(3*c + d*x))/(d*(c + d*x))) + E^(I*(b/d + (2*a)/(c + d*x))))*x*Sin[b/d]
+ 2*d*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*Cos[b/d]*Sin[(-(b*c) +
a*d)/(d*(c + d*x))]))/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 2*(b*c
- a*d)*Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))])/(2*d^2)
```

Maple [A] time = 0.013, size = 142, normalized size = 1.4

$$-(ad-cb)\left(-\frac{1}{d}\sin\left(\frac{b}{d}+\frac{ad-cb}{d\,(dx+c)}\right)\left(d\left(\frac{b}{d}+\frac{ad-cb}{d\,(dx+c)}\right)-b\right)^{-1}+\frac{1}{d}\left(-\frac{1}{d}\mathrm{Si}\left(\frac{ad-cb}{d\,(dx+c)}\right)\sin\left(\frac{b}{d}\right)+\frac{1}{d}\mathrm{Ci}\left(\frac{ad-cb}{d\,(dx+c)}\right)\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c)),x)
```

```
[Out] -(a*d-b*c)*(-sin(b/d+(a*d-b*c)/d/(d*x+c))/(d*(b/d+(a*d-b*c)/d/(d*x+c))-b)/d
+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{bx+a}{dx+c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c)), x)
```

Fricas [A] time = 2.2517, size = 323, normalized size = 3.23

$$-\frac{2\,(bc-ad)\sin\left(\frac{b}{d}\right)\mathrm{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-\left((bc-ad)\,\mathrm{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\,\mathrm{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cos\left(\frac{b}{d}\right)-2\left(d^2x+cd\right)\sin\left(\frac{bx+a}{dx+c}\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*c - a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - ((b
*c - a*d)*cos_integral((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integra
l(-(b*c - a*d)/(d^2*x + c*d)))*cos(b/d) - 2*(d^2*x + c*d)*sin((b*x + a)/(d*
x + c)))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{bx+a}{dx+c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c)), x)
```

3.37 $\int \sin^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((b*c - a*d)*CosIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 +
 ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.192004, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4563, 3313, 12, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*CosIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 +
 ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 4563

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sin^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cos\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\sin\left(\frac{2b}{d}\right)\right.}{}$$

$$= \frac{(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 7.27871, size = 401, normalized size = 3.75

$$-\frac{\left(acd-bc^2\right)\left(\frac{\left(-1+e^{\frac{4ib}{d}}\right)\left(e^{\frac{4ibc}{d(c+dx)}}-e^{\frac{4ia}{c+dx}}\right)\exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)}{8(bc-ad)}-\frac{\left(1+e^{\frac{4ib}{d}}\right)\left(e^{\frac{4ia}{c+dx}}+e^{\frac{4ibc}{d(c+dx)}}\right)\exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)}{8(bc-ad)}\right)}{d} + \frac{-2ad\sin\left(\frac{2b}{d}\right)\text{CosInteg}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] -(((-(b*c^2) + a*c*d)*(((-1 + E^(((4*I)*b)/d))*(-E^(((4*I)*a)/(c + d*x)) +
E^(((4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d
*x))/(d*(c + d*x)))) - ((1 + E^(((4*I)*b)/d))*(E^(((4*I)*a)/(c + d*x)) + E^
(((4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d*x
))/(d*(c + d*x))))))/d) - (x*Cos[(2*b)/d]*Cos[(2*(-(b*c) + a*d))/(d*(c + d*
x))])/2 + (x*Sin[(2*b)/d]*Sin[(2*(-(b*c) + a*d))/(d*(c + d*x))])/2 + (d^2*x
 + 2*b*c*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] - 2*a*d
*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] + 2*b*c*Cos[(2*
b)/d]*SinIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))] - 2*a*d*Cos[(2*b)/d]*Si
nIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

Maple [A] time = 0.015, size = 195, normalized size = 1.8

$$-\frac{ad-cb}{d^2}\left(-\frac{d}{2}\left(d\left(\frac{b}{d}+\frac{ad-cb}{d\,(dx+c)}\right)-b\right)^{-1}-\frac{d^2}{4}\left(-2\,\frac{1}{d}\cos\left(2\,\frac{ad-cb}{d\,(dx+c)}+2\,\frac{b}{d}\right)\left(d\left(\frac{b}{d}+\frac{ad-cb}{d\,(dx+c)}\right)-b\right)^{-1}-2\,\frac{1}{d}\left(2\,\frac{1}{d}\mathrm{Si}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/d^2*(a*d-b*c)*(-1/2*d/(d*(b/d+(a*d-b*c)/d/(d*x+c))-b)-1/4*d^2*(-2*cos(2*
(a*d-b*c)/d/(d*x+c)+2*b/d)/(d*(b/d+(a*d-b*c)/d/(d*x+c))-b)/d-2*(2*Si(2*(a*d
-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*(a*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,x-\frac{1}{2}\int\cos\left(\frac{2\,(bx+a)}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

Fricas [A] time = 2.25056, size = 351, normalized size = 3.28

$$\frac{2\,d^2x-2\left(d^2x+cd\right)\cos\left(\frac{bx+a}{dx+c}\right)^2+2\,(bc-ad)\cos\left(\frac{2\,b}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)+\left((bc-ad)\,\mathrm{Ci}\left(\frac{2\,(bc-ad)}{d^2x+cd}\right)+(bc-ad)\,\mathrm{Ci}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right.\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*x - 2*(d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + 2*(b*c - a*d)*c
os(2*b/d)*sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) + ((b*c - a*d)*cos_int
egral(2*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-2*(b*c - a*d
```

```
)/(d^2*x + c*d)))*sin(2*b/d))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{bx+a}{dx+c}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c))^2, x)
```

3.38 $\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=194

$$\frac{3\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\cos\left(\frac{3b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}$$

```
[Out] (3*(b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (
3*(b*c - a*d)*Cos[(3*b)/d]*CosIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d
^2) + ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sin[b/d]*Si
nIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sin[(3*b)/d]
*SinIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rubi [A] time = 0.322127, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4563, 3313, 3303, 3299, 3302}

$$\frac{3\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\cos\left(\frac{3b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (3*(b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (
3*(b*c - a*d)*Cos[(3*b)/d]*CosIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d
^2) + ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sin[b/d]*Si
nIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sin[(3*b)/d]
*SinIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rule 4563

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
```

```
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sin^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad))\,\text{Subst}\left(\int\left(-\frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x}+\frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \cdots\right.}{4\cdots}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left(3(bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{\left(3(bc-ad)\cos\left(\frac{3b}{d}\cdots\right.\right.}{\cdots}$$

$$= \frac{3(bc-ad)\cos\left(\frac{b}{d}\right)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\cos\left(\frac{3b}{d}\right)\text{Ci}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{3(bc-\cdots}{\cdots}$$

Mathematica [C] time = 7.72116, size = 657, normalized size = 3.39

$$-\frac{3\left(acd-bc^2\right)\left(\frac{i\left(1+e^{\frac{2ib}{d}}\right)\left(e^{\frac{2ibc}{d(c+dx)}}-e^{\frac{2ia}{c+dx}}\right)\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)}{4(bc-ad)}-\frac{i\left(-1+e^{\frac{2ib}{d}}\right)\left(e^{\frac{2ia}{c+dx}}+e^{\frac{2ibc}{d(c+dx)}}\right)\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)}{4(bc-ad)}\right)}{4d}+\frac{3\left(acd-bc^2\right)\left(\frac{i\left(1+e^{\frac{6ib}{d}}\right)\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*(-(b*c^2) + a*c*d)*(((I/4)*(1 + E^(((2*I)*b)/d))*(-E^(((2*I)*a)/(c + d*
x)) + E^(((2*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^((I*(2*b*c + a*d + b*d
*x))/(d*(c + d*x)))) - ((I/4)*(-1 + E^(((2*I)*b)/d))*(E^(((2*I)*a)/(c + d*x
)) + E^(((2*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^((I*(2*b*c + a*d + b*d*
x))/(d*(c + d*x))))))/(4*d) + (3*(-(b*c^2) + a*c*d)*(((I/12)*(1 + E^(((6*I)
*b)/d))*(-E^(((6*I)*a)/(c + d*x)) + E^(((6*I)*b*c)/(d*(c + d*x)))))/((b*c -
 a*d)*E^(((3*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))) - ((I/12)*(-1 + E^((
(6*I)*b)/d))*(E^(((6*I)*a)/(c + d*x)) + E^(((6*I)*b*c)/(d*(c + d*x)))))/((b
*c - a*d)*E^(((3*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x))))))/(4*d) + (3*x*C
os[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d])/4 - (x*Cos[(3*(-(b*c) + a*d))/(d
*(c + d*x))]*Sin[(3*b)/d])/4 + (3*x*Cos[b/d]*Sin[(-(b*c) + a*d)/(d*(c + d*x
))])/4 - (x*Cos[(3*b)/d]*Sin[(3*(-(b*c) + a*d))/(d*(c + d*x))])/4 + (3*(-(b
*c) + a*d)*(-(Cos[b/d]*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))]) + Cos[(3*
b)/d]*CosIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))] + Sin[b/d]*SinIntegral[
(-(b*c) + a*d)/(d*(c + d*x))] - Sin[(3*b)/d]*SinIntegral[(3*(-(b*c) + a*d))
/(d*(c + d*x))]))/(4*d^2)
```

Maple [A] time = 0.015, size = 295, normalized size = 1.5

$$-\frac{ad-cb}{d^2}\left(-\frac{d^2}{12}\left(-3\frac{1}{d}\sin\left(3\frac{ad-cb}{d(dx+c)}+3\frac{b}{d}\right)\left(d\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)-b\right)^{-1}+3\frac{1}{d}\left(-3\frac{1}{d}\mathrm{Si}\left(3\frac{ad-cb}{d(dx+c)}\right)\sin\left(3\frac{b}{d}\right)+3\frac{1}{d}\mathrm{C}\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c))^3,x)
```

```
[Out] -1/d^2*(a*d-b*c)*(-1/12*d^2*(-3*sin(3*(a*d-b*c)/d/(d*x+c)+3*b/d)/(d*(b/d+(a
*d-b*c)/d/(d*x+c))-b)/d+3*(-3*Si(3*(a*d-b*c)/d/(d*x+c))*sin(3*b/d)/d+3*Ci(3
*(a*d-b*c)/d/(d*x+c))*cos(3*b/d)/d)/d)+3/4*d^2*(-sin(b/d+(a*d-b*c)/d/(d*x+c
```

```
))/(d*(b/d+(a*d-b*c)/d/(d*x+c))-b)/d+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+C
i((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{bx+a}{dx+c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c))^3, x)
```

Fricas [A] time = 2.47593, size = 651, normalized size = 3.36

$$-\frac{6\,(bc-ad)\sin\left(\frac{b}{d}\right)\mathrm{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-6\,(bc-ad)\sin\left(\frac{3\,b}{d}\right)\mathrm{Si}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)+3\left((bc-ad)\,\mathrm{Ci}\left(\frac{3\,(bc-ad)}{d^2x+cd}\right)+(bc-ad)\,\mathrm{Ci}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - 6*(
b*c - a*d)*sin(3*b/d)*sin_integral(-3*(b*c - a*d)/(d^2*x + c*d)) + 3*((b*c
- a*d)*cos_integral(3*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral
(-3*(b*c - a*d)/(d^2*x + c*d)))*cos(3*b/d) - 3*((b*c - a*d)*cos_integral((b
*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c
*d)))*cos(b/d) - 8*(d^2*x - (d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + c*d)
*sin((b*x + a)/(d*x + c)))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{bx+a}{dx+c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c))^3, x)
```

3.39 $\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) + SinIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rubi [A] time = 0.110703, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3299}

$$\frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) + SinIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4x} - \frac{\sin(3x)}{4x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} \\
&= -\frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} + \frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0961445, size = 53, normalized size = 0.91

$$\frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right) - 3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + SinIntegral[(3*Sqrt[1 - a*x]
)/Sqrt[1 + a*x]])/(4*a)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\sin\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(
a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.40 $\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]/(2*a)
```

Rubi [A] time = 0.0817573, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3302}

$$\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]/(2*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} \\
&= \frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0749969, size = 57, normalized size = 0.98

$$
\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(ax+1)}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[1 - a*x]/(4*a) + L
og[1 + a*x]/(4*a)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$
\int \frac{1}{-a^2x^2+1}\left(\sin\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^2 dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int\frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx+a\int\frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{\left(a^2x^2-1\right)\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2+\left(a^2x^2-1\right)\sin\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx+\log\left(ax+1\right)-\log\left(ax-1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x
) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*co
s(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/sq
rt(a*x + 1))^2), x) + log(a*x + 1) - log(a*x - 1))/a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2-1}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.41 $\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi [A] time = 0.0397187, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3299}

$$-\frac{\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.0413151, size = 26, normalized size = 1.

$$-\frac{\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1} \sin\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] integral(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.42 $\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=39

$$\text{Unintegrable}\left(\frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.0369904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csc[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\csc(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 5.54207, size = 0, normalized size = 0.

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\sin\left(\sqrt{-ax+1}\,\frac{1}{\sqrt{ax+1}}\right)\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="ma
xima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="fr
icas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^2x^2\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sin(sqrt(-a*x +
1)/sqrt(a*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="gi
ac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.43 $\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$

Optimal. Leaf size=41

$$\text{Unintegrable}\left(\frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.0806222, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csc[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a
)
```

Rubi steps

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{\csc^2(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 18.7101, size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\sin\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{a^2x^2-\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
fricas")
```

```
[Out] integral(-1/(a^2*x^2 - (a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 -
1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.44 $\int (x+\cos(x))^2 \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x}{2}+2x\sin(x)+2\cos(x)+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + x^3/3 + 2*Cos[x] + 2*x*Sin[x] + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0344333, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2638, 2635, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2x\sin(x)+2\cos(x)+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cos[x])^2,x]
```

```
[Out] x/2 + x^3/3 + 2*Cos[x] + 2*x*Sin[x] + (Cos[x]*Sin[x])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (x+\cos(x))^2\,dx &= \int \left(x^2+2x\cos(x)+\cos^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\cos(x)\,dx+\int \cos^2(x)\,dx\\ &= \frac{x^3}{3}+2x\sin(x)+\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}-2\int \sin(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0688514, size = 26, normalized size = 0.87

$$\frac{1}{6}\left(x\left(2x^2+12\sin(x)+3\right)+3(\sin(x)+4)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cos[x])^2,x]
```

```
[Out] (3*Cos[x]*(4 + Sin[x]) + x*(3 + 2*x^2 + 12*Sin[x]))/6
```

Maple [A] time = 0.008, size = 25, normalized size = 0.8

$$\frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cos(x))^2,x)
```

```
[Out] 1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.12573, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 + 2x\sin(x) + \frac{1}{2}x + 2\cos(x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.2988, size = 76, normalized size = 2.53

$$\frac{1}{3}x^3 + \frac{1}{2}(4x + \cos(x))\sin(x) + \frac{1}{2}x + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*(4*x + cos(x))*sin(x) + 1/2*x + 2*cos(x)
```

Sympy [A] time = 0.209608, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x\sin^2(x)}{2} + 2x\sin(x) + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))**2,x)
```

```
[Out] x**3/3 + x*sin(x)**2/2 + 2*x*sin(x) + x*cos(x)**2/2 + sin(x)*cos(x)/2 + 2*c
os(x)
```

Giac [A] time = 1.10982, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 + 2x\sin(x) + \frac{1}{2}x + 2\cos(x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)
```

3.45 $\int (x+\cos(x))^3\,dx$

Optimal. Leaf size=56

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sin(x)-\frac{\sin^3(x)}{3}-5\sin(x)+\frac{3\cos^2(x)}{4}+6x\cos(x)+\frac{3}{2}x\sin(x)\cos(x)$$

```
[Out] (3*x^2)/4 + x^4/4 + 6*x*Cos[x] + (3*Cos[x]^2)/4 - 5*Sin[x] + 3*x^2*Sin[x] +
 (3*x*Cos[x]*Sin[x])/2 - Sin[x]^3/3
```

Rubi [A] time = 0.0694327, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6742, 3296, 2637, 3310, 30, 2633}

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sin(x)-\frac{\sin^3(x)}{3}-5\sin(x)+\frac{3\cos^2(x)}{4}+6x\cos(x)+\frac{3}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cos[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 + 6*x*Cos[x] + (3*Cos[x]^2)/4 - 5*Sin[x] + 3*x^2*Sin[x] +
 (3*x*Cos[x]*Sin[x])/2 - Sin[x]^3/3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int (x+\cos(x))^3\,dx &= \int \left(x^3+3x^2\cos(x)+3x\cos^2(x)+\cos^3(x)\right)\,dx \\
&= \frac{x^4}{4}+3\int x^2\cos(x)\,dx+3\int x\cos^2(x)\,dx+\int \cos^3(x)\,dx \\
&= \frac{x^4}{4}+\frac{3\cos^2(x)}{4}+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)+\frac{3\int x\,dx}{2}-6\int x\sin(x)\,dx-\text{Subst}\left(\int \left(1-x^2\right)\right. \\
&= \frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}+\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x)}{3}-6\int \cos(x \\
&= \frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}-5\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.103939, size = 51, normalized size = 0.91

$$\frac{1}{12}\left(3x^4+9x^2+9\left(4x^2-7\right)\sin(x)+9x\sin(2x)+\sin(3x)\right)+6x\cos(x)+\frac{3}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cos[x])^3,x]
```

```
[Out] 6*x*Cos[x] + (3*Cos[2*x])/8 + (9*x^2 + 3*x^4 + 9*(-7 + 4*x^2)*Sin[x] + 9*x*
Sin[2*x] + Sin[3*x])/12
```

Maple [A] time = 0.032, size = 57, normalized size = 1.

$$\frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}+3x\left(1/2\cos(x)\sin(x)+x/2\right)-\frac{3x^2}{4}-\frac{3(\sin(x))^2}{4}+3x^2\sin(x)-6\sin(x)+6x\cos(x)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cos(x))^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)+3*x*(1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2-3/4*sin(x)^2+
3*x^2*sin(x)-6*sin(x)+6*x*cos(x)+1/4*x^4
```

Maxima [A] time = 1.1555, size = 62, normalized size = 1.11

$$\frac{1}{4}x^4-\frac{1}{3}\sin(x)^3+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}x\sin(2x)+3\left(x^2-2\right)\sin(x)+\frac{3}{8}\cos(2x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/3*sin(x)^3 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3*(x^2 - 2
)*sin(x) + 3/8*cos(2*x) + sin(x)
```

Fricas [A] time = 2.32278, size = 135, normalized size = 2.41

$$\frac{1}{4}x^4+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}\cos(x)^2+\frac{1}{6}\left(18x^2+9x\cos(x)+2\cos(x)^2-32\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*cos(x)^2 + 1/6*(18*x^2 + 9*x*cos(x) +
2*cos(x)^2 - 32)*sin(x)
```

Sympy [A] time = 0.5201, size = 85, normalized size = 1.52

$$\frac{x^4}{4} + \frac{3x^2\sin^2(x)}{4} + 3x^2\sin(x) + \frac{3x^2\cos^2(x)}{4} + \frac{3x\sin(x)\cos(x)}{2} + 6x\cos(x) + \frac{2\sin^3(x)}{3} + \sin(x)\cos^2(x) - 6\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*sin(x) + 3*x**2*cos(x)**2/4 + 3*x*sin(
x)*cos(x)/2 + 6*x*cos(x) + 2*sin(x)**3/3 + sin(x)*cos(x)**2 - 6*sin(x) + 3*
cos(x)**2/4
```

Giac [A] time = 1.12398, size = 62, normalized size = 1.11

$$\frac{1}{4}\,x^4 + \frac{3}{4}\,x^2 + 6\,x\cos(x) + \frac{3}{4}\,x\sin(2\,x) + \frac{3}{4}\left(4\,x^2 - 7\right)\sin(x) + \frac{3}{8}\,\cos(2\,x) + \frac{1}{12}\,\sin(3\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3/4*(4*x^2 - 7)*sin(x) +
3/8*cos(2*x) + 1/12*sin(3*x)
```

3.46 $\int \frac{\cos(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}+$$

```
[Out] (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*
Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])
/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a + (b*Sqrt[-c])/Sqrt[d]]*SinI
ntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a - (b*Sqr
t[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d
])
```

Rubi [A] time = 0.3079, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3334, 3303, 3299, 3302}

$$\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*
Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])
/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a + (b*Sqrt[-c])/Sqrt[d]]*SinI
ntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a - (b*Sqr
t[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d
])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\cos(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\cos(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$= -\frac{\int \frac{\cos(a+bx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\int \frac{\cos(a+bx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} + \frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}}{2\sqrt{-c}}$$

$$= \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} +$$

Mathematica [C] time = 0.302956, size = 172, normalized size = 0.81

$$-\frac{i\left(\cos\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{CosIntegral}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)-\cos\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{CosIntegral}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+\sin\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right.\right.}{2\sqrt{c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-I/2)*(Cos[a + (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[b*(((-I)*Sqrt[c])/Sqrt[
d] + x)] - Cos[a - (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[b*((I*Sqrt[c])/Sqrt[d
] + x)] + Sin[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[b*((I*Sqrt[c])/Sqrt[d]
 + x)] + Sin[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(I*b*Sqrt[c])/Sqrt[d] -
 b*x]))/(Sqrt[c]*Sqrt[d])
```

Maple [A] time = 0.016, size = 229, normalized size = 1.1

$$b\left(\frac{1}{2d}\left(-\mathrm{Si}\left(bx+a-\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)\sin\left(\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)+\mathrm{Ci}\left(bx+a-\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)\cos\left(\frac{1}{d}\left(b\sqrt{-cd}+ad\right)\right)\right)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x^2+c),x)
```

```
[Out] b*(1/2/d/((b*(-c*d)^(1/2)+a*d)/d-a)*(-Si(b*x+a-(b*(-c*d)^(1/2)+a*d)/d)*sin(
(b*(-c*d)^(1/2)+a*d)/d)+Ci(b*x+a-(b*(-c*d)^(1/2)+a*d)/d)*cos((b*(-c*d)^(1/2
)+a*d)/d))+1/2/d/(-(b*(-c*d)^(1/2)-a*d)/d-a)*(Si(b*x+a+(b*(-c*d)^(1/2)-a*d)
/d)*sin((b*(-c*d)^(1/2)-a*d)/d)+Ci(b*x+a+(b*(-c*d)^(1/2)-a*d)/d)*cos((b*(-c
*d)^(1/2)-a*d)/d)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(d*x^2 + c), x)
```

Fricas [C] time = 2.49606, size = 398, normalized size = 1.87

$$\frac{2i\sqrt{\frac{b^2c}{d}}\mathrm{Ei}\left(ibx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(ia+\sqrt{\frac{b^2c}{d}}\right)}-2i\sqrt{\frac{b^2c}{d}}\mathrm{Ei}\left(ibx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(ia-\sqrt{\frac{b^2c}{d}}\right)}-2i\sqrt{\frac{b^2c}{d}}\mathrm{Ei}\left(-ibx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(-ia+\sqrt{\frac{b^2c}{d}}\right)}+2i\sqrt{\frac{b^2c}{d}}}{8bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) -
2*I*sqrt(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) - 2*I*s
qrt(b^2*c/d)*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) + 2*I*sqrt
(b^2*c/d)*Ei(-I*b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x^2 + c), x)
```

3.47 $\int \frac{\cos(a+bx)}{c+dx+ex^2} dx$

Optimal. Leaf size=271

$$\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a\right.}{}$$

```
[Out] (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 -
4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt[d^2 - 4*c*
e]))/(2*e)]*CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2
- 4*c*e] - (Sin[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d -
Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] + (Sin[a - (b*(d + Sqrt
[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]
)/Sqrt[d^2 - 4*c*e]
```

Rubi [A] time = 0.56233, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6728, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x + e*x^2),x]
```


```
[Out] (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 -
4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt[d^2 - 4*c*
e]))/(2*e)]*CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2
- 4*c*e] - (Sin[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d -
Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] + (Sin[a - (b*(d + Sqrt
[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]
)/Sqrt[d^2 - 4*c*e]
```


Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx &= \int \left(\frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)}-\frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx\\
&= \frac{(2e)\int \frac{\cos(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}-\frac{(2e)\int \frac{\cos(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}\\
&= \frac{\left(2e\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cos\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}-\frac{\left(2e\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cos\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d+\sqrt{d^2-4ce}+2ex}
\\
&= \frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Ci}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin}{}
\end{aligned}
$$

Mathematica [A] time = 0.541585, size = 236, normalized size = 0.87

$$
\frac{\cos\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\mathrm{CosIntegral}\left(\frac{b\left(-\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right)-\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right)+\sin\left(a\right.}{\sqrt{d^2-4ce}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cos[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)] - Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIn
tegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] + Sin[a + (b*(-d + Sqrt[d
^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x]
+ Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)])/Sqrt[d^2 - 4*c*e]
```

Maple [A] time = 0.017, size = 320, normalized size = 1.2

$$b\left(\left(-\mathrm{Si}\left(bx+a-\frac{1}{2e}\left(2ae-db+\sqrt{-4b^2ce+b^2d^2}\right)\right)\sin\left(\frac{1}{2e}\left(2ae-db+\sqrt{-4b^2ce+b^2d^2}\right)\right)+\mathrm{Ci}\left(bx+a-\frac{1}{2e}\left(2ae-d\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(b*x+a-1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2
*d^2)^(1/2)))*sin(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(b*x+a-1/
2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))*cos(1/2/e*(2*a*e-d*b+(-4*b^2*c*
e+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(b*x+a+1/2*(-2*a*e+d*b+
(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/
2))/e)+Ci(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(1/2*(-2*
a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)
```

Fricas [C] time = 2.64289, size = 963, normalized size = 3.55

$$-\frac{-i e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\,ae+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)}+i\,e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\cdots}{\cdots}\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(-I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e*
sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b^2
*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-
2*I*b*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d -
2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2 - 4*b
^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)
)/e)*e^(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - I*
e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sqrt(-(b^2
*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*
b^2*c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)
```

3.48 $\int \frac{x \cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=10

$$\sin\left(\sqrt{x^2+1}\right)$$

```
[Out] Sin[Sqrt[1 + x^2]]
```

Rubi [A] time = 0.135869, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6715, 3432, 15, 2637}

$$\sin\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] Sin[Sqrt[1 + x^2]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
```

```
&& !IntegerQ[m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\cos\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx, x, x^2\right)\\ &= \text{Subst}\left(\int \frac{x\cos(x)}{\sqrt{x^2}}\,dx, x, \sqrt{1+x^2}\right)\\ &= 1\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{1+x^2}\right)\\ &= \sin\left(\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0324673, size = 10, normalized size = 1.

$$\sin\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] Sin[Sqrt[1 + x^2]]
```

Maple [A] time = 0.011, size = 9, normalized size = 0.9

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

```
[Out] sin((x^2+1)^(1/2))
```

Maxima [A] time = 1.12438, size = 11, normalized size = 1.1

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sin(sqrt(x^2 + 1))
```

Fricas [A] time = 2.30427, size = 27, normalized size = 2.7

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sin(sqrt(x^2 + 1))
```

Sympy [A] time = 0.902063, size = 8, normalized size = 0.8

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

```
[Out] sin(sqrt(x**2 + 1))
```

Giac [A] time = 1.09923, size = 11, normalized size = 1.1

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sin(sqrt(x^2 + 1))
```

3.49 $\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}}$$

```
[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]
```

Rubi [A] time = 0.189288, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6715, 3432, 15, 2637}

$$\frac{\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]
```

```
[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\cos\left(\sqrt{3}\sqrt{2+x}\right)}{\sqrt{2+x}}\,dx, x, x^2\right)\\ &= \mathrm{Subst}\left(\int \frac{x\cos\left(\sqrt{3}x\right)}{\sqrt{x^2}}\,dx, x, \sqrt{2+x^2}\right)\\ &= 1\,\mathrm{Subst}\left(\int \cos\left(\sqrt{3}x\right)\,dx, x, \sqrt{2+x^2}\right)\\ &= \frac{\sin\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0526874, size = 22, normalized size = 1.

$$\frac{\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]
```

```
[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]
```

Maple [A] time = 0.015, size = 18, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x)
```

```
[Out] 1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.59016, size = 23, normalized size = 1.05

$$\frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

Fricas [B] time = 2.33569, size = 112, normalized size = 5.09

$$\frac{2\sqrt{3}\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)}{3\left(\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*tan(1/2*sqrt(3)*sqrt(x^2 + 2))/(tan(1/2*sqrt(3)*sqrt(x^2 + 2))^
2 + 1)
```

Sympy [A] time = 1.58456, size = 20, normalized size = 0.91

$$\frac{\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3**(1/2)*(x**2+2)**(1/2))/(x**2+2)**(1/2),x)
```

```
[Out] sqrt(3)*sin(sqrt(3)*sqrt(x**2 + 2))/3
```

Giac [A] time = 1.09903, size = 23, normalized size = 1.05

$$\frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

3.50 $\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx$

Optimal. Leaf size=24

$$\frac{1}{6}\sin\left(\sqrt{3}\sqrt{(2x-1)^2+2}\right)$$

```
[Out] Sin[Sqrt[3]*Sqrt[2 + (-1 + 2*x)^2]]/6
```

Rubi [A] time = 0.493079, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6715, 3432, 15, 2637}

$$\frac{1}{6}\sin\left(\sqrt{3}\sqrt{(2x-1)^2+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^2],x]
```

```
[Out] Sin[Sqrt[3]*Sqrt[2 + (-1 + 2*x)^2]]/6
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
```

```
&& !IntegerQ[m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x\cos\left(\sqrt{6+3x^2}\right)}{\sqrt{6+3x^2}}\,dx, x, -1+2x\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{\cos\left(\sqrt{6+3x}\right)}{\sqrt{6+3x}}\,dx, x, (-1+2x)^2\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{x\cos(x)}{\sqrt{x^2}}\,dx, x, \sqrt{3}\sqrt{2+(-1+2x)^2}\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{3}\sqrt{2+(-1+2x)^2}\right)\\
&= \frac{1}{6}\sin\left(\sqrt{3}\sqrt{2+(-1+2x)^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.15551, size = 20, normalized size = 0.83

$$\frac{1}{6}\sin\left(\sqrt{3(1-2x)^2+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^
2],x]
```

```
[Out] Sin[Sqrt[6 + 3*(1 - 2*x)^2]]/6
```

Maple [A] time = 0.025, size = 16, normalized size = 0.7

$$\frac{1}{6}\sin\left(\sqrt{12\,x^2-12\,x+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x)
```

```
[Out] 1/6*sin((12*x^2-12*x+9)^(1/2))
```

Maxima [A] time = 1.13861, size = 22, normalized size = 0.92

$$\frac{1}{6}\sin\left(\sqrt{3(2x-1)^2+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="maxima")
```

```
[Out] 1/6*sin(sqrt(3*(2*x - 1)^2 + 6))
```

Fricas [A] time = 2.1526, size = 46, normalized size = 1.92

$$\frac{1}{6}\sin\left(\sqrt{12x^2-12x+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/6*sin(sqrt(12*x^2 - 12*x + 9))
```

Sympy [A] time = 7.19953, size = 15, normalized size = 0.62

$$\frac{\sin\left(\sqrt{3(2x-1)^2+6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)**2)**(1/2))/(6+3*(-1+2*x)**2)**(1/2),x
)
```

```
[Out] sin(sqrt(3*(2*x - 1)**2 + 6))/6
```

Giac [A] time = 1.09801, size = 26, normalized size = 1.08

$$\frac{1}{6}\sin\left(\sqrt{3}\sqrt{4x^2-4x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="giac")
```

```
[Out] 1/6*sin(sqrt(3)*sqrt(4*x^2 - 4*x + 3))
```

3.51 $\int \cos\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$-\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*CosIntegral[(b*c - a*
d)/(d*(c + d*x))]*Sin[b/d])/d^2 + ((b*c - a*d)*Cos[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.132663, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4564, 3297, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*CosIntegral[(b*c - a*
d)/(d*(c + d*x))]*Sin[b/d])/d^2 + ((b*c - a*d)*Cos[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rule 4564

```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Cos[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sin\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad)\sin\left(\frac{b}{d}\right)\right)\text{Su}}{}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)}{d^2} + \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 5.1352, size = 260, normalized size = 2.57

$$\frac{-4\sin\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{ad-bc}{d(c+dx)}\right) + d\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)\left(2c\left(e^{2i\left(\frac{a}{c+dx}+\frac{b}{d}\right)} + e^{\frac{2ibc}{d(c+dx)}}\right) + dx\left(1+e^{\frac{2ib}{d}}\right)\left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+d}}\right.\right.\right.}{4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[(a + b*x)/(c + d*x)],x]
```

```
[Out] (-4*(b*c - a*d)*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d] + (d*(2*
c*(E^(((2*I)*b*c)/(d*(c + d*x))) + E^((2*I)*(b/d + a/(c + d*x)))) + d*(1 +
E^(((2*I)*b)/d))*(E^(((2*I)*a)/(c + d*x)) + E^(((2*I)*b*c)/(d*(c + d*x))))*
x - 4*d*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*Sin[b/d]*Sin[(-(b*c)
+ a*d)/(d*(c + d*x))]))/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 4*(b*
c - a*d)*Cos[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))])/(4*d^2)
```

Maple [A] time = 0.016, size = 142, normalized size = 1.4

$$-(ad-cb)\left(-\frac{1}{d}\cos\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)\left(d\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)-b\right)^{-1}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)+\frac{1}{d}\mathrm{Ci}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)/(d*x+c)),x)
```

```
[Out] -(a*d-b*c)*(-cos(b/d+(a*d-b*c)/d/(d*x+c))/(d*(b/d+(a*d-b*c)/d/(d*x+c))-b)/d
-(Si((a*d-b*c)/d/(d*x+c))*cos(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*sin(b/d)/d)/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos((b*x + a)/(d*x + c)), x)
```

Fricas [A] time = 2.47045, size = 323, normalized size = 3.2

$$-\frac{2(bc-ad)\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-2\left(d^2x+cd\right)\cos\left(\frac{bx+a}{dx+c}\right)+\left((bc-ad)\,\mathrm{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\,\mathrm{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\sin\left(\frac{b}{d}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*c - a*d)*cos(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - 2*(
d^2*x + c*d)*cos((b*x + a)/(d*x + c)) + ((b*c - a*d)*cos_integral((b*c - a*
d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c*d)))*s
in(b/d))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos((b*x + a)/(d*x + c)), x)
```

3.52 $\int \cos^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$-\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*CosIntegral[(2*(b*c
 - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 + ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.160648, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4564, 3313, 12, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*CosIntegral[(2*(b*c
 - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 + ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 4564

```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Cos[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\cos^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad))\,\text{Subst}\left(\int -\frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad)\sin\left(\frac{2\ldots}{\ldots}\right.\right.}{\ldots}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 6.11297, size = 400, normalized size = 3.74

$$\frac{\left(acd-bc^2\right)\left(\frac{\left(-1+e^{\frac{4ib}{d}}\right)\left(e^{\frac{4ibc}{d(c+dx)}}-e^{\frac{4ia}{c+dx}}\right)\exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)}{8(bc-ad)} - \frac{\left(1+e^{\frac{4ib}{d}}\right)\left(e^{\frac{4ia}{c+dx}}+e^{\frac{4ibc}{d(c+dx)}}\right)\exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)}{8(bc-ad)}\right)}{d} + \frac{2ad\sin\left(\frac{2b}{d}\right)\text{CosIntegr}\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((-(b*c^2) + a*c*d)*(((-1 + E^(((4*I)*b)/d))*(-E^(((4*I)*a)/(c + d*x)) + E^
(((4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d*x
))/(d*(c + d*x)))) - ((1 + E^(((4*I)*b)/d))*(E^(((4*I)*a)/(c + d*x)) + E^((
(4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d*x))
/(d*(c + d*x))))))/d + (x*Cos[(2*b)/d]*Cos[(2*(-(b*c) + a*d))/(d*(c + d*x))
])/2 - (x*Sin[(2*b)/d]*Sin[(2*(-(b*c) + a*d))/(d*(c + d*x))])/2 + (d^2*x -
2*b*c*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] + 2*a*d*Co
sIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] - 2*b*c*Cos[(2*b)/
d]*SinIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))] + 2*a*d*Cos[(2*b)/d]*SinIn
tegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

Maple [A] time = 0.02, size = 195, normalized size = 1.8

$$-\frac{ad-cb}{d^2}\left(\frac{d^2}{4}\left(-2\,\frac{1}{d}\cos\left(2\,\frac{ad-cb}{d\,(dx+c)}+2\,\frac{b}{d}\right)\left(d\left(\frac{b}{d}+\frac{ad-cb}{d\,(dx+c)}\right)-b\right)^{-1}-2\,\frac{1}{d}\left(2\,\frac{1}{d}\mathrm{Si}\left(2\,\frac{ad-cb}{d\,(dx+c)}\right)\cos\left(2\,\frac{b}{d}\right)+2\,\frac{1}{d}\mathrm{Ci}\left(2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/d^2*(a*d-b*c)*(1/4*d^2*(-2*cos(2*(a*d-b*c)/d/(d*x+c)+2*b/d)/(d*(b/d+(a*d
-b*c)/d/(d*x+c))-b)/d-2*(2*Si(2*(a*d-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*(a
*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/d)-1/2*d/(d*(b/d+(a*d-b*c)/d/(d*x+c))-b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,x+\frac{1}{2}\int\cos\left(\frac{2\,(bx+a)}{dx+c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

Fricas [A] time = 2.583, size = 338, normalized size = 3.16

$$\frac{2\left(d^2x+cd\right)\cos\left(\frac{bx+a}{dx+c}\right)^2-2\,(bc-ad)\cos\left(\frac{2\,b}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)-\left((bc-ad)\,\mathrm{Ci}\left(\frac{2\,(bc-ad)}{d^2x+cd}\right)+(bc-ad)\,\mathrm{Ci}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\right)\sin\left(\frac{2\,b}{d}\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 - 2*(b*c - a*d)*cos(2*b/d)*
sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) - ((b*c - a*d)*cos_integral(2*(b
*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-2*(b*c - a*d)/(d^2*x +
```

```
c*d)))*sin(2*b/d))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\frac{bx+a}{dx+c}\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos((b*x + a)/(d*x + c))^2, x)
```

3.53 $\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] (-3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CosIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rubi [A] time = 0.110671, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3302}

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CosIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x}+\frac{\cos(3x)}{4x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}\\
&= -\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0235487, size = 53, normalized size = 0.91

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] -(3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + CosIntegral[(3*Sqrt[1 - a*x]
)/Sqrt[1 + a*x]])/(4*a)
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cos\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cos^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.54 $\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]]/(2*a)
```

Rubi [A] time = 0.0799495, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3302}

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]]/(2*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cos(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} \\
&= -\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0381715, size = 51, normalized size = 0.88

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + Log[Sqrt[1 - a*x]/Sqrt[1 +
 a*x]])/(2*a)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cos\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\int\frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx+a\int\frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{\left(a^2x^2-1\right)\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2+\left(a^2x^2-1\right)\sin\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx-\log\left(ax+1\right)+\log\left(ax-1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1),
x) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*c
os(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/s
qrt(a*x + 1))^2), x) - log(a*x + 1) + log(a*x - 1))/a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.55 $\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi [A] time = 0.0377898, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3302}

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.0064529, size = 26, normalized size = 1.

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\cos\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.56 $\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=39

$$\text{Unintegrable}\left(\frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.0375202, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sec[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sec(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 2.8971, size = 0, normalized size = 0.

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cos\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="ma
xima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="fr
icas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^2x^2\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cos(sqrt(-a*x +
1)/sqrt(a*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="gi
ac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.57 $\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=41

$$\text{Unintegrable}\left(\frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.0842014, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sec[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a
)
```

Rubi steps

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sec^2(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 10.9928, size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cos\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2x^2-1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.58 $\int \frac{\tan(\sqrt{x})}{\sqrt{x}} \, dx$

Optimal. Leaf size=9

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

```
[Out] -2*Log[Cos[Sqrt[x]]]
```

Rubi [A] time = 0.0097141, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3747, 3475}

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[Sqrt[x]]/Sqrt[x],x]

[Out] -2*Log[Cos[Sqrt[x]]]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}} \, dx &= 2\,\text{Subst}\left(\int \tan(x) \, dx, x, \sqrt{x}\right)\\ &= -2\log\left(\cos\left(\sqrt{x}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0120247, size = 9, normalized size = 1.

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Log[Cos[Sqrt[x]]]
```

Maple [A] time = 0.003, size = 8, normalized size = 0.9

$$-2\ln\left(\cos\left(\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*ln(cos(x^(1/2)))
```

Maxima [A] time = 1.01361, size = 9, normalized size = 1.

$$2\log\left(\sec\left(\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*log(sec(sqrt(x)))
```

Fricas [A] time = 2.3592, size = 41, normalized size = 4.56

$$-\log\left(\frac{1}{\tan\left(\sqrt{x}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -log(1/(tan(sqrt(x))^2 + 1))
```

Sympy [A] time = 0.991418, size = 10, normalized size = 1.11

$$\log\left(\tan^2\left(\sqrt{x}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x**(1/2))/x**(1/2),x)
```

```
[Out] log(tan(sqrt(x))**2 + 1)
```

Giac [A] time = 1.10323, size = 11, normalized size = 1.22

$$-2\log\left(\left|\cos\left(\sqrt{x}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(cos(sqrt(x))))
```

3.59 $\int \frac{\tan^2(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=16

$$2\tan\left(\sqrt{x}\right) - 2\sqrt{x}$$

```
[Out] -2*Sqrt[x] + 2*Tan[Sqrt[x]]
```

Rubi [A] time = 0.0178689, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3747, 3473, 8}

$$2\tan\left(\sqrt{x}\right) - 2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] + 2*Tan[Sqrt[x]]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \tan^2(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\tan\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.034028, size = 18, normalized size = 1.12

$$2\tan\left(\sqrt{x}\right) - 2\tan^{-1}\left(\tan\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*ArcTan[Tan[Sqrt[x]]] + 2*Tan[Sqrt[x]]
```

Maple [A] time = 0.007, size = 13, normalized size = 0.8

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))^2/x^(1/2),x)
```

```
[Out] -2*x^(1/2)+2*tan(x^(1/2))
```

Maxima [A] time = 1.47961, size = 16, normalized size = 1.

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

Fricas [A] time = 2.28983, size = 39, normalized size = 2.44

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

Sympy [A] time = 1.02012, size = 14, normalized size = 0.88

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x**(1/2))**2/x**(1/2),x)
```

```
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

Giac [A] time = 1.11438, size = 16, normalized size = 1.

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

3.60 $\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=70

$$2i\sqrt{x}\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)-\text{PolyLog}\left(3,-e^{2i\sqrt{x}}\right)+\frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)$$

```
[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[
2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

Rubi [A] time = 0.0912281, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3747, 3719, 2190, 2531, 2282, 6589}

$$2i\sqrt{x}\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)-\text{PolyLog}\left(3,-e^{2i\sqrt{x}}\right)+\frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Tan[Sqrt[x]],x]
```

```
[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[
2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx &= 2\,\text{Subst}\left(\int x^2\tan(x)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2} - 4i\,\text{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2} - 2x\log\left(1+e^{2i\sqrt{x}}\right) + 4\,\text{Subst}\left(\int x\log\left(1+e^{2ix}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2} - 2x\log\left(1+e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right) - 2i\,\text{Subst}\left(\int \text{Li}_2\left(-e^{2ix}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2} - 2x\log\left(1+e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right) - \text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x}\,dx, x, e^{2i\sqrt{x}}\right)\\
&= \frac{2}{3}ix^{3/2} - 2x\log\left(1+e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right) - \text{Li}_3\left(-e^{2i\sqrt{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0205173, size = 70, normalized size = 1.

$$2i\sqrt{x}\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)-\text{PolyLog}\left(3,-e^{2i\sqrt{x}}\right)+\frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Tan[Sqrt[x]],x]
```

```
[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[
2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int\sqrt{x}\tan\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*tan(x^(1/2)),x)
```

```
[Out] int(x^(1/2)*tan(x^(1/2)),x)
```

Maxima [A] time = 1.52134, size = 108, normalized size = 1.54

$$-2i\,x\arctan\left(\sin\left(2\sqrt{x}\right),\cos\left(2\sqrt{x}\right)+1\right)-x\log\left(\cos\left(2\sqrt{x}\right)^2+\sin\left(2\sqrt{x}\right)^2+2\cos\left(2\sqrt{x}\right)+1\right)+\frac{2}{3}i\,x^{\frac{3}{2}}+2i\sqrt{x}\text{Li}_2\left(-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*I*x*arctan2(sin(2*sqrt(x)), cos(2*sqrt(x)) + 1) - x*log(cos(2*sqrt(x))^2
 + sin(2*sqrt(x))^2 + 2*cos(2*sqrt(x)) + 1) + 2/3*I*x^(3/2) + 2*I*sqrt(x)*d
ilog(-e^(2*I*sqrt(x))) - polylog(3, -e^(2*I*sqrt(x)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x}\tan\left(\sqrt{x}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)*tan(sqrt(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\tan\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*tan(x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)*tan(sqrt(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*tan(sqrt(x)), x)
```

3.61 $\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)dx$

Optimal. Leaf size=19

$$-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

```
[Out] -Log[Cos[a + b*x + c*x^2]]/(2*c)
```

Rubi [A] time = 0.0176489, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {3763}

$$-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]

[Out] -Log[Cos[a + b*x + c*x^2]]/(2*c)
```

Rule 3763

```
Int[((d_.) + (e_.)*(x_))*Tan[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Log[Cos[a + b*x + c*x^2]])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c)
, Int[Tan[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*
d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)dx&=\frac{b\int\tan\left(a+bx+cx^2\right)dx}{2c}+\int x\tan\left(a+bx+cx^2\right)dx\\&=-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.676132, size = 18, normalized size = 0.95

$$-\frac{\log(\cos(a+x(b+cx)))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]
```

```
[Out] -Log[Cos[a + x*(b + c*x)]]/(2*c)
```

Maple [A] time = 0.03, size = 18, normalized size = 1.

$$-\frac{\ln\left(\cos\left(cx^2+bx+a\right)\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x)
```

```
[Out] -1/2*ln(cos(c*x^2+b*x+a))/c
```

Maxima [B] time = 1.20718, size = 112, normalized size = 5.89

$$-\frac{\log\left(\cos\left(2\,cx^2\right)^2+2\,\cos\left(2\,cx^2\right)\cos\left(2\,bx+2\,a\right)+\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,cx^2\right)^2-2\,\sin\left(2\,cx^2\right)\sin\left(2\,bx+2\,a\right)+\sin\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="maxima"
)
```

```
[Out] -1/4*log(cos(2*c*x^2)^2 + 2*cos(2*c*x^2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a
)^2 + sin(2*c*x^2)^2 - 2*sin(2*c*x^2)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2
)/c
```

Fricas [A] time = 2.37484, size = 59, normalized size = 3.11

$$-\frac{\log\left(\frac{1}{\tan\left(cx^2+bx+a\right)^2+1}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="fricas"
)
```

```
[Out] -1/4*log(1/(tan(c*x^2 + b*x + a)^2 + 1))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int b\tan\left(a+bx+cx^2\right)dx+\int 2cx\tan\left(a+bx+cx^2\right)dx}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x**2+b*x+a)/c+x*tan(c*x**2+b*x+a),x)
```

```
[Out] (Integral(b*tan(a + b*x + c*x**2), x) + Integral(2*c*x*tan(a + b*x + c*x**2
), x))/(2*c)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\tan\left(cx^2+bx+a\right)+\frac{b\tan\left(cx^2+bx+a\right)}{2\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*tan(c*x^2 + b*x + a) + 1/2*b*tan(c*x^2 + b*x + a)/c, x)
```

3.62 $\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=16

$$-2\sqrt{x}-2\cot\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x] - 2*Cot[Sqrt[x]]
```

Rubi [A] time = 0.0184914, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3748, 3473, 8}

$$-2\sqrt{x}-2\cot\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] - 2*Cot[Sqrt[x]]
```

Rule 3748

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \cot^2(x)\,dx, x, \sqrt{x}\right)$$
$$= -2\cot\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

Mathematica [C] time = 0.0443974, size = 26, normalized size = 1.62

$$-2\cot\left(\sqrt{x}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*Cot[Sqrt[x]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[Sqrt[x]]^2]
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$-2\cot\left(\sqrt{x}\right) + \pi - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))^2/x^(1/2),x)
```

```
[Out] -2*cot(x^(1/2))+Pi-2*x^(1/2)
```

Maxima [A] time = 1.49848, size = 19, normalized size = 1.19

$$-2\sqrt{x} - \frac{2}{\tan\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x) - 2/tan(sqrt(x))
```

Fricas [B] time = 2.38831, size = 88, normalized size = 5.5

$$-\frac{2\left(\sqrt{x}\sin\left(2\sqrt{x}\right)+\cos\left(2\sqrt{x}\right)+1\right)}{\sin\left(2\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(x)*sin(2*sqrt(x)) + cos(2*sqrt(x)) + 1)/sin(2*sqrt(x))
```

Sympy [A] time = 0.709173, size = 15, normalized size = 0.94

$$-2\sqrt{x}-2\cot\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x**(1/2))**2/x**(1/2),x)
```

```
[Out] -2*sqrt(x) - 2*cot(sqrt(x))
```

Giac [A] time = 1.12142, size = 30, normalized size = 1.88

$$-2\sqrt{x}-\frac{1}{\tan\left(\frac{1}{2}\sqrt{x}\right)}+\tan\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(x) - 1/tan(1/2*sqrt(x)) + tan(1/2*sqrt(x))
```

3.63 $\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{a+b\sec(c+dx)}E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}}$$

```
[Out] (EllipticE[ArcSin[Tan[c + d*x]/(1 + Sec[c + d*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(a + b*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))])
```

Rubi [A] time = 0.164096, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2829, 3968}

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{a+b\sec(c+dx)}E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[c + d*x]/(1 + Sec[c + d*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(a + b*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))])
```

Rule 2829

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_)*((a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(m_.), x_Symbol] :> Int[((b + a*Csc[e + f*x])^m*(c + d*Csc[e +
f*x])^n)/Csc[e + f*x]^m, x] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !Integer
Q[n] && IntegerQ[m]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
```

```
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{1+\sec(c+dx)}\,dx$$
$$= \frac{E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}}$$

Mathematica [A] time = 1.56502, size = 85, normalized size = 0.92

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{a+b\sec(c+dx)}E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x
])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))])
```

Maple [A] time = 0.305, size = 150, normalized size = 1.6

$$-\frac{(-a-b)(\cos(dx+c)-1)(1+\cos(dx+c))^2}{d(a\cos(dx+c)+b)(\sin(dx+c))^2}\mathrm{EllipticE}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{a\cos(dx+c)+b}{(a+b)(1+\cos(dx+c))}}\sqrt{\frac{\cos}{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x)
```

```
[Out] -1/d*(-a-b)*EllipticE((cos(d*x+c)-1)/sin(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+
b)*(a*cos(d*x+c)+b)/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)-1)*((a*cos(d*x+c)+b)/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(a*cos
(d*x+c)+b)/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos(c+dx)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/(1+cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/(cos(c + d*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

3.64 $\int \sec(a+bx)\sec(2a+2bx)\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

Rubi [A] time = 0.0333557, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4364, 1093, 207}

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Sec[2*a + 2*b*x],x]
```



```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```


Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec(a+bx)\sec(2a+2bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sin(a+bx)\right)}{b} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(a+bx)\right)}{b} \\
&= -\frac{\tanh^{-1}(\sin(a+bx))}{b} + \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0369994, size = 35, normalized size = 1.

$$
\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b} - \frac{\tanh^{-1}(\sin(a+bx))}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[2*a + 2*b*x],x]
```

```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

Maple [A] time = 0.045, size = 48, normalized size = 1.4

$$
\frac{\text{Artanh}\left(\sin(bx+a)\sqrt{2}\right)\sqrt{2}}{b} - \frac{\ln(1+\sin(bx+a))}{2b} + \frac{\ln(\sin(bx+a)-1)}{2b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b-1/2/b*ln(1+sin(b*x+a))+1/2/b*ln(sin(b
*x+a)-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.4085, size = 193, normalized size = 5.51

$$\frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^2-2\sqrt{2}\sin(bx+a)-3}{2\cos(bx+a)^2-1}\right)-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(-\sin\left(bx+a\right)+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*
x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(a+bx\right)\sec\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

Giac [B] time = 2.54691, size = 1280, normalized size = 36.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2
*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12
*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*t
an(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*ta
n(1/2*a) - 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*t
an(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)/abs(2*tan(1/2*
b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)
^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
- 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 + 2*
sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*
x + 2*a) + 12*tan(1/2*a) + 2)) + sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1
/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*
b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a
)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^
3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2
*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*ta
n(1/2*a) - 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)
*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*ta
n(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1
/2*a) + 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(
1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)) - 2*log(abs(tan(
1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a
)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a)
 + 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1
/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
 + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

3.65 $\int \sec(a+bx)\sec(2(a+bx))\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

Rubi [A] time = 0.0325854, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4364, 1093, 207}

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Sec[2*(a + b*x)],x]
```

```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec(a+bx)\sec(2(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sin(a+bx)\right)}{b} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(a+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\sin(a+bx))}{b} + \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0245152, size = 35, normalized size = 1.

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b} - \frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[2*(a + b*x)],x]
```

```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

Maple [A] time = 0.002, size = 48, normalized size = 1.4

$$\frac{\text{Artanh}\left(\sin(bx+a)\sqrt{2}\right)\sqrt{2}}{b} - \frac{\ln(1+\sin(bx+a))}{2b} + \frac{\ln(\sin(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b-1/2/b*ln(1+sin(b*x+a))+1/2/b*ln(sin(b
*x+a)-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.48587, size = 193, normalized size = 5.51

$$\frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^2-2\sqrt{2}\sin(bx+a)-3}{2\cos(bx+a)^2-1}\right)-\log(\sin(bx+a)+1)+\log(-\sin(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*
x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(a+bx)\sec(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

Giac [B] time = 2.52323, size = 1280, normalized size = 36.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2
*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12
*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*t
an(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*ta
n(1/2*a) - 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*t
an(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)/abs(2*tan(1/2*
b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)
^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
- 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 + 2*
sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*
x + 2*a) + 12*tan(1/2*a) + 2)) + sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1
/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*
b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a
)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^
3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2
*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*ta
n(1/2*a) - 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)
*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*ta
n(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1
/2*a) + 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(
1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)) - 2*log(abs(tan(
1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a
)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a)
 + 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1
/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
 + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

3.66 $\int \sin(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Rubi [A] time = 0.0088231, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(2x)\,dx = \frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.0052901, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Maple [A] time = 0.01, size = 7, normalized size = 0.5

$$\frac{2 (\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x),x)
```

```
[Out] 2/3*sin(x)^3
```

Maxima [A] time = 0.960009, size = 15, normalized size = 1.

$$-\frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A] time = 2.30363, size = 38, normalized size = 2.53

$$-\frac{2}{3} \left(\cos(x)^2 - 1\right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.549111, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3}+\frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x)
```

```
[Out] -2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

Giac [A] time = 1.12975, size = 15, normalized size = 1.

$$-\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

3.67 $\int \sin(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

[Out] Sin[2*x]/4 - Sin[4*x]/8

Rubi [A] time = 0.0082572, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[3*x],x]

[Out] Sin[2*x]/4 - Sin[4*x]/8

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(3x)\,dx=\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

Mathematica [A] time = 0.0065166, size = 17, normalized size = 1.

$$\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[3*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[4*x]/8
```

Maple [A] time = 0.026, size = 14, normalized size = 0.8

$$\frac{\sin(2x)}{4} - \frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(3*x),x)
```

```
[Out] 1/4*sin(2*x)-1/8*sin(4*x)
```

Maxima [A] time = 0.970992, size = 18, normalized size = 1.06

$$-\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.32462, size = 39, normalized size = 2.29

$$-\left(\cos(x)^3 - \cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -(cos(x)^3 - cos(x))*sin(x)
```

Sympy [A] time = 0.774595, size = 20, normalized size = 1.18

$$-\frac{3\sin(x)\cos(3x)}{8} + \frac{\sin(3x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x)
```

```
[Out] -3*sin(x)*cos(3*x)/8 + sin(3*x)*cos(x)/8
```

Giac [A] time = 1.10946, size = 18, normalized size = 1.06

$$-\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/8*sin(4*x) + 1/4*sin(2*x)
```

3.68 $\int \sin(x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

```
[Out] Sin[3*x]/6 - Sin[5*x]/10
```

Rubi [A] time = 0.0076908, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[4*x],x]

[Out] Sin[3*x]/6 - Sin[5*x]/10
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(4x)\,dx=\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

Mathematica [A] time = 0.0068608, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[4*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[5*x]/10
```

Maple [A] time = 0.029, size = 14, normalized size = 0.8

$$\frac{\sin(3x)}{6} - \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(4*x),x)
```

```
[Out] 1/6*sin(3*x)-1/10*sin(5*x)
```

Maxima [A] time = 1.01932, size = 18, normalized size = 1.06

$$-\frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/10*sin(5*x) + 1/6*sin(3*x)
```

Fricas [A] time = 2.23181, size = 59, normalized size = 3.47

$$-\frac{4}{15}\left(6\cos(x)^4 - 7\cos(x)^2 + 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -4/15*(6*cos(x)^4 - 7*cos(x)^2 + 1)*sin(x)
```

Sympy [A] time = 1.77609, size = 20, normalized size = 1.18

$$-\frac{4\sin(x)\cos(4x)}{15}+\frac{\sin(4x)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x)
```

```
[Out] -4*sin(x)*cos(4*x)/15 + sin(4*x)*cos(x)/15
```

Giac [A] time = 1.10356, size = 18, normalized size = 1.06

$$-\frac{1}{10}\sin(5x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/10*sin(5*x) + 1/6*sin(3*x)
```

3.69 $\int \sin(x)\sin(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((m+1)x)}{2(m+1)}$$

```
[Out] Sin[(1 - m)*x]/(2*(1 - m)) - Sin[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.0308315, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4569, 2637}

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[m*x],x]
```

```
[Out] Sin[(1 - m)*x]/(2*(1 - m)) - Sin[(1 + m)*x]/(2*(1 + m))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin(x)\sin(mx)\,dx &= \int \left(\frac{1}{2}\cos((1-m)x)-\frac{1}{2}\cos((1+m)x)\right)dx\\
&= \frac{1}{2}\int \cos((1-m)x)\,dx-\frac{1}{2}\int \cos((1+m)x)\,dx\\
&= \frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((1+m)x)}{2(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0433652, size = 25, normalized size = 0.71

$$\frac{\cos(x)\sin(mx) - m\sin(x)\cos(mx)}{m^2 - 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[m*x],x]
```

```
[Out] (-(m*Cos[m*x]*Sin[x]) + Cos[x]*Sin[m*x])/(-1 + m^2)
```

Maple [A] time = 0.013, size = 28, normalized size = 0.8

$$\frac{\sin\left((m-1)\,x\right)}{2\,m-2} - \frac{\sin\left((1+m)\,x\right)}{2+2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(m*x),x)
```

```
[Out] 1/2/(m-1)*sin((m-1)*x)-1/2*sin((1+m)*x)/(1+m)
```

Maxima [A] time = 0.968175, size = 38, normalized size = 1.09

$$-\frac{\sin\left((m+1)x\right)}{2\,(m+1)} - \frac{\sin\left(-(m-1)x\right)}{2\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="maxima")
```

```
[Out] -1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

Fricas [A] time = 2.27251, size = 68, normalized size = 1.94

$$-\frac{m\cos\left(mx\right)\sin\left(x\right) - \cos\left(x\right)\sin\left(mx\right)}{m^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="fricas")
```

```
[Out] -(m*cos(m*x)*sin(x) - cos(x)*sin(m*x))/(m^2 - 1)
```

Sympy [A] time = 5.02167, size = 78, normalized size = 2.23

$$\begin{cases} -\frac{x\sin^2(x)}{2} - \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = -1 \\ \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} - \frac{\sin(x)\cos(x)}{2} & \text{for } m = 1 \\ -\frac{m\sin(x)\cos(mx)}{m^2-1} + \frac{\sin(mx)\cos(x)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x)
```

```
[Out] Piecewise((-x*sin(x)**2/2 - x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1)), (x
*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2, Eq(m, 1)), (-m*sin(x)*cos(m
*x)/(m**2 - 1) + sin(m*x)*cos(x)/(m**2 - 1), True))
```

Giac [A] time = 1.11365, size = 39, normalized size = 1.11

$$-\frac{\sin(mx + x)}{2(m + 1)} + \frac{\sin(mx - x)}{2(m - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="giac")
```

```
[Out] -1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

3.70 $\int \cos(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

```
[Out] Cos[x]/2 - Cos[3*x]/6
```

Rubi [A] time = 0.0081673, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sin[x],x]
```

```
[Out] Cos[x]/2 - Cos[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(2x)\sin(x)\,dx=\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Mathematica [A] time = 0.0051229, size = 15, normalized size = 1.

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sin[x],x]
```

```
[Out] Cos[x]/2 - Cos[3*x]/6
```

Maple [A] time = 0.013, size = 12, normalized size = 0.8

$$\frac{\cos(x)}{2} - \frac{\cos(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(x),x)
```

```
[Out] 1/2*cos(x)-1/6*cos(3*x)
```

Maxima [A] time = 0.984009, size = 15, normalized size = 1.

$$-\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*x) + 1/2*cos(x)
```

Fricas [A] time = 2.22676, size = 32, normalized size = 2.13

$$-\frac{2}{3}\cos(x)^3 + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -2/3*cos(x)^3 + cos(x)
```

Sympy [A] time = 1.86378, size = 20, normalized size = 1.33

$$\frac{2\sin(x)\sin(2x)}{3} + \frac{\cos(x)\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x)
```

```
[Out] 2*sin(x)*sin(2*x)/3 + cos(x)*cos(2*x)/3
```

Giac [A] time = 1.10839, size = 15, normalized size = 1.

$$-\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*cos(3*x) + 1/2*cos(x)
```

3.71 $\int \cos(3x)\sin(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] Cos[2*x]/4 - Cos[4*x]/8
```

Rubi [A] time = 0.0081178, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Sin[x],x]
```

```
[Out] Cos[2*x]/4 - Cos[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\sin(x)\,dx=\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Mathematica [A] time = 0.0055286, size = 17, normalized size = 1.

$$\frac{\cos^2(x)}{2}-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Sin[x],x]
```

```
[Out] Cos[x]^2/2 - Cos[4*x]/8
```

Maple [A] time = 0.035, size = 14, normalized size = 0.8

$$\frac{\cos(2x)}{4} - \frac{\cos(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(x),x)
```

```
[Out] 1/4*cos(2*x)-1/8*cos(4*x)
```

Maxima [A] time = 0.985178, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x) + \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) + 1/4*cos(2*x)
```

Fricas [A] time = 2.2507, size = 35, normalized size = 2.06

$$-\cos(x)^4 + \frac{3}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 3/2*cos(x)^2
```

Sympy [A] time = 0.896868, size = 20, normalized size = 1.18

$$\frac{3\sin(x)\sin(3x)}{8} + \frac{\cos(x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x)
```

```
[Out] 3*sin(x)*sin(3*x)/8 + cos(x)*cos(3*x)/8
```

Giac [A] time = 1.14821, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x) + \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/8*cos(4*x) + 1/4*cos(2*x)
```

3.72 $\int \cos(4x)\sin(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

```
[Out] Cos[3*x]/6 - Cos[5*x]/10
```

Rubi [A] time = 0.0080587, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sin[x],x]
```

```
[Out] Cos[3*x]/6 - Cos[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(4x)\sin(x)\,dx = \frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

Mathematica [A] time = 0.0061143, size = 17, normalized size = 1.

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sin[x],x]
```

```
[Out] Cos[3*x]/6 - Cos[5*x]/10
```

Maple [A] time = 0.035, size = 14, normalized size = 0.8

$$\frac{\cos(3x)}{6} - \frac{\cos(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sin(x),x)
```

```
[Out] 1/6*cos(3*x)-1/10*cos(5*x)
```

Maxima [A] time = 1.00624, size = 18, normalized size = 1.06

$$-\frac{1}{10}\cos(5x) + \frac{1}{6}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/6*cos(3*x)
```

Fricas [A] time = 2.28441, size = 53, normalized size = 3.12

$$-\frac{8}{5}\cos(x)^5 + \frac{8}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 8/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.957047, size = 20, normalized size = 1.18

$$\frac{4\sin(x)\sin(4x)}{15} + \frac{\cos(x)\cos(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x)
```

```
[Out] 4*sin(x)*sin(4*x)/15 + cos(x)*cos(4*x)/15
```

Giac [A] time = 1.09016, size = 18, normalized size = 1.06

$$-\frac{1}{10}\cos(5x) + \frac{1}{6}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/6*cos(3*x)
```

3.73 $\int \cos(mx)\sin(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

```
[Out] -Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.0271964, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[m*x]*Sin[x],x]
```

```
[Out] -Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(mx)\sin(x)\,dx &= \int \left(\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((1+m)x)\right)dx\\ &= \frac{1}{2}\int \sin((1-m)x)\,dx+\frac{1}{2}\int \sin((1+m)x)\,dx\\ &= -\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0401192, size = 24, normalized size = 0.69

$$\frac{m\sin(x)\sin(mx)+\cos(x)\cos(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[m*x]*Sin[x],x]
```

```
[Out] (Cos[x]*Cos[m*x] + m*Sin[x]*Sin[m*x])/(-1 + m^2)
```

Maple [A] time = 0.011, size = 28, normalized size = 0.8

$$\frac{\cos\left((m-1)\,x\right)}{2\,m-2}-\frac{\cos\left((1+m)\,x\right)}{2+2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(m*x)*sin(x),x)
```

```
[Out] 1/2*cos((m-1)*x)/(m-1)-1/2*cos((1+m)*x)/(1+m)
```

Maxima [A] time = 0.979889, size = 38, normalized size = 1.09

$$-\frac{\cos\left((m+1)x\right)}{2\,(m+1)}+\frac{\cos\left(-(m-1)x\right)}{2\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos((m + 1)*x)/(m + 1) + 1/2*cos(-(m - 1)*x)/(m - 1)
```

Fricas [A] time = 2.35943, size = 66, normalized size = 1.89

$$\frac{m\sin\left(mx\right)\sin\left(x\right)+\cos\left(mx\right)\cos\left(x\right)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="fricas")
```

```
[Out] (m*sin(m*x)*sin(x) + cos(m*x)*cos(x))/(m^2 - 1)
```

Sympy [A] time = 3.7095, size = 39, normalized size = 1.11

$$\begin{cases} -\frac{\cos^2(x)}{2} & \text{for } m = -1 \vee m = 1 \\ \frac{m\sin(x)\sin(mx)}{m^2-1} + \frac{\cos(x)\cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x)
```

```
[Out] Piecewise((-cos(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sin(x)*sin(m*x)/(m**2 -
1) + cos(x)*cos(m*x)/(m**2 - 1), True))
```

Giac [A] time = 1.14272, size = 39, normalized size = 1.11

$$-\frac{\cos(mx+x)}{2(m+1)} + \frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(m*x + x)/(m + 1) + 1/2*cos(m*x - x)/(m - 1)
```

3.74 $\int \sin(x)\tan(2x)\,dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-\sin(x)$$

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

Rubi [A] time = 0.0230994, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2}{1-2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^2}{1-2x^2}\,dx, x, \sin(x)\right)\\ &= -\sin(x) + \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)\end{aligned}$$

Mathematica [A] time = 0.0134385, size = 20, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$-\sin(x) + \frac{\text{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(2*x),x)
```

```
[Out] -sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.53618, size = 190, normalized size = 9.5

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(
x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sq
rt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*s
qrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - sin(x)
```

Fricas [B] time = 2.36381, size = 109, normalized size = 5.45

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - si
n(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x)
```

```
[Out] Integral(sin(x)*tan(2*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(2*x), x)
```

3.75 $\int \sin(x) \tan(3x) \, dx$

Optimal. Leaf size=47

$$-\sin(x) - \frac{1}{6}\log(1-2\sin(x)) - \frac{1}{6}\log(1-\sin(x)) + \frac{1}{6}\log(\sin(x)+1) + \frac{1}{6}\log(2\sin(x)+1)$$

```
[Out] -Log[1 - 2*Sin[x]]/6 - Log[1 - Sin[x]]/6 + Log[1 + Sin[x]]/6 + Log[1 + 2*Si
n[x]]/6 - Sin[x]
```

Rubi [A] time = 0.0519936, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1279, 1161, 616, 31}

$$-\sin(x) - \frac{1}{6}\log(1-2\sin(x)) - \frac{1}{6}\log(1-\sin(x)) + \frac{1}{6}\log(\sin(x)+1) + \frac{1}{6}\log(2\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[3*x],x]
```

```
[Out] -Log[1 - 2*Sin[x]]/6 - Log[1 - Sin[x]]/6 + Log[1 + Sin[x]]/6 + Log[1 + 2*Si
n[x]]/6 - Sin[x]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sin(x)\tan(3x)\,dx &= \text{Subst}\left(\int \frac{x^2\left(3-4x^2\right)}{1-5x^2+4x^4}\,dx, x, \sin(x)\right)\\
&= -\sin(x)-\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx, x, \sin(x)\right)\\
&= -\sin(x)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx, x, \sin(x)\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx, x, \sin(x)\right)\\
&= -\sin(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \sin(x)\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx, x, \sin(x)\right)+\frac{1}{6}\text{Subst}\Bigg(\int\\
&= -\frac{1}{6}\log(1-2\sin(x))-\frac{1}{6}\log(1-\sin(x))+\frac{1}{6}\log(1+\sin(x))+\frac{1}{6}\log(1+2\sin(x))-\sin(x)
\end{aligned}
$$

Mathematica [A] time = 0.0284489, size = 21, normalized size = 0.45

$$-\sin(x)+\frac{1}{3}\tanh^{-1}(\sin(x))+\frac{1}{3}\tanh^{-1}(2\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[3*x],x]
```

```
[Out] ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - Sin[x]
```

Maple [A] time = 0.094, size = 38, normalized size = 0.8

$$\frac{\ln(1+\sin(x))}{6}-\frac{\ln(\sin(x)-1)}{6}+\frac{\ln(1+2\sin(x))}{6}-\frac{\ln(-1+2\sin(x))}{6}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(3*x),x)
```

```
[Out] 1/6*ln(1+sin(x))-1/6*ln(sin(x)-1)+1/6*ln(1+2*sin(x))-1/6*ln(-1+2*sin(x))-si
n(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(\cos(3x)+\cos(x))\cos(4x)-(\cos(2x)-1)\cos(3x)-\cos(2x)\cos(x)+(\sin(3x)+\sin(x))\sin(4x)-\sin(3x)\sin}{3\left(2(\cos(2x)-1)\cos(4x)-\cos(4x)^2-\cos(2x)^2-\sin(4x)^2+2\sin(4x)\sin(2x)-\sin(2x)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*((cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - co
s(2*x)*cos(x) + (sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)
*sin(x) + cos(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/6*log
(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 - 2*sin(
x) + 1) - sin(x)
```

Fricas [A] time = 2.54101, size = 138, normalized size = 2.94

$$\frac{1}{6}\log(2\sin(x)+1)+\frac{1}{6}\log(\sin(x)+1)-\frac{1}{6}\log(-\sin(x)+1)-\frac{1}{6}\log(-2\sin(x)+1)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*sin(x) + 1) + 1/6*log(sin(x) + 1) - 1/6*log(-sin(x) + 1) - 1/6*lo
g(-2*sin(x) + 1) - sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x)
```

```
[Out] Integral(sin(x)*tan(3*x), x)
```

Giac [B] time = 1.25093, size = 491, normalized size = 10.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="giac")
```

```
[Out] 1/12*(log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 + 8*tan(1/2*x) +
 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - log((tan(1/2*x)^4 -
 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan
(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(t
an(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/
(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*
tan(1/2*x)^2 + 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) - log
((tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(
1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)
/(tan(1/2*x)^2 + 1)) - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1)) - 24*tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

3.76 $\int \sin(x)\tan(4x)\,dx$

Optimal. Leaf size=71

$$-\sin(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/4 - Sin[x]
```

Rubi [A] time = 0.109319, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1279, 1166, 207}

$$-\sin(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/4 - Sin[x]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(4x)\,dx &= \text{Subst}\left(\int \frac{x^2\left(4-8x^2\right)}{1-8x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= -\sin(x)-\frac{1}{8}\text{Subst}\left(\int \frac{-8+32x^2}{1-8x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= -\sin(x)-\left(2-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \sin(x)\right)-\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+\dots}\right.\\ &= \frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)-\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0688958, size = 69, normalized size = 0.97

$$\frac{1}{4}\left(-4\sin(x)+\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]] + Sqrt[2 + Sqrt[2]
]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]] - 4*Sin[x])/4
```

Maple [B] time = 0.166, size = 115, normalized size = 1.6

$$\frac{\left(\sqrt{2}-2\right)\sqrt{2}}{4\sqrt{2-\sqrt{2}}}\text{Artanh}\left(2\frac{\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{\sqrt{2}\sqrt{2+\sqrt{2}}}{4}\text{Artanh}\left(2\frac{\sin(x)}{\sqrt{2+\sqrt{2}}}\right)-\sin(x)+\frac{\sqrt{2}}{4\sqrt{2-\sqrt{2}}}\text{Artanh}\left(2\frac{\sin(x)}{\sqrt{2-\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(4*x),x)
```

```
[Out] 1/4*(2^(1/2)-2)*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2
))+1/4*(2+2^(1/2))^(1/2)*2^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))-sin(x)
+1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-1/4*2^(1
/2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(7x) + \cos(x))\cos(8x) + (\sin(7x) + \sin(x))\sin(8x) + \cos(7x) + \cos(x)}{\cos(8x)^2 + \sin(8x)^2 + 2\cos(8x) + 1}\, dx - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="maxima")
```

```
[Out] integrate(((cos(7*x) + cos(x))*cos(8*x) + (sin(7*x) + sin(x))*sin(8*x) + co
s(7*x) + cos(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x) - sin(x)
```

Fricas [A] time = 2.45544, size = 329, normalized size = 4.63

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2} + 2\sin(x)\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2} - 2\sin(x)\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2} + 2\sin(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(sqrt(2)
+ 2)*log(sqrt(sqrt(2) + 2) - 2*sin(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-s
qrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2
*sin(x)) - sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(4x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(4*x), x)
```

3.77 $\int \sin(x)\tan(5x)\,dx$

Optimal. Leaf size=112

$$-\sin(x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\sin(x)-\sqrt{}\right.$$

```
[Out] ArcTanh[Sin[x]]/5 - ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/20 - ((1 +
Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/20 + ((1 - Sqrt[5])*Log[1 - Sqrt[5] +
 4*Sin[x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]])/20 - Sin[x]
```

Rubi [A] time = 0.169567, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2075, 207, 632, 31}

$$-\sin(x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\sin(x)-\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[5*x],x]
```


```
[Out] ArcTanh[Sin[x]]/5 - ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/20 - ((1 +
Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/20 + ((1 - Sqrt[5])*Log[1 - Sqrt[5] +
 4*Sin[x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]])/20 - Sin[x]
```


Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(5x)\,dx &= \text{Subst}\left(\int \frac{x^2\left(5-20x^2+16x^4\right)}{1-13x^2+28x^4-16x^6}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)dx, x, \sin(x)\right)\\ &= -\sin(x)-\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)-\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \sin(x)\right)+\frac{2}{5}\text{Su}\\ &= \frac{1}{5}\tanh^{-1}(\sin(x))-\sin(x)+\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \sin(x)\right)-\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subs}\\ &= \frac{1}{5}\tanh^{-1}(\sin(x))-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)+\end{aligned}$$

Mathematica [A] time = 0.150762, size = 100, normalized size = 0.89

$$\frac{1}{20}\left(-20\sin(x)+\left(\sqrt{5}-1\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)-\left(\sqrt{5}-1\right)\log\left(4\sin(x)-\sqrt{5}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[5*x],x]
```

```
[Out] (4*ArcTanh[Sin[x]] + (-1 + Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]] - (1 + Sqrt
[5])*Log[1 + Sqrt[5] - 4*Sin[x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Sin[x
]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]] - 20*Sin[x])/20
```

Maple [A] time = 0.141, size = 84, normalized size = 0.8

$$-\frac{\ln\left(4\,(\sin(x))^2-2\,\sin(x)-1\right)}{20}+\frac{\sqrt{5}}{10}\text{Artanh}\left(\frac{(8\,\sin(x)-2)\sqrt{5}}{10}\right)+\frac{\ln(1+\sin(x))}{10}-\frac{\ln(\sin(x)-1)}{10}+\frac{\ln\left(4\,(\sin(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(5*x),x)
```

```
[Out] -1/20*ln(4*sin(x)^2-2*sin(x)-1)+1/10*5^(1/2)*arctanh(1/10*(8*sin(x)-2)*5^(1
/2))+1/10*ln(1+sin(x))-1/10*ln(sin(x)-1)+1/20*ln(4*sin(x)^2+2*sin(x)-1)+1/1
0*5^(1/2)*arctanh(1/10*(8*sin(x)+2)*5^(1/2))-sin(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="maxima")
```

```
[Out] integrate(-1/5*((3*cos(7*x) - cos(5*x) - cos(3*x) + 3*cos(x))*cos(8*x) - 3*
(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(7*x) + (cos(5*x) + cos(3*x) - 3*co
s(x))*cos(6*x) - (cos(4*x) - cos(2*x) + 1)*cos(5*x) - (cos(3*x) - 3*cos(x))
*cos(4*x) + (cos(2*x) - 1)*cos(3*x) - 3*cos(2*x)*cos(x) + (3*sin(7*x) - sin
(5*x) - sin(3*x) + 3*sin(x))*sin(8*x) - 3*(sin(6*x) - sin(4*x) + sin(2*x))*
sin(7*x) + (sin(5*x) + sin(3*x) - 3*sin(x))*sin(6*x) - (sin(4*x) - sin(2*x)
)*sin(5*x) - (sin(3*x) - 3*sin(x))*sin(4*x) + sin(3*x)*sin(2*x) - 3*sin(2*x
)*sin(x) + 3*cos(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos
(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) -
 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))
*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/10*lo
g(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/10*log(cos(x)^2 + sin(x)^2 - 2*si
n(x) + 1) - sin(x)
```

Fricas [A] time = 2.73607, size = 451, normalized size = 4.03

$$\frac{1}{20}\sqrt{5}\log\left(\frac{8\cos(x)^2 - 4\left(\sqrt{5}-1\right)\sin(x) + \sqrt{5} - 11}{4\cos(x)^2 + 2\sin(x) - 3}\right) + \frac{1}{20}\sqrt{5}\log\left(-\frac{8\cos(x)^2 - 4\left(\sqrt{5}+1\right)\sin(x) - \sqrt{5} - 11}{4\cos(x)^2 - 2\sin(x) - 3}\right) - \frac{1}{20}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*co
s(x)^2 + 2*sin(x) - 3)) + 1/20*sqrt(5)*log(-(8*cos(x)^2 - 4*(sqrt(5) + 1)*s
in(x) - sqrt(5) - 11)/(4*cos(x)^2 - 2*sin(x) - 3)) - 1/20*log(4*cos(x)^2 +
2*sin(x) - 3) + 1/20*log(4*cos(x)^2 - 2*sin(x) - 3) + 1/10*log(sin(x) + 1)
- 1/10*log(-sin(x) + 1) - sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(5x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(5*x), x)
```

3.78 $\int \sin(x)\tan(6x)\,dx$

Optimal. Leaf size=89

$$-\sin(x)+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqr
t[3]]])/6 - Sin[x]
```

Rubi [A] time = 0.270688, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 207, 1166}

$$-\sin(x)+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[6*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqr
t[3]]])/6 - Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
```

```
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \sin(x)\tan(6x)\,dx &= \text{Subst}\left(\int \frac{2x^2\left(3-16x^2+16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2\left(3-16x^2+16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{2}+\frac{1-12x^2+16x^4}{2\left(1-18x^2+48x^4-32x^6\right)}\right)dx, x, \sin(x)\right)\\
&= -\sin(x)+\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= -\sin(x)+\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}-\frac{2\left(-1+8x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \sin(x)\right)\\
&= -\sin(x)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(x)\right)-\frac{2}{3}\text{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx, x, \sin(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}-\sin(x)-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx, x, \sin(x)\right)-\frac{1}{3}\left(4\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)-\sin(x
\end{aligned}$$

Mathematica [A] time = 0.133556, size = 84, normalized size = 0.94

$$\frac{1}{6}\left(-6\sin(x)+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(x)\right)+\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[6*x],x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqr
t[2 - Sqrt[3]]] + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]] -
 6*Sin[x])/6
```

Maple [B] time = 0.281, size = 256, normalized size = 2.9

$$\frac{\left(-3+2\sqrt{3}\right)\sqrt{3}}{6\sqrt{6}-6\sqrt{2}}\mathrm{Artanh}\left(8\frac{\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)+\frac{\left(3+2\sqrt{3}\right)\sqrt{3}}{6\sqrt{6}+6\sqrt{2}}\mathrm{Artanh}\left(8\frac{\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)+\frac{\mathrm{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{6}-\frac{}{6\sqrt{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(6*x),x)
```

```
[Out] 1/3*(-3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2
)-2*2^(1/2)))+1/3*(3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin
(x)/(2*6^(1/2)+2*2^(1/2)))+1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)-4/3/(2*6^(1/
2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))-4/3/(2*6^(1/2)+2*2^(1
/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))-sin(x)+1/9*(3+2*3^(1/2))*3^(1/
2)/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))+1/9*(-3+2*
3^(1/2))*3^(1/2)/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((2*cos(7*x) - c
os(5*x) - cos(3*x) + 2*cos(x))*cos(8*x) - 2*(cos(4*x) - 1)*cos(7*x) + (cos(
4*x) - 1)*cos(5*x) + (cos(3*x) - 2*cos(x))*cos(4*x) + (2*sin(7*x) - sin(5*x
) - sin(3*x) + 2*sin(x))*sin(8*x) + (sin(3*x) - 2*sin(x))*sin(4*x) - 2*sin(
7*x)*sin(4*x) + sin(5*x)*sin(4*x) - cos(3*x) + 2*cos(x))/(2*(cos(4*x) - 1)*
cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin
(4*x)^2 + 2*cos(4*x) - 1), x) - sin(x)
```

Fricas [B] time = 2.78035, size = 435, normalized size = 4.89

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\sin(x)\right)-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\sin(x)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(sqrt(3
) + 2)*log(sqrt(sqrt(3) + 2) - 2*sin(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt
(-sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2)
 - 2*sin(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos
(x)^2 - 1)) - sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(6*x), x)
```

3.79 $\int \sin(x)\tan(nx)\,dx$

Optimal. Leaf size=105

$$-ie^{-ix}\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2inx}\right)-ie^{ix}\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(\frac{1}{n}+2\right),-e^{2inx}\right)+\frac{1}{2}ie^{-ix}+$$

```
[Out] (I/2)/E^(I*x) + (I/2)*E^(I*x) - (I*Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*
n), -E^((2*I)*n*x)])/E^(I*x) - I*E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -E^((2*I)*n*x)]
```

Rubi [A] time = 0.0773561, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4557, 2194, 2251}

$$-ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)-ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2inx}\right)+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[n*x],x]
```


```
[Out] (I/2)/E^(I*x) + (I/2)*E^(I*x) - (I*Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*
n), -E^((2*I)*n*x)])/E^(I*x) - I*E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -E^((2*I)*n*x)]
```


Rule 4557

```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(nx)\,dx &= \int \left(\frac{e^{-ix}}{2}-\frac{e^{ix}}{2}-\frac{e^{-ix}}{1+e^{2inx}}+\frac{e^{ix}}{1+e^{2inx}}\right)dx\\ &= \frac{1}{2}\int e^{-ix}\,dx-\frac{1}{2}\int e^{ix}\,dx-\int \frac{e^{-ix}}{1+e^{2inx}}\,dx+\int \frac{e^{ix}}{1+e^{2inx}}\,dx\\ &= \frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}-ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)-ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2inx}\right)\end{aligned}$$

Mathematica [A] time = 0.164693, size = 200, normalized size = 1.9

$$-\frac{ie^{-2ix}\left((2n+1)e^{i(2nx+x)}\text{Hypergeometric2F1}\left(1,1-\frac{1}{2n},2-\frac{1}{2n},-e^{2inx}\right)+(2n-1)\left((2n+1)e^{ix}\left(\text{Hypergeometric2F1}\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[n*x],x]
```

```
[Out] ((-I/2)*(E^(I*(x + 2*n*x))*(1 + 2*n)*Hypergeometric2F1[1, 1 - 1/(2*n), 2 -
1/(2*n), -E^((2*I)*n*x)] + (-1 + 2*n)*(-(E^(I*(3 + 2*n)*x)*Hypergeometric2F
1[1, 1 + 1/(2*n), 2 + 1/(2*n), -E^((2*I)*n*x)]) + E^(I*x)*(1 + 2*n)*(Hyperg
eometric2F1[1, -1/(2*n), 1 - 1/(2*n), -E^((2*I)*n*x)] + E^((2*I)*x)*Hyperge
ometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^((2*I)*n*x)]))))/(E^((2*I)*x)*(-1 +
4*n^2))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \sin(x)\tan(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(n*x),x)
```

```
[Out] int(sin(x)*tan(n*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(nx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)*tan(n*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sin(x)\tan(nx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="fricas")
```

```
[Out] integral(sin(x)*tan(n*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(nx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x)
```

```
[Out] Integral(sin(x)*tan(n*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\tan(nx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(n*x), x)
```

3.80 $\int \cot(2x)\sin(x)\,dx$

Optimal. Leaf size=10

$$\sin(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/2 + Sin[x]
```

Rubi [A] time = 0.0218386, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\sin(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + Sin[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot(2x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1-2x^2}{2-2x^2}\,dx, x, \sin(x)\right)$$
$$= \sin(x) - \text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sin(x)\right)$$
$$= -\frac{1}{2}\tanh^{-1}(\sin(x)) + \sin(x)$$

Mathematica [A] time = 0.0090133, size = 10, normalized size = 1.

$$\sin(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + Sin[x]
```

Maple [A] time = 0.026, size = 12, normalized size = 1.2

$$\sin(x) - \frac{\ln(\sec(x) + \tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*sin(x),x)
```

```
[Out] sin(x)-1/2*ln(sec(x)+tan(x))
```

Maxima [B] time = 1.49586, size = 50, normalized size = 5.

$$-\frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*sin(x) + 1) + sin(x)
```

Fricas [B] time = 2.41989, size = 73, normalized size = 7.3

$$-\frac{1}{4}\log\left(\sin\left(x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

Sympy [B] time = 1.20567, size = 19, normalized size = 1.9

$$\frac{\log\left(\sin\left(x\right)-1\right)}{4}-\frac{\log\left(\sin\left(x\right)+1\right)}{4}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 + sin(x)
```

Giac [B] time = 1.12964, size = 26, normalized size = 2.6

$$-\frac{1}{4}\log\left(\sin\left(x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

3.81 $\int \cot(3x)\sin(x)\,dx$

Optimal. Leaf size=20

$$\sin(x)-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Rubi [A] time = 0.026517, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\sin(x)-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[3*x]*Sin[x],x]
```

```
[Out] -(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(3x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \sin(x)\right)\\ &= \sin(x) - 2\,\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sin(x)\end{aligned}$$

Mathematica [A] time = 0.0168605, size = 20, normalized size = 1.

$$\sin(x) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[3*x]*Sin[x],x]
```

```
[Out] -(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Maple [A] time = 0.046, size = 17, normalized size = 0.9

$$\sin(x) - \frac{\sqrt{3}}{3}\text{Artanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*sin(x),x)
```

```
[Out] sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

Maxima [B] time = 1.58707, size = 171, normalized size = 8.55

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 + \frac{4}{3}\sqrt{3}\sin(x) + \frac{4}{3}\cos(x) + \frac{4}{3}\right) - \frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 + \frac{4}{3}\sqrt{3}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + sin(x)
```

Fricas [B] time = 2.32617, size = 109, normalized size = 5.45

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{4\cos(x)^2 + 4\sqrt{3}\sin(x) - 7}{4\cos(x)^2 - 1}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + si
n(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\cot(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(3*x), x)
```

Giac [B] time = 1.15718, size = 46, normalized size = 2.3

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3} + 8\sin(x)\right|}{\left|4\sqrt{3} + 8\sin(x)\right|}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + sin
(x)
```

3.82 $\int \cot(4x)\sin(x)\,dx$

Optimal. Leaf size=28

$$\sin(x) - \frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

```
[Out] -ArcTanh[Sin[x]]/4 - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

Rubi [A] time = 0.0511705, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\sin(x) - \frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cot(4x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(1-\frac{3-4x^2}{4-12x^2+8x^4}\right)dx, x, \sin(x)\right)\\
&= \sin(x) - \text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\
&= \sin(x) + 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right) + 2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)
\end{aligned}
$$

Mathematica [A] time = 0.0320893, size = 28, normalized size = 1.

$$\sin(x) - \frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

Maple [A] time = 0.047, size = 30, normalized size = 1.1

$$\sin(x) - \frac{\text{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4} - \frac{\ln(1+\sin(x))}{8} + \frac{\ln(\sin(x)-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(4*x)*sin(x),x)
```

```
[Out] sin(x)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))+1/8*ln(sin(x)-1
)
```

Maxima [B] time = 1.57866, size = 234, normalized size = 8.36

$$-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2
*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 +
2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

Fricas [B] time = 2.50194, size = 170, normalized size = 6.07

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin(x)\cot(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(4*x), x)
```

Giac [B] time = 1.16234, size = 68, normalized size = 2.43

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/8
*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

3.83 $\int \cot(5x)\sin(x)\,dx$

Optimal. Leaf size=82

$$\sin(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]])/5 - (Sqrt[
(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Sin[x]])/5 + Sin[x]
```

Rubi [A] time = 0.196014, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\sin(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[5*x]*Sin[x],x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]])/5 - (Sqrt[
(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Sin[x]])/5 + Sin[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(5x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(1-\frac{4\left(1-2x^2\right)}{5-20x^2+16x^4}\right)dx, x, \sin(x)\right)\\
&= \sin(x) - 4\,\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \sin(x)\right)\\
&= \sin(x) + \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \sin(x)\right) + \frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int\right.\\
&= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right) +
\end{aligned}$$

Mathematica [A] time = 0.224122, size = 76, normalized size = 0.93

$$\frac{1}{10}\left(10\sin(x) - \sqrt{10-2\sqrt{5}}\tanh^{-1}\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sin(x)\right) - \sqrt{2\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[5*x]*Sin[x],x]
```

```
[Out] (-(Sqrt[10 - 2*Sqrt[5]]*ArcTanh[Sqrt[2 + 2/Sqrt[5]]*Sin[x]]) - Sqrt[2*(5 +
Sqrt[5])]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]] + 10*Sin[x])/10
```

Maple [A] time = 0.103, size = 70, normalized size = 0.9

$$\sin(x) - \frac{\left(\sqrt{5}-1\right)\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\text{Artanh}\left(4\frac{\sin(x)}{\sqrt{10-2\sqrt{5}}}\right) - \frac{\left(\sqrt{5}+1\right)\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\text{Artanh}\left(4\frac{\sin(x)}{\sqrt{10+2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*sin(x),x)
```

```
[Out] sin(x)-1/5*(5^(1/2)-1)*5^(1/2)/(10-2*5^(1/2))^(1/2)*arctanh(4*sin(x)/(10-2*
5^(1/2))^(1/2))-1/5*(5^(1/2)+1)*5^(1/2)/(10+2*5^(1/2))^(1/2)*arctanh(4*sin(
x)/(10+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*((cos(3*x) + cos(2*x) + cos(x))*cos(4*x) + (2*cos(2*x) + 2*c
os(x) + 1)*cos(3*x) + cos(3*x)^2 + (2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + c
os(x)^2 + (sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + 2*(sin(2*x) + sin(x))*s
in(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + cos(x))/
(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) +
 cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 +
 cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin
(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + si
n(x)^2 + 2*cos(x) + 1), x) - integrate(-1/2*((cos(3*x) - cos(2*x) + cos(x))
*cos(4*x) + (2*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - cos(3*x)^2 + (2*cos(x) -
 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + (sin(3*x) - sin(2*x) + sin(x))*sin(4
*x) + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)
*sin(x) - sin(x)^2 + cos(x))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x)
 - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x)
 - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*s
in(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*
x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + sin(x)
```

Fricas [B] time = 2.70209, size = 423, normalized size = 5.16

$$-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\sin\left(x\right)\right)+\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\sin\left(x\right)\right)-\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*sin(x)) +
 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*sin(x)) -
 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*sin(x))
 + 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*sin(x
)) + sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29034, size = 150, normalized size = 1.83

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)-\frac{1}{20}\sqrt{-2\sqrt{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(abs(1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + sin(x)
)) + 1/20*sqrt(2*sqrt(5) + 10)*log(abs(-1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + s
in(x))) - 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(-1/8*sqrt(5) + 5/8) + sin
(x))) + 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(-1/8*sqrt(5) + 5/8) + sin(
x))) + sin(x)
```

3.84 $\int \cot(6x)\sin(x)\,dx$

Optimal. Leaf size=38

$$\sin(x) - \frac{1}{6}\tanh^{-1}(\sin(x)) - \frac{1}{6}\tanh^{-1}(2\sin(x)) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[Sin[x]]/6 - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*S
qrt[3]) + Sin[x]
```

Rubi [A] time = 0.0823428, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 207}

$$\sin(x) - \frac{1}{6}\tanh^{-1}(\sin(x)) - \frac{1}{6}\tanh^{-1}(2\sin(x)) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[6*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/6 - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*S
qrt[3]) + Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot(6x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1-18x^2+48x^4-32x^6}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1-18x^2+48x^4-32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(2+\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}+\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \sin(x)\right)$$
$$= \sin(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \sin(x)\right)+\text{Subst}\left(\int\right.$$
$$= -\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sin(x)$$

Mathematica [A] time = 0.0667443, size = 38, normalized size = 1.

$$\sin(x)-\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[6*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/6 - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*S
qrt[3]) + Sin[x]
```

Maple [A] time = 0.069, size = 49, normalized size = 1.3

$$\sin(x)-\frac{\ln(1+\sin(x))}{12}+\frac{\ln(\sin(x)-1)}{12}-\frac{\ln(1+2\sin(x))}{12}+\frac{\ln(-1+2\sin(x))}{12}-\frac{\sqrt{3}}{6}\text{Artanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*sin(x),x)
```

```
[Out] sin(x)-1/12*ln(1+sin(x))+1/12*ln(sin(x)-1)-1/12*ln(1+2*sin(x))+1/12*ln(-1+2
*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) - integrate(-1/6*((cos(3
*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3
*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(
cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/12*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) + 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

Fricas [B] time = 2.71913, size = 243, normalized size = 6.39

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)-\frac{1}{12}\log(2\sin(x)+1)-\frac{1}{12}\log(\sin(x)+1)+\frac{1}{12}\log(-\sin(x)+1)+\frac{1}{12}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) - 1
/12*log(2*sin(x) + 1) - 1/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) + 1/12
*log(-2*sin(x) + 1) + sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15072, size = 95, normalized size = 2.5

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin\left(x\right)\right|}{\left|4\sqrt{3}+8\sin\left(x\right)\right|}\right)-\frac{1}{12}\log\left(\sin\left(x\right)+1\right)+\frac{1}{12}\log\left(-\sin\left(x\right)+1\right)-\frac{1}{12}\log\left(\left|2\sin\left(x\right)+1\right|\right)+\frac{1}{12}\log\left(\left|2\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) - 1/
12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) - 1/12*log(abs(2*sin(x) + 1)) +
1/12*log(abs(2*sin(x) - 1)) + sin(x)
```

3.85 $\int \sec(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rubi [A] time = 0.0153169, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4357, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec(2x)\sin(x)\,dx = -\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right) = \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.367907, size = 174, normalized size = 11.6

$$\frac{4\tanh^{-1}\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right)+\log\left(-\sqrt{2}\sin(x)+\sqrt{2}\cos(x)+2\right)+2i\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right)-(\sqrt{}}{\left(1+\sqrt{2}\right)\cos}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - Log[2 - Sqrt[2
]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*S
qrt[2])
```

Maple [A] time = 0.028, size = 13, normalized size = 0.9

$$\frac{\text{Artanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*x)*sin(x),x)
```

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.71228, size = 174, normalized size = 11.6

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+2\sqrt{2}\,\text{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1)
```

Fricas [B] time = 2.45967, size = 97, normalized size = 6.47

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^{2}+2\sqrt{2}\cos\left(x\right)+1}{2\cos\left(x\right)^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin\left(x\right)\sec\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(2*x), x)
```

Giac [B] time = 1.26431, size = 66, normalized size = 4.4

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2\left(\cos\left(x\right)-1\right)}{\cos\left(x\right)+1}-6\right|}{\left|4\sqrt{2}-\frac{2\left(\cos\left(x\right)-1\right)}{\cos\left(x\right)+1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqr
t(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

3.86 $\int \sec(3x)\sin(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\cos(x))-\frac{1}{6}\log\left(3-4\cos^2(x)\right)$$

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/6
```

Rubi [A] time = 0.0273773, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4357, 266, 36, 29, 31}

$$\frac{1}{3}\log(\cos(x))-\frac{1}{6}\log\left(3-4\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[3*x]*Sin[x],x]
```

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/6
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec(3x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(-3+4x^2\right)}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-3+4x)}\,dx, x, \cos^2(x)\right)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^2(x)\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3+4x}\,dx, x, \cos^2(x)\right)\\
&= \frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3-4\cos^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0081411, size = 17, normalized size = 0.81

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(8\sin^2(x)-5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[3*x]*Sin[x],x]
```

```
[Out] -ArcTanh[(-5 + 8*Sin[x]^2)/3]/3
```

Maple [A] time = 0.061, size = 18, normalized size = 0.9

$$\frac{\ln(\cos(x))}{3} - \frac{\ln\left(4\,(\cos(x))^2-3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*sin(x),x)
```

```
[Out] 1/3*ln(cos(x))-1/6*ln(4*cos(x)^2-3)
```

Maxima [B] time = 1.47328, size = 109, normalized size = 5.19

$$-\frac{1}{12}\log\left(-2\left(\cos\left(2x\right)-1\right)\cos\left(4x\right)+\cos\left(4x\right)^{2}+\cos\left(2x\right)^{2}+\sin\left(4x\right)^{2}-2\sin\left(4x\right)\sin\left(2x\right)+\sin\left(2x\right)^{2}-2\cos\left(2x\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/12*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 2.53631, size = 61, normalized size = 2.9

$$-\frac{1}{6}\log\left(4\cos\left(x\right)^{2}-3\right)+\frac{1}{3}\log\left(-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/6*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin\left(x\right)\sec\left(3x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(3*x), x)
```

Giac [B] time = 1.23202, size = 68, normalized size = 3.24

$$-\frac{1}{6}\log\left(\left|\frac{14\,(\cos(x)-1)}{\cos(x)+1}+\frac{(\cos(x)-1)^2}{(\cos(x)+1)^2}+1\right|\right)+\frac{1}{3}\log\left(\left|-\frac{\cos(x)-1}{\cos(x)+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(14*(cos(x) - 1)/(cos(x) + 1) + (cos(x) - 1)^2/(cos(x) + 1)^2 +
 1)) + 1/3*log(abs(-(cos(x) - 1)/(cos(x) + 1) - 1))
```

3.87 $\int \sec(4x)\sin(x)\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

```
[Out] -ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) + ArcTanh[
(2*Cos[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rubi [A] time = 0.0621501, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4357, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) + ArcTanh[
(2*Cos[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec(4x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\\
&= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \cos(x)\right)\right)+\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cos(x)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}
\end{aligned}$$

Mathematica [C] time = 56.6215, size = 4845, normalized size = 68.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[4*x]*Sin[x],x]
```

```
[Out] ((-2*(-1)^(3/8)*(1 + Sqrt[2])*x - (2*(-1)^(1/4)*(-2 - (1 - I)*(-1)^(5/8) +
(-1)^(5/8)*Sqrt[2])*ArcTan[(-Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(3/8) +
 Cos[x] - Sqrt[2]*Cos[x] + Sin[x])])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) -
(2*(1 - I)^(3/2)*2^(1/4)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (2 +
2*I)*(-1)^(3/8)*Sqrt[2] + 2*(-1)^(5/8)*Sqrt[2])*ArcTan[((1 + I) + I*Sqrt[2]
 + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])/(Sqrt[1 - I]*2^(3/4))])/((
-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) + 2*(-1)^(3/8)*Log[Sec[x/2]^2] + ((-1)^(3
/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*Log[-(Sec[x/2]^4*(-2 + (
1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Cos[2*x] - 2*
(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x]))])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]
))*((-1/2 - I/2)/(((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1
 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos
[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - Sin[x]/(Sqrt[-1 -
 I]*(1 - I)^(1/4)*(1 + I)^(1/4)*((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1
 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*S
qrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - ((I
```

```
/2)*Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(1/4)*Sin[x])/(((-1 + I) + Sqrt[1 -
I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Co
s[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt
[1 + I]*Sin[x]))))/(-2*(-1)^(3/8)*(1 + Sqrt[2]) - (2*(-1)^(1/4)*(-2 - (1 -
I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*(((1 + Sqrt[2])*Cos[x] + Sin[x])/(2*(-1
)^(3/8) + Cos[x] - Sqrt[2]*Cos[x] + Sin[x]) - ((Cos[x] - Sin[x] + Sqrt[2]*S
in[x])*(-Cos[x] + (1 + Sqrt[2])*Sin[x]))/(2*(-1)^(3/8) + Cos[x] - Sqrt[2]*C
os[x] + Sin[x])^2))/(((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*(1 + (-Cos[x] + (1
 + Sqrt[2])*Sin[x])^2/(2*(-1)^(3/8) + Cos[x] - Sqrt[2]*Cos[x] + Sin[x])^2))
 + 2*(-1)^(3/8)*Tan[x/2] - ((-1)^(3/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8
)*Sqrt[2])*Cos[x/2]^4*(-(Sec[x/2]^4*(-2*(-1)^(3/8)*Cos[x] + 2*Sqrt[2]*Cos[2
*x] - 2*(-1)^(3/8)*(-1 + Sqrt[2])*Sin[x] - 2*Sqrt[2]*Sin[2*x])) - 2*Sec[x/2
]^4*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Co
s[2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x])*Tan[x/2]))/(((-1 + I) + 2*
(-1)^(3/8) + Sqrt[2])*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*C
os[x] + Sqrt[2]*Cos[2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x])) - ((1 -
 I)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (2 + 2*I)*(-1)^(3/8)*Sqrt[
2] + 2*(-1)^(5/8)*Sqrt[2])*Sec[x/2]^2)/(Sqrt[2]*(1 + ((1/4 + I/4)*((1 + I)
+ I*Sqrt[2] + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])^2)/Sqrt[2]))) +
 ((-4*Sqrt[-1 - I]*(-1 + Sqrt[2])*ArcTanh[((-I)*((1 + I) + Sqrt[2]) + ((1 +
 I) + 2*(-1)^(5/8) - Sqrt[2])*Tan[x/2])/(Sqrt[-1 - I]*2^(3/4))] + (-1)^(1/8
)*2^(1/4)*(2*ArcTan[(Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(5/8) + (-1 + S
qrt[2])*Cos[x] + Sin[x])] - I*(2*(1 + Sqrt[2])*x + 2*Log[Sec[x/2]^2] - Log[
Sec[x/2]^4*(2 - (1 + I)*Sqrt[2] + 2*(-1)^(5/8)*(-1 + Sqrt[2])*Cos[x] - Sqrt
[2]*Cos[2*x] + 2*(-1)^(5/8)*Sin[x] + Sqrt[2]*Sin[2*x])])))*(2 + I*Sqrt[-1 +
 I]*2^(1/4)*((1 + I) + Sqrt[2])*Sin[x]))/(2^(1/4)*(4*Sqrt[-1 + I]*2^(1/4)*(
(-1 - I) + Sqrt[2]) - 8*(-1 + Sqrt[2])*Cos[x] - 8*Sin[x])*((2*(-1)^(1/8)*(-
2 - (1 + I)*Sqrt[2] + (-1)^(1/8)*((1 + I) + I*Sqrt[2])*Cos[x] + (2*I)*(1 +
Sqrt[2])*Cos[2*x] + (-1)^(1/8)*Sin[x] - (-1)^(5/8)*Sin[x] + 3*(-1)^(1/8)*Sq
rt[2]*Sin[x] - (2*I)*Sin[2*x]))/(2 - (1 + I)*Sqrt[2] + 2*(-1)^(5/8)*(-1 + S
qrt[2])*Cos[x] - Sqrt[2]*Cos[2*x] + 2*(-1)^(5/8)*Sin[x] + Sqrt[2]*Sin[2*x])
 - (((1 + I) + 2*(-1)^(5/8) - Sqrt[2])*(-1 + Sqrt[2])*Sec[x/2]^2)/(1 + ((1/
4 - I/4)*(I*((1 + I) + Sqrt[2]) + ((-1 - I) - 2*(-1)^(5/8) + Sqrt[2])*Tan[x
/2])^2)/Sqrt[2]))) + ((-2*(-1)^(3/8)*Sqrt[2]*(1 + (-1)^(1/4))*x + (2*(-2*I
+ 2*(-1)^(3/4) + 2*(-1)^(1/8)*Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(7/8)*Sqr
t[2])*ArcTan[Cos[x]/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] + (1 + (-1)^
(1/4))*Sin[x])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) - ((4 + 4*I)*(-1)^(
5/8)*((3 - 3*I) - (2 - 2*I)*Sqrt[2] + (-1)^(1/8)*Sqrt[2] - (-1)^(3/8)*Sqrt[
2] + (1 - I)*(-1)^(5/8)*Sqrt[2] + (1 + I)*(-1)^(7/8)*Sqrt[2])*ArcTanh[(1/2
+ I/2)*(-1)^(5/8)*(-1 - (-1)^(1/4) + (-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])
*Tan[x/2])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) - 2*(-1)^(7/8)*Sqrt[2]*
(-1 + (-1)^(1/4))*Log[Sec[x/2]^2] - ((-1 + (-1)^(1/4))*(2 - (-1)^(3/8)*Sqrt
[2] + (-1)^(5/8)*Sqrt[2])*Log[(1/4 + I/4)*Sec[x/2]^4*((2 - 2*I) + 6*Sqrt[2]
 - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[2])*Cos[2*x] - (
4 - 4*I)*(-1)^(1/8)*Sqrt[2]*Sin[x] - (4 - 4*I)*(-1)^(3/8)*Sqrt[2]*Sin[x] -
```

```
(2 - 2*I)*Sin[2*x] + (2*I)*Sqrt[2]*Sin[2*x])])/(-I + (-1)^(3/4) + (-1)^(1/8
)*Sqrt[2]))*(I/(Sqrt[1 - I]*((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])^2*(Sqrt[-1
 - I]*(1 - I)^(3/4)*(1 + I)^(1/4) + Sqrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x]
 + I*Sqrt[1 - I]*Sin[x] + I*Sqrt[1 + I]*Sin[x])) + 1/(Sqrt[1 + I]*((-1 + I)
 + Sqrt[1 - I]*Sqrt[1 + I])^2*(Sqrt[-1 - I]*(1 - I)^(3/4)*(1 + I)^(1/4) + S
qrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] + I*Sqrt[1 - I]*Sin[x] + I*Sqrt[1 +
I]*Sin[x])) - (2*Sin[x])/(Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(3/4)*((-1 + I
) + Sqrt[1 - I]*Sqrt[1 + I])^2*(Sqrt[-1 - I]*(1 - I)^(3/4)*(1 + I)^(1/4) +
Sqrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] + I*Sqrt[1 - I]*Sin[x] + I*Sqrt[1 +
 I]*Sin[x]))))/(-2*(-1)^(3/8)*Sqrt[2]*(1 + (-1)^(1/4)) + (2*(-2*I + 2*(-1)^
(3/4) + 2*(-1)^(1/8)*Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(7/8)*Sqrt[2])*(-(
(Cos[x]*((1 + (-1)^(1/4))*Cos[x] - (-1)^(3/4)*Sin[x]))/(-((-1)^(1/8)*Sqrt[2
]) + (-1)^(3/4)*Cos[x] + (1 + (-1)^(1/4))*Sin[x])^2) - Sin[x]/(-((-1)^(1/8)
*Sqrt[2]) + (-1)^(3/4)*Cos[x] + (1 + (-1)^(1/4))*Sin[x])))/((-I + (-1)^(3/4
) + (-1)^(1/8)*Sqrt[2])*(1 + Cos[x]^2/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*C
os[x] + (1 + (-1)^(1/4))*Sin[x])^2)) - 2*(-1)^(7/8)*Sqrt[2]*(-1 + (-1)^(1/4
))*Tan[x/2] - ((2 - 2*I)*(-1 + (-1)^(1/4))*(2 - (-1)^(3/8)*Sqrt[2] + (-1)^(
5/8)*Sqrt[2])*Cos[x/2]^4*((1/4 + I/4)*Sec[x/2]^4*((-4 + 4*I)*(-1)^(1/8)*Sqr
t[2]*Cos[x] - (4 - 4*I)*(-1)^(3/8)*Sqrt[2]*Cos[x] - (4 - 4*I)*Cos[2*x] + (4
*I)*Sqrt[2]*Cos[2*x] + (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Sin[x] + 4*((1 + I) + S
qrt[2])*Sin[2*x]) + (1/2 + I/2)*Sec[x/2]^4*((2 - 2*I) + 6*Sqrt[2] - (4 - 4*
I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[2])*Cos[2*x] - (4 - 4*I)*(
-1)^(1/8)*Sqrt[2]*Sin[x] - (4 - 4*I)*(-1)^(3/8)*Sqrt[2]*Sin[x] - (2 - 2*I)*
Sin[2*x] + (2*I)*Sqrt[2]*Sin[2*x])*Tan[x/2]))/((-I + (-1)^(3/4) + (-1)^(1/8
)*Sqrt[2])*((2 - 2*I) + 6*Sqrt[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2
*((1 + I) + Sqrt[2])*Cos[2*x] - (4 - 4*I)*(-1)^(1/8)*Sqrt[2]*Sin[x] - (4 -
4*I)*(-1)^(3/8)*Sqrt[2]*Sin[x] - (2 - 2*I)*Sin[2*x] + (2*I)*Sqrt[2]*Sin[2*x
])) + (2*(-1)^(3/4)*((3 - 3*I) - (2 - 2*I)*Sqrt[2] + (-1)^(1/8)*Sqrt[2] - (
-1)^(3/8)*Sqrt[2] + (1 - I)*(-1)^(5/8)*Sqrt[2] + (1 + I)*(-1)^(7/8)*Sqrt[2]
)*Sec[x/2]^2)/(1 + ((-1)^(3/4)*(-1 - (-1)^(1/4) + (-I + (-1)^(3/4) + (-1)^(
1/8)*Sqrt[2])*Tan[x/2])^2)/2)) + ((2*((-1)^(1/8) + (-1)^(3/8))*x - (2*(-1)^
(7/8)*(2 - Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*ArcTan[Cos[x]
/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] - (1 + (-1)^(1/4))*Sin[x])])/(-
I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) - ((4 + 4*I)*(-1)^(5/8)*(3*I + (-1)^(1
/8) - (-1)^(3/8) - (1 + I)*(-1)^(5/8) - (2*I)*Sqrt[2] + (1 + I)*(-1)^(5/8)*
Sqrt[2])*ArcTanh[(1/2 + I/2)*(-1)^(5/8)*(1 + (-1)^(1/4) + (-I + (-1)^(3/4)
+ (-1)^(1/8)*Sqrt[2])*Tan[x/2])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) +
2*(-1)^(3/8)*(-I + (-1)^(1/4))*Log[Sec[x/2]^2] + ((-1)^(3/8)*(2 - Sqrt[2] -
 (-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*Log[(1/4 + I/4)*Sec[x/2]^4*((2 -
2*I) + 6*Sqrt[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[
2])*Cos[2*x] + (4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Sin[x] + (2 - 2*I)*
Sin[2*x] - (2*I)*Sqrt[2]*Sin[2*x])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])
)*(1/(Sqrt[1 - I]*((-1 - I) + Sqrt[1 - I]*Sqrt[1 + I])^2*(-(Sqrt[-1 + I]*(1
 - I)^(1/4)*(1 + I)^(3/4)) + Sqrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] - I*Sq
rt[1 - I]*Sin[x] - I*Sqrt[1 + I]*Sin[x])) - I/(Sqrt[1 + I]*((-1 - I) + Sqrt
```

```
[1 - I]*Sqrt[1 + I])^2*(-(Sqrt[-1 + I]*(1 - I)^(1/4)*(1 + I)^(3/4)) + Sqrt[
1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] - I*Sqrt[1 - I]*Sin[x] - I*Sqrt[1 + I]*S
in[x])) + (2*Sin[x])/(Sqrt[-1 + I]*(1 - I)^(3/4)*(1 + I)^(1/4)*((-1 - I) +
Sqrt[1 - I]*Sqrt[1 + I])^2*(-(Sqrt[-1 + I]*(1 - I)^(1/4)*(1 + I)^(3/4)) + S
qrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] - I*Sqrt[1 - I]*Sin[x] - I*Sqrt[1 +
I]*Sin[x]))))/(2*((-1)^(1/8) + (-1)^(3/8)) - (2*(-1)^(7/8)*(2 - Sqrt[2] - (
-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*(-((Cos[x]*(-((1 + (-1)^(1/4))*Cos[
x]) - (-1)^(3/4)*Sin[x]))/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] - (1 +
 (-1)^(1/4))*Sin[x])^2) - Sin[x]/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x]
 - (1 + (-1)^(1/4))*Sin[x])))/((-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])*(1 +
Cos[x]^2/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] - (1 + (-1)^(1/4))*Sin[
x])^2)) + 2*(-1)^(3/8)*(-I + (-1)^(1/4))*Tan[x/2] + ((2 - 2*I)*(-1)^(3/8)*(
2 - Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*Cos[x/2]^4*((1/4 + I
/4)*Sec[x/2]^4*((4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Cos[x] + (4 - 4*I)
*Cos[2*x] - (4*I)*Sqrt[2]*Cos[2*x] + (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Sin[x] +
4*((1 + I) + Sqrt[2])*Sin[2*x]) + (1/2 + I/2)*Sec[x/2]^4*((2 - 2*I) + 6*Sqr
t[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[2])*Cos[2*x]
 + (4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Sin[x] + (2 - 2*I)*Sin[2*x] - (
2*I)*Sqrt[2]*Sin[2*x])*Tan[x/2]))/((-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])*(
(2 - 2*I) + 6*Sqrt[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) +
Sqrt[2])*Cos[2*x] + (4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Sin[x] + (2 -
2*I)*Sin[2*x] - (2*I)*Sqrt[2]*Sin[2*x])) + (2*(-1)^(3/4)*(3*I + (-1)^(1/8)
- (-1)^(3/8) - (1 + I)*(-1)^(5/8) - (2*I)*Sqrt[2] + (1 + I)*(-1)^(5/8)*Sqrt
[2])*Sec[x/2]^2)/(1 + ((-1)^(3/4)*(1 + (-1)^(1/4) + (-I + (-1)^(3/4) + (-1)
^(1/8)*Sqrt[2])*Tan[x/2])^2)/2))
```

Maple [A] time = 0.066, size = 54, normalized size = 0.8

$$-\frac{\sqrt{2}}{4\sqrt{2-\sqrt{2}}}\text{Artanh}\left(2\frac{\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{\sqrt{2}}{4\sqrt{2+\sqrt{2}}}\text{Artanh}\left(2\frac{\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(4*x)*sin(x),x)
```

```
[Out] -1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))+1/4*2^(1
/2)/(2+2^(1/2))^(1/2)*arctanh(2*cos(x)/(2+2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(4x)\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] integrate(sec(4*x)*sin(x), x)
```

Fricas [B] time = 2.88998, size = 394, normalized size = 5.55

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\cos(x)\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\cos(x)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*cos(x)) + 1/
8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*cos(x)) + 1/8*s
qrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*cos(x)) - 1/8*sq
rt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\sec(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(4*x), x)
```

Giac [B] time = 1.29793, size = 147, normalized size = 2.07

$$2\sqrt{-\frac{1}{256}\sqrt{2}+\frac{1}{128}}\log\left(\left|124864\sqrt{\sqrt{2}+2}+249728\cos(x)\right|\right)-2\sqrt{-\frac{1}{256}\sqrt{2}+\frac{1}{128}}\log\left(\left|-124864\sqrt{\sqrt{2}+2}+24972\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] 2*sqrt(-1/256*sqrt(2) + 1/128)*log(abs(124864*sqrt(sqrt(2) + 2) + 249728*co
s(x))) - 2*sqrt(-1/256*sqrt(2) + 1/128)*log(abs(-124864*sqrt(sqrt(2) + 2) +
 249728*cos(x))) - 1/8*sqrt(sqrt(2) + 2)*log(abs(507968*sqrt(-sqrt(2) + 2)
+ 1015936*cos(x))) + 1/8*sqrt(sqrt(2) + 2)*log(abs(-507968*sqrt(-sqrt(2) +
2) + 1015936*cos(x)))
```

3.88 $\int \sec(5x)\sin(x)\,dx$

Optimal. Leaf size=62

$$\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cos^2(x)-\sqrt{5}+5\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cos^2(x)+\sqrt{5}+5\right)-\frac{1}{5}\log(\cos(x))$$

```
[Out] -Log[Cos[x]]/5 + ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cos[x]^2])/20 + ((1 - S
qrt[5])*Log[5 + Sqrt[5] - 8*Cos[x]^2])/20
```

Rubi [A] time = 0.0725292, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4357, 1114, 705, 29, 632, 31}

$$\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cos^2(x)-\sqrt{5}+5\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cos^2(x)+\sqrt{5}+5\right)-\frac{1}{5}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[5*x]*Sin[x],x]
```


```
[Out] -Log[Cos[x]]/5 + ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cos[x]^2])/20 + ((1 - S
qrt[5])*Log[5 + Sqrt[5] - 8*Cos[x]^2])/20
```


Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec(5x)\sin(x)\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^2(x)\right)\right) - \frac{1}{10}\operatorname{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \cos^2(x)\right)\\
&= -\frac{1}{5}\log(\cos(x)) + \frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \cos^2(x)\right) + \frac{1}{5}\left(4\left(1+\sqrt{5}\right)\right)\operatorname{Sub}\\
&= -\frac{1}{5}\log(\cos(x)) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cos^2(x)\right) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\cos^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0969927, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(-4\log(\cos(x)) - \left(\sqrt{5}-1\right)\log\left(4\cos(2x)-\sqrt{5}-1\right) + \left(1+\sqrt{5}\right)\log\left(4\cos(2x)+\sqrt{5}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[5*x]*Sin[x],x]
```

```
[Out] (-4*Log[Cos[x]] - (-1 + Sqrt[5])*Log[-1 - Sqrt[5] + 4*Cos[2*x]] + (1 + Sqrt
[5])*Log[-1 + Sqrt[5] + 4*Cos[2*x]])/20
```

Maple [A] time = 0.079, size = 43, normalized size = 0.7

$$-\frac{\ln(\cos(x))}{5}+\frac{\ln\left(16\,(\cos(x))^4-20\,(\cos(x))^2+5\right)}{20}+\frac{\sqrt{5}}{10}\mathrm{Artanh}\left(\frac{\left(32\,(\cos(x))^2-20\right)\sqrt{5}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(5*x)*sin(x),x)
```

```
[Out] -1/5*ln(cos(x))+1/20*ln(16*cos(x)^4-20*cos(x)^2+5)+1/10*5^(1/2)*arctanh(1/2
0*(32*cos(x)^2-20)*5^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/5*integrate(-(cos(4*x)*sin(8*x) - cos(8*x)*sin(4*x) + cos(3/2*arctan2(sin
(4*x), cos(4*x)))*sin(4*x) + cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x)
- cos(4*x)*sin(3/2*arctan2(sin(4*x), cos(4*x))) - cos(4*x)*sin(1/2*arctan2(
sin(4*x), cos(4*x))) - sin(4*x))/(2*(cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 +
cos(4*x)^2 - 2*(cos(8*x) + cos(4*x) - cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ 1)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + cos(3/2*arctan2(sin(4*x), cos(4
*x)))^2 - 2*(cos(8*x) + cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(8*x)^2 + 2*sin(8*x)*sin(4*x)
 + sin(4*x)^2 - 2*(sin(8*x) + sin(4*x) - sin(1/2*arctan2(sin(4*x), cos(4*x)
)))*sin(3/2*arctan2(sin(4*x), cos(4*x))) + sin(3/2*arctan2(sin(4*x), cos(4*
x)))^2 - 2*(sin(8*x) + sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x))) + sin
(1/2*arctan2(sin(4*x), cos(4*x)))^2 + 2*cos(4*x) + 1), x) + 4/5*integrate(-
(cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x) - cos(8*x)*sin(2
*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))/(2*(cos(6*x) - cos(
```

```
4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*co
s(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 +
 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - s
in(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x
)^2 + 2*cos(2*x) - 1), x) - 2/5*integrate(-(cos(4/3*arctan2(sin(6*x), cos(6
*x)))*sin(6*x) + cos(2/3*arctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(1/3*ar
ctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(6*x)*sin(4/3*arctan2(sin(6*x), co
s(6*x))) - cos(6*x)*sin(2/3*arctan2(sin(6*x), cos(6*x))) + cos(6*x)*sin(1/3
*arctan2(sin(6*x), cos(6*x))) + sin(6*x))/(cos(6*x)^2 - 2*(cos(6*x) - cos(2
/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), cos(6*x))) - 1)
*cos(4/3*arctan2(sin(6*x), cos(6*x))) + cos(4/3*arctan2(sin(6*x), cos(6*x))
)^2 - 2*(cos(6*x) + cos(1/3*arctan2(sin(6*x), cos(6*x))) - 1)*cos(2/3*arcta
n2(sin(6*x), cos(6*x))) + cos(2/3*arctan2(sin(6*x), cos(6*x)))^2 + 2*(cos(6
*x) - 1)*cos(1/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), c
os(6*x)))^2 + sin(6*x)^2 - 2*(sin(6*x) - sin(2/3*arctan2(sin(6*x), cos(6*x)
)) + sin(1/3*arctan2(sin(6*x), cos(6*x))))*sin(4/3*arctan2(sin(6*x), cos(6*
x))) + sin(4/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*(sin(6*x) + sin(1/3*arcta
n2(sin(6*x), cos(6*x))))*sin(2/3*arctan2(sin(6*x), cos(6*x))) + sin(2/3*arc
tan2(sin(6*x), cos(6*x)))^2 + 2*sin(6*x)*sin(1/3*arctan2(sin(6*x), cos(6*x)
)) + sin(1/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*cos(6*x) + 1), x) - 2/5*int
egrate(-(sin(8*x) - sin(6*x) + sin(4*x) - sin(2*x))/(2*(cos(6*x) - cos(4*x)
 + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*
x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(
sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2
*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2
+ 2*cos(2*x) - 1), x) - 1/10*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 2.78257, size = 230, normalized size = 3.71

$$\frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4+8\left(\sqrt{5}-5\right)\cos(x)^2-5\sqrt{5}+15}{16\cos(x)^4-20\cos(x)^2+5}\right)+\frac{1}{20}\log\left(16\cos(x)^4-20\cos(x)^2+5\right)-\frac{1}{5}\log\left(-\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 5)*cos(x)^2 - 5*sqrt(5) + 15)/
(16*cos(x)^4 - 20*cos(x)^2 + 5)) + 1/20*log(16*cos(x)^4 - 20*cos(x)^2 + 5)
- 1/5*log(-cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\sec(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(5*x), x)
```

Giac [B] time = 1.26142, size = 159, normalized size = 2.56

$$\frac{1}{20}\sqrt{5}\log\left(\left|2560\cos(x)^2+320\sqrt{5}-1600\right|\right)-\frac{1}{20}\sqrt{5}\log\left(\left|2560\cos(x)^2-320\sqrt{5}-1600\right|\right)+\frac{1}{20}\log\left(\left|\frac{44(\cos(x)-1}{\cos(x)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(2560*cos(x)^2 + 320*sqrt(5) - 1600)) - 1/20*sqrt(5)*lo
g(abs(2560*cos(x)^2 - 320*sqrt(5) - 1600)) + 1/20*log(abs(44*(cos(x) - 1)/(
cos(x) + 1) + 166*(cos(x) - 1)^2/(cos(x) + 1)^2 + 44*(cos(x) - 1)^3/(cos(x)
 + 1)^3 + (cos(x) - 1)^4/(cos(x) + 1)^4 + 1)) - 1/5*log(abs(-(cos(x) - 1)/(
cos(x) + 1) - 1))
```

3.89 $\int \sec(6x)\sin(x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -ArcTanh[Sqrt[2]*Cos[x]]/(3*Sqrt[2]) + ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rubi [A] time = 0.0631543, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4357, 2057, 207, 1166}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[6*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sqrt[2]*Cos[x]]/(3*Sqrt[2]) + ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
```

```
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \sec(6x)\sin(x)\,dx &= -\operatorname{Subst}\left(\int \frac{1}{-1+18x^2-48x^4+32x^6}\,dx, x, \cos(x)\right) \\
&= -\operatorname{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}+\frac{4\left(-1+2x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \cos(x)\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)-\frac{4}{3}\operatorname{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \cos(x)\right) \\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}-\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \cos(x)\right)-\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+ \right. \\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}
$$

Mathematica [C] time = 9.28038, size = 678, normalized size = 7.98

$$
\left(\frac{1}{6}+\frac{i}{6}\right)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\sec\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)-\left(\frac{1}{6}+\frac{i}{6}\right)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{x}{2}\right)\left(\mathrm{co}\right.\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[6*x]*Sin[x],x]
```

```
[Out] (1/6 + I/6)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*Sec[x/2]*(Cos[x/2] + S
in[x/2])] - (1/6 + I/6)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[x/2]*
(Cos[x/2] - Sin[x/2])] + ((1 + Sqrt[2])*(x + 2*Sqrt[3]*ArcTanh[(2 + (2 + Sq
rt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[2] - 2
*Cos[x] + 2*Sin[x]))]))/(12*(2 + Sqrt[2])) - (x - 2*Sqrt[3]*ArcTanh[(Sqrt[2
] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]^2] + Log[Sec[x/2]^2*(1
 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])])/(12*Sqrt[2]) + ((2*(Sqrt[2] + Sqrt[3]
)*ArcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x - Log[Se
c[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(1 + Sqrt[
6]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x]))/(12
*((12 + 5*Sqrt[6])*Cos[2*x] + 2*Cos[x]*(5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]) -
 2*(12 + 5*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Sin[x] - 6*Sin[2*x]))) + ((-2*(-2 +
Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*Sqrt[2] - 2*S
qrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sqrt[2]*Cos[x] -
 Sqrt[2]*Sin[x]))]))*(Sqrt[2] - 2*Sqrt[3]*Sin[x])*(-3 + Sqrt[6] - (-2 + Sqr
t[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/(24*((-12 + 5*Sqrt[6])*Cos[2*x] + 2*
Cos[x]*(-5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]) - 2*(-12 + 5*Sqrt[6] + 4*(-5 + 2
*Sqrt[6])*Sin[x] + 6*Sin[2*x])))
```

Maple [A] time = 0.096, size = 80, normalized size = 0.9

$$\frac{2}{6\sqrt{6}-6\sqrt{2}}\operatorname{Artanh}\left(8\frac{\cos(x)}{2\sqrt{6}-2\sqrt{2}}\right)+\frac{2}{6\sqrt{6}+6\sqrt{2}}\operatorname{Artanh}\left(8\frac{\cos(x)}{2\sqrt{6}+2\sqrt{2}}\right)-\frac{\operatorname{Artanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(6*x)*sin(x),x)
```

```
[Out] 2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^
(1/2)+2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)+2*2^(1/2)))-1/6*arctanh(cos(x)
*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) + 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*co
s(x) + 1) - integrate(1/3*((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*
x) - (sin(3*x) - sin(x))*cos(4*x) - (cos(7*x) - cos(5*x) + cos(3*x) - cos(x
))*sin(8*x) - (cos(4*x) - 1)*sin(7*x) + (cos(4*x) - 1)*sin(5*x) + (cos(3*x)
 - cos(x))*sin(4*x) + cos(7*x)*sin(4*x) - cos(5*x)*sin(4*x) + sin(3*x) - si
n(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2
*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x)
```

Fricas [B] time = 2.99748, size = 498, normalized size = 5.86

$$-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\cos\left(x\right)\right)+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\cos\left(x\right)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*cos(x)) + 1
/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*cos(x)) + 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*cos(x)) - 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*cos(x)) + 1/1
2*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(x\right)\sec\left(6x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(6*x), x)
```

Giac [B] time = 1.31898, size = 208, normalized size = 2.45

$$\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|8\sqrt{6}-8\sqrt{2}+32\cos\left(x\right)\right|\right)+\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|\sqrt{6}+\sqrt{2}+4\cos\left(x\right)\right|\right)-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|-\sqrt{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + sqrt(2))*log(abs(8*sqrt(6) - 8*sqrt(2) + 32*cos(x))) + 1/24
*(sqrt(6) - sqrt(2))*log(abs(sqrt(6) + sqrt(2) + 4*cos(x))) - 1/24*(sqrt(6)
 - sqrt(2))*log(abs(-sqrt(6) - sqrt(2) + 4*cos(x))) - 1/24*(sqrt(6) + sqrt(
2))*log(abs(-8*sqrt(6) + 8*sqrt(2) + 32*cos(x))) - 1/12*sqrt(2)*log(abs(-4*
sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqrt(2) - 2*(cos(x) - 1)/(
cos(x) + 1) - 6))
```

3.90 $\int \csc(2x)\sin(x)\,dx$

Optimal. Leaf size=7

$$\frac{1}{2}\tanh^{-1}(\sin(x))$$

[Out] ArcTanh[Sin[x]]/2

Rubi [A] time = 0.0119241, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4288, 3770}

$$\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[2*x]*Sin[x],x]

[Out] ArcTanh[Sin[x]]/2

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)\sin(x)\,dx &= \frac{1}{2}\int \sec(x)\,dx\\ &= \frac{1}{2}\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0062136, size = 37, normalized size = 5.29

$$\frac{1}{2}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*Sin[x],x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/2
```

Maple [A] time = 0.018, size = 9, normalized size = 1.3

$$\frac{\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*sin(x),x)
```

```
[Out] 1/2*ln(sec(x)+tan(x))
```

Maxima [B] time = 1.54641, size = 47, normalized size = 6.71

$$\frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos(x)^2 + sin(x)^2 -
 2*sin(x) + 1)
```

Fricas [B] time = 2.3467, size = 59, normalized size = 8.43

$$\frac{1}{4}\log\left(\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

Sympy [B] time = 6.40099, size = 15, normalized size = 2.14

$$-\frac{\log(\sin(x)-1)}{4}+\frac{\log(\sin(x)+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4
```

Giac [B] time = 1.13954, size = 34, normalized size = 4.86

$$\frac{1}{8}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\frac{1}{8}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(1/sin(x) + sin(x) + 2)) - 1/8*log(abs(1/sin(x) + sin(x) - 2))
```

3.91 $\int \csc(3x)\sin(x)\,dx$

Optimal. Leaf size=45

$$\frac{\log\left(\sin(x)+\sqrt{3}\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}$$

```
[Out] -Log[Sqrt[3]*Cos[x] - Sin[x]]/(2*Sqrt[3]) + Log[Sqrt[3]*Cos[x] + Sin[x]]/(2
*Sqrt[3])
```

Rubi [A] time = 0.0399368, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {206}

$$\frac{\log\left(\sin(x)+\sqrt{3}\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[3*x]*Sin[x],x]
```


```
[Out] -Log[Sqrt[3]*Cos[x] - Sin[x]]/(2*Sqrt[3]) + Log[Sqrt[3]*Cos[x] + Sin[x]]/(2
*Sqrt[3])
```


Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(3x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \tan(x)\right)\\ &= -\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01982, size = 15, normalized size = 0.33

$$\frac{\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[3*x]*Sin[x],x]
```

```
[Out] ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]
```

Maple [A] time = 0.064, size = 14, normalized size = 0.3

$$\frac{\sqrt{3}}{3}\operatorname{Artanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(3*x)*sin(x),x)
```

```
[Out] 1/3*3^(1/2)*arctanh(1/3*tan(x)*3^(1/2))
```

Maxima [B] time = 1.57434, size = 169, normalized size = 3.76

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3)
```

Fricas [A] time = 2.34947, size = 177, normalized size = 3.93

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4 - 16\cos(x)^2 - 4\left(2\sqrt{3}\cos(x)^3 + \sqrt{3}\cos(x)\right)\sin(x) - 1}{16\cos(x)^4 - 8\cos(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(8*cos(x)^4 - 16*cos(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt(
3)*cos(x))*sin(x) - 1)/(16*cos(x)^4 - 8*cos(x)^2 + 1))
```

Sympy [A] time = 4.89941, size = 76, normalized size = 1.69

$$\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) + \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) + \sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x)
```

```
[Out] sqrt(3)*log(tan(x/2) - sqrt(3))/6 - sqrt(3)*log(tan(x/2) - sqrt(3)/3)/6 + s
qrt(3)*log(tan(x/2) + sqrt(3)/3)/6 - sqrt(3)*log(tan(x/2) + sqrt(3))/6
```

Giac [A] time = 1.20102, size = 42, normalized size = 0.93

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3} + 2\tan(x)\right|}{\left|2\sqrt{3} + 2\tan(x)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x)))
```

3.92 $\int \csc(4x)\sin(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0250421, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(4x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)$$
$$= -\frac{1}{4}\tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.022676, size = 26, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} - \frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Maple [A] time = 0.068, size = 28, normalized size = 1.1

$$\frac{\text{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4} - \frac{\ln(1+\sin(x))}{8} + \frac{\ln(\sin(x)-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(4*x)*sin(x),x)
```

```
[Out] 1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))+1/8*ln(sin(x)-1)
```

Maxima [B] time = 1.55964, size = 231, normalized size = 8.88

$$\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 2.58287, size = 158, normalized size = 6.08

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16074, size = 65, normalized size = 2.5

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.93 $\int \csc(5x)\sin(x)\,dx$

Optimal. Leaf size=165

$$-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}}\left(5-\right.$$

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] - Sin[x]])/10 + (Sqr
t[(5 + Sqrt[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] - Sin[x]])/10 + (Sqrt[(5
- Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] + Sin[x]])/10 - (Sqrt[(5 + Sqr
t[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] + Sin[x]])/10
```

Rubi [A] time = 0.140952, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 207}

$$-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}}\left(5-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Csc[5*x]*Sin[x],x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] - Sin[x]])/10 + (Sqr
t[(5 + Sqrt[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] - Sin[x]])/10 + (Sqrt[(5
- Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] + Sin[x]])/10 - (Sqrt[(5 + Sqr
t[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] + Sin[x]])/10
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(5x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1+x^2}{5-10x^2+x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{10}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5+2\sqrt{5}+x^2}\,dx, x, \tan(x)\right) + \frac{1}{10}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5-2\sqrt{5}+x^2}\right.$$
$$= -\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\sin\right.$$

Mathematica [A] time = 0.109661, size = 84, normalized size = 0.51

$$\frac{\sqrt{5+\sqrt{5}}\tanh^{-1}\left(\frac{\left(\sqrt{5}-3\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tanh^{-1}\left(\frac{\left(3+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[5*x]*Sin[x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((-3 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTanh[((3 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [A] time = 0.107, size = 66, normalized size = 0.4

$$-\frac{\left(3+\sqrt{5}\right)\sqrt{5}}{10\sqrt{5+2\sqrt{5}}}\text{Artanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)-\frac{\left(\sqrt{5}-3\right)\sqrt{5}}{10\sqrt{5-2\sqrt{5}}}\text{Artanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(5*x)*sin(x),x)
```

```
[Out] -1/10*(3+5^(1/2))*5^(1/2)/(5+2*5^(1/2))^(1/2)*arctanh(tan(x)/(5+2*5^(1/2))^
(1/2))-1/10*(5^(1/2)-3)*5^(1/2)/(5-2*5^(1/2))^(1/2)*arctanh(tan(x)/(5-2*5^(
1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(5x)\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] integrate(csc(5*x)*sin(x), x)
```

Fricas [B] time = 2.73042, size = 759, normalized size = 4.6

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(-\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5
) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) + 1/40*sqrt(
2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*cos
(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) - 1/40*sqrt(2)*sqrt(-s
qrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x
) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) - 3) + 1/40*sqrt(2)*sqrt(-sqrt(5) +
5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x) + 2*(s
qrt(5) - 1)*cos(x)^2 - sqrt(5) - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\csc(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*csc(5*x), x)
```

Giac [A] time = 1.35647, size = 142, normalized size = 0.86

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{2\sqrt{5}+5}+\tan\left(x\right)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{2\sqrt{5}+5}+\tan\left(x\right)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(abs(sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sq
rt(2*sqrt(5) + 10)*log(abs(-sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sqrt(-2*s
qrt(5) + 10)*log(abs(sqrt(-2*sqrt(5) + 5) + tan(x))) - 1/20*sqrt(-2*sqrt(5)
 + 10)*log(abs(-sqrt(-2*sqrt(5) + 5) + tan(x)))
```

3.94 $\int \csc(6x)\sin(x)\,dx$

Optimal. Leaf size=36

$$\frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTanh[Sin[x]]/6 + ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rubi [A] time = 0.0463048, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 207}

$$\frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[6*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sin[x]]/6 + ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(6x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \sin(x)\right)\\
&= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \sin(x)\right)+\text{Subst}\left(\int \frac{1}{-3+4
\end{aligned}$$

$$= \frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.0374529, size = 30, normalized size = 0.83

$$\frac{1}{6}\left(\tanh^{-1}(\sin(x))+\tanh^{-1}(2\sin(x))-\sqrt{3}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[6*x]*Sin[x],x]
```

```
[Out] (ArcTanh[Sin[x]] + ArcTanh[2*Sin[x]] - Sqrt[3]*ArcTanh[(2*Sin[x])/Sqrt[3]])
/6
```

Maple [A] time = 0.093, size = 47, normalized size = 1.3

$$\frac{\ln(1+\sin(x))}{12}-\frac{\ln(\sin(x)-1)}{12}+\frac{\ln(1+2\sin(x))}{12}-\frac{\ln(-1+2\sin(x))}{12}-\frac{\sqrt{3}}{6}\text{Artanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(6*x)*sin(x),x)
```

```
[Out] 1/12*ln(1+sin(x))-1/12*ln(sin(x)-1)+1/12*ln(1+2*sin(x))-1/12*ln(-1+2*sin(x)
)-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + integrate(-1/6*((cos(3
*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3
*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(
cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/12*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 2.73993, size = 231, normalized size = 6.42

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)+\frac{1}{12}\log(2\sin(x)+1)+\frac{1}{12}\log(\sin(x)+1)-\frac{1}{12}\log(-\sin(x)+1)-\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + 1
/12*log(2*sin(x) + 1) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) - 1/12
*log(-2*sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin(x)\csc(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*csc(6*x), x)
```

Giac [B] time = 1.19834, size = 92, normalized size = 2.56

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin\left(x\right)\right|}{\left|4\sqrt{3}+8\sin\left(x\right)\right|}\right)+\frac{1}{12}\log\left(\sin\left(x\right)+1\right)-\frac{1}{12}\log\left(-\sin\left(x\right)+1\right)+\frac{1}{12}\log\left(\left|2\sin\left(x\right)+1\right|\right)-\frac{1}{12}\log\left(\left|2\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + 1/
12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) + 1/12*log(abs(2*sin(x) + 1)) -
1/12*log(abs(2*sin(x) - 1))
```

3.95 $\int \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=8

$$x + 2\sin(x)\cos(x)$$

```
[Out] x + 2*Cos[x]*Sin[x]
```

Rubi [A] time = 0.0303075, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {385, 203}

$$x + 2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sin[3*x],x]
```

```
[Out] x + 2*Cos[x]*Sin[x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(x)\sin(3x)\,dx = \text{Subst}\left(\int \frac{3-x^2}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= 2\cos(x)\sin(x) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= x + 2\cos(x)\sin(x)$$

Mathematica [A] time = 0.0116813, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sin[3*x],x]
```

```
[Out] x + Sin[2*x]
```

Maple [A] time = 0.032, size = 9, normalized size = 1.1

$$x + 2\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x),x)
```

```
[Out] x+2*cos(x)*sin(x)
```

Maxima [A] time = 1.02185, size = 8, normalized size = 1.

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] x + sin(2*x)
```

Fricas [A] time = 2.42448, size = 28, normalized size = 3.5

$$2 \cos(x)\sin(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 2*cos(x)*sin(x) + x
```

Sympy [A] time = 1.68294, size = 5, normalized size = 0.62

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x)
```

```
[Out] x + sin(2*x)
```

Giac [A] time = 1.09955, size = 8, normalized size = 1.

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] x + sin(2*x)
```

3.96 $\int \csc(3x)\sin(6x)\,dx$

Optimal. Leaf size=8

$$\frac{2}{3}\sin(3x)$$

```
[Out] (2*Sin[3*x])/3
```

Rubi [A] time = 0.0135801, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4288, 2637}

$$\frac{2}{3}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[3*x]*Sin[6*x],x]
```

```
[Out] (2*Sin[3*x])/3
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(3x)\sin(6x)\,dx &= 2\int \cos(3x)\,dx\\ &= \frac{2}{3}\sin(3x)\end{aligned}$$

Mathematica [A] time = 0.0030499, size = 8, normalized size = 1.

$$\frac{2}{3}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[3*x]*Sin[6*x],x]
```

```
[Out] (2*Sin[3*x])/3
```

Maple [A] time = 0.012, size = 9, normalized size = 1.1

$$\frac{2}{3\csc(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(3*x)*sin(6*x),x)
```

```
[Out] 2/3/csc(3*x)
```

Maxima [A] time = 0.992151, size = 8, normalized size = 1.

$$\frac{2}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="maxima")
```

```
[Out] 2/3*sin(3*x)
```

Fricas [A] time = 2.47485, size = 19, normalized size = 2.38

$$\frac{2}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="fricas")
```

```
[Out] 2/3*sin(3*x)
```

Sympy [A] time = 4.92115, size = 7, normalized size = 0.88

$$\frac{2\sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x)
```

```
[Out] 2*sin(3*x)/3
```

Giac [A] time = 1.15465, size = 8, normalized size = 1.

$$\frac{2}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="giac")
```

```
[Out] 2/3*sin(3*x)
```

3.97 $\int \cos(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

```
[Out] -Cos[x]/2 - Cos[3*x]/6
```

Rubi [A] time = 0.0086686, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[2*x],x]
```

```
[Out] -Cos[x]/2 - Cos[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(2x)\,dx=-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Mathematica [A] time = 0.0050567, size = 15, normalized size = 1.

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[2*x],x]
```

```
[Out] -Cos[x]/2 - Cos[3*x]/6
```

Maple [A] time = 0.009, size = 7, normalized size = 0.5

$$-\frac{2\,(\cos(x))^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(2*x),x)
```

```
[Out] -2/3*cos(x)^3
```

Maxima [A] time = 0.994978, size = 15, normalized size = 1.

$$-\frac{1}{6}\cos(3\,x) - \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*x) - 1/2*cos(x)
```

Fricas [A] time = 2.29816, size = 20, normalized size = 1.33

$$-\frac{2}{3}\cos(x)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*cos(x)^3
```

Sympy [A] time = 0.891557, size = 22, normalized size = 1.47

$$-\frac{\sin(x)\sin(2x)}{3} - \frac{2\cos(x)\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x)
```

```
[Out] -sin(x)*sin(2*x)/3 - 2*cos(x)*cos(2*x)/3
```

Giac [A] time = 1.12472, size = 15, normalized size = 1.

$$-\frac{1}{6}\cos(3x) - \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/6*cos(3*x) - 1/2*cos(x)
```

3.98 $\int \cos(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rubi [A] time = 0.0084583, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(3x)\,dx=-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Mathematica [A] time = 0.0056631, size = 17, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[3*x],x]
```

```
[Out] -Cos[x]^2/2 - Cos[4*x]/8
```

Maple [A] time = 0.034, size = 14, normalized size = 0.8

$$-(\cos(x))^4 + \frac{(\cos(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(3*x),x)
```

```
[Out] -cos(x)^4+1/2*cos(x)^2
```

Maxima [A] time = 1.00115, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x) - \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

Fricas [A] time = 2.47686, size = 35, normalized size = 2.06

$$-\cos(x)^4 + \frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

Sympy [A] time = 0.618337, size = 22, normalized size = 1.29

$$-\frac{\sin(x)\sin(3x)}{8} - \frac{3\cos(x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x)
```

```
[Out] -sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

Giac [A] time = 1.08209, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x) - \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

3.99 $\int \cos(x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

```
[Out] -Cos[3*x]/6 - Cos[5*x]/10
```

Rubi [A] time = 0.0082679, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[4*x],x]
```

```
[Out] -Cos[3*x]/6 - Cos[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(4x)\,dx=-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Mathematica [A] time = 0.0057735, size = 17, normalized size = 1.

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[4*x],x]
```

```
[Out] -Cos[3*x]/6 - Cos[5*x]/10
```

Maple [A] time = 0.02, size = 14, normalized size = 0.8

$$-\frac{8\,\left(\cos\left(x\right)\right)^{5}}{5}+\frac{4\,\left(\cos\left(x\right)\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(4*x),x)
```

```
[Out] -8/5*cos(x)^5+4/3*cos(x)^3
```

Maxima [A] time = 1.00275, size = 18, normalized size = 1.06

$$-\frac{1}{10}\,\cos\left(5\,x\right)-\frac{1}{6}\,\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) - 1/6*cos(3*x)
```

Fricas [A] time = 2.50008, size = 41, normalized size = 2.41

$$-\frac{8}{5}\,\cos\left(x\right)^{5}+\frac{4}{3}\,\cos\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 4/3*cos(x)^3
```

Sympy [A] time = 0.689863, size = 22, normalized size = 1.29

$$-\frac{\sin(x)\sin(4x)}{15}-\frac{4\cos(x)\cos(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x)
```

```
[Out] -sin(x)*sin(4*x)/15 - 4*cos(x)*cos(4*x)/15
```

Giac [A] time = 1.11087, size = 18, normalized size = 1.06

$$-\frac{1}{10}\cos(5x)-\frac{1}{6}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) - 1/6*cos(3*x)
```

3.100 $\int \cos(x)\sin(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

```
[Out] Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.0302323, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[m*x],x]
```

```
[Out] Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(mx)\,dx &= \int \left(-\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((1+m)x)\right)dx\\ &= -\left(\frac{1}{2}\int \sin((1-m)x)\,dx\right)+\frac{1}{2}\int \sin((1+m)x)\,dx\\ &= \frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0491642, size = 26, normalized size = 0.74

$$\frac{\sin(x)\sin(mx)+m\cos(x)\cos(mx)}{1-m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[m*x],x]
```

```
[Out] (m*Cos[x]*Cos[m*x] + Sin[x]*Sin[m*x])/(1 - m^2)
```

Maple [A] time = 0.008, size = 28, normalized size = 0.8

$$-\frac{\cos\left((m-1)\,x\right)}{2\,m-2}-\frac{\cos\left((1+m)\,x\right)}{2+2\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(m*x),x)
```

```
[Out] -1/2*cos((m-1)*x)/(m-1)-1/2*cos((1+m)*x)/(1+m)
```

Maxima [A] time = 0.977066, size = 36, normalized size = 1.03

$$-\frac{\cos\left((m+1)x\right)}{2\,(m+1)}-\frac{\cos\left((m-1)x\right)}{2\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="maxima")
```

```
[Out] -1/2*cos((m + 1)*x)/(m + 1) - 1/2*cos((m - 1)*x)/(m - 1)
```

Fricas [A] time = 2.52271, size = 68, normalized size = 1.94

$$-\frac{m\cos\left(mx\right)\cos\left(x\right)+\sin\left(mx\right)\sin\left(x\right)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="fricas")
```

```
[Out] -(m*cos(m*x)*cos(x) + sin(m*x)*sin(x))/(m^2 - 1)
```

Sympy [A] time = 1.24566, size = 44, normalized size = 1.26

$$\begin{cases} \frac{\cos^2(x)}{2} & \text{for } m = -1 \\ -\frac{\cos^2(x)}{2} & \text{for } m = 1 \\ -\frac{m\cos(x)\cos(mx)}{m^2-1} - \frac{\sin(x)\sin(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x)
```

```
[Out] Piecewise((cos(x)**2/2, Eq(m, -1)), (-cos(x)**2/2, Eq(m, 1)), (-m*cos(x)*co
s(m*x)/(m**2 - 1) - sin(x)*sin(m*x)/(m**2 - 1), True))
```

Giac [A] time = 1.10258, size = 39, normalized size = 1.11

$$-\frac{\cos(mx+x)}{2(m+1)} - \frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="giac")
```

```
[Out] -1/2*cos(m*x + x)/(m + 1) - 1/2*cos(m*x - x)/(m - 1)
```

3.101 $\int \cos(x)\cos(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Rubi [A] time = 0.008827, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x],x]
```

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(2x)\,dx=\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.0048753, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x],x]
```

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Maple [A] time = 0.018, size = 12, normalized size = 0.8

$$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x),x)
```

```
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

Maxima [A] time = 0.979609, size = 15, normalized size = 1.

$$\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A] time = 2.33741, size = 39, normalized size = 2.6

$$\frac{1}{3}\left(2\cos(x)^2+1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A] time = 0.786458, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3} + \frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x)
```

```
[Out] -sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

Giac [A] time = 1.10561, size = 15, normalized size = 1.

$$\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

3.102 $\int \cos(x)\cos(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

```
[Out] Sin[2*x]/4 + Sin[4*x]/8
```

Rubi [A] time = 0.0088595, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[3*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[4*x]/8
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

Mathematica [A] time = 0.0057414, size = 17, normalized size = 1.

$$\frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[3*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[4*x]/8
```

Maple [A] time = 0.021, size = 14, normalized size = 0.8

$$\frac{\sin(2x)}{4} + \frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(3*x),x)
```

```
[Out] 1/4*sin(2*x)+1/8*sin(4*x)
```

Maxima [A] time = 0.986717, size = 18, normalized size = 1.06

$$\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/8*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.36121, size = 23, normalized size = 1.35

$$\cos(x)^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] cos(x)^3*sin(x)
```

Sympy [A] time = 0.813564, size = 20, normalized size = 1.18

$$-\frac{\sin(x)\cos(3x)}{8}+\frac{3\sin(3x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x)
```

```
[Out] -sin(x)*cos(3*x)/8 + 3*sin(3*x)*cos(x)/8
```

Giac [A] time = 1.10823, size = 18, normalized size = 1.06

$$\frac{1}{8}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/8*sin(4*x) + 1/4*sin(2*x)
```

3.103 $\int \cos(x) \cos(4x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rubi [A] time = 0.0083703, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[4*x],x]
```

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x) \cos(4x) \, dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

Mathematica [A] time = 0.0052487, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[4*x],x]
```

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Maple [A] time = 0.041, size = 14, normalized size = 0.8

$$\frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(4*x),x)
```

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Maxima [A] time = 1.00794, size = 18, normalized size = 1.06

$$\frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

Fricas [A] time = 2.26851, size = 59, normalized size = 3.47

$$\frac{1}{15}\left(24\cos(x)^4 - 8\cos(x)^2 - 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.558477, size = 20, normalized size = 1.18

$$-\frac{\sin(x)\cos(4x)}{15}+\frac{4\sin(4x)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x)
```

```
[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

Giac [A] time = 1.1551, size = 18, normalized size = 1.06

$$\frac{1}{10}\sin(5x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

3.104 $\int \cos(x)\cos(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((m+1)x)}{2(m+1)}$$

```
[Out] Sin[(1 - m)*x]/(2*(1 - m)) + Sin[(1 + m)*x]/(2*(1 + m))
```

Rubi [A] time = 0.0288736, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4570, 2637}

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[m*x],x]
```

```
[Out] Sin[(1 - m)*x]/(2*(1 - m)) + Sin[(1 + m)*x]/(2*(1 + m))
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(mx)\,dx &= \int \left(\frac{1}{2}\cos((1-m)x)+\frac{1}{2}\cos((1+m)x)\right)dx\\ &= \frac{1}{2}\int \cos((1-m)x)\,dx+\frac{1}{2}\int \cos((1+m)x)\,dx\\ &= \frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0394965, size = 25, normalized size = 0.71

$$\frac{m\cos(x)\sin(mx) - \sin(x)\cos(mx)}{m^2 - 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[m*x],x]
```

```
[Out] (-(Cos[m*x]*Sin[x]) + m*Cos[x]*Sin[m*x])/(-1 + m^2)
```

Maple [A] time = 0.012, size = 28, normalized size = 0.8

$$\frac{\sin((m-1)x)}{2m-2} + \frac{\sin((1+m)x)}{2+2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(m*x),x)
```

```
[Out] 1/2/(m-1)*sin((m-1)*x)+1/2*sin((1+m)*x)/(1+m)
```

Maxima [A] time = 1.01591, size = 38, normalized size = 1.09

$$\frac{\sin((m+1)x)}{2(m+1)} - \frac{\sin(-(m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="maxima")
```

```
[Out] 1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

Fricas [A] time = 2.42915, size = 66, normalized size = 1.89

$$\frac{m\cos(x)\sin(mx) - \cos(mx)\sin(x)}{m^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="fricas")
```

```
[Out] (m*cos(x)*sin(m*x) - cos(m*x)*sin(x))/(m^2 - 1)
```

Sympy [A] time = 4.91142, size = 56, normalized size = 1.6

$$\begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = -1 \vee m = 1 \\ \frac{m\sin(mx)\cos(x)}{m^2-1} - \frac{\sin(x)\cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x)
```

```
[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1) | Eq(
m, 1)), (m*sin(m*x)*cos(x)/(m**2 - 1) - sin(x)*cos(m*x)/(m**2 - 1), True))
```

Giac [A] time = 1.11229, size = 39, normalized size = 1.11

$$\frac{\sin(mx+x)}{2(m+1)} + \frac{\sin(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="giac")
```

```
[Out] 1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

3.105 $\int \cos(x)\tan(2x)\,dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}-\cos(x)$$

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2] - Cos[x]
```

Rubi [A] time = 0.0254347, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2] - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\tan(2x)\,dx &= -\operatorname{Subst}\left(\int \frac{2x^2}{-1+2x^2}\,dx, x, \cos(x)\right)\\
&= -\left(2\operatorname{Subst}\left(\int \frac{x^2}{-1+2x^2}\,dx, x, \cos(x)\right)\right)\\
&= -\cos(x) - \operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}} - \cos(x)
\end{aligned}
$$

Mathematica [C] time = 0.23959, size = 183, normalized size = 9.15

$$
\frac{-4\sqrt{2}\cos(x) + 4\tanh^{-1}\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}\right) - \log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right) + \log\left(-\sqrt{2}\sin(x)+\sqrt{2}\cos(x)+2\right) + 2i\,\text{ta}}{4\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Tan[2*x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - 4*Sqrt[2]*Cos[
x] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sq
rt[2]*Sin[x]])/(4*Sqrt[2])
```

Maple [A] time = 0.015, size = 18, normalized size = 0.9

$$
-\cos(x) + \frac{\operatorname{Artanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(2*x),x)
```

```
[Out] -cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.57688, size = 180, normalized size = 9.

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin\left(2x\right)\sin\left(x\right)+2\left(\sqrt{2}\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+2\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}+2\sin\left(x\right)^{2}+2\sqrt{2}\,\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1) - cos(x)
```

Fricas [B] time = 2.82491, size = 109, normalized size = 5.45

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^{2}+2\sqrt{2}\cos\left(x\right)+1}{2\cos\left(x\right)^{2}-1}\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - co
s(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(x\right)\tan\left(2x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x)
```

```
[Out] Integral(cos(x)*tan(2*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(2*x), x)
```

3.106 $\int \cos(x)\tan(3x)\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

```
[Out] ArcTanh[(2*Cos[x])/Sqrt[3]]/Sqrt[3] - Cos[x]
```

Rubi [A] time = 0.0242572, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[3*x],x]
```

```
[Out] ArcTanh[(2*Cos[x])/Sqrt[3]]/Sqrt[3] - Cos[x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(x)\tan(3x)\,dx = -\text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \cos(x)\right)$$
$$= -\cos(x) + 2\,\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \cos(x)\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

Mathematica [B] time = 0.0527161, size = 48, normalized size = 2.29

$$-\cos(x) - \frac{\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-2}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Tan[3*x],x]
```

```
[Out] -(ArcTanh[(-2 + Tan[x/2])/Sqrt[3]]/Sqrt[3]) + ArcTanh[(2 + Tan[x/2])/Sqrt[3
]]/Sqrt[3] - Cos[x]
```

Maple [A] time = 0.022, size = 19, normalized size = 0.9

$$-\cos(x) + \frac{\sqrt{3}}{3}\text{Artanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(3*x),x)
```

```
[Out] -cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\cos(x) - \int \frac{(\sin(3x) - \sin(x))\cos(4x) - (\cos(3x) - \cos(x))\sin(4x) - (\cos(2x) - 1)\sin(3x) + \cos(3x)\sin(2x) - }{2(\cos(2x) - 1)\cos(4x) - \cos(4x)^2 - \cos(2x)^2 - \sin(4x)^2 + 2\sin(4x)\sin(2x) - \sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin
(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos
(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2
 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

Fricas [B] time = 2.85551, size = 109, normalized size = 5.19

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\cos(x)+3}{4\cos(x)^2-3}\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - co
s(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\tan(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x)
```

```
[Out] Integral(cos(x)*tan(3*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\tan(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(3*x), x)
```

3.107 $\int \cos(x)\tan(4x)\,dx$

Optimal. Leaf size=71

$$-\cos(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[2]]])/4 - Cos[x]
```

Rubi [A] time = 0.0831244, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1279, 1166, 207}

$$-\cos(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[2]]])/4 - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\tan(4x)\,dx &= -\text{Subst}\left(\int \frac{4x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\\
&= -\left(4\,\text{Subst}\left(\int \frac{x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\right)\\
&= -\cos(x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{2-8x^2}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\\
&= -\cos(x)+\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cos(x)\right)-\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}}\right.\\
&= \frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)-\cos(x)
\end{aligned}$$

Mathematica [C] time = 58.3466, size = 6196, normalized size = 87.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[4*x],x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.043, size = 68, normalized size = 1.

$$-\cos(x)+\frac{\sqrt{2}\left(\sqrt{2}-1\right)}{4\sqrt{2-\sqrt{2}}}\operatorname{Artanh}\left(2\frac{\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{\left(1+\sqrt{2}\right)\sqrt{2}}{4\sqrt{2+\sqrt{2}}}\operatorname{Artanh}\left(2\frac{\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(4*x),x)
```

```
[Out] -cos(x)+1/4*2^(1/2)*(2^(1/2)-1)/(2-2^(1/2))^(1/2)*arctanh(2*cos(x)/(2-2^(1/
2))^(1/2))+1/4*(1+2^(1/2))*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*cos(x)/(2+2^
(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\cos(x)-\int-\frac{(\sin(7x)-\sin(x))\cos(8x)-(\cos(7x)-\cos(x))\sin(8x)+\sin(7x)-\sin(x)}{\cos(8x)^2+\sin(8x)^2+2\cos(8x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(-((sin(7*x) - sin(x))*cos(8*x) - (cos(7*x) - cos(x))*si
n(8*x) + sin(7*x) - sin(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x)
```

Fricas [A] time = 2.78908, size = 329, normalized size = 4.63

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\cos(x)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\cos(x)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+2\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(sqrt(2)
+ 2)*log(sqrt(sqrt(2) + 2) - 2*cos(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-s
qrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2
```

```
*cos(x)) - cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\tan(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(4*x), x)
```

3.108 $\int \cos(x)\tan(5x)\,dx$

Optimal. Leaf size=84

$$-\cos(x)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)$$

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cos[x]])/5 + (Sqrt[(
5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cos[x]])/5 - Cos[x]
```

Rubi [A] time = 0.0979321, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$-\cos(x)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[5*x],x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cos[x]])/5 + (Sqrt[(
5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cos[x]])/5 - Cos[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\tan(5x)\,dx &= -\operatorname{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx,x,\cos(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(1-\frac{4\left(1-2x^2\right)}{5-20x^2+16x^4}\right)dx,x,\cos(x)\right)\\ &= -\cos(x)+4\operatorname{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx,x,\cos(x)\right)\\ &= -\cos(x)-\frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx,x,\cos(x)\right)-\frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)\operatorname{Subst}\left(\int\right.\\ &= \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)-\mathrm{c}\end{aligned}$$

Mathematica [B] time = 0.597829, size = 215, normalized size = 2.56

$$-\cos(x)+\frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\frac{4-\left(\sqrt{5}-1\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\left(5+\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\frac{\left(\sqrt{5}-1\right)\tan\left(\frac{x}{2}\right)+4}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\left(5+\sqrt{5}\right)}}+\frac{\left(\sqrt{5}-1\right)\tanh^{-1}\left(\frac{4-\left(1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{50-10\sqrt{5}}}+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[5*x],x]
```

```
[Out] ((1 + Sqrt[5])*ArcTanh[(4 - (-1 + Sqrt[5])*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]]
)/Sqrt[10*(5 + Sqrt[5])] + ((1 + Sqrt[5])*ArcTanh[(4 + (-1 + Sqrt[5])*Tan[x
/2])/Sqrt[2*(5 + Sqrt[5])]])/Sqrt[10*(5 + Sqrt[5])] + ((-1 + Sqrt[5])*ArcTa
nh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]]])/Sqrt[50 - 10*Sqrt[5]
] + ((-1 + Sqrt[5])*ArcTanh[(4 + (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5
]]])/Sqrt[50 - 10*Sqrt[5]] - Cos[x]
```

Maple [A] time = 0.036, size = 72, normalized size = 0.9

$$-\cos(x)+\frac{(\sqrt{5}-1)\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\operatorname{Artanh}\left(4\frac{\cos(x)}{\sqrt{10-2\sqrt{5}}}\right)+\frac{(\sqrt{5}+1)\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\operatorname{Artanh}\left(4\frac{\cos(x)}{\sqrt{10+2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(5*x),x)
```

```
[Out] -cos(x)+1/5*(5^(1/2)-1)*5^(1/2)/(10-2*5^(1/2))^(1/2)*arctanh(4*cos(x)/(10-2
*5^(1/2))^(1/2))+1/5*(5^(1/2)+1)*5^(1/2)/(10+2*5^(1/2))^(1/2)*arctanh(4*cos
(x)/(10+2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*x) + (
sin(6*x) - sin(4*x) + sin(2*x))*cos(7*x) + (sin(5*x) - sin(3*x) + sin(x))*c
os(6*x) + (sin(4*x) - sin(2*x))*cos(5*x) + (sin(3*x) - sin(x))*cos(4*x) - (
cos(7*x) - cos(5*x) + cos(3*x) - cos(x))*sin(8*x) - (cos(6*x) - cos(4*x) +
cos(2*x) - 1)*sin(7*x) - (cos(5*x) - cos(3*x) + cos(x))*sin(6*x) - (cos(4*x
) - cos(2*x) + 1)*sin(5*x) - (cos(3*x) - cos(x))*sin(4*x) - (cos(2*x) - 1)*
sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)*sin(x) - sin(x))/
(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x)
 - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*
x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^
2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)
*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

Fricas [B] time = 2.63353, size = 421, normalized size = 5.01

$$\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\cos(x)\right)-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\cos(x)\right)+\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*cos(x)) -
1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*cos(x)) +
1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*cos(x))
- 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*cos(x)
) - cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\tan(5x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(5*x), x)
```

3.109 $\int \cos(x)\tan(6x)\,dx$

Optimal. Leaf size=89

$$-\cos(x)+\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cos[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqr
t[3]]])/6 - Cos[x]
```

Rubi [A] time = 0.238226, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 207, 1166}

$$-\cos(x)+\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[6*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cos[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqr
t[3]]])/6 - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
```

```
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\tan(6x)\,dx &= -\operatorname{Subst}\left(\int \frac{2x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)\\
&= -\left(2\operatorname{Subst}\left(\int \frac{x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)\right)\\
&= -\left(2\operatorname{Subst}\left(\int \left(\frac{1}{2}-\frac{1-12x^2+16x^4}{2\left(1-18x^2+48x^4-32x^6\right)}\right)dx, x, \cos(x)\right)\right)\\
&= -\cos(x)+\operatorname{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)\\
&= -\cos(x)+\operatorname{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}-\frac{2\left(-1+8x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \cos(x)\right)\\
&= -\cos(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx, x, \cos(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}-\cos(x)-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx, x, \cos(x)\right)-\frac{1}{3}\left(4\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)-\cos(x)
\end{aligned}
$$

Mathematica [C] time = 8.92697, size = 679, normalized size = 7.63

$$-\cos(x) + \left(-\frac{1}{6} - \frac{i}{6}\right)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt[4]{-1}\sec\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right) + \left(\frac{1}{6} + \frac{i}{6}\right)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[6*x],x]
```

```
[Out] (-1/6 - I/6)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*Sec[x/2]*(Cos[x/2] +
Sin[x/2])] + (1/6 + I/6)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[x/2]
*(Cos[x/2] - Sin[x/2])] - Cos[x] - ((1 + Sqrt[2])*(x - 2*Sqrt[3]*ArcTanh[(2
 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(S
qrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(12*(2 + Sqrt[2])) + (x + 2*Sqrt[3]*ArcTa
nh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]^2] + Log[Sec
[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])])/(12*Sqrt[2]) - ((2*(-2 + Sq
rt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*Sqrt[2] - 2*Sqr
t[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sqrt[2]*Cos[x] - S
qrt[2]*Sin[x]))]))*(Sqrt[2] - Sqrt[3]*Sin[x])*(-3 + Sqrt[6] - (-2 + Sqrt[6]
)*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/(12*(-36 + 15*Sqrt[6] + (20 - 8*Sqrt[6])
*Cos[x] + (12 - 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] + 20*Sqrt[6]*Sin[x] + 12*Si
n[2*x] - 5*Sqrt[6]*Sin[2*x])) + ((-2*(Sqrt[2] + Sqrt[3])*ArcTanh[(2 + (2 +
Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x - Log[Sec[x/2]^2] + Log[-(Se
c[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(2 + Sqrt[6]*Sin[x])*(3 + Sqrt
[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x]))/(12*(-36 - 15*Sqrt[6] +
 4*(5 + 2*Sqrt[6])*Cos[x] + (12 + 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] - 20*Sqrt
[6]*Sin[x] + 12*Sin[2*x] + 5*Sqrt[6]*Sin[2*x]))
```

Maple [A] time = 0.049, size = 104, normalized size = 1.2

$$-\cos(x) + \frac{\left(-6+4\sqrt{3}\right)\sqrt{3}}{18\sqrt{6}-18\sqrt{2}}\mathrm{Artanh}\left(8\frac{\cos(x)}{2\sqrt{6}-2\sqrt{2}}\right) + \frac{\left(6+4\sqrt{3}\right)\sqrt{3}}{18\sqrt{6}+18\sqrt{2}}\mathrm{Artanh}\left(8\frac{\cos(x)}{2\sqrt{6}+2\sqrt{2}}\right) + \frac{\mathrm{Artanh}\left(\cos(x)\sqrt{2}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(6*x),x)
```

```
[Out] -cos(x)+2/9*(-3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)-2*2^(1/2))*arctanh(8*cos(x)/(
2*6^(1/2)-2*2^(1/2)))+2/9*(3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)+2*2^(1/2))*arcta
```

```
nh(8*cos(x)/(2*6^(1/2)+2*2^(1/2)))+1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+2\sqrt{2}\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
1) - 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*c
os(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 1) - cos(x) - integrate(1/3*((2*sin(7*x) + sin(5*x) - sin(3*x) - 2*si
n(x))*cos(8*x) + (sin(3*x) + 2*sin(x))*cos(4*x) - (2*cos(7*x) + cos(5*x) -
cos(3*x) - 2*cos(x))*sin(8*x) - 2*(cos(4*x) - 1)*sin(7*x) - (cos(4*x) - 1)*
sin(5*x) - (cos(3*x) + 2*cos(x))*sin(4*x) + 2*cos(7*x)*sin(4*x) + cos(5*x)*
sin(4*x) - sin(3*x) - 2*sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - c
os(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1)
, x)
```

Fricas [B] time = 2.75038, size = 435, normalized size = 4.89

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\cos(x)\right)-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\cos(x)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\,\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(sqrt(3
) + 2)*log(sqrt(sqrt(3) + 2) - 2*cos(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt
(-sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2)
 - 2*cos(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos
(x)^2 - 1)) - cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\tan(6x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(6*x), x)
```

3.110 $\int \cos(x)\cot(2x)\,dx$

Optimal. Leaf size=10

$$\cos(x) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]/2 + Cos[x]
```

Rubi [A] time = 0.0203756, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 388, 206}

$$\cos(x) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[2*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(2x)\,dx &= -\operatorname{Subst}\left(\int \frac{-1+2x^2}{2\left(1-x^2\right)}\,dx, x, \cos(x)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+2x^2}{1-x^2}\,dx, x, \cos(x)\right)\right)\\ &= \cos(x) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x)) + \cos(x)\end{aligned}$$

Mathematica [B] time = 0.0149474, size = 25, normalized size = 2.5

$$\cos(x) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[2*x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2
```

Maple [A] time = 0.025, size = 14, normalized size = 1.4

$$\cos(x) + \frac{\ln(\csc(x) - \cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(2*x),x)
```

```
[Out] cos(x)+1/2*ln(csc(x)-cot(x))
```

Maxima [B] time = 0.991714, size = 50, normalized size = 5.

$$\cos(x) - \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="maxima")
```

```
[Out] cos(x) - 1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1)
```

Fricas [B] time = 2.51684, size = 88, normalized size = 8.8

$$\cos\left(x\right) - \frac{1}{4} \, \log\left(\frac{1}{2} \, \cos\left(x\right) + \frac{1}{2}\right) + \frac{1}{4} \, \log\left(-\frac{1}{2} \, \cos\left(x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 1.48852, size = 19, normalized size = 1.9

$$\frac{\log\left(\cos\left(x\right) - 1\right)}{4} - \frac{\log\left(\cos\left(x\right) + 1\right)}{4} + \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)
```

Giac [B] time = 1.13916, size = 26, normalized size = 2.6

$$\cos\left(x\right) - \frac{1}{4} \, \log\left(\cos\left(x\right) + 1\right) + \frac{1}{4} \, \log\left(-\cos\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

3.111 $\int \cos(x)\cot(3x)\,dx$

Optimal. Leaf size=45

$$\cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(\cos(x)+1)-\frac{1}{6}\log(2\cos(x)+1)$$

```
[Out] Cos[x] + Log[1 - 2*Cos[x]]/6 + Log[1 - Cos[x]]/6 - Log[1 + Cos[x]]/6 - Log[
1 + 2*Cos[x]]/6
```

Rubi [A] time = 0.0529215, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1279, 1161, 616, 31}

$$\cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(\cos(x)+1)-\frac{1}{6}\log(2\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[3*x],x]
```

```
[Out] Cos[x] + Log[1 - 2*Cos[x]]/6 + Log[1 - Cos[x]]/6 - Log[1 + Cos[x]]/6 - Log[
1 + 2*Cos[x]]/6
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\cot(3x)\,dx &= -\text{Subst}\left(\int \frac{x^2\left(3-4x^2\right)}{1-5x^2+4x^4}\,dx,x,\cos(x)\right)\\
&= \cos(x)+\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx,x,\cos(x)\right)\\
&= \cos(x)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx,x,\cos(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx,x,\cos(x)\right)\\
&= \cos(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\cos(x)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx,x,\cos(x)\right)-\frac{1}{6}\text{Subst}\left(\int \frac{}{\frac{1}{2}}\right.\\
&= \cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(1+\cos(x))-\frac{1}{6}\log(1+2\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.0189685, size = 47, normalized size = 1.04

$$\cos(x)+\frac{1}{3}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cos(x))-\frac{1}{6}\log(2\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[3*x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]]/3 + Log[1 - 2*Cos[x]]/6 - Log[1 + 2*Cos[x]]/6 + Log[
Sin[x/2]]/3
```

Maple [A] time = 0.101, size = 36, normalized size = 0.8

$$-\frac{\ln(1+\cos(x))}{6}+\frac{\ln(-1+\cos(x))}{6}-\frac{\ln(1+2\cos(x))}{6}+\frac{\ln(2\cos(x)-1)}{6}+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(3*x),x)
```

```
[Out] -1/6*ln(1+cos(x))+1/6*ln(-1+cos(x))-1/6*ln(1+2*cos(x))+1/6*ln(2*cos(x)-1)+c
os(x)
```

Maxima [B] time = 1.50595, size = 177, normalized size = 3.93

$$\cos(x)-\frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x)+\cos(2x)^2+\cos(x)^2+\sin(2x)^2+2\sin(2x)\sin(x)+\sin(x)^2+2\cos(x)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="maxima")
```

```
[Out] cos(x) - 1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x
)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(-2*(cos(x) -
1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(
x)^2 - 2*cos(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*lo
g(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 2.52134, size = 155, normalized size = 3.44

$$\cos(x)-\frac{1}{6}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{6}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{6}\log(-2\cos(x)+1)-\frac{1}{6}\log(-2\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/6*log(1/2*cos(x) + 1/2) + 1/6*log(-1/2*cos(x) + 1/2) + 1/6*log(-
2*cos(x) + 1) - 1/6*log(-2*cos(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\cot(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x)
```

```
[Out] Integral(cos(x)*cot(3*x), x)
```

Giac [A] time = 1.14194, size = 53, normalized size = 1.18

$$\cos(x) - \frac{1}{6}\log(\cos(x)+1) + \frac{1}{6}\log(-\cos(x)+1) - \frac{1}{6}\log(|2\cos(x)+1|) + \frac{1}{6}\log(|2\cos(x)-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/6*log(cos(x) + 1) + 1/6*log(-cos(x) + 1) - 1/6*log(abs(2*cos(x)
+ 1)) + 1/6*log(abs(2*cos(x) - 1))
```

3.112 $\int \cos(x)\cot(4x)\,dx$

Optimal. Leaf size=28

$$\cos(x)-\frac{1}{4}\tanh^{-1}(\cos(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

```
[Out] -ArcTanh[Cos[x]]/4 - ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) + Cos[x]
```

Rubi [A] time = 0.0485803, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\cos(x)-\frac{1}{4}\tanh^{-1}(\cos(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[4*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/4 - ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) + Cos[x]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\cot(4x)\,dx &= -\text{Subst}\left(\int \frac{-1+8x^2-8x^4}{4-12x^2+8x^4}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(-1+\frac{3-4x^2}{4-12x^2+8x^4}\right)dx, x, \cos(x)\right)\\
&= \cos(x) - \text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx, x, \cos(x)\right)\\
&= \cos(x) + 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \cos(x)\right) + 2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \cos(x)\right)\\
&= -\frac{1}{4}\tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}} + \cos(x)
\end{aligned}
$$

Mathematica [C] time = 0.0662992, size = 73, normalized size = 2.61

$$
\frac{1}{4}\left(4\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)-(1-i)\sqrt[4]{-1}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[4*x],x]
```

```
[Out] ((-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (1 - I)*(-1)^(1/4)*
ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + 4*Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]])
/4
```

Maple [A] time = 0.134, size = 30, normalized size = 1.1

$$
-\frac{\text{Artanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln(1+\cos(x))}{8}+\frac{\ln(-1+\cos(x))}{8}+\cos(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(4*x),x)
```

```
[Out] -1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*ln(1+cos(x))+1/8*ln(-1+cos(x))+cos
(x)
```

Maxima [B] time = 1.54073, size = 223, normalized size = 7.96

$$-\frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+2\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) + 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*co
s(x) + 1) + cos(x) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [B] time = 2.57622, size = 185, normalized size = 6.61

$$\frac{1}{8}\sqrt{2}\log\left(\frac{2\cos(x)^2-2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)+\cos(x)-\frac{1}{8}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{8}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) + cos
(x) - 1/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos(x)\cot(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x)
```

```
[Out] Integral(cos(x)*cot(4*x), x)
```

Giac [B] time = 1.13957, size = 68, normalized size = 2.43

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\cos\left(x\right)\right|}{\left|2\sqrt{2}+4\cos\left(x\right)\right|}\right)+\cos\left(x\right)-\frac{1}{8}\log\left(\cos\left(x\right)+1\right)+\frac{1}{8}\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) + cos
(x) - 1/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

3.113 $\int \cos(x)\cot(5x)\,dx$

Optimal. Leaf size=110

$$\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cos(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cos(x)-\right.$$

```
[Out] -ArcTanh[Cos[x]]/5 + Cos[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[x]])/2
0 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]])/20 - ((1 - Sqrt[5])*Log[1 -
 Sqrt[5] + 4*Cos[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[x]])/20
```

Rubi [A] time = 0.15549, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2075, 207, 632, 31}

$$\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cos(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cos(x)-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[5*x],x]
```


```
[Out] -ArcTanh[Cos[x]]/5 + Cos[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[x]])/2
0 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]])/20 - ((1 - Sqrt[5])*Log[1 -
 Sqrt[5] + 4*Cos[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[x]])/20
```


Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(5x)\,dx &= -\text{Subst}\left(\int \frac{x^2\left(5-20x^2+16x^4\right)}{1-13x^2+28x^4-16x^6}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)dx, x, \cos(x)\right)\\ &= \cos(x)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \cos(x)\right)-\frac{2}{5}\text{Sub}\\ &= -\frac{1}{5}\tanh^{-1}(\cos(x))+\cos(x)-\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \cos(x)\right)+\frac{1}{5}\left(1-\sqrt{5}\right)\text{Sub}\\ &= -\frac{1}{5}\tanh^{-1}(\cos(x))+\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-\right.\end{aligned}$$

Mathematica [A] time = 0.125386, size = 133, normalized size = 1.21

$$\frac{1}{100}\left(100\cos(x)+20\log\left(\sin\left(\frac{x}{2}\right)\right)-20\log\left(\cos\left(\frac{x}{2}\right)\right)+\sqrt{5}\left(\sqrt{5}-5\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\sqrt{5}\left(5+\sqrt{5}\right)\log\left(-4\,\text{co}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[5*x],x]
```

```
[Out] (100*Cos[x] - 20*Log[Cos[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] - 4
*Cos[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]] - Sqrt[5]*(-5
+ Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt
[5] + 4*Cos[x]] + 20*Log[Sin[x/2]])/100
```

Maple [A] time = 0.153, size = 82, normalized size = 0.8

$$\frac{\ln\left(4\ (\cos(x))^2-2\ \cos(x)-1\right)}{20}-\frac{\sqrt{5}}{10}\mathrm{Artanh}\left(\frac{(8\ \cos(x)-2)\sqrt{5}}{10}\right)-\frac{\ln(1+\cos(x))}{10}+\frac{\ln(-1+\cos(x))}{10}-\frac{\ln\left(4\ (\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(5*x),x)
```

```
[Out] 1/20*ln(4*cos(x)^2-2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*(8*cos(x)-2)*5^(1/
2))-1/10*ln(1+cos(x))+1/10*ln(-1+cos(x))-1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/1
0*5^(1/2)*arctanh(1/10*(8*cos(x)+2)*5^(1/2))+cos(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="maxima")
```

```
[Out] cos(x) + 1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*a
rctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))
*sin(2*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/
2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos
(4*x)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), c
os(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2
*x), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), c
os(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x
)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x)
, cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*
x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*
x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*
integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x)
, cos(2*x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos
(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2
*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2
*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*
cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))
^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos
(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + si
```

```
n(2*x)^2 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*s
in(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^
2 + 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*
arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(
x)*sin(4*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x
)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*
cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2
*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + s
in(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2
 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) - 1/10*inte
grate(-(cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(
x) + cos(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) +
cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - co
s(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) -
sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) -
 sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x)
 + 3/10*integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3
*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)
))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(
2/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3
*x))) + sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos
(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x
), cos(3*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(
1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x)))
 + cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2
(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2
 + 2*(sin(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin
(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(s
in(3*x), cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))
*sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x))
)^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin
(3*x), cos(3*x)))^2 + 2*cos(3*x) + 1), x) + 3/10*integrate(-(cos(4/3*arctan
2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))*sin(
3*x) - cos(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - cos(3*x)*sin(4/3*arc
tan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) +
 cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*
(cos(3*x) - cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x)
, cos(3*x))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(si
n(3*x), cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))) -
 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*
x)))^2 + 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*ar
ctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2
(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x
) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x
```

```
))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*cos(3*x) +
 1), x) + 1/5*integrate((sin(4*x) + sin(3*x) + sin(2*x) + sin(x))/(2*(cos(3
*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) +
 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2
 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + s
in(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 +
2*cos(x) + 1), x) - 1/5*integrate(-(sin(4*x) - sin(3*x) + sin(2*x) - sin(x)
)/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x)
 - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2
 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(s
in(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) -
sin(x)^2 + 2*cos(x) - 1), x) - 1/10*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
 + 1/10*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 2.60458, size = 466, normalized size = 4.24

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}-1\right)\cos\left(x\right)-8\cos\left(x\right)^{2}+\sqrt{5}-3}{4\cos\left(x\right)^{2}+2\cos\left(x\right)-1}\right)+\frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}+1\right)\cos\left(x\right)-8\cos\left(x\right)^{2}-\sqrt{5}-3}{4\cos\left(x\right)^{2}-2\cos\left(x\right)-1}\right)+\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log(-(4*(sqrt(5) - 1)*cos(x) - 8*cos(x)^2 + sqrt(5) - 3)/(4*co
s(x)^2 + 2*cos(x) - 1)) + 1/20*sqrt(5)*log(-(4*(sqrt(5) + 1)*cos(x) - 8*cos
(x)^2 - sqrt(5) - 3)/(4*cos(x)^2 - 2*cos(x) - 1)) + cos(x) - 1/20*log(4*cos
(x)^2 + 2*cos(x) - 1) + 1/20*log(4*cos(x)^2 - 2*cos(x) - 1) - 1/10*log(1/2*
cos(x) + 1/2) + 1/10*log(-1/2*cos(x) + 1/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19695, size = 158, normalized size = 1.44

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos\left(x\right)+2\right|}{\left|2\sqrt{5}+8\cos\left(x\right)+2\right|}\right)+\frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos\left(x\right)-2\right|}{\left|2\sqrt{5}+8\cos\left(x\right)-2\right|}\right)+\cos\left(x\right)-\frac{1}{10}\log\left(\cos\left(x\right)+1\right)+\frac{1}{10}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) + 2)/abs(2*sqrt(5) + 8*cos(x) +
2)) + 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) - 2)/abs(2*sqrt(5) + 8*cos
(x) - 2)) + cos(x) - 1/10*log(cos(x) + 1) + 1/10*log(-cos(x) + 1) - 1/20*lo
g(abs(4*cos(x)^2 + 2*cos(x) - 1)) + 1/20*log(abs(4*cos(x)^2 - 2*cos(x) - 1)
)
```

3.114 $\int \cos(x)\cot(6x)\,dx$

Optimal. Leaf size=38

$$\cos(x)-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[Cos[x]]/6 - ArcTanh[2*Cos[x]]/6 - ArcTanh[(2*Cos[x])/Sqrt[3]]/(2*S
qrt[3]) + Cos[x]
```

Rubi [A] time = 0.0712837, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 207}

$$\cos(x)-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[6*x],x]
```


```
[Out] -ArcTanh[Cos[x]]/6 - ArcTanh[2*Cos[x]]/6 - ArcTanh[(2*Cos[x])/Sqrt[3]]/(2*S
qrt[3]) + Cos[x]
```


Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(6x)\,dx &= -\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx,x,\cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{3-19x^2+32x^4-16x^6}\,dx,x,\cos(x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int\left(-2-\frac{1}{3\left(-1+x^2\right)}-\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx,x,\cos(x)\right)\right)\\ &= \cos(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\cos(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx,x,\cos(x)\right)+\text{Subst}\left(\int \frac{}{}\right.\\ &= -\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\cos(x)\end{aligned}$$

Mathematica [B] time = 0.0860789, size = 87, normalized size = 2.29

$$\frac{1}{12}\left(12\cos(x)+2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(1-2\cos(x))-\log(2\cos(x)+1)+2\sqrt{3}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-2}{\sqrt{3}}\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[6*x],x]
```

```
[Out] (2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + Tan[x/
2])/Sqrt[3]] + 12*Cos[x] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*
Cos[x]] + 2*Log[Sin[x/2]])/12
```

Maple [A] time = 0.2, size = 49, normalized size = 1.3

$$-\frac{\ln(1+\cos(x))}{12}+\frac{\ln(-1+\cos(x))}{12}-\frac{\ln(1+2\cos(x))}{12}+\frac{\ln(2\cos(x)-1)}{12}-\frac{\sqrt{3}}{6}\text{Artanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(6*x),x)
```

```
[Out] -1/12*ln(1+cos(x))+1/12*ln(-1+cos(x))-1/12*ln(1+2*cos(x))+1/12*ln(2*cos(x)-
1)-1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)+cos(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\cos(x) + \int \frac{(\sin(3x) - \sin(x))\cos(4x) - (\cos(3x) - \cos(x))\sin(4x) - (\cos(2x) - 1)\sin(3x) + \cos(3x)\sin(2x) - \cdots}{2\left(2(\cos(2x) - 1)\cos(4x) - \cos(4x)^2 - \cos(2x)^2 - \sin(4x)^2 + 2\sin(4x)\sin(2x) - \mathrm{si}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="maxima")
```

```
[Out] cos(x) + integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*
sin(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) +
cos(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x
)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) -
 1/24*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [B] time = 2.61589, size = 259, normalized size = 6.82

$$\frac{1}{12}\sqrt{3}\log\left(\frac{4\cos(x)^2 - 4\sqrt{3}\cos(x) + 3}{4\cos(x)^2 - 3}\right) + \cos(x) - \frac{1}{12}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log(-2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((4*cos(x)^2 - 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) + co
s(x) - 1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(
-2*cos(x) + 1) - 1/12*log(-2*cos(x) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14683, size = 95, normalized size = 2.5

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos\left(x\right)\right|}{\left|4\sqrt{3}+8\cos\left(x\right)\right|}\right)+\cos\left(x\right)-\frac{1}{12}\log\left(\cos\left(x\right)+1\right)+\frac{1}{12}\log\left(-\cos\left(x\right)+1\right)-\frac{1}{12}\log\left(\left|2\cos\left(x\right)+1\right|\right)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) + co
s(x) - 1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x)
 + 1)) + 1/12*log(abs(2*cos(x) - 1))
```

3.115 $\int \cos(x)\cot(nx)\,dx$

Optimal. Leaf size=92

$$e^{-ix}\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2inx}\right)-e^{ix}\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(\frac{1}{n}+2\right),e^{2inx}\right)-\frac{e^{-ix}}{2}+\frac{e^{ix}}{2}$$

```
[Out] -1/(2*E^(I*x)) + E^(I*x)/2 + Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), E^
((2*I)*n*x)]/E^(I*x) - E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, E^((2*I)*n*x)]
```

Rubi [A] time = 0.0908161, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4558, 2194, 2251}

$$e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)-\frac{e^{-ix}}{2}+\frac{e^{ix}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[n*x],x]
```


```
[Out] -1/(2*E^(I*x)) + E^(I*x)/2 + Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), E^
((2*I)*n*x)]/E^(I*x) - E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, E^((2*I)*n*x)]
```


Rule 4558

```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I/(E^
(I*(a + b*x))*2) + (I*E^(I*(a + b*x)))/2 - I/(E^(I*(a + b*x))*(1 - E^(2*I*(
c + d*x)))) - (I*E^(I*(a + b*x)))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(nx)\,dx &= \int \left(\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}-\frac{ie^{-ix}}{1-e^{2inx}}-\frac{ie^{ix}}{1-e^{2inx}}\right)dx\\ &= \frac{1}{2}i\int e^{-ix}\,dx+\frac{1}{2}i\int e^{ix}\,dx-i\int \frac{e^{-ix}}{1-e^{2inx}}\,dx-i\int \frac{e^{ix}}{1-e^{2inx}}\,dx\\ &= -\frac{1}{2}e^{-ix}+\frac{e^{ix}}{2}+e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)\end{aligned}$$

Mathematica [A] time = 0.190771, size = 179, normalized size = 1.95

$$\frac{1}{2}e^{-2ix}\left(-\frac{e^{i(2nx+x)}\text{Hypergeometric2F1}\left(1,1-\frac{1}{2n},2-\frac{1}{2n},e^{2inx}\right)}{2n-1}-\frac{e^{i(2n+3)x}\text{Hypergeometric2F1}\left(1,\frac{1}{2n}+1,\frac{1}{2n}+2,e^{2inx}\right)}{2n+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[n*x],x]
```

```
[Out] (-((E^(I*(x + 2*n*x))*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), E^((2*
I)*n*x)])/(-1 + 2*n)) - (E^(I*(3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*n)
, 2 + 1/(2*n), E^((2*I)*n*x)])/(1 + 2*n) + E^(I*x)*Hypergeometric2F1[1, -1/
(2*n), 1 - 1/(2*n), E^((2*I)*n*x)] - E^((3*I)*x)*Hypergeometric2F1[1, 1/(2*
n), 1 + 1/(2*n), E^((2*I)*n*x)])/(2*E^((2*I)*x))
```

Maple [F] time = 0.223, size = 0, normalized size = 0.

$$\int \cos(x)\cot(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(n*x),x)
```

```
[Out] int(cos(x)*cot(n*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\cot(nx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*cot(n*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos(x)\cot(nx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="fricas")
```

```
[Out] integral(cos(x)*cot(n*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\cot(nx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x)
```

```
[Out] Integral(cos(x)*cot(n*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\cot(nx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*cot(n*x), x)
```

3.116 $\int \cos(x)\sec(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Rubi [A] time = 0.0147158, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4356, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(x)\sec(2x)\,dx = \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.007397, size = 15, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Maple [A] time = 0.029, size = 13, normalized size = 0.9

$$\frac{\text{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(2*x),x)
```

```
[Out] 1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.55053, size = 185, normalized size = 12.33

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(
x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sq
rt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*s
qrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2)
```

Fricas [B] time = 2.2895, size = 97, normalized size = 6.47

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\sec(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x)
```

```
[Out] Integral(cos(x)*sec(2*x), x)
```

Giac [B] time = 1.13511, size = 42, normalized size = 2.8

$$\frac{1}{4}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2}+\sin(x)\right|\right) - \frac{1}{4}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(1/2*sqrt(2) + sin(x))) - 1/4*sqrt(2)*log(abs(-1/2*sqrt(
2) + sin(x)))
```

3.117 $\int \cos(x)\sec(3x)\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(\sqrt{3}\sin(x)+\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

```
[Out] -Log[Cos[x] - Sqrt[3]*Sin[x]]/(2*Sqrt[3]) + Log[Cos[x] + Sqrt[3]*Sin[x]]/(2
*Sqrt[3])
```

Rubi [A] time = 0.0362144, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {206}

$$\frac{\log\left(\sqrt{3}\sin(x)+\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[3*x],x]
```

```
[Out] -Log[Cos[x] - Sqrt[3]*Sin[x]]/(2*Sqrt[3]) + Log[Cos[x] + Sqrt[3]*Sin[x]]/(2
*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(3x)\,dx &= \text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \tan(x)\right)\\ &= -\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0165691, size = 15, normalized size = 0.34

$$\frac{\tanh^{-1}\left(\sqrt{3}\tan(x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[3*x],x]
```

```
[Out] ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]
```

Maple [A] time = 0.054, size = 13, normalized size = 0.3

$$\frac{\sqrt{3}\text{Artanh}\left(\tan(x)\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(3*x),x)
```

```
[Out] 1/3*3^(1/2)*arctanh(tan(x)*3^(1/2))
```

Maxima [B] time = 1.54449, size = 103, normalized size = 2.34

$$\frac{1}{12}\sqrt{3}\left(\log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin(2x)^2 + \frac{4}{3}\sqrt{3}\sin(2x) - \frac{4}{3}\cos(2x) + \frac{4}{3}\right) - \log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin(2x)^2 - \frac{4}{3}\sqrt{3}\,\text{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*(log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 + 4/3*sqrt(3)*sin(2*x) -
4/3*cos(2*x) + 4/3) - log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 - 4/3*sqrt(3)*sin
(2*x) - 4/3*cos(2*x) + 4/3))
```

Fricas [A] time = 2.48839, size = 162, normalized size = 3.68

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4+4\left(2\sqrt{3}\cos(x)^3-3\sqrt{3}\cos(x)\right)\sin(x)-9}{16\cos(x)^4-24\cos(x)^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(8*cos(x)^4 + 4*(2*sqrt(3)*cos(x)^3 - 3*sqrt(3)*cos(x))*s
in(x) - 9)/(16*cos(x)^4 - 24*cos(x)^2 + 9))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos(x)\sec(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x)
```

```
[Out] Integral(cos(x)*sec(3*x), x)
```

Giac [A] time = 1.15658, size = 42, normalized size = 0.95

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+6\tan(x)\right|}{\left|2\sqrt{3}+6\tan(x)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(x))/abs(2*sqrt(3) + 6*tan(x)))
```

3.118 $\int \cos(x)\sec(4x)\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[(
2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rubi [A] time = 0.0457058, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4356, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[4*x],x]
```

```
[Out] ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[(
2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(4x)\,dx &= \text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \sin(x)\right) - \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [A] time = 0.106529, size = 67, normalized size = 0.94

$$\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[4*x],x]
```

```
[Out] (Sqrt[2 + Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTanh[(2*Si
n[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Maple [A] time = 0.083, size = 54, normalized size = 0.8

$$\frac{\sqrt{2}}{4\sqrt{2-\sqrt{2}}}\text{Artanh}\left(2\,\frac{\sin(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{\sqrt{2}}{4\sqrt{2+\sqrt{2}}}\text{Artanh}\left(2\,\frac{\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(4*x),x)
```

```
[Out] 1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-1/4*2^(1/
2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\sec(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*sec(4*x), x)
```

Fricas [B] time = 2.60697, size = 393, normalized size = 5.54

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\sin(x)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\sin(x)\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*sin(x)) - 1/8
*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*sin(x)) - 1/8*sq
rt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*sin(x)) + 1/8*sqr
t(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\sec(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x)
```

```
[Out] Integral(cos(x)*sec(4*x), x)
```

Giac [B] time = 1.30961, size = 134, normalized size = 1.89

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|\sqrt{-\frac{1}{4}\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-sqrt(2) + 2)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt
(-sqrt(2) + 2)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(sqrt(2)
 + 2)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/8*sqrt(sqrt(2) + 2)*l
og(abs(-sqrt(-1/4*sqrt(2) + 1/2) + sin(x)))
```

3.119 $\int \cos(x)\sec(5x)\,dx$

Optimal. Leaf size=163

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\sin(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\sin(x)+\cos(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right.}$$

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt
[(5 - Sqrt[5])/2]*Log[Cos[x] + Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt[(5 +
 Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10 + (Sqrt[(5 + Sqrt
[5])/2]*Log[Cos[x] + Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10
```

Rubi [A] time = 0.129449, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 207}

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\sin(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\sin(x)+\cos(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[5*x],x]
```


```
[Out] (Sqrt[(5 - Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt
[(5 - Sqrt[5])/2]*Log[Cos[x] + Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt[(5 +
 Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10 + (Sqrt[(5 + Sqrt
[5])/2]*Log[Cos[x] + Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10
```


Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(x)\sec(5x)\,dx = \text{Subst}\left(\int \frac{1+x^2}{1-10x^2+5x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5+2\sqrt{5}+5x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\left(1+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5-2\sqrt{5}+5x^2}\,dx\right.$$
$$= \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\sin(x)\right) - \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)+\sqrt{5-2\sqrt{5}}\sin(\right.$$

Mathematica [A] time = 0.103674, size = 84, normalized size = 0.52

$$\frac{\sqrt{5+\sqrt{5}}\tanh^{-1}\left(\frac{\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tanh^{-1}\left(\frac{\left(\sqrt{5}-5\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[5*x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((5 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] + S
qrt[5 - Sqrt[5]]*ArcTanh[((-5 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [A] time = 0.1, size = 68, normalized size = 0.4

$$-\frac{\left(5+\sqrt{5}\right)\sqrt{5}}{10\sqrt{25+10\sqrt{5}}}\text{Artanh}\left(5\frac{\tan(x)}{\sqrt{25+10\sqrt{5}}}\right)-\frac{\left(\sqrt{5}-5\right)\sqrt{5}}{10\sqrt{25-10\sqrt{5}}}\text{Artanh}\left(5\frac{\tan(x)}{\sqrt{25-10\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(5*x),x)
```

```
[Out] -1/10*(5+5^(1/2))*5^(1/2)/(25+10*5^(1/2))^(1/2)*arctanh(5*tan(x)/(25+10*5^(
1/2))^(1/2))-1/10*(5^(1/2)-5)*5^(1/2)/(25-10*5^(1/2))^(1/2)*arctanh(5*tan(x
)/(25-10*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\sec(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*sec(5*x), x)
```

Fricas [B] time = 2.80953, size = 759, normalized size = 4.66

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}-5\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(-\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5
) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) + 1/40*sqrt(
2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*cos
(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) - 1/40*sqrt(2)*sqrt(-s
qrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x
) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) + 5) + 1/40*sqrt(2)*sqrt(-sqrt(5) +
5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x) + 2*(s
qrt(5) - 1)*cos(x)^2 - sqrt(5) + 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\sec(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x)
```

```
[Out] Integral(cos(x)*sec(5*x), x)
```

Giac [A] time = 1.35687, size = 142, normalized size = 0.87

$$-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan\left(x\right)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan\left(x\right)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20
*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20*sqr
t(2*sqrt(5) + 10)*log(abs(sqrt(-2/5*sqrt(5) + 1) + tan(x))) - 1/20*sqrt(2*s
qrt(5) + 10)*log(abs(-sqrt(-2/5*sqrt(5) + 1) + tan(x)))
```

3.120 $\int \cos(x)\sec(6x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -ArcTanh[Sqrt[2]*Sin[x]]/(3*Sqrt[2]) + ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rubi [A] time = 0.0606861, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4356, 2057, 207, 1166}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[6*x],x]
```

```
[Out] -ArcTanh[Sqrt[2]*Sin[x]]/(3*Sqrt[2]) + ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
```

```
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\sec(6x)\,dx &= \text{Subst}\left(\int \frac{1}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{3\left(-1+2x^2\right)}-\frac{4\left(-1+2x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \sin(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(x)\right)-\frac{4}{3}\text{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \sin(x)\right)\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \sin(x)\right)-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+1}\right.\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}$$

Mathematica [A] time = 0.0870621, size = 81, normalized size = 0.95

$$\frac{1}{6}\left(-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(x)\right)+\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[6*x],x]
```

```
[Out] (-(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]]) + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/
Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]
])/6
```

Maple [A] time = 0.111, size = 80, normalized size = 0.9

$$\frac{2}{6\sqrt{6}-6\sqrt{2}}\operatorname{Artanh}\left(8\frac{\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)+\frac{2}{6\sqrt{6}+6\sqrt{2}}\operatorname{Artanh}\left(8\frac{\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)-\frac{\operatorname{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(6*x),x)
```

```
[Out] 2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^
(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))-1/6*arctanh(sin(x)
*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)+\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2
*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((cos(7*x) + co
s(5*x) + cos(3*x) + cos(x))*cos(8*x) - (cos(4*x) - 1)*cos(7*x) - (cos(4*x)
- 1)*cos(5*x) - (cos(3*x) + cos(x))*cos(4*x) + (sin(7*x) + sin(5*x) + sin(3
*x) + sin(x))*sin(8*x) - (sin(3*x) + sin(x))*sin(4*x) - sin(7*x)*sin(4*x) -
 sin(5*x)*sin(4*x) + cos(3*x) + cos(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*
x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4
```

```
*x) - 1), x)
```

Fricas [B] time = 2.79687, size = 500, normalized size = 5.88

$$-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\sin\left(x\right)\right)+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\sin\left(x\right)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*sin(x)) + 1
/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*sin(x)) + 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*sin(x)) - 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*sin(x)) + 1/1
2*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(x\right)\sec\left(6x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x)
```

```
[Out] Integral(cos(x)*sec(6*x), x)
```

Giac [A] time = 1.33534, size = 178, normalized size = 2.09

$$\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)+\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|-\frac{1}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - sqrt(2))*log(abs(1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) + 1/2
4*(sqrt(6) + sqrt(2))*log(abs(1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) - 1/24*(
sqrt(6) + sqrt(2))*log(abs(-1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) - 1/24*(sq
rt(6) - sqrt(2))*log(abs(-1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) + 1/12*sqrt(
2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)))
```

3.121 $\int \cos(2x)\sec(x)\,dx$

Optimal. Leaf size=10

$$2\sin(x) - \tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]] + 2*Sin[x]
```

Rubi [A] time = 0.0179098, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4364, 388, 206}

$$2\sin(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sec[x],x]

[Out] -ArcTanh[Sin[x]] + 2*Sin[x]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\sec(x)\,dx &= \text{Subst}\left(\int \frac{1-2x^2}{1-x^2}\,dx, x, \sin(x)\right)\\ &= 2\sin(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\ &= -\tanh^{-1}(\sin(x)) + 2\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0074482, size = 10, normalized size = 1.

$$2\sin(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sec[x],x]
```

```
[Out] -ArcTanh[Sin[x]] + 2*Sin[x]
```

Maple [A] time = 0.026, size = 14, normalized size = 1.4

$$2\,\sin(x) - \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sec(x),x)
```

```
[Out] 2*sin(x)-ln(sec(x)+tan(x))
```

Maxima [A] time = 0.995195, size = 26, normalized size = 2.6

$$-\frac{1}{2}\,\log(\sin(x) + 1) + \frac{1}{2}\,\log(\sin(x) - 1) + 2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="maxima")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) + 2*sin(x)
```

Fricas [A] time = 2.36127, size = 76, normalized size = 7.6

$$-\frac{1}{2}\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)+2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

Sympy [B] time = 2.57399, size = 20, normalized size = 2.

$$\frac{\log\left(\sin\left(x\right)-1\right)}{2}-\frac{\log\left(\sin\left(x\right)+1\right)}{2}+2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x)
```

```
[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/2 + 2*sin(x)
```

Giac [A] time = 1.09479, size = 28, normalized size = 2.8

$$-\frac{1}{2}\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)+2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="giac")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

3.122 $\int \cos(4x)\sec(2x)\,dx$

Optimal. Leaf size=14

$$\sin(2x) - \frac{1}{2}\tanh^{-1}(\sin(2x))$$

```
[Out] -ArcTanh[Sin[2*x]]/2 + Sin[2*x]
```

Rubi [A] time = 0.0199355, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4364, 388, 206}

$$\sin(2x) - \frac{1}{2}\tanh^{-1}(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[2*x],x]
```

```
[Out] -ArcTanh[Sin[2*x]]/2 + Sin[2*x]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(4x)\sec(2x)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1-2x^2}{1-x^2}\,dx, x, \sin(2x)\right)\\ &= \sin(2x) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(2x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\sin(2x)) + \sin(2x)\end{aligned}$$

Mathematica [A] time = 0.008197, size = 14, normalized size = 1.

$$\sin(2x) - \frac{1}{2}\tanh^{-1}(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[2*x],x]
```

```
[Out] -ArcTanh[Sin[2*x]]/2 + Sin[2*x]
```

Maple [A] time = 0.034, size = 18, normalized size = 1.3

$$-\frac{\ln(\sec(2x) + \tan(2x))}{2} + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sec(2*x),x)
```

```
[Out] -1/2*ln(sec(2*x)+tan(2*x))+sin(2*x)
```

Maxima [B] time = 1.54558, size = 174, normalized size = 12.43

$$\frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\,\text{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2)
- 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2
) - 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) +
 2) + 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x)
 + 2) + sin(2*x)
```

Fricas [B] time = 2.31288, size = 81, normalized size = 5.79

$$-\frac{1}{4}\log\left(\sin\left(2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2x\right)+1\right)+\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

Sympy [B] time = 20.5348, size = 427, normalized size = 30.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x)
```

```
[Out] -4*x + 32*x*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 64*x*tan(x/2
)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*x/(8*tan(x/2)**4 + 16*tan(x/
2)**2 + 8) - 3*log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 + 3*log(tan(x/2)**2 + 2*
tan(x/2) - 1)/2 + 8*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(8*tan(x/
2)**4 + 16*tan(x/2)**2 + 8) + 16*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)
**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 8*log(tan(x/2)**2 - 2*tan(x/2) -
 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(tan(x/2)**2 + 2*tan(x/2) -
 1)*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 16*log(tan(x/2)**2 +
 2*tan(x/2) - 1)*tan(x/2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(t
an(x/2)**2 + 2*tan(x/2) - 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 32*tan(
x/2)**3/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*tan(x/2)/(8*tan(x/2)**4 +
 16*tan(x/2)**2 + 8)
```

Giac [B] time = 1.1391, size = 34, normalized size = 2.43

$$-\frac{1}{4}\log\left(\sin\left(2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2x\right)+1\right)+\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="giac")
```

```
[Out] -1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

3.123 $\int \cos(x)\csc(2x)\,dx$

Optimal. Leaf size=7

$$-\frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]/2
```

Rubi [A] time = 0.0112712, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4287, 3770}

$$-\frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[2*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/2
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(2x)\,dx &= \frac{1}{2}\int \csc(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.0031234, size = 21, normalized size = 3.

$$\frac{1}{2}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[2*x],x]
```

```
[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]])/2
```

Maple [A] time = 0.017, size = 11, normalized size = 1.6

$$\frac{\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(2*x),x)
```

```
[Out] 1/2*ln(csc(x)-cot(x))
```

Maxima [B] time = 0.988998, size = 47, normalized size = 6.71

$$-\frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)+\frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1)
```

Fricas [B] time = 2.48182, size = 77, normalized size = 11.

$$-\frac{1}{4}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 7.94328, size = 15, normalized size = 2.14

$$\frac{\log\left(\cos\left(x\right)-1\right)}{4}-\frac{\log\left(\cos\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4
```

Giac [A] time = 1.11464, size = 11, normalized size = 1.57

$$\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*x)))
```

3.124 $\int \cos(x) \csc(3x) \, dx$

Optimal. Leaf size=21

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log\left(3 - 4 \sin^2(x)\right)$$

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rubi [A] time = 0.0257293, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4356, 266, 36, 31, 29}

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log\left(3 - 4 \sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[3*x],x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\csc(3x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(3-4x^2\right)}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3-4x)x}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{3-4x}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0084318, size = 21, normalized size = 1.

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[3*x],x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(3*x),x)
```

```
[Out] int(cos(x)*csc(3*x),x)
```

Maxima [B] time = 1.52045, size = 174, normalized size = 8.29

$$-\frac{1}{12}\log\left(2\left(\cos\left(x\right)+1\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^{2}+\cos\left(x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\sin\left(2\,x\right)\sin\left(x\right)+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)-\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(cos(x)
^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 2.53787, size = 65, normalized size = 3.1

$$-\frac{1}{6}\log\left(4\,\cos\left(x\right)^{2}-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\,\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="fricas")
```

```
[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

Sympy [A] time = 4.88602, size = 17, normalized size = 0.81

$$-\frac{\log\left(4\sin^{2}\left(x\right)-3\right)}{6}+\frac{\log\left(\sin\left(x\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x)
```

```
[Out] -log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

Giac [A] time = 1.11439, size = 41, normalized size = 1.95

$$-\frac{1}{6}\log\left(\left|-\frac{3(\cos(x)+1)}{\cos(x)-1}-\frac{3(\cos(x)-1)}{\cos(x)+1}-10\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(-3*(cos(x) + 1)/(cos(x) - 1) - 3*(cos(x) - 1)/(cos(x) + 1) - 1
0))
```

3.125 $\int \cos(x)\csc(4x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]/4 + ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0257339, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[4*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/4 + ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(4x)\,dx &= -\,\text{Subst}\left(\int \frac{1}{-4+12x^2-8x^4}\,dx, x, \cos(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{4-8x^2}\,dx, x, \cos(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{8-8x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\cos(x)) + \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0578497, size = 66, normalized size = 2.54

$$\frac{1}{4}\left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + (1+i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right) + \sqrt{2}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[4*x],x]
```

```
[Out] ((1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + Sqrt[2]*ArcTanh[(1 +
 Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[Sin[x/2]])/4
```

Maple [A] time = 0.045, size = 28, normalized size = 1.1

$$\frac{\text{Artanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{4} - \frac{\ln(1+\cos(x))}{8} + \frac{\ln(-1+\cos(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(4*x),x)
```

```
[Out] 1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*ln(1+cos(x))+1/8*ln(-1+cos(x))
```

Maxima [B] time = 1.5529, size = 220, normalized size = 8.46

$$\frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x)+1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
1) - 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*c
os(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 1) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1)
```

Fricas [B] time = 2.47266, size = 174, normalized size = 6.69

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)-\frac{1}{8}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{8}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - 1/
8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17051, size = 88, normalized size = 3.38

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)+\frac{1}{8}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqr
t(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) + 1/8*log(-(cos(x) - 1)/(cos(x) +
1))
```

3.126 $\int \cos(x)\csc(5x)\,dx$

Optimal. Leaf size=62

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\sin^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\sin^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sin(x))$$

```
[Out] Log[Sin[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Sin[x]^2])/20 - ((1 - Sq
rt[5])*Log[5 + Sqrt[5] - 8*Sin[x]^2])/20
```

Rubi [A] time = 0.0699113, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4356, 1114, 705, 29, 632, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\sin^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\sin^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[5*x],x]
```


```
[Out] Log[Sin[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Sin[x]^2])/20 - ((1 - Sq
rt[5])*Log[5 + Sqrt[5] - 8*Sin[x]^2])/20
```


Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(5x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)+\frac{1}{10}\text{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{5}\log(\sin(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \sin^2(x)\right)-\frac{1}{5}\left(4\left(1+\sqrt{5}\right)\right)\text{Sub}\\ &= \frac{1}{5}\log(\sin(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\sin^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\sin^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0623529, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(4\log(\sin(x))-\left(1+\sqrt{5}\right)\log\left(4\cos(2x)-\sqrt{5}+1\right)+\left(\sqrt{5}-1\right)\log\left(4\cos(2x)+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[5*x],x]
```

```
[Out] (-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[2*x]]) + (-1 + Sqrt[5])*Log[1 + Sq
rt[5] + 4*Cos[2*x]] + 4*Log[Sin[x]])/20
```

Maple [A] time = 0.097, size = 80, normalized size = 1.3

$$-\frac{\ln\left(4\,(\cos(x))^2-2\,\cos(x)-1\right)}{20}+\frac{\sqrt{5}}{10}\mathrm{Artanh}\left(\frac{(8\,\cos(x)-2)\,\sqrt{5}}{10}\right)+\frac{\ln(1+\cos(x))}{10}+\frac{\ln(-1+\cos(x))}{10}-\frac{\ln\left(4\,(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(5*x),x)
```

```
[Out] -1/20*ln(4*cos(x)^2-2*cos(x)-1)+1/10*5^(1/2)*arctanh(1/10*(8*cos(x)-2)*5^(1
/2))+1/10*ln(1+cos(x))+1/10*ln(-1+cos(x))-1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/
10*5^(1/2)*arctanh(1/10*(8*cos(x)+2)*5^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="maxima")
```

```
[Out] -1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*arctan2(s
in(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x
) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/2*arctan
2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2
+ cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x))
) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos
(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))
) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*
x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x), cos(2*
x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(
2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + s
in(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*integrat
e((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x), cos(2*
```

```
x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(2*x)*si
n(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2*x), cos
(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 +
2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*cos(3/2*
arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(
cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos(1/2*arc
tan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2
 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*sin(3/2*a
rctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(s
in(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*arctan2(
sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(x)*sin(4
*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x)*sin(x)
 - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x)
 + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x)
 + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*s
in(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*
x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) + 1/10*integrate(-(
cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) + cos
(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) + cos(x) -
 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2
 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x)
 + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x
)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + 3/10*
integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2
(sin(3*x), cos(3*x)))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3
*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arct
an2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) +
sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))
+ cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x), cos(3
*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(1/3*arct
an2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/
3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2(sin(3*x
), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 + 2*(si
n(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), c
os(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x),
 cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3
*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*
sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), c
os(3*x)))^2 + 2*cos(3*x) + 1), x) - 3/10*integrate(-(cos(4/3*arctan2(sin(3*
x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - c
os(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin
(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) + cos(3*x
)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*(cos(3*x
) - cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*
x))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(sin(3*x),
```

```
cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))) - 1)*cos(
2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 +
 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(si
n(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(sin(3*x),
 cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x
), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x) + sin(
1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x))) + si
n(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x),
 cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*cos(3*x) + 1), x)
+ 1/5*integrate((sin(4*x) + sin(3*x) + sin(2*x) + sin(x))/(2*(cos(3*x) + co
s(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(
3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(si
n(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*s
in(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x)
 + 1), x) + 1/5*integrate(-(sin(4*x) - sin(3*x) + sin(2*x) - sin(x))/(2*(co
s(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x
) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x
)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x)
- sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2
 + 2*cos(x) - 1), x) + 1/10*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/10*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 2.56656, size = 232, normalized size = 3.74

$$\frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4+8\left(\sqrt{5}-3\right)\cos(x)^2-3\sqrt{5}+7}{16\cos(x)^4-12\cos(x)^2+1}\right)-\frac{1}{20}\log\left(16\cos(x)^4-12\cos(x)^2+1\right)+\frac{1}{5}\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 3)*cos(x)^2 - 3*sqrt(5) + 7)/(
16*cos(x)^4 - 12*cos(x)^2 + 1)) - 1/20*log(16*cos(x)^4 - 12*cos(x)^2 + 1) +
 1/5*log(1/2*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos(x)\csc(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x)
```

```
[Out] Integral(cos(x)*csc(5*x), x)
```

Giac [B] time = 1.20177, size = 177, normalized size = 2.85

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|-16\sqrt{5}-\frac{10(\cos(x)+1)}{\cos(x)-1}-\frac{10(\cos(x)-1)}{\cos(x)+1}-60\right|}{\left|16\sqrt{5}-\frac{10(\cos(x)+1)}{\cos(x)-1}-\frac{10(\cos(x)-1)}{\cos(x)+1}-60\right|}\right)-\frac{1}{20}\log\left(\left|5\left(\frac{\cos(x)+1}{\cos(x)-1}+\frac{\cos(x)-1}{\cos(x)+1}\right)^2+\frac{60(\cos(x)+1)}{\cos(x)-1}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(5)*log(abs(-16*sqrt(5) - 10*(cos(x) + 1)/(cos(x) - 1) - 10*(cos(
x) - 1)/(cos(x) + 1) - 60)/abs(16*sqrt(5) - 10*(cos(x) + 1)/(cos(x) - 1) -
10*(cos(x) - 1)/(cos(x) + 1) - 60)) - 1/20*log(abs(5*((cos(x) + 1)/(cos(x)
- 1) + (cos(x) - 1)/(cos(x) + 1))^2 + 60*(cos(x) + 1)/(cos(x) - 1) + 60*(co
s(x) - 1)/(cos(x) + 1) + 116))
```

3.127 $\int \cos(x)\csc(6x)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[Cos[x]]/6 - ArcTanh[2*Cos[x]]/6 + ArcTanh[(2*Cos[x])/Sqrt[3]]/(2*S
qrt[3])
```

Rubi [A] time = 0.0414389, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 207}

$$-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[6*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/6 - ArcTanh[2*Cos[x]]/6 + ArcTanh[(2*Cos[x])/Sqrt[3]]/(2*S
qrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\csc(6x)\,dx &= -\operatorname{Subst}\left(\int \frac{1}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \cos(x)\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \cos(x)\right)\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cos(x)\right)-\operatorname{Subst}\left(\int \frac{1}{-3+4x^2}\right.\\
&= -\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}
$$

Mathematica [B] time = 0.0727779, size = 83, normalized size = 2.31

$$
\frac{1}{12}\left(2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(1-2\cos(x))-\log(2\cos(x)+1)-2\sqrt{3}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-2}{\sqrt{3}}\right)+2\sqrt{3}\tanh\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[6*x],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + Tan[x
/2])/Sqrt[3]] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*Cos[x]] + 2
*Log[Sin[x/2]])/12
```

Maple [A] time = 0.059, size = 47, normalized size = 1.3

$$
-\frac{\ln(1+\cos(x))}{12}+\frac{\ln(-1+\cos(x))}{12}-\frac{\ln(1+2\cos(x))}{12}+\frac{\ln(2\cos(x)-1)}{12}+\frac{\sqrt{3}}{6}\operatorname{Artanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(6*x),x)
```

```
[Out] -1/12*ln(1+cos(x))+1/12*ln(-1+cos(x))-1/12*ln(1+2*cos(x))+1/12*ln(2*cos(x)-
1)+1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(\sin(3x) - \sin(x))\cos(4x) - (\cos(3x) - \cos(x))\sin(4x) - (\cos(2x) - 1)\sin(3x) + \cos(3x)\sin(2x) - \cos(x)\sin}{2\left(2(\cos(2x) - 1)\cos(4x) - \cos(4x)^2 - \cos(2x)^2 - \sin(4x)^2 + 2\sin(4x)\sin(2x) - \sin(2x)^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x)
 - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)
*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/24*lo
g(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*sin(2*x)
*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2*x) + co
s(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2*cos(x)
+ 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1)
```

Fricas [B] time = 2.65906, size = 248, normalized size = 6.89

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2 + 4\sqrt{3}\cos(x) + 3}{4\cos(x)^2 - 3}\right) - \frac{1}{12}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log(-2\cos(x) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - 1
/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(-2*cos(x
) + 1) - 1/12*log(-2*cos(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\csc(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x)
```

```
[Out] Integral(cos(x)*csc(6*x), x)
```

Giac [B] time = 1.178, size = 136, normalized size = 3.78

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-8\sqrt{3}-\frac{2(\cos(x)-1)}{\cos(x)+1}-14\right|}{\left|8\sqrt{3}-\frac{2(\cos(x)-1)}{\cos(x)+1}-14\right|}\right)+\frac{1}{12}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)-\frac{1}{12}\log\left(\left|-\frac{\cos(x)-1}{\cos(x)+1}-3\right|\right)+\frac{1}{12}\log\left(\left|-\frac{3(\cos(x)-}{\cos(x)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-8*sqrt(3) - 2*(cos(x) - 1)/(cos(x) + 1) - 14)/abs(8*s
qrt(3) - 2*(cos(x) - 1)/(cos(x) + 1) - 14)) + 1/12*log(-(cos(x) - 1)/(cos(x
) + 1)) - 1/12*log(abs(-(cos(x) - 1)/(cos(x) + 1) - 3)) + 1/12*log(abs(-3*(
cos(x) - 1)/(cos(x) + 1) - 1))
```

3.128 $\int \cos^3(6x)\sin(x)\,dx$

Optimal. Leaf size=33

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

```
[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Rubi [A] time = 0.0314779, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2638}

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[6*x]^3*Sin[x],x]
```

```
[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(6x)\sin(x)\,dx &= \int \left(-\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)+\frac{1}{8}\sin(19x)\right)dx\\ &= -\left(\frac{1}{8}\int \sin(17x)\,dx\right)+\frac{1}{8}\int \sin(19x)\,dx-\frac{3}{8}\int \sin(5x)\,dx+\frac{3}{8}\int \sin(7x)\,dx\\ &= \frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)\end{aligned}$$

Mathematica [A] time = 0.0172737, size = 33, normalized size = 1.

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[6*x]^3*Sin[x],x]
```

```
[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Maple [A] time = 0.092, size = 26, normalized size = 0.8

$$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(x),x)
```

```
[Out] 3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

Maxima [A] time = 0.99366, size = 34, normalized size = 1.03

$$-\frac{1}{152}\cos(19x)+\frac{1}{136}\cos(17x)-\frac{3}{56}\cos(7x)+\frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="maxima")
```

```
[Out] -1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

Fricas [B] time = 2.796, size = 234, normalized size = 7.09

$$-\frac{32768}{19}\cos(x)^{19}+\frac{147456}{17}\cos(x)^{17}-18432\cos(x)^{15}+21504\cos(x)^{13}-14976\cos(x)^{11}+6336\cos(x)^{9}-\frac{11112}{7}\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="fricas")
```

```
[Out] -32768/19*cos(x)^19 + 147456/17*cos(x)^17 - 18432*cos(x)^15 + 21504*cos(x)^
13 - 14976*cos(x)^11 + 6336*cos(x)^9 - 11112/7*cos(x)^7 + 1116/5*cos(x)^5 -
 18*cos(x)^3 + cos(x)
```

Sympy [B] time = 11.4344, size = 63, normalized size = 1.91

$$\frac{1296\sin(x)\sin^3(6x)}{11305}+\frac{1926\sin(x)\sin(6x)\cos^2(6x)}{11305}+\frac{216\sin^2(6x)\cos(x)\cos(6x)}{11305}+\frac{251\cos(x)\cos^3(6x)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)**3*sin(x),x)
```

```
[Out] 1296*sin(x)*sin(6*x)**3/11305 + 1926*sin(x)*sin(6*x)*cos(6*x)**2/11305 + 21
6*sin(6*x)**2*cos(x)*cos(6*x)/11305 + 251*cos(x)*cos(6*x)**3/11305
```

Giac [A] time = 1.11997, size = 34, normalized size = 1.03

$$-\frac{1}{152}\cos(19x)+\frac{1}{136}\cos(17x)-\frac{3}{56}\cos(7x)+\frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="giac")
```

```
[Out] -1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

3.129 $\int \cos^3(6x)\sin(9x)\,dx$

Optimal. Leaf size=33

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

```
[Out] -Cos[3*x]/8 + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Rubi [A] time = 0.0328019, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4354, 2638}

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[6*x]^3*Sin[9*x],x]
```

```
[Out] -Cos[3*x]/8 + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cos^3(6x)\sin(9x)\,dx = \int \left(\frac{3}{8}\sin(3x) - \frac{1}{8}\sin(9x) + \frac{3}{8}\sin(15x) + \frac{1}{8}\sin(27x)\right)dx$$
$$= -\left(\frac{1}{8}\int \sin(9x)\,dx\right) + \frac{1}{8}\int \sin(27x)\,dx + \frac{3}{8}\int \sin(3x)\,dx + \frac{3}{8}\int \sin(15x)\,dx$$
$$= -\frac{1}{8}\cos(3x) + \frac{1}{72}\cos(9x) - \frac{1}{40}\cos(15x) - \frac{1}{216}\cos(27x)$$

Mathematica [A] time = 0.0173943, size = 33, normalized size = 1.

$$-\frac{1}{8}\cos(3x) + \frac{1}{72}\cos(9x) - \frac{1}{40}\cos(15x) - \frac{1}{216}\cos(27x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[6*x]^3*Sin[9*x],x]
```

```
[Out] -Cos[3*x]/8 + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Maple [A] time = 0.038, size = 26, normalized size = 0.8

$$-\frac{\cos(3x)}{8} + \frac{\cos(9x)}{72} - \frac{\cos(15x)}{40} - \frac{\cos(27x)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(9*x),x)
```

```
[Out] -1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

Maxima [A] time = 1.01214, size = 34, normalized size = 1.03

$$-\frac{1}{216}\cos(27x) - \frac{1}{40}\cos(15x) + \frac{1}{72}\cos(9x) - \frac{1}{8}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="maxima")
```

```
[Out] -1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

Fricas [A] time = 2.44613, size = 117, normalized size = 3.55

$$-\frac{32}{27}\cos(3x)^9 + \frac{8}{3}\cos(3x)^7 - \frac{12}{5}\cos(3x)^5 + \frac{10}{9}\cos(3x)^3 - \frac{1}{3}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="fricas")
```

```
[Out] -32/27*cos(3*x)^9 + 8/3*cos(3*x)^7 - 12/5*cos(3*x)^5 + 10/9*cos(3*x)^3 - 1/
3*cos(3*x)
```

Sympy [B] time = 23.3911, size = 71, normalized size = 2.15

$$-\frac{16\sin^3(6x)\sin(9x)}{135} - \frac{8\sin^2(6x)\cos(6x)\cos(9x)}{45} - \frac{2\sin(6x)\sin(9x)\cos^2(6x)}{45} - \frac{19\cos^3(6x)\cos(9x)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)**3*sin(9*x),x)
```

```
[Out] -16*sin(6*x)**3*sin(9*x)/135 - 8*sin(6*x)**2*cos(6*x)*cos(9*x)/45 - 2*sin(6
*x)*sin(9*x)*cos(6*x)**2/45 - 19*cos(6*x)**3*cos(9*x)/135
```

Giac [A] time = 1.12231, size = 34, normalized size = 1.03

$$-\frac{1}{216}\cos(27x) - \frac{1}{40}\cos(15x) + \frac{1}{72}\cos(9x) - \frac{1}{8}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="giac")
```



```
[Out] -1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

3.130 $\int \cos(2x) \sin^2(6x)\, dx$

Optimal. Leaf size=25

$$\frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

```
[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Rubi [A] time = 0.027917, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4354, 2637}

$$\frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sin[6*x]^2,x]
```

```
[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(2x)\sin^2(6x)\,dx &= \int \left(\frac{1}{2}\cos(2x) - \frac{1}{4}\cos(10x) - \frac{1}{4}\cos(14x)\right)dx \\
&= -\left(\frac{1}{4}\int \cos(10x)\,dx\right) - \frac{1}{4}\int \cos(14x)\,dx + \frac{1}{2}\int \cos(2x)\,dx \\
&= \frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)
\end{aligned}
$$

Mathematica [A] time = 0.0163452, size = 25, normalized size = 1.

$$\frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sin[6*x]^2,x]
```

```
[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Maple [A] time = 0.048, size = 20, normalized size = 0.8

$$\frac{\sin(2x)}{4} - \frac{\sin(10x)}{40} - \frac{\sin(14x)}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(6*x)^2,x)
```

```
[Out] 1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

Maxima [A] time = 0.982074, size = 26, normalized size = 1.04

$$-\frac{1}{56}\sin(14x) - \frac{1}{40}\sin(10x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.36754, size = 92, normalized size = 3.68

$$-\frac{1}{70} \left(80 \cos(2x)^{6} - 72 \cos(2x)^{4} + 9 \cos(2x)^{2} - 17\right) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="fricas")
```

```
[Out] -1/70*(80*cos(2*x)^6 - 72*cos(2*x)^4 + 9*cos(2*x)^2 - 17)*sin(2*x)
```

Sympy [B] time = 12.7746, size = 48, normalized size = 1.92

$$\frac{17 \sin(2x) \sin^{2}(6x)}{70} + \frac{9 \sin(2x) \cos^{2}(6x)}{35} - \frac{3 \sin(6x) \cos(2x) \cos(6x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)**2,x)
```

```
[Out] 17*sin(2*x)*sin(6*x)**2/70 + 9*sin(2*x)*cos(6*x)**2/35 - 3*sin(6*x)*cos(2*x
)*cos(6*x)/35
```

Giac [A] time = 1.12103, size = 26, normalized size = 1.04

$$-\frac{1}{56} \sin(14x) - \frac{1}{40} \sin(10x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="giac")
```

```
[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

3.131 $\int \cos(x)\sin^2(6x)\,dx$

Optimal. Leaf size=23

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

```
[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Rubi [A] time = 0.0263351, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2637}

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[6*x]^2,x]
```

```
[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\sin^2(6x)\,dx = \int \left(\frac{\cos(x)}{2} - \frac{1}{4}\cos(11x) - \frac{1}{4}\cos(13x)\right)dx$$
$$= -\left(\frac{1}{4}\int \cos(11x)\,dx\right) - \frac{1}{4}\int \cos(13x)\,dx + \frac{1}{2}\int \cos(x)\,dx$$
$$= \frac{\sin(x)}{2} - \frac{1}{44}\sin(11x) - \frac{1}{52}\sin(13x)$$

Mathematica [A] time = 0.0119901, size = 23, normalized size = 1.

$$\frac{\sin(x)}{2} - \frac{1}{44}\sin(11x) - \frac{1}{52}\sin(13x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[6*x]^2,x]
```

```
[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Maple [A] time = 0.051, size = 18, normalized size = 0.8

$$\frac{\sin(x)}{2} - \frac{\sin(11x)}{44} - \frac{\sin(13x)}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(6*x)^2,x)
```

```
[Out] 1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

Maxima [A] time = 1.00829, size = 23, normalized size = 1.

$$-\frac{1}{52}\sin(13x) - \frac{1}{44}\sin(11x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

Fricas [B] time = 2.48903, size = 154, normalized size = 6.7

$$-\frac{4}{143}\left(2816\cos(x)^{12}-6912\cos(x)^{10}+6048\cos(x)^{8}-2240\cos(x)^{6}+315\cos(x)^{4}-9\cos(x)^{2}-18\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="fricas")
```

```
[Out] -4/143*(2816*cos(x)^12 - 6912*cos(x)^10 + 6048*cos(x)^8 - 2240*cos(x)^6 + 3
15*cos(x)^4 - 9*cos(x)^2 - 18)*sin(x)
```

Sympy [B] time = 12.8496, size = 42, normalized size = 1.83

$$\frac{71\sin(x)\sin^2(6x)}{143}+\frac{72\sin(x)\cos^2(6x)}{143}-\frac{12\sin(6x)\cos(x)\cos(6x)}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)**2,x)
```

```
[Out] 71*sin(x)*sin(6*x)**2/143 + 72*sin(x)*cos(6*x)**2/143 - 12*sin(6*x)*cos(x)*
cos(6*x)/143
```

Giac [A] time = 1.11927, size = 23, normalized size = 1.

$$-\frac{1}{52}\sin(13x)-\frac{1}{44}\sin(11x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="giac")
```

```
[Out] -1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

3.132 $\int \cos(x)\sin^3(6x)\,dx$

Optimal. Leaf size=33

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

```
[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Rubi [A] time = 0.031273, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2638}

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[6*x]^3,x]
```

```
[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin^3(6x)\,dx &= \int \left(\frac{3}{8}\sin(5x) + \frac{3}{8}\sin(7x) - \frac{1}{8}\sin(17x) - \frac{1}{8}\sin(19x)\right) dx \\ &= -\left(\frac{1}{8}\int \sin(17x)\,dx\right) - \frac{1}{8}\int \sin(19x)\,dx + \frac{3}{8}\int \sin(5x)\,dx + \frac{3}{8}\int \sin(7x)\,dx \\ &= -\frac{3}{40}\cos(5x) - \frac{3}{56}\cos(7x) + \frac{1}{136}\cos(17x) + \frac{1}{152}\cos(19x)\end{aligned}$$

Mathematica [A] time = 0.0145286, size = 33, normalized size = 1.

$$-\frac{3}{40}\cos(5x) - \frac{3}{56}\cos(7x) + \frac{1}{136}\cos(17x) + \frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[6*x]^3,x]
```

```
[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Maple [A] time = 0.059, size = 26, normalized size = 0.8

$$-\frac{3\cos(5x)}{40} - \frac{3\cos(7x)}{56} + \frac{\cos(17x)}{136} + \frac{\cos(19x)}{152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(6*x)^3,x)
```

```
[Out] -3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

Maxima [A] time = 1.02419, size = 34, normalized size = 1.03

$$\frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) - \frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) - 3/40*cos(5*x)
```

Fricas [A] time = 2.76163, size = 197, normalized size = 5.97

$$\frac{32768}{19}\cos(x)^{19} - \frac{131072}{17}\cos(x)^{17} + 14336\cos(x)^{15} - 14336\cos(x)^{13} + 8320\cos(x)^{11} - 2816\cos(x)^{9} + \frac{3672}{7}\cos(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="fricas")
```

```
[Out] 32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^1
3 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

Sympy [B] time = 16.1038, size = 65, normalized size = 1.97

$$-\frac{251\sin(x)\sin^3(6x)}{11305} - \frac{216\sin(x)\sin(6x)\cos^2(6x)}{11305} - \frac{1926\sin^2(6x)\cos(x)\cos(6x)}{11305} - \frac{1296\cos(x)\cos^3(6x)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)**3,x)
```

```
[Out] -251*sin(x)*sin(6*x)**3/11305 - 216*sin(x)*sin(6*x)*cos(6*x)**2/11305 - 192
6*sin(6*x)**2*cos(x)*cos(6*x)/11305 - 1296*cos(x)*cos(6*x)**3/11305
```

Giac [A] time = 1.1403, size = 34, normalized size = 1.03

$$\frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) - \frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="giac")
```



```
[Out] 1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) - 3/40*cos(5*x)
```

3.133 $\int \cos(7x) \sin^3(6x) \, dx$

Optimal. Leaf size=31

$$\frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

```
[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Rubi [A] time = 0.0299518, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4354, 2638}

$$\frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[7*x]*Sin[6*x]^3,x]
```

```
[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cos(7x)\sin^3(6x)\,dx = \int \left(-\frac{3\sin(x)}{8} - \frac{1}{8}\sin(11x) + \frac{3}{8}\sin(13x) - \frac{1}{8}\sin(25x)\right)dx$$
$$= -\left(\frac{1}{8}\int \sin(11x)\,dx\right) - \frac{1}{8}\int \sin(25x)\,dx - \frac{3}{8}\int \sin(x)\,dx + \frac{3}{8}\int \sin(13x)\,dx$$
$$= \frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

Mathematica [A] time = 0.0153507, size = 31, normalized size = 1.

$$\frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[7*x]*Sin[6*x]^3,x]
```

```
[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Maple [A] time = 0.18, size = 24, normalized size = 0.8

$$\frac{3\cos(x)}{8} + \frac{\cos(11x)}{88} - \frac{3\cos(13x)}{104} + \frac{\cos(25x)}{200}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(7*x)*sin(6*x)^3,x)
```

```
[Out] 3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

Maxima [A] time = 1.00676, size = 31, normalized size = 1.

$$\frac{1}{200}\cos(25x) - \frac{3}{104}\cos(13x) + \frac{1}{88}\cos(11x) + \frac{3}{8}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

Fricas [B] time = 3.11891, size = 297, normalized size = 9.58

$$\frac{2097152}{25}\cos(x)^{25} - 524288\cos(x)^{23} + 1441792\cos(x)^{21} - 2293760\cos(x)^{19} + 2334720\cos(x)^{17} - \frac{7938048}{5}\cos(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="fricas")
```

```
[Out] 2097152/25*cos(x)^25 - 524288*cos(x)^23 + 1441792*cos(x)^21 - 2293760*cos(x
)^19 + 2334720*cos(x)^17 - 7938048/5*cos(x)^15 + 9503232/13*cos(x)^13 - 248
4992/11*cos(x)^11 + 45248*cos(x)^9 - 5400*cos(x)^7 + 1512/5*cos(x)^5
```

Sympy [B] time = 11.8116, size = 70, normalized size = 2.26

$$\frac{1421\sin^3(6x)\sin(7x)}{3575} + \frac{1062\sin^2(6x)\cos(6x)\cos(7x)}{3575} + \frac{1512\sin(6x)\sin(7x)\cos^2(6x)}{3575} + \frac{1296\cos^3(6x)\cos(7x)}{3575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)**3,x)
```

```
[Out] 1421*sin(6*x)**3*sin(7*x)/3575 + 1062*sin(6*x)**2*cos(6*x)*cos(7*x)/3575 +
1512*sin(6*x)*sin(7*x)*cos(6*x)**2/3575 + 1296*cos(6*x)**3*cos(7*x)/3575
```

Giac [A] time = 1.1035, size = 31, normalized size = 1.

$$\frac{1}{200}\cos(25x) - \frac{3}{104}\cos(13x) + \frac{1}{88}\cos(11x) + \frac{3}{8}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="giac")
```


```
[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

3.134 $\int \cos^2(3x)\sin^3(2x)\,dx$

Optimal. Leaf size=41

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

```
[Out] (-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[1
2*x]/192
```

Rubi [A] time = 0.0433548, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4354, 2638}

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]^2*Sin[2*x]^3,x]
```


```
[Out] (-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[1
2*x]/192
```


Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cos^2(3x)\sin^3(2x)\,dx = \int \left(\frac{3}{8}\sin(2x) - \frac{3}{16}\sin(4x) - \frac{1}{8}\sin(6x) + \frac{3}{16}\sin(8x) - \frac{1}{16}\sin(12x)\right)dx$$
$$= -\left(\frac{1}{16}\int \sin(12x)\,dx\right) - \frac{1}{8}\int \sin(6x)\,dx - \frac{3}{16}\int \sin(4x)\,dx + \frac{3}{16}\int \sin(8x)\,dx + \frac{3}{8}\int \sin(2x)$$
$$= -\frac{3}{16}\cos(2x) + \frac{3}{64}\cos(4x) + \frac{1}{48}\cos(6x) - \frac{3}{128}\cos(8x) + \frac{1}{192}\cos(12x)$$

Mathematica [A] time = 0.0185465, size = 41, normalized size = 1.

$$-\frac{3}{16}\cos(2x) + \frac{3}{64}\cos(4x) + \frac{1}{48}\cos(6x) - \frac{3}{128}\cos(8x) + \frac{1}{192}\cos(12x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]^2*Sin[2*x]^3,x]
```

```
[Out] (-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[1
2*x]/192
```

Maple [A] time = 0.066, size = 32, normalized size = 0.8

$$-\frac{3\cos(2x)}{16} + \frac{3\cos(4x)}{64} + \frac{\cos(6x)}{48} - \frac{3\cos(8x)}{128} + \frac{\cos(12x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)^2*sin(2*x)^3,x)
```

```
[Out] -3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

Maxima [A] time = 1.0121, size = 42, normalized size = 1.02

$$\frac{1}{192}\cos(12x) - \frac{3}{128}\cos(8x) + \frac{1}{48}\cos(6x) + \frac{3}{64}\cos(4x) - \frac{3}{16}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos
(2*x)
```

Fricas [A] time = 2.39628, size = 80, normalized size = 1.95

$$\frac{32}{3}\cos(x)^{12} - 32\cos(x)^{10} + 33\cos(x)^{8} - 12\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="fricas")
```

```
[Out] 32/3*cos(x)^12 - 32*cos(x)^10 + 33*cos(x)^8 - 12*cos(x)^6
```

Sympy [B] time = 52.9172, size = 228, normalized size = 5.56

$$-\frac{x\sin^{3}(2x)\sin^{2}(3x)}{16} + \frac{x\sin^{3}(2x)\cos^{2}(3x)}{16} - \frac{3x\sin^{2}(2x)\sin(3x)\cos(2x)\cos(3x)}{8} + \frac{3x\sin(2x)\sin^{2}(3x)\cos^{2}(2x)}{16} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)**2*sin(2*x)**3,x)
```

```
[Out] -x*sin(2*x)**3*sin(3*x)**2/16 + x*sin(2*x)**3*cos(3*x)**2/16 - 3*x*sin(2*x)
**2*sin(3*x)*cos(2*x)*cos(3*x)/8 + 3*x*sin(2*x)*sin(3*x)**2*cos(2*x)**2/16
- 3*x*sin(2*x)*cos(2*x)**2*cos(3*x)**2/16 + x*sin(3*x)*cos(2*x)**3*cos(3*x)
/8 - sin(2*x)**2*sin(3*x)**2*cos(2*x)/32 - 15*sin(2*x)**2*cos(2*x)*cos(3*x)
**2/32 + 9*sin(2*x)*sin(3*x)*cos(2*x)**2*cos(3*x)/16 - 13*sin(3*x)**2*cos(2
*x)**3/48 - cos(2*x)**3*cos(3*x)**2/16
```

Giac [A] time = 1.16051, size = 42, normalized size = 1.02

$$\frac{1}{192}\cos(12x) - \frac{3}{128}\cos(8x) + \frac{1}{48}\cos(6x) + \frac{3}{64}\cos(4x) - \frac{3}{16}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="giac")
```

```
[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos
(2*x)
```

3.135 $\int \sin(a+bx)\sin(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

```
[Out] (x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.0243191, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4569, 2637}

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + b*x],x]
```

```
[Out] (x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin(c+bx)\,dx &= \int \left(\frac{1}{2}\cos(a-c)-\frac{1}{2}\cos(a+c+2bx)\right)dx\\ &= \frac{1}{2}x\cos(a-c)-\frac{1}{2}\int \cos(a+c+2bx)\,dx\\ &= \frac{1}{2}x\cos(a-c)-\frac{\sin(a+c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0476423, size = 26, normalized size = 0.96

$$-\frac{\sin(a+2bx+c)-2bx\cos(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + b*x],x]
```

```
[Out] -(-2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/(4*b)
```

Maple [A] time = 0.011, size = 24, normalized size = 0.9

$$\frac{x\cos(a-c)}{2}-\frac{\sin(2bx+a+c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(b*x+c),x)
```

```
[Out] 1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Maxima [A] time = 0.999916, size = 31, normalized size = 1.15

$$\frac{1}{2}x\cos(-a+c)-\frac{\sin(2bx+a+c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(-a + c) - 1/4*sin(2*b*x + a + c)/b
```

Fricas [B] time = 2.4508, size = 127, normalized size = 4.7

$$\frac{bx\cos(-a+c)-\cos(bx+c)\cos(-a+c)\sin(bx+c)+\cos(bx+c)^2\sin(-a+c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(-a + c) - cos(b*x + c)*cos(-a + c)*sin(b*x + c) + cos(b*x + c)
^2*sin(-a + c))/b
```

Sympy [A] time = 1.39209, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} - \frac{\sin(bx+c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 -
sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)*sin(c), True))
```

Giac [A] time = 1.13145, size = 31, normalized size = 1.15

$$\frac{1}{2}x\cos(a-c) - \frac{\sin(2bx+a+c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a - c) - 1/4*sin(2*b*x + a + c)/b
```

3.136 $\int \sin(c-bx)\sin(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sin(a+2bx-c)}{4b}-\frac{1}{2}x\cos(a+c)$$

```
[Out] -(x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.025399, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4569, 2637}

$$\frac{\sin(a+2bx-c)}{4b}-\frac{1}{2}x\cos(a+c)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c - b*x]*Sin[a + b*x],x]
```

```
[Out] -(x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin(c-bx)\sin(a+bx)\,dx &= \int \left(-\frac{1}{2}\cos(a+c)+\frac{1}{2}\cos(a-c+2bx)\right)dx\\
&= -\frac{1}{2}x\cos(a+c)+\frac{1}{2}\int \cos(a-c+2bx)\,dx\\
&= -\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.0338703, size = 26, normalized size = 0.96

$$\frac{\sin(a+2bx-c)-2bx\cos(a+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c - b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

Maple [A] time = 0.01, size = 24, normalized size = 0.9

$$-\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(b*x-c)*sin(b*x+a),x)
```

```
[Out] -1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Maxima [A] time = 1.00554, size = 31, normalized size = 1.15

$$-\frac{1}{2}x\cos(a+c)+\frac{\sin(2bx+a-c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Fricas [A] time = 2.4156, size = 124, normalized size = 4.59

$$-\frac{bx\cos(a+c)-\cos(bx+a)\cos(a+c)\sin(bx+a)+\cos(bx+a)^2\sin(a+c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*cos(a + c) - cos(b*x + a)*cos(a + c)*sin(b*x + a) + cos(b*x + a)^
2*sin(a + c))/b
```

Sympy [A] time = 1.21596, size = 61, normalized size = 2.26

$$-\begin{cases} \frac{x\sin{(a+bx)}\sin{(bx-c)}}{2}+\frac{x\cos{(a+bx)}\cos{(bx-c)}}{2}-\frac{\sin{(bx-c)}\cos{(a+bx)}}{2b} & \text{for}\ b\neq 0\\ -x\sin{(a)}\sin{(c)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x)
```

```
[Out] -Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 -
 sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (-x*sin(a)*sin(c), True))
```

Giac [A] time = 1.11759, size = 31, normalized size = 1.15

$$-\frac{1}{2}\,x\cos{(a+c)}+\frac{\sin{(2\,bx+a-c)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

3.137 $\int \cos(a+bx)\cos(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

```
[Out] (x*Cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.017493, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4570, 2637}

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + b*x],x]
```

```
[Out] (x*Cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos(c+bx)\,dx &= \int \left(\frac{1}{2}\cos(a-c)+\frac{1}{2}\cos(a+c+2bx)\right)dx\\ &= \frac{1}{2}x\cos(a-c)+\frac{1}{2}\int \cos(a+c+2bx)\,dx\\ &= \frac{1}{2}x\cos(a-c)+\frac{\sin(a+c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0248488, size = 26, normalized size = 0.96

$$\frac{\sin(a+2bx+c)+2bx\cos(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + b*x],x]
```

```
[Out] (2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/(4*b)
```

Maple [A] time = 0.013, size = 24, normalized size = 0.9

$$\frac{x\cos(a-c)}{2}+\frac{\sin(2bx+a+c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(b*x+c),x)
```

```
[Out] 1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Maxima [A] time = 0.981778, size = 31, normalized size = 1.15

$$\frac{1}{2}x\cos(-a+c)+\frac{\sin(2bx+a+c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(-a + c) + 1/4*sin(2*b*x + a + c)/b
```

Fricas [B] time = 2.2965, size = 127, normalized size = 4.7

$$\frac{bx\cos(-a+c)+\cos(bx+c)\cos(-a+c)\sin(bx+c)-\cos(bx+c)^2\sin(-a+c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(-a + c) + cos(b*x + c)*cos(-a + c)*sin(b*x + c) - cos(b*x + c)
^2*sin(-a + c))/b
```

Sympy [A] time = 1.0933, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x \sin(a+bx) \sin(bx+c)}{2} + \frac{x \cos(a+bx) \cos(bx+c)}{2} + \frac{\sin(bx+c) \cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x \cos(a) \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 +
sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

Giac [A] time = 1.10714, size = 31, normalized size = 1.15

$$\frac{1}{2} x \cos(a - c) + \frac{\sin(2 bx + a + c)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a - c) + 1/4*sin(2*b*x + a + c)/b
```

3.138 $\int \cos(c-bx)\cos(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sin(a+2bx-c)}{4b}+\frac{1}{2}x\cos(a+c)$$

```
[Out] (x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)
```

Rubi [A] time = 0.0186565, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4570, 2637}

$$\frac{\sin(a+2bx-c)}{4b}+\frac{1}{2}x\cos(a+c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c - b*x]*Cos[a + b*x],x]
```

```
[Out] (x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(c-bx)\cos(a+bx)\,dx &= \int \left(\frac{1}{2}\cos(a+c)+\frac{1}{2}\cos(a-c+2bx)\right)dx\\
&= \frac{1}{2}x\cos(a+c)+\frac{1}{2}\int \cos(a-c+2bx)\,dx\\
&= \frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.0231336, size = 26, normalized size = 0.96

$$\frac{\sin(a+2bx-c)+2bx\cos(a+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c - b*x]*Cos[a + b*x],x]
```

```
[Out] (2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

Maple [A] time = 0.012, size = 24, normalized size = 0.9

$$\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x-c)*cos(b*x+a),x)
```

```
[Out] 1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Maxima [A] time = 1.01705, size = 31, normalized size = 1.15

$$\frac{1}{2}x\cos(a+c)+\frac{\sin(2bx+a-c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Fricas [A] time = 2.38264, size = 123, normalized size = 4.56

$$\frac{bx\cos(a+c)+\cos(bx+a)\cos(a+c)\sin(bx+a)-\cos(bx+a)^2\sin(a+c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(a + c) + cos(b*x + a)*cos(a + c)*sin(b*x + a) - cos(b*x + a)^2
*sin(a + c))/b
```

Sympy [A] time = 3.50541, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin(a+bx)\sin(bx-c)}{2} + \frac{x\cos(a+bx)\cos(bx-c)}{2} + \frac{\sin(bx-c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 +
sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

Giac [A] time = 1.11017, size = 31, normalized size = 1.15

$$\frac{1}{2}\,x\cos(a+c) + \frac{\sin(2\,bx+a-c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

3.139 $\int \tan(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(bx+c))}{b}-x$$

```
[Out] -x - (Cot[a - c]*Log[Cos[a + b*x]])/b + (Cot[a - c]*Log[Cos[c + b*x]])/b
```

Rubi [A] time = 0.065661, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4612, 4610, 3475}

$$-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(bx+c))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -x - (Cot[a - c]*Log[Cos[a + b*x]])/b + (Cot[a - c]*Log[Cos[c + b*x]])/b
```

Rule 4612

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[(b*Cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(a+bx)\tan(c+bx)\,dx = -x + \cos(a-c)\int \sec(a+bx)\sec(c+bx)\,dx$$
$$= -x + \cot(a-c)\int \tan(a+bx)\,dx - \cot(a-c)\int \tan(c+bx)\,dx$$
$$= -x - \frac{\cot(a-c)\log(\cos(a+bx))}{b} + \frac{\cot(a-c)\log(\cos(c+bx))}{b}$$

Mathematica [A] time = 0.503351, size = 31, normalized size = 0.79

$$\frac{\cot(a-c)(\log(\cos(bx+c)) - \log(\cos(a+bx)))}{b} - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -x + (Cot[a - c]*(-Log[Cos[a + b*x]] + Log[Cos[c + b*x]]))/b
```

Maple [C] time = 0.074, size = 173, normalized size = 4.4

$$-x - \frac{i\ln\left(e^{2\,i(bx+a)}+1\right)e^{2\,ia}}{b\left(e^{2\,ia}-e^{2\,ic}\right)} - \frac{i\ln\left(e^{2\,i(bx+a)}+1\right)e^{2\,ic}}{b\left(e^{2\,ia}-e^{2\,ic}\right)} + \frac{i\ln\left(e^{2\,i(bx+a)}+e^{2\,i(a-c)}\right)e^{2\,ia}}{b\left(e^{2\,ia}-e^{2\,ic}\right)} + \frac{i\ln\left(e^{2\,i(bx+a)}+e^{2\,i(a-c)}\right)e^{2\,ic}}{b\left(e^{2\,ia}-e^{2\,ic}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)*tan(b*x+c),x)
```

```
[Out] -x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+1)*exp(2*I*a)-I/b/(exp(2
*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+1)*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*c)
```

Maxima [B] time = 1.13055, size = 501, normalized size = 12.85

$$-\frac{\left(2\,b\cos\left(2\,a\right)\cos\left(2\,c\right) - b\cos\left(2\,c\right)^{2} + 2\,b\sin\left(2\,a\right)\sin\left(2\,c\right) - b\sin\left(2\,c\right)^{2} - \left(\cos\left(2\,a\right)^{2} + \sin\left(2\,a\right)^{2}\right)b\right)x + \left(\cos\left(2\,a\right)^{2} - \mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c
)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - (co
s(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2
*c), cos(2*b*x) + cos(2*c)) - (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(c
os(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b
*x)*sin(2*a) + sin(2*a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(co
s(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*
x)*sin(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(
2*a)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] time = 2.46691, size = 387, normalized size = 9.92

$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\left(\cos(-2\,a+2\,c)-1\right)\tan(bx+c)^2-2\,\sin(-2\,a+2\,c)\tan(bx+c)-\cos(-2\,a+2\,c)-1}{\left(\cos(-2\,a+2\,c)+1\right)\tan(bx+c)^2+\cos(-2\,a+2\,c)+1}\right)+\left(\cos\left(-2\right.\right.}{2\,b\sin\left(-2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-((cos(-2*a + 2*c)
- 1)*tan(b*x + c)^2 - 2*sin(-2*a + 2*c)*tan(b*x + c) - cos(-2*a + 2*c) - 1)
/((cos(-2*a + 2*c) + 1)*tan(b*x + c)^2 + cos(-2*a + 2*c) + 1)) + (cos(-2*a
+ 2*c) + 1)*log(1/(tan(b*x + c)^2 + 1)))/(b*sin(-2*a + 2*c))
```

Sympy [B] time = 10.5871, size = 7713, normalized size = 197.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (b*x*tan(c)**4*tan(b*x)/(b*t
an(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 +
```

```
b*tan(c)**2*tan(b*x) - b*tan(c)) - b*x*tan(c)**3/(b*tan(c)**6*tan(b*x) - b*
tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) -
 b*tan(c)) - b*x*tan(c)**2*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2
*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) +
b*x*tan(c)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2
*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + 2*log(tan(b*x) - 1/tan(c)
)*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - 2*log(tan(b*x)
- 1/tan(c))*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*t
an(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - log(tan(b*x)**
2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)*
*4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + log(tan(b*
x)**2 + 1)*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - tan(c)**2/(b*ta
n(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b
*tan(c)**2*tan(b*x) - b*tan(c)) - 1/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2
*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)), E
q(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (0, E
q(b, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(b*x) - 1/
tan(c))/(2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*
tan(c)**3 + 2*b*tan(c)), Eq(a, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**3 + 2*b*tan
(a)) - 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3 + 2*b*tan(a)) - log(tan(b*
x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3 + 2*b*tan(a)), Eq(c, 0)), (2*b*x*tan(a)
**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)*
*3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)*
*3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*ta
n(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c
)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*
x*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3
 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3
 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(c)**2/(2*b*tan(a)**3*tan(c)
**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*
tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x
) - 1/tan(a))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(
c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(
c)**3 - 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(ta
n(b*x) - 1/tan(c))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2
*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b
*tan(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b
```

```
*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)
**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) -
2*b*tan(c)**3 - 2*b*tan(c)), True)) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & E
q(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**
4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 4
*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x
) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))
*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3
*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1
/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*
tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/
tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c
)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x)
 - 1/tan(c))/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*
x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)*tan(
c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(
b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*ta
n(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) -
4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)*ta
n(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4
*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c
)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*
b*tan(c)*tan(b*x) - 2*b) - 2*tan(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)
**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) -
 2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*t
an(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b), Eq(a, atan(tan(c)) + pi*floor((c - p
i/2)/pi) + pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (-2*b*x*tan(c)/(2*b*tan(c
)**2 + 2*b) - 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**2 + 2*b) + log(tan(b*
x)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**2
+ 2*b) - 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**2
 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-2*b*x*tan(a)**2/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan
(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(ta
n(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*ta
n(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a
) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2*tan(
c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b
*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b
*tan(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b
*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*ta
```

```
n(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*t
an(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*ta
n(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*ta
n(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(t
an(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a
)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) -
 2*b*tan(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*ta
n(a) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-4*b*x*tan(c)**2*tan
(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*
b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 4*b*x*tan(c)/(2*b*tan(c)**5*tan(
b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)
**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)**2/(2*b*tan(c)*
*5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*
tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)*tan(b*x)/(2*b*ta
n(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 +
 2*b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**5*tan
(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)
*tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*t
an(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(
c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x)
 - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(
b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x)
- 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b
*x) - 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 +
 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*ta
n(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) -
4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2/(2*b*tan(c)**5*tan(b*x) - 2*
b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x)
- 2*b), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2)
)), (0, Eq(b, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x) -
1/tan(c))/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)/(2*b*tan(c)**2 + 2*b
), Eq(a, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**2 + 2*b) - 2*log(tan(b*x) - 1/tan
(a))/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq
(c, 0)), (-2*b*x*tan(a)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*t
an(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(
a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(c)**2/(2*b*tan(a)**3*tan(c)**2
 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan
(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) -
 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*
```

```
tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan
(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(a)/(2*b*
tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)*
*2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c))
 + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2
*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*
tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b*x) - 1/t
an(c))*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(
c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(
c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*
tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)
 + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan
(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2
*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan
(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*
tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) -
log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*
tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan
(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*tan(c) + Piecewise((x, Eq(a, 0) &
 Eq(b, 0) & Eq(c, 0)), (-b*x*tan(c)**3*tan(b*x)/(b*tan(c)**5*tan(b*x) - b*t
an(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b)
 + b*x*tan(c)**2/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x
) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + b*x*tan(c)*tan(b*x)/(b*tan(c)*
*5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(
c)*tan(b*x) - b) - b*x/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) - 2*log(tan(b*x) - 1/tan(
c))*tan(c)**2*tan(b*x)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + 2*log(tan(b*x) - 1/tan(
c))*tan(c)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2
*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + log(tan(b*x)**2 + 1)*tan(c)**2*tan(
b*x)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan
(c)**2 + b*tan(c)*tan(b*x) - b) - log(tan(b*x)**2 + 1)*tan(c)/(b*tan(c)**5*
tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) - b) - tan(c)**3/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)*
*3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) - tan(c)/(b*tan(c)**5*
tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) - b), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi
 - 1/2))), (x, Eq(b, 0)), (2*b*x/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x) - 1
/tan(c))*tan(c)/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(c)/(2*b*ta
n(c)**2 + 2*b), Eq(a, 0)), (2*b*x/(2*b*tan(a)**2 + 2*b) - 2*log(tan(b*x) -
1/tan(a))*tan(a)/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(a)/(2*b*t
an(a)**2 + 2*b), Eq(c, 0)), (-2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)*
*2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)*t
```

```
an(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3
 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3
 - 2*b*tan(c)) + 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*
b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)**2
 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan
(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) -
 1/tan(a))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2
*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(a)**2
/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*t
an(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*t
an(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(
c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*
b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b
*x) - 1/tan(c))*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*ta
n(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a
) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - lo
g(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b
*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*tan(a)*tan(c)
```

Giac [B] time = 1.16272, size = 109, normalized size = 2.79

$$-x-\frac{\left(\tan\left(a\right)^{2}\tan\left(c\right)+\tan\left(a\right)\right)\log\left(\left|\tan\left(bx\right)\tan\left(a\right)-1\right|\right)}{b\tan\left(a\right)^{2}-b\tan\left(a\right)\tan\left(c\right)}+\frac{\left(\tan\left(a\right)\tan\left(c\right)^{2}+\tan\left(c\right)\right)\log\left(\left|\tan\left(bx\right)\tan\left(c\right)-1\right|\right)}{b\tan\left(a\right)\tan\left(c\right)-b\tan\left(c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] -x - (tan(a)^2*tan(c) + tan(a))*log(abs(tan(b*x)*tan(a) - 1))/(b*tan(a)^2 -
 b*tan(a)*tan(c)) + (tan(a)*tan(c)^2 + tan(c))*log(abs(tan(b*x)*tan(c) - 1)
)/(b*tan(a)*tan(c) - b*tan(c)^2)
```

3.140 $\int \tan(c - bx)\tan(a + bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}+x$$

```
[Out] x - (Cot[a + c]*Log[Cos[c - b*x]])/b + (Cot[a + c]*Log[Cos[a + b*x]])/b
```

Rubi [A] time = 0.0678364, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4612, 4610, 3475}

$$-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c - b*x]*Tan[a + b*x],x]
```

```
[Out] x - (Cot[a + c]*Log[Cos[c - b*x]])/b + (Cot[a + c]*Log[Cos[a + b*x]])/b
```

Rule 4612

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[(b*Cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan(c-bx)\tan(a+bx)\,dx &= x-\cos(a+c)\int \sec(c-bx)\sec(a+bx)\,dx\\ &= x-\cot(a+c)\int \tan(c-bx)\,dx-\cot(a+c)\int \tan(a+bx)\,dx\\ &= x-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.516919, size = 28, normalized size = 0.82

$$\frac{\cot(a+c)(\log(\cos(a+bx))-\log(\cos(c-bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c - b*x]*Tan[a + b*x],x]
```

```
[Out] x + (Cot[a + c]*(-Log[Cos[c - b*x]] + Log[Cos[a + b*x]]))/b
```

Maple [C] time = 0.067, size = 145, normalized size = 4.3

$$x+\frac{i\ln\left(e^{2i(bx+a)}+1\right)e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}+\frac{i\ln\left(e^{2i(bx+a)}+1\right)}{b\left(e^{2i(a+c)}-1\right)}-\frac{i\ln\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}-\frac{i\ln\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(a+c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(b*x-c)*tan(b*x+a),x)
```

```
[Out] x+I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(b*x+a))+1)*exp(2*I*(a+c))+I/b/(exp(2*I
*(a+c))-1)*ln(exp(2*I*(b*x+a))+1)-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+
exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+e
xp(2*I*(b*x+a)))
```

Maxima [B] time = 1.08495, size = 392, normalized size = 11.53

$$\frac{\left(b\cos(2a+2c)^2+b\sin(2a+2c)^2-2b\cos(2a+2c)+b\right)x-\left(\cos(2a+2c)^2+\sin(2a+2c)^2-1\right)\arctan(\sin(2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (co
s(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2*b*x) - sin(2*a), cos(2
*b*x) + cos(2*a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2
*b*x) + sin(2*c), cos(2*b*x) + cos(2*c)) + log(cos(2*b*x)^2 + 2*cos(2*b*x)*
cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)*
sin(2*a + 2*c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + si
n(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(2*a + 2*c))/(b*cos(2*a
 + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

Fricas [B] time = 2.57033, size = 374, normalized size = 11.

$$\frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{\left(\cos(2\,a+2\,c)-1\right)\tan(bx+a)^{2}-2\,\sin(2\,a+2\,c)\tan(bx+a)-\cos(2\,a+2\,c)-1}{\left(\cos(2\,a+2\,c)+1\right)\tan(bx+a)^{2}+\cos(2\,a+2\,c)+1}\right)+\left(\cos\left(2\,a+2\,c\right)\right.}{2\,b\sin\left(2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-((cos(2*a + 2*c) - 1)
*tan(b*x + a)^2 - 2*sin(2*a + 2*c)*tan(b*x + a) - cos(2*a + 2*c) - 1)/((cos
(2*a + 2*c) + 1)*tan(b*x + a)^2 + cos(2*a + 2*c) + 1)) + (cos(2*a + 2*c) +
1)*log(1/(tan(b*x + a)^2 + 1)))/(b*sin(2*a + 2*c))
```

Sympy [B] time = 10.1553, size = 7720, normalized size = 227.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**
2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*x)*
*2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*t
```

```
an(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2
+ 2*b*tan(c)*tan(b*x) + 2*b) - 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*t
an(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2
*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x)
 + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(
b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x)
+ 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b
*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(
b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**5*tan(b*x) + 2*b*
tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) +
2*b) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*
b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x)
+ 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c
)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b)
- log(tan(b*x)**2 + 1)*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)
**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) -
 log(tan(b*x)**2 + 1)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)*
*3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*tan(c)**2/(2*b
*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**
2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4
+ 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b), Eq(a
, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(
b, 0)), (2*b*x*tan(a)/(2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x) - 1/tan(a))/(2
*b*tan(a)**2 + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0))
, (2*b*x*tan(a)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2
*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b
*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*t
an(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(
c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a
))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log
(tan(b*x) + 1/tan(c))*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a
)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**
2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*t
an(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2
 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a
)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
```

```
 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(
b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)), True))*tan(a) - Piecewise((0, Eq(a, 0) & Eq(b, 0
) & Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**2 + 2*b) + 2*log(tan(b*x) + 1/ta
n(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)**2 + 2*b), E
q(a, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**
4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 4
*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x
) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))
*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3
*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1
/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*
tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/
tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c
)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x)
 + 1/tan(c))/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*
x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(
c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(
b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*ta
n(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) +
4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)*tan(c)*ta
n(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4
*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c
)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*
b*tan(c)*tan(b*x) + 2*b) - 2*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)
**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) -
 2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*t
an(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b), Eq(a, -atan(tan(c)) - pi*floor((c -
pi/2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (2*b*x*tan(a)/(2*b*tan(a
)**2 + 2*b) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) - log(tan(b*
x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (2*b*x*tan(a)**2/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x
*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)*
*3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)*
*3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*lo
g(tan(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(a)**2*
tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +
```

```
 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)/(2*b*tan(a)**3*tan(c)**2 +
 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a
)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 +
1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - l
og(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*t
an(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(
a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)
)*tan(c) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (2*b*x*tan(c)/(2*
b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**3 + 2*b
*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**3 + 2*b*tan(c)), Eq(
a, 0)), (-b*x*tan(c)**4*tan(b*x)/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*
tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) - b*x
*tan(c)**3/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2
*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) + b*x*tan(c)**2*tan(b*x)/(b
*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3
+ b*tan(c)**2*tan(b*x) + b*tan(c)) + b*x*tan(c)/(b*tan(c)**6*tan(b*x) + b*t
an(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) +
b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(
b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*t
an(b*x) + b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(b*tan(c)**6*tan
(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*
tan(b*x) + b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**6
*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)
**2*tan(b*x) + b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**2/(b*tan(c)**6*tan(
b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*t
an(b*x) + b*tan(c)) + tan(c)**2/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*t
an(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) + 1/(b
*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3
+ b*tan(c)**2*tan(b*x) + b*tan(c)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/
2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (2*b*x*tan(a)/(2*b*tan(a)**
3 + 2*b*tan(a)) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3 + 2*b*tan(a)) +
 log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3 + 2*b*tan(a)), Eq(c, 0)), (2
*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)
**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)*
*2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)/(
2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan
```

```
(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan
(c)) + 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(c)**2/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*
log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan
(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a)
 + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(b*x) + 1/tan(c))*tan(a)**2/(2*b*
tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)*
*2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
 - 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*t
an(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**
2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
- log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)) - Piecewise((-x, Eq(a, 0) &
Eq(b, 0) & Eq(c, 0)), (-2*b*x/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x) + 1/ta
n(c))*tan(c)/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(c
)**2 + 2*b), Eq(a, 0)), (b*x*tan(c)**3*tan(b*x)/(b*tan(c)**5*tan(b*x) + b*t
an(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b)
+ b*x*tan(c)**2/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x
) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - b*x*tan(c)*tan(b*x)/(b*tan(c)*
*5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(
c)*tan(b*x) + b) - b*x/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - 2*log(tan(b*x) + 1/tan(
c))*tan(c)**2*tan(b*x)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - 2*log(tan(b*x) + 1/tan(
c))*tan(c)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2
*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) + log(tan(b*x)**2 + 1)*tan(c)**2*tan(
b*x)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan
(c)**2 + b*tan(c)*tan(b*x) + b) + log(tan(b*x)**2 + 1)*tan(c)/(b*tan(c)**5*
tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) + b) + tan(c)**3/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)*
*3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) + tan(c)/(b*tan(c)**5*
tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) + b), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/p
i - 1/2))), (-x, Eq(b, 0)), (-2*b*x/(2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x)
- 1/tan(a))*tan(a)/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2 + 1)*tan(a)/(2*b
*tan(a)**2 + 2*b), Eq(c, 0)), (-2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c
)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b
*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)
*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)*
```

```
*3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)*
*3 + 2*b*tan(c)) - 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b
*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)*
*2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x)
 - 1/tan(a))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)*
*2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b
*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b
*tan(c)) - 2*log(tan(b*x) + 1/tan(c))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan
(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*
tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan
(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan
(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*
tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) +
log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2
*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True))*tan(a)*tan(c)
```

Giac [B] time = 1.16567, size = 109, normalized size = 3.21

$$x-\frac{\left(\tan\left(a\right)^{2}\tan\left(c\right)-\tan\left(a\right)\right)\log\left(\left|\tan\left(bx\right)\tan\left(a\right)-1\right|\right)}{b\tan\left(a\right)^{2}+b\tan\left(a\right)\tan\left(c\right)}+\frac{\left(\tan\left(a\right)\tan\left(c\right)^{2}-\tan\left(c\right)\right)\log\left(\left|\tan\left(bx\right)\tan\left(c\right)+1\right|\right)}{b\tan\left(a\right)\tan\left(c\right)+b\tan\left(c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] x - (tan(a)^2*tan(c) - tan(a))*log(abs(tan(b*x)*tan(a) - 1))/(b*tan(a)^2 +
b*tan(a)*tan(c)) + (tan(a)*tan(c)^2 - tan(c))*log(abs(tan(b*x)*tan(c) + 1))
/(b*tan(a)*tan(c) + b*tan(c)^2)
```

3.141 $\int \cot(a+bx)\cot(c+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(bx+c))}{b}-x$$

```
[Out] -x - (Cot[a - c]*Log[Sin[a + b*x]])/b + (Cot[a - c]*Log[Sin[c + b*x]])/b
```

Rubi [A] time = 0.0321744, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4613, 4611, 3475}

$$-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(bx+c))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -x - (Cot[a - c]*Log[Sin[a + b*x]])/b + (Cot[a - c]*Log[Sin[c + b*x]])/b
```

Rule 4613

```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(a+bx)\cot(c+bx)\,dx = -x+\cos(a-c)\int \csc(a+bx)\csc(c+bx)\,dx$$
$$= -x-\cot(a-c)\int \cot(a+bx)\,dx+\cot(a-c)\int \cot(c+bx)\,dx$$
$$= -x-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(c+bx))}{b}$$

Mathematica [A] time = 0.508159, size = 31, normalized size = 0.79

$$\frac{\cot(a-c)(\log(\sin(bx+c))-\log(\sin(a+bx)))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -x + (Cot[a - c]*(-Log[Sin[a + b*x]] + Log[Sin[c + b*x]]))/b
```

Maple [C] time = 0.076, size = 177, normalized size = 4.5

$$-x+\frac{i\ln\left(e^{2\,i(bx+a)}-e^{2\,i(a-c)}\right)e^{2\,ia}}{b\left(e^{2\,ia}-e^{2\,ic}\right)}+\frac{i\ln\left(e^{2\,i(bx+a)}-e^{2\,i(a-c)}\right)e^{2\,ic}}{b\left(e^{2\,ia}-e^{2\,ic}\right)}-\frac{i\ln\left(e^{2\,i(bx+a)}-1\right)e^{2\,ia}}{b\left(e^{2\,ia}-e^{2\,ic}\right)}-\frac{i\ln\left(e^{2\,i(bx+a)}-1\right)e^{2\,ic}}{b\left(e^{2\,ia}-e^{2\,ic}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)*cot(b*x+c),x)
```

```
[Out] -x+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*
a)+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*
c)-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*a)-I/b/(exp(2
*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*c)
```

Maxima [B] time = 1.16627, size = 741, normalized size = 19.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

```
[Out] -((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c
)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + (cos(2*a)^2
 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(a), cos(b*x
) + cos(a)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(s
in(b*x) + sin(c), cos(b*x) - cos(c)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(c), cos(b*x) + cos(c)) - (cos(2*c)*s
in(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(2*c)*sin(2*a) - cos(2*a
)*sin(2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*
sin(b*x)*sin(a) + sin(a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(c
os(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
 sin(c)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 - 2*cos
(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2))/(2*b*
cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c)^2 - (
cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] time = 2.52468, size = 315, normalized size = 8.08

$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos\left(2\,bx+2\,c\right)\cos\left(-2\,a+2\,c\right)+\sin\left(2\,bx+2\,c\right)\sin\left(-2\,a+2\,c\right)-1}{\cos\left(-2\,a+2\,c\right)+1}\right)+\left(\cos\left(-2\,a+2\,c\right)+1\right)}{2\,b\sin\left(-2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*c)*
cos(-2*a + 2*c) + sin(2*b*x + 2*c)*sin(-2*a + 2*c) - 1)/(cos(-2*a + 2*c) +
1)) + (cos(-2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*c) + 1/2))/(b*sin(-2*a +
 2*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2426, size = 470, normalized size = 12.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*ta
n(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 - 4*tan(1/2*a)*tan(
1/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*tan
(1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)
^3 - 2*tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) + tan
(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan
(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*c
) + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) - 2*tan(1/
2*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^2*
tan(1/2*c) + 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a) + tan(1/
2*c)))/b
```

3.142 $\int \cot(c-bx)\cot(a+bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}+x$$

```
[Out] x - (Cot[a + c]*Log[Sin[c - b*x]])/b + (Cot[a + c]*Log[Sin[a + b*x]])/b
```

Rubi [A] time = 0.034246, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4613, 4611, 3475}

$$-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c - b*x]*Cot[a + b*x],x]
```

```
[Out] x - (Cot[a + c]*Log[Sin[c - b*x]])/b + (Cot[a + c]*Log[Sin[a + b*x]])/b
```

Rule 4613

```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(c-bx)\cot(a+bx)\,dx = x+\cos(a+c)\int \csc(c-bx)\csc(a+bx)\,dx$$
$$= x+\cot(a+c)\int \cot(c-bx)\,dx+\cot(a+c)\int \cot(a+bx)\,dx$$
$$= x-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.488814, size = 30, normalized size = 0.88

$$\frac{\cot(a+c)(\log(-\sin(a+bx))-\log(\sin(c-bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c - b*x]*Cot[a + b*x],x]
```

```
[Out] x + (Cot[a + c]*(-Log[Sin[c - b*x]] + Log[-Sin[a + b*x]]))/b
```

Maple [C] time = 0.08, size = 149, normalized size = 4.4

$$x+\frac{i\ln\left(e^{2i(bx+a)}-1\right)e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}+\frac{i\ln\left(e^{2i(bx+a)}-1\right)}{b\left(e^{2i(a+c)}-1\right)}-\frac{i\ln\left(-e^{2i(a+c)}+e^{2i(bx+a)}\right)e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}-\frac{i\ln\left(-e^{2i(a+c)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(a+c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(b*x-c)*cot(b*x+a),x)
```

```
[Out] x+I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)*exp(2*I*(a+c))+I/b/(exp(2*I
*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))
+exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))
+exp(2*I*(b*x+a)))
```

Maxima [B] time = 1.1565, size = 583, normalized size = 17.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (co
s(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) + sin(a), cos(b*x)
- cos(a)) - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) - si
n(a), cos(b*x) + cos(a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan
2(sin(b*x) + sin(c), cos(b*x) + cos(c)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c
)^2 - 1)*arctan2(sin(b*x) - sin(c), cos(b*x) - cos(c)) + log(cos(b*x)^2 + 2
*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2)*si
n(2*a + 2*c) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(a) + sin(a)^2)*sin(2*a + 2*c) - log(cos(b*x)^2 + 2*cos(b*x)
*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(2*a + 2
*c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(c) + sin(c)^2)*sin(2*a + 2*c))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c
)^2 - 2*b*cos(2*a + 2*c) + b)
```

Fricas [B] time = 2.53066, size = 304, normalized size = 8.94

$$\frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos(2\,bx+2\,a)\cos(2\,a+2\,c)+\sin(2\,bx+2\,a)\sin(2\,a+2\,c)-1}{\cos(2\,a+2\,c)+1}\right)+\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{1}{2}\right.}{2\,b\sin\left(2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*a)*cos
(2*a + 2*c) + sin(2*b*x + 2*a)*sin(2*a + 2*c) - 1)/(cos(2*a + 2*c) + 1)) +
(cos(2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2))/(b*sin(2*a + 2*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20349, size = 466, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x - (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 - 4*tan(1/2*a)^3*tan
(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*tan(
1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^
3 - 2*tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) + tan(
1/2*c)) + (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 - 4*tan(
1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c)
 + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) + 2*tan(1/2
*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^2*t
an(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*a) + tan(1/2
*c)))/b
```

3.143 $\int \sec(a+bx)\sec(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

```
[Out] -((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos[c + b*x]])/b
```

Rubi [A] time = 0.019024, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4610, 3475}

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Sec[c + b*x],x]
```

```
[Out] -((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos[c + b*x]])/b
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\sec(c+bx)\,dx &= \csc(a-c)\int \tan(a+bx)\,dx-\csc(a-c)\int \tan(c+bx)\,dx\\ &= -\frac{\csc(a-c)\log(\cos(a+bx))}{b}+\frac{\csc(a-c)\log(\cos(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.224254, size = 28, normalized size = 0.78

$$-\frac{\csc(a-c)(\log(\cos(a+bx))-\log(\cos(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[c + b*x],x]
```

```
[Out] -((Csc[a - c]*(Log[Cos[a + b*x]] - Log[Cos[c + b*x]]))/b)
```

Maple [A] time = 0.231, size = 55, normalized size = 1.5

$$-\frac{\ln\left(-\tan\left(bx+a\right)\cos\left(a\right)\sin\left(c\right)+\tan\left(bx+a\right)\sin\left(a\right)\cos\left(c\right)+\cos\left(a\right)\cos\left(c\right)+\sin\left(a\right)\sin\left(c\right)\right)}{b\left(\cos\left(a\right)\sin\left(c\right)-\sin\left(a\right)\cos\left(c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(b*x+c),x)
```

```
[Out] -1/b/(cos(a)*sin(c)-sin(a)*cos(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*
sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))
```

Maxima [B] time = 1.05778, size = 471, normalized size = 13.08

$$-\frac{2\left(\left(\cos\left(2a\right)-\cos\left(2c\right)\right)\cos\left(a+c\right)+\left(\sin\left(2a\right)-\sin\left(2c\right)\right)\sin\left(a+c\right)\right)\arctan\left(\sin\left(2bx\right)-\sin\left(2a\right),\cos\left(2bx\right)+\cos\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

```
[Out] -(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*a
rctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*((cos(2*a) - cos(2
*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(2*b*x) - si
n(2*c), cos(2*b*x) + cos(2*c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2
*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(
2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + ((sin(2*a) -
sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 +
```

```
 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c)
+ sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c
) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] time = 2.60332, size = 273, normalized size = 7.58

$$-\frac{\log\left(\cos\left(bx+c\right)^2\right)-\log\left(\frac{4\left(2\,\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\,\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2+1\right)}{\cos(-a+c)^2+2\,\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(log(cos(b*x + c)^2) - log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*
sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(co
s(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(a+bx\right)\sec\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x)
```

```
[Out] Integral(sec(a + b*x)*sec(b*x + c), x)
```

Giac [B] time = 1.24713, size = 231, normalized size = 6.42

$$\frac{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)^2+\tan\left(\frac{1}{2}\,c\right)^2+1\right)\log\left(\left|2\,\tan\left(bx+a\right)\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)-2\,\tan\left(bx+a\right)\tan\left(\frac{1}{2}\,a\right)\right.\right.}{2\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(2
*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c
)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2
- 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/
((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*
c))*b)
```

3.144 $\int \sec(c - bx)\sec(a + bx)\,dx$

Optimal. Leaf size=33

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b} - \frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

```
[Out] (Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a + b*x]])/b
```

Rubi [A] time = 0.018378, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4610, 3475}

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b} - \frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c - b*x]*Sec[a + b*x],x]
```

```
[Out] (Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a + b*x]])/b
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c-bx)\sec(a+bx)\,dx &= \csc(a+c)\int \tan(c-bx)\,dx + \csc(a+c)\int \tan(a+bx)\,dx\\ &= \frac{\csc(a+c)\log(\cos(c-bx))}{b} - \frac{\csc(a+c)\log(\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.228994, size = 26, normalized size = 0.79

$$\frac{\csc(a+c)(\log(\cos(c-bx))-\log(\cos(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c - b*x]*Sec[a + b*x],x]
```

```
[Out] (Csc[a + c]*(Log[Cos[c - b*x]] - Log[Cos[a + b*x]]))/b
```

Maple [A] time = 0.221, size = 53, normalized size = 1.6

$$\frac{\ln(\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}{b(\sin(a)\cos(c)+\cos(a)\sin(c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x-c)*sec(b*x+a),x)
```

```
[Out] 1/b/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*si
n(a)*cos(c)+cos(a)*cos(c)-sin(a)*sin(c))
```

Maxima [B] time = 1.09119, size = 435, normalized size = 13.18

$$2(\cos(2a+2c)\cos(a+c)+\sin(2a+2c)\sin(a+c)-\cos(a+c))\arctan(\sin(2bx)-\sin(2a),\cos(2bx)+\cos(2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arc
tan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*(cos(2*a + 2*c)*cos(
a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(2*b*x) + sin(2
*c), cos(2*b*x) + cos(2*c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*s
in(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)
^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (cos(a + c)*sin(2
*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*co
```

```
s(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin
(2*c)^2))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b
)
```

Fricas [B] time = 2.52903, size = 263, normalized size = 7.97

$$-\frac{\log\left(\cos\left(bx+a\right)^2\right)-\log\left(\frac{4\left(2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^2-1\right)\cos(bx+a)^2-\cos(a+c)^2+1\right)}{\cos(a+c)^2+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(cos(b*x + a)^2) - log(4*(2*cos(b*x + a)*cos(a + c)*sin(b*x + a)*s
in(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2 + 1)/(cos(a
+ c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sec\left(a+bx\right)\sec\left(bx-c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)*sec(b*x - c), x)
```

Giac [B] time = 1.24222, size = 228, normalized size = 6.91

$$-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+2\tan\left(bx+a\right)\tan\left(\frac{1}{2}\right.\right.\right.}{2\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(
2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*
c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + tan(1/2*a)^2
 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1))
/((tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2
*c))*b)
```

3.145 $\int \csc(a+bx)\csc(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\csc(a-c)\log(\sin(bx+c))}{b}-\frac{\csc(a-c)\log(\sin(a+bx))}{b}$$

```
[Out] -((Csc[a - c]*Log[Sin[a + b*x]])/b) + (Csc[a - c]*Log[Sin[c + b*x]])/b
```

Rubi [A] time = 0.0184961, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4611, 3475}

$$\frac{\csc(a-c)\log(\sin(bx+c))}{b}-\frac{\csc(a-c)\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[c + b*x],x]
```

```
[Out] -((Csc[a - c]*Log[Sin[a + b*x]])/b) + (Csc[a - c]*Log[Sin[c + b*x]])/b
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\csc(c+bx)\,dx &= -\left(\csc(a-c)\int \cot(a+bx)\,dx\right)+\csc(a-c)\int \cot(c+bx)\,dx\\ &= -\frac{\csc(a-c)\log(\sin(a+bx))}{b}+\frac{\csc(a-c)\log(\sin(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.241302, size = 28, normalized size = 0.78

$$-\frac{\csc(a-c)(\log(\sin(a+bx))-\log(\sin(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[c + b*x],x]
```

```
[Out] -((Csc[a - c]*(Log[Sin[a + b*x]] - Log[Sin[c + b*x]]))/b)
```

Maple [B] time = 0.234, size = 169, normalized size = 4.7

$$\frac{\ln(\tan(bx+a))}{b(\cos(a)\sin(c)-\sin(a)\cos(c))}-\frac{\ln(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a}{b(\cos(a)\sin(c)-\sin(a)\cos(c))(\cos(a)\cos(c)+\sin(a)\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(b*x+c),x)
```

```
[Out] 1/b/(cos(a)*sin(c)-sin(a)*cos(c))*ln(tan(b*x+a))-1/b/(cos(a)*sin(c)-sin(a)*
cos(c))/(cos(a)*cos(c)+sin(a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a
)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))*cos(a)*cos(c)-1/b/(cos(a)*sin(
c)-sin(a)*cos(c))/(cos(a)*cos(c)+sin(a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)
+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))*sin(a)*sin(c)
```

Maxima [B] time = 1.11522, size = 761, normalized size = 21.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

```
[Out] -(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*a
rctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*((cos(2*a) - cos(2*c))*cos
(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(a), cos(
b*x) + cos(a)) - 2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c)
)*sin(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - 2*((cos(2*a)
```

```
- cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x)
 - sin(c), cos(b*x) + cos(c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2*
a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - ((sin(2*a) - sin(2*c))*cos(a +
 c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + ((sin(2*a) - sin(
2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos
(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + ((si
n(2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b
*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin
(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*s
in(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] time = 2.75295, size = 281, normalized size = 7.81

$$-\frac{\log\left(-\frac{1}{4}\cos\left(bx+c\right)^2+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2}{\cos(-a+c)^2+2\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(log(-1/4*cos(b*x + c)^2 + 1/4) - log(-(2*cos(b*x + c)*cos(-a + c)*sin
(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^
2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\csc\left(a+bx\right)\csc\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*csc(b*x + c), x)
```

Giac [B] time = 1.25554, size = 535, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^
4 + 4*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 +
4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2
 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan
(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^
2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^3
 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan
(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2
*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^
3 + tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c) - tan(1/2
*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)))/b
```

3.146 $\int \csc(c-bx)\csc(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\csc(a+c)\log(\sin(a+bx))}{b}-\frac{\csc(a+c)\log(\sin(c-bx))}{b}$$

```
[Out] -((Csc[a + c]*Log[Sin[c - b*x]])/b) + (Csc[a + c]*Log[Sin[a + b*x]])/b
```

Rubi [A] time = 0.0184097, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4611, 3475}

$$\frac{\csc(a+c)\log(\sin(a+bx))}{b}-\frac{\csc(a+c)\log(\sin(c-bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c - b*x]*Csc[a + b*x],x]
```

```
[Out] -((Csc[a + c]*Log[Sin[c - b*x]])/b) + (Csc[a + c]*Log[Sin[a + b*x]])/b
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c-bx)\csc(a+bx)\,dx &= \csc(a+c)\int \cot(c-bx)\,dx+\csc(a+c)\int \cot(a+bx)\,dx\\ &= -\frac{\csc(a+c)\log(\sin(c-bx))}{b}+\frac{\csc(a+c)\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.21866, size = 29, normalized size = 0.88

$$-\frac{\csc(a+c)(\log(\sin(c-bx))-\log(-\sin(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c - b*x]*Csc[a + b*x],x]
```

```
[Out] -((Csc[a + c]*(Log[Sin[c - b*x]] - Log[-Sin[a + b*x]]))/b)
```

Maple [B] time = 0.232, size = 81, normalized size = 2.5

$$\frac{\ln(\tan(bx+a))}{b(\sin(a)\cos(c)+\cos(a)\sin(c))}-\frac{\ln(\tan(bx+a)\cos(a)\cos(c)-\tan(bx+a)\sin(a)\sin(c)-\cos(a)\sin(c)-\sin(a}{b(\sin(a)\cos(c)+\cos(a)\sin(c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csc(b*x-c)*csc(b*x+a),x)
```

```
[Out] 1/b/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a))-1/b/(sin(a)*cos(c)+cos(a)*
sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)-tan(b*x+a)*sin(a)*sin(c)-cos(a)*sin(c)-
sin(a)*cos(c))
```

Maxima [B] time = 1.11456, size = 724, normalized size = 21.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -(2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*ar
ctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*(cos(2*a + 2*c)*cos(a + c)
+ sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - sin(a), cos(b*
x) + cos(a)) - 2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - c
os(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) - 2*(cos(2*a + 2*c
)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - s
in(c), cos(b*x) - cos(c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*sin
```

```
(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(a + c)*sin(2*a + 2*c) - cos(2*
a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(
a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(a + c)*sin(2*a + 2
*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*c
os(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(a + c)
*sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 -
2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2))/
(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

Fricas [B] time = 2.66961, size = 270, normalized size = 8.18

$$\frac{\log\left(-\frac{1}{4}\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^{2}-1\right)\cos(bx+a)^{2}-\cos(a+c)^{2}}{\cos(a+c)^{2}+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(-1/4*cos(b*x + a)^2 + 1/4) - log(-(2*cos(b*x + a)*cos(a + c)*sin(b
*x + a)*sin(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2)/(c
os(a + c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24936, size = 536, normalized size = 16.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^
4 - 4*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 -
4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2
 - tan(b*x + a)*tan(1/2*a)^2 - 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) + 2*tan
(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^
2 + tan(b*x + a) - 2*tan(1/2*a) - 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^3
 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan(1/2*a)^3*tan
(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 + tan(1/2
*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^
3 - tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c) + tan(1/2
*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c)))/b
```

3.147 $\int \sqrt{\sin(x)\tan(x)}\,dx$

Optimal. Leaf size=13

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Rubi [A] time = 0.0321317, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4400, 2589}

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[x]*Tan[x]],x]

[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sin(x)\tan(x)}\,dx &= \frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\ &= -2\cot(x)\sqrt{\sin(x)\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.0817373, size = 13, normalized size = 1.

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[x]*Tan[x]],x]
```

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Maple [B] time = 0.217, size = 177, normalized size = 13.6

$$\frac{\sqrt{4}\,(-1+\cos(x))\cos(x)}{4\,(\sin(x))^3}\left(4\,\cos(x)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}+4\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\ln\left(-\frac{1}{(\sin(x))^2}\left(2\,(\cos(x))^2\sqrt{-\frac{\cos(}{(1+\cos}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(1/2),x)
```

```
[Out] 1/4*4^(1/2)*(-1+cos(x))*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+4*(-cos(x)/(
1+cos(x))^2)^(1/2)-ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*
cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+ln(-2*(2*cos(x)^2*(-cos(
x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/
sin(x)^2))*cos(x)*(-(-1+cos(x)^2)/cos(x))^(1/2)/sin(x)^3/(-cos(x)/(1+cos(x)
)^2)^(1/2)
```

Maxima [B] time = 1.53871, size = 77, normalized size = 5.92

$$\frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)
/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))
```

Fricas [A] time = 2.43167, size = 63, normalized size = 4.85

$$-\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sin(x)\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)*tan(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sin(x)\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)*tan(x)), x)
```

3.148 $\int (\sin(x)\tan(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rubi [A] time = 0.0534873, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4400, 2598, 2589}

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]*Tan[x])^(3/2),x]
```

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
```

```
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= -\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(4\sqrt{\sin(x)\tan(x)}\right)\int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}\,dx}{3\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= \frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Mathematica [A] time = 0.0409342, size = 23, normalized size = 0.74

$$\frac{2}{3}\sin(x)\left(4\csc^2(x)-1\right)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]*Tan[x])^(3/2),x]
```

```
[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Maple [B] time = 0.13, size = 587, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(3/2),x)
```

```
[Out] 1/12*4^(1/2)*(-1+cos(x))^2*(3*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(
2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos
(x))^2)^(1/2)-1)/sin(x)^2)-3*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*c
os(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x)
)^2)^(1/2)-1)/sin(x)^2)+9*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*co
s(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))
^2)^(1/2)-1)/sin(x)^2)-9*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x
```

```
)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)
^(1/2)-1)/sin(x)^2)+9*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2
*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1
/2)-1)/sin(x)^2)-9*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-co
s(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1
)/sin(x)^2)+3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+co
s(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2
)-3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/
2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+4*cos(x)^3
+12*cos(x))*(1+cos(x))^2*(-(-1+cos(x)^2)/cos(x))^(3/2)/sin(x)^7
```

Maxima [B] time = 1.55491, size = 77, normalized size = 2.48

$$-\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x
)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 2.41241, size = 76, normalized size = 2.45

$$\frac{2\left(\cos(x)^2+3\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{3\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sin(x)\tan(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sin(x)*tan(x))^(3/2), x)
```

3.149 $\int (\sin(x)\tan(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rubi [A] time = 0.0753787, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4400, 2598, 2594, 2589}

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]*Tan[x])^(5/2),x]
```

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int (\sin(x)\tan(x))^{5/2}\,dx &= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}} \\
&= -\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(8\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin(x)}\tan^{\frac{5}{2}}(x)\,dx}{5\sqrt{\sin(x)}\sqrt{\tan(x)}} \\
&= \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{\left(32\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin(x)}\sqrt{t}}{15\sqrt{\sin(x)}\sqrt{\tan(x)}} \\
&= \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0991568, size = 29, normalized size = 0.58

$$\frac{2}{15}\tan(x)\sqrt{\sin(x)\tan(x)}\left(3\cos^2(x) + 32\cot^2(x) + 5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]*Tan[x])^(5/2),x]
```

```
[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

Maple [B] time = 0.167, size = 324, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(5/2),x)
```

```
[Out] -1/30*4^(1/2)*(-1+cos(x))^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^
(1/2)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-co
s(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1
/2)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-co
s(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-15*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1
+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x
)^2)*(-cos(x)/(1+cos(x))^2)^(1/2)+15*cos(x)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+c
os(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^
2)*(-cos(x)/(1+cos(x))^2)^(1/2)-60*cos(x)^2-10)*cos(x)*(1+cos(x))^2*(-(-1+c
os(x)^2)/cos(x))^(5/2)/sin(x)^9
```

Maxima [B] time = 1.60706, size = 111, normalized size = 2.22

$$-\frac{32\left(\frac{5\sin(x)^4}{(\cos(x)+1)^4}-\frac{5\sin(x)^6}{(\cos(x)+1)^6}+\frac{2\sin(x)^{10}}{(\cos(x)+1)^{10}}-2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10
/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1
) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 2.40271, size = 112, normalized size = 2.24

$$-\frac{2\left(3\cos(x)^4-30\cos(x)^2-5\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*s
in(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sin(x)\tan(x))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((sin(x)*tan(x))^(5/2), x)
```

3.150 $\int \sqrt{\cos(x)\cot(x)}\, dx$

Optimal. Leaf size=13

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rubi [A] time = 0.0381709, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4400, 2589}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]*Cot[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(x)\cot(x)}\, dx &= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}\, dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\ &= 2\sqrt{\cos(x)\cot(x)}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0687166, size = 13, normalized size = 1.

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]*Cot[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Maple [A] time = 0.132, size = 20, normalized size = 1.5

$$2\,\frac{\sin(x)}{\cos(x)}\sqrt{\frac{(\cos(x))^2}{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(1/2),x)
```

```
[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)
```

Maxima [B] time = 1.79664, size = 254, normalized size = 19.54

$$\frac{\left(\left(\cos\left(\frac{3}{2}\,x\right)-\cos\left(\frac{1}{2}\,x\right)+\sin\left(\frac{3}{2}\,x\right)+\sin\left(\frac{1}{2}\,x\right)\right)\cos\left(\frac{1}{2}\,\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}\,x\right)-\cos\left(\frac{1}{2}\,x\right)-\sin\left(\frac{3}{2}\,x\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
```

/4))

Fricas [A] time = 2.38723, size = 53, normalized size = 4.08

$$\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(x)\cot(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x)*cot(x)), x)
```

Giac [A] time = 1.14641, size = 16, normalized size = 1.23

$$2\,\mathrm{sgn}(\cos(x))\,\mathrm{sgn}(\sin(x))\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sgn(cos(x))*sgn(sin(x))*sqrt(sin(x))
```

3.151 $\int (\cos(x)\cot(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Rubi [A] time = 0.0683295, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4400, 2598, 2589}

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cot[x])^(3/2),x]
```

```
[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
```

```
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(4\sqrt{\cos(x)\cot(x)}\right)\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}\,dx}{3\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)$$

Mathematica [A] time = 0.0384839, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cos^2(x)-4\right)\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cot[x])^(3/2),x]
```

```
[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Maple [A] time = 0.117, size = 26, normalized size = 0.8

$$\frac{\left(2\,(\cos(x))^2-8\right)\sin(x)}{3\,(\cos(x))^3}\left(\frac{(\cos(x))^2}{\sin(x)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(3/2),x)
```

```
[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3
```

Maxima [B] time = 1.8546, size = 424, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos
(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) -
 sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(s
in(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2
*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arct
an2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*
x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x
) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos
(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(
5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*
sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 +
1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

Fricas [A] time = 2.39589, size = 66, normalized size = 2.13

$$\frac{2\left(\cos\left(x\right)^{2}-4\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{3\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12894, size = 26, normalized size = 0.84

$$-\frac{2}{3}\left(\sin(x)^{\frac{3}{2}}+\frac{3}{\sqrt{\sin(x)}}\right)\operatorname{sgn}(\cos(x))\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(sin(x)^(3/2) + 3/sqrt(sin(x)))*sgn(cos(x))*sgn(sin(x))
```

3.152 $\int (\cos(x)\cot(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rubi [A] time = 0.0944576, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4400, 2598, 2594, 2589}

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cot[x])^(5/2),x]
```

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}
\int (\cos(x)\cot(x))^{5/2}\,dx &= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(8\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)\,dx}{5\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{\left(32\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(x}}{15\sqrt{\cos(x)}\sqrt{\cot(x}}\\
&= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.0997359, size = 29, normalized size = 0.58

$$-\frac{2}{15}\tan(x)\sqrt{\cos(x)\cot(x)}\left(3\cos^2(x)+5\cot^2(x)+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cot[x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

Maple [A] time = 0.155, size = 34, normalized size = 0.7

$$\frac{\left(6\,(\cos(x))^4+48\,(\cos(x))^2-64\right)\sin(x)}{15\,(\cos(x))^5}\left(\frac{(\cos(x))^2}{\sin(x)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(5/2),x)
```

```
[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5
```

Maxima [B] time = 1.90874, size = 576, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*sin
(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan2(s
in(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3
*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/
2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*sin
(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) + 1)) - (
(3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) + 410*cos(
3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*sin(7/2*x)
- 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2(sin(x), c
os(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2
*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 4
10*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*sin(5/2*arc
tan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^4
 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))
```

Fricas [A] time = 2.54345, size = 103, normalized size = 2.06

$$\frac{2\left(3\cos\left(x\right)^{4}+24\cos\left(x\right)^{2}-32\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{15\cos\left(x\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12164, size = 36, normalized size = 0.72

$$\frac{2}{15} \left(3 \sin(x)^{\frac{5}{2}} - 30 \sqrt{\sin(x)} - \frac{5}{\sin(x)^{\frac{3}{2}}}\right) \operatorname{sgn}(\cos(x)) \operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*sin(x)^(5/2) - 30*sqrt(sin(x)) - 5/sin(x)^(3/2))*sgn(cos(x))*sgn(si
n(x))
```

3.153 $\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=58

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2]) - x/(b*(a
+ b*Sin[x]))
```

Rubi [A] time = 0.0776166, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4422, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[x])/(a + b*Sin[x])^2,x]
```


```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2]) - x/(b*(a
+ b*Sin[x]))
```


Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx &= -\frac{x}{b(a+b\sin(x))}+\frac{\int \frac{1}{a+b\sin(x)}\,dx}{b}\\ &= -\frac{x}{b(a+b\sin(x))}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= -\frac{x}{b(a+b\sin(x))}-\frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}\end{aligned}$$

Mathematica [A] time = 0.140019, size = 56, normalized size = 0.97

$$\frac{\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{x}{a+b\sin(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x])/(a + b*Sin[x])^2,x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - x/(a + b*Si
n[x]))/b
```

Maple [C] time = 0.247, size = 159, normalized size = 2.7

$$\frac{-2\,ix\mathrm{e}^{ix}}{b\left(b\mathrm{e}^{2\,ix}-b+2\,ia\mathrm{e}^{ix}\right)}-\frac{1}{b}\ln\left(\mathrm{e}^{ix}+\frac{1}{b}\left(ia\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}\right)\frac{1}{\sqrt{-a^{2}+b^{2}}}\right)\frac{1}{\sqrt{-a^{2}+b^{2}}}+\frac{1}{b}\ln\left(\mathrm{e}^{ix}+\frac{1}{b}\left(ia\sqrt{-a^{2}+b^{2}}+a^{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)/(a+b*sin(x))^2,x)
```

```
[Out] -2*I*x*exp(I*x)/b/(b*exp(2*I*x)-b+2*I*a*exp(I*x))-1/(-a^2+b^2)^(1/2)/b*ln(e
xp(I*x)+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/(-a^2+b^2)^(1/
2)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.58181, size = 531, normalized size = 9.16

$$\left[-\frac{\sqrt{-a^{2}+b^{2}}(b\sin\left(x\right)+a)\log\left(\frac{\left(2\,a^{2}-b^{2}\right)\cos\left(x\right)^{2}-2\,ab\sin\left(x\right)-a^{2}-b^{2}+2\left(a\cos\left(x\right)\sin\left(x\right)+b\cos\left(x\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(x\right)^{2}-2\,ab\sin\left(x\right)-a^{2}-b^{2}}\right)+2\left(a^{2}-b^{2}\right)x}{2\left(a^{3}b-ab^{3}+\left(a^{2}b^{2}-b^{4}\right)\sin\left(x\right)\right)},-\frac{\sqrt{a^{2}-b^{2}}(b\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*sin(x) + a)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (
a^2*b^2 - b^4)*sin(x)), -(sqrt(a^2 - b^2)*(b*sin(x) + a)*arctan(-(a*sin(x)
+ b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 -
```

```
b^4)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(x)}{(b\sin(x)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)/(b*sin(x) + a)^2, x)
```

3.154 $\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}-\frac{x}{2b(a+b\sin(x))^2}$$

```
[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) - x/(2*b
*(a + b*Sin[x])^2) + Cos[x]/(2*(a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.105745, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4422, 2664, 12, 2660, 618, 204}

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}-\frac{x}{2b(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[x])/(a + b*Sin[x])^3,x]
```


```
[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) - x/(2*b
*(a + b*Sin[x])^2) + Cos[x]/(2*(a^2 - b^2)*(a + b*Sin[x]))
```


Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx = -\frac{x}{2b(a+b\sin(x))^2} + \frac{\int \frac{1}{(a+b\sin(x))^2}\,dx}{2b}$$

$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{a}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} + \frac{a\int \frac{1}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b\left(a^2-b^2\right)}$$

$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right.}{b\left(a^2-b^2\right)}$$

$$= \frac{a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} - \frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}$$

Mathematica [A] time = 0.253827, size = 84, normalized size = 0.99

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{\frac{\cos(x)(a+b\sin(x))}{(a-b)(a+b)} - \frac{x}{b}}{2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x])/(a + b*Sin[x])^3,x]
```

```
[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) + (-(x/b
) + (Cos[x]*(a + b*Sin[x]))/((a - b)*(a + b)))/(2*(a + b*Sin[x])^2)
```

Maple [C] time = 0.412, size = 257, normalized size = 3.

$$\frac{2\,ia^2e^{2\,ix} + ib^2e^{2\,ix} + 2\,xa^2e^{2\,ix} + bae^{3\,ix} - 2\,b^2xe^{2\,ix} - ib^2 - 3\,abe^{ix}}{\left(be^{2\,ix} - b + 2\,iae^{ix}\right)^2\left(a^2-b^2\right)b} - \frac{a}{(2\,a+2\,b)(a-b)\,b}\ln\left(e^{ix} + \frac{1}{b}\left(ia\sqrt{-a^2+b^2} - a^2 + b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)/(a+b*sin(x))^3,x)
```

```
[Out] (2*I*a^2*exp(2*I*x)+I*b^2*exp(2*I*x)+2*x*a^2*exp(2*I*x)+b*a*exp(3*I*x)-2*b^
2*x*exp(2*I*x)-I*b^2-3*a*b*exp(I*x))/(b*exp(2*I*x)-b+2*I*a*exp(I*x))^2/(a^2
-b^2)/b-1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)^(1
/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(ex
p(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55744, size = 1013, normalized size = 11.92

$$\left[\frac{2\left(a^{2}b^{2}-b^{4}\right)\cos\left(x\right)\sin\left(x\right)-\left(ab^{2}\cos\left(x\right)^{2}-2\,a^{2}b\sin\left(x\right)-a^{3}-ab^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(-\frac{\left(2\,a^{2}-b^{2}\right)\cos(x)^{2}-2\,ab\sin(x)-a^{2}-b^{2}-2\,(a}{b^{2}\cos(x)^{2}-2\,ab\sin(}\right.}{4\left(a^{6}b-a^{4}b^{3}-a^{2}b^{5}+b^{7}-\left(a^{4}b^{3}-2\,a^{2}b^{5}+b^{7}\right)\cos\left(x\right)^{2}+2\left(a^{5}b^{2}-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*b^2 - b^4)*cos(x)*sin(x) - (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) -
a^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) -
 a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2
 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 2*(a^3*b - a*
b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)
*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*sin(x)), 1/2*((a^2*b^2 - b^4)*c
os(x)*sin(x) + (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) - a^3 - a*b^2)*sqrt(a^2 - b
^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^4 - 2*a^2*b^2 + b
^4)*x + (a^3*b - a*b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3
 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(x)}{(b\sin(x)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)/(b*sin(x) + a)^3, x)
```

3.155 $\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx$

Optimal. Leaf size=59

$$\frac{x}{b(a+b\cos(x))}-\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (-2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b])
 + x/(b*(a + b*Cos[x]))
```

Rubi [A] time = 0.0592501, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4423, 2659, 205}

$$\frac{x}{b(a+b\cos(x))}-\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[x])/(a + b*Cos[x])^2,x]
```



```
[Out] (-2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b])
 + x/(b*(a + b*Cos[x]))
```


Rule 4423

```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Cos[c + d*
x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
 - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx &= \frac{x}{b(a+b\cos(x))} - \frac{\int \frac{1}{a+b\cos(x)}\,dx}{b} \\
&= \frac{x}{b(a+b\cos(x))} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b} \\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}} + \frac{x}{b(a+b\cos(x))}
\end{aligned}
$$

Mathematica [A] time = 0.103584, size = 58, normalized size = 0.98

$$
\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\sqrt{b^2-a^2}} + \frac{x}{b(a+b\cos(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^2,x]
```

```
[Out] (2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) + x/(
b*(a + b*Cos[x]))
```

Maple [C] time = 0.125, size = 154, normalized size = 2.6

$$
2\,\frac{xe^{ix}}{b\left(be^{2\,ix}+2\,ae^{ix}+b\right)} - \frac{i}{b}\ln\left(e^{ix}+\frac{1}{b}\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\frac{1}{\sqrt{a^2-b^2}}\right)\frac{1}{\sqrt{a^2-b^2}} + \frac{i}{b}\ln\left(e^{ix}+\frac{1}{b}\left(a\sqrt{a^2-b^2}-a^2+b^2\right)\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/(a+b*cos(x))^2,x)
```

```
[Out] 2*x*exp(I*x)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)-I/(a^2-b^2)^(1/2)/b*ln(exp(I*x
)+(a*(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)+I/(a^2-b^2)^(1/2)/b*ln(exp
(I*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52556, size = 516, normalized size = 8.75

$$\left[-\frac{\sqrt{-a^2+b^2}(b\cos{(x)}+a)\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2-2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)-2\left(a^2-b^2\right)x}{2\left(a^3b-ab^3+\left(a^2b^2-b^4\right)\cos{(x)}\right)},-\frac{\sqrt{a^2-b^2}(b\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*cos(x) + a)*log((2*a*b*cos(x) + (2*a^2 - b^2)*co
s(x)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x
)^2 + 2*a*b*cos(x) + a^2)) - 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b
^4)*cos(x)), -(sqrt(a^2 - b^2)*(b*cos(x) + a)*arctan(-(a*cos(x) + b)/(sqrt(
a^2 - b^2)*sin(x))) - (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(x)}{(b\cos(x)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)/(b*cos(x) + a)^2, x)
```

3.156 $\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx$

Optimal. Leaf size=88

$$\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}+\frac{x}{2b(a+b\cos(x))^2}-\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -((a*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2))) + x/(2*b*(a + b*Cos[x])^2) + Sin[x]/(2*(a^2 - b^2)*(a + b*Cos[x]))
```

Rubi [A] time = 0.103267, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4423, 2664, 12, 2659, 205}

$$\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}+\frac{x}{2b(a+b\cos(x))^2}-\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[x])/(a + b*Cos[x])^3,x]
```

```
[Out] -((a*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2))) + x/(2*b*(a + b*Cos[x])^2) + Sin[x]/(2*(a^2 - b^2)*(a + b*Cos[x]))
```

Rule 4423

```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Cos[c + d*
x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
 - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx &= \frac{x}{2b(a+b\cos(x))^2}-\frac{\int \frac{1}{(a+b\cos(x))^2}\,dx}{2b}\\
&= \frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}-\frac{\int \frac{a}{a+b\cos(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= \frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}-\frac{a\int \frac{1}{a+b\cos(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= \frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b\left(a^2-b^2\right)}\\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}+\frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}
\end{aligned}
$$

Mathematica [A] time = 0.315517, size = 85, normalized size = 0.97

$$
\frac{\frac{\sin(x)(a+b\cos(x))}{(a-b)(a+b)}+\frac{x}{b}}{2(a+b\cos(x))^2}-\frac{a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\left(b^2-a^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^3,x]
```

```
[Out] -((a*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*(-a^2 + b^2)^(3/2)))
+ (x/b + ((a + b*Cos[x])*Sin[x])/((a - b)*(a + b)))/(2*(a + b*Cos[x])^2)
```

Maple [C] time = 0.231, size = 250, normalized size = 2.8

$$\frac{i\left(-2\,ia^2xe^{2\,ix}+2\,ib^2xe^{2\,ix}+bae^{3\,ix}+2\,a^2e^{2\,ix}+b^2e^{2\,ix}+3\,abe^{ix}+b^2\right)}{b\left(be^{2\,ix}+2\,ae^{ix}+b\right)^2\left(a^2-b^2\right)}-\frac{\frac{i}{2}a}{(a+b)\,(a-b)\,b}\ln\left(e^{ix}+\frac{1}{b}\left(a\sqrt{a^2-b^2}+a^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/(a+b*cos(x))^3,x)
```

```
[Out] I*(-2*I*a^2*x*exp(2*I*x)+2*I*b^2*x*exp(2*I*x)+b*a*exp(3*I*x)+2*a^2*exp(2*I*
x)+b^2*exp(2*I*x)+3*a*b*exp(I*x)+b^2)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)^2/(a^
2-b^2)-1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1/2)
+a^2-b^2)/(a^2-b^2)^(1/2)/b)+1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I
*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.68733, size = 944, normalized size = 10.73

$$\left[\frac{\left(ab^2\cos\left(x\right)^2+2\,a^2b\cos\left(x\right)+a^3\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)+2\left(a^4-2\,a^2\right.}{4\left(a^6b-2\,a^4b^3+a^2b^5+\left(a^4b^3-2\,a^2b^5+b^7\right)\cos\left(x\right)^2+2\left(a^5b^2-2\,a^3b^4+ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(-a^2 + b^2)*log((2*a*b*c
os(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) -
 a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) + 2*(a^4 - 2*a^2*b^2 + b
^4)*x + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*b
^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^
4 + a*b^6)*cos(x)), -1/2*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(a^2
- b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - (a^4 - 2*a^2*b^2
+ b^4)*x - (a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*
b^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b
^4 + a*b^6)*cos(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(x)}{(b \cos(x) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)/(b*cos(x) + a)^3, x)
```

3.157 $\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx$

Optimal. Leaf size=50

$$\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}$$

```
[Out] (a*x)/(b*(a^2 + b^2)) + Log[a*Cos[x] + b*Sin[x]]/(a^2 + b^2) - x/(b*(a + b*
Tan[x]))
```

Rubi [A] time = 0.0826108, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4424, 3484, 3530}

$$\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

```
[Out] (a*x)/(b*(a^2 + b^2)) + Log[a*Cos[x] + b*Sin[x]]/(a^2 + b^2) - x/(b*(a + b*
Tan[x]))
```

Rule 4424

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^2*((a_) + (b_.)*Tan[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Tan[c + d
*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(
m - 1)*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
 x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx &= -\frac{x}{b(a+b\tan(x))}+\frac{\int \frac{1}{a+b\tan(x)}\,dx}{b}\\ &= \frac{ax}{b\left(a^2+b^2\right)}-\frac{x}{b(a+b\tan(x))}+\frac{\int \frac{b-a\tan(x)}{a+b\tan(x)}\,dx}{a^2+b^2}\\ &= \frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}\end{aligned}$$

Mathematica [A] time = 0.176216, size = 48, normalized size = 0.96

$$\frac{a\log(a\cos(x)+b\sin(x))-bx}{a^3+ab^2}+\frac{x\sin(x)}{a^2\cos(x)+ab\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

```
[Out] (-(b*x) + a*Log[a*Cos[x] + b*Sin[x]])/(a^3 + a*b^2) + (x*Sin[x])/(a^2*Cos[x
] + a*b*Sin[x])
```

Maple [C] time = 0.164, size = 86, normalized size = 1.7

$$\frac{-2\,ix}{a^2+b^2}+\frac{2\,ix}{\left(-ib{\rm e}^{2\,ix}+a{\rm e}^{2\,ix}+ib+a\right)(-ib+a)}+\frac{1}{a^2+b^2}\ln\left({\rm e}^{2\,ix}-\frac{ib+a}{ib-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)^2/(a+b*tan(x))^2,x)
```

```
[Out] -2*I/(a^2+b^2)*x+2*I*x/(-I*b*exp(2*I*x)+a*exp(2*I*x)+I*b+a)/(-I*b+a)+1/(a^2
+b^2)*ln(exp(2*I*x)-(I*b+a)/(I*b-a))
```

Maxima [B] time = 1.039, size = 338, normalized size = 6.76

$$-\frac{8\,abx\cos(2\,x) - 4\left(a^2 - b^2\right)x\sin(2\,x) - \left(\left(a^2 + b^2\right)\cos(2\,x)^2 + 4\,ab\sin(2\,x) + \left(a^2 + b^2\right)\sin(2\,x)^2 + a^2 + b^2 + 2\left(a^2 - b^2\right)\right.}{2\left(a^4 + 2\,a^2b^2 + b^4 + \left(a^4 + 2\,a^2b^2 + b^4\right)\cos(2\,x)^2 + \left(a^4 + 2\,a^2b^2 + b^4\right)\sin(2\,x)^2 + 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*b*x*cos(2*x) - 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^2
 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(
2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2
 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4
 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2
+ 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

Fricas [A] time = 2.5764, size = 216, normalized size = 4.32

$$-\frac{2\,bx\cos(x) - 2\,ax\sin(x) - (a\cos(x) + b\sin(x))\log\left(2\,ab\cos(x)\sin(x) + \left(a^2 - b^2\right)\cos(x)^2 + b^2\right)}{2\left(\left(a^3 + ab^2\right)\cos(x) + \left(a^2b + b^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*cos(x) - 2*a*x*sin(x) - (a*cos(x) + b*sin(x))*log(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)
*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sec^2(x)}{(a + b\tan(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)**2/(a+b*tan(x))**2,x)
```

```
[Out] Integral(x*sec(x)**2/(a + b*tan(x))**2, x)
```

Giac [B] time = 1.362, size = 435, normalized size = 8.7

$$-\frac{2\,bx\tan\left(\frac{1}{2}\,x\right)^{2}-a\log\left(\frac{4\left(a^{2}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,ab\tan\left(\frac{1}{2}\,x\right)^{3}-2\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ax\tan\left(\frac{1}{2}\,x\right)+}{2\left(a^{3}\tan\left(\frac{1}{2}\,x\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*tan(1/2*x)^2 - a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 -
 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2
*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*a*x*tan(1/2*x) + 2*b*log(4*(a
^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x
)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*
x) - 2*b*x + a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*tan(1/2
*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(
1/2*x)^2 + 1)))/(a^3*tan(1/2*x)^2 + a*b^2*tan(1/2*x)^2 - 2*a^2*b*tan(1/2*x)
 - 2*b^3*tan(1/2*x) - a^3 - a*b^2)
```

3.158 $\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx$

Optimal. Leaf size=50

$$-\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\sin(x)+b\cos(x))}{a^2+b^2}+\frac{x}{b(a+b\cot(x))}$$

```
[Out] -((a*x)/(b*(a^2 + b^2))) + x/(b*(a + b*Cot[x])) + Log[b*Cos[x] + a*Sin[x]]/
(a^2 + b^2)
```

Rubi [A] time = 0.0817342, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4425, 3484, 3530}

$$-\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\sin(x)+b\cos(x))}{a^2+b^2}+\frac{x}{b(a+b\cot(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

```
[Out] -((a*x)/(b*(a^2 + b^2))) + x/(b*(a + b*Cot[x])) + Log[b*Cos[x] + a*Sin[x]]/
(a^2 + b^2)
```

Rule 4425

```
Int[Csc[(c_.) + (d_.)*(x_)]^2*(Cot[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*
((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Cot[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Cot[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx &= \frac{x}{b(a+b\cot(x))}-\frac{\int \frac{1}{a+b\cot(x)}\,dx}{b}\\ &= -\frac{ax}{b\left(a^2+b^2\right)}+\frac{x}{b(a+b\cot(x))}+\frac{\int \frac{-b+a\cot(x)}{a+b\cot(x)}\,dx}{a^2+b^2}\\ &= -\frac{ax}{b\left(a^2+b^2\right)}+\frac{x}{b(a+b\cot(x))}+\frac{\log(b\cos(x)+a\sin(x))}{a^2+b^2}\end{aligned}$$

Mathematica [A] time = 0.182409, size = 48, normalized size = 0.96

$$\frac{b\log(a\sin(x)+b\cos(x))-ax}{a^2b+b^3}+\frac{x\sin(x)}{ab\sin(x)+b^2\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

```
[Out] (-(a*x) + b*Log[b*Cos[x] + a*Sin[x]])/(a^2*b + b^3) + (x*Sin[x])/(b^2*Cos[x
] + a*b*Sin[x])
```

Maple [C] time = 0.147, size = 87, normalized size = 1.7

$$\frac{-2\,ix}{a^2+b^2}-\frac{2\,ix}{\left(ib\mathrm{e}^{2\,ix}+a\mathrm{e}^{2\,ix}+ib-a\right)(ib+a)}+\frac{1}{a^2+b^2}\ln\left(\mathrm{e}^{2\,ix}+\frac{ib-a}{ib+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)^2/(a+b*cot(x))^2,x)
```

```
[Out] -2*I/(a^2+b^2)*x-2*I*x/(I*b*exp(2*I*x)+a*exp(2*I*x)+I*b-a)/(I*b+a)+1/(a^2+b
^2)*ln(exp(2*I*x)+(I*b-a)/(I*b+a))
```

Maxima [B] time = 1.05116, size = 338, normalized size = 6.76

$$-\frac{8\,abx\cos\left(2\,x\right)+4\,\left(a^{2}-b^{2}\right)x\sin\left(2\,x\right)-\left(\left(a^{2}+b^{2}\right)\cos\left(2\,x\right)^{2}+4\,ab\sin\left(2\,x\right)+\left(a^{2}+b^{2}\right)\sin\left(2\,x\right)^{2}+a^{2}+b^{2}-2\,\left(a^{2}-b\right.\right.}{2\,\left(a^{4}+2\,a^{2}b^{2}+b^{4}+\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\cos\left(2\,x\right)^{2}+\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sin\left(2\,x\right)^{2}-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*b*x*cos(2*x) + 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^2
 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(
2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2
 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4
 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2
- 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

Fricas [A] time = 2.64612, size = 216, normalized size = 4.32

$$-\frac{2\,ax\cos\left(x\right)-2\,bx\sin\left(x\right)-\left(b\cos\left(x\right)+a\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)-\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+a^{2}\right)}{2\,\left(\left(a^{2}b+b^{3}\right)\cos\left(x\right)+\left(a^{3}+ab^{2}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*cos(x) - 2*b*x*sin(x) - (b*cos(x) + a*sin(x))*log(2*a*b*cos(x)*
sin(x) - (a^2 - b^2)*cos(x)^2 + a^2))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)
*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\csc^{2}\left(x\right)}{\left(a+b\cot\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2/(a+b*cot(x))**2,x)
```

```
[Out] Integral(x*csc(x)**2/(a + b*cot(x))**2, x)
```

Giac [B] time = 1.30335, size = 435, normalized size = 8.7

$$-\frac{2\,ax\tan\left(\frac{1}{2}\,x\right)^{2} - b\log\left(\frac{4\left(b^{2}\tan\left(\frac{1}{2}\,x\right)^{4} - 4\,ab\tan\left(\frac{1}{2}\,x\right)^{3} + 4\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2} - 2\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2} + 4\,ab\tan\left(\frac{1}{2}\,x\right) + b^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4} + 2\,\tan\left(\frac{1}{2}\,x\right)^{2} + 1}\right)\tan\left(\frac{1}{2}\,x\right)^{2} + 4\,bx\tan\left(\frac{1}{2}\,x\right) +}{2\left(a^{2}b\tan\left(\frac{1}{2}\,x\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*x*tan(1/2*x)^2 - b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 +
 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2
*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*b*x*tan(1/2*x) + 2*a*log(4*(b
^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x
)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*
x) - 2*a*x + b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*tan(1/2
*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(
1/2*x)^2 + 1)))/(a^2*b*tan(1/2*x)^2 + b^3*tan(1/2*x)^2 - 2*a^3*tan(1/2*x) -
 2*a*b^2*tan(1/2*x) - a^2*b - b^3)
```

3.159 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] time = 0.0547793, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Mathematica [A] time = 0.0965612, size = 32, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A] time = 0.065, size = 24, normalized size = 0.8

$$\frac{1}{d}\arctan\left(\tan(dx+c)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88463, size = 487, normalized size = 15.22

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)}{4\,abd}, -\frac{\sqrt{ab}\arctan\left(\frac{\left((a+b)\cos\right.}{2\,ab\cos(d}\right.}{2\,abd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*
cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin
(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*
x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2
 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.4661, size = 54, normalized size = 1.69

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(s
qrt(a*b)*d)
```

3.160 $\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=211

$$-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2}+\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d}$$

```
[Out] ((-I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])/(Sq
rt[a]*Sqrt[b]*d) + ((I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a]
+ Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*
x)))/(Sqrt[a] - Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2) + PolyLog[2, -(((a - b
)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2)
```

Rubi [A] time = 0.528379, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4588, 3321, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2}+\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```


```
[Out] ((-I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])/(Sq
rt[a]*Sqrt[b]*d) + ((I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a]
+ Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*
x)))/(Sqrt[a] - Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2) + PolyLog[2, -(((a - b
)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2)
```


Rule 4588

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b + c + (b
- c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0
]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = 2\int \frac{x}{a+b+(a-b)\cos(2c+2dx)}\,dx$$
$$= 4\int \frac{e^{i(2c+2dx)}x}{a-b+2(a+b)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$
$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{-4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{i\int\log\left(1+\frac{2(a-b)e^{i(2c+2dx)}}{-4\sqrt{a}\sqrt{b}+2(a+b)}\right)dx}{2\sqrt{a}\sqrt{b}d} - \frac{i\int \mathrm{lo}}{}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2(a-b)x}{-4\sqrt{a}\sqrt{b}+2(a+b)}\right)}{x}dx, x, e^{i(2c}\right.}{4\sqrt{a}\sqrt{b}d^2}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2} + \frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right.}{4\sqrt{a}\sqrt{b}d^2}$$

Mathematica [B] time = 6.53475, size = 512, normalized size = 2.43

$$\frac{x\left(-\sqrt{a}\text{PolyLog}\left(2,\frac{\sqrt{b}(1-i\tan(c+dx))}{\sqrt{b}+i\sqrt{-a}}\right)-\sqrt{a}\text{PolyLog}\left(2,\frac{\sqrt{b}(1+i\tan(c+dx))}{\sqrt{b}+i\sqrt{-a}}\right)+\sqrt{a}\text{PolyLog}\left(2,-\frac{\sqrt{b}(\tan(c+dx)-i)}{\sqrt{-a}+i\sqrt{b}}\right)+\sqrt{a}\text{PolyLo}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (x*((4*I)*Sqrt[-a]*c*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Log[1
 + I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt
[b])] + Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x
])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a]
+ Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt[b])] + Sqrt[a]*Log[1 + I*Tan[c +
 d*x]]*Log[(Sqrt[-a] + Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt
[a]*PolyLog[2, (Sqrt[b]*(1 - I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] - Sqr
t[a]*PolyLog[2, (Sqrt[b]*(1 + I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] + Sq
rt[a]*PolyLog[2, -((Sqrt[b]*(-I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b]))] +
 Sqrt[a]*PolyLog[2, (Sqrt[b]*(I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b])]))/
(2*Sqrt[-a^2]*Sqrt[b]*d*((2*I)*c + Log[1 - I*Tan[c + d*x]] - Log[1 + I*Tan[
```

```
c + d*x]]))
```

Maple [B] time = 0.157, size = 1003, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/2/d^2/(a*b)^(1/2)*c^2-1/d^2/(-2*(a*b)^(1/2)-a-b)*c^2-1/2/d^2/(-2*(a*b)^(
1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))-1/4/d^2/(a
*b)^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))-1/d/(a*b)^(
1/2)*c*x-2/d/(-2*(a*b)^(1/2)-a-b)*c*x-1/2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*
a*x^2-1/2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*x^2-1/d/(a*b)^(1/2)/(-2*(a*b)^
(1/2)-a-b)*a*c*x-1/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c*x-1/2*I/d/(a*b)^(
1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))
*a*x-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(
-2*(a*b)^(1/2)-a-b))*b*x-1/2*I/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a
-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*a*c-1/2*I/d^2/(a*b)^(1/2)/(-2*(a
*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b*c-1/2/(a
*b)^(1/2)*x^2-1/(-2*(a*b)^(1/2)-a-b)*x^2-1/2*I/d^2/(a*b)^(1/2)*ln(1-(a-b)*e
xp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*c-I/d^2/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b
)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c-I/d^2*c/(a*b)^(1/2)*arctanh(1/4*
(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b)/(a*b)^(1/2))-1/2*I/d/(a*b)^(1/2)*ln(1-(a
-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*x-I/d/(-2*(a*b)^(1/2)-a-b)*ln(1-(
a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)
^(1/2)-a-b)*a*c^2-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c^2-1/4/d^2/(a
*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(
1/2)-a-b))*a-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2
*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.23159, size = 8015, normalized size = 37.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(-4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*s
qrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d
*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) - 2*I*s
in(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(
a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) + 2
*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-
(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c)
 - 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sq
rt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x +
 c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*
x + c) - 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*lo
g(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos
(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c
*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*
cos(d*x + c) - 2*I*sin(d*x + c)) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*
dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a -
 b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a +
2*b)/(a - b) + 1) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*
(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x +
c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) +
 1) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b)*cos(d*x
 + c) + (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(
a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 4*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b)*cos(d*x + c) - (-2*I*
a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c
))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b +
b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2
```

- 2*a*b + b^2))*dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d
x + c) + 4((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2
- 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a
- b)) - 2*a + 2*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*
dilog(-1/2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a
- b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2
b)/(a - b) + 1) - 4(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a
+ b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c
) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b) + 1
) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b)*cos(d*x
+ c) - (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a
^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 4*(I*(a -
b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*co
s(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a
+ I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(
a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a
- b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*
cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I
*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqr
t(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*
(a - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a +
b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) -
(I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*
sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(
I*(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a +
b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c)
+ (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*
(-I*(a - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(
a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c
) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b
)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4
(I(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a
+ b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c)
- (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*
(I*(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a
+ b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c
) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) +
4*(-I*(a - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*
(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x +

```
 c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b)))
/(a*b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sec(dx+c)^2}{b\tan(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

3.161 $\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=337

$$-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2}+\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2}+\frac{i\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3}-\frac{i\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3}$$

```
[Out] ((-I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])/(
Sqrt[a]*Sqrt[b]*d) + ((I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt
[a] + Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I)*
(c + d*x)))/(Sqrt[a] - Sqrt[b])^2)])/(2*Sqrt[a]*Sqrt[b]*d^2) + (x*PolyLog[2
, -(((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)])/(2*Sqrt[a]*Sqrt[
b]*d^2) + ((I/4)*PolyLog[3, -(((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sq
rt[a] + Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^3) - ((I/4)*PolyLog[3, -(((Sqrt[a] +
 Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^3)
```

Rubi [A] time = 0.900404, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4588, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2}+\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2}+\frac{i\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3}-\frac{i\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])/(
Sqrt[a]*Sqrt[b]*d) + ((I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt
[a] + Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I)*
(c + d*x)))/(Sqrt[a] - Sqrt[b])^2)])/(2*Sqrt[a]*Sqrt[b]*d^2) + (x*PolyLog[2
, -(((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)])/(2*Sqrt[a]*Sqrt[
b]*d^2) + ((I/4)*PolyLog[3, -(((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sq
rt[a] + Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^3) - ((I/4)*PolyLog[3, -(((Sqrt[a] +
 Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^3)
```

Rule 4588

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b + c + (b
- c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0
]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = 2\int \frac{x^2}{a+b+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x^2}{a-b+2(a+b)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{-4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{i\int x\log\left(1+\frac{2(a-b)e^{i(2c+2dx)}}{-4\sqrt{a}\sqrt{b}+2(a+b)}\right)dx}{\sqrt{a}\sqrt{b}d} - \frac{i\int$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c}}{(\sqrt{a}+\sqrt{}}\right.}{2\sqrt{a}\sqrt{b}d^2}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c}}{(\sqrt{a}+\sqrt{}}\right.}{2\sqrt{a}\sqrt{b}d^2}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c}}{(\sqrt{a}+\sqrt{}}\right.}{2\sqrt{a}\sqrt{b}d^2}$$

Mathematica [A] time = 1.0536, size = 294, normalized size = 0.87

$$\frac{i\left(-2idx\text{PolyLog}\left(2,\frac{\left(\sqrt{b}-\sqrt{a}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)+2idx\text{PolyLog}\left(2,-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)+\text{PolyLog}\left(3,\frac{\left(\sqrt{b}-\sqrt{a}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)-\text{PolyLog}\left(3,-\right.\right.}{4\sqrt{a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((I/4)*(2*d^2*x^2*Log[1 + ((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a
] + Sqrt[b])] - 2*d^2*x^2*Log[1 + ((Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))
/(Sqrt[a] - Sqrt[b])] - (2*I)*d*x*PolyLog[2, ((-Sqrt[a] + Sqrt[b])*E^((2*I)
*(c + d*x)))/(Sqrt[a] + Sqrt[b])] + (2*I)*d*x*PolyLog[2, -(((Sqrt[a] + Sqrt
[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))] + PolyLog[3, ((-Sqrt[a] + S
qrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])] - PolyLog[3, -(((Sqrt[a]
+ Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))]))/(Sqrt[a]*Sqrt[b]*d^
3)
```

Maple [B] time = 0.144, size = 1251, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c^2*x-1/2/d^2/(a*b)^(1/2)/(-2*(a*b
)^(1/2)-a-b)*a*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x-1/2
/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-
2*(a*b)^(1/2)-a-b))*x+1/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c^2*x-1/4*I/
d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2
*(a*b)^(1/2)-a-b))-1/4*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*polylog(3,(
a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))+2/3/d^3/(a*b)^(1/2)*c^3+4/3/d^3
/(-2*(a*b)^(1/2)-a-b)*c^3-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*ln(1-(
a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x^2-1/2*I/d/(a*b)^(1/2)/(-2*(a*
b)^(1/2)-a-b)*b*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x^2+1/2*I
/d^3*c^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(
a*b)^(1/2)-a-b))*a+1/2*I/d^3*c^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b
)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b-1/3/(a*b)^(1/2)*x^3-2/3/(-2*(a*b
)^(1/2)-a-b)*x^3-I/d/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(
a*b)^(1/2)-a-b))*x^2-1/2*I/d/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*
b)^(1/2)-a-b))*x^2+I/d^3/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(
-2*(a*b)^(1/2)-a-b))*c^2+1/2*I/d^3/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/
(2*(a*b)^(1/2)-a-b))*c^2+2/3/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c^3+2/3
/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c^3+I/d^3*c^2/(a*b)^(1/2)*arctanh(1
/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b)/(a*b)^(1/2))-1/3/(a*b)^(1/2)/(-2*(a*b
)^(1/2)-a-b)*a*x^3-1/3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*x^3+2/d^2/(-2*(a*
b)^(1/2)-a-b)*c^2*x-1/2/d^2/(a*b)^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2
*(a*b)^(1/2)-a-b))*x-1/d^2/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*
x+c))/(-2*(a*b)^(1/2)-a-b))*x+1/d^2/(a*b)^(1/2)*c^2*x-1/4*I/d^3/(a*b)^(1/2)
*polylog(3,(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))-1/2*I/d^3/(-2*(a*b)^
```

```
(1/2)-a-b)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.12958, size = 11173, normalized size = 33.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(1/2*((2*(a + b)*cos
(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a
+ I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a
*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-1/2*((2*(a + b)*cos(d*x + c)
- (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin
(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 -
2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 8*(a - b)*sqrt(
a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(1/2*((2*(a + b)*cos(d*x + c) + (-2*I*a -
 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*
sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2
)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a - b)*sqrt(a*b/(a^2 -
2*a*b + b^2))*d*x*dilog(-1/2*((2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*si
n(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(
a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b
)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^
2))*d*x*dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) +
 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b
+ b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) -
2*a + 2*b)/(a - b) + 1) - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog
```

```
(-1/2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*
cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqr
t((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(
a - b) + 1) - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(1/2*((2*(a
+ b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c)
 + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b
)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b) + 1)
 - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-1/2*((2*(a + b)*cos(d
*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a +
 I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b
/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 4*I*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2
 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) - 4
*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a - b)*sqrt(a*
b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c
)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a - b)*s
qrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d
*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) - 2*I
*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt((
2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x + c)
+ 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*s
qrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x
+ c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*lo
g(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos
(d*x + c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c
^2*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) -
2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*
sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*cos(d*x + c) + (2*I*a +
2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*
sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2
)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a -
 b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*cos(d*x + c) - (
2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*
x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a
*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2
+ I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*cos(d*x
 + c) + (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(
a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*
d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*
cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (I
*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqr
t(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*
(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2
```

```
*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x +
 c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
 b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) +
 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/
2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(
d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2
*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a -
b)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*l
og(-1/2*((2*(a + b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a -
b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*
b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b
+ b^2))*log(1/2*((2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) +
4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b
+ b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) +
2*a - 2*b)/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polyl
og(3, -1/2*(2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a -
 b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b))/(a - b))
 + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a +
b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) +
(-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*
sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*
b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b)*cos(d*x + c) +
(-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d
*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*
a*b + b^2)) + a + b)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 -
2*a*b + b^2))*polylog(3, 1/2*(2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin
(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a
^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)
/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polyl
og(3, -1/2*(2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a -
 b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b))/(a - b)) +
 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b)*
cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*
a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(
a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqr
t(a*b/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b)*cos(d*x + c) + (-2*I*
a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x +
c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b +
b^2)) - a - b)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b
 + b^2))*polylog(3, 1/2*(2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x
+ c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 -
 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a -
```

```
b))/(a - b)))/(a*b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sec^2(c + dx)}{a + b \tan^2(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sec(dx + c)^2}{b \tan(dx + c)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

3.162 $\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\sqrt{b+c}}$$

```
[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

Rubi [A] time = 0.605453, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\sqrt{b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+c+(b+c)x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c}\sqrt{b+cd}}\end{aligned}$$

Mathematica [A] time = 0.263189, size = 40, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\sqrt{b+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

Maple [A] time = 0.084, size = 34, normalized size = 0.9

$$\frac{1}{d}\arctan\left((b+c)\tan(dx+c)\frac{1}{\sqrt{(a+c)(b+c)}}\right)\frac{1}{\sqrt{(a+c)(b+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/((a+c)*(b+c))^(1/2)*arctan((b+c)*tan(d*x+c)/((a+c)*(b+c))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.93774, size = 747, normalized size = 18.68

$$\left[-\frac{\sqrt{-ab-(a+b)c-c^2}\log\left(\frac{\left(a^2+6\,ab+b^2+8\,(a+b)c+8\,c^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2+(3\,a+5\,b)c+4\,c^2\right)\cos(dx+c)^2+4\left((a+b+2\,c)\cos(dx+c)^3-(b+c)\cos(d\right.}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2+(a-b)c\right)\cos(dx+c)^2+b^2+2\,bc+c^2}\right.}{4\left(ab+(a+b)c+c^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] [-1/4*sqrt(-a*b - (a + b)*c - c^2)*log(((a^2 + 6*a*b + b^2 + 8*(a + b)*c +
8*c^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2 + (3*a + 5*b)*c + 4*c^2)*cos(d*x + c
)^2 + 4*((a + b + 2*c)*cos(d*x + c)^3 - (b + c)*cos(d*x + c))*sqrt(-a*b - (
a + b)*c - c^2)*sin(d*x + c) + b^2 + 2*b*c + c^2)/((a^2 - 2*a*b + b^2)*cos(
d*x + c)^4 + 2*(a*b - b^2 + (a - b)*c)*cos(d*x + c)^2 + b^2 + 2*b*c + c^2))
/((a*b + (a + b)*c + c^2)*d), -1/2*arctan(1/2*((a + b + 2*c)*cos(d*x + c)^2
 - b - c)/(sqrt(a*b + (a + b)*c + c^2)*cos(d*x + c)*sin(d*x + c)))/(sqrt(a*
b + (a + b)*c + c^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

Giac [B] time = 1.42797, size = 103, normalized size = 2.58

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,b+2\,c\right)+\arctan\left(\frac{b\tan(dx+c)+c\tan(dx+c)}{\sqrt{ab+ac+bc+c^2}}\right)}{\sqrt{ab+ac+bc+c^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(2*b + 2*c) + arctan((b*tan(d*x + c) + c*t
an(d*x + c))/sqrt(a*b + a*c + b*c + c^2)))/(sqrt(a*b + a*c + b*c + c^2)*d)
```

3.163 $\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=267

$$-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}+\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\sqrt{b+c}+c\right)+a+b}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\sqrt{b+c}}+\frac{ix\log\left(1+\right.}{2d\sqrt{}}$$

```
[Out] ((-I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c
]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x*Log[1 + ((a - b)*E^
((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a + c]
*Sqrt[b + c]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c -
 2*Sqrt[a + c]*Sqrt[b + c]))]/(4*Sqrt[a + c]*Sqrt[b + c]*d^2) + PolyLog[2,
-(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c])))]
/(4*Sqrt[a + c]*Sqrt[b + c]*d^2)
```

Rubi [A] time = 0.718662, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4589, 3321, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}+\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\sqrt{b+c}+c\right)+a+b}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\sqrt{b+c}}+\frac{ix\log\left(1+\right.}{2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```



```
[Out] ((-I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c
]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x*Log[1 + ((a - b)*E^
((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a + c]
*Sqrt[b + c]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c -
 2*Sqrt[a + c]*Sqrt[b + c]))]/(4*Sqrt[a + c]*Sqrt[b + c]*d^2) + PolyLog[2,
-(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c])))]
/(4*Sqrt[a + c]*Sqrt[b + c]*d^2)
```


Rule 4589

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> D
ist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
```

```
FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c
, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = 2\int \frac{x}{a+b+2c+(a-b)\cos(2c+2dx)}\,dx$$
$$= 4\int \frac{e^{i(2c+2dx)}x}{a-b+2(a+b+2c)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$
$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{-4\sqrt{a+c}\sqrt{b+c}+2(a+b+2c)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a+c}\sqrt{b+c}} - \frac{(2(a-b))\int \frac{}{4\sqrt{a+c}\sqrt{b+c}+2}}{\sqrt{a+c}}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{i\int\log\left(\right.}{}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+cd}} - \frac{\text{Li}_2\left(-\frac{}{a+}\right.}{4\sqrt{a}}$$

Mathematica [B] time = 4.34411, size = 751, normalized size = 2.81

$$\frac{x\left(\sqrt{a+c}-\sqrt{-b-c}\tan(c+dx)\right)\left(\sqrt{a+c}+\sqrt{-b-c}\tan(c+dx)\right)\left(i\sqrt{b+c}\text{PolyLog}\left(2,\frac{\sqrt{-b-c}(1-i\tan(c+dx))}{\sqrt{-b-c}-i\sqrt{a+c}}\right)-i\sqrt{b+c}\text{Po}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] (x*(4*Sqrt[-b - c]*c*ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]] - I*Sqr
t[b + c]*Log[1 + I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c +
 d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*Log[1 - I*Tan[c + d
*x]]*Log[(I*(-Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*S
qrt[a + c])] + I*Sqrt[b + c]*Log[1 + I*Tan[c + d*x]]*Log[((-I)*(Sqrt[a + c]
 + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b +
 c]*Log[1 - I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]
))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]
*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b + c]*Poly
Log[2, (Sqrt[-b - c]*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])]
+ I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1 + I*Tan[c + d*x]))/(Sqrt[-b - c
```

```
] - I*Sqrt[a + c])] - I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1 + I*Tan[c +
 d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])])*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c
+ d*x])*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(2*Sqrt[a + c]*Sqrt[-(b
+ c)^2]*d*(2*c - I*Log[1 - I*Tan[c + d*x]] + I*Log[1 + I*Tan[c + d*x]])*(a
+ c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2))
```

Maple [B] time = 0.201, size = 1670, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c*x-1/d/((a+c)*
(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c*x-I/d^2/((a+c)*(b+c))^(1/
2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*
(b+c))^(1/2)-a-b-2*c))*c^2-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1
/2)-a-b-2*c)*a*c^2-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-
2*c)*b*c^2-1/4/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*pol
ylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a-1/4/d^2/(
(a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I
*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*b-1/2/d^2/((a+c)*(b+c))^(1/2)/(
-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)
*(b+c))^(1/2)-a-b-2*c))*c-2/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a
-b-2*c)*c^2*x-I/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*
x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c-1/2*I/d^2/((a+c)*(b+c))^(1/2)*ln(
1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c-I/d^2*c/(a*b+a*
c+b*c+c^2)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b+4*c)/(a*b+a*
c+b*c+c^2)^(1/2))-I/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(
d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-1/2*I/d/((a+c)*(b+c))^(1/2)*ln(
1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-1/((a+c)*(b+c))
^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*x^2-1/2/((a+c)*(b+c))^(1/2)/(-2*(
(a+c)*(b+c))^(1/2)-a-b-2*c)*a*x^2-1/2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))
^(1/2)-a-b-2*c)*b*x^2-2/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*x-1/2/d^2/((a+
c)*(b+c))^(1/2)*c^2-1/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^2-1/2/d^2/(-2*
((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b
+c))^(1/2)-a-b-2*c))-1/4/d^2/((a+c)*(b+c))^(1/2)*polylog(2,(a-b)*exp(2*I*(d
*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/2/((a+c)*(b+c))^(1/2)*x^2-1/(-2*(
(a+c)*(b+c))^(1/2)-a-b-2*c)*x^2-1/2*I/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b
+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b
-2*c))*a*c-1/2*I/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*l
```

```
n(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*b*c-1/2*I/d/((
a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+
c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a*x-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*(
(a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(
1/2)-a-b-2*c))*b*x-I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)
*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c*x-1/d/((a+
c)*(b+c))^(1/2)*c*x-1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2
*c)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.02899, size = 10267, normalized size = 38.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(-4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(
2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a +
b + 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((
a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos
(d*x + c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x +
 c)) - 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(
2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a +
b + 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt
```

((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(
d*x + c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(
a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c
)) - 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*
sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b -
 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b
+ (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x
 + c) - 2*I*sin(d*x + c)) + 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*
I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b
 + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*
b)/(a - b) + 1) + 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2
))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(
d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b +
 (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b
)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b
) + 1) + 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(
1/2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c)
- 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c
 + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)
/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 4
*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(
a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a -
 b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(
a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 4*(a - b)*
sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*
c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x
 + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
 + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))
 - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 4*(a - b)*sqrt((a*b +
(a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x
+ c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*
a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*s
qrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b -
2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 4*(a - b)*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I
*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*s
in(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
- b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a -
b)) - 2*a + 2*b)/(a - b) + 1) - 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2

```
 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*
b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x +
c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a
 - 2*b)/(a - b) + 1) + 4*(I*(a - b)*d*x + I*(a - b)*c)*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*
I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*
sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(
a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a
- b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*d*x - I*(a - b)*c)*sqrt((a*b +
(a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*cos(d*x + c
) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a
+ I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqr
t(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2
*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d*x - I*(a - b)*c)*sqrt(
(a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b + 2*c)*cos
(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c)
 - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a
 + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d*x + I*(a - b)*c
)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*
c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*
x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*
b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2
)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d*x - I*(a
 - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a
+ b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)
*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
 + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d*x +
 I*(a - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2
*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a
- b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(I*(a - b)*d
*x + I*(a - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/
2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) +
4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c
+ c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(
a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(
a - b)*d*x - I*(a - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))
*log(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x
+ c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a
+ b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b)))/(
```

```
(a*b + (a + b)*c + c^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sec(dx+c)^2}{c\sec(dx+c)^2+b\tan(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

3.164 $\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=407

$$-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}+\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\sqrt{b+c}+c\right)+a+b}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}-\frac{i\text{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}}+\frac{i\text{Pol}}{}$$

```
[Out] ((-I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a +
 c]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x^2*Log[1 + ((a - b
)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a
+ c]*Sqrt[b + c]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b
+ 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) + (x*
PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt
[b + c])))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) - ((I/4)*PolyLog[3, -(((a - b)
*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a
+ c]*Sqrt[b + c]*d^3) + ((I/4)*PolyLog[3, -(((a - b)*E^((2*I)*(c + d*x)))/(
a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c])))])/(Sqrt[a + c]*Sqrt[b + c]*d^3)
```

Rubi [A] time = 1.0751, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4589, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}+\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\sqrt{b+c}+c\right)+a+b}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}-\frac{i\text{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}}+\frac{i\text{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a +
 c]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x^2*Log[1 + ((a - b
)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a
+ c]*Sqrt[b + c]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b
+ 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) + (x*
PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt
[b + c])))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) - ((I/4)*PolyLog[3, -(((a - b)
*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a
+ c]*Sqrt[b + c]*d^3) + ((I/4)*PolyLog[3, -(((a - b)*E^((2*I)*(c + d*x)))/(
```

```
a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))))])/(Sqrt[a + c]*Sqrt[b + c]*d^3)
```

Rule 4589

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> D
ist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c
, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = 2\int \frac{x^2}{a+b+2c+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x^2}{a-b+2(a+b+2c)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{-4\sqrt{a+c}\sqrt{b+c}+2(a+b+2c)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a+c}\sqrt{b+c}} - \frac{(2(a-b))\int \frac{}{4\sqrt{a+c}\sqrt{b+c}+2}}{\sqrt{a+c}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{i\int x\,\mathrm{l}}{}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+cd}} - \frac{x\mathrm{Li}_2\left(\right.}{2}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+cd}} - \frac{x\mathrm{Li}_2\left(\right.}{2}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+cd}} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+cd}} - \frac{x\mathrm{Li}_2\left(\right.}{2}$$

Mathematica [A] time = 2.19379, size = 499, normalized size = 1.23

$$-\frac{ie^{2ic}\left(-2idx\mathrm{PolyLog}\left(2,\frac{(b-a)e^{2i(2c+dx)}}{-2\sqrt{e^{4ic}(a+c)(b+c)}+ae^{2ic}+be^{2ic}+2ce^{2ic}}\right)+2idx\mathrm{PolyLog}\left(2,\frac{(b-a)e^{2i(2c+dx)}}{2\sqrt{e^{4ic}(a+c)(b+c)}+ae^{2ic}+be^{2ic}+2ce^{2ic}}\right)+\mathrm{PolyLog}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-I/4)*E^((2*I)*c)*(2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a*E
^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4
*I)*c)])] - 2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c
) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])]
 - (2*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b
*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] + (2
*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((
2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] + PolyLog
[3, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E
^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] - PolyLog[3, ((-a + b)*E
^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) + 2*
Sqrt[(a + c)*(b + c)*E^((4*I)*c)])]))/(d^3*Sqrt[(a + c)*(b + c)*E^((4*I)*c)
])
```

Maple [B] time = 0.173, size = 2061, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)
```

```
[Out] -1/3/((a+c)*(b+c))^(1/2)*x^3-2/3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x^3-1/d^2
/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*polylog(2,(a-b)*exp
(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+1/d^2/((a+c)*(b+c))^(1/2)
/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c^2*x+1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a
+c)*(b+c))^(1/2)-a-b-2*c)*b*c^2*x-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b
+c))^(1/2)-a-b-2*c)*a*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1
/2)-a-b-2*c))*x-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c
)*b*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+I/
d^3*c^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp
(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/4*I/d^3/((a+c)*(b+c))^(1/
2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*
((a+c)*(b+c))^(1/2)-a-b-2*c))-1/4*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c
))^(1/2)-a-b-2*c)*b*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2
)-a-b-2*c))-1/2*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*
c*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))+2/d^2*
c^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x+2/3/d^3/((a+c)*(
```

```
b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c^3+2/3/d^3/((a+c)*(b+c))^(1
/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c^3-I/d/(-2*((a+c)*(b+c))^(1/2)-a-b-
2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2+I/d^
3*c^2/(a*b+a*c+b*c+c^2)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b
+4*c)/(a*b+a*c+b*c+c^2)^(1/2))-I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1
/2)-a-b-2*c)*c*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)
)*x^2+1/2*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*ln(1
-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2+1/2*I/d^3/((a
+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*ln(1-(a-b)*exp(2*I*(d*x
+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*
((a+c)*(b+c))^(1/2)-a-b-2*c)*b*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c)
)^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a
-b-2*c)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2
+2/3/d^3*c^3/((a+c)*(b+c))^(1/2)+4/3/d^3*c^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*
c)-1/2*I/d/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))
^(1/2)-a-b-2*c))*x^2-1/3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*
c)*a*x^3-1/3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*x^3-2/3
/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*x^3-1/2/d^2/((a+c)*
(b+c))^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*
c))*x-1/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c
))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+1/d^2*c^2/((a+c)*(b+c))^(1/2)*x+2/d^
2*c^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x+4/3/d^3*c^4/((a+c)*(b+c))^(1/2)/(-
2*((a+c)*(b+c))^(1/2)-a-b-2*c)-1/2*I/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*p
olylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/4*I/d^3
/((a+c)*(b+c))^(1/2)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2
)-a-b-2*c))+1/2*I/d^3*c^2/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(
2*((a+c)*(b+c))^(1/2)-a-b-2*c))+I/d^3*c^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*
ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.74332, size = 14288, normalized size = 35.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/16*(8*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog
(1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c)
- 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2
)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) +
8*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*
((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*(
(a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c
^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^
2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 8*(a
- b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a
 + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a -
b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*
a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a - b)*d
*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b
+ 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*co
s(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 -
2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)
*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x +
 c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) -
 a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b
+ (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*
x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (
I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b
- 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)*cos(d*x + c) +
 (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a +
I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(
(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)
/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b + (a + b)*c +
```

c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +

```
b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) +
 a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a
 - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(
a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a -
b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a
*b + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d^2
*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log
(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c)
+ 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c
 + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(I*(
a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*s
in(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*
b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a +
 b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a -
 b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I
*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x +
 c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqr
t((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*
a - 2*b)/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2))*polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b
 + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c
))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))/(a -
b)) + 4*(-2*I*a + 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*
polylog(3, 1/2*(2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(
d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b +
 (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b
)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))/(a - b)) + 4*(-2*I*
a + 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, -1/
2*(2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4
*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c +
c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqr
t((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b + 2
*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d
*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a
*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)) + a + b + 2*c)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x +
 c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (I*a
 - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sq
```

```
rt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2
*c)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*
I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x +
 c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqr
t((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a -
 b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*
polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*si
n(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b
 + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a - b)) + 4*(-2*I
*a + 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, 1/
2*(2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4
*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a - b)))/((a*b + (a + b)*c + c^
2)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sec(dx+c)^2}{c\sec(dx+c)^2+b\tan(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

3.165 $\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=155

$$\frac{3x^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-\frac{6\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^4}-\frac{6x\tan(e+fx)\sqrt{a-a\sin(e+fx)}}{f^3}$$

```
[Out] (-6*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*x^2*Sqrt[a
- a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (6*x*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^3*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rubi [A] time = 0.198727, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4604, 3296, 2638}

$$\frac{3x^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-\frac{6\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^4}-\frac{6x\tan(e+fx)\sqrt{a-a\sin(e+fx)}}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```



```
[Out] (-6*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*x^2*Sqrt[a
- a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (6*x*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^3*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int x^3\cos(e+fx)\,dx$$
$$= \frac{x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f} - \frac{\left(3\sec(e+fx)\sqrt{a\right.}{}$$
$$= \frac{3x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,\mathrm{s}}}{f}$$
$$= \frac{3x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{6x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,\mathrm{s}}}{f^3}$$
$$= -\frac{6\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^4} + \frac{3x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c}}{f^2}$$

Mathematica [A] time = 0.511689, size = 61, normalized size = 0.39

$$\frac{\left(fx\left(f^2x^2-6\right)\tan(e+fx)+3f^2x^2-6\right)\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-6 + 3*f^2*x^2 + f*x*
(-6 + f^2*x^2)*Tan[e + f*x]))/f^4
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int x^3\sqrt{a-a\sin\left(fx+e\right)}\sqrt{c+c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: AttributeError
```

3.166 $\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$\frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-\frac{2\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{x^2\tan(e+fx)\sqrt{a-a\text{s}}}{}$$

```
[Out] (2*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rubi [A] time = 0.176069, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4604, 3296, 2637}

$$\frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-\frac{2\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{x^2\tan(e+fx)\sqrt{a-a\text{s}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```



```
[Out] (2*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx &= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int x^2\cos(e+fx)\,dx\\ &= \frac{x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f}-\frac{\left(2\sec(e+fx)\sqrt{a}\right.}{}\\ &= \frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}+\frac{x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,\mathrm{si}}}{f}\\ &= \frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}{f^3}\end{aligned}$$

Mathematica [A] time = 0.332446, size = 54, normalized size = 0.46

$$\frac{\left(\left(f^2x^2-2\right)\tan(e+fx)+2fx\right)\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(2*f*x + (-2 + f^2*x^2
)*Tan[e + f*x]))/f^3
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int x^2\sqrt{a-a\sin\left(fx+e\right)}\sqrt{c+c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^2, x)
```

3.167 $\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f}$$

```
[Out] (Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (x*Sqrt[a - a*Sin
[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rubi [A] time = 0.110354, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4604, 3296, 2638}

$$\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (x*Sqrt[a - a*Sin
[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int x\cos(e+fx)\,dx$$
$$= \frac{x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sec(e+fx)\sqrt{a-a\right.}{}$$
$$= \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}}{f}$$

Mathematica [A] time = 0.206221, size = 44, normalized size = 0.59

$$\frac{(fx\tan(e+fx)+1)\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(1 + f*x*Tan[e + f*x])
)/f^2
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int x\sqrt{a-a\sin\left(fx+e\right)}\sqrt{c+c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: AttributeError
```

3.168 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx$

Optimal. Leaf size=86

$$\cos(e)\mathbf{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\sin(e)\mathbf{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{}$$

```
[Out] Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Si
n[e + f*x]] - Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e
 + f*x]]*SinIntegral[f*x]
```

Rubi [A] time = 0.183241, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4604, 3303, 3299, 3302}

$$\cos(e)\mathbf{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\sin(e)\mathbf{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```


```
[Out] Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Si
n[e + f*x]] - Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e
 + f*x]]*SinIntegral[f*x]
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)}{x}\,dx$$
$$= \left(\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(fx)}{x}\,dx -$$
$$= \cos(e)\mathbf{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)} - \sec(e+f$$

Mathematica [A] time = 0.222457, size = 52, normalized size = 0.6

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}(\cos(e)\text{CosIntegral}(fx)-\sin(e)\text{Si}(fx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```

```
[Out] Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e]*Co
sIntegral[f*x] - Sin[e]*SinIntegral[f*x])
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a-a\sin\left(fx+e\right)}\sqrt{c+c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x, x)
```

3.169 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx$

Optimal. Leaf size=123

$$-f\sin(e)\mathbf{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-f\cos(e)\mathbf{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+$$

```
[Out] -((Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) - f*CosIntegral[f*
x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] -
f*Cos[e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin
Integral[f*x]
```

Rubi [A] time = 0.198508, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4604, 3297, 3303, 3299, 3302}

$$-f\sin(e)\mathbf{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-f\cos(e)\mathbf{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

```
[Out] -((Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) - f*CosIntegral[f*
x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] -
f*Cos[e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin
Integral[f*x]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)}{x^2}\,dx$$
$$= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x} - \left(f\sec(e+fx)\sqrt{a-a\sin(e+fx)}\right.$$
$$= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x} - \left(f\cos(e)\sec(e+fx)\sqrt{a-a\sin(e\right.$$
$$= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x} - f\text{Ci}(fx)\sec(e+fx)\sin(e)\sqrt{a-a}$$

Mathematica [A] time = 0.247739, size = 65, normalized size = 0.53

$$-\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}(fx\sin(e)\text{CosIntegral}(fx)+fx\cos(e)\text{Si}(fx)+\cos(e+fx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

```
[Out] -((Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e
+ f*x] + f*x*CosIntegral[f*x]*Sin[e] + f*x*Cos[e]*SinIntegral[f*x]))/x)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a-a\sin\left(fx+e\right)}\sqrt{c+c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^2, x)
```

3.170 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$

Optimal. Leaf size=176

$$-\frac{1}{2}f^2\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+\frac{1}{2}f^2\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\,\text{si}}$$

```
[Out] -(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*x^2) - (f^2*Cos[e]*
CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]])/2 + (f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*SinIntegral[f*x])/2 + (f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*Tan[e + f*x])/(2*x)
```

Rubi [A] time = 0.225648, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4604, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}f^2\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+\frac{1}{2}f^2\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\,\text{si}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

```
[Out] -(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*x^2) - (f^2*Cos[e]*
CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]])/2 + (f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*SinIntegral[f*x])/2 + (f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*Tan[e + f*x])/(2*x)
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx &= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)}{x^3}\,dx \\
&= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{1}{2}\left(f\sec(e+fx)\sqrt{a-a\sin(e+f\right. \\
&= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} + \frac{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}}{2x} \\
&= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} + \frac{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}}{2x} \\
&= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{1}{2}f^2\cos(e)\mathbf{Ci}(fx)\sec(e+fx)\sqrt{a}
\end{aligned}$$

Mathematica [A] time = 0.290416, size = 87, normalized size = 0.49

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}\left(-f^2x^2\cos(e)\mathbf{CosIntegral}(fx)+f^2x^2\sin(e)\mathbf{Si}(fx)+fx\sin(e+fx)-\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

```
[Out] (Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-Cos[e +
 f*x] - f^2*x^2*Cos[e]*CosIntegral[f*x] + f*x*Sin[e + f*x] + f^2*x^2*Sin[e]
*SinIntegral[f*x]))/(2*x^2)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a - a\sin\left(fx+e\right)}\sqrt{c + c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^3, x)
```

3.171 $\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=393

$$\frac{3cx^2\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{3cx^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-3c\sin(e+fx)\sqrt{a}$$

```
[Out] (-6*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*c*x^2*Sqr
t[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (3*c*x*Sec[e + f*x]*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^3) - (3*c*x^3*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) - (3*c*Sin[
e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^4) + (3*c*
x^2*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2)
 + (x^3*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(
2*c*f) - (6*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f
*x])/f^3 - (3*c*x*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]]*Tan[e + f*x])/(4*f^3)
```

Rubi [A] time = 0.375268, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4604, 4422, 3317, 3296, 2638, 3311, 30, 2635, 8}

$$\frac{3cx^2\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{3cx^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-3c\sin(e+fx)\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-6*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*c*x^2*Sqr
t[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (3*c*x*Sec[e + f*x]*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^3) - (3*c*x^3*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) - (3*c*Sin[
e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^4) + (3*c*
x^2*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2)
 + (x^3*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(
2*c*f) - (6*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f
*x])/f^3 - (3*c*x*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]]*Tan[e + f*x])/(4*f^3)
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int x^3\cos(e+fx)($$

$$=\frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf}-\frac{\left(3\sec(e+f\right.}{}$$

$$=\frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf}-\frac{\left(3\sec(e+f\right.}{}$$

$$=-\frac{cx^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2f}+\frac{x^3\sec(e+f}{}$$

$$=\frac{3cx^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{cx^3\sec(e+fx)\sqrt{a-a\,\mathrm{si}}}{}$$

$$=\frac{3cx^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{3cx^3\sec(e+fx)\sqrt{a-a}}{}$$

$$=-\frac{6c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^4}+\frac{3cx^2\sqrt{a-a\sin(e+fx)}\sqrt{}}{f^2}$$

Mathematica [A] time = 1.15899, size = 113, normalized size = 0.29

$$\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}\left(\left(6f^2x^2-3\right)\sin(e+fx)+8\left(fx\left(f^2x^2-6\right)\tan(e+fx)+3f^2x^2-6\right)-fx\left(2f^2\right.\right.}{8f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-(f*x*(-3 + 2*f^2*x
^2)*Cos[2*(e + f*x)]*Sec[e + f*x]) + (-3 + 6*f^2*x^2)*Sin[e + f*x] + 8*(-6
+ 3*f^2*x^2 + f*x*(-6 + f^2*x^2)*Tan[e + f*x])))/(8*f^4)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^{3} \left(c+c \sin \left(f x+e\right)\right)^{\frac{3}{2}} \sqrt{a-a \sin \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a \sin \left(f x+e\right)+a} \left(c \sin \left(f x+e\right)+c\right)^{\frac{3}{2}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: AttributeError
```

3.172 $\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=265

$$\frac{2cx\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{cx\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{2f^2}-\frac{c\sin(e+fx)\tan(e+f}{}$$

```
[Out] (2*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x^2*Se
c[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*x*
Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*f^2) + (
x^2*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*
f) - (2*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
^3 - (c*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[
e + f*x])/(4*f^3)
```

Rubi [A] time = 0.273629, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4604, 4422, 3317, 3296, 2637, 3310, 30}

$$\frac{2cx\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{cx\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{2f^2}-\frac{c\sin(e+fx)\tan(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```


```
[Out] (2*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x^2*Se
c[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*x*
Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*f^2) + (
x^2*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*
f) - (2*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
^3 - (c*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[
e + f*x])/(4*f^3)
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
```

```
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int x^2\cos(e+fx)(c$$
$$= \frac{x^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{\left(\sec(e+fx)\right.}{}$$
$$= \frac{x^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{\left(\sec(e+fx)\right.}{}$$
$$= -\frac{cx^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2f} + \frac{x^2\sec(e+fx}{}$$
$$= \frac{2cx\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{cx^2\sec(e+fx)\sqrt{a-a\sin(}}{2}$$
$$= \frac{2cx\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{3cx^2\sec(e+fx)\sqrt{a-a\sin}}{4}$$

Mathematica [A] time = 0.823209, size = 95, normalized size = 0.36

$$\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}\left(8\left(f^2x^2-2\right)\tan(e+fx)-\left(2f^2x^2-1\right)\cos(2(e+fx))\sec(e+fx)+4fx\sin(e+\right.}{8f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(16*f*x - (-1 + 2*f^
2*x^2)*Cos[2*(e + f*x)]*Sec[e + f*x] + 4*f*x*Sin[e + f*x] + 8*(-2 + f^2*x^2
)*Tan[e + f*x]))/(8*f^3)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int x^2\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] sage2
```

3.173 $\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=168

$$\frac{c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\sec(e+fx)\sqrt{a-a\sin(}{}$$

```
[Out] (c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x*Sec[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*Sin[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2) + (x*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*f)
```

Rubi [A] time = 0.143336, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4604, 4422, 2644}

$$\frac{c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\sec(e+fx)\sqrt{a-a\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```


```
[Out] (c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x*Sec[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*Sin[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2) + (x*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*f)
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
```

```
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int x\cos(e+fx)(c+$$

$$= \frac{x\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{\left(\sec(e+fx)\sqrt{a}\right.}{}$$

$$= \frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{3cx\sec(e+fx)\sqrt{a-a\sin(e+}}{4f}$$

Mathematica [A] time = 0.620265, size = 73, normalized size = 0.43

$$\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}(\sin(e+fx)+4fx\tan(e+fx)-fx\cos(2(e+fx))\sec(e+fx)+4)}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(4 - f*x*Cos[2*(e +
f*x)]*Sec[e + f*x] + Sin[e + f*x] + 4*f*x*Tan[e + f*x]))/(4*f^2)
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int x\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sin(fx+e)+a}\,(c\sin(fx+e)+c)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a \sin \left(fx+e\right)+a}\left(c \sin \left(fx+e\right)+c\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x, x)
```

3.174 $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx$

Optimal. Leaf size=186

$$\frac{1}{2}c\sin(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+c\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{}$$

```
[Out] c*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]] + (c*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[2*e]*Sqrt[a - a*Sin[
e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Si
n[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] + (c*Cos[2*e]*Sec[e +
 f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x])
/2
```

Rubi [A] time = 0.661755, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4604, 6741, 12, 6742, 3303, 3299, 3302}

$$\frac{1}{2}c\sin(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+c\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

```
[Out] c*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]] + (c*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[2*e]*Sqrt[a - a*Sin[
e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Si
n[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] + (c*Cos[2*e]*Sec[e +
 f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x])
/2
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)(c+c}{x}$$
$$= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{c\cos(e+fx)(1+}{x}$$
$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)(1+}{x}$$
$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \left(\frac{\cos(e+fx)}{x}+\right.$$
$$= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\sin(2e+2fx}{x}$$
$$= \left(c\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(fx)}{x}$$
$$= c\cos(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}+\frac{1}{2}c\text{C}$$

Mathematica [C] time = 1.24243, size = 150, normalized size = 0.81

$$\frac{ce^{-i(e-fx)}\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\left(2e^{ie}\text{ExpIntegralEi}(-ifx)+2e^{3ie}\text{ExpIntegralEi}(ifx)+i\left(\text{ExpIntegralEi}(-2ifx\right.\right.}{2\sqrt{2}\left(1+e^{2i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

```
[Out] (c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(2*E^(I*e)*ExpInt
egralEi[(-I)*f*x] + 2*E^((3*I)*e)*ExpIntegralEi[I*f*x] + I*(ExpIntegralEi[(
-2*I)*f*x] - E^((4*I)*e)*ExpIntegralEi[(2*I)*f*x]))*Sqrt[a - a*Sin[e + f*x]
])/(2*Sqrt[2]*E^(I*(e - f*x))*(1 + E^((2*I)*(e + f*x))))
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x, x)
```

3.175 $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=273

$-cf\sin(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+cf\cos(2e)\text{CosIntegral}(2fx)\sec(e+fx)$

```
[Out] -((c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) + c*f*Cos[2*e]*C
osIntegral[2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]] - c*f*CosIntegral[f*x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*S
qrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(2*x) - c*f*Cos[e]*Sec[e + f*x]*Sqrt[a -
 a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] - c*f*Sec[e + f*
x]*Sin[2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2
*f*x]
```

Rubi [A] time = 0.664795, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4604, 6741, 12, 6742, 3297, 3303, 3299, 3302}

$-cf\sin(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+cf\cos(2e)\text{CosIntegral}(2fx)\sec(e+fx)$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```


```
[Out] -((c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) + c*f*Cos[2*e]*C
osIntegral[2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]] - c*f*CosIntegral[f*x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*S
qrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(2*x) - c*f*Cos[e]*Sec[e + f*x]*Sqrt[a -
 a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] - c*f*Sec[e + f*
x]*Sin[2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2
*f*x]
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
```

```
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)(c+c\,s}{x^2}$$

$$= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{c\cos(e+fx)(1+s}{x^2}$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)(1+s}{x^2}$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int\left(\frac{\cos(e+fx)}{x^2}+\right.$$

$$= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\sin(2e+2fx)}{x^2}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x} - \frac{c\sec(e+fx)\sqrt{a-a\sin(e+f}}{}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x} - \frac{c\sec(e+fx)\sqrt{a-a\sin(e+f}}{}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x} + cf\cos(2e)\mathrm{Ci}(2fx)\sec(e+fx$$

Mathematica [C] time = 1.48438, size = 231, normalized size = 0.85

$$\frac{ce^{-i(e+fx)}\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\left(-2ifxe^{i(e+2fx)}\mathrm{ExpIntegralEi}(-ifx)+2ifxe^{3ie+2ifx}\mathrm{ExpIntegralEi}(ifx)+2fxe^{2i(2e}\right.}{2\sqrt{2}x\left(1+e\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

```
[Out] (c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(-I - 2*E^(I*(e +
 f*x)) - 2*E^((3*I)*(e + f*x)) + I*E^((4*I)*(e + f*x)) - (2*I)*E^(I*(e + 2*
f*x))*f*x*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f*x*ExpIn
tegralEi[I*f*x] + 2*E^((2*I)*f*x)*f*x*ExpIntegralEi[(-2*I)*f*x] + 2*E^((2*I
)*(2*e + f*x))*f*x*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*Sin[e + f*x]])/(2*S
qrt[2]*E^(I*(e + f*x))*(1 + E^((2*I)*(e + f*x)))*x)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^2, x)
```

3.176 $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx$

Optimal. Leaf size=385

$$-cf^2\sin(2e)\mathbf{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\frac{1}{2}cf^2\cos(e)\mathbf{CosIntegral}(fx)\sec(e-$$

```
[Out] -(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*x^2) - (c*f*Cos[2
*e + 2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
/(2*x) - (c*f^2*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*f^2*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[
2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(4*x^2) +
 (c*f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x
]]*SinIntegral[f*x])/2 - c*f^2*Cos[2*e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x] + (c*f*Sqrt[a - a*Sin[e + f*
x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/(2*x)
```

Rubi [A] time = 0.741289, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4604, 6741, 12, 6742, 3297, 3303, 3299, 3302}

$$-cf^2\sin(2e)\mathbf{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\frac{1}{2}cf^2\cos(e)\mathbf{CosIntegral}(fx)\sec(e-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```


```
[Out] -(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*x^2) - (c*f*Cos[2
*e + 2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
/(2*x) - (c*f^2*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*f^2*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[
2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(4*x^2) +
 (c*f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x
]]*SinIntegral[f*x])/2 - c*f^2*Cos[2*e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x] + (c*f*Sqrt[a - a*Sin[e + f*
x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/(2*x)
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)(c+c}{x^3}$$
$$= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{c\cos(e+fx)(1+}{x^3}$$
$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx)(1+}{x^3}$$
$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \left(\frac{\cos(e+fx)}{x^3}+\right.$$
$$= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\right)\int \frac{\sin(2e+2fx}{x^3}$$
$$= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{c\sec(e+fx)\sqrt{a-a\sin(e+}}{}$$
$$= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e+2fx)\sec(e+fx}{}$$
$$= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e+2fx)\sec(e+fx}{}$$
$$= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e+2fx)\sec(e+fx}{}$$

Mathematica [C] time = 1.81722, size = 317, normalized size = 0.82

$$\frac{c^2e^{-2i(e+fx)}\left(e^{i(e+fx)}+i\right)\left(2if^2x^2e^{i(e+2fx)}\text{ExpIntegralEi}(-ifx)+2if^2x^2e^{3ie+2ifx}\text{ExpIntegralEi}(ifx)+4f^2x^2e^{2i(2e+fx)}\text{Ex}\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```

```
[Out] (c^2*(I + E^(I*(e + f*x)))*(-1 + (2*I)*E^(I*(e + f*x)) + (2*I)*E^((3*I)*(e
+ f*x)) + E^((4*I)*(e + f*x)) + (2*I)*f*x + 2*E^(I*(e + f*x))*f*x - 2*E^((3
*I)*(e + f*x))*f*x + (2*I)*E^((4*I)*(e + f*x))*f*x + (2*I)*E^(I*(e + 2*f*x)
)*f^2*x^2*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f^2*x^2*E
xpIntegralEi[I*f*x] - 4*E^((2*I)*f*x)*f^2*x^2*ExpIntegralEi[(-2*I)*f*x] + 4
*E^((2*I)*(2*e + f*x))*f^2*x^2*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*Sin[e +
```

```
 f*x]])/(4*Sqrt[2]*E^((2*I)*(e + f*x))*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*
(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*x^2)
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} \left(c + c\sin\left(fx+e\right)\right)^{\frac{3}{2}} \sqrt{a - a\sin\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^3, x)
```

3.177 $\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=767

$$-\frac{6ah^2(g+hx)\cos(e+fx)\text{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{6ah^2(g+hx)\cos(e+fx)\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{3ah^2(g+hx)\cos(e}{2f^3\sqrt{a-a\sin(}}$$

```
[Out] ((-I/4)*a*(g + h*x)^4*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^3*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])
/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^3*Cos
[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c
+ c*Sin[e + f*x]]) + ((3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, (-I)*E^
(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (
(3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt
[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (((3*I)/2)*a*h*(g + h*x)^2
*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]) - (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3, (
-I)*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]
]) + (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*h^2*(g + h*x)*Cos[e
 + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3*Sqrt[a - a*Sin[e + f*x]]*S
qrt[c + c*Sin[e + f*x]]) - ((6*I)*a*h^3*Cos[e + f*x]*PolyLog[4, (-I)*E^(I*(
e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + ((6*I
)*a*h^3*Cos[e + f*x]*PolyLog[4, I*E^(I*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e +
 f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (((3*I)/4)*a*h^3*Cos[e + f*x]*PolyLog[4,
 -E^((2*I)*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]])
```

Rubi [A] time = 1.35139, antiderivative size = 767, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {4604, 6741, 12, 6742, 4181, 2531, 6609, 2282, 6589, 3719, 2190}

$$-\frac{6ah^2(g+hx)\cos(e+fx)\text{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{6ah^2(g+hx)\cos(e+fx)\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{3ah^2(g+hx)\cos(e}{2f^3\sqrt{a-a\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-I/4)*a*(g + h*x)^4*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^3*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])
```

```
/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^3*Cos
[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c
+ c*Sin[e + f*x]]) + ((3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, (-I)*E^
(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (
(3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt
[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (((3*I)/2)*a*h*(g + h*x)^2
*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]) - (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3, (
-I)*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]
]) + (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*h^2*(g + h*x)*Cos[e
 + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3*Sqrt[a - a*Sin[e + f*x]]*S
qrt[c + c*Sin[e + f*x]]) - ((6*I)*a*h^3*Cos[e + f*x]*PolyLog[4, (-I)*E^(I*(
e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + ((6*I
)*a*h^3*Cos[e + f*x]*PolyLog[4, I*E^(I*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e +
 f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (((3*I)/4)*a*h^3*Cos[e + f*x]*PolyLog[4,
 -E^((2*I)*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]])
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int (g+hx)^3\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a(g+hx)^3\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)^3\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left((g+hx)^3\sec(e+fx)-(g+hx)^3\tan(e+fx)\right)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)^3\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int (g+hx)^3\tan(e+fx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+f}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+f}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+f}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+f}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+f}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+f}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

Mathematica [A] time = 2.75239, size = 247, normalized size = 0.32

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{1}{2}i(e+fx)}\left(e^{i(e+fx)}+i\right)\sqrt{a-a\sin(e+fx)}\left(\frac{24h\left(f^2(g+hx)^2\text{PolyLog}\left(2,-ie^{-i(e+fx)}\right)-2h\left(if(g+hx)\text{PolyLog}\left(3,-ie^{-i(e+fx)}\right)+h\text{PolyLog}\left(4,-ie\right.\right.\right.}{f^4}\right.}{\sqrt{2}\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x
]
```

```
[Out] ((1/4 + I/4)*(I + E^(I*(e + f*x)))*((g + h*x)^4/h - ((8*I)*(g + h*x)^3*Log[
1 + I/E^(I*(e + f*x))])/f + (24*h*(f^2*(g + h*x)^2*PolyLog[2, (-I)/E^(I*(e
+ f*x))] - 2*h*(I*f*(g + h*x)*PolyLog[3, (-I)/E^(I*(e + f*x))] + h*PolyLog[
4, (-I)/E^(I*(e + f*x))])))/f^4)*Sqrt[a - a*Sin[e + f*x]])/(Sqrt[2]*E^((I/2
)*(e + f*x))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (hx+g)^3\sqrt{a-a\sin(fx+e)}\frac{1}{\sqrt{c+c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(hx+g)^3\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\left(g+hx\right)^{3}}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**3/sqrt(c*(sin(e + f*x) + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(hx+g\right)^{3}\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

3.178 $\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=555

$$\frac{2iah(g+hx)\cos(e+fx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2iah(g+hx)\cos(e+fx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah(g+hx)\cos(e+}{f^2\sqrt{a-a\sin(e-}}$$

```
[Out] ((-I/3)*a*(g + h*x)^3*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^2*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])
/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^2*Cos
[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c
+ c*Sin[e + f*x]]) + ((2*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, (-I)*E^(I
*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((2
*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a -
 a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*(g + h*x)*Cos[e + f*x]*
PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - (2*a*h^2*Cos[e + f*x]*PolyLog[3, (-I)*E^(I*(e + f*x))])/(f
^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*h^2*Cos[e + f*
x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) + (a*h^2*Cos[e + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f
^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi [A] time = 0.876905, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {4604, 6741, 12, 6742, 4181, 2531, 2282, 6589, 3719, 2190}

$$\frac{2iah(g+hx)\cos(e+fx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2iah(g+hx)\cos(e+fx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah(g+hx)\cos(e+}{f^2\sqrt{a-a\sin(e-}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-I/3)*a*(g + h*x)^3*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^2*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])
/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^2*Cos
[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c
+ c*Sin[e + f*x]]) + ((2*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, (-I)*E^(I
*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((2
*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a -
```

```
a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*(g + h*x)*Cos[e + f*x]*
PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - (2*a*h^2*Cos[e + f*x]*PolyLog[3, (-I)*E^(I*(e + f*x))])/(f
^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*h^2*Cos[e + f*
x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) + (a*h^2*Cos[e + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f
^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int(g+hx)^2\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a(g+hx)^2\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int(g+hx)^2\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int\left((g+hx)^2\sec(e+fx)-(g+hx)^2\tan(e+fx)\right)dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int(g+hx)^2\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int(g+hx)^2\tan(e+fx)\,d}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

Mathematica [A] time = 1.92293, size = 194, normalized size = 0.35

$$\frac{\sqrt{2}\left(e^{i(e+fx)}+i\right)\sqrt{a-a\sin(e+fx)}\left(12fh^2(g+hx)\text{PolyLog}\left(2,-ie^{-i(e+fx)}\right)-12ih^3\text{PolyLog}\left(3,-ie^{-i(e+fx)}\right)+f^2(g+hx)^2\right.}{3f^3h\left(e^{i(e+fx)}-i\right)\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x
]
```

```
[Out] (Sqrt[2]*(I + E^(I*(e + f*x)))*(f^2*(g + h*x)^2*(f*(g + h*x) - (6*I)*h*Log[
1 + I/E^(I*(e + f*x))]) + 12*f*h^2*(g + h*x)*PolyLog[2, (-I)/E^(I*(e + f*x)
```

```
)] - (12*I)*h^3*PolyLog[3, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*x]])
/(3*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e +
f*x))]*f^3*h)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (hx+g)^2 \sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{c+c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(hx+g)^2\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\left(g+hx\right)^{2}}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**2/sqrt(c*(sin(e + f*x) + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(hx+g\right)^{2}\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

3.179 $\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=355

$$\frac{iah\cos(e+fx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\cos(e+fx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\cos(e+fx)\text{PolyLog}\left(2,-e^{2i(\cdots}\right.}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+\cdots}}$$

```
[Out] ((-I/2)*a*(g + h*x)^2*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])/(
f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)*Cos[e +
 f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) + (I*a*h*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(f^2
*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*Cos[e + f*x]*P
olyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]) - ((I/2)*a*h*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2
*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi [A] time = 0.511259, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4604, 6741, 12, 6742, 4181, 2279, 2391, 3719, 2190}

$$\frac{iah\cos(e+fx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\cos(e+fx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\cos(e+fx)\text{PolyLog}\left(2,-e^{2i(\cdots}\right.}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```


```
[Out] ((-I/2)*a*(g + h*x)^2*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])/(
f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)*Cos[e +
 f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) + (I*a*h*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(f^2
*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*Cos[e + f*x]*P
olyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]) - ((I/2)*a*h*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2
*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int (g+hx)\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a(g+hx)\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int ((g+hx)\sec(e+fx)-(g+hx)\tan(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int (g+hx)\tan(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

Mathematica [A] time = 1.24138, size = 154, normalized size = 0.43

$$\frac{\left(e^{i(e+fx)}+i\right)\sqrt{a-a\sin(e+fx)}\left(4h\text{PolyLog}\left(2,-ie^{-i(e+fx)}\right)+f\left(fx(2g+hx)-4i(g+hx)\log\left(1+ie^{-i(e+fx)}\right)\right)\right)}{\sqrt{2}f^2\left(e^{i(e+fx)}-i\right)\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((I + E^(I*(e + f*x)))*(f*(f*x*(2*g + h*x) - (4*I)*(g + h*x)*Log[1 + I/E^(I
*(e + f*x))]) + 4*h*PolyLog[2, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*
x]])/(Sqrt[2]*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/
E^(I*(e + f*x))]*f^2)
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int (hx+g)\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{c+c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(hx+g\right)\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\left(g+hx\right)}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)/sqrt(c*(sin(e + f*x) + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(hx+g\right)\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

3.180 $\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=108

$$\frac{a\cos(e+fx)\text{Unintegrable}\left(\frac{\sec(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{a\cos(e+fx)\text{Unintegrable}\left(\frac{\tan(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

```
[Out] (a*Cos[e + f*x]*Unintegrable[Sec[e + f*x]/(g + h*x), x])/(Sqrt[a - a*Sin[e
+ f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Unintegrable[Tan[e + f*
x]/(g + h*x), x])/(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi [A] time = 0.646446, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
```

```
[Out] (a*Cos[e + f*x]*Defer[Int][Sec[e + f*x]/(g + h*x), x])/(Sqrt[a - a*Sin[e +
f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Defer[Int][Tan[e + f*x]/(
g + h*x), x])/(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \frac{\sec(e+fx)(a-a\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int \frac{a\sec(e+fx)(1-\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \frac{\sec(e+fx)(1-\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left(\frac{\sec(e+fx)}{g+hx}-\frac{\tan(e+fx)}{g+hx}\right)dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \frac{\sec(e+fx)}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int \frac{\tan(e+fx)}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

Mathematica [A] time = 3.5796, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
```

```
[Out] Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]), x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{hx+g}\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{c+c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{\left(hx+g\right)\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x
)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)/(h*x+g)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))/(sqrt(c*(sin(e + f*x) + 1))*(g + h*x))
, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{\left(hx+g\right)\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x
)
```

3.181 $\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=536

$$\frac{6ia\cos(e+fx)\mathrm{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{6ia\cos(e+fx)\mathrm{PolyLog}\left(2,ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{3ia\cos(e+fx)\mathrm{PolyLog}\left(2,-e^{2i\cdots}\right.}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+\cdots}}$$

```
[Out] (-3*a*x^2)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((3*
I)*a*x^2*Cos[e + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]]) - ((12*I)*a*x*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])/(c*f^3*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (6*a*x*Cos[e + f*x]*Log[1 + E^((
2*I)*(e + f*x))])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
 + ((6*I)*a*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(c*f^4*Sqrt[a -
a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((6*I)*a*Cos[e + f*x]*PolyLog[2
, I*E^(I*(e + f*x))])/(c*f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]]) - ((3*I)*a*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(c*f^4*Sqrt[
a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^3*Sec[e + f*x])/(c*f*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*x^2*Sin[e + f*x])/
(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^3*Tan[e +
f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi [A] time = 3.54974, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {4604, 6741, 12, 6742, 4186, 4181, 2279, 2391, 2531, 6609, 2282, 6589, 3757, 4184, 3719, 2190, 4413}

$$\frac{6ia\cos(e+fx)\mathrm{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{6ia\cos(e+fx)\mathrm{PolyLog}\left(2,ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{3ia\cos(e+fx)\mathrm{PolyLog}\left(2,-e^{2i\cdots}\right.}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*a*x^2)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((3*
I)*a*x^2*Cos[e + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]]) - ((12*I)*a*x*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])/(c*f^3*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (6*a*x*Cos[e + f*x]*Log[1 + E^((
2*I)*(e + f*x))])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
 + ((6*I)*a*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(c*f^4*Sqrt[a -
a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((6*I)*a*Cos[e + f*x]*PolyLog[2
```

```
, I*E^(I*(e + f*x))])/(c*f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]]) - ((3*I)*a*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(c*f^4*Sqrt[
a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^3*Sec[e + f*x])/(c*f*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*x^2*Sin[e + f*x])/
(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^3*Tan[e +
f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x^3\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a^2x^3\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^3\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left(x^3\sec^3(e+fx)-2x^3\sec^2(e+fx)\tan(e+fx)+x^3\sec(e+fx)\tan^2(e+\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^3\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x^3\sec(e+fx)\tan^2(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{3ax^2}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax^3\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + $$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{6iax\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + $$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - $$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - $$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - $$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - $$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - $$

Mathematica [A] time = 2.05024, size = 193, normalized size = 0.36

$$-\frac{\sqrt{a-a\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(12i(\sin(e+fx)+1)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)+fx\left(-12\log\left(1-ie^{i\cdots}\right.\right.\right.}{f^4(c(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((12*I)*P
olyLog[2, I*E^(I*(e + f*x))]*(1 + Sin[e + f*x]) + f*x*((3*I)*f*x + f^2*x^2
+ 3*f*x*Cos[e + f*x] - 12*Log[1 - I*E^(I*(e + f*x))] + (3*I)*(f*x + (4*I)*L
og[1 - I*E^(I*(e + f*x))])*Sin[e + f*x])))/(f^4*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int x^{3} \sqrt{a - a\sin\left(fx+e\right)}\left(c + c\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right) + a}x^{3}}{\left(c\sin\left(fx+e\right) + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a\sin\left(fx+e\right) + a}\sqrt{c\sin\left(fx+e\right) + c}x^{3}}{c^{2}\cos\left(fx+e\right)^{2} - 2\,c^{2}\sin\left(fx+e\right) - 2\,c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3/(c^2*cos(f
*x + e)^2 - 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}x^3}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

3.182 $\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{2ax\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2a\cos(e+fx)\log(\cos(e+}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+}$$

```
[Out] (-2*a*x)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Log[Cos[e + f*x]])/(c*f^3*Sqrt[a - a*
Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^2*Sec[e + f*x])/(c*f*Sqrt[a
- a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*x*Sin[e + f*x])/(c*f^2*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^2*Tan[e + f*x])/(c
*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi [A] time = 2.1552, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {4604, 6741, 12, 6742, 4186, 3770, 4181, 2531, 2282, 6589, 3757, 4184, 3475, 4413}

$$\frac{2ax\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2a\cos(e+fx)\log(\cos(e+}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-2*a*x)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Log[Cos[e + f*x]])/(c*f^3*Sqrt[a - a*
Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^2*Sec[e + f*x])/(c*f*Sqrt[a
- a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*x*Sin[e + f*x])/(c*f^2*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^2*Tan[e + f*x])/(c
*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```


Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
```

```
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x^2\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a^2x^2\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^2\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left(x^2\sec^3(e+fx)-2x^2\sec^2(e+fx)\tan(e+fx)+x^2\sec(e+fx)\tan^2(e+\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^2\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x^2\sec(e+fx)\tan^2(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= -\frac{ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax^2\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{2c}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{iax^2\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{cf}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{c}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{c}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{c}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{c}$$

Mathematica [C] time = 1.59998, size = 154, normalized size = 0.55

$$-\frac{\sqrt{a-a\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-4\log\left(e^{i(e+fx)}+i\right)+2fx\cos(e+fx)+\left(2ifx-4\log\left(e^{i(e+fx)}\right.\right.\right.}{f^3(c(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((2*I)*f*
x + f^2*x^2 + 2*f*x*Cos[e + f*x] - 4*Log[I + E^(I*(e + f*x))] + ((2*I)*f*x
- 4*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(f^3*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int x^{2}\sqrt{a - a\sin\left(fx + e\right)}\left(c + c\sin\left(fx + e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx + e\right) + a}x^{2}}{\left(c\sin\left(fx + e\right) + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}x^2}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)
```

3.183 $\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{a\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{ax\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+\cdots}}$$

```
[Out] -(a/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])) - (a*x*Sec[e
 + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*Sin[e
 + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x*T
an[e + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi [A] time = 0.972936, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {4604, 6741, 12, 6742, 4185, 4181, 2279, 2391, 3757, 3767, 8, 4413}

$$\frac{a\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{ax\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])) - (a*x*Sec[e
 + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*Sin[e
 + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x*T
an[e + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
```

```
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a^2x\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left(x\sec^3(e+fx)-2x\sec^2(e+fx)\tan(e+fx)+x\sec(e+fx)\tan^2(e+fx\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x\sec(e+fx)\tan^2(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} -$$

$$= -\frac{a}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{2}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{iax\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{}{cf}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{cf}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ia\cos(e+fx)\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{2}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{}{cf}$$

Mathematica [A] time = 0.621979, size = 150, normalized size = 0.88

$$-\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}\left(fx\sin\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}+fx\right)+\cos\left(\frac{e}{2}\right)(fx-1)+\cos\left(\frac{e}{2}+fx\right)+\sin\left(\frac{e}{2}\right)\right)}{c^2f^2\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((((-1 + f*x)*Cos[e/2] + Cos[e/2 + f*x] + Sin[e/2] + f*x*Sin[e/2] - Sin[e/
2 + f*x])*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]])/(c^2*f^2*(Co
s[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
```

```
+ Sin[(e + f*x)/2])^3))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x\sqrt{a - a\sin\left(fx + e\right)}\left(c + c\sin\left(fx + e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx + e\right) + a}x}{\left(c\sin\left(fx + e\right) + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 2.3572, size = 180, normalized size = 1.05

$$-\frac{\left(fx + \cos\left(fx + e\right)\right)\sqrt{-a\sin\left(fx + e\right) + a}\sqrt{c\sin\left(fx + e\right) + c}}{c^{2}f^{2}\cos\left(fx + e\right)\sin\left(fx + e\right) + c^{2}f^{2}\cos\left(fx + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] -(f*x + cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/(c
^2*f^2*cos(f*x + e)*sin(f*x + e) + c^2*f^2*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}x}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)
```

3.184 $\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz$

Optimal. Leaf size=300

$$\frac{2iz\sin(z)\text{PolyLog}\left(2,-e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2iz\sin(z)\text{PolyLog}\left(2,e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{iz\sin(z)\text{PolyLog}\left(2,e^{2iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2\sin(z)\text{PolyLog}\left(3,-e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}+\cdots$$

```
[Out] ((-I/3)*z^3*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*z^2*ArcTanh[E^
(I*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (z^2*Log[1 - E^((2*I)*
z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + ((2*I)*z*PolyLog[2, -E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - ((2*I)*z*PolyLog[2, E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (I*z*PolyLog[2, E^((2*I)
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*PolyLog[3, -E^(I*z)]*
Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (2*PolyLog[3, E^(I*z)]*Sin[z]
)/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (PolyLog[3, E^((2*I)*z)]*Sin[z])/(2
*Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]])
```

Rubi [A] time = 0.438342, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4605, 6742, 3717, 2190, 2531, 2282, 6589, 4183}

$$\frac{2iz\sin(z)\text{PolyLog}\left(2,-e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2iz\sin(z)\text{PolyLog}\left(2,e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{iz\sin(z)\text{PolyLog}\left(2,e^{2iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2\sin(z)\text{PolyLog}\left(3,-e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

```
[Out] ((-I/3)*z^3*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*z^2*ArcTanh[E^
(I*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (z^2*Log[1 - E^((2*I)*
z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + ((2*I)*z*PolyLog[2, -E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - ((2*I)*z*PolyLog[2, E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (I*z*PolyLog[2, E^((2*I)
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*PolyLog[3, -E^(I*z)]*
Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (2*PolyLog[3, E^(I*z)]*Sin[z]
)/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (PolyLog[3, E^((2*I)*z)]*Sin[z])/(2
*Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]])
```

Rule 4605

```
Int[(Cos[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(Cos[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_)*((g_.) + (h_.)*(x_))^(p_.), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Cos[e + f*x])^FracPart[m]*(c + d*Cos[e + f*x])^FracPa
rt[m])/Sin[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Sin[e + f*x]^(2*m)*(c
+ d*Cos[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz &= \frac{\sin(z)\int z^2(1+\cos(z))\csc(z)\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} \\
&= \frac{\sin(z)\int \left(z^2\cot(z)+z^2\csc(z)\right)dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} \\
&= \frac{\sin(z)\int z^2\cot(z)\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{\sin(z)\int z^2\csc(z)\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} \\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{(2i\sin(z))\int \frac{e^{2iz}z^2}{1-e^{2iz}}\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{(2\sin(z))\int}{\sqrt{1-\cos}} \\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{2iz\mathrm{Li}_2\left(-\right.}{\sqrt{1-\cos(z}} \\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{2iz\mathrm{Li}_2\left(-\right.}{\sqrt{1-\cos(z}} \\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{2iz\mathrm{Li}_2\left(-\right.}{\sqrt{1-\cos(z}} \\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{2iz\mathrm{Li}_2\left(-\right.}{\sqrt{1-\cos(z}}
\end{aligned}
$$

Mathematica [A] time = 0.0838063, size = 85, normalized size = 0.28

$$
\frac{\sqrt{\cos(z)+1}\tan\left(\frac{z}{2}\right)\left(12iz\mathrm{PolyLog}\left(2,e^{-iz}\right)+12\mathrm{PolyLog}\left(3,e^{-iz}\right)+iz^3+6z^2\log\left(1-e^{-iz}\right)-i\pi^3\right)}{3\sqrt{1-\cos(z)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

```
[Out] (Sqrt[1 + Cos[z]]*((-I)*Pi^3 + I*z^3 + 6*z^2*Log[1 - E^((-I)*z)] + (12*I)*z
*PolyLog[2, E^((-I)*z)] + 12*PolyLog[3, E^((-I)*z)])*Tan[z/2])/(3*Sqrt[1 -
Cos[z]])
```

Maple [A] time = 0.067, size = 154, normalized size = 0.5

$$\frac{\left(e^{iz}-1\right)z^{3}}{3\,e^{iz}+3}\sqrt{\left(e^{iz}+1\right)^{2}e^{-iz}}\frac{1}{\sqrt{-\left(e^{iz}-1\right)^{2}e^{-iz}}}+\frac{2\,i\left(e^{iz}-1\right)\left(\frac{i}{3}z^{3}-z^{2}\ln\left(1-e^{iz}\right)+2\,iz\mathrm{polylog}\left(2,e^{iz}\right)-2\,\mathrm{polylog}\left(3,e^{iz}\right.\right.}{e^{iz}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z)
```

```
[Out] 1/3*((exp(I*z)+1)^2*exp(-I*z))^(1/2)/(exp(I*z)+1)/(-(exp(I*z)-1)^2*exp(-I*z
))^(1/2)*(exp(I*z)-1)*z^3+2*I*((exp(I*z)+1)^2*exp(-I*z))^(1/2)/(exp(I*z)+1)
/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1)*(1/3*I*z^3-z^2*ln(1-exp(I*z
))+2*I*z*polylog(2,exp(I*z))-2*polylog(3,exp(I*z)))
```

Maxima [A] time = 1.56118, size = 76, normalized size = 0.25

$$\frac{1}{3}i\,z^{3}+2i\,z^{2}\arctan\left(\sin\left(z\right),-\cos\left(z\right)+1\right)-z^{2}\log\left(\cos\left(z\right)^{2}+\sin\left(z\right)^{2}-2\,\cos\left(z\right)+1\right)+4i\,z\mathrm{Li}_{2}\left(e^{(i\,z)}\right)-4\,\mathrm{Li}_{3}(e^{(i\,z)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="maxima")
```

```
[Out] 1/3*I*z^3 + 2*I*z^2*arctan2(sin(z), -cos(z) + 1) - z^2*log(cos(z)^2 + sin(z
)^2 - 2*cos(z) + 1) + 4*I*z*dilog(e^(I*z)) - 4*polylog(3, e^(I*z))
```

Fricas [C] time = 2.43017, size = 271, normalized size = 0.9

$$z^{2}\log\left(-\cos\left(z\right)+i\,\sin\left(z\right)+1\right)+z^{2}\log\left(-\cos\left(z\right)-i\,\sin\left(z\right)+1\right)-2i\,z\mathrm{Li}_{2}\left(\cos\left(z\right)+i\,\sin\left(z\right)\right)+2i\,z\mathrm{Li}_{2}\left(\cos\left(z\right)-i\,\sin\left(z\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="fricas")
```

```
[Out] z^2*log(-cos(z) + I*sin(z) + 1) + z^2*log(-cos(z) - I*sin(z) + 1) - 2*I*z*d
ilog(cos(z) + I*sin(z)) + 2*I*z*dilog(cos(z) - I*sin(z)) + 2*polylog(3, cos
(z) + I*sin(z)) + 2*polylog(3, cos(z) - I*sin(z))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{z^2\sqrt{\cos(z)+1}}{\sqrt{1-\cos(z)}}\,dz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z**2*(1+cos(z))**(1/2)/(1-cos(z))**(1/2),z)
```

```
[Out] Integral(z**2*sqrt(cos(z) + 1)/sqrt(1 - cos(z)), z)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{z^2\sqrt{\cos(z)+1}}{\sqrt{-\cos(z)+1}}\,dz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="giac")
```

```
[Out] integrate(z^2*sqrt(cos(z) + 1)/sqrt(-cos(z) + 1), z)
```

3.185 $\int (a + a\cos(x))(A + B\sec(x))\,dx$

Optimal. Leaf size=18

$$ax(A + B) + aA\sin(x) + aB\tanh^{-1}(\sin(x))$$

```
[Out] a*(A + B)*x + a*B*ArcTanh[Sin[x]] + a*A*Sin[x]
```

Rubi [A] time = 0.0983354, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2828, 2968, 3023, 2735, 3770}

$$ax(A + B) + aA\sin(x) + aB\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])*(A + B*Sec[x]),x]
```

```
[Out] a*(A + B)*x + a*B*ArcTanh[Sin[x]] + a*A*Sin[x]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(x))(A+B\sec(x))\,dx &= \int (a+a\cos(x))(B+A\cos(x))\sec(x)\,dx\\ &= \int \left(aB+(aA+aB)\cos(x)+aA\cos^2(x)\right)\sec(x)\,dx\\ &= aA\sin(x)+\int (aB+a(A+B)\cos(x))\sec(x)\,dx\\ &= a(A+B)x+aA\sin(x)+(aB)\int \sec(x)\,dx\\ &= a(A+B)x+aB\tanh^{-1}(\sin(x))+aA\sin(x)\end{aligned}$$

Mathematica [B] time = 0.0152075, size = 51, normalized size = 2.83

$$aAx+aA\sin(x)+aBx-aB\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+aB\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])*(A + B*Sec[x]),x]
```

```
[Out] a*A*x + a*B*x - a*B*Log[Cos[x/2] - Sin[x/2]] + a*B*Log[Cos[x/2] + Sin[x/2]]
 + a*A*Sin[x]
```

Maple [A] time = 0.038, size = 24, normalized size = 1.3

$$aA\sin(x)+Bax+aAx+Ba\ln(\sec(x)+\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))*(A+B*sec(x)),x)
```

```
[Out] a*A*sin(x)+B*a*x+a*A*x+B*a*ln(sec(x)+tan(x))
```

Maxima [A] time = 0.97508, size = 31, normalized size = 1.72

$$Aax + Bax + Ba\log\left(\sec\left(x\right) + \tan\left(x\right)\right) + Aa\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] A*a*x + B*a*x + B*a*log(sec(x) + tan(x)) + A*a*sin(x)
```

Fricas [A] time = 2.53537, size = 107, normalized size = 5.94

$$(A + B)ax + \frac{1}{2}Ba\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}Ba\log\left(-\sin\left(x\right) + 1\right) + Aa\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] (A + B)*a*x + 1/2*B*a*log(sin(x) + 1) - 1/2*B*a*log(-sin(x) + 1) + A*a*sin(
x)
```

Sympy [A] time = 3.57851, size = 27, normalized size = 1.5

$$Aax + Aa\sin\left(x\right) + Bax + Ba\log\left(\tan\left(x\right) + \sec\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x)
```

```
[Out] A*a*x + A*a*sin(x) + B*a*x + B*a*log(tan(x) + sec(x))
```

Giac [B] time = 1.17056, size = 69, normalized size = 3.83

$$Ba\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+(Aa+Ba)x+\frac{2\,Aa\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a*log(abs(tan(1/2*x) + 1)) - B*a*log(abs(tan(1/2*x) - 1)) + (A*a + B*a)*x
 + 2*A*a*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

3.186 $\int (a+a\cos(x))^2(A+B\sec(x))\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}a^2x(3A+4B)+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)+a^2B\tanh^{-1}(\sin(x))$$

```
[Out] (a^2*(3*A + 4*B)*x)/2 + a^2*B*ArcTanh[Sin[x]] + (a^2*(3*A + 2*B)*Sin[x])/2
+ (A*(a^2 + a^2*Cos[x])*Sin[x])/2
```

Rubi [A] time = 0.201573, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{2}a^2x(3A+4B)+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)+a^2B\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

```
[Out] (a^2*(3*A + 4*B)*x)/2 + a^2*B*ArcTanh[Sin[x]] + (a^2*(3*A + 2*B)*Sin[x])/2
+ (A*(a^2 + a^2*Cos[x])*Sin[x])/2
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(x))^2(A+B\sec(x))\,dx &= \int (a+a\cos(x))^2(B+A\cos(x))\sec(x)\,dx \\
&= \frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\frac{1}{2}\int (a+a\cos(x))(2aB+a(3A+2B)\cos(x))\sec(x) \\
&= \frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\frac{1}{2}\int \left(2a^2B+\left(2a^2B+a^2(3A+2B)\right)\cos(x)+a^2(3A \\
&= \frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\frac{1}{2}\int \left(2a^2B+a^2(3A+4B)\mathrm{co}\right. \\
&= \frac{1}{2}a^2(3A+4B)x+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\left(a^2B\right)\int \mathrm{s} \\
&= \frac{1}{2}a^2(3A+4B)x+a^2B\tanh^{-1}(\sin(x))+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+a^2\cos(x\right.
\end{aligned}
$$

Mathematica [A] time = 0.0823791, size = 67, normalized size = 1.18

$$\frac{1}{4}a^2\left(4(2A+B)\sin(x)+6Ax+A\sin(2x)+8Bx-4B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+4B\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

```
[Out] (a^2*(6*A*x + 8*B*x - 4*B*Log[Cos[x/2] - Sin[x/2]] + 4*B*Log[Cos[x/2] + Sin
[x/2]] + 4*(2*A + B)*Sin[x] + A*Sin[2*x]))/4
```

Maple [A] time = 0.046, size = 52, normalized size = 0.9

$$\frac{a^2A\sin(x)\cos(x)}{2}+\frac{3\,a^2Ax}{2}+a^2B\sin(x)+2\,a^2A\sin(x)+2\,a^2Bx+a^2B\ln(\sec(x)+\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^2*(A+B*sec(x)),x)
```

```
[Out] 1/2*a^2*A*sin(x)*cos(x)+3/2*a^2*A*x+a^2*B*sin(x)+2*a^2*A*sin(x)+2*a^2*B*x+a
^2*B*ln(sec(x)+tan(x))
```

Maxima [A] time = 1.01363, size = 73, normalized size = 1.28

$$\frac{1}{4}\,Aa^2(2\,x+\sin(2\,x))+Aa^2x+2\,Ba^2x+Ba^2\log(\sec(x)+\tan(x))+2\,Aa^2\sin(x)+Ba^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/4*A*a^2*(2*x + sin(2*x)) + A*a^2*x + 2*B*a^2*x + B*a^2*log(sec(x) + tan(x
)) + 2*A*a^2*sin(x) + B*a^2*sin(x)
```

Fricas [A] time = 2.57262, size = 170, normalized size = 2.98

$$\frac{1}{2}\,(3\,A+4\,B)a^2x+\frac{1}{2}\,Ba^2\log(\sin(x)+1)-\frac{1}{2}\,Ba^2\log(-\sin(x)+1)+\frac{1}{2}\left(Aa^2\cos(x)+2\,(2\,A+B)a^2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*A + 4*B)*a^2*x + 1/2*B*a^2*log(sin(x) + 1) - 1/2*B*a^2*log(-sin(x) +
 1) + 1/2*(A*a^2*cos(x) + 2*(2*A + B)*a^2)*sin(x)
```

Sympy [A] time = 8.7605, size = 61, normalized size = 1.07

$$\frac{3Aa^2x}{2} + 2Aa^2\sin(x) + \frac{Aa^2\sin(2x)}{4} + 2Ba^2x + Ba^2\log(\tan(x) + \sec(x)) + Ba^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**2*(A+B*sec(x)),x)
```

```
[Out] 3*A*a**2*x/2 + 2*A*a**2*sin(x) + A*a**2*sin(2*x)/4 + 2*B*a**2*x + B*a**2*lo
g(tan(x) + sec(x)) + B*a**2*sin(x)
```

Giac [A] time = 1.18303, size = 135, normalized size = 2.37

$$Ba^2\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right) - Ba^2\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right) + \frac{1}{2}\left(3\,Aa^2 + 4\,Ba^2\right)x + \frac{3\,Aa^2\tan\left(\frac{1}{2}x\right)^3 + 2\,Ba^2\tan\left(\frac{1}{2}x\right)^3 + 5\,A}{\left(\tan\left(\frac{1}{2}x\right)^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^2*log(abs(tan(1/2*x) + 1)) - B*a^2*log(abs(tan(1/2*x) - 1)) + 1/2*(3*A*
a^2 + 4*B*a^2)*x + (3*A*a^2*tan(1/2*x)^3 + 2*B*a^2*tan(1/2*x)^3 + 5*A*a^2*t
an(1/2*x) + 2*B*a^2*tan(1/2*x))/(tan(1/2*x)^2 + 1)^2
```

3.187 $\int (a+a\cos(x))^3(A+B\sec(x))\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}a^3x(5A+7B)+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{6}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)+a^3B\tanh^{-1}(\sin(x))+\frac{1}{3}aA\sin(x)(a\cos(x)$$

```
[Out] (a^3*(5*A + 7*B)*x)/2 + a^3*B*ArcTanh[Sin[x]] + (5*a^3*(A + B)*Sin[x])/2 +
(a*A*(a + a*Cos[x])^2*Sin[x])/3 + ((5*A + 3*B)*(a^3 + a^3*Cos[x])*Sin[x])/6
```

Rubi [A] time = 0.297154, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{2}a^3x(5A+7B)+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{6}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)+a^3B\tanh^{-1}(\sin(x))+\frac{1}{3}aA\sin(x)(a\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

```
[Out] (a^3*(5*A + 7*B)*x)/2 + a^3*B*ArcTanh[Sin[x]] + (5*a^3*(A + B)*Sin[x])/2 +
(a*A*(a + a*Cos[x])^2*Sin[x])/3 + ((5*A + 3*B)*(a^3 + a^3*Cos[x])*Sin[x])/6
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(x))^3(A+B\sec(x))\,dx &= \int (a+a\cos(x))^3(B+A\cos(x))\sec(x)\,dx \\
&= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{3}\int (a+a\cos(x))^2(3aB+a(5A+3B)\cos(x))\sec(x) \\
&= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)+\frac{1}{6}\int (a+a\cos \\
&= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)+\frac{1}{6}\int \Big(6a^3B+\Big(\\
&= \frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin \\
&= \frac{1}{2}a^3(5A+7B)x+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\Big(\\
&= \frac{1}{2}a^3(5A+7B)x+a^3B\tanh^{-1}(\sin(x))+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2
\end{aligned}$$

Mathematica [A] time = 0.103826, size = 80, normalized size = 1.07

$$\frac{1}{12}a^3\left(9(5A+4B)\sin(x)+3(3A+B)\sin(2x)+30Ax+A\sin(3x)+42Bx-12B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+12B\log\left(\sin\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

```
[Out] (a^3*(30*A*x + 42*B*x - 12*B*Log[Cos[x/2] - Sin[x/2]] + 12*B*Log[Cos[x/2] +
 Sin[x/2]] + 9*(5*A + 4*B)*Sin[x] + 3*(3*A + B)*Sin[2*x] + A*Sin[3*x]))/12
```

Maple [A] time = 0.054, size = 77, normalized size = 1.

$$\frac{Aa^3\left(2+(\cos(x))^2\right)\sin(x)}{3}+\frac{Ba^3\sin(x)\cos(x)}{2}+\frac{7\,Ba^3x}{2}+\frac{3\,Aa^3\sin(x)\cos(x)}{2}+\frac{5\,Aa^3x}{2}+3\,Ba^3\sin(x)+3\,Aa^3\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^3*(A+B*sec(x)),x)
```

```
[Out] 1/3*A*a^3*(2+cos(x)^2)*sin(x)+1/2*B*a^3*sin(x)*cos(x)+7/2*B*a^3*x+3/2*A*a^3
*sin(x)*cos(x)+5/2*A*a^3*x+3*B*a^3*sin(x)+3*A*a^3*sin(x)+B*a^3*ln(sec(x)+ta
n(x))
```

Maxima [A] time = 0.991514, size = 113, normalized size = 1.51

$$-\frac{1}{3}\left(\sin(x)^3-3\,\sin(x)\right)Aa^3+\frac{3}{4}\,Aa^3(2\,x+\sin(2\,x))+\frac{1}{4}\,Ba^3(2\,x+\sin(2\,x))+Aa^3x+3\,Ba^3x+Ba^3\log(\sec(x)+\tan($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))*A*a^3 + 3/4*A*a^3*(2*x + sin(2*x)) + 1/4*B*a^3*(
2*x + sin(2*x)) + A*a^3*x + 3*B*a^3*x + B*a^3*log(sec(x) + tan(x)) + 3*A*a^
3*sin(x) + 3*B*a^3*sin(x)
```

Fricas [A] time = 2.5886, size = 213, normalized size = 2.84

$$\frac{1}{2}\,(5\,A+7\,B)a^3x+\frac{1}{2}\,Ba^3\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\,Ba^3\log\left(-\sin\left(x\right)+1\right)+\frac{1}{6}\,\left(2\,Aa^3\cos\left(x\right)^2+3\,(3\,A+B)a^3\cos\left(x\right)+2\,(11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(5*A + 7*B)*a^3*x + 1/2*B*a^3*log(sin(x) + 1) - 1/2*B*a^3*log(-sin(x) +
 1) + 1/6*(2*A*a^3*cos(x)^2 + 3*(3*A + B)*a^3*cos(x) + 2*(11*A + 9*B)*a^3)*
sin(x)
```

Sympy [A] time = 37.1643, size = 92, normalized size = 1.23

$$\frac{5Aa^3x}{2}-\frac{Aa^3\sin^3\left(x\right)}{3}+4Aa^3\sin\left(x\right)+\frac{3Aa^3\sin\left(2x\right)}{4}+\frac{7Ba^3x}{2}+Ba^3\log\left(\tan\left(x\right)+\sec\left(x\right)\right)+\frac{Ba^3\sin\left(x\right)\cos\left(x\right)}{2}+3B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**3*(A+B*sec(x)),x)
```

```
[Out] 5*A*a**3*x/2 - A*a**3*sin(x)**3/3 + 4*A*a**3*sin(x) + 3*A*a**3*sin(2*x)/4 +
 7*B*a**3*x/2 + B*a**3*log(tan(x) + sec(x)) + B*a**3*sin(x)*cos(x)/2 + 3*B*
a**3*sin(x)
```

Giac [A] time = 1.15883, size = 169, normalized size = 2.25

$$Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)-Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)+\frac{1}{2}\,\left(5\,Aa^3+7\,Ba^3\right)x+\frac{15\,Aa^3\tan\left(\frac{1}{2}\,x\right)^5+15\,Ba^3\tan\left(\frac{1}{2}\,x\right)^5+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^3*log(abs(tan(1/2*x) + 1)) - B*a^3*log(abs(tan(1/2*x) - 1)) + 1/2*(5*A*
a^3 + 7*B*a^3)*x + 1/3*(15*A*a^3*tan(1/2*x)^5 + 15*B*a^3*tan(1/2*x)^5 + 40*
A*a^3*tan(1/2*x)^3 + 36*B*a^3*tan(1/2*x)^3 + 33*A*a^3*tan(1/2*x) + 21*B*a^3
*tan(1/2*x))/(tan(1/2*x)^2 + 1)^3
```

3.188 $\int (a + a\cos(x))^4 (A + B\sec(x))\,dx$

Optimal. Leaf size=104

$$\frac{1}{8}a^4x(35A+48B)+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{12}(7A+4B)\sin(x)\left(a^2\cos(x)+a^2\right)^2+\frac{1}{24}(35A+32B)\sin(x)\left(a^4\cos(x)+a\right.$$

```
[Out] (a^4*(35*A + 48*B)*x)/8 + a^4*B*ArcTanh[Sin[x]] + (5*a^4*(7*A + 8*B)*Sin[x]
)/8 + (a*A*(a + a*Cos[x])^3*Sin[x])/4 + ((7*A + 4*B)*(a^2 + a^2*Cos[x])^2*S
in[x])/12 + ((35*A + 32*B)*(a^4 + a^4*Cos[x])*Sin[x])/24
```

Rubi [A] time = 0.402811, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{8}a^4x(35A+48B)+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{12}(7A+4B)\sin(x)\left(a^2\cos(x)+a^2\right)^2+\frac{1}{24}(35A+32B)\sin(x)\left(a^4\cos(x)+a\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

```
[Out] (a^4*(35*A + 48*B)*x)/8 + a^4*B*ArcTanh[Sin[x]] + (5*a^4*(7*A + 8*B)*Sin[x]
)/8 + (a*A*(a + a*Cos[x])^3*Sin[x])/4 + ((7*A + 4*B)*(a^2 + a^2*Cos[x])^2*S
in[x])/12 + ((35*A + 32*B)*(a^4 + a^4*Cos[x])*Sin[x])/24
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(x))^4(A+B\sec(x))\,dx = \int (a+a\cos(x))^4(B+A\cos(x))\sec(x)\,dx$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x) + \frac{1}{4}\int (a+a\cos(x))^3(4aB+a(7A+4B)\cos(x))\sec(x)\,d$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x) + \frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x) + \frac{1}{12}\int (a+a\,\mathrm{co}$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x) + \frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x) + \frac{1}{24}(35A+32B$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x) + \frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x) + \frac{1}{24}(35A+32B$$
$$= \frac{5}{8}a^4(7A+8B)\sin(x) + \frac{1}{4}aA(a+a\cos(x))^3\sin(x) + \frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2$$
$$= \frac{1}{8}a^4(35A+48B)x + \frac{5}{8}a^4(7A+8B)\sin(x) + \frac{1}{4}aA(a+a\cos(x))^3\sin(x) + \frac{1}{12}(7A+4$$
$$= \frac{1}{8}a^4(35A+48B)x + a^4B\tanh^{-1}(\sin(x)) + \frac{5}{8}a^4(7A+8B)\sin(x) + \frac{1}{4}aA(a+a\cos(x$$

Mathematica [A] time = 0.126385, size = 97, normalized size = 0.93

$$\frac{1}{96}a^4\Big(24(28A+27B)\sin(x)+24(7A+4B)\sin(2x)+420Ax+32A\sin(3x)+3A\sin(4x)+576Bx+8B\sin(3x)-96B\,\mathrm{lo}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

```
[Out] (a^4*(420*A*x + 576*B*x - 96*B*Log[Cos[x/2] - Sin[x/2]] + 96*B*Log[Cos[x/2]
 + Sin[x/2]] + 24*(28*A + 27*B)*Sin[x] + 24*(7*A + 4*B)*Sin[2*x] + 32*A*Sin
[3*x] + 8*B*Sin[3*x] + 3*A*Sin[4*x]))/96
```

Maple [A] time = 0.088, size = 103, normalized size = 1.

$$\frac{Aa^4\sin(x)(\cos(x))^3}{4}+\frac{27\,Aa^4\sin(x)\cos(x)}{8}+\frac{35\,Aa^4x}{8}+\frac{Ba^4\left(2+(\cos(x))^2\right)\sin(x)}{3}+\frac{4\,Aa^4\left(2+(\cos(x))^2\right)\sin(x)}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^4*(A+B*sec(x)),x)
```

```
[Out] 1/4*A*a^4*sin(x)*cos(x)^3+27/8*A*a^4*sin(x)*cos(x)+35/8*A*a^4*x+1/3*B*a^4*(
2+cos(x)^2)*sin(x)+4/3*A*a^4*(2+cos(x)^2)*sin(x)+2*B*a^4*sin(x)*cos(x)+6*B*
```

```
a^4*x+6*B*a^4*sin(x)+4*A*a^4*sin(x)+B*a^4*ln(sec(x)+tan(x))
```

Maxima [A] time = 1.00779, size = 159, normalized size = 1.53

$$-\frac{4}{3}\left(\sin(x)^3 - 3\sin(x)\right)Aa^4 - \frac{1}{3}\left(\sin(x)^3 - 3\sin(x)\right)Ba^4 + \frac{1}{32}Aa^4(12x + \sin(4x) + 8\sin(2x)) + \frac{3}{2}Aa^4(2x + \sin(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] -4/3*(sin(x)^3 - 3*sin(x))*A*a^4 - 1/3*(sin(x)^3 - 3*sin(x))*B*a^4 + 1/32*A
*a^4*(12*x + sin(4*x) + 8*sin(2*x)) + 3/2*A*a^4*(2*x + sin(2*x)) + B*a^4*(2
*x + sin(2*x)) + A*a^4*x + 4*B*a^4*x + B*a^4*log(sec(x) + tan(x)) + 4*A*a^4
*sin(x) + 6*B*a^4*sin(x)
```

Fricas [A] time = 2.54186, size = 255, normalized size = 2.45

$$\frac{1}{8}(35A + 48B)a^4x + \frac{1}{2}Ba^4\log(\sin(x) + 1) - \frac{1}{2}Ba^4\log(-\sin(x) + 1) + \frac{1}{24}\left(6Aa^4\cos(x)^3 + 8(4A + B)a^4\cos(x)^2 +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(35*A + 48*B)*a^4*x + 1/2*B*a^4*log(sin(x) + 1) - 1/2*B*a^4*log(-sin(x)
 + 1) + 1/24*(6*A*a^4*cos(x)^3 + 8*(4*A + B)*a^4*cos(x)^2 + 3*(27*A + 16*B)
*a^4*cos(x) + 160*(A + B)*a^4)*sin(x)
```

Sympy [A] time = 174.516, size = 116, normalized size = 1.12

$$\frac{35Aa^4x}{8} - \frac{4Aa^4\sin^3(x)}{3} + 8Aa^4\sin(x) + \frac{7Aa^4\sin(2x)}{4} + \frac{Aa^4\sin(4x)}{32} + 6Ba^4x + Ba^4\log(\tan(x) + \sec(x)) - \frac{Ba^4\sin}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**4*(A+B*sec(x)),x)
```

```
[Out] 35*A*a**4*x/8 - 4*A*a**4*sin(x)**3/3 + 8*A*a**4*sin(x) + 7*A*a**4*sin(2*x)/
4 + A*a**4*sin(4*x)/32 + 6*B*a**4*x + B*a**4*log(tan(x) + sec(x)) - B*a**4*
sin(x)**3/3 + 2*B*a**4*sin(x)*cos(x) + 7*B*a**4*sin(x)
```

Giac [A] time = 1.16871, size = 201, normalized size = 1.93

$$Ba^4 \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right) - Ba^4 \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right) + \frac{1}{8}\left(35\,Aa^4 + 48\,Ba^4\right)x + \frac{105\,Aa^4 \tan\left(\frac{1}{2}x\right)^7 + 120\,Ba^4 \tan\left(\frac{1}{2}x\right)^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^4*log(abs(tan(1/2*x) + 1)) - B*a^4*log(abs(tan(1/2*x) - 1)) + 1/8*(35*A
*a^4 + 48*B*a^4)*x + 1/12*(105*A*a^4*tan(1/2*x)^7 + 120*B*a^4*tan(1/2*x)^7
+ 385*A*a^4*tan(1/2*x)^5 + 424*B*a^4*tan(1/2*x)^5 + 511*A*a^4*tan(1/2*x)^3
+ 520*B*a^4*tan(1/2*x)^3 + 279*A*a^4*tan(1/2*x) + 216*B*a^4*tan(1/2*x))/(ta
n(1/2*x)^2 + 1)^4
```

3.189 $\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=25

$$\frac{(A-B)\sin(x)}{a\cos(x)+a}+\frac{B\tanh^{-1}(\sin(x))}{a}$$

```
[Out] (B*ArcTanh[Sin[x]])/a + ((A - B)*Sin[x])/(a + a*Cos[x])
```

Rubi [A] time = 0.0928542, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{(A-B)\sin(x)}{a\cos(x)+a}+\frac{B\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x]),x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a + ((A - B)*Sin[x])/(a + a*Cos[x])
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{a+a\cos(x)}\,dx\\ &= \frac{(A-B)\sin(x)}{a+a\cos(x)}+\frac{\int aB\sec(x)\,dx}{a^2}\\ &= \frac{(A-B)\sin(x)}{a+a\cos(x)}+\frac{B\int \sec(x)\,dx}{a}\\ &= \frac{B\tanh^{-1}(\sin(x))}{a}+\frac{(A-B)\sin(x)}{a+a\cos(x)}\end{aligned}$$

Mathematica [B] time = 0.0846522, size = 71, normalized size = 2.84

$$-\frac{2\cos\left(\frac{x}{2}\right)\left((B-A)\sin\left(\frac{x}{2}\right)+B\cos\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)\right)}{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x]),x]
```

```
[Out] (-2*Cos[x/2]*(B*Cos[x/2]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2
]]) + (-A + B)*Sin[x/2]))/(a*(1 + Cos[x]))
```

Maple [A] time = 0.03, size = 46, normalized size = 1.8

$$\frac{A}{a}\tan\left(\frac{x}{2}\right)-\frac{B}{a}\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{B}{a}\ln\left(1+\tan\left(\frac{x}{2}\right)\right)-\frac{B}{a}\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x)),x)
```

```
[Out] 1/a*A*tan(1/2*x)-1/a*B*ln(tan(1/2*x)-1)+1/a*B*ln(1+tan(1/2*x))-1/a*B*tan(1/
2*x)
```

Maxima [B] time = 0.992415, size = 85, normalized size = 3.4

$$B\left(\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a}-\frac{\sin\left(x\right)}{a(\cos\left(x\right)+1)}\right)+\frac{A\sin\left(x\right)}{a(\cos\left(x\right)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] B*(log(sin(x)/(cos(x) + 1) + 1)/a - log(sin(x)/(cos(x) + 1) - 1)/a - sin(x)
/(a*(cos(x) + 1))) + A*sin(x)/(a*(cos(x) + 1))
```

Fricas [A] time = 2.46082, size = 143, normalized size = 5.72

$$\frac{(B\cos\left(x\right)+B)\log\left(\sin\left(x\right)+1\right)-(B\cos\left(x\right)+B)\log\left(-\sin\left(x\right)+1\right)+2\left(A-B\right)\sin\left(x\right)}{2\left(a\cos\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((B*cos(x) + B)*log(sin(x) + 1) - (B*cos(x) + B)*log(-sin(x) + 1) + 2*(
A - B)*sin(x))/(a*cos(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\cos\left(x\right)+1}\,dx+\int\frac{B\sec\left(x\right)}{\cos\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x)
```

```
[Out] (Integral(A/(cos(x) + 1), x) + Integral(B*sec(x)/(cos(x) + 1), x))/a
```

Giac [A] time = 1.17919, size = 62, normalized size = 2.48

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a}+\frac{A\tan\left(\frac{1}{2}\,x\right)-B\tan\left(\frac{1}{2}\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a - B*log(abs(tan(1/2*x) - 1))/a + (A*tan(1/2*x)
 - B*tan(1/2*x))/a
```

3.190 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx$

Optimal. Leaf size=48

$$\frac{(A-4B)\sin(x)}{3a^2(\cos(x)+1)}+\frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

```
[Out] (B*ArcTanh[Sin[x]])/a^2 + ((A - 4*B)*Sin[x])/(3*a^2*(1 + Cos[x])) + ((A - B
)*Sin[x])/(3*(a + a*Cos[x])^2)
```

Rubi [A] time = 0.184609, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{(A-4B)\sin(x)}{3a^2(\cos(x)+1)}+\frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^2 + ((A - 4*B)*Sin[x])/(3*a^2*(1 + Cos[x])) + ((A - B
)*Sin[x])/(3*(a + a*Cos[x])^2)
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^2}\,dx \\
&= \frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}+\frac{\int \frac{(3aB+a(A-B)\cos(x))\sec(x)}{a+a\cos(x)}\,dx}{3a^2} \\
&= \frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}+\frac{\int 3a^2B\sec(x)\,dx}{3a^4} \\
&= \frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}+\frac{B\int \sec(x)\,dx}{a^2} \\
&= \frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}
\end{aligned}$$

Mathematica [A] time = 0.203533, size = 76, normalized size = 1.58

$$\frac{\sin(x)((A-4B)\cos(x)+2A-5B)-12B\cos^4\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{3a^2(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
```

```
[Out] (-12*B*Cos[x/2]^4*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + (
2*A - 5*B + (A - 4*B)*Cos[x])*Sin[x])/(3*a^2*(1 + Cos[x])^2)
```

Maple [A] time = 0.032, size = 71, normalized size = 1.5

$$\frac{A}{6a^2}\left(\tan\left(\frac{x}{2}\right)\right)^3-\frac{B}{6a^2}\left(\tan\left(\frac{x}{2}\right)\right)^3+\frac{A}{2a^2}\tan\left(\frac{x}{2}\right)-\frac{3B}{2a^2}\tan\left(\frac{x}{2}\right)-\frac{B}{a^2}\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{B}{a^2}\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^2,x)
```

```
[Out] 1/6/a^2*tan(1/2*x)^3*A-1/6/a^2*tan(1/2*x)^3*B+1/2/a^2*A*tan(1/2*x)-3/2/a^2*
B*tan(1/2*x)-1/a^2*B*ln(tan(1/2*x)-1)+1/a^2*B*ln(1+tan(1/2*x))
```

Maxima [B] time = 1.02993, size = 126, normalized size = 2.62

$$-\frac{1}{6}B\left(\frac{\frac{9\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^2}\right)+\frac{A\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*B*((9*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2 - 6*log(sin(x
)/(cos(x) + 1) + 1)/a^2 + 6*log(sin(x)/(cos(x) + 1) - 1)/a^2) + 1/6*A*(3*si
n(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2
```

Fricas [A] time = 2.43617, size = 248, normalized size = 5.17

$$\frac{3\left(B\cos\left(x\right)^2+2B\cos\left(x\right)+B\right)\log\left(\sin\left(x\right)+1\right)-3\left(B\cos\left(x\right)^2+2B\cos\left(x\right)+B\right)\log\left(-\sin\left(x\right)+1\right)+2\left(\left(A-4B\right)\cos\left(x\right)+2A-5B\right)\sin\left(x\right)}{6\left(a^2\cos\left(x\right)^2+2a^2\cos\left(x\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(B*cos(x)^2 + 2*B*cos(x) + B)*log(sin(x) + 1) - 3*(B*cos(x)^2 + 2*B*
cos(x) + B)*log(-sin(x) + 1) + 2*((A - 4*B)*cos(x) + 2*A - 5*B)*sin(x))/(a^
2*cos(x)^2 + 2*a^2*cos(x) + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\cos^2(x)+2\cos(x)+1}\,dx + \int \frac{B\sec(x)}{\cos^2(x)+2\cos(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**2,x)
```

```
[Out] (Integral(A/(cos(x)**2 + 2*cos(x) + 1), x) + Integral(B*sec(x)/(cos(x)**2 +
 2*cos(x) + 1), x))/a**2
```

Giac [A] time = 1.19135, size = 104, normalized size = 2.17

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a^2} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a^2} + \frac{Aa^4\tan\left(\frac{1}{2}x\right)^3 - Ba^4\tan\left(\frac{1}{2}x\right)^3 + 3\,Aa^4\tan\left(\frac{1}{2}x\right) - 9\,Ba^4\tan\left(\frac{1}{2}x\right)}{6\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^2 - B*log(abs(tan(1/2*x) - 1))/a^2 + 1/6*(A*a^
4*tan(1/2*x)^3 - B*a^4*tan(1/2*x)^3 + 3*A*a^4*tan(1/2*x) - 9*B*a^4*tan(1/2*
x))/a^6
```

3.191 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx$

Optimal. Leaf size=75

$$\frac{2(A-11B)\sin(x)}{15\left(a^3\cos(x)+a^3\right)}+\frac{B\tanh^{-1}(\sin(x))}{a^3}+\frac{(2A-7B)\sin(x)}{15a(a\cos(x)+a)^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

```
[Out] (B*ArcTanh[Sin[x]])/a^3 + ((A - B)*Sin[x])/(5*(a + a*Cos[x])^3) + ((2*A - 7
*B)*Sin[x])/(15*a*(a + a*Cos[x])^2) + (2*(A - 11*B)*Sin[x])/(15*(a^3 + a^3*
Cos[x]))
```

Rubi [A] time = 0.310955, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{2(A-11B)\sin(x)}{15\left(a^3\cos(x)+a^3\right)}+\frac{B\tanh^{-1}(\sin(x))}{a^3}+\frac{(2A-7B)\sin(x)}{15a(a\cos(x)+a)^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^3 + ((A - B)*Sin[x])/(5*(a + a*Cos[x])^3) + ((2*A - 7
*B)*Sin[x])/(15*a*(a + a*Cos[x])^2) + (2*(A - 11*B)*Sin[x])/(15*(a^3 + a^3*
Cos[x]))
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^3}\,dx \\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{\int \frac{(5aB+2a(A-B)\cos(x))\sec(x)}{(a+a\cos(x))^2}\,dx}{5a^2} \\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{\int \frac{\left(15a^2B+a^2(2A-7B)\cos(x)\right)\sec(x)}{a+a\cos(x)}\,dx}{15a^4} \\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)} + \frac{\int 15a^3B\sec(x)\,dx}{15a^6} \\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)} + \frac{B\int \sec(x)\,dx}{a^3} \\
&= \frac{B\tanh^{-1}(\sin(x))}{a^3} + \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.351139, size = 88, normalized size = 1.17

$$
\frac{\sin(x)((6A-51B)\cos(x)+(A-11B)\cos(2x)+8A-43B)-120B\cos^6\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\right.\right.}{15a^3(\cos(x)+1)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

```
[Out] (-120*B*Cos[x/2]^6*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
(8*A - 43*B + (6*A - 51*B)*Cos[x] + (A - 11*B)*Cos[2*x])*Sin[x])/(15*a^3*(1
 + Cos[x])^3)
```

Maple [A] time = 0.035, size = 95, normalized size = 1.3

$$\frac{A}{20\,a^3}\left(\tan\left(\frac{x}{2}\right)\right)^5-\frac{B}{20\,a^3}\left(\tan\left(\frac{x}{2}\right)\right)^5+\frac{A}{6\,a^3}\left(\tan\left(\frac{x}{2}\right)\right)^3-\frac{B}{3\,a^3}\left(\tan\left(\frac{x}{2}\right)\right)^3+\frac{B}{a^3}\ln\left(1+\tan\left(\frac{x}{2}\right)\right)-\frac{B}{a^3}\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^3,x)
```

```
[Out] 1/20/a^3*tan(1/2*x)^5*A-1/20/a^3*tan(1/2*x)^5*B+1/6/a^3*tan(1/2*x)^3*A-1/3/
a^3*tan(1/2*x)^3*B+1/a^3*B*ln(1+tan(1/2*x))-1/a^3*B*ln(tan(1/2*x)-1)+1/4/a^
3*A*tan(1/2*x)-7/4/a^3*B*tan(1/2*x)
```

Maxima [A] time = 1.0112, size = 161, normalized size = 2.15

$$-\frac{1}{60}\,B\left(\frac{\frac{105\,\sin(x)}{\cos(x)+1}+\frac{20\,\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\,\sin(x)^5}{(\cos(x)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^3}\right)+\frac{A\left(\frac{15\,\sin(x)}{\cos(x)+1}+\frac{10\,\sin(x)^3}{(\cos(x)+1)^3}+\cdots\right)}{60\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="maxima")
```

```
[Out] -1/60*B*((105*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5
/(cos(x) + 1)^5)/a^3 - 60*log(sin(x)/(cos(x) + 1) + 1)/a^3 + 60*log(sin(x)/
(cos(x) + 1) - 1)/a^3) + 1/60*A*(15*sin(x)/(cos(x) + 1) + 10*sin(x)^3/(cos(
x) + 1)^3 + 3*sin(x)^5/(cos(x) + 1)^5)/a^3
```

Fricas [A] time = 2.43628, size = 356, normalized size = 4.75

$$\frac{15\left(B\cos(x)^3+3\,B\cos(x)^2+3\,B\cos(x)+B\right)\log(\sin(x)+1)-15\left(B\cos(x)^3+3\,B\cos(x)^2+3\,B\cos(x)+B\right)\log(-\sin\cdots}{30\left(a^3\cos(x)^3+3\,a^3\cos(x)^2+3\,a^3\cos(x)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(sin(x) + 1) - 15*
(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(-sin(x) + 1) + 2*(2*(A - 1
1*B)*cos(x)^2 + 3*(2*A - 17*B)*cos(x) + 7*A - 32*B)*sin(x))/(a^3*cos(x)^3 +
 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx + \int \frac{B\sec(x)}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**3,x)
```

```
[Out] (Integral(A/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x) + Integral(B*sec(x
)/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x))/a**3
```

Giac [A] time = 1.16466, size = 138, normalized size = 1.84

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a^3} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a^3} + \frac{3\,Aa^{12}\tan\left(\frac{1}{2}x\right)^5 - 3\,Ba^{12}\tan\left(\frac{1}{2}x\right)^5 + 10\,Aa^{12}\tan\left(\frac{1}{2}x\right)^3 - 20\,Ba^{12}}{60\,a^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^3 - B*log(abs(tan(1/2*x) - 1))/a^3 + 1/60*(3*A
*a^12*tan(1/2*x)^5 - 3*B*a^12*tan(1/2*x)^5 + 10*A*a^12*tan(1/2*x)^3 - 20*B*
a^12*tan(1/2*x)^3 + 15*A*a^12*tan(1/2*x) - 105*B*a^12*tan(1/2*x))/a^15
```

3.192 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx$

Optimal. Leaf size=96

$$\frac{2(3A-80B)\sin(x)}{105a^4(\cos(x)+1)}+\frac{(6A-55B)\sin(x)}{105a^4(\cos(x)+1)^2}+\frac{B\tanh^{-1}(\sin(x))}{a^4}+\frac{(3A-10B)\sin(x)}{35a(a\cos(x)+a)^3}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

```
[Out] (B*ArcTanh[Sin[x]])/a^4 + ((6*A - 55*B)*Sin[x])/(105*a^4*(1 + Cos[x])^2) +
(2*(3*A - 80*B)*Sin[x])/(105*a^4*(1 + Cos[x])) + ((A - B)*Sin[x])/(7*(a + a
*Cos[x])^4) + ((3*A - 10*B)*Sin[x])/(35*a*(a + a*Cos[x])^3)
```

Rubi [A] time = 0.414411, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{2(3A-80B)\sin(x)}{105a^4(\cos(x)+1)}+\frac{(6A-55B)\sin(x)}{105a^4(\cos(x)+1)^2}+\frac{B\tanh^{-1}(\sin(x))}{a^4}+\frac{(3A-10B)\sin(x)}{35a(a\cos(x)+a)^3}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^4 + ((6*A - 55*B)*Sin[x])/(105*a^4*(1 + Cos[x])^2) +
(2*(3*A - 80*B)*Sin[x])/(105*a^4*(1 + Cos[x])) + ((A - B)*Sin[x])/(7*(a + a
*Cos[x])^4) + ((3*A - 10*B)*Sin[x])/(35*a*(a + a*Cos[x])^3)
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^4}\,dx \\
&= \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{\int \frac{(7aB+3a(A-B)\cos(x))\sec(x)}{(a+a\cos(x))^3}\,dx}{7a^2} \\
&= \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{\int \frac{\left(35a^2B+2a^2(3A-10B)\cos(x)\right)\sec(x)}{(a+a\cos(x))^2}\,dx}{35a^4} \\
&= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{\int \frac{\left(105a^3B+a^3(6A-55B)\cos(x)\right)\sec(x)}{a+a\cos(x)}\,dx}{105a^6} \\
&= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3A-80B)\sin(x)}{105\left(a^4+a^4\cos(x)\right)} + \frac{\int 105a^4B\,s}{105} \\
&= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3A-80B)\sin(x)}{105\left(a^4+a^4\cos(x)\right)} + \frac{B\int \sec(x)}{a^4} \\
&= \frac{B\tanh^{-1}(\sin(x))}{a^4} + \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3A-80B)}{105\left(a^4+a^4\cos\right.}
\end{aligned}$$

Mathematica [A] time = 0.805181, size = 104, normalized size = 1.08

$$\frac{\sin(x)((87A-1480B)\cos(x)+(24A-535B)\cos(2x)+3A\cos(3x)+96A-80B\cos(3x)-1055B)-3360B\cos^8\left(\frac{x}{2}\right)\left(\mathrm{lo}\right.}{210a^4(\cos(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
```

```
[Out] (-3360*B*Cos[x/2]^8*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
 (96*A - 1055*B + (87*A - 1480*B)*Cos[x] + (24*A - 535*B)*Cos[2*x] + 3*A*Co
s[3*x] - 80*B*Cos[3*x])*Sin[x])/(210*a^4*(1 + Cos[x])^4)
```

Maple [A] time = 0.037, size = 119, normalized size = 1.2

$$\frac{A}{56\,a^4}\left(\tan\left(\frac{x}{2}\right)\right)^7-\frac{B}{56\,a^4}\left(\tan\left(\frac{x}{2}\right)\right)^7+\frac{3\,A}{40\,a^4}\left(\tan\left(\frac{x}{2}\right)\right)^5-\frac{B}{8\,a^4}\left(\tan\left(\frac{x}{2}\right)\right)^5+\frac{A}{8\,a^4}\left(\tan\left(\frac{x}{2}\right)\right)^3-\frac{11\,B}{24\,a^4}\left(\tan\left(\frac{x}{2}\right)\right)^3+\frac{B}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^4,x)
```

```
[Out] 1/56/a^4*tan(1/2*x)^7*A-1/56/a^4*tan(1/2*x)^7*B+3/40/a^4*tan(1/2*x)^5*A-1/8
/a^4*tan(1/2*x)^5*B+1/8/a^4*tan(1/2*x)^3*A-11/24/a^4*tan(1/2*x)^3*B+1/a^4*B
*ln(1+tan(1/2*x))-1/a^4*B*ln(tan(1/2*x)-1)+1/8/a^4*A*tan(1/2*x)-15/8/a^4*B*
tan(1/2*x)
```

Maxima [A] time = 1.09966, size = 193, normalized size = 2.01

$$-\frac{1}{168}\,B\left(\frac{\frac{315\,\sin(x)}{\cos(x)+1}+\frac{77\,\sin(x)^3}{(\cos(x)+1)^3}+\frac{21\,\sin(x)^5}{(\cos(x)+1)^5}+\frac{3\,\sin(x)^7}{(\cos(x)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^4}\right)+\frac{A\left(\frac{35\,\sin(x)}{\cos(x)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="maxima")
```

```
[Out] -1/168*B*((315*sin(x)/(cos(x) + 1) + 77*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x)
^5/(cos(x) + 1)^5 + 3*sin(x)^7/(cos(x) + 1)^7)/a^4 - 168*log(sin(x)/(cos(x)
 + 1) + 1)/a^4 + 168*log(sin(x)/(cos(x) + 1) - 1)/a^4) + 1/280*A*(35*sin(x)
/(cos(x) + 1) + 35*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x)^5/(cos(x) + 1)^5 + 5
*sin(x)^7/(cos(x) + 1)^7)/a^4
```

Fricas [A] time = 2.53879, size = 464, normalized size = 4.83

$$\frac{105\left(B\cos\left(x\right)^{4}+4\,B\cos\left(x\right)^{3}+6\,B\cos\left(x\right)^{2}+4\,B\cos\left(x\right)+B\right)\log\left(\sin\left(x\right)+1\right)-105\left(B\cos\left(x\right)^{4}+4\,B\cos\left(x\right)^{3}+6\,B\cos\left(\right.\right.}{210\left(a^{4}\cos\left(x\right)^{4}+4\,a^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="fricas")
```

```
[Out] 1/210*(105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) + B)*log(
sin(x) + 1) - 105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) +
B)*log(-sin(x) + 1) + 2*(2*(3*A - 80*B)*cos(x)^3 + (24*A - 535*B)*cos(x)^2
+ (39*A - 620*B)*cos(x) + 36*A - 260*B)*sin(x))/(a^4*cos(x)^4 + 4*a^4*cos(x
)^3 + 6*a^4*cos(x)^2 + 4*a^4*cos(x) + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15574, size = 170, normalized size = 1.77

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a^{4}}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a^{4}}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,x\right)^{7}-15\,Ba^{24}\tan\left(\frac{1}{2}\,x\right)^{7}+63\,Aa^{24}\tan\left(\frac{1}{2}\,x\right)^{5}-105\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^4 - B*log(abs(tan(1/2*x) - 1))/a^4 + 1/840*(15
*A*a^24*tan(1/2*x)^7 - 15*B*a^24*tan(1/2*x)^7 + 63*A*a^24*tan(1/2*x)^5 - 10
5*B*a^24*tan(1/2*x)^5 + 105*A*a^24*tan(1/2*x)^3 - 385*B*a^24*tan(1/2*x)^3 +
 105*A*a^24*tan(1/2*x) - 1575*B*a^24*tan(1/2*x))/a^28
```

3.193 $\int (a + a\cos(x))^{5/2}(A + B\sec(x))\,dx$

Optimal. Leaf size=98

$$\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a\cos(x)+a}} + \frac{2}{15}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a} + 2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right) + \frac{2}{5}aA\sin(x)(a\cos(x)$$

```
[Out] 2*a^(5/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^3*(32*A + 3
5*B)*Sin[x])/(15*Sqrt[a + a*Cos[x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[x]
]*Sin[x])/15 + (2*a*A*(a + a*Cos[x])^(3/2)*Sin[x])/5
```

Rubi [A] time = 0.431637, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2828, 2976, 2981, 2773, 206}

$$\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a\cos(x)+a}} + \frac{2}{15}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a} + 2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right) + \frac{2}{5}aA\sin(x)(a\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

```
[Out] 2*a^(5/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^3*(32*A + 3
5*B)*Sin[x])/(15*Sqrt[a + a*Cos[x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[x]
]*Sin[x])/15 + (2*a*A*(a + a*Cos[x])^(3/2)*Sin[x])/5
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx = \int (a+a\cos(x))^{5/2}(B+A\cos(x))\sec(x)\,dx$$

$$= \frac{2}{5}aA(a+a\cos(x))^{3/2}\sin(x) + \frac{2}{5}\int (a+a\cos(x))^{3/2}\left(\frac{5aB}{2}+\frac{1}{2}a(8A+5B)\cos(x)\right)\mathrm{s}$$

$$= \frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\sin(x) + \frac{2}{5}aA(a+a\cos(x))^{3/2}\sin(x) + \frac{4}{15}\int \sqrt{a+a}$$

$$= \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}} + \frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\sin(x) + \frac{2}{5}aA(a+a\cos(x)$$

$$= \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}} + \frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\sin(x) + \frac{2}{5}aA(a+a\cos(x)$$

$$= 2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right) + \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}} + \frac{2}{15}a^2(8A+5B)\sqrt{a+a}$$

Mathematica [A] time = 0.164598, size = 78, normalized size = 0.8

$$\frac{1}{30}a^2\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}\left(2\sin\left(\frac{x}{2}\right)(2(14A+5B)\cos(x)+3A\cos(2x)+89A+80B)+30\sqrt{2}B\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(30*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]]
+ 2*(89*A + 80*B + 2*(14*A + 5*B)*Cos[x] + 3*A*Cos[2*x])*Sin[x/2]))/30
```

Maple [B] time = 2.795, size = 228, normalized size = 2.3

$$\frac{1}{15}a^{\frac{3}{2}}\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin\left(\frac{x}{2}\right)\right)^2}\left(24\,A\sqrt{2}\sqrt{a\,(\sin\,(x/2))^2}\sqrt{a}\,(\sin\,(x/2))^4-20\,\sqrt{2}\sqrt{a\,(\sin\,(x/2))^2}\sqrt{a}\,(4\,A+B)\,(\sin\,(x/2))^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(5/2)*(A+B*sec(x)),x)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(24*A*2^(1/2)*(a*sin(1/2*x)^
2)^(1/2)*a^(1/2)*sin(1/2*x)^4-20*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)*(4*
A+B)*sin(1/2*x)^2+120*A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+90*B*2^(1/2)
*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+15*B*ln(-4/(-2*cos(1/2*x)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)*cos(1/2*x)+2*a))*a+15*B*ln(4/(2*c
os(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(
1/2)+2*a))*a)/sin(1/2*x)/(cos(1/2*x)^2*a)^(1/2)
```

Maxima [A] time = 1.73421, size = 58, normalized size = 0.59

$$\frac{1}{30}\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}x\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}x\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}x\right)\right)A\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*x) + 25*sqrt(2)*a^2*sin(3/2*x) + 150*sqrt(2)*a^
2*sin(1/2*x))*A*sqrt(a)
```

Fricas [A] time = 2.48419, size = 354, normalized size = 3.61

$$\frac{15\left(Ba^2\cos(x)+Ba^2\right)\sqrt{a}\log\left(\frac{a\cos(x)^3-7a\cos(x)^2-4\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8a}{\cos(x)^3+\cos(x)^2}\right)+4\left(3Aa^2\cos(x)^2+(14A+5B)a^2\cos(x)\cdots\right.}{30(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(B*a^2*cos(x) + B*a^2)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*
sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)
) + 4*(3*A*a^2*cos(x)^2 + (14*A + 5*B)*a^2*cos(x) + (43*A + 40*B)*a^2)*sqrt
(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(5/2)*(A+B*sec(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.30931, size = 244, normalized size = 2.49

$$\frac{Ba^{\frac{7}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}x\right)-\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}x\right)-\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{2\left(60\sqrt{2}Aa^5+45\sqrt{2}Ba^5+\left(80\sqrt{2}Aa^5+80\sqrt{2}Ba^5+\left(32\sqrt{2}Aa^5+\cdots\right.\right.\right.}{15\left(a\tan\left(\frac{1}{2}x\right)^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^(7/2)*log(abs(2*(sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 - 4*s
qrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^
2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 2/15*(60*sqrt(2)*A*a^5 + 45*sqrt(2)*B
*a^5 + (80*sqrt(2)*A*a^5 + 80*sqrt(2)*B*a^5 + (32*sqrt(2)*A*a^5 + 35*sqrt(2
)*B*a^5)*tan(1/2*x)^2)*tan(1/2*x)^2)*tan(1/2*x)/(a*tan(1/2*x)^2 + a)^(5/2)
```

3.194 $\int (a + a\cos(x))^{3/2}(A + B\sec(x))\,dx$

Optimal. Leaf size=72

$$\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a\cos(x)+a}} + 2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right) + \frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

```
[Out] 2*a^(3/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^2*(4*A + 3*
B)*Sin[x])/(3*Sqrt[a + a*Cos[x]]) + (2*a*A*Sqrt[a + a*Cos[x]]*Sin[x])/3
```

Rubi [A] time = 0.294085, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2828, 2976, 2981, 2773, 206}

$$\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a\cos(x)+a}} + 2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right) + \frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

```
[Out] 2*a^(3/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^2*(4*A + 3*
B)*Sin[x])/(3*Sqrt[a + a*Cos[x]]) + (2*a*A*Sqrt[a + a*Cos[x]]*Sin[x])/3
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
```

```
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx &= \int (a+a\cos(x))^{3/2}(B+A\cos(x))\sec(x)\,dx \\
&= \frac{2}{3}aA\sqrt{a+a\cos(x)}\sin(x)+\frac{2}{3}\int \sqrt{a+a\cos(x)}\left(\frac{3aB}{2}+\frac{1}{2}a(4A+3B)\cos(x)\right)\text{sec} \\
&= \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}}+\frac{2}{3}aA\sqrt{a+a\cos(x)}\sin(x)+(aB)\int \sqrt{a+a\cos(x)}\sec(x) \\
&= \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}}+\frac{2}{3}aA\sqrt{a+a\cos(x)}\sin(x)-\left(2a^2B\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,\right. \\
&= 2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}}+\frac{2}{3}aA\sqrt{a+a\cos(x)}\sin
\end{aligned}
$$

Mathematica [A] time = 0.104619, size = 62, normalized size = 0.86

$$
\frac{1}{3}a\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}\left(2\sin\left(\frac{x}{2}\right)(A\cos(x)+5A+3B)+3\sqrt{2}B\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(3*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2
*(5*A + 3*B + A*Cos[x])*Sin[x/2]))/3
```

Maple [B] time = 2.882, size = 199, normalized size = 2.8

$$\frac{1}{3}\sqrt{a}\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin\left(\frac{x}{2}\right)\right)^2}\left(-4\,A\sqrt{2}\sqrt{a\,(\sin(x/2))^2}\sqrt{a}\,(\sin(x/2))^2+12\,A\sqrt{2}\sqrt{a\,(\sin(x/2))^2}\sqrt{a}+6\,B\sqrt{2}\sqrt{a\,(\sin(x/2))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)*(A+B*sec(x)),x)
```

```
[Out] 1/3*a^(1/2)*cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(-4*A*2^(1/2)*(a*sin(1/2*x)^2
)^(1/2)*a^(1/2)*sin(1/2*x)^2+12*A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+6*
B*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+3*B*ln(-4/(-2*cos(1/2*x)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)*cos(1/2*x)+2*a))*a+3*B*ln
(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*
x)^2)^(1/2)+2*a))*a)/sin(1/2*x)/(cos(1/2*x)^2*a)^(1/2)
```

Maxima [A] time = 1.69244, size = 35, normalized size = 0.49

$$\frac{1}{3}\left(\sqrt{2}a\sin\left(\frac{3}{2}\,x\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,x\right)\right)A\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*x) + 9*sqrt(2)*a*sin(1/2*x))*A*sqrt(a)
```

Fricas [A] time = 2.48128, size = 297, normalized size = 4.12

$$\frac{3\,(Ba\cos{(x)}+Ba)\sqrt{a}\log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)+4\,(Aa\cos{(x)}+(5\,A+3\,B)a)\sqrt{a\cos{(x)}+}}{6\,(\cos{(x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(B*a*cos(x) + B*a)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a
*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*
(A*a*cos(x) + (5*A + 3*B)*a)*sqrt(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)*(A+B*sec(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.23506, size = 209, normalized size = 2.9

$$\frac{Ba^{\frac{5}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,x\right)-\sqrt{a\tan\left(\frac{1}{2}\,x\right)^2+a}\right)^2-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,x\right)-\sqrt{a\tan\left(\frac{1}{2}\,x\right)^2+a}\right)^2+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(6\,\sqrt{2}Aa^3+3\,\sqrt{2}Ba^3+\left(4\,\sqrt{2}Aa^3+3\,\sqrt{2}Ba^3\right)\tan\left(\frac{1}{2}\,x\right)^2\right)\tan\left(\frac{1}{2}\right.}{3\left(a\tan\left(\frac{1}{2}\,x\right)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^(5/2)*log(abs(2*(sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 - 4*s
qrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^
```

```
2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 2/3*(6*sqrt(2)*A*a^3 + 3*sqrt(2)*B*a^
3 + (4*sqrt(2)*A*a^3 + 3*sqrt(2)*B*a^3)*tan(1/2*x)^2)*tan(1/2*x)/(a*tan(1/2
*x)^2 + a)^(3/2)
```

3.195 $\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx$

Optimal. Leaf size=44

$$\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}+2\sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)$$

```
[Out] 2*Sqrt[a]*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a*A*Sin[x])/S
qrt[a + a*Cos[x]]
```

Rubi [A] time = 0.160331, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2828, 2981, 2773, 206}

$$\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}+2\sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

```
[Out] 2*Sqrt[a]*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a*A*Sin[x])/S
qrt[a + a*Cos[x]]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx &= \int \sqrt{a+a\cos(x)}(B+A\cos(x))\sec(x)\,dx\\ &= \frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}+B\int \sqrt{a+a\cos(x)}\sec(x)\,dx\\ &= \frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}-(2aB)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)\\ &= 2\sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}\end{aligned}$$

Mathematica [A] time = 0.0370911, size = 47, normalized size = 1.07

$$\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}\left(2A\sin\left(\frac{x}{2}\right)+\sqrt{2}B\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

```
[Out] Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2*A*Si
n[x/2])
```

Maple [B] time = 2.702, size = 152, normalized size = 3.5

$$\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin\left(\frac{x}{2}\right)\right)^2}\left(2A\sqrt{2}\sqrt{a(\sin(x/2))^2}\sqrt{a}+B\ln\left(-4\frac{\sqrt{a}\sqrt{2}\sqrt{a(\sin(x/2))^2}-a\sqrt{2}\cos(x/2)+2a}{-2\cos(x/2)+\sqrt{2}}\right)a+B\ln\left(4\frac{a\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)*(A+B*sec(x)),x)
```

```
[Out] 1/a^(1/2)*cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(2*A*2^(1/2)*(a*sin(1/2*x)^2)^(
1/2)*a^(1/2)+B*ln(-4/(-2*cos(1/2*x)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*x)
^2)^(1/2)-a*2^(1/2)*cos(1/2*x)+2*a))*a+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(
1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*a)/sin(1/2*x)/
(cos(1/2*x)^2*a)^(1/2)
```

Maxima [A] time = 1.66974, size = 18, normalized size = 0.41

$$2\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*A*sqrt(a)*sin(1/2*x)
```

Fricas [B] time = 2.43054, size = 252, normalized size = 5.73

$$\frac{(B\cos(x)+B)\sqrt{a}\log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)+4\,\sqrt{a\cos(x)+a}A\sin(x)}{2\,(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x
) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*sqrt(a
*cos(x) + a)*A*sin(x))/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\cos(x)+1)}\,(A+B\sec(x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)*(A+B*sec(x)),x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))*(A + B*sec(x)), x)
```

Giac [B] time = 3.22241, size = 155, normalized size = 3.52

$$\frac{2\sqrt{2}Aa\tan\left(\frac{1}{2}x\right)}{\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}}+\frac{Ba^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}x\right)-\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}x\right)-\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*A*a*tan(1/2*x)/sqrt(a*tan(1/2*x)^2 + a) + B*a^(3/2)*log(abs(2*(sq
rt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/ab
s(2*(sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 + 4*sqrt(2)*abs(a) -
6*a))/abs(a)
```

3.196 $\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/Sqrt[a] + (Sqrt[2]*(A -
B)*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/Sqrt[a]
```

Rubi [A] time = 0.195222, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2828, 2985, 2649, 206, 2773}

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/Sqrt[a] + (Sqrt[2]*(A -
B)*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/Sqrt[a]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{\sqrt{a+a\cos(x)}}\,dx\\ &= \frac{B\int \sqrt{a+a\cos(x)}\sec(x)\,dx}{a}-(-A+B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx\\ &= -\left((2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)\right)-(2B)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin}{\sqrt{a+a}}\right.\\ &= \frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0506302, size = 52, normalized size = 0.76

$$\frac{2\cos\left(\frac{x}{2}\right)\left((A-B)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)+\sqrt{2}B\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a(\cos(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
```

```
[Out] (2*((A - B)*ArcTanh[Sin[x/2]] + Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]])*Cos[x/
2])/Sqrt[a*(1 + Cos[x])]
```

Maple [B] time = 3.422, size = 192, normalized size = 2.8

$$\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin\left(\frac{x}{2}\right)\right)^2}\left(\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(x/2\right)\right)^2+a}}{\cos\left(x/2\right)}\right)A-\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(x/2\right)\right)^2+a}}{\cos\left(x/2\right)}\right)B+B\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\,(\mathrm{s}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(1/2),x)
```

```
[Out] cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(a*sin(
1/2*x)^2)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(a*sin(1/2*x)^2)^(1/
2)+a))*B+B*ln(-4/(-2*cos(1/2*x)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^
(1/2)-a*2^(1/2)*cos(1/2*x)+2*a))+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*c
os(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a)))/a^(1/2)/sin(1/2*x)/
(cos(1/2*x)^2*a)^(1/2)
```

Maxima [A] time = 1.70862, size = 78, normalized size = 1.15

$$\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,x\right)^2+\sin\left(\frac{1}{2}\,x\right)^2+2\,\sin\left(\frac{1}{2}\,x\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,x\right)^2+\sin\left(\frac{1}{2}\,x\right)^2-2\,\sin\left(\frac{1}{2}\,x\right)+1\right)\right)A}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*x)^2 + sin(1/2*x)^2 + 2*sin(1/2*x) + 1) - sqrt(2)*
log(cos(1/2*x)^2 + sin(1/2*x)^2 - 2*sin(1/2*x) + 1))*A/sqrt(a)
```

Fricas [B] time = 2.60085, size = 354, normalized size = 5.21

$$-\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(x)^2+\frac{2\,\sqrt{2}\sqrt{a\cos(x)+a}\sin(x)}{\sqrt{a}}-2\,\cos(x)-3}{\cos(x)^2+2\,\cos(x)+1}\right)-B\sqrt{a}\log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)
*sin(x)/sqrt(a) - 2*cos(x) - 3)/(cos(x)^2 + 2*cos(x) + 1)) - B*sqrt(a)*log(
(a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(
x) + 8*a)/(cos(x)^3 + cos(x)^2)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(x)}{\sqrt{a(\cos(x) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral((A + B*sec(x))/sqrt(a*(cos(x) + 1)), x)
```

Giac [B] time = 2.23482, size = 180, normalized size = 2.65

$$-\frac{\sqrt{2}\left(A\sqrt{a} - B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,x\right) - \sqrt{a\tan\left(\frac{1}{2}\,x\right)^2 + a}\right)^2\right)}{2\,a} + \frac{B\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,x\right) - \sqrt{a\tan\left(\frac{1}{2}\,x\right)^2 + a}\right)^2 - a\left(2\sqrt{2}\cdots\right.\right.\right.}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1
/2*x)^2 + a))^2)/a + B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 +
a))^2 - a*(2*sqrt(2) + 3)))/sqrt(a) - B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(
a*tan(1/2*x)^2 + a))^2 + a*(2*sqrt(2) - 3)))/sqrt(a)
```

3.197 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{3/2}}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(3/2) + ((A - 5*B)*Arc
Tanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(2*Sqrt[2]*a^(3/2)) +
((A - B)*Sin[x])/(2*(a + a*Cos[x])^(3/2))
```

Rubi [A] time = 0.335956, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2828, 2978, 2985, 2649, 206, 2773}

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{3/2}}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(3/2) + ((A - 5*B)*Arc
Tanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(2*Sqrt[2]*a^(3/2)) +
((A - B)*Sin[x])/(2*(a + a*Cos[x])^(3/2))
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$$

$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} + \frac{\int \frac{\left(2aB+\frac{1}{2}a(A-B)\cos(x)\right)\sec(x)}{\sqrt{a+a\cos(x)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} + \frac{(A-5B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx}{4a} + \frac{B\int \sqrt{a+a\cos(x)}\sec(x)\,dx}{a^2}$$

$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} - \frac{(A-5B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{2a} - \frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\right.}{a}$$

$$= \frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{3/2}} + \frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{2\sqrt{2}a^{3/2}} + \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}}$$

Mathematica [A] time = 0.372451, size = 73, normalized size = 0.79

$$\frac{\frac{1}{2}(A-B)\sin(x) + (A-5B)\cos^3\left(\frac{x}{2}\right)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right) + 4\sqrt{2}B\cos^3\left(\frac{x}{2}\right)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)}{(a(\cos(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

```
[Out] ((A - 5*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x
/2]]*Cos[x/2]^3 + ((A - B)*Sin[x])/2)/(a*(1 + Cos[x]))^(3/2)
```

Maple [B] time = 3.653, size = 270, normalized size = 2.9

$$\frac{1}{4}\sqrt{a\left(\sin\left(\frac{x}{2}\right)\right)^2}\left(A\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin(x/2)\right)^2+2a}}{\cos(x/2)}\right)\sqrt{2}\left(\cos\left(\frac{x}{2}\right)\right)^2 a - 5B\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin(x/2)\right)^2+2a}}{\cos(x/2)}\right)(\cos(x/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(3/2),x)
```

```
[Out] 1/4/a^(5/2)/cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(A*ln(2*(2*a^(1/2)*(a*sin(1/2
*x)^2)^(1/2)+2*a)/cos(1/2*x))*2^(1/2)*cos(1/2*x)^2*a-5*B*2^(1/2)*ln(2*(2*a^
(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^2*a+4*B*ln(-4*(a*2
^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-2*a)/(2*cos(1/2*x)
-2^(1/2)))*cos(1/2*x)^2*a+4*B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/
2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*cos(1/2*x)^2*a+A*2^(1/2)*
(a*sin(1/2*x)^2)^(1/2)*a^(1/2)-B*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2))/si
n(1/2*x)/(cos(1/2*x)^2*a)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.48969, size = 559, normalized size = 6.08

$$-\frac{\sqrt{2}\Big((A-5\,B)\cos{(x)}^2+2\,(A-5\,B)\cos{(x)}+A-5\,B\Big)\sqrt{a}\log\left(-\frac{a\cos(x)^2+2\,\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{a}\sin(x)-2\,a\cos(x)-3\,a}{\cos(x)^2+2\,\cos(x)+1}\right)-4\left(B\cos{(x)}\right.}{8\left(a^2\cos{(x)}^2+2\,a^2\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((A - 5*B)*cos(x)^2 + 2*(A - 5*B)*cos(x) + A - 5*B)*sqrt(a)*l
og(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(a)*sin(x) - 2*a*cos(x)
- 3*a)/(cos(x)^2 + 2*cos(x) + 1)) - 4*(B*cos(x)^2 + 2*B*cos(x) + B)*sqrt(a)
*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)
*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) - 4*sqrt(a*cos(x) + a)*(A - B)*sin(x)
)/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(x)}{(a(\cos(x) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral((A + B*sec(x))/(a*(cos(x) + 1))**(3/2), x)
```

Giac [B] time = 2.29119, size = 227, normalized size = 2.47

$$-\frac{\sqrt{2}\left(A\sqrt{a} - 5B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}x\right) - \sqrt{a\tan\left(\frac{1}{2}x\right)^2 + a}\right)^2\right)}{8a^2} + \frac{B\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}x\right) - \sqrt{a\tan\left(\frac{1}{2}x\right)^2 + a}\right)^2 - a\left(2\right.\right.}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(A*sqrt(a) - 5*B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan
(1/2*x)^2 + a))^2)/a^2 + B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^
2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(3/2) - B*log(abs((sqrt(a)*tan(1/2*x) - s
qrt(a*tan(1/2*x)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) + 1/4*sqrt(a*tan(1
/2*x)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*x)/a^3
```

3.198 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{16\sqrt{2}a^{5/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a\cos(x)+a)^{3/2}}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(5/2) + ((3*A - 43*B)*
ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(16*Sqrt[2]*a^(5/2)
) + ((A - B)*Sin[x])/(4*(a + a*Cos[x])^(5/2)) + ((3*A - 11*B)*Sin[x])/(16*a
*(a + a*Cos[x])^(3/2))
```

Rubi [A] time = 0.481825, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2828, 2978, 2985, 2649, 206, 2773}

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{16\sqrt{2}a^{5/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a\cos(x)+a)^{3/2}}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(5/2) + ((3*A - 43*B)*
ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(16*Sqrt[2]*a^(5/2)
) + ((A - B)*Sin[x])/(4*(a + a*Cos[x])^(5/2)) + ((3*A - 11*B)*Sin[x])/(16*a
*(a + a*Cos[x])^(3/2))
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx=\int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{\int \frac{\left(4aB+\frac{3}{2}a(A-B)\cos(x)\right)\sec(x)}{(a+a\cos(x))^{3/2}}\,dx}{4a^2}$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}}+\frac{\int \frac{\left(8a^2B+\frac{1}{4}a^2(3A-11B)\cos(x)\right)\sec(x)}{\sqrt{a+a\cos(x)}}\,dx}{8a^4}$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}}+\frac{(3A-43B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx}{32a^2}+\frac{B\int \sqrt{a+a\cos(x)}\,\mathrm{sec}}{a^3}$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}}-\frac{(3A-43B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{16a^2}-\frac{(2}{}$$

$$=\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{5/2}}+\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{16\sqrt{2}a^{5/2}}+\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-11}{16a(a+a\,\mathrm{c}}$$

Mathematica [A] time = 0.499836, size = 95, normalized size = 0.79

$$\frac{\tan\left(\frac{x}{2}\right)(3A\cos(x)+7A-11B\cos(x)-15B)+2(3A-43B)\cos^3\left(\frac{x}{2}\right)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)+64\sqrt{2}B\cos^3\left(\frac{x}{2}\right)\tanh^{-1}\left(\sqrt{2}\sin\right.}{16a(a(\cos(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

```
[Out] (2*(3*A - 43*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 64*Sqrt[2]*B*ArcTanh[Sqrt[2]
*Sin[x/2]]*Cos[x/2]^3 + (7*A - 15*B + 3*A*Cos[x] - 11*B*Cos[x])*Tan[x/2])/(
16*a*(a*(1 + Cos[x]))^(3/2))
```

Maple [B] time = 3.66, size = 322, normalized size = 2.7

$$\frac{1}{32}\sqrt{a\left(\sin\left(\frac{x}{2}\right)\right)^2}\left(3A\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(x/2))^2}+2a}{\cos(x/2)}\right)\sqrt{2}(\cos(x/2))^4a-43B\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(x/2))^2}+2a}{\cos(x/2)}\right)a(\mathrm{cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(5/2),x)
```

```
[Out] 1/32/a^(7/2)/cos(1/2*x)^3*(a*sin(1/2*x)^2)^(1/2)*(3*A*ln(2*(2*a^(1/2)*(a*si
n(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*2^(1/2)*cos(1/2*x)^4*a-43*B*2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*a*cos(1/2*x)^4+32*B*ln(
-4*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-2*a)/(2*cos
(1/2*x)-2^(1/2)))*a*cos(1/2*x)^4+32*B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2
)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*a*cos(1/2*x)^4+3*
A*a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*cos(1/2*x)^2-11*B*a^(1/2)*2^(1/2)*
(a*sin(1/2*x)^2)^(1/2)*cos(1/2*x)^2+2*A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1
/2)-2*B*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2))/sin(1/2*x)/(cos(1/2*x)^2*a)
^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.6152, size = 689, normalized size = 5.74

$$-\frac{\sqrt{2}\left((3\,A-43\,B)\cos\left(x\right)^{3}+3\,(3\,A-43\,B)\cos\left(x\right)^{2}+3\,(3\,A-43\,B)\cos\left(x\right)+3\,A-43\,B\right)\sqrt{a}\log\left(-\frac{a\cos(x)^{2}+2\,\sqrt{2}\sqrt{a\cos(x)+}}{\cos(x)^{2}+2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*((3*A - 43*B)*cos(x)^3 + 3*(3*A - 43*B)*cos(x)^2 + 3*(3*A -
43*B)*cos(x) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(
x) + a)*sqrt(a)*sin(x) - 2*a*cos(x) - 3*a)/(cos(x)^2 + 2*cos(x) + 1)) - 32*
(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*
cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^
3 + cos(x)^2)) - 4*((3*A - 11*B)*cos(x) + 7*A - 15*B)*sqrt(a*cos(x) + a)*si
```

```
n(x))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.43033, size = 269, normalized size = 2.24

$$\frac{1}{32}\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}x\right)^2}{a^8}+\frac{\sqrt{2}\left(5Aa^5-13Ba^5\right)}{a^8}\right)\tan\left(\frac{1}{2}x\right)-\frac{\sqrt{2}\left(3A\sqrt{a}-43B\sqrt{a}\right)\log\left(\left(\sqrt{a}\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(a*tan(1/2*x)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*x)^2/a^8 +
 sqrt(2)*(5*A*a^5 - 13*B*a^5)/a^8)*tan(1/2*x) - 1/64*sqrt(2)*(3*A*sqrt(a) -
 43*B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2)/a^3 +
 B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 - a*(2*sqrt(2)
 + 3)))/a^(5/2) - B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))
^2 + a*(2*sqrt(2) - 3)))/a^(5/2)
```

3.199 $\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=25

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

```
[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

Rubi [A] time = 0.0531446, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4592, 2668, 31}

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]
```

```
[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

Rule 4592

```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sin[(c_.) + (d_.)*(x_)]))/((a_) + (
b_.)*Sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> -Simp[(B*(e + f*x)*Cos[c + d*
x])/(a*d*(a + b*Sin[c + d*x])), x] + Dist[(B*f)/(a*d), Int[Cos[c + d*x]/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*A
- b*B, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = -\frac{x\cos(x)}{a+b\sin(x)}+\int \frac{\cos(x)}{a+b\sin(x)}\,dx$$
$$= -\frac{x\cos(x)}{a+b\sin(x)}+\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}$$
$$= \frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

Mathematica [A] time = 0.196906, size = 25, normalized size = 1.

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]
```

```
[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

Maple [B] time = 0.337, size = 80, normalized size = 3.2

$$\left(x\left(\tan\left(\frac{x}{2}\right)\right)^4-x\right)\left(1+\left(\tan\left(\frac{x}{2}\right)\right)^2\right)^{-1}\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2+2\,b\tan(x/2)+a\right)^{-1}+\frac{1}{b}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2+2\,b\tan(x/2)+a\right)-\frac{1}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*sin(x))/(a+b*sin(x))^2,x)
```

```
[Out] (x*tan(1/2*x)^4-x)/(1+tan(1/2*x)^2)/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+1/b*l
n(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)-1/b*ln(1+tan(1/2*x)^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.53441, size = 93, normalized size = 3.72

$$-\frac{bx\cos(x) - (b\sin(x) + a)\log(b\sin(x) + a)}{b^2\sin(x) + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(b*x*cos(x) - (b*sin(x) + a)*log(b*sin(x) + a))/(b^2*sin(x) + a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29717, size = 382, normalized size = 15.28

$$\frac{4\,bx\tan\left(\frac{1}{2}\,x\right)^2 + a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4+4\,ab\tan\left(\frac{1}{2}\,x\right)^3+2\,a^2\tan\left(\frac{1}{2}\,x\right)^2+4\,b^2\tan\left(\frac{1}{2}\,x\right)^2+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2 + 2\,b\log\left(\frac{4\left(a^2\tan\left(\right.\right.}{}\right.}{2\left(ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b*x*tan(1/2*x)^2 + a*log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 +
2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*
x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*b*log(4*(a^2*tan(1/2*x)^4 + 4*
a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*
x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - 4*b*x + a*log(4
*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/
2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b
*tan(1/2*x)^2 + 2*b^2*tan(1/2*x) + a*b)
```

3.200 $\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx$

Optimal. Leaf size=24

$$\frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

```
[Out] Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

Rubi [A] time = 0.0555215, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4593, 2668, 31}

$$\frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
```

```
[Out] Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

Rule 4593

```
Int[((Cos[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cos[(c_.
) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[(B*(e + f*x)*Sin[c + d*x
])/(a*d*(a + b*Cos[c + d*x])), x] - Dist[(B*f)/(a*d), Int[Sin[c + d*x]/(a +
 b*Cos[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*A -
 b*B, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{x\sin(x)}{a+b\cos(x)} - \int \frac{\sin(x)}{a+b\cos(x)}\,dx$$
$$= \frac{x\sin(x)}{a+b\cos(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}$$
$$= \frac{\log(a+b\cos(x))}{b} + \frac{x\sin(x)}{a+b\cos(x)}$$

Mathematica [A] time = 0.125291, size = 24, normalized size = 1.

$$\frac{\log(a+b\cos(x))}{b} + \frac{x\sin(x)}{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
```

```
[Out] Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

Maple [B] time = 0.166, size = 91, normalized size = 3.8

$$\left(2\,x\tan(x/2) + 2\,x(\tan(x/2))^3\right)\left(1 + \left(\tan\left(\frac{x}{2}\right)\right)^2\right)^{-1}\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2 - b\left(\tan\left(\frac{x}{2}\right)\right)^2 + a + b\right)^{-1} + \frac{1}{b}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2 - b\left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*cos(x))/(a+b*cos(x))^2,x)
```

```
[Out] (2*x*tan(1/2*x)+2*x*tan(1/2*x)^3)/(1+tan(1/2*x)^2)/(a*tan(1/2*x)^2-b*tan(1/
2*x)^2+a+b)+1/b*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+a+b)-1/b*ln(1+tan(1/2*x)^2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52855, size = 93, normalized size = 3.88

$$\frac{bx\sin(x) + (b\cos(x) + a)\log(-b\cos(x) - a)}{b^2\cos(x) + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="fricas")
```

```
[Out] (b*x*sin(x) + (b*cos(x) + a)*log(-b*cos(x) - a))/(b^2*cos(x) + a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32801, size = 536, normalized size = 22.33

$$\frac{a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4 - 2ab\tan\left(\frac{1}{2}x\right)^4 + b^2\tan\left(\frac{1}{2}x\right)^4 + 2a^2\tan\left(\frac{1}{2}x\right)^2 - 2b^2\tan\left(\frac{1}{2}x\right)^2 + a^2 + 2ab + b^2\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)\tan\left(\frac{1}{2}x\right)^2 - b\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4 - 2ab\tan\left(\frac{1}{2}\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*
a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 +
2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1
/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2
+ 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 8*b*x*ta
n(1/2*x) + a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^
4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x
)^4 + 2*tan(1/2*x)^2 + 1)) + b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4
 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b
 + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b*tan(1/2*x)^2 - b^2*tan(1
/2*x)^2 + a*b + b^2)
```

3.201 $\int \frac{1+\sin^2(x)}{1-\sin^2(x)} \, dx$

Optimal. Leaf size=8

$$2\tan(x) - x$$

```
[Out] -x + 2*Tan[x]
```

Rubi [A] time = 0.0414272, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$2\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
```

```
[Out] -x + 2*Tan[x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx &= -x+2\int \frac{1}{1-\sin^2(x)}\,dx \\
&= -x+2\int \sec^2(x)\,dx \\
&= -x-2\,\text{Subst}\left(\int 1\,dx,x,-\tan(x)\right) \\
&= -x+2\tan(x)
\end{aligned}
$$

Mathematica [A] time = 0.0079257, size = 8, normalized size = 1.

$$2\tan(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
```

```
[Out] -x + 2*Tan[x]
```

Maple [A] time = 0.022, size = 9, normalized size = 1.1

$$-x+2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x)^2)/(1-sin(x)^2),x)
```

```
[Out] -x+2*tan(x)
```

Maxima [A] time = 1.49172, size = 11, normalized size = 1.38

$$-x+2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="maxima")
```

```
[Out] -x + 2*tan(x)
```

Fricas [A] time = 2.21415, size = 42, normalized size = 5.25

$$-\frac{x\cos(x) - 2\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - 2*sin(x))/cos(x)
```

Sympy [B] time = 1.50302, size = 41, normalized size = 5.12

$$-\frac{x\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} + \frac{x}{\tan^2\left(\frac{x}{2}\right) - 1} - \frac{4\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)**2)/(1-sin(x)**2),x)
```

```
[Out] -x*tan(x/2)**2/(tan(x/2)**2 - 1) + x/(tan(x/2)**2 - 1) - 4*tan(x/2)/(tan(x/
2)**2 - 1)
```

Giac [A] time = 1.11664, size = 11, normalized size = 1.38

$$-x + 2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="giac")
```

```
[Out] -x + 2*tan(x)
```

3.202 $\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$\sqrt{2}x-x+\sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)$$

```
[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]
```

Rubi [A] time = 0.0410113, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 203}

$$\sqrt{2}x-x+\sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
```

```
[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx &= -x+2\int \frac{1}{1+\sin^2(x)}\,dx\\ &= -x+2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= -x+\sqrt{2}x+\sqrt{2}\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.0286231, size = 24, normalized size = 0.67

$$-2\left(\frac{x}{2}-\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2])
```

Maple [A] time = 0.022, size = 16, normalized size = 0.4

$$\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x)^2)/(1+sin(x)^2),x)
```

```
[Out] 2^(1/2)*arctan(tan(x)*2^(1/2))-x
```

Maxima [A] time = 1.48226, size = 20, normalized size = 0.56

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

Fricas [A] time = 2.39607, size = 107, normalized size = 2.97

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) -
 x
```

Sympy [B] time = 77.8847, size = 416, normalized size = 11.56

$$-\frac{58x\sqrt{3-2\sqrt{2}}}{41\sqrt{2}\sqrt{3-2\sqrt{2}}+58\sqrt{3-2\sqrt{2}}}-\frac{41\sqrt{2}x\sqrt{3-2\sqrt{2}}}{41\sqrt{2}\sqrt{3-2\sqrt{2}}+58\sqrt{3-2\sqrt{2}}}+\frac{24\sqrt{2}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{41\sqrt{2}\sqrt{3-2\sqrt{2}}+58\sqrt{3-2\sqrt{2}}}+\frac{34\left(\operatorname{atan}\left(\cdots\right.\right.}{41\sqrt{2}\sqrt{3-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)**2)/(1+sin(x)**2),x)
```

```
[Out] -58*x*sqrt(3 - 2*sqrt(2))/(41*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 58*sqrt(3 - 2*s
qrt(2))) - 41*sqrt(2)*x*sqrt(3 - 2*sqrt(2))/(41*sqrt(2)*sqrt(3 - 2*sqrt(2))
 + 58*sqrt(3 - 2*sqrt(2))) + 24*sqrt(2)*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 58*sqrt(3 -
 2*sqrt(2))) + 34*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/
2)/pi))/(41*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 58*sqrt(3 - 2*sqrt(2))) + 24*sqrt
(2)*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) +
 3)) + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*sqrt(3 - 2*sqrt(2)) + 58*sqrt
(3 - 2*sqrt(2))) + 34*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2
)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*sqrt(3 - 2*
sqrt(2)) + 58*sqrt(3 - 2*sqrt(2)))
```

Giac [A] time = 1.11781, size = 66, normalized size = 1.83

$$\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - 2\sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - 2\cos(2x) + 2}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sq
rt(2) - 2*cos(2*x) + 2))) - x
```

3.203 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=8

$$-x-2\cot(x)$$

```
[Out] -x - 2*Cot[x]
```

Rubi [A] time = 0.0410208, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx &= -x+2\int \frac{1}{1-\cos^2(x)}\,dx\\ &= -x+2\int \csc^2(x)\,dx\\ &= -x-2\,\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -x-2\cot(x)\end{aligned}$$

Mathematica [A] time = 0.0076072, size = 8, normalized size = 1.

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Maple [A] time = 0.037, size = 11, normalized size = 1.4

$$-2\,(\tan(x))^{-1}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)/(1-cos(x)^2),x)
```

```
[Out] -2/tan(x)-x
```

Maxima [A] time = 1.5106, size = 14, normalized size = 1.75

$$-x-\frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
```

```
[Out] -x - 2/tan(x)
```

Fricas [A] time = 2.2177, size = 42, normalized size = 5.25

$$-\frac{x \sin(x) + 2 \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + 2*cos(x))/sin(x)
```

Sympy [A] time = 2.48755, size = 12, normalized size = 1.5

$$-x + \tan\left(\frac{x}{2}\right) - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(1-cos(x)**2),x)
```

```
[Out] -x + tan(x/2) - 1/tan(x/2)
```

Giac [A] time = 1.15266, size = 22, normalized size = 2.75

$$-x - \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="giac")
```

```
[Out] -x - 1/tan(1/2*x) + tan(1/2*x)
```

3.204 $\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=37

$$\sqrt{2}x - x - \sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x) + \sqrt{2} + 1}\right)$$

```
[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]
```

Rubi [A] time = 0.0383074, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 203}

$$\sqrt{2}x - x - \sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x) + \sqrt{2} + 1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]

[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = -x + 2\int \frac{1}{1+\cos^2(x)}\,dx$$
$$= -x - 2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(x)\right)$$
$$= -x + \sqrt{2}x - \sqrt{2}\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)$$

Mathematica [A] time = 0.0315964, size = 23, normalized size = 0.62

$$2\left(\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]
```

```
[Out] 2*(-x/2 + ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2])
```

Maple [A] time = 0.03, size = 17, normalized size = 0.5

$$\sqrt{2}\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)/(1+cos(x)^2),x)
```

```
[Out] 2^(1/2)*arctan(1/2*tan(x)*2^(1/2))-x
```

Maxima [A] time = 1.48758, size = 22, normalized size = 0.59

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

Fricas [A] time = 2.50551, size = 104, normalized size = 2.81

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x
```

Sympy [A] time = 5.6742, size = 61, normalized size = 1.65

$$-x+\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)+\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)/(1+cos(x)**2),x)
```

```
[Out] -x + sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi)) + sqr
t(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))
```

Giac [A] time = 1.16333, size = 66, normalized size = 1.78

$$\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt
(2) - cos(2*x) + 1))) - x
```

3.205 $\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=14

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

```
[Out] (c*x)/d^2 - Sin[x]/d
```

Rubi [A] time = 0.127796, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4397, 3016, 2637}

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (c*x)/d^2 - Sin[x]/d
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx &= \int \frac{\frac{c^2}{d^2}-\cos^2(x)}{c+d\cos(x)}\,dx \\
&= -\frac{\int(-c+d\cos(x))\,dx}{d^2} \\
&= \frac{cx}{d^2}-\frac{\int \cos(x)\,dx}{d} \\
&= \frac{cx}{d^2}-\frac{\sin(x)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0100972, size = 14, normalized size = 1.

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (c*x)/d^2 - Sin[x]/d
```

Maple [B] time = 0.029, size = 32, normalized size = 2.3

$$-2\,\frac{\tan(x/2)}{d\left(1+(\tan(x/2))^2\right)}+2\,\frac{c\arctan(\tan(x/2))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x)
```

```
[Out] -2/d*tan(1/2*x)/(1+tan(1/2*x)^2)+2/d^2*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38779, size = 30, normalized size = 2.14

$$\frac{cx - d\sin(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] (c*x - d*sin(x))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+sin(x)**2)/(c+d*cos(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15006, size = 35, normalized size = 2.5

$$\frac{cx}{d^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] c*x/d^2 - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d)
```

3.206 $\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=105

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}+\frac{bcx}{d^2}-\frac{2b\sqrt{c-d}\sqrt{c+d}\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}-\frac{b\sin(x)}{d}$$

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(Sqrt[c - d]
*Sqrt[c + d]) - (2*b*Sqrt[c - d]*Sqrt[c + d]*ArcTan[(Sqrt[c - d]*Tan[x/2])/
Sqrt[c + d]])/d^2 - (b*Sin[x])/d
```

Rubi [A] time = 0.259909, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4401, 2659, 205, 2695, 2735}

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}+\frac{bcx}{d^2}-\frac{2b\sqrt{c-d}\sqrt{c+d}\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}-\frac{b\sin(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(Sqrt[c - d]
*Sqrt[c + d]) - (2*b*Sqrt[c - d]*Sqrt[c + d]*ArcTan[(Sqrt[c - d]*Tan[x/2])/
Sqrt[c + d]])/d^2 - (b*Sin[x])/d
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx &= \int \left(\frac{a}{c+d\cos(x)}+\frac{b\sin^2(x)}{c+d\cos(x)}\right)dx\\
&= a\int \frac{1}{c+d\cos(x)}\,dx + b\int \frac{\sin^2(x)}{c+d\cos(x)}\,dx\\
&= -\frac{b\sin(x)}{d} + (2a)\,\mathrm{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{b\int \frac{-d-c\cos(x)}{c+d\cos(x)}\,dx}{d}\\
&= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}} - \frac{b\sin(x)}{d} + \frac{\left(b\left(-c^2+d^2\right)\right)\int \frac{1}{c+d\cos(x)}\,dx}{d^2}\\
&= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}} - \frac{b\sin(x)}{d} + \frac{\left(2b\left(-c^2+d^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{d^2}\\
&= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}} - \frac{2b\sqrt{c-d}\sqrt{c+d}\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2} - \frac{b\sin(x)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.153592, size = 73, normalized size = 0.7

$$\frac{-\frac{2\left(ad^2+b\left(d^2-c^2\right)\right)\tanh^{-1}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)}{\sqrt{d^2-c^2}}+bcx-bd\sin(x)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (b*c*x - (2*(a*d^2 + b*(-c^2 + d^2))*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 +
 d^2]])/Sqrt[-c^2 + d^2] - b*d*Sin[x])/d^2
```

Maple [A] time = 0.042, size = 148, normalized size = 1.4

$$2\frac{a}{\sqrt{(c+d)(c-d)}}\arctan\left(\frac{(c-d)\tan(x/2)}{\sqrt{(c+d)(c-d)}}\right)-2\frac{bc^2}{d^2\sqrt{(c+d)(c-d)}}\arctan\left(\frac{(c-d)\tan(x/2)}{\sqrt{(c+d)(c-d)}}\right)+2\frac{b}{\sqrt{(c+d)(c-d)}}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+d*cos(x)),x)
```

```
[Out] 2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*a-2/d^2/
((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*b*c^2+2/((
c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*b-2*b/d*tan(
1/2*x)/(1+tan(1/2*x)^2)+2*b/d^2*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.72175, size = 558, normalized size = 5.31

$$\left[\frac{\left(bc^2-(a+b)d^2\right)\sqrt{-c^2+d^2}\log\left(\frac{2\,cd\cos(x)+\left(2\,c^2-d^2\right)\cos(x)^2+2\,\sqrt{-c^2+d^2}(c\cos(x)+d)\sin(x)-c^2+2\,d^2}{d^2\cos(x)^2+2\,cd\cos(x)+c^2}\right)+2\left(bc^3-bcd^2\right)x-2\left(bc^2d-bd^3\right.}{2\left(c^2d^2-d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log((2*c*d*cos(x) + (2*c^2 - d
^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d^2
*cos(x)^2 + 2*c*d*cos(x) + c^2)) + 2*(b*c^3 - b*c*d^2)*x - 2*(b*c^2*d - b*d
^3)*sin(x))/(c^2*d^2 - d^4), -((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d^2)*arctan
(-(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x))) - (b*c^3 - b*c*d^2)*x + (b*c^2*d
 - b*d^3)*sin(x))/(c^2*d^2 - d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14017, size = 149, normalized size = 1.42

$$\frac{bcx}{d^2}-\frac{2\,b\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)d}+\frac{2\left(bc^2-ad^2-bd^2\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,c+2\,d\right)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,x\right)-d\tan\left(\frac{1}{2}\,x\right)}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] b*c*x/d^2 - 2*b*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d) + 2*(b*c^2 - a*d^2 - b*d^
2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*ta
n(1/2*x))/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2)
```

3.207 $\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+2b)}{\sqrt{2}c}-\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c}-\frac{bx}{c}$$

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) - ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Cos[x]^2)])/(Sqrt[2]*c)
```

Rubi [A] time = 0.134189, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 1166, 203}

$$\frac{x(a+2b)}{\sqrt{2}c}-\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c}-\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) - ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Cos[x]^2)])/(Sqrt[2]*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{a+(a+b)x^2}{c\left(2+3x^2+x^4\right)}\,dx, x, \tan(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{2+3x^2+x^4}\,dx, x, \tan(x)\right)}{c}\\
&= -\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{c} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(x)\right)}{c}\\
&= -\frac{bx}{c} + \frac{(a+2b)x}{\sqrt{2}c} - \frac{(a+2b)\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}c}
\end{aligned}$$

Mathematica [A] time = 0.0885643, size = 34, normalized size = 0.6

$$-\frac{(-a-2b)\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}c} - \frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

```
[Out] -((b*x)/c) - ((-a - 2*b)*ArcTan[Tan[x]/Sqrt[2]])/(Sqrt[2]*c)
```

Maple [A] time = 0.032, size = 44, normalized size = 0.8

$$\frac{\sqrt{2}a}{2\,c}\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right) + \frac{\sqrt{2}b}{c}\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right) - \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+c*cos(x)^2),x)
```

```
[Out] 1/2/c*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))*a+1/c*2^(1/2)*arctan(1/2*tan(x)*2^
(1/2))*b-1/c*b*arctan(tan(x))
```

Maxima [A] time = 1.49859, size = 39, normalized size = 0.68

$$\frac{\sqrt{2}(a+2\,b)\arctan\left(\frac{1}{2}\,\sqrt{2}\tan\left(x\right)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(1/2*sqrt(2)*tan(x))/c - b*x/c
```

Fricas [A] time = 2.56949, size = 128, normalized size = 2.25

$$-\frac{\sqrt{2}(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*s
in(x))) + 4*b*x)/c
```

Sympy [B] time = 27.9418, size = 143, normalized size = 2.51

$$\frac{\sqrt{2}a\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}+\frac{\sqrt{2}a\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}-\frac{bx}{c}+\frac{\sqrt{2}b\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+c*cos(x)**2),x)
```

[Out] sqrt(2)*a*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/(2*c) + sqrt(2)*a*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(2*c) - b*x/c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/c

---

Giac [A] time = 1.09982, size = 84, normalized size = 1.47

$$\frac{\sqrt{2}(a+2b)\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)}{2c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="giac")

[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1)))/c - b*x/c

3.208 $\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{bx}{c}-\frac{a\cot(x)}{c}$$

```
[Out] (b*x)/c - (a*Cot[x])/c
```

Rubi [A] time = 0.0890223, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {453, 205}

$$\frac{bx}{c}-\frac{a\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

```
[Out] (b*x)/c - (a*Cot[x])/c
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{a+(a+b)x^2}{x^2\left(c+cx^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{a\cot(x)}{c} + b\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \tan(x)\right)$$
$$= \frac{bx}{c} - \frac{a\cot(x)}{c}$$

Mathematica [A] time = 0.0129729, size = 15, normalized size = 1.

$$\frac{bx}{c} - \frac{a\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

```
[Out] (b*x)/c - (a*Cot[x])/c
```

Maple [A] time = 0.031, size = 20, normalized size = 1.3

$$-\frac{a}{c\tan(x)} + \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c-c*cos(x)^2),x)
```

```
[Out] -1/c*a/tan(x)+1/c*b*arctan(tan(x))
```

Maxima [A] time = 1.56041, size = 23, normalized size = 1.53

$$\frac{bx}{c} - \frac{a}{c\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b*x/c - a/(c*tan(x))
```

Fricas [A] time = 2.34039, size = 49, normalized size = 3.27

$$\frac{bx\sin(x) - a\cos(x)}{c\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] (b*x*sin(x) - a*cos(x))/(c*sin(x))
```

Sympy [B] time = 1.87029, size = 24, normalized size = 1.6

$$\frac{a\tan\left(\frac{x}{2}\right)}{2c} - \frac{a}{2c\tan\left(\frac{x}{2}\right)} + \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c-c*cos(x)**2),x)
```

```
[Out] a*tan(x/2)/(2*c) - a/(2*c*tan(x/2)) + b*x/c
```

Giac [A] time = 1.16905, size = 39, normalized size = 2.6

$$\frac{bx}{c} + \frac{a\tan\left(\frac{1}{2}\,x\right)}{2\,c} - \frac{a}{2\,c\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="giac")
```

```
[Out] b*x/c + 1/2*a*tan(1/2*x)/c - 1/2*a/(c*tan(1/2*x))
```

3.209 $\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx$

Optimal. Leaf size=49

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}d\sqrt{c+d}}-\frac{bx}{d}$$

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rubi [A] time = 0.149867, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {522, 203, 205}

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}d\sqrt{c+d}}-\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
```

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = \operatorname{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)(c+d+cx^2)}\,dx, x, \tan(x)\right)$$
$$= -\frac{b\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{d} + \frac{(ad+b(c+d))\operatorname{Subst}\left(\int \frac{1}{c+d+cx^2}\,dx, x, \tan(x)\right)}{d}$$
$$= -\frac{bx}{d} + \frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}d\sqrt{c+d}}$$

Mathematica [A] time = 0.159684, size = 47, normalized size = 0.96

$$\frac{\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\sqrt{c+d}} - bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
```

```
[Out] (-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt[c]
*Sqrt[c + d]))/d
```

Maple [A] time = 0.038, size = 78, normalized size = 1.6

$$a\arctan\left(\tan(x)\,c\frac{1}{\sqrt{(c+d)\,c}}\right)\frac{1}{\sqrt{(c+d)\,c}} + \frac{cb}{d}\arctan\left(\tan(x)\,c\frac{1}{\sqrt{(c+d)\,c}}\right)\frac{1}{\sqrt{(c+d)\,c}} + b\arctan\left(\tan(x)\,c\frac{1}{\sqrt{(c+d)\,c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+d*cos(x)^2),x)
```

```
[Out] 1/((c+d)*c)^(1/2)*arctan(tan(x)*c/((c+d)*c)^(1/2))*a+1/d/((c+d)*c)^(1/2)*ar
ctan(tan(x)*c/((c+d)*c)^(1/2))*c*b+1/((c+d)*c)^(1/2)*arctan(tan(x)*c/((c+d)
```

```
*c)^(1/2))*b-b/d*arctan(tan(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87929, size = 547, normalized size = 11.16

$$\left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\log\left(\frac{\left(8c^2+8cd+d^2\right)\cos(x)^4-2\left(4c^2+3cd\right)\cos(x)^2+4\left((2c+d)\cos(x)^3-c\cos(x)\right)\sqrt{-c^2-cd}\sin(x)+c^2}{d^2\cos(x)^4+2cd\cos(x)^2+c^2}\right)+4\left(bc^2+bcd\right.}{4\left(c^2d+cd^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)
^4 - 2*(4*c^2 + 3*c*d)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - c*cos(x))*sqrt(-c
^2 - c*d)*sin(x) + c^2)/(d^2*cos(x)^4 + 2*c*d*cos(x)^2 + c^2)) + 4*(b*c^2 +
 b*c*d)*x)/(c^2*d + c*d^2), -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(
1/2*((2*c + d)*cos(x)^2 - c)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 +
b*c*d)*x)/(c^2*d + c*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)**2),x)
```

[Out] Timed out

Giac [A] time = 1.14524, size = 78, normalized size = 1.59

$$-\frac{bx}{d}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(c\right)+\arctan\left(\frac{c\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/d + (pi*floor(x/pi + 1/2)*sgn(c) + arctan(c*tan(x)/sqrt(c^2 + c*d)))*(
b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

3.210 $\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=13

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

```
[Out] (c*x)/d^2 + Cos[x]/d
```

Rubi [A] time = 0.11933, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4397, 3016, 2638}

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (c*x)/d^2 + Cos[x]/d
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx = \int \frac{\frac{c^2}{d^2}-\sin^2(x)}{c+d\sin(x)}\,dx$$
$$= -\frac{\int(-c+d\sin(x))\,dx}{d^2}$$
$$= \frac{cx}{d^2}-\frac{\int\sin(x)\,dx}{d}$$
$$= \frac{cx}{d^2}+\frac{\cos(x)}{d}$$

Mathematica [A] time = 0.0100794, size = 13, normalized size = 1.

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (c*x)/d^2 + Cos[x]/d
```

Maple [B] time = 0.044, size = 28, normalized size = 2.2

$$2\,\frac{1}{d\left(1+(\tan(x/2))^2\right)}+2\,\frac{c\arctan(\tan(x/2))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x)
```

```
[Out] 2/d/(1+tan(1/2*x)^2)+2/d^2*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.41321, size = 30, normalized size = 2.31

$$\frac{cx + d\cos(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

```
[Out] (c*x + d*cos(x))/d^2
```

Sympy [B] time = 152.837, size = 76, normalized size = 5.85

$$\frac{cx\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right) + d^2} + \frac{cx}{d^2\tan^2\left(\frac{x}{2}\right) + d^2} - \frac{d\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right) + d^2} + \frac{d}{d^2\tan^2\left(\frac{x}{2}\right) + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+cos(x)**2)/(c+d*sin(x)),x)
```

```
[Out] c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2) -
 d*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + d/(d**2*tan(x/2)**2 + d**2)
```

Giac [A] time = 1.1409, size = 30, normalized size = 2.31

$$\frac{cx}{d^2} + \frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

```
[Out] c*x/d^2 + 2/((tan(1/2*x)^2 + 1)*d)
```

3.211 $\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=100

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{bcx}{d^2}+\frac{b\cos(x)}{d}$$

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2
] - (2*b*Sqrt[c^2 - d^2]*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/d^2 + (b
*Cos[x])/d
```

Rubi [A] time = 0.240105, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4401, 2660, 618, 204, 2695, 2735}

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{bcx}{d^2}+\frac{b\cos(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2
] - (2*b*Sqrt[c^2 - d^2]*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/d^2 + (b
*Cos[x])/d
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx = \int \left(\frac{a}{c+d\sin(x)}+\frac{b\cos^2(x)}{c+d\sin(x)}\right)dx$$
$$= a\int \frac{1}{c+d\sin(x)}\,dx + b\int \frac{\cos^2(x)}{c+d\sin(x)}\,dx$$
$$= \frac{b\cos(x)}{d} + (2a)\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + \frac{b\int \frac{d+c\sin(x)}{c+d\sin(x)}\,dx}{d}$$
$$= \frac{bcx}{d^2} + \frac{b\cos(x)}{d} - (4a)\,\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d+2c\tan\left(\frac{x}{2}\right)\right) - \frac{\left(b\left(c^2-d^2\right)\right)\int \frac{1}{c+d\sin(}$$
$$= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{b\cos(x)}{d} - \frac{\left(2b\left(c^2-d^2\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{d^2}$$
$$= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{b\cos(x)}{d} + \frac{\left(4b\left(c^2-d^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d+2c\tan\left(\frac{x}{2}\right.\right.}{d^2}$$
$$= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{2b\sqrt{c^2-d^2}\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{d^2} + \frac{b\cos(x)}{d}$$

Mathematica [A] time = 0.189639, size = 72, normalized size = 0.72

$$\frac{\frac{2\left(ad^2+b\left(d^2-c^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b(cx+d\cos(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] ((2*(a*d^2 + b*(-c^2 + d^2))*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt
[c^2 - d^2] + b*(c*x + d*Cos[x]))/d^2
```

Maple [A] time = 0.033, size = 153, normalized size = 1.5

$$2\frac{a}{\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan(x/2)+2\,d}{\sqrt{c^2-d^2}}\right) - 2\,\frac{c^2b}{d^2\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan(x/2)+2\,d}{\sqrt{c^2-d^2}}\right) + 2\,\frac{b}{\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+d*sin(x)),x)
```

```
[Out] 2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*a-2/d^2/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*c^2*b+2/(c
^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*b+2*b/d/(1+t
an(1/2*x)^2)+2*b/d^2*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63432, size = 571, normalized size = 5.71

$$\left[\frac{\left(bc^2-(a+b)d^2\right)\sqrt{-c^2+d^2}\log\left(\frac{\left(2c^2-d^2\right)\cos(x)^2-2cd\sin(x)-c^2-d^2+2\left(c\cos(x)\sin(x)+d\cos(x)\right)\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2cd\sin(x)-c^2-d^2}\right)+2\left(bc^3-bcd^2\right)x+2\left(bc^2d\right.}{2\left(c^2d^2-d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 -
2*c*d*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))
/(d^2*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2)) + 2*(b*c^3 - b*c*d^2)*x + 2*(b*
c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4), ((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d
^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x))) + (b*c^3 - b*c*d^2)*x
+ (b*c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16553, size = 126, normalized size = 1.26

$$\frac{bcx}{d^2} - \frac{2\left(bc^2 - ad^2 - bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}\left(c\right) + \arctan\left(\frac{c\tan\left(\frac{1}{2}x\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2 - d^2}d^2} + \frac{2\,b}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

```
[Out] b*c*x/d^2 - 2*(b*c^2 - a*d^2 - b*d^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(c) + ar
ctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2) + 2*b/((tan
(1/2*x)^2 + 1)*d)
```

3.212 $\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx$

Optimal. Leaf size=56

$$\frac{x(a+2b)}{\sqrt{2}c}+\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c}-\frac{bx}{c}$$

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) + ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Sin[x]^2)])/(Sqrt[2]*c)
```

Rubi [A] time = 0.196645, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1166, 205}

$$\frac{x(a+2b)}{\sqrt{2}c}+\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c}-\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) + ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Sin[x]^2)])/(Sqrt[2]*c)
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{a+b+ax^2}{c+3cx^2+2cx^4}\,dx, x, \tan(x)\right)$$
$$= -\left((2b)\,\text{Subst}\left(\int \frac{1}{2c+2cx^2}\,dx, x, \tan(x)\right)\right) + (a+2b)\,\text{Subst}\left(\int \frac{1}{c+2cx^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{bx}{c} + \frac{(a+2b)x}{\sqrt{2}c} + \frac{(a+2b)\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}c}$$

Mathematica [A] time = 0.0830549, size = 31, normalized size = 0.55

$$\frac{(a+2b)\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}c} - \frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*ArcTan[Sqrt[2]*Tan[x]])/(Sqrt[2]*c)
```

Maple [A] time = 0.038, size = 42, normalized size = 0.8

$$\frac{\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)a}{2c} + \frac{\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)b}{c} - \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+c*sin(x)^2),x)
```

```
[Out] 1/2/c*2^(1/2)*arctan(tan(x)*2^(1/2))*a+1/c*2^(1/2)*arctan(tan(x)*2^(1/2))*b
-1/c*b*arctan(tan(x))
```

Maxima [A] time = 1.49939, size = 38, normalized size = 0.68

$$\frac{\sqrt{2}(a+2\,b)\arctan\left(\sqrt{2}\tan(x)\right)}{2\,c} - \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(sqrt(2)*tan(x))/c - b*x/c
```

Fricas [A] time = 2.5734, size = 131, normalized size = 2.34

$$-\frac{\sqrt{2}(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\cos(x)^2-2\,\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)
*sin(x))) + 4*b*x)/c
```

Sympy [B] time = 98.6209, size = 782, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+c*sin(x)**2),x)
```

```
[Out] 12*sqrt(2)*a*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi
))/(41*sqrt(2)*c*sqrt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt(2))) + 17*a*(at
an(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*c
*sqrt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt(2))) + 12*sqrt(2)*a*sqrt(3 - 2*
sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor
((x/2 - pi/2)/pi))/(41*sqrt(2)*c*sqrt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt
(2))) + 17*a*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*
sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*c*sqrt(3 - 2*sqrt(2)
) + 58*c*sqrt(3 - 2*sqrt(2))) - 58*b*x*sqrt(3 - 2*sqrt(2))/(41*sqrt(2)*c*sq
rt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt(2))) - 41*sqrt(2)*b*x*sqrt(3 - 2*s
qrt(2))/(41*sqrt(2)*c*sqrt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt(2))) + 24*
sqrt(2)*b*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/
```

```
(41*sqrt(2)*c*sqrt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt(2))) + 34*b*(atan(
tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*c*sq
rt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt(2))) + 24*sqrt(2)*b*sqrt(3 - 2*sqr
t(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x
/2 - pi/2)/pi))/(41*sqrt(2)*c*sqrt(3 - 2*sqrt(2)) + 58*c*sqrt(3 - 2*sqrt(2)
)) + 34*b*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqr
t(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*c*sqrt(3 - 2*sqrt(2)) +
 58*c*sqrt(3 - 2*sqrt(2)))
```

Giac [A] time = 1.15276, size = 84, normalized size = 1.5

$$\frac{\sqrt{2}(a+2\,b)\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2\,x)-2\,\sin(2\,x)}{\sqrt{2}\cos(2\,x)+\sqrt{2}-2\,\cos(2\,x)+2}\right)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)
*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2)))/c - b*x/c
```

3.213 $\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{a\tan(x)}{c}+\frac{bx}{c}$$

```
[Out] (b*x)/c + (a*Tan[x])/c
```

Rubi [A] time = 0.0576156, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3175, 3012, 8}

$$\frac{a\tan(x)}{c}+\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]
```

```
[Out] (b*x)/c + (a*Tan[x])/c
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx = \frac{\int \left(a+b\cos^2(x)\right)\sec^2(x)\,dx}{c}$$
$$= \frac{a\tan(x)}{c} + \frac{b\int 1\,dx}{c}$$
$$= \frac{bx}{c} + \frac{a\tan(x)}{c}$$

Mathematica [A] time = 0.0121171, size = 14, normalized size = 1.

$$\frac{a\tan(x)}{c} + \frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]
```

```
[Out] (b*x)/c + (a*Tan[x])/c
```

Maple [A] time = 0.029, size = 17, normalized size = 1.2

$$\frac{a\tan(x)}{c} + \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c-c*sin(x)^2),x)
```

```
[Out] a*tan(x)/c+1/c*b*arctan(tan(x))
```

Maxima [A] time = 1.52015, size = 19, normalized size = 1.36

$$\frac{bx}{c} + \frac{a\tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b*x/c + a*tan(x)/c
```

Fricas [A] time = 2.44106, size = 49, normalized size = 3.5

$$\frac{bx\cos(x) + a\sin(x)}{c\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] (b*x*cos(x) + a*sin(x))/(c*cos(x))
```

Sympy [B] time = 2.20449, size = 51, normalized size = 3.64

$$-\frac{2a\tan\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right) - c} + \frac{bx\tan^2\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right) - c} - \frac{bx}{c\tan^2\left(\frac{x}{2}\right) - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c-c*sin(x)**2),x)
```

```
[Out] -2*a*tan(x/2)/(c*tan(x/2)**2 - c) + b*x*tan(x/2)**2/(c*tan(x/2)**2 - c) - b
*x/(c*tan(x/2)**2 - c)
```

Giac [A] time = 1.13976, size = 31, normalized size = 2.21

$$\frac{b\arctan\left(\frac{|c|\tan(x)}{c}\right)}{|c|} + \frac{a\tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(abs(c)*tan(x)/c)/abs(c) + a*tan(x)/c
```

3.214 $\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx$

Optimal. Leaf size=49

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}d\sqrt{c+d}}-\frac{bx}{d}$$

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rubi [A] time = 0.160941, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {522, 203, 205}

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}d\sqrt{c+d}}-\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{a+b+ax^2}{\left(1+x^2\right)\left(c+(c+d)x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{d} + \frac{(-ac+(a+b)(c+d))\,\text{Subst}\left(\int \frac{1}{c+(c+d)x^2}\,dx, x, \tan(x)\right)}{d}$$
$$= -\frac{bx}{d} + \frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}d\sqrt{c+d}}$$

Mathematica [A] time = 0.148626, size = 47, normalized size = 0.96

$$\frac{\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{c+d}} - bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

```
[Out] (-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt[c]
*Sqrt[c + d]))/d
```

Maple [B] time = 0.039, size = 84, normalized size = 1.7

$$a\arctan\left((c+d)\tan(x)\frac{1}{\sqrt{(c+d)c}}\right)\frac{1}{\sqrt{(c+d)c}} + \frac{cb}{d}\arctan\left((c+d)\tan(x)\frac{1}{\sqrt{(c+d)c}}\right)\frac{1}{\sqrt{(c+d)c}} + b\arctan\left((c+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+d*sin(x)^2),x)
```

```
[Out] 1/((c+d)*c)^(1/2)*arctan((c+d)*tan(x)/((c+d)*c)^(1/2))*a+1/d/((c+d)*c)^(1/2
)*arctan((c+d)*tan(x)/((c+d)*c)^(1/2))*c*b+1/((c+d)*c)^(1/2)*arctan((c+d)*t
```

```
an(x)/((c+d)*c)^(1/2))*b-b/d*arctan(tan(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73243, size = 617, normalized size = 12.59

$$\left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\log\left(\frac{\left(8c^2+8cd+d^2\right)\cos(x)^4-2\left(4c^2+5cd+d^2\right)\cos(x)^2+4\left((2c+d)\cos(x)^3-(c+d)\cos(x)\right)\sqrt{-c^2-cd}\sin(x)+c^2+2cd+d^2}{d^2\cos(x)^4-2\left(cd+d^2\right)\cos(x)^2+c^2+2cd+d^2}\right)+}{4\left(c^2d+cd^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)
^4 - 2*(4*c^2 + 5*c*d + d^2)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - (c + d)*cos
(x))*sqrt(-c^2 - c*d)*sin(x) + c^2 + 2*c*d + d^2)/(d^2*cos(x)^4 - 2*(c*d +
d^2)*cos(x)^2 + c^2 + 2*c*d + d^2)) + 4*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^2),
 -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(1/2*((2*c + d)*cos(x)^2 - c
 - d)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^
2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17064, size = 95, normalized size = 1.94

$$-\frac{bx}{d}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,c+2\,d)+\arctan\left(\frac{c\tan(x)+d\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/d + (pi*floor(x/pi + 1/2)*sgn(2*c + 2*d) + arctan((c*tan(x) + d*tan(x)
)/sqrt(c^2 + c*d)))*(b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

3.215 $\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=74

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}$$

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c
- d]*Sqrt[c + d]) - (b*d*ArcTanh[Sin[x]])/c^2 + (b*Tan[x])/c
```

Rubi [A] time = 0.245693, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4234, 3056, 3001, 3770, 2659, 205}

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c
- d]*Sqrt[c + d]) - (b*d*ArcTanh[Sin[x]])/c^2 + (b*Tan[x])/c
```

Rule 4234

```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Cos[a + b*x]^2))/Cos[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx &= \int \frac{\left(b+a\cos^2(x)\right)\sec^2(x)}{c+d\cos(x)}\,dx\\
&= \frac{b\tan(x)}{c}+\frac{\int \frac{(-bd+ac\cos(x))\sec(x)}{c+d\cos(x)}\,dx}{c}\\
&= \frac{b\tan(x)}{c}-\frac{(bd)\int \sec(x)\,dx}{c^2}+\left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\cos(x)}\,dx\\
&= -\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}+\left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\
&= \frac{2\left(a+\frac{bd^2}{c^2}\right)\tan^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}
\end{aligned}$$

Mathematica [A] time = 0.442426, size = 98, normalized size = 1.32

$$\frac{-\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)}{\sqrt{d^2-c^2}}+bc\tan(x)+bd\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] ((-2*(a*c^2 + b*d^2)*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 + d^2]])/Sqrt[-c^
2 + d^2] + b*d*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + b*c*
Tan[x])/c^2
```

Maple [B] time = 0.036, size = 135, normalized size = 1.8

$$2\,\frac{a}{\sqrt{(c+d)(c-d)}}\arctan\left(\frac{(c-d)\tan(x/2)}{\sqrt{(c+d)(c-d)}}\right)+2\,\frac{bd^2}{c^2\sqrt{(c+d)(c-d)}}\arctan\left(\frac{(c-d)\tan(x/2)}{\sqrt{(c+d)(c-d)}}\right)-\frac{b}{c}\left(1+\tan\left(\frac{x}{2}\right)\right)^{-1}-\frac{db}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(x)^2)/(c+d*cos(x)),x)
```

```
[Out] 2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*a+2/c^2/
((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*b*d^2-b/c/
```

```
(1+tan(1/2*x))-d*b/c^2*ln(1+tan(1/2*x))-b/c/(tan(1/2*x)-1)+d*b/c^2*ln(tan(1
/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.93377, size = 768, normalized size = 10.38

$$\left[-\frac{\left(ac^2+bd^2\right)\sqrt{-c^2+d^2}\cos\left(x\right)\log\left(\frac{2\,cd\cos(x)+\left(2\,c^2-d^2\right)\cos(x)^2+2\,\sqrt{-c^2+d^2}(c\cos(x)+d)\sin(x)-c^2+2\,d^2}{d^2\cos(x)^2+2\,cd\cos(x)+c^2}\right)+\left(bc^2d-bd^3\right)\cos\left(x\right)\log\left(s\right.}{2\left(c^4-c^2d^2\right)\cos\left(x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*cos(x)*log((2*c*d*cos(x) + (2*c^2 -
 d^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d
^2*cos(x)^2 + 2*c*d*cos(x) + c^2)) + (b*c^2*d - b*d^3)*cos(x)*log(sin(x) +
1) - (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) - 2*(b*c^3 - b*c*d^2)*sin(x)
)/((c^4 - c^2*d^2)*cos(x)), 1/2*(2*(a*c^2 + b*d^2)*sqrt(c^2 - d^2)*arctan(-
(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x)))*cos(x) - (b*c^2*d - b*d^3)*cos(x)*
log(sin(x) + 1) + (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) + 2*(b*c^3 - b*
c*d^2)*sin(x))/((c^4 - c^2*d^2)*cos(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\sec^2\left(x\right)}{c+d\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)**2)/(c+d*cos(x)),x)
```

```
[Out] Integral((a + b*sec(x)**2)/(c + d*cos(x)), x)
```

Giac [A] time = 1.18005, size = 169, normalized size = 2.28

$$-\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{c^2}+\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{c^2}-\frac{2\,b\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2-1\right)c}-\frac{2\left(ac^2+bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\mathrm{a}\right.}{\sqrt{c^2-d^2}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] -b*d*log(abs(tan(1/2*x) + 1))/c^2 + b*d*log(abs(tan(1/2*x) - 1))/c^2 - 2*b*
tan(1/2*x)/((tan(1/2*x)^2 - 1)*c) - 2*(a*c^2 + b*d^2)*(pi*floor(1/2*x/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*tan(1/2*x))/sqrt(c^2 - d^2
)))/(sqrt(c^2 - d^2)*c^2)
```

3.216 $\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=72

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{c^2\sqrt{c^2-d^2}}+\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/(c^2*Sqrt[c^2
- d^2]) + (b*d*ArcTanh[Cos[x]])/c^2 - (b*Cot[x])/c
```

Rubi [A] time = 0.237838, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4233, 3056, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{c^2\sqrt{c^2-d^2}}+\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/(c^2*Sqrt[c^2
- d^2]) + (b*d*ArcTanh[Cos[x]])/c^2 - (b*Cot[x])/c
```

Rule 4233

```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Sin[a + b*x]^2))/Sin[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx=\int \frac{\csc^2(x)\left(b+a\sin^2(x)\right)}{c+d\sin(x)}\,dx$$

$$=-\frac{b\cot(x)}{c}+\frac{\int \frac{\csc(x)(-bd+ac\sin(x))}{c+d\sin(x)}\,dx}{c}$$

$$=-\frac{b\cot(x)}{c}-\frac{(bd)\int \csc(x)\,dx}{c^2}+\left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\sin(x)}\,dx$$

$$=\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}+\left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)$$

$$=\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}-\left(4\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\tan\left(\frac{x}{2}\right.\right.$$

$$=\frac{2\left(a+\frac{bd^2}{c^2}\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

Mathematica [A] time = 0.518584, size = 102, normalized size = 1.42

$$\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(\frac{2\sin(x)\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-b\left(c\cos(x)+d\sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)\right)\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (Csc[x/2]*Sec[x/2]*((2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d
^2]]*Sin[x])/Sqrt[c^2 - d^2] - b*(c*Cos[x] + d*(-Log[Cos[x/2]] + Log[Sin[x/
2]])*Sin[x])))/(2*c^2)
```

Maple [A] time = 0.038, size = 120, normalized size = 1.7

$$\frac{b}{2\,c}\tan\left(\frac{x}{2}\right)+2\,\frac{a}{\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan(x/2)+2\,d}{\sqrt{c^2-d^2}}\right)+2\,\frac{bd^2}{c^2\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan(x/2)+2\,d}{\sqrt{c^2-d^2}}\right)-\frac{b}{2\,c}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(x)^2)/(c+d*sin(x)),x)
```

```
[Out] 1/2*b/c*tan(1/2*x)+2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d
^2)^(1/2))*a+2/c^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2
)^(1/2))*b*d^2-1/2*b/c/tan(1/2*x)-1/c^2*d*b*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.01471, size = 817, normalized size = 11.35

$$\left[-\frac{\left(ac^2+bd^2\right)\sqrt{-c^2+d^2}\log\left(\frac{\left(2c^2-d^2\right)\cos(x)^2-2cd\sin(x)-c^2-d^2+2\left(c\cos(x)\sin(x)+d\cos(x)\right)\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2cd\sin(x)-c^2-d^2}\right)\sin(x)-\left(bc^2d-bd^3\right)\log\left(\frac{1}{2}\cos\right.}{2\left(c^4-c^2d^2\right)\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 - 2*c*d
*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))/(d^2
*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2))*sin(x) - (b*c^2*d - b*d^3)*log(1/2*c
os(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(
b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x)), -1/2*(2*(a*c^2 + b*d^2)*
sqrt(c^2 - d^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x)))*sin(x) - (
b*c^2*d - b*d^3)*log(1/2*cos(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*
cos(x) + 1/2)*sin(x) + 2*(b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\csc^2(x)}{c + d\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)**2)/(c+d*sin(x)),x)
```

```
[Out] Integral((a + b*csc(x)**2)/(c + d*sin(x)), x)
```

Giac [A] time = 1.16477, size = 149, normalized size = 2.07

$$-\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{c^2} + \frac{b\tan\left(\frac{1}{2}x\right)}{2c} + \frac{2\left(ac^2 + bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(c) + \arctan\left(\frac{c\tan\left(\frac{1}{2}x\right) + d}{\sqrt{c^2 - d^2}}\right)\right)}{\sqrt{c^2 - d^2}c^2} + \frac{2\,bd\tan\left(\frac{1}{2}x\right) - bc}{2\,c^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

```
[Out] -b*d*log(abs(tan(1/2*x)))/c^2 + 1/2*b*tan(1/2*x)/c + 2*(a*c^2 + b*d^2)*(pi*
floor(1/2*x/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/
(sqrt(c^2 - d^2)*c^2) + 1/2*(2*b*d*tan(1/2*x) - b*c)/(c^2*tan(1/2*x))
```

3.217 $\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=136

$$-\frac{\sin\left(-\tan^{-1}(a,b)+c+dx\right)(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\cos^{n+1}\left(-\tan^{-1}(a,b)+c+dx\right)\text{Hyp}}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a,b)+c+dx\right)}}$$

```
[Out] -((Cos[c + d*x - ArcTan[a, b]]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3
 + n)/2, Cos[c + d*x - ArcTan[a, b]]^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^n
*Sin[c + d*x - ArcTan[a, b]])/(d*(1 + n)*((a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2])^n*Sqrt[Sin[c + d*x - ArcTan[a, b]]^2]))
```

Rubi [A] time = 0.0581775, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3078, 2643}

$$-\frac{\sin\left(-\tan^{-1}(a,b)+c+dx\right)(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\cos^{n+1}\left(-\tan^{-1}(a,b)+c+dx\right)\,{}_2F_1}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a,b)+c+dx\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x - ArcTan[a, b]]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3
 + n)/2, Cos[c + d*x - ArcTan[a, b]]^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^n
*Sin[c + d*x - ArcTan[a, b]])/(d*(1 + n)*((a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2])^n*Sqrt[Sin[c + d*x - ArcTan[a, b]]^2]))
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx = \left((a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\right)\int \cos^n\left(c+\right.$$

$$= -\frac{\cos^{1+n}\left(c+dx-\tan^{-1}(a,b)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\tan^{-1}(a,b)\right)\right)(a}{d(1+n)\sqrt{\sin^2\left(c\right.}}$$

Mathematica [A] time = 0.228257, size = 94, normalized size = 0.69

$$-\frac{\sin\left(2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\right)\sin^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)^{-\frac{n}{2}-\frac{1}{2}}(a\cos(c+dx)+b\sin(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[a/b]]^2]*(a*C
os[c + d*x] + b*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[a/b]]^2)^(-1/2 - n/2)
*Sin[2*(c + d*x + ArcTan[a/b])])/(2*d)
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)
```

```
[Out] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\cos(dx+c)+b\sin(dx+c))^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

3.218 $\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx$

Optimal. Leaf size=95

$$-\frac{13^{n/2}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\cos^{n+1}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(n+1)\sqrt{\sin^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

```
[Out] -((13^(n/2)*Cos[c + d*x - ArcTan[3/2]]^(1 + n)*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/
(d*(1 + n)*Sqrt[Sin[c + d*x - ArcTan[3/2]]^2]))
```

Rubi [A] time = 0.0489555, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3077, 2643}

$$-\frac{13^{n/2}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\cos^{n+1}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(n+1)\sqrt{\sin^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
```

```
[Out] -((13^(n/2)*Cos[c + d*x - ArcTan[3/2]]^(1 + n)*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/
(d*(1 + n)*Sqrt[Sin[c + d*x - ArcTan[3/2]]^2]))
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = 13^{n/2}\int \cos^n\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)dx$$
$$= -\frac{13^{n/2}\cos^{1+n}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(1+n)\sqrt{\sin^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

Mathematica [A] time = 0.176881, size = 88, normalized size = 0.93

$$-\frac{\sin\left(2\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\right)\sin^2\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)^{-\frac{n}{2}-\frac{1}{2}}(3\sin(c+dx)+2\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[2/3]]^2]*(2*C
os[c + d*x] + 3*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[2/3]]^2)^(-1/2 - n/2)
*Sin[2*(c + d*x + ArcTan[2/3])])/(2*d)
```

Maple [F] time = 0.416, size = 0, normalized size = 0.

$$\int (2\cos(dx+c)+3\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

```
[Out] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((2 \cos(dx + c) + 3 \sin(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

3.219 $\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx$

Optimal. Leaf size=127

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^5}{5d}+\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))^3}{d}-\frac{\left(a^2+b^2\right)^3(b\cos(c+dx)-a}{d}$$

```
[Out] -(((a^2 + b^2)^3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + ((a^2 + b^2)^2*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/d - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Si
n[c + d*x])^5)/(5*d) + (b*Cos[c + d*x] - a*Sin[c + d*x])^7/(7*d)
```

Rubi [A] time = 0.0778204, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3072, 194}

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^5}{5d}+\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))^3}{d}-\frac{\left(a^2+b^2\right)^3(b\cos(c+dx)-a}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]
```

```
[Out] -(((a^2 + b^2)^3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + ((a^2 + b^2)^2*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/d - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Si
n[c + d*x])^5)/(5*d) + (b*Cos[c + d*x] - a*Sin[c + d*x])^7/(7*d)
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx = -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)^3\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{3a^4b^2+3a^2b^4+b^6}{a^6}\right)-3a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)x^2+3a^2\left(1+\frac{b^2}{a^2}\right)x^4-x^6\right.}{d}$$

$$= -\frac{\left(a^2+b^2\right)^3(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c}{d}$$

Mathematica [A] time = 1.0378, size = 246, normalized size = 1.94

$$\frac{1225a\left(a^2+b^2\right)^3\sin(c+dx)+245a\left(a^2-3b^2\right)\left(a^2+b^2\right)^2\sin(3(c+dx))+49a\left(-9a^4b^2-5a^2b^4+a^6+5b^6\right)\sin(5(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]
```

```
[Out] (-1225*b*(a^2 + b^2)^3*Cos[c + d*x] + 245*b*(-3*a^2 + b^2)*(a^2 + b^2)^2*Co
s[3*(c + d*x)] - 49*b*(5*a^6 - 5*a^4*b^2 - 9*a^2*b^4 + b^6)*Cos[5*(c + d*x)
] + 5*b*(-7*a^6 + 35*a^4*b^2 - 21*a^2*b^4 + b^6)*Cos[7*(c + d*x)] + 1225*a*
(a^2 + b^2)^3*Sin[c + d*x] + 245*a*(a^2 - 3*b^2)*(a^2 + b^2)^2*Sin[3*(c + d
*x)] + 49*a*(a^6 - 9*a^4*b^2 - 5*a^2*b^4 + 5*b^6)*Sin[5*(c + d*x)] + 5*a*(a
^6 - 21*a^4*b^2 + 35*a^2*b^4 - 7*b^6)*Sin[7*(c + d*x)])/(2240*d)
```

Maple [B] time = 0.169, size = 321, normalized size = 2.5

$$\frac{1}{d}\left(-\frac{b^7\cos(dx+c)}{7}\left(\frac{16}{5}+(\sin(dx+c))^6+\frac{6(\sin(dx+c))^4}{5}+\frac{8(\sin(dx+c))^2}{5}\right)+ab^6(\sin(dx+c))^7+21\,a^2b^5\left(-1/7\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^7,x)
```

```
[Out] 1/d*(-1/7*b^7*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+a*b^6*sin(d*x+c)^7+21*a^2*b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(
d*x+c)^2*cos(d*x+c)^3-8/105*cos(d*x+c)^3)+35*a^3*b^4*(-1/7*sin(d*x+c)^3*cos
(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))+35
```

```
*a^4*b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+21*a^5*b^2*(-1/
7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))-a^6*b*cos(d*x+c)^7+1/7*a^7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(
d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.00866, size = 347, normalized size = 2.73

$$-\frac{35\,a^6b\cos\left(dx+c\right)^7-35\,ab^6\sin\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^7-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="maxima")
```

```
[Out] -1/35*(35*a^6*b*cos(d*x + c)^7 - 35*a*b^6*sin(d*x + c)^7 + (5*sin(d*x + c)^
7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^7 - 7*(15*si
n(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5*b^2 - 35*(5*cos(d
*x + c)^7 - 7*cos(d*x + c)^5)*a^4*b^3 + 35*(5*sin(d*x + c)^7 - 7*sin(d*x +
c)^5)*a^3*b^4 + 7*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^
3)*a^2*b^5 - (5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35
*cos(d*x + c))*b^7)/d
```

Fricas [B] time = 3.08474, size = 583, normalized size = 4.59

$$-\frac{35\,b^7\cos\left(dx+c\right)+5\left(7\,a^6b-35\,a^4b^3+21\,a^2b^5-b^7\right)\cos\left(dx+c\right)^7+7\left(35\,a^4b^3-42\,a^2b^5+3\,b^7\right)\cos\left(dx+c\right)^5+35\left(7\,\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^7*cos(d*x + c) + 5*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*co
s(d*x + c)^7 + 7*(35*a^4*b^3 - 42*a^2*b^5 + 3*b^7)*cos(d*x + c)^5 + 35*(7*a
^2*b^5 - b^7)*cos(d*x + c)^3 - (16*a^7 + 56*a^5*b^2 + 70*a^3*b^4 + 35*a*b^6
 + 5*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^6 + (6*a^7 + 21
*a^5*b^2 - 280*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^4 + (8*a^7 + 28*a^5*b^2 +
35*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 10.1715, size = 461, normalized size = 3.63

$$\begin{cases} \frac{16a^7\sin^7(c+dx)}{35d} + \frac{8a^7\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^7\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^7\sin(c+dx)\cos^6(c+dx)}{d} - \frac{a^6b\cos^7(c+dx)}{d} + \frac{8a^5b^2\sin^7(c+dx)}{5d} + 2 \\ x\left(a\cos(c) + b\sin(c)\right)^7 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**7,x)
```

```
[Out] Piecewise((16*a**7*sin(c + d*x)**7/(35*d) + 8*a**7*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**7*sin(c + d*x)**3*cos(c + d*x)**4/d + a**7*sin(c + d*x
)*cos(c + d*x)**6/d - a**6*b*cos(c + d*x)**7/d + 8*a**5*b**2*sin(c + d*x)**
7/(5*d) + 28*a**5*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 7*a**5*b**2*
sin(c + d*x)**3*cos(c + d*x)**4/d - 7*a**4*b**3*sin(c + d*x)**2*cos(c + d*x
)**5/d - 2*a**4*b**3*cos(c + d*x)**7/d + 2*a**3*b**4*sin(c + d*x)**7/d + 7*
a**3*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - 7*a**2*b**5*sin(c + d*x)**4*c
os(c + d*x)**3/d - 28*a**2*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 8*a
**2*b**5*cos(c + d*x)**7/(5*d) + a*b**6*sin(c + d*x)**7/d - b**7*sin(c + d*
x)**6*cos(c + d*x)/d - 2*b**7*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**7*si
n(c + d*x)**2*cos(c + d*x)**5/(5*d) - 16*b**7*cos(c + d*x)**7/(35*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**7, True))
```

Giac [B] time = 1.13206, size = 427, normalized size = 3.36

$$-\frac{\left(7\,a^6b - 35\,a^4b^3 + 21\,a^2b^5 - b^7\right)\cos\left(7\,dx + 7\,c\right)}{448\,d} - \frac{7\left(5\,a^6b - 5\,a^4b^3 - 9\,a^2b^5 + b^7\right)\cos\left(5\,dx + 5\,c\right)}{320\,d} - \frac{7\left(3\,a^6b + 5\,a^4b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="giac")
```

```
[Out] -1/448*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*cos(7*d*x + 7*c)/d - 7/320
*(5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*cos(5*d*x + 5*c)/d - 7/64*(3*a^6*b
 + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(3*d*x + 3*c)/d - 35/64*(a^6*b + 3*a^4*b^3
 + 3*a^2*b^5 + b^7)*cos(d*x + c)/d + 1/448*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 -
 7*a*b^6)*sin(7*d*x + 7*c)/d + 7/320*(a^7 - 9*a^5*b^2 - 5*a^3*b^4 + 5*a*b^6
)*sin(5*d*x + 5*c)/d + 7/64*(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*sin(3*d*x
 + 3*c)/d + 35/64*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(d*x + c)/d
```

3.220 $\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$

Optimal. Leaf size=161

$$-\frac{5\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{24d}-\frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))(a}{16d}$$

```
[Out] (5*(a^2 + b^2)^3*x)/16 - (5*(a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(16*d) - (5*(a^2 + b^2)*(b*Cos[c + d*x]
 - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(24*d) - ((b*Cos[c
+ d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)/(6*d)
```

Rubi [A] time = 0.0789943, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$-\frac{5\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{24d}-\frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))(a}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

```
[Out] (5*(a^2 + b^2)^3*x)/16 - (5*(a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(16*d) - (5*(a^2 + b^2)*(b*Cos[c + d*x]
 - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(24*d) - ((b*Cos[c
+ d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)/(6*d)
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx = -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}+\frac{1}{6}\Big(5\left(a^2+b^2\right.$$
$$= -\frac{5\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{24d}-\frac{(b}{}$$
$$= -\frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{16d}-\frac{5(}{}$$
$$= \frac{5}{16}\left(a^2+b^2\right)^3x-\frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\,\mathrm{s}}{16d}$$

Mathematica [A] time = 0.707346, size = 192, normalized size = 1.19

$$\frac{60\left(a^2+b^2\right)^3(c+dx)+45\left(a^2-b^2\right)\left(a^2+b^2\right)^2\sin(2(c+dx))+9\left(-5a^4b^2-5a^2b^4+a^6+b^6\right)\sin(4(c+dx))+\left(-15a^4b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

```
[Out] (60*(a^2 + b^2)^3*(c + d*x) - 90*a*b*(a^2 + b^2)^2*Cos[2*(c + d*x)] - 36*a*
b*(a^4 - b^4)*Cos[4*(c + d*x)] - 2*a*b*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Cos[6*(
c + d*x)] + 45*(a^2 - b^2)*(a^2 + b^2)^2*Sin[2*(c + d*x)] + 9*(a^6 - 5*a^4*
b^2 - 5*a^2*b^4 + b^6)*Sin[4*(c + d*x)] + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 -
b^6)*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.141, size = 285, normalized size = 1.8

$$\frac{1}{d}\left(b^6\left(-\frac{\cos(dx+c)}{6}\left((\sin(dx+c))^5+\frac{5(\sin(dx+c))^3}{4}+\frac{15\sin(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+ab^5(\sin(dx+c))^6+15a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^6,x)
```

```
[Out] 1/d*(b^6*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5
/16*d*x+5/16*c)+a*b^5*sin(d*x+c)^6+15*a^2*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)
^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c)+
20*a^3*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+15*a^4*b^2*(-
```

```
1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1
/16*d*x+1/16*c)-a^5*b*cos(d*x+c)^6+a^6*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.02943, size = 321, normalized size = 1.99

$$-\frac{192\,a^5b\cos\left(dx+c\right)^6-192\,ab^5\sin\left(dx+c\right)^6+\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] -1/192*(192*a^5*b*cos(d*x + c)^6 - 192*a*b^5*sin(d*x + c)^6 + (4*sin(2*d*x
+ 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^6 -
15*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4*b^2 + 32
0*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3*b^3 + 15*(4*sin(2*d*x + 2*c)^3
- 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2*b^4 - (4*sin(2*d*x + 2*c)^3 + 60*
d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^6)/d
```

Fricas [A] time = 2.91479, size = 501, normalized size = 3.11

$$-\frac{144\,ab^5\cos\left(dx+c\right)^2+16\left(3\,a^5b-10\,a^3b^3+3\,ab^5\right)\cos\left(dx+c\right)^6+48\left(5\,a^3b^3-3\,ab^5\right)\cos\left(dx+c\right)^4-15\left(a^6+3\,a^4b^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] -1/48*(144*a*b^5*cos(d*x + c)^2 + 16*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*cos(d
*x + c)^6 + 48*(5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 - 15*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*d*x - (8*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*cos(d*x +
c)^5 + 2*(5*a^6 + 15*a^4*b^2 - 105*a^2*b^4 + 13*b^6)*cos(d*x + c)^3 + 3*(5*
a^6 + 15*a^4*b^2 + 15*a^2*b^4 - 11*b^6)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 6.39096, size = 821, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

```
[Out] Piecewise((5*a**6*x*sin(c + d*x)**6/16 + 15*a**6*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**6*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**6*x*cos(c
+ d*x)**6/16 + 5*a**6*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**6*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**6*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - a**5*b*cos(c + d*x)**6/d + 15*a**4*b**2*x*sin(c + d*x)**6/16 + 45*a**4*b
**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 45*a**4*b**2*x*sin(c + d*x)**2*c
os(c + d*x)**4/16 + 15*a**4*b**2*x*cos(c + d*x)**6/16 + 15*a**4*b**2*sin(c
+ d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*b**2*sin(c + d*x)**3*cos(c + d*x)**3
/(2*d) - 15*a**4*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 5*a**3*b**3*sin
(c + d*x)**2*cos(c + d*x)**4/d - 5*a**3*b**3*cos(c + d*x)**6/(3*d) + 15*a**
2*b**4*x*sin(c + d*x)**6/16 + 45*a**2*b**4*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 45*a**2*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 15*a**2*b**4*x*c
os(c + d*x)**6/16 + 15*a**2*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*a*
*2*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 15*a**2*b**4*sin(c + d*x)*c
os(c + d*x)**5/(16*d) - 3*a*b**5*sin(c + d*x)**4*cos(c + d*x)**2/d - 3*a*b*
*5*sin(c + d*x)**2*cos(c + d*x)**4/d - a*b**5*cos(c + d*x)**6/d + 5*b**6*x*
sin(c + d*x)**6/16 + 15*b**6*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**6
*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**6*x*cos(c + d*x)**6/16 - 11*b*
*6*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*b**6*sin(c + d*x)**3*cos(c + d*x
)**3/(6*d) - 5*b**6*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*c
os(c) + b*sin(c))**6, True))
```

Giac [A] time = 1.1269, size = 317, normalized size = 1.97

$$\frac{5}{16}\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)x-\frac{\left(3\,a^5b-10\,a^3b^3+3\,ab^5\right)\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{3\left(a^5b-ab^5\right)\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{15\left(a^5b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

```
[Out] 5/16*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 1/96*(3*a^5*b - 10*a^3*b^3 + 3
*a*b^5)*cos(6*d*x + 6*c)/d - 3/16*(a^5*b - a*b^5)*cos(4*d*x + 4*c)/d - 15/3
2*(a^5*b + 2*a^3*b^3 + a*b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^6 - 15*a^4*b^2
+ 15*a^2*b^4 - b^6)*sin(6*d*x + 6*c)/d + 3/64*(a^6 - 5*a^4*b^2 - 5*a^2*b^4
+ b^6)*sin(4*d*x + 4*c)/d + 15/64*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*sin(2*d*x
 + 2*c)/d
```

3.221 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=94

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rubi [A] time = 0.0452859, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3072, 194}

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)^2\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)-2a^2\left(1+\frac{b^2}{a^2}\right)x^2+x^4\right)\,dx, x, b\cos(c+dx)-a\sin\right.}{d}$$

$$= -\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c}{3d}$$

Mathematica [A] time = 0.466708, size = 156, normalized size = 1.66

$$\frac{150a\left(a^2+b^2\right)^2\sin(c+dx)+25a\left(-2a^2b^2+a^4-3b^4\right)\sin(3(c+dx))+3a\left(-10a^2b^2+a^4+5b^4\right)\sin(5(c+dx))-150b\,(}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3*
(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2 +
 b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*
a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

Maple [A] time = 0.105, size = 175, normalized size = 1.9

$$\frac{1}{d}\left(-\frac{b^5\cos(dx+c)}{5}\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4\,(\sin(dx+c))^2}{3}\right)+ab^4(\sin(dx+c))^5+10\,a^2b^3\left(-1/5\,(\sin(dx+c))^2(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-1/5*b^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+
c)^5+10*a^2*b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+10*a^3*b
^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-a^4*b*co
s(d*x+c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.997787, size = 232, normalized size = 2.47

$$-\frac{a^4 b \cos(dx+c)^5}{d} + \frac{ab^4 \sin(dx+c)^5}{d} + \frac{\left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c)\right)a^5}{15\,d} - \frac{2\left(3 \sin(dx+c)^5 - 5\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*sin
(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b^3/
d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

Fricas [A] time = 2.49648, size = 354, normalized size = 3.77

$$-\frac{15\,b^5 \cos(dx+c) + 3\left(5\,a^4 b - 10\,a^2 b^3 + b^5\right)\cos(dx+c)^5 + 10\left(5\,a^2 b^3 - b^5\right)\cos(dx+c)^3 - \left(8\,a^5 + 20\,a^3 b^2 + 15\,ab^4 + 3\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
+ 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 3*
(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*a*b
^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.97084, size = 267, normalized size = 2.84

$$\begin{cases} \frac{8a^5 \sin^5(c+dx)}{15d} + \frac{4a^5 \sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^5 \sin(c+dx)\cos^4(c+dx)}{d} - \frac{a^4 b \cos^5(c+dx)}{d} + \frac{4a^3b^2 \sin^5(c+dx)}{3d} + \frac{10a^3b^2 \sin^3(c+dx)\cos^2(c+dx)}{3d} - 1 \\ x\left(a\cos(c) + b\sin(c)\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)**5
```

```
/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 4*a
**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c + d*
x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**
5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, True))
```

Giac [B] time = 1.12192, size = 252, normalized size = 2.68

$$-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\left(3\,a^4b+2\,a^2b^3-b^5\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{5\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)}{8\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*a
^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x +
c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5 - 2
*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*sin(
d*x + c)/d
```

3.222 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=108

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(c+dx)}{}$$

```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```

Rubi [A] time = 0.0444016, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```


Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}+\frac{1}{4}\left(3\left(a^2+b^2\right.\right.$$
$$= -\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}-\frac{(b\,c}{}$$
$$= \frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin}{8d}$$

Mathematica [A] time = 0.317601, size = 107, normalized size = 0.99

$$\frac{12\left(a^2+b^2\right)^2(c+dx)+8\left(a^4-b^4\right)\sin(2(c+dx))+\left(-6a^2b^2+a^4+b^4\right)\sin(4(c+dx))-16ab\left(a^2+b^2\right)\cos(2(c+dx))-}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*(
a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a^2
*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.085, size = 153, normalized size = 1.4

$$\frac{1}{d}\left(b^4\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+ab^3(\sin(dx+c))^4+6a^2b^2\left(-1/4\sin(dx+c)(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*b^
3*sin(d*x+c)^4+6*a^2*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d
*x+c)+1/8*d*x+1/8*c)-cos(d*x+c)^4*a^3*b+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+
c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.01215, size = 184, normalized size = 1.7

$$-\frac{a^3b\cos(dx+c)^4}{d}+\frac{ab^3\sin(dx+c)^4}{d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d}+\frac{3\,(4\,dx+4\,c-\sin(4\,}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*x
+ 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x +
2*c))*b^4/d
```

Fricas [A] time = 2.49203, size = 273, normalized size = 2.53

$$-\frac{16\,ab^3\cos\left(dx+c\right)^2 + 8\left(a^3b-ab^3\right)\cos\left(dx+c\right)^4 - 3\left(a^4+2\,a^2b^2+b^4\right)dx - \left(2\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3 + \left(3\,a^4+\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4 +
 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^4
+ 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.74196, size = 406, normalized size = 3.76

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^4x\cos^4(c+dx)}{8} + \frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a^3b\cos^4(c+dx)}{d} + \frac{3a}{} \\ x\left(a\cos(c)+b\sin(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)**4/
d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*a*b
**3*sin(c + d*x)**2*cos(c + d*x)**2/d - a*b**3*cos(c + d*x)**4/d + 3*b**4*x
```

```
*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**4*x*
cos(c + d*x)**4/8 - 5*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*b**4*sin(
c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**4, Tru
e))
```

Giac [A] time = 1.13133, size = 165, normalized size = 1.53

$$\frac{3}{8}\left(a^4+2\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(4\,dx+4\,c\right)}{8\,d}-\frac{\left(a^3b+ab^3\right)\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1/
2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4*d
*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

3.223 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=58

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rubi [A] time = 0.0236529, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3072}

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx&=-\frac{\operatorname{Subst}\left(\int\left(a^2+b^2-x^2\right)dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\&=-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}\end{aligned}$$

Mathematica [A] time = 0.336525, size = 81, normalized size = 1.4

$$\frac{-9b\left(a^2+b^2\right)\cos(c+dx)+\left(b^3-3a^2b\right)\cos(3(c+dx))+2a\sin(c+dx)\left(\left(a^2-3b^2\right)\cos(2(c+dx))+5a^2+3b^2\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.055, size = 75, normalized size = 1.3

$$\frac{1}{d}\left(-\frac{b^3\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}+ab^2(\sin(dx+c))^3-a^2b(\cos(dx+c))^3+\frac{a^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-a^2*b*cos(d*x+
c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.988474, size = 113, normalized size = 1.95

$$-\frac{a^2b\cos(dx+c)^3}{d}+\frac{ab^2\sin(dx+c)^3}{d}-\frac{\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3}{3\,d}+\frac{\left(\cos(dx+c)^3-3\cos(dx+c)\right)b^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3*
sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d
```

Fricas [A] time = 2.63253, size = 173, normalized size = 2.98

$$-\frac{3\,b^3\cos\left(dx+c\right)+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^3-\left(2\,a^3+3\,ab^2+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^
2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 0.722125, size = 117, normalized size = 2.02

$$\begin{cases} \frac{2a^3\sin^3\left(c+dx\right)}{3d}+\frac{a^3\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}-\frac{a^2b\cos^3\left(c+dx\right)}{d}+\frac{ab^2\sin^3\left(c+dx\right)}{d}-\frac{b^3\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{2b^3\cos^3\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^3 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d*x
)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**3, True))
```

Giac [A] time = 1.13192, size = 123, normalized size = 2.12

$$-\frac{\left(3\,a^2b-b^3\right)\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\left(a^2b+b^3\right)\cos\left(dx+c\right)}{4\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{3\left(a^3+ab^2\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/d
 + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x + c)
/d
```

3.224 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rubi [A] time = 0.0192334, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}+\frac{1}{2}\left(a^2+b^2\right)\int\\ &= \frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\end{aligned}$$

Mathematica [A] time = 0.102152, size = 52, normalized size = 0.95

$$\frac{2\left(a^2+b^2\right)(c+dx)+\left(a^2-b^2\right)\sin(2(c+dx))-2ab\cos(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
d*x)])/(4*d)
```

Maple [A] time = 0.041, size = 70, normalized size = 1.3

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-(\cos(dx+c))^2ab+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-cos(d*x+c)^2*a*b+a^2*(1
/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.970316, size = 92, normalized size = 1.67

$$-\frac{ab\cos(dx+c)^2}{d}+\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2}{4\,d}+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(2
*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

Fricas [A] time = 2.59547, size = 120, normalized size = 2.18

$$-\frac{2\,ab\cos\left(dx+c\right)^{2}-\left(a^{2}+b^{2}\right)dx-\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

Sympy [A] time = 0.339938, size = 128, normalized size = 2.33

$$\begin{cases}\frac{a^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{a^{2}x\cos^{2}\left(c+dx\right)}{2}+\frac{a^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{ab\sin^{2}\left(c+dx\right)}{d}+\frac{b^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{b^{2}x\cos^{2}\left(c+dx\right)}{2}-\frac{b^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} & \text{fo} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^{2} & \text{ot}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) + a*b*sin(c + d*x)**2/d + b**2*x*sin(c + d*x)**2
/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

Giac [A] time = 1.10991, size = 68, normalized size = 1.24

$$\frac{1}{2}\left(a^{2}+b^{2}\right)x-\frac{ab\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
+ 2*c)/d
```

3.225 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0142294, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2637, 2638}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\,dx+b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0121349, size = 46, normalized size = 1.92

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.008, size = 25, normalized size = 1.

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Maxima [A] time = 0.98092, size = 32, normalized size = 1.33

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Fricas [A] time = 1.92775, size = 51, normalized size = 2.12

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Sympy [A] time = 0.179612, size = 31, normalized size = 1.29

$$a\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d \neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for } d \neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-c
os(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

Giac [A] time = 1.13581, size = 32, normalized size = 1.33

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.226 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rubi [A] time = 0.0272597, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Mathematica [A] time = 0.0596009, size = 45, normalized size = 0.96

$$\frac{2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Maple [A] time = 0.075, size = 43, normalized size = 0.9

$$2\,\frac{1}{d\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,\tan(1/2\,dx+c/2)\,a-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11999, size = 312, normalized size = 6.64

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.2816, size = 100, normalized size = 2.13

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.227 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0164173, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3075}

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Mathematica [A] time = 0.0400738, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A] time = 0.128, size = 21, normalized size = 0.7

$$-\frac{1}{db\left(a+b\tan\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/b/(a+b*tan(d*x+c))
```

Maxima [A] time = 0.981473, size = 28, normalized size = 0.88

$$-\frac{1}{\left(b^{2}\tan\left(dx+c\right)+ab\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b^2*tan(d*x + c) + a*b)*d)
```

Fricas [A] time = 2.15933, size = 132, normalized size = 4.12

$$-\frac{b\cos\left(dx+c\right)-a\sin\left(dx+c\right)}{\left(a^{3}+ab^{2}\right)d\cos\left(dx+c\right)+\left(a^{2}b+b^{3}\right)d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-2), x)
```

Giac [A] time = 1.11441, size = 27, normalized size = 0.84

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

3.228 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^
(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0567913, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3076, 3074, 206}

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] -ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^
(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(\right.}{2\left(a^2+b^2\right.}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Mathematica [C] time = 0.268526, size = 132, normalized size = 1.28

$$\frac{\left(a^2+b^2\right)(a\sin(c+dx)-b\cos(c+dx))+2\sqrt{a^2+b^2}(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{2d(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.157, size = 191, normalized size = 1.9

$$\frac{1}{d}\left(-2\frac{1}{\left(a(\tan(1/2\,dx+c/2))^2-2\,b\tan(1/2\,dx+c/2)-a\right)^2}\left(-1/2\,\frac{\left(a^2+2\,b^2\right)(\tan(1/2\,dx+c/2))^3}{\left(a^2+b^2\right)a}-1/2\,\frac{b\left(a^2-2\,b^2\right)(\text{ta}}{\left(a^2+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/a/(a^2+b^2)*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(1/
2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.2617, size = 679, normalized size = 6.59

$$\frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\cdots}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos\cdots}\right.}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\cos\left(dx+c\right.\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25005, size = 298, normalized size = 2.89

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^2\,\mathrm{t}\right.}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.229 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x]))
```

Rubi [A] time = 0.041788, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3076, 3075}

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x]))
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{3\left(a^2+b^2\right)}$$
$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\sin(c+dx)}{3a\left(a^2+b^2\right)d(a\cos(c+dx)+}$$

Mathematica [A] time = 0.290424, size = 85, normalized size = 0.87

$$\frac{\sin(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+2a^2+b^2\right)-ab\cos(3(c+dx))}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Sin
[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A] time = 0.181, size = 64, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{1}{b^3\,(a+b\tan(dx+c))}-\frac{a^2+b^2}{3\,b^3\,(a+b\tan(dx+c))^3}+\frac{a}{b^3\,(a+b\tan(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-1/b^3/(a+b*tan(d*x+c))-1/3*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^3+a/b^3/(a+
b*tan(d*x+c))^2)
```

Maxima [A] time = 1.01743, size = 115, normalized size = 1.17

$$-\frac{3\,b^2\tan(dx+c)^2+3\,ab\tan(dx+c)+a^2+b^2}{3\left(b^6\tan(dx+c)^3+3\,ab^5\tan(dx+c)^2+3\,a^2b^4\tan(dx+c)+a^3b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
+ c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

Fricas [B] time = 2.21456, size = 470, normalized size = 4.8

$$-\frac{2\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{3}-3\left(a^{2}b-b^{3}\right)\cos\left(dx+c\right)-\left(a^{3}+3\,ab^{2}+2\left(a^{3}-3\,ab^{2}\right)\cos\left(d\right.\right.}{3\left(\left(a^{7}-a^{5}b^{2}-5\,a^{3}b^{4}-3\,ab^{6}\right)d\cos\left(dx+c\right)^{3}+3\left(a^{5}b^{2}+2\,a^{3}b^{4}+ab^{6}\right)d\cos\left(dx+c\right)+\left(\left(3\,a^{6}b+5\,a^{4}b^{3}+a^{2}b^{5}-b^{7}\right)d\,\mathrm{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a^
3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5*b
^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c)^2
+ (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17313, size = 68, normalized size = 0.69

$$-\frac{3\,b^{2}\tan\left(dx+c\right)^{2}+3\,ab\tan\left(dx+c\right)+a^{2}+b^{2}}{3\left(b\tan\left(dx+c\right)+a\right)^{3}b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
c) + a)^3*b^3*d)
```

3.230 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx$

Optimal. Leaf size=156

$$-\frac{3(b\cos(c+dx)-a\sin(c+dx))}{8d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin}{\sqrt{a^2+b^2}}\right.}{8d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] (-3*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(8*(a^2 + b
^2)^(5/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(4*(a^2 + b^2)*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) - (3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(8*(a^2
 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0860091, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3076, 3074, 206}

$$-\frac{3(b\cos(c+dx)-a\sin(c+dx))}{8d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin}{\sqrt{a^2+b^2}}\right.}{8d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

```
[Out] (-3*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(8*(a^2 + b
^2)^(5/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(4*(a^2 + b^2)*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) - (3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(8*(a^2
 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx}{4\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3(b\cos(c+dx)-a\sin(}{8\left(a^2+b^2\right)^2 d(a\cos(c+dx)+}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3(b\cos(c+dx)-a\sin(}{8\left(a^2+b^2\right)^2 d(a\cos(c+dx)+}$$

$$= -\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{8\left(a^2+b^2\right)^{5/2} d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}$$

Mathematica [A] time = 1.12765, size = 157, normalized size = 1.01

$$\frac{\frac{-11b\left(a^2+b^2\right)\cos(c+dx)+\left(3b^3-9a^2b\right)\cos(3(c+dx))+2a\sin(c+dx)\left(3\left(a^2-3b^2\right)\cos(2(c+dx))+7a^2+b^2\right)}{4\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{6\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

```
[Out] ((6*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
 (-11*b*(a^2 + b^2)*Cos[c + d*x] + (-9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*
a*(7*a^2 + b^2 + 3*(a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(4*(a^2 +
b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4))/(8*d)
```

Maple [B] time = 0.222, size = 514, normalized size = 3.3

$$\frac{1}{d}\left(-2\,\frac{1}{\left(a\,(\tan(1/2\,dx+c/2))^{2}-2\,b\tan(1/2\,dx+c/2)-a\right)^{4}}\left(-1/8\,\frac{\left(5\,a^{4}+16\,a^{2}b^{2}+8\,b^{4}\right)(\tan(1/2\,dx+c/2))^{7}}{a\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}+3/8\,\frac{b\left(a^{4}\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-2*(-1/8*(5*a^4+16*a^2*b^2+8*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/
2*c)^7+3/8*b*(a^4+16*a^2*b^2+8*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2
*c)^6-1/8/a^3*(3*a^6-36*a^4*b^2+56*a^2*b^4+32*b^6)/(a^4+2*a^2*b^2+b^4)*tan(
1/2*d*x+1/2*c)^5+1/8/a^4*b*(15*a^6-114*a^4*b^2-8*a^2*b^4+16*b^6)/(a^4+2*a^2
*b^2+b^4)*tan(1/2*d*x+1/2*c)^4-1/8/a^3*(3*a^6+84*a^4*b^2-56*a^2*b^4-32*b^6)
/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-1/8*b*(23*a^4-64*a^2*b^2-24*b^4)/
a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^2-1/8*(5*a^4-24*a^2*b^2-8*b^4)/a
/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)+1/8*b*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+
b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^4+3/4/(a^4+2*a^2*b^
2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(
1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53538, size = 1216, normalized size = 7.79

$$-\frac{6\left(3\,a^{4}b+2\,a^{2}b^{3}-b^{5}\right)\cos(dx+c)^{3}-3\left(\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\cos(dx+c)^{4}+b^{4}+2\left(3\,a^{2}b^{2}-b^{4}\right)\cos(dx+c)^{2}+4\left(ab^{3}\cos\ldots\right.\right.}{16\left(\left(a^{10}-3\,a^{8}b^{2}-14\,a^{6}b^{4}-14\,a^{4}b^{6}-3\,a^{2}b^{8}+b^{10}\right)d\cos(dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(3*a^4*b + 2*a^2*b^3 - b^5)*cos(d*x + c)^3 - 3*((a^4 - 6*a^2*b^2 +
 b^4)*cos(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^3*
cos(d*x + c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a^2 + b^2
)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*(4*a^4*b - a^
2*b^3 - 5*b^5)*cos(d*x + c) - 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + 3*(a^5 - 2*a
^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 - 3*a^8*b^2 - 14*a^6
*b^4 - 14*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^4 + 2*(3*a^8*b^2 + 8*a
^6*b^4 + 6*a^4*b^6 - b^10)*d*cos(d*x + c)^2 + (a^6*b^4 + 3*a^4*b^6 + 3*a^2*
b^8 + b^10)*d + 4*((a^9*b + 2*a^7*b^3 - 2*a^3*b^7 - a*b^9)*d*cos(d*x + c)^3
 + (a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34513, size = 794, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/8*(3*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*
sqrt(a^2 + b^2)) - 2*(5*a^7*tan(1/2*d*x + 1/2*c)^7 + 16*a^5*b^2*tan(1/2*d*x
 + 1/2*c)^7 + 8*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*a^6*b*tan(1/2*d*x + 1/2*
c)^6 - 48*a^4*b^3*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^
6 + 3*a^7*tan(1/2*d*x + 1/2*c)^5 - 36*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 56*a
```

```
^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a^6*b*
tan(1/2*d*x + 1/2*c)^4 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2*b^5*tan
(1/2*d*x + 1/2*c)^4 - 16*b^7*tan(1/2*d*x + 1/2*c)^4 + 3*a^7*tan(1/2*d*x + 1
/2*c)^3 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 - 32*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 23*a^6*b*tan(1/2*d*x + 1/2*c)^2 -
64*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 + 5*a
^7*tan(1/2*d*x + 1/2*c) - 24*a^5*b^2*tan(1/2*d*x + 1/2*c) - 8*a^3*b^4*tan(1
/2*d*x + 1/2*c) - 5*a^6*b - 2*a^4*b^3)/((a^8 + 2*a^6*b^2 + a^4*b^4)*(a*tan(
1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^4))/d
```

3.231 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx$

Optimal. Leaf size=151

$$\frac{8\sin(c+dx)}{15ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}-\frac{4(b\cos(c+dx)-a\sin(c+dx))}{15d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{b\cos(c+dx)}{5d\left(a^2+b^2\right)(a\cos(c}$$

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^5) - (4*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(15*(a^2 + b^2)^2*d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (8*Sin[c + d*x])/(15*a*(a^2 + b^2)^2*
d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0693007, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3076, 3075}

$$\frac{8\sin(c+dx)}{15ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}-\frac{4(b\cos(c+dx)-a\sin(c+dx))}{15d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{b\cos(c+dx)}{5d\left(a^2+b^2\right)(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^5) - (4*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(15*(a^2 + b^2)^2*d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (8*Sin[c + d*x])/(15*a*(a^2 + b^2)^2*
d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{4\int\frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx}{5\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5}-\frac{4(b\cos(c+dx)-a\sin(c}{15\left(a^2+b^2\right)^2 d(a\cos(c+dx)+}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5}-\frac{4(b\cos(c+dx)-a\sin(c}{15\left(a^2+b^2\right)^2 d(a\cos(c+dx)+}$$

Mathematica [A] time = 0.531786, size = 182, normalized size = 1.21

$$\frac{20a^2b^2\sin(c+dx)-6a^2b^2\sin(5(c+dx))-10ab\left(a^2+b^2\right)\cos(3(c+dx))+\left(4ab^3-4a^3b\right)\cos(5(c+dx))+10a^4\sin(c+}{30ad\left(a^2+b^2\right)^2(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

```
[Out] (-10*a*b*(a^2 + b^2)*Cos[3*(c + d*x)] + (-4*a^3*b + 4*a*b^3)*Cos[5*(c + d*x
)] + 10*a^4*Sin[c + d*x] + 20*a^2*b^2*Sin[c + d*x] + 10*b^4*Sin[c + d*x] +
5*a^4*Sin[3*(c + d*x)] - 5*b^4*Sin[3*(c + d*x)] + a^4*Sin[5*(c + d*x)] - 6*
a^2*b^2*Sin[5*(c + d*x)] + b^4*Sin[5*(c + d*x)])/(30*a*(a^2 + b^2)^2*d*(a*C
os[c + d*x] + b*Sin[c + d*x])^5)
```

Maple [A] time = 0.225, size = 125, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a\left(a^2+b^2\right)}{b^5\left(a+b\tan(dx+c)\right)^4}-\frac{a^4+2a^2b^2+b^4}{5b^5\left(a+b\tan(dx+c)\right)^5}-\frac{1}{b^5\left(a+b\tan(dx+c)\right)}-\frac{6a^2+2b^2}{3b^5\left(a+b\tan(dx+c)\right)^3}+2\frac{}{b^5(a+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x)
```

```
[Out] 1/d*(a*(a^2+b^2)/b^5/(a+b*tan(d*x+c))^4-1/5*(a^4+2*a^2*b^2+b^4)/b^5/(a+b*ta
n(d*x+c))^5-1/b^5/(a+b*tan(d*x+c))-1/3*(6*a^2+2*b^2)/b^5/(a+b*tan(d*x+c))^3
+2*a/b^5/(a+b*tan(d*x+c))^2)
```

Maxima [A] time = 1.04956, size = 235, normalized size = 1.56

$$-\frac{15\,b^4\tan\left(dx+c\right)^4+30\,ab^3\tan\left(dx+c\right)^3+3\,a^4+a^2b^2+3\,b^4+10\left(3\,a^2b^2+b^4\right)\tan\left(dx+c\right)^2+5\left(3\,a^3b+ab^3\right)\tan\left(a\right.}{15\left(b^{10}\tan\left(dx+c\right)^5+5\,ab^9\tan\left(dx+c\right)^4+10\,a^2b^8\tan\left(dx+c\right)^3+10\,a^3b^7\tan\left(dx+c\right)^2+5\,a^4b^6\tan\left(dx+c\right)+a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 3*a^4 + a^2*b^2 +
3*b^4 + 10*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 5*(3*a^3*b + a*b^3)*tan(d*x +
 c))/((b^10*tan(d*x + c)^5 + 5*a*b^9*tan(d*x + c)^4 + 10*a^2*b^8*tan(d*x +
c)^3 + 10*a^3*b^7*tan(d*x + c)^2 + 5*a^4*b^6*tan(d*x + c) + a^5*b^5)*d)
```

Fricas [B] time = 2.77556, size = 984, normalized size = 6.52

$$-\frac{8\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^5-20\left(a^4b-6\,a^2b^3+b\right.}{15\left(\left(a^{11}-7\,a^9b^2-22\,a^7b^4-14\,a^5b^6+5\,a^3b^8+5\,ab^{10}\right)d\cos\left(dx+c\right)^5+10\left(a^9b^2+2\,a^7b^4-2\,a^3b^8-ab^{10}\right)d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5 - 20*(a^4*b - 6*a^2*b^
3 + b^5)*cos(d*x + c)^3 - 5*(a^4*b + 6*a^2*b^3 - 3*b^5)*cos(d*x + c) - (3*a
^5 + 10*a^3*b^2 + 15*a*b^4 + 8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4
+ 4*(a^5 + 10*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 - 7*
a^9*b^2 - 22*a^7*b^4 - 14*a^5*b^6 + 5*a^3*b^8 + 5*a*b^10)*d*cos(d*x + c)^5
+ 10*(a^9*b^2 + 2*a^7*b^4 - 2*a^3*b^8 - a*b^10)*d*cos(d*x + c)^3 + 5*(a^7*b
^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((5*a^10*b + 5*a^8*b^
3 - 14*a^6*b^5 - 22*a^4*b^7 - 7*a^2*b^9 + b^11)*d*cos(d*x + c)^4 + 2*(5*a^8
*b^3 + 14*a^6*b^5 + 12*a^4*b^7 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^6*
b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12876, size = 159, normalized size = 1.05

$$-\frac{15\, b^{4} \tan\left(dx + c\right)^{4} + 30\, ab^{3} \tan\left(dx + c\right)^{3} + 30\, a^{2} b^{2} \tan\left(dx + c\right)^{2} + 10\, b^{4} \tan\left(dx + c\right)^{2} + 15\, a^{3} b \tan\left(dx + c\right) + 5\, ab^{3} \tan}{15\, \left(b \tan\left(dx + c\right) + a\right)^{5} b^{5} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 30*a^2*b^2*tan(d*x
 + c)^2 + 10*b^4*tan(d*x + c)^2 + 15*a^3*b*tan(d*x + c) + 5*a*b^3*tan(d*x +
 c) + 3*a^4 + a^2*b^2 + 3*b^4)/((b*tan(d*x + c) + a)^5*b^5*d)
```

3.232 $\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=186

$$\frac{10\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,b)+c+dx\right),2\right)}{21d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx}{21d}$$

```
[Out] (-10*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*
Sin[c + d*x]])/(21*d) - (2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x
] + b*Sin[c + d*x])^(5/2))/(7*d) + (10*(a^2 + b^2)^2*EllipticF[(c + d*x - A
rcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(21*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rubi [A] time = 0.0958722, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3078, 2641}

$$\frac{10\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{21d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-10*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*
Sin[c + d*x]])/(21*d) - (2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x
] + b*Sin[c + d*x])^(5/2))/(7*d) + (10*(a^2 + b^2)^2*EllipticF[(c + d*x - A
rcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(21*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}+\frac{1}{7}\left(5\left(a^2+\right.\right.$$

$$= -\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{21d}-\frac{2(}{}$$

$$= -\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{21d}-\frac{2(}{}$$

$$= -\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{21d}-\frac{2(}{}$$

Mathematica [C] time = 1.85859, size = 205, normalized size = 1.1

$$\frac{\frac{20\left(a^2+b^2\right)^2\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\,\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},\sin^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\right)}{\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}}+\sqrt{a\cos(c+dx)+b\sin(c+d}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

```
[Out] (Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(-23*b*(a^2 + b^2)*Cos[c + d*x] + (-
9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*a*(13*a^2 + 7*b^2 + 3*(a^2 - 3*b^2)*C
os[2*(c + d*x)])*Sin[c + d*x]) + (20*(a^2 + b^2)^2*Sqrt[Cos[c + d*x + ArcTa
n[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^
2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTa
```

```
n[a/b]]])/(42*d)
```

Maple [A] time = 1.891, size = 183, normalized size = 1.

$$\frac{\left(a^2+b^2\right)^2}{21\cos\left(dx+c-\arctan\left(-a,b\right)\right)d}\left(6\left(\sin\left(dx+c-\arctan\left(-a,b\right)\right)\right)^5+5\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\sqrt{-2\sin\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/21*(a^2+b^2)^2*(6*sin(d*x+c-arctan(-a,b))^5+5*(1+sin(d*x+c-arctan(-a,b)))
^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2
)*EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))+4*sin(d*x+c-arct
an(-a,b))^3-10*sin(d*x+c-arctan(-a,b)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-
arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3\,ab^2\cos\left(dx+c\right)+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+\left(b^3+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\sqrt{a\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((3*a*b^2*cos(d*x + c) + (a^3 - 3*a*b^2)*cos(d*x + c)^3 + (b^3 + (3
*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a*cos(d*x + c) + b*sin(d*x
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c) + b\sin(dx+c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

3.233 $\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{6\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx$$

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/
2))/(5*d) + (6*(a^2 + b^2)*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*
Cos[c + d*x] + b*Sin[c + d*x]])/(5*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2]])
```

Rubi [A] time = 0.057692, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3078, 2639}

$$\frac{6\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/
2))/(5*d) + (6*(a^2 + b^2)*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*
Cos[c + d*x] + b*Sin[c + d*x]])/(5*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2]])
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}+\frac{1}{5}\left(3\left(a^2+\right.\right.$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}+\frac{\left(3\left(a^2+b^2\right.\right.}{}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}+\frac{6\left(a^2+b^2\right)}{}$$

Mathematica [C] time = 1.61262, size = 256, normalized size = 1.95

$$\sqrt{a\cos(c+dx)+b\sin(c+dx)}\left(b\left(a^2-b^2\right)\sin(2(c+dx))+6a\left(a^2+b^2\right)-2ab^2\cos(2(c+dx))\right)-\frac{3\left(a^2+b^2\right)^2\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(6*a*(a^2 + b^2) - 2*a*b^2*Cos[2*(c
+ d*x)] + b*(a^2 - b^2)*Sin[2*(c + d*x)]) - (3*(a^2 + b^2)^2*Cos[c + d*x -
ArcTan[b/a]]*(b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan
[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] + Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(
2*a*Cos[c + d*x - ArcTan[b/a]] - b*Sin[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1
 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a
```

```
]]^2]))/(5*b*d)
```

Maple [A] time = 1.464, size = 246, normalized size = 1.9

$$-\frac{1}{5\cos\left(dx+c-\arctan\left(-a,b\right)\right)d}\left(a^2+b^2\right)^{\frac{3}{2}}\left(6\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\sqrt{-2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/5*(a^2+b^2)^(3/2)*(6*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arc
tan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticE((1+sin(d*x+c
-arctan(-a,b)))^(1/2),1/2*2^(1/2))-3*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*
sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF
((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^4
+2*sin(d*x+c-arctan(-a,b))^2)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,
b))*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2\right)\sqrt{a\cos(dx+c)+b\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^
2)*sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

3.234 $\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{2\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,b)+c+dx\right),2\right)}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/(3*d) + (2*(a^2 + b^2)*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*C
os[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*d*Sqrt[a*Cos[c + d*x] +
b*Sin[c + d*x]])
```

Rubi [A] time = 0.0580733, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3078, 2641}

$$\frac{2\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/(3*d) + (2*(a^2 + b^2)*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*C
os[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*d*Sqrt[a*Cos[c + d*x] +
b*Sin[c + d*x]])
```


Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
```

```
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}+\frac{1}{3}\left(a^2+b^2\right)\int$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}+\frac{\left(\left(a^2+b^2\right)\sqrt{}\right.}{}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}+\frac{2\left(a^2+b^2\right)F}{}$$

Mathematica [C] time = 1.32645, size = 143, normalized size = 1.09

$$\frac{2\left(\frac{\left(a^2+b^2\right)\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\,\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},\sin^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\right)}{\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}}+\sqrt{a\cos(c+dx)+b\sin(c+d}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*
x]] + ((a^2 + b^2)*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/S
qrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*d)
```

Maple [A] time = 1.221, size = 163, normalized size = 1.2

$$\frac{a^2+b^2}{3\cos\left(dx+c-\arctan\left(-a,b\right)\right)d}\left(\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\sqrt{-2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\sqrt{-\sin\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/3*(a^2+b^2)*((1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b)
)+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-
a,b)))^(1/2),1/2*2^(1/2))+2*sin(d*x+c-arctan(-a,b))^3-2*sin(d*x+c-arctan(-a
,b)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

3.235 $\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

```
[Out] (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/(d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rubi [A] time = 0.0296799, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3078, 2639}

$$\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/(d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}\,dx}{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

$$= \frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

Mathematica [C] time = 1.14195, size = 268, normalized size = 3.57

$$\frac{\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\left(\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(b\left(a^2+b^2\right)\sin\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)-2a\left(a^2+b^2\right)\cos\left(-\mathrm{t}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (Cos[c + d*x - ArcTan[b/a]]*(-(b*(a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]]) + Sqrt[S
in[c + d*x - ArcTan[b/a]]^2]*(-2*a*(a^2 + b^2)*Cos[c + d*x - ArcTan[b/a]] +
 2*a^2*Sqrt[1 + b^2/a^2]*Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]
]]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + b*(a^2 + b^2)*Sin[c + d*x - ArcT
an[b/a]])))/(b*d*(a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqr
t[Sin[c + d*x - ArcTan[b/a]]^2])
```

Maple [A] time = 1.204, size = 159, normalized size = 2.1

$$-\frac{1}{\cos\left(dx+c-\arctan\left(-a,b\right)\right)d}\sqrt{a^2+b^2}\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\sqrt{-2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\sqrt{-\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -(a^2+b^2)^(1/2)*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,
b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*(2*EllipticE((1+sin(d*x+c-arc
tan(-a,b)))^(1/2),1/2*2^(1/2))-EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),
1/2*2^(1/2)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1
/2))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(dx+c)+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\cos(dx+c)+b\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(dx+c)+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

3.236 $\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,b)+c+dx\right),2\right)}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

```
[Out] (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c
+ d*x])/Sqrt[a^2 + b^2]])/(d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rubi [A] time = 0.0296477, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3078, 2641}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c
+ d*x])/Sqrt[a^2 + b^2]])/(d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}}\,dx}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Mathematica [C] time = 0.186312, size = 92, normalized size = 1.23

$$\frac{2\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},\sin^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\right)}{d\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/(d*Sqrt[Sqrt[1 +
a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]])
```

Maple [A] time = 1.072, size = 121, normalized size = 1.6

$$\frac{1}{\cos(dx+c-\arctan(-a,b))\,d}\sqrt{1+\sin(dx+c-\arctan(-a,b))}\sqrt{-2\sin(dx+c-\arctan(-a,b))+2}\sqrt{-\sin(dx+c-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-si
n(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/
2*2^(1/2))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2)
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{a\cos(dx+c)+b\sin(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

3.237 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/((a^2 + b^2)*d*Sqrt[a*Cos[c + d*x] +
 b*Sin[c + d*x]]) - (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[
c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c +
 d*x])/Sqrt[a^2 + b^2]])
```

Rubi [A] time = 0.0576028, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3078, 2639}

$$-\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/((a^2 + b^2)*d*Sqrt[a*Cos[c + d*x] +
 b*Sin[c + d*x]]) - (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[
c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c +
 d*x])/Sqrt[a^2 + b^2]])
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{\left(a^2+b^2\right)d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}}{a^2+b^2}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{\left(a^2+b^2\right)d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int}{\left(a^2+b^2\right)\sqrt{\frac{a\cos}{}}}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{\left(a^2+b^2\right)d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{\left(a^2+b^2\right)d\sqrt{\frac{a\cos}{}}}$$

Mathematica [C] time = 3.11977, size = 219, normalized size = 1.59

$$\frac{\frac{\tan\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\sqrt{a\sqrt{\frac{b^2}{a^2}+1}\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\,\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\right)}{\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}} - \tan\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+\right.}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((-2*b*Cos[c + d*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + (2*a*Sin[c + d
*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] - Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c
 + d*x - ArcTan[b/a]]]*Tan[c + d*x - ArcTan[b/a]] + (Sqrt[a*Sqrt[1 + b^2/a^
2]*Cos[c + d*x - ArcTan[b/a]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c
 + d*x - ArcTan[b/a]]^2]*Tan[c + d*x - ArcTan[b/a]])/Sqrt[Sin[c + d*x - Arc
```

```
Tan[b/a]]^2])/((a^2 + b^2)*d)
```

Maple [A] time = 1.742, size = 228, normalized size = 1.7

$$\frac{1}{\cos\left(dx+c-\arctan\left(-a,b\right)\right)d}\left(2\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\sqrt{-2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\sqrt{-\sin\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x)
```

```
[Out] (2*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(
-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticE((1+sin(d*x+c-arctan(-a,b)))^(1/2)
,1/2*2^(1/2))-(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))
+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-a
,b)))^(1/2),1/2*2^(1/2))-2*cos(d*x+c-arctan(-a,b))^2)/(a^2+b^2)^(1/2)/cos(d
*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}}{2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

3.238 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,b)+c+dx\right),2\right)}{3d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(3/2)) + (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a
*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*(a^2 + b^2)*d*Sqrt[a*C
os[c + d*x] + b*Sin[c + d*x]])
```

Rubi [A] time = 0.0551344, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3078, 2641}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{3d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(3/2)) + (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a
*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*(a^2 + b^2)*d*Sqrt[a*C
os[c + d*x] + b*Sin[c + d*x]])
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{}{\sqrt{\cos}}}{3\left(a^2+b^2\right)\sqrt{a\cos(c+dx)}}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\right|}{3\left(a^2+b^2\right)d\sqrt{a\cos(c+}}$$

Mathematica [C] time = 1.68659, size = 145, normalized size = 1.02

$$\frac{2\left(\frac{\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{2}\right\},\left\{\frac{5}{4}\right\},\sin^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\right)}{\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}}+\frac{a\sin(c+dx)-b\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\right)}{3d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x])^
(3/2) + (Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1
 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*(a^2 + b^2)*d)
```

Maple [A] time = 1.589, size = 178, normalized size = 1.3

$$\frac{1}{3\sin\left(dx+c-\arctan\left(-a,b\right)\right)\left(a^2+b^2\right)\cos\left(dx+c-\arctan\left(-a,b\right)\right)d}\left(\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\sqrt{-2\sin\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/3/sin(d*x+c-arctan(-a,b))/(a^2+b^2)*((1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-
2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*Ellipti
cF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))*sin(d*x+c-arctan(-a,b))-2
*cos(d*x+c-arctan(-a,b))^2)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b)
)*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}}{3\,ab^2\cos\left(dx+c\right)+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+\left(b^3+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/(3*a*b^2*cos(d*x + c) + (a^3
 - 3*a*b^2)*cos(d*x + c)^3 + (b^3 + (3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x
```

```
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

3.239 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{6\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{6(b\cos(c+dx)-a\sin(c+dx))}{5d\left(a^2+b^2\right)^2\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{}{5d\left(a\right.}$$

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(5/2)) - (6*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2
)^2*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]) - (6*EllipticE[(c + d*x - ArcT
an[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])/(5*(a^2 + b^2)^2*d*S
qrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rubi [A] time = 0.0859234, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3078, 2639}

$$-\frac{6\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{6(b\cos(c+dx)-a\sin(c+dx))}{5d\left(a^2+b^2\right)^2\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{}{5d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(5/2)) - (6*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2
)^2*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]) - (6*EllipticE[(c + d*x - ArcT
an[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])/(5*(a^2 + b^2)^2*d*S
qrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}+\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx}{5\left(a^2+b^2\right)}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}-\frac{6(b\cos(c+dx)-a\sin}{5\left(a^2+b^2\right)^2 d\sqrt{a\cos(c+dx)}}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}-\frac{6(b\cos(c+dx)-a\sin}{5\left(a^2+b^2\right)^2 d\sqrt{a\cos(c+dx)}}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}-\frac{6(b\cos(c+dx)-a\sin}{5\left(a^2+b^2\right)^2 d\sqrt{a\cos(c+dx)}}$$

Mathematica [C] time = 2.42064, size = 277, normalized size = 1.41

$$\frac{\frac{\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\left(3b\sin\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\right)-3\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(b\sin\left(-\tan^{-1}\right.\right.\right.}{\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(a\sqrt{\frac{b^2}{a^2}+1}\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\right)^{3/2}}}{5bd\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

```
[Out] ((-2*(3*a^2*Cos[c + d*x]^3 - a*b*Sin[c + d*x] + 6*a*b*Cos[c + d*x]^2*Sin[c
+ d*x] + b^2*Cos[c + d*x]*(1 + 3*Sin[c + d*x]^2)))/(a*Cos[c + d*x] + b*Sin[
c + d*x])^(5/2) + (Cos[c + d*x - ArcTan[b/a]]*(3*b*HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] - 3
*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(-2*a*Cos[c + d*x - ArcTan[b/a]] + b*Si
n[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]
])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]))/(5*b*(a^2 + b^2)*d)
```

Maple [A] time = 2.812, size = 309, normalized size = 1.6

$$\frac{1}{5\left(\sin\left(dx+c-\arctan\left(-a,b\right)\right)\right)^{2}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\cos\left(dx+c-\arctan\left(-a,b\right)\right)d}\sqrt{a^{2}+b^{2}}\left(6\sqrt{1+\sin\left(dx+c-\mathrm{arcta}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/5/sin(d*x+c-arctan(-a,b))^2*(a^2+b^2)^(1/2)*(6*(1+sin(d*x+c-arctan(-a,b))
)^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/
2)*sin(d*x+c-arctan(-a,b))^2*EllipticE((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/
2*2^(1/2))-3*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+
2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*sin(d*x+c-arctan(-a,b))^2*Ellipti
cF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))+6*sin(d*x+c-arctan(-a,b))
^4-4*sin(d*x+c-arctan(-a,b))^2-2)/(a^4+2*a^2*b^2+b^4)/cos(d*x+c-arctan(-a,b
))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos(dx+c)+b\sin(dx+c)}}{\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^4+b^4+2\left(3\,a^2b^2-b^4\right)\cos(dx+c)^2+4\left(ab^3\cos(dx+c)+\left(a^3b-ab^3\right)\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/((a^4 - 6*a^2*b^2 + b^4)*cos
(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^3*cos(d*x +
 c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

3.240 $\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=120

$$\frac{130\,13^{3/4}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right),2\right)}{21d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}-13$$

```
[Out] (130*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(21*d) - (130*(3*Cos
[c + d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(21*d) -
 (2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/
2))/(7*d)
```

Rubi [A] time = 0.0699545, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3077, 2641}

$$-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}-\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]
```

```
[Out] (130*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(21*d) - (130*(3*Cos
[c + d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(21*d) -
 (2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/
2))/(7*d)
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
```

```
b^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{5/2}}{7d}+\frac{65}{7}\int (2\cos$$

$$= -\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{21d}-\frac{2(3\cos(c}{}$$

$$= -\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{21d}-\frac{2(3\cos(c}{}$$

$$= \frac{130\ 13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{21d}-\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2}}{21d}$$

Mathematica [C] time = 0.514321, size = 153, normalized size = 1.27

$$\frac{260\ 13^{3/4}\sqrt{-\left(\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)-1\right)\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\sec\left(c+dx+\tan^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-(Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(897*Cos[c + d*x] + 27*Cos[3*(c +
d*x)] - 598*Sin[c + d*x] + 138*Sin[3*(c + d*x)])) + 260*13^(3/4)*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcT
an[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x + ArcTan[2/3]
])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(42*d)
```

Maple [A] time = 1.334, size = 128, normalized size = 1.1

$$\frac{1}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)d}\left(\frac{338\sin\left(dx+c+\arctan\left(2/3\right)\right)\left(\cos\left(dx+c+\arctan\left(2/3\right)\right)\right)^{4}}{7}+\frac{845}{21}\sqrt{1+\sin\left(dx+c+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x)
```

```
[Out] (338/7*sin(d*x+c+arctan(2/3))*cos(d*x+c+arctan(2/3))^4+845/21*(1+sin(d*x+c+
arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(
2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))-2704/2
1*sin(d*x+c+arctan(2/3))*cos(d*x+c+arctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(
13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\cos(dx+c)+3\sin(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(46\cos(dx+c)^{3}-9\left(\cos(dx+c)^{2}+3\right)\sin(dx+c)-54\cos(dx+c)\right)\sqrt{2\cos(dx+c)+3\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(46*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 + 3)*sin(d*x + c) - 54*cos
(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

3.241 $\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{78\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

```
[Out] (78*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(5*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))/(5*d)
```

Rubi [A] time = 0.0445689, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3077, 2639}

$$\frac{78\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

```
[Out] (78*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(5*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3/2}}{5d}+\frac{39}{5}\int \sqrt{2\,c}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3/2}}{5d}+\frac{1}{5}\left(39\sqrt[4]{13}\right.$$

$$= \frac{78\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c}{5d}$$

Mathematica [C] time = 0.876895, size = 199, normalized size = 2.65

$$\frac{-\frac{39\sqrt[4]{13}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{\sqrt{-\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}}+\sqrt{3\sin(c+dx)+2\cos(c+dx)}(-5\sin(2(c+dx}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(52 - 12*Cos[2*(c + d*x)] - 5*Sin[2*
(c + d*x)]) - (13*13^(1/4)*(4*Cos[c + d*x - ArcTan[3/2]] - 3*Sin[c + d*x -
ArcTan[3/2]]))/Sqrt[Cos[c + d*x - ArcTan[3/2]]] - (39*13^(1/4)*Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcT
an[3/2]])/(Sqrt[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/
2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[3/2]]]))/(5*d)
```

Maple [A] time = 1.238, size = 174, normalized size = 2.3

$$-\frac{13\sqrt{13}}{5\cos(dx+c+\arctan(2/3))\,d}\left(6\sqrt{1+\sin(dx+c+\arctan(2/3))}\sqrt{-2\sin(dx+c+\arctan(2/3))+2}\sqrt{-\sin(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x)
```

```
[Out] -13/5*13^(1/2)*(6*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3
))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((1+sin(d*x+c+arctan(2
/3)))^(1/2),1/2*2^(1/2))-3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+a
rctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c
+arctan(2/3)))^(1/2),1/2*2^(1/2))-2*sin(d*x+c+arctan(2/3))^4+2*sin(d*x+c+ar
ctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2 \cos (dx+c)+3 \sin (dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(5 \cos (dx+c)^2-12 \cos (dx+c)\sin (dx+c)-9\right)\sqrt{2 \cos (dx+c)+3 \sin (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(5*cos(d*x + c)^2 - 12*cos(d*x + c)*sin(d*x + c) - 9)*sqrt(2*cos(
d*x + c) + 3*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\cos(dx+c)+3\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

3.242 $\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{2\,13^{3/4}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right),2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

```
[Out] (2*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(3*d) - (2*(3*Cos[c +
d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.0426062, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3077, 2641}

$$\frac{2\,13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(3*d) - (2*(3*Cos[c +
d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(3*d)
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d} + \frac{13}{3}\int \frac{}{\sqrt{2\cos}}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d} + \frac{1}{3}13^{3/4}\int$$

$$= \frac{2\,13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c}}{3d}$$

Mathematica [C] time = 0.317872, size = 133, normalized size = 1.77

$$\frac{2\,13^{3/4}\sqrt{-\left(\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)-1\right)\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\sec\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(-3*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]
 + 2*13^(3/4)*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3
]]^2]*Sec[c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*S
in[c + d*x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(3*d)
```

Maple [A] time = 1.186, size = 108, normalized size = 1.4

$$\frac{1}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)d}\left(\frac{13}{3}\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\sqrt{-2\sin\left(dx+c+\arctan(2/3)\right)+2}\sqrt{-\sin\left(dx+c+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x)
```

```
[Out] (13/3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*
(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),
1/2*2^(1/2))-26/3*sin(d*x+c+arctan(2/3))*cos(d*x+c+arctan(2/3))^2)/cos(d*x+
c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

3.243 $\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{2\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

```
[Out] (2*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/d
```

Rubi [A] time = 0.0227896, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3077, 2639}

$$\frac{2\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/d
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx = \sqrt[4]{13}\int \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\,dx$$
$$= \frac{2\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

Mathematica [C] time = 0.864092, size = 184, normalized size = 6.81

$$\frac{-\frac{3\sqrt[4]{13}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{\sqrt{-\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}}+4\sqrt{3\sin(c+dx)+2\cos(c+dx)}-4\sqrt[4]{13}\sqrt{\cos\left(\right.}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (-4*13^(1/4)*Sqrt[Cos[c + d*x - ArcTan[3/2]]] + 4*Sqrt[2*Cos[c + d*x] + 3*S
in[c + d*x]] + (3*13^(1/4)*Sin[c + d*x - ArcTan[3/2]])/Sqrt[Cos[c + d*x - A
rcTan[3/2]]] - (3*13^(1/4)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d
*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(Sqrt[-((-1 + Cos[c + d*x
- ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[
3/2]]]))/(3*d)
```

Maple [A] time = 0.935, size = 112, normalized size = 4.2

$$-\frac{\sqrt{13}}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)d}\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\sqrt{-2\sin\left(dx+c+\arctan(2/3)\right)+2}\sqrt{-\sin\left(dx+c+\text{arc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x)
```

```
[Out] -13^(1/2)*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1
/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*(2*EllipticE((1+sin(d*x+c+arctan(2/3)))
^(1/2),1/2*2^(1/2))-EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))
)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3 \sin(c + dx) + 2 \cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

3.244 $\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{2\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right),2\right)}{\sqrt[4]{13}d}$$

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(13^(1/4)*d)
```

Rubi [A] time = 0.0240906, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3077, 2641}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(13^(1/4)*d)
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}\,dx}{\sqrt[4]{13}} = \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}d}$$

Mathematica [C] time = 0.109203, size = 88, normalized size = 3.26

$$\frac{2\sqrt{-\left(\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)-1\right)\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\sec\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\text{Hy}}{\sqrt[4]{13}d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c
 + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x
+ ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(13^(1/4)*d)
```

Maple [A] time = 0.687, size = 85, normalized size = 3.2

$$\frac{1}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)d}\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\sqrt{-2\,\sin\left(dx+c+\arctan\left(2/3\right)\right)+2}\sqrt{-\sin\left(dx+c+\text{arcta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(
d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^
(1/2))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\cos(dx+c)+3\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\cos(dx+c)+3\sin(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\sin(c+dx)+2\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\cos(dx+c)+3\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

3.245 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}$$

```
[Out] (-2*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(13^(3/4)*d) - (2*(3*Cos[c + d
*x] - 2*Sin[c + d*x]))/(13*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])
```

Rubi [A] time = 0.0413938, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3077, 2639}

$$-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```


```
[Out] (-2*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(13^(3/4)*d) - (2*(3*Cos[c + d
*x] - 2*Sin[c + d*x]))/(13*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])
```


Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{1}{13}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{\int \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\,dx}{13^{3/4}}$$

$$= -\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

Mathematica [C] time = 1.11188, size = 190, normalized size = 2.6

$$\frac{\frac{3\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{13^{3/4}\sqrt{-\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}} - \frac{2\cos(c+dx)}{\sqrt{3\sin(c+dx)+2\cos(c+dx)}} + \frac{4\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}{13^{3/4}} - \frac{3\sin(\ldots}{13^{3/4}\sqrt{c\ldots}}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) - (2*Cos[c + d*x])/Sqrt[2*Co
s[c + d*x] + 3*Sin[c + d*x]] - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqr
t[Cos[c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-(
(-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos
[c + d*x - ArcTan[3/2]]]))/(3*d)
```

Maple [A] time = 1.253, size = 162, normalized size = 2.2

$$\frac{\sqrt{13}}{13\cos(dx+c+\arctan(2/3))\,d}\left(2\sqrt{1+\sin(dx+c+\arctan(2/3))}\sqrt{-2\sin(dx+c+\arctan(2/3))+2}\sqrt{-\sin(dx+c+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/13*13^(1/2)*(2*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3)
)+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((1+sin(d*x+c+arctan(2/
3)))^(1/2),1/2*2^(1/2))-(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arct
an(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+ar
ctan(2/3)))^(1/2),1/2*2^(1/2))-2*cos(d*x+c+arctan(2/3))^2)/cos(d*x+c+arctan
(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2 \cos\left(dx+c\right)+3 \sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{2 \cos\left(dx+c\right)+3 \sin\left(dx+c\right)}}{5 \cos\left(dx+c\right)^{2}-12 \cos\left(dx+c\right)\sin\left(dx+c\right)-9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(5*cos(d*x + c)^2 - 12*cos(
d*x + c)*sin(d*x + c) - 9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2 \cos\left(dx+c\right)+3 \sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

3.246 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right),2\right)}{39\sqrt[4]{13}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(39*13^(1/4)*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x]))/(39*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0421979, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3077, 2641}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(39*13^(1/4)*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x]))/(39*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}+\frac{1}{39}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}\,dx}{39\sqrt[4]{13}}$$

$$= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.741956, size = 157, normalized size = 2.09

$$\frac{\sqrt{2}13^{3/4}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}(3\sin(c+dx)+2\cos(c+dx))^{3/2}\sec\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\sqrt{2\sin\left(c+dx+\tan^{-1}\right.}}{507d(3\sin}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-78*Cos[c + d*x] + 52*Sin[c + d*x] + Sqrt[2]*13^(3/4)*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcTan[2/3]]*(
2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2)*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]]
*Sqrt[-1 + Cos[2*(c + d*x + ArcTan[2/3])] + 2*Sin[c + d*x + ArcTan[2/3]]])/
(507*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

Maple [A] time = 1.101, size = 118, normalized size = 1.6

$$\frac{1}{39\sin(dx+c+\arctan(2/3))\cos(dx+c+\arctan(2/3))d}\left(\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\sqrt{-2\sin(dx+c+\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/39/sin(d*x+c+arctan(2/3))*((1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c
+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x
+c+arctan(2/3)))^(1/2),1/2*2^(1/2))*sin(d*x+c+arctan(2/3))-2*cos(d*x+c+arct
an(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{2\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)}}{46\,\cos\left(dx+c\right)^{3}-9\left(\cos\left(dx+c\right)^{2}+3\right)\sin\left(dx+c\right)-54\,\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(46*cos(d*x + c)^3 - 9*(cos
(d*x + c)^2 + 3)*sin(d*x + c) - 54*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

3.247 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}-\frac{6E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\ 13^{3/4}d}$$

```
[Out] (-6*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(65*13^(3/4)*d) - (2*(3*Cos[c
+ d*x] - 2*Sin[c + d*x]))/(65*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2)) -
(6*(3*Cos[c + d*x] - 2*Sin[c + d*x]))/(845*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c
+ d*x]])
```

Rubi [A] time = 0.0636797, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3077, 2639}

$$-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}-\frac{6E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\ 13^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

```
[Out] (-6*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(65*13^(3/4)*d) - (2*(3*Cos[c
+ d*x] - 2*Sin[c + d*x]))/(65*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2)) -
(6*(3*Cos[c + d*x] - 2*Sin[c + d*x]))/(845*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c
+ d*x]])
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
```

```
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}+\frac{3}{65}\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3}}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

$$= -\frac{6E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\ 13^{3/4}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}-\frac{6}{845}$$

Mathematica [C] time = 2.10189, size = 224, normalized size = 1.87

$$\frac{\frac{3\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{13^{3/4}\sqrt{-\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}}+\frac{-4(\sin(c+dx)+3\sin(3(c+dx)))-33\cos(c+dx)+5\cos(3(c+dx))}{2(3\sin(c+dx)+2\cos(c+dx))^{5/2}}+\frac{4\sqrt{c}}{}}{65d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) + (-33*Cos[c + d*x] + 5*Cos[
3*(c + d*x)] - 4*(Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(2*(2*Cos[c + d*x] +
3*Sin[c + d*x])^(5/2)) - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[Cos[
c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c
+ d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-((-1 +
Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d
*x - ArcTan[3/2]]]))/(65*d)
```

Maple [A] time = 1.348, size = 205, normalized size = 1.7

$$\frac{\sqrt{13}}{845\,(\sin(dx+c+\arctan(2/3)))^2\cos(dx+c+\arctan(2/3))\,d}\left(6\sqrt{1+\sin(dx+c+\arctan(2/3))}\sqrt{-2\sin(dx+c+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/845*13^(1/2)/sin(d*x+c+arctan(2/3))^2*(6*(1+sin(d*x+c+arctan(2/3)))^(1/2)
*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*sin(d*
x+c+arctan(2/3))^2*EllipticE((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))-
3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-si
n(d*x+c+arctan(2/3)))^(1/2)*sin(d*x+c+arctan(2/3))^2*EllipticF((1+sin(d*x+c
+arctan(2/3)))^(1/2),1/2*2^(1/2))+6*sin(d*x+c+arctan(2/3))^4-4*sin(d*x+c+ar
ctan(2/3))^2-2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{2\cos(dx+c)+3\sin(dx+c)}}{119\cos(dx+c)^4-54\cos(dx+c)^2+24\left(5\cos(dx+c)^3-9\cos(dx+c)\right)\sin(dx+c)-81},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(119*cos(d*x + c)^4 - 54*co
s(d*x + c)^2 + 24*(5*cos(d*x + c)^3 - 9*cos(d*x + c))*sin(d*x + c) - 81), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

3.248 $\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=32

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

```
[Out] ((-I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n)/(d*n)
```

Rubi [A] time = 0.0154417, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
```

```
[Out] ((-I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n)/(d*n)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

Mathematica [A] time = 0.0840538, size = 31, normalized size = 0.97

$$-\frac{i(a(\cos(c+dx)+i\sin(c+dx)))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
```

```
[Out] ((-I)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d*n)
```

Maple [A] time = 0.029, size = 31, normalized size = 1.

$$\frac{-i\,(a\cos(dx+c)+ia\sin(dx+c))^{n}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x)
```

```
[Out] -I*(a*cos(d*x+c)+I*a*sin(d*x+c))^n/d/n
```

Maxima [B] time = 1.53916, size = 80, normalized size = 2.5

$$-\frac{i\,a^{n}e^{\left(-n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)+n\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -I*a^n*e^(-n*log(sin(d*x + c)/(cos(d*x + c) + 1) + I) + n*log(-sin(d*x + c)
/(cos(d*x + c) + 1) + I))/(d*n)
```

Fricas [A] time = 2.02777, size = 43, normalized size = 1.34

$$-\frac{i\,\left(ae^{(i\,dx+i\,c)}\right)^{n}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -I*(a*e^(I*d*x + I*c))^n/(d*n)
```

Sympy [A] time = 7.14932, size = 116, normalized size = 3.62

$$\begin{cases} x & \text{for } d = 0 \wedge n = 0 \\ x (ia \sin(c) + a \cos(c))^n & \text{for } d = 0 \\ x & \text{for } n = 0 \\ \frac{(ia \sin(c+dx)+a \cos(c+dx))^n \sin(c+dx)}{idn \sin(c+dx)+dn \cos(c+dx)} - \frac{i(ia \sin(c+dx)+a \cos(c+dx))^n \cos(c+dx)}{idn \sin(c+dx)+dn \cos(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(I*a*sin(c) + a*cos(c))**n, Eq(d, 0)
), (x, Eq(n, 0)), ((I*a*sin(c + d*x) + a*cos(c + d*x))**n*sin(c + d*x)/(I*d
*n*sin(c + d*x) + d*n*cos(c + d*x)) - I*(I*a*sin(c + d*x) + a*cos(c + d*x))
**n*cos(c + d*x)/(I*d*n*sin(c + d*x) + d*n*cos(c + d*x)), True))
```

Giac [A] time = 1.64344, size = 31, normalized size = 0.97

$$-\frac{i\, e^{(i\, dnx + i\, cn + n \log(a))}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)
```

3.249 $\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

```
[Out] ((-I/4)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

Rubi [A] time = 0.0181858, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

```
[Out] ((-I/4)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Mathematica [A] time = 0.113335, size = 31, normalized size = 1.

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

```
[Out] ((-I/4)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

Maple [B] time = 0.06, size = 151, normalized size = 4.9

$$\frac{1}{d}\left(a^4\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)-ia^4(\sin(dx+c))^4-6a^4\left(-1/4\sin(dx+c)(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-I*a^
4*sin(d*x+c)^4-6*a^4*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c
)+1/8*d*x+1/8*c)-I*a^4*cos(d*x+c)^4+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*
sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [B] time = 1.00341, size = 178, normalized size = 5.74

$$-\frac{i a^4 \cos(dx+c)^4}{d}-\frac{i a^4 \sin(dx+c)^4}{d}+\frac{(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^4}{32d}+\frac{(12dx+12c+\sin(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -I*a^4*cos(d*x + c)^4/d - I*a^4*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 1/32*(12*d*x + 12*c + sin(4*d*
x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d - 3/16*(4*d*x + 4*c - sin(4*d*x + 4*c)
)*a^4/d
```

Fricas [A] time = 2.02668, size = 46, normalized size = 1.48

$$-\frac{i a^4 e^{(4i dx+4i c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/4*I*a^4*e^(4*I*d*x + 4*I*c)/d
```

Sympy [A] time = 0.171032, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{ia^4e^{4ic}e^{4idx}}{4d} & \text{for } 4d \neq 0 \\ a^4xe^{4ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-I*a**4*exp(4*I*c)*exp(4*I*d*x)/(4*d), Ne(4*d, 0)), (a**4*x*exp(
4*I*c), True))
```

Giac [B] time = 1.15405, size = 70, normalized size = 2.26

$$-\frac{i\,a^4e^{(4i\,dx+4i\,c)}}{8\,d}-\frac{i\,a^4e^{(-4i\,dx-4i\,c)}}{8\,d}+\frac{a^4\sin\left(4\,dx+4\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*I*a^4*e^(4*I*d*x + 4*I*c)/d - 1/8*I*a^4*e^(-4*I*d*x - 4*I*c)/d + 1/4*a
^4*sin(4*d*x + 4*c)/d
```

3.250 $\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

```
[Out] ((-I/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

Rubi [A] time = 0.0162244, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Mathematica [A] time = 0.0821072, size = 31, normalized size = 1.

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

Maple [B] time = 0.058, size = 76, normalized size = 2.5

$$\frac{1}{d}\left(\frac{i}{3}a^3\left(2+(\sin(dx+c))^2\right)\cos(dx+c)-a^3(\sin(dx+c))^3-ia^3(\cos(dx+c))^3+\frac{a^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*I*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)-a^3*sin(d*x+c)^3-I*a^3*cos(d*x+c
)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.01103, size = 112, normalized size = 3.61

$$-\frac{i\,a^3\cos(dx+c)^3}{d}-\frac{a^3\sin(dx+c)^3}{d}-\frac{i\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)a^3}{3\,d}-\frac{\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -I*a^3*cos(d*x + c)^3/d - a^3*sin(d*x + c)^3/d - 1/3*I*(cos(d*x + c)^3 - 3*
cos(d*x + c))*a^3/d - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d
```

Fricas [A] time = 2.03724, size = 46, normalized size = 1.48

$$-\frac{i\,a^3e^{(3i\,dx+3i\,c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

Sympy [A] time = 0.166956, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{ia^3e^{3ic}e^{3idx}}{3d} & \text{for } 3d \neq 0 \\ a^3xe^{3ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(3*d, 0)), (a**3*x*exp(
3*I*c), True))
```

Giac [B] time = 1.10807, size = 70, normalized size = 2.26

$$-\frac{i\,a^3e^{(3i\,dx+3i\,c)}}{6\,d}-\frac{i\,a^3e^{(-3i\,dx-3i\,c)}}{6\,d}+\frac{a^3\sin\left(3\,dx+3\,c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*I*a^3*e^(3*I*d*x + 3*I*c)/d - 1/6*I*a^3*e^(-3*I*d*x - 3*I*c)/d + 1/3*a
^3*sin(3*d*x + 3*c)/d
```

3.251 $\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

```
[Out] ((-I/2)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

Rubi [A] time = 0.0143722, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-I/2)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Mathematica [A] time = 0.054802, size = 31, normalized size = 1.

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-I/2)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

Maple [B] time = 0.047, size = 73, normalized size = 2.4

$$\frac{1}{d}\left(-a^{2}\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ia^{2}(\cos(dx+c))^{2}+a^{2}\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-I*a^2*cos(d*x+c)^2+a^2
*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [B] time = 0.999395, size = 93, normalized size = 3.

$$-\frac{i\,a^{2}\cos(dx+c)^{2}}{d}+\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^{2}}{4\,d}-\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^{2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -I*a^2*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d - 1/4*
(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d
```

Fricas [A] time = 2.18604, size = 46, normalized size = 1.48

$$-\frac{i\,a^{2}e^{(2i\,dx+2i\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Sympy [A] time = 0.160782, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } 2d \neq 0 \\ a^2xe^{2ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(2*d, 0)), (a**2*x*exp(
2*I*c), True))
```

Giac [B] time = 1.16402, size = 70, normalized size = 2.26

$$-\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{4\,d} - \frac{i\,a^2e^{(-2i\,dx-2i\,c)}}{4\,d} + \frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/4*I*a^2*e^(2*I*d*x + 2*I*c)/d - 1/4*I*a^2*e^(-2*I*d*x - 2*I*c)/d + 1/2*a
^2*sin(2*d*x + 2*c)/d
```

3.252 $\int (a\cos(c+dx)+ia\sin(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0140323, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2637, 2638}

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+ia\sin(c+dx))\,dx &= (ia)\int \sin(c+dx)\,dx+a\int \cos(c+dx)\,dx\\ &= -\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0118994, size = 51, normalized size = 1.96

$$\frac{ia\sin(c)\sin(dx)}{d}-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

```
[Out] ((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
+ (I*a*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.001, size = 26, normalized size = 1.

$$\frac{-ia\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+I*a*sin(d*x+c),x)
```

```
[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Maxima [A] time = 0.98745, size = 32, normalized size = 1.23

$$-\frac{i\,a\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Fricas [A] time = 2.16157, size = 32, normalized size = 1.23

$$-\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -I*a*e^(I*d*x + I*c)/d
```

Sympy [A] time = 0.144124, size = 26, normalized size = 1.

$$\begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x)
```

```
[Out] Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

Giac [A] time = 1.12907, size = 32, normalized size = 1.23

$$-\frac{i\,a\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.253 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rubi [A] time = 0.0147709, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Mathematica [A] time = 0.0349311, size = 29, normalized size = 1.

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Maple [A] time = 0.074, size = 23, normalized size = 0.8

$$2\,\frac{1}{ad\left(\tan\left(1/2\,dx+c/2\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 2/d/a/(tan(1/2*d*x+1/2*c)-I)
```

Maxima [A] time = 0.991109, size = 39, normalized size = 1.34

$$\frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A] time = 2.10801, size = 35, normalized size = 1.21

$$\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [A] time = 0.160911, size = 31, normalized size = 1.07

$$\begin{cases} \frac{ie^{-ic}e^{-idx}}{ad} & \text{for } ade^{ic} \neq 0 \\ \frac{xe^{-ic}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*c
)/a, True))
```

Giac [A] time = 1.12504, size = 28, normalized size = 0.97

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

3.254 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0155649, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Mathematica [A] time = 0.0453233, size = 31, normalized size = 1.

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Maple [A] time = 0.092, size = 23, normalized size = 0.7

$$\frac{i}{a^{2}d\left(i\tan\left(dx+c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] I/d/a^2/(I*tan(d*x+c)+1)
```

Maxima [A] time = 0.982692, size = 30, normalized size = 0.97

$$\frac{1}{\left(a^{2}\tan\left(dx+c\right)-i\,a^{2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/((a^2*tan(d*x + c) - I*a^2)*d)
```

Fricas [A] time = 1.99282, size = 49, normalized size = 1.58

$$\frac{i\,e^{\left(-2i\,dx-2i\,c\right)}}{2\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [A] time = 0.179153, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for } 2a^2de^{2ic} \neq 0 \\ \frac{xe^{-2ic}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(2*a**2*d*exp(2*I*c),
0)), (x*exp(-2*I*c)/a**2, True))
```

Giac [A] time = 1.11583, size = 41, normalized size = 1.32

$$-\frac{2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^2 d \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

3.255 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=31

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rubi [A] time = 0.0153039, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Mathematica [A] time = 0.0468053, size = 31, normalized size = 1.

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Maple [B] time = 0.108, size = 57, normalized size = 1.8

$$2\,\frac{1}{da^3}\left(\frac{2\,i}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^2}+\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-1}-4/3\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(2*I/(tan(1/2*d*x+1/2*c)-I)^2+1/(tan(1/2*d*x+1/2*c)-I)-4/3/(tan(1/2
*d*x+1/2*c)-I)^3)
```

Maxima [A] time = 1.03654, size = 39, normalized size = 1.26

$$\frac{i\,\cos\left(3\,dx+3\,c\right)+\sin\left(3\,dx+3\,c\right)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Fricas [A] time = 1.99961, size = 49, normalized size = 1.58

$$\frac{i\,e^{\left(-3i\,dx-3i\,c\right)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [A] time = 0.181533, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for } 3a^3de^{3ic} \neq 0 \\ \frac{xe^{-3ic}}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(3*a**3*d*exp(3*I*c),
0)), (x*exp(-3*I*c)/a**3, True))
```

Giac [A] time = 1.10743, size = 49, normalized size = 1.58

$$\frac{2\left(3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)}{3a^3d\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

3.256 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx$

Optimal. Leaf size=31

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

```
[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

Rubi [A] time = 0.0148451, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
```

```
[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = \frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Mathematica [A] time = 0.0486514, size = 31, normalized size = 1.

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
```

```
[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

Maple [A] time = 0.115, size = 36, normalized size = 1.2

$$\frac{1}{da^4}\left(\frac{-i}{(\tan(dx+c)-i)^2}-(\tan(dx+c)-i)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(-I/(tan(d*x+c)-I)^2-1/(tan(d*x+c)-I))
```

Maxima [A] time = 1.03061, size = 39, normalized size = 1.26

$$\frac{i\cos(4\,dx+4\,c)+\sin(4\,dx+4\,c)}{4\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(I*cos(4*d*x + 4*c) + sin(4*d*x + 4*c))/(a^4*d)
```

Fricas [A] time = 2.06525, size = 49, normalized size = 1.58

$$\frac{i\,e^{(-4i\,dx-4i\,c)}}{4\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [A] time = 0.194294, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-4ic}e^{-4idx}}{4a^4d} & \text{for } 4a^4de^{4ic} \neq 0 \\ \frac{xe^{-4ic}}{a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((I*exp(-4*I*c)*exp(-4*I*d*x)/(4*a**4*d), Ne(4*a**4*d*exp(4*I*c),
0)), (x*exp(-4*I*c)/a**4, True))
```

Giac [A] time = 1.14093, size = 59, normalized size = 1.9

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1/
2*c) - I)^4)
```

3.257 $\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

```
[Out] (((-2*I)/5)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.015664, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx=-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

Mathematica [A] time = 0.0304076, size = 32, normalized size = 0.97

$$-\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))/d
```

Maple [A] time = 0.055, size = 28, normalized size = 0.9

$$\frac{-\frac{2i}{5}}{d}\left(a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/5*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)/d
```

Maxima [B] time = 1.57343, size = 69, normalized size = 2.09

$$-\frac{2i\,a^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*a^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(5/2))
```

Fricas [A] time = 2.00607, size = 57, normalized size = 1.73

$$-\frac{2i\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.14632, size = 23, normalized size = 0.7

$$-\frac{2 i a^{\frac{5}{2}} e^{\left(\frac{5}{2} i d x + \frac{5}{2} i c\right)}}{5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

3.258 $\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

```
[Out] (((-2*I)/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.0160221, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx=-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

Mathematica [A] time = 0.0294467, size = 32, normalized size = 0.97

$$-\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))/d
```

Maple [A] time = 0.03, size = 28, normalized size = 0.9

$$\frac{-\frac{2i}{3}}{d}\left(a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)/d
```

Maxima [B] time = 1.54804, size = 69, normalized size = 2.09

$$-\frac{2i\,a^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*a^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(3/2))
```

Fricas [A] time = 1.9168, size = 57, normalized size = 1.73

$$-\frac{2i\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*I*a^(3/2)*e^(3/2*I*d*x + 3/2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15549, size = 34, normalized size = 1.03

$$-\frac{2i\,(a\cos\left(dx+c\right)+i\,a\sin\left(dx+c\right))^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*I*(a*cos(d*x + c) + I*a*sin(d*x + c))^(3/2)/d
```

3.259 $\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

```
[Out] ((-2*I)*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])/d
```

Rubi [A] time = 0.0160164, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx=-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

Mathematica [A] time = 0.0220753, size = 30, normalized size = 0.97

$$-\frac{2i\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])/d
```

Maple [A] time = 0.033, size = 28, normalized size = 0.9

$$\frac{-2\,i}{d}\sqrt{a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)/d
```

Maxima [B] time = 1.54232, size = 69, normalized size = 2.23

$$-\frac{2i\,\sqrt{a}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(a)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + I)/(d*sqrt(sin(d*x + c
)/(cos(d*x + c) + 1) + I))
```

Fricas [A] time = 2.01866, size = 54, normalized size = 1.74

$$-\frac{2i\,\sqrt{a}e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*I*sqrt(a)*e^(1/2*I*d*x + 1/2*I*c)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia\sin(c+dx)+a\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

Giac [A] time = 1.6811, size = 23, normalized size = 0.74

$$-\frac{2i\sqrt{a}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*sqrt(a)*e^(1/2*I*d*x + 1/2*I*c)/d
```

3.260 $\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

```
[Out] (2*I)/(d*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])
```

Rubi [A] time = 0.017475, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] (2*I)/(d*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx = \frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

Mathematica [A] time = 0.0350954, size = 30, normalized size = 0.97

$$\frac{2i}{d\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] (2*I)/(d*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])
```

Maple [A] time = 0.029, size = 28, normalized size = 0.9

$$\frac{2i}{d}\frac{1}{\sqrt{a\cos(dx+c)+ia\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)
```

Maxima [B] time = 1.54175, size = 69, normalized size = 2.23

$$\frac{2i\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{\sqrt{a}d\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + I)/(sqrt(a)*d*sqrt(-sin(d*x + c)
/(cos(d*x + c) + 1) + I))
```

Fricas [A] time = 1.94906, size = 57, normalized size = 1.84

$$\frac{2i\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*I*e^(-1/2*I*d*x - 1/2*I*c)/(sqrt(a)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ia\sin{(c+dx)}+a\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

Giac [A] time = 1.70704, size = 50, normalized size = 1.61

$$\frac{2i}{d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*I/(d*sqrt(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

3.261 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

```
[Out] ((2*I)/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0161539, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx=\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0327103, size = 32, normalized size = 0.97

$$\frac{2i}{3d(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)/3)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.03, size = 28, normalized size = 0.9

$$\frac{\frac{2i}{3}}{d}\left(a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)
```

Maxima [B] time = 1.62867, size = 69, normalized size = 2.09

$$\frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(a^(3/2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + I)^(3/2))
```

Fricas [A] time = 2.08097, size = 59, normalized size = 1.79

$$\frac{2i\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*I*e^(-3/2*I*d*x - 3/2*I*c)/(a^(3/2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.70679, size = 50, normalized size = 1.52

$$\frac{2i}{3\,d\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*I/(d*(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I))^(3/2)
)
```

3.262 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

```
[Out] ((2*I)/5)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))
```

Rubi [A] time = 0.0163566, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] ((2*I)/5)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx=\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.036128, size = 32, normalized size = 0.97

$$\frac{2i}{5d(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] ((2*I)/5)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.029, size = 28, normalized size = 0.9

$$\frac{\frac{2i}{5}}{d}(a\cos(dx+c)+ia\sin(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)
```

Maxima [B] time = 1.58497, size = 69, normalized size = 2.09

$$\frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5a^{\frac{5}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(a^(5/2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + I)^(5/2))
```

Fricas [A] time = 1.92852, size = 59, normalized size = 1.79

$$\frac{2ie^{\left(-\frac{5}{2}idx-\frac{5}{2}ic\right)}}{5a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*I*e^(-5/2*I*d*x - 5/2*I*c)/(a^(5/2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.79466, size = 50, normalized size = 1.52

$$\frac{2i}{5\,d\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*I/(d*(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I))^(5/2)
)
```

3.263 $\int (a\sec(x)+b\tan(x))^5\,dx$

Optimal. Leaf size=149

$$-\frac{1}{8}ab^4\left(7-\frac{3a^2}{b^2}\right)\sin(x)-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(\sin(x)+1)+$$

```
[Out] -((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]])/16 + ((a - b)^3*(3*a^2
 + 9*a*b + 8*b^2)*Log[1 + Sin[x]])/16 - (a*(7 - (3*a^2)/b^2)*b^4*Sin[x])/8
+ (Sec[x]^4*(b + a*Sin[x])*(a + b*Sin[x])^4)/4 + (Sec[x]^2*(a + b*Sin[x])^2
*(2*b*(a^2 - 2*b^2) + a*(3*a^2 - 5*b^2)*Sin[x]))/8
```

Rubi [A] time = 0.211375, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4391, 2668, 739, 819, 774, 633, 31}

$$-\frac{1}{8}ab^4\left(7-\frac{3a^2}{b^2}\right)\sin(x)-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(\sin(x)+1)+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^5,x]
```

```
[Out] -((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]])/16 + ((a - b)^3*(3*a^2
 + 9*a*b + 8*b^2)*Log[1 + Sin[x]])/16 - (a*(7 - (3*a^2)/b^2)*b^4*Sin[x])/8
+ (Sec[x]^4*(b + a*Sin[x])*(a + b*Sin[x])^4)/4 + (Sec[x]^2*(a + b*Sin[x])^2
*(2*b*(a^2 - 2*b^2) + a*(3*a^2 - 5*b^2)*Sin[x]))/8
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a\sec(x)+b\tan(x))^5\,dx=\int \sec^5(x)(a+b\sin(x))^5\,dx$$
$$=b^5\,\text{Subst}\left(\int \frac{(a+x)^5}{\left(b^2-x^2\right)^3}\,dx,x,b\sin(x)\right)$$
$$=\frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4-\frac{1}{4}b^3\,\text{Subst}\left(\int \frac{(a+x)^3\left(-3a^2+4b^2+ax\right)}{\left(b^2-x^2\right)^2}\,dx,x,b\,\text{s}$$
$$=\frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4+\frac{1}{8}\sec^2(x)(a+b\sin(x))^2\left(2b\left(a^2-2b^2\right)+a\left(3a^2-5\right.\right.$$
$$=\frac{1}{8}ab^2\left(3a^2-7b^2\right)\sin(x)+\frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4+\frac{1}{8}\sec^2(x)(a+b\sin(x))^2\left(\right.$$
$$=\frac{1}{8}ab^2\left(3a^2-7b^2\right)\sin(x)+\frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4+\frac{1}{8}\sec^2(x)(a+b\sin(x))^2\left(\right.$$
$$=-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(1+\sin(x$$

Mathematica [B] time = 1.22735, size = 303, normalized size = 2.03

$$-\frac{-2ab^6\left(3a^2+5b^2\right)\sin^5(x)+4b^5\left(-12a^2b^2-9a^4+b^4\right)\sin^4(x)-10ab^4\left(8a^2b^2+9a^4-b^4\right)\sin^3(x)+8b^3\left(-4a^4b^2-2a^2b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^5,x]
```

```
[Out] -((a^2 - b^2)^2*((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]] - (a - b
)^3*(3*a^2 + 9*a*b + 8*b^2)*Log[1 + Sin[x]]) - 10*a*b^2*(9*a^6 - 6*a^4*b^2
+ 8*a^2*b^4 - 3*b^6)*Sin[x] + 8*b^3*(-15*a^6 - 4*a^4*b^2 - 2*a^2*b^4 + b^6)
*Sin[x]^2 - 10*a*b^4*(9*a^4 + 8*a^2*b^2 - b^4)*Sin[x]^3 + 4*b^5*(-9*a^4 - 1
2*a^2*b^2 + b^4)*Sin[x]^4 - 2*a*b^6*(3*a^2 + 5*b^2)*Sin[x]^5 + 4*(-a^2 + b^
2)*Sec[x]^4*(-b + a*Sin[x])*(a + b*Sin[x])^6 + 2*Sec[x]^2*(a + b*Sin[x])^6*
(6*a^2*b + 2*b^3 - 3*a^3*Sin[x] - 5*a*b^2*Sin[x]))/(16*(a^2 - b^2)^2)
```

Maple [A] time = 0.075, size = 199, normalized size = 1.3

$$\frac{a^5\tan(x)(\sec(x))^3}{4}+\frac{3a^5\sec(x)\tan(x)}{8}+\frac{3a^5\ln(\sec(x)+\tan(x))}{8}+\frac{5a^4b}{4(\cos(x))^4}+\frac{5a^3b^2(\sin(x))^3}{2(\cos(x))^4}+\frac{5a^3b^2(\sin(x)}{4(\cos(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^5,x)
```

```
[Out] 1/4*a^5*tan(x)*sec(x)^3+3/8*a^5*sec(x)*tan(x)+3/8*a^5*ln(sec(x)+tan(x))+5/4
*a^4*b/cos(x)^4+5/2*a^3*b^2*sin(x)^3/cos(x)^4+5/4*a^3*b^2*sin(x)^3/cos(x)^2
+5/4*sin(x)*a^3*b^2-5/4*a^3*b^2*ln(sec(x)+tan(x))+5/2*a^2*b^3*sin(x)^4/cos(
x)^4+5/4*a*b^4*sin(x)^5/cos(x)^4-5/8*a*b^4*sin(x)^5/cos(x)^2-5/8*a*b^4*sin(
x)^3-15/8*sin(x)*a*b^4+15/8*a*b^4*ln(sec(x)+tan(x))+1/4*b^5*tan(x)^4-1/2*b^
5*tan(x)^2-b^5*ln(cos(x))
```

Maxima [A] time = 1.10693, size = 275, normalized size = 1.85

$$\frac{5}{2}\,a^2b^3\tan\left(x\right)^4+\frac{5}{16}\,ab^4\left(\frac{2\left(5\,\sin\left(x\right)^3-3\,\sin\left(x\right)\right)}{\sin\left(x\right)^4-2\,\sin\left(x\right)^2+1}+3\,\log\left(\sin\left(x\right)+1\right)-3\,\log\left(\sin\left(x\right)-1\right)\right)-\frac{1}{16}\,a^5\left(\frac{2\left(3\,\sin\left(x\right)^3-5\right.}{\sin\left(x\right)^4-2\,\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*a^2*b^3*tan(x)^4 + 5/16*a*b^4*(2*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*
sin(x)^2 + 1) + 3*log(sin(x) + 1) - 3*log(sin(x) - 1)) - 1/16*a^5*(2*(3*sin
(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 3*log(sin(x) + 1) + 3*log(s
in(x) - 1)) + 5/8*a^3*b^2*(2*(sin(x)^3 + sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1
) - log(sin(x) + 1) + log(sin(x) - 1)) + 1/4*b^5*((4*sin(x)^2 - 3)/(sin(x)^
4 - 2*sin(x)^2 + 1) - 2*log(sin(x)^2 - 1)) + 5/4*a^4*b/(sin(x)^2 - 1)^2
```

Fricas [A] time = 2.39677, size = 405, normalized size = 2.72

$$\frac{\left(3\,a^5-10\,a^3b^2+15\,ab^4-8\,b^5\right)\cos\left(x\right)^4\log\left(\sin\left(x\right)+1\right)-\left(3\,a^5-10\,a^3b^2+15\,ab^4+8\,b^5\right)\cos\left(x\right)^4\log\left(-\sin\left(x\right)+1\right)+2}{16\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

```
[Out] 1/16*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cos(x)^4*log(sin(x) + 1) - (3
*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cos(x)^4*log(-sin(x) + 1) + 20*a^4*b
```

```
+ 40*a^2*b^3 + 4*b^5 - 16*(5*a^2*b^3 + b^5)*cos(x)^2 + 2*(2*a^5 + 20*a^3*b^
2 + 10*a*b^4 + (3*a^5 - 10*a^3*b^2 - 25*a*b^4)*cos(x)^2)*sin(x))/cos(x)^4
```

Sympy [B] time = 8.31164, size = 308, normalized size = 2.07

$$-\frac{3a^5\log(\sin(x)-1)}{16}+\frac{3a^5\log(\sin(x)+1)}{16}-\frac{3a^5\sin^3(x)}{8\sin^4(x)-16\sin^2(x)+8}+\frac{5a^5\sin(x)}{8\sin^4(x)-16\sin^2(x)+8}+\frac{5a^4b\sec^4(x)}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**5,x)
```

```
[Out] -3*a**5*log(sin(x) - 1)/16 + 3*a**5*log(sin(x) + 1)/16 - 3*a**5*sin(x)**3/(
8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2
 + 8) + 5*a**4*b*sec(x)**4/4 + 5*a**3*b**2*log(sin(x) - 1)/8 - 5*a**3*b**2*
log(sin(x) + 1)/8 + 10*a**3*b**2*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8)
 + 10*a**3*b**2*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**2*b**3*tan(x
)**4/2 - 15*a*b**4*log(sin(x) - 1)/16 + 15*a*b**4*log(sin(x) + 1)/16 + 25*a
*b**4*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*a*b**4*sin(x)/(8*sin(
x)**4 - 16*sin(x)**2 + 8) + b**5*log(sec(x)**2)/2 + b**5*sec(x)**4/4 - b**5
*sec(x)**2
```

Giac [A] time = 1.11954, size = 240, normalized size = 1.61

$$\frac{1}{16}\left(3a^5-10a^3b^2+15ab^4-8b^5\right)\log(\sin(x)+1)-\frac{1}{16}\left(3a^5-10a^3b^2+15ab^4+8b^5\right)\log(-\sin(x)+1)+\frac{6b^5\sin(x)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*log(sin(x) + 1) - 1/16*(3*a^5
- 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*log(-sin(x) + 1) + 1/8*(6*b^5*sin(x)^4 - 3
*a^5*sin(x)^3 + 10*a^3*b^2*sin(x)^3 + 25*a*b^4*sin(x)^3 + 40*a^2*b^3*sin(x)
^2 - 4*b^5*sin(x)^2 + 5*a^5*sin(x) + 10*a^3*b^2*sin(x) - 15*a*b^4*sin(x) +
10*a^4*b - 20*a^2*b^3)/(sin(x)^2 - 1)^2
```

3.264 $\int (a\sec(x)+b\tan(x))^4\,dx$

Optimal. Leaf size=100

$$\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\sin(x)\cos(x)-\frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)+\frac{1}{3}\sec^3(x)(a$$

```
[Out] b^4*x + (4*a*b*(a^2 - 2*b^2)*Cos[x])/3 + (b^2*(2*a^2 - 3*b^2)*Cos[x]*Sin[x]
)/3 + (Sec[x]^3*(b + a*Sin[x])*(a + b*Sin[x])^3)/3 - (Sec[x]*(a + b*Sin[x])
^2*(a*b - (2*a^2 - 3*b^2)*Sin[x]))/3
```

Rubi [A] time = 0.196253, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2691, 2861, 2734}

$$\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\sin(x)\cos(x)-\frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)+\frac{1}{3}\sec^3(x)(a$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^4,x]
```

```
[Out] b^4*x + (4*a*b*(a^2 - 2*b^2)*Cos[x])/3 + (b^2*(2*a^2 - 3*b^2)*Cos[x]*Sin[x]
)/3 + (Sec[x]^3*(b + a*Sin[x])*(a + b*Sin[x])^3)/3 - (Sec[x]*(a + b*Sin[x])
^2*(a*b - (2*a^2 - 3*b^2)*Sin[x]))/3
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a\sec(x)+b\tan(x))^4\,dx = \int \sec^4(x)(a+b\sin(x))^4\,dx$$
$$= \frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3 - \frac{1}{3}\int \sec^2(x)(a+b\sin(x))^2\left(-2a^2+3b^2+ab\sin(x)\right.$$
$$= \frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3 - \frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)+$$
$$= b^4x+\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\cos(x)\sin(x)+\frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b$$

Mathematica [A] time = 0.192781, size = 96, normalized size = 0.96

$$\frac{1}{12}\sec^3(x)\left(18a^2b^2\sin(x)-6a^2b^2\sin(3x)+16a^3b+6a^4\sin(x)+2a^4\sin(3x)-24ab^3\cos(2x)-8ab^3-4b^4\sin(3x)+9b^4x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^4,x]
```

```
[Out] (Sec[x]^3*(16*a^3*b - 8*a*b^3 + 9*b^4*x*Cos[x] - 24*a*b^3*Cos[2*x] + 3*b^4*
x*Cos[3*x] + 6*a^4*Sin[x] + 18*a^2*b^2*Sin[x] + 2*a^4*Sin[3*x] - 6*a^2*b^2*
Sin[3*x] - 4*b^4*Sin[3*x]))/12
```

Maple [A] time = 0.06, size = 96, normalized size = 1.

$$-a^4\left(-\frac{2}{3}-\frac{(\sec(x))^2}{3}\right)\tan(x)+\frac{4a^3b}{3(\cos(x))^3}+2\frac{a^2b^2(\sin(x))^3}{(\cos(x))^3}+4ab^3\left(1/3\frac{(\sin(x))^4}{(\cos(x))^3}-1/3\frac{(\sin(x))^4}{\cos(x)}-1/3\left(2+(\sin(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^4,x)
```

```
[Out] -a^4*(-2/3-1/3*sec(x)^2)*tan(x)+4/3*a^3*b/cos(x)^3+2*a^2*b^2*sin(x)^3/cos(x
)^3+4*a*b^3*(1/3*sin(x)^4/cos(x)^3-1/3*sin(x)^4/cos(x)-1/3*(2+sin(x)^2)*cos
(x))+b^4*(1/3*tan(x)^3-tan(x)+x)
```

Maxima [A] time = 1.60153, size = 97, normalized size = 0.97

$$2a^2b^2\tan(x)^3+\frac{1}{3}\left(\tan(x)^3+3\tan(x)\right)a^4+\frac{1}{3}\left(\tan(x)^3+3x-3\tan(x)\right)b^4-\frac{4\left(3\cos(x)^2-1\right)ab^3}{3\cos(x)^3}+\frac{4a^3b}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

```
[Out] 2*a^2*b^2*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*a^4 + 1/3*(tan(x)^3 + 3*x -
3*tan(x))*b^4 - 4/3*(3*cos(x)^2 - 1)*a*b^3/cos(x)^3 + 4/3*a^3*b/cos(x)^3
```

Fricas [A] time = 2.18665, size = 196, normalized size = 1.96

$$\frac{3b^4x\cos(x)^3-12ab^3\cos(x)^2+4a^3b+4ab^3+\left(a^4+6a^2b^2+b^4+2\left(a^4-3a^2b^2-2b^4\right)\cos(x)^2\right)\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x*cos(x)^3 - 12*a*b^3*cos(x)^2 + 4*a^3*b + 4*a*b^3 + (a^4 + 6*a^
2*b^2 + b^4 + 2*(a^4 - 3*a^2*b^2 - 2*b^4)*cos(x)^2)*sin(x))/cos(x)^3
```

Sympy [A] time = 4.69919, size = 97, normalized size = 0.97

$$\frac{a^4\tan^3(x)}{3}+a^4\tan(x)+\frac{4a^3b\sec^3(x)}{3}+2a^2b^2\tan^3(x)+\frac{4ab^3\sec^3(x)}{3}-4ab^3\sec(x)+b^4x+\frac{b^4\sin^3(x)}{3\cos^3(x)}-\frac{b^4\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**4,x)
```

```
[Out] a**4*tan(x)**3/3 + a**4*tan(x) + 4*a**3*b*sec(x)**3/3 + 2*a**2*b**2*tan(x)*
*3 + 4*a*b**3*sec(x)**3/3 - 4*a*b**3*sec(x) + b**4*x + b**4*sin(x)**3/(3*co
s(x)**3) - b**4*sin(x)/cos(x)
```

Giac [A] time = 1.12689, size = 177, normalized size = 1.77

$$b^4x-\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,x\right)^5-3\,b^4\tan\left(\frac{1}{2}\,x\right)^5+12\,a^3b\tan\left(\frac{1}{2}\,x\right)^4-2\,a^4\tan\left(\frac{1}{2}\,x\right)^3+24\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3+10\,b^4\tan\left(\frac{1}{2}\,x\right)^3+\cdots\right.}{3\left(\tan\left(\frac{1}{2}\,x\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

```
[Out] b^4*x - 2/3*(3*a^4*tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a^3*b*tan(1/2*x)^
4 - 2*a^4*tan(1/2*x)^3 + 24*a^2*b^2*tan(1/2*x)^3 + 10*b^4*tan(1/2*x)^3 + 24
*a*b^3*tan(1/2*x)^2 + 3*a^4*tan(1/2*x) - 3*b^4*tan(1/2*x) + 4*a^3*b - 8*a*b
^3)/(tan(1/2*x)^2 - 1)^3
```

3.265 $\int (a\sec(x)+b\tan(x))^3\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}ab^2\sin(x)+\frac{1}{4}(a+2b)(a-b)^2\log(\sin(x)+1)-\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{2}\sec^2(x)(a\sin(x)+b)(a+b\sin(x$$

```
[Out] -((a - 2*b)*(a + b)^2*Log[1 - Sin[x]])/4 + ((a - b)^2*(a + 2*b)*Log[1 + Sin
[x]])/4 + (a*b^2*Sin[x])/2 + (Sec[x]^2*(b + a*Sin[x])*(a + b*Sin[x])^2)/2
```

Rubi [A] time = 0.136336, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4391, 2668, 739, 774, 633, 31}

$$\frac{1}{2}ab^2\sin(x)+\frac{1}{4}(a+2b)(a-b)^2\log(\sin(x)+1)-\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{2}\sec^2(x)(a\sin(x)+b)(a+b\sin(x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^3,x]
```

```
[Out] -((a - 2*b)*(a + b)^2*Log[1 - Sin[x]])/4 + ((a - b)^2*(a + 2*b)*Log[1 + Sin
[x]])/4 + (a*b^2*Sin[x])/2 + (Sec[x]^2*(b + a*Sin[x])*(a + b*Sin[x])^2)/2
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
```

```
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a\sec(x)+b\tan(x))^3\,dx = \int \sec^3(x)(a+b\sin(x))^3\,dx$$

$$= b^3\,\mathrm{Subst}\left(\int \frac{(a+x)^3}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(x)\right)$$

$$= \frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2 - \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{(a+x)\left(-a^2+2b^2+ax\right)}{b^2-x^2}\,dx, x, b\sin(\right.$$

$$= \frac{1}{2}ab^2\sin(x) + \frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2 + \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{-ab^2-a\left(-a^2+2b^2\right)}{b^2-x^2}\right.$$

$$= \frac{1}{2}ab^2\sin(x) + \frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2 + \frac{1}{4}\left((a-2b)(a+b)^2\right)\mathrm{Subst}\left(\int \frac{1}{b-x}\right.$$

$$= -\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x)) + \frac{1}{4}(a-b)^2(a+2b)\log(1+\sin(x)) + \frac{1}{2}ab^2\sin(x) + \frac{1}{2}\mathrm{se}$$

Mathematica [A] time = 0.57476, size = 123, normalized size = 1.64

$$\frac{\left(4a^2b^3-8a^4b+2b^5\right)\tan^2(x)+\left(a^2-b^2\right)\left((a-2b)(a+b)^2\log(1-\sin(x))-(a-b)^2(a+2b)\log(\sin(x)+1)\right)-2a\left(2a^2b\right.}{4\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^3,x]
```

```
[Out] ((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[x]] - (a - b)^2*(a + 2*b)*Log
[1 + Sin[x]]) + 2*a^4*b*Sec[x]^2 - 2*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sec[x]*Tan
[x] + (-8*a^4*b + 4*a^2*b^3 + 2*b^5)*Tan[x]^2)/(4*(-a^2 + b^2))
```

Maple [A] time = 0.063, size = 82, normalized size = 1.1

$$\frac{a^3\sec(x)\tan(x)}{2}+\frac{a^3\ln(\sec(x)+\tan(x))}{2}+\frac{3a^2b}{2(\cos(x))^2}+\frac{3ab^2(\sin(x))^3}{2(\cos(x))^2}+\frac{3ab^2\sin(x)}{2}-\frac{3ab^2\ln(\sec(x)+\tan(x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^3,x)
```

```
[Out] 1/2*a^3*sec(x)*tan(x)+1/2*a^3*ln(sec(x)+tan(x))+3/2*a^2*b/cos(x)^2+3/2*a*b^
2*sin(x)^3/cos(x)^2+3/2*a*b^2*sin(x)-3/2*a*b^2*ln(sec(x)+tan(x))+1/2*b^3*ta
n(x)^2+b^3*ln(cos(x))
```

Maxima [A] time = 1.09997, size = 128, normalized size = 1.71

$$\frac{3}{2}a^2b\tan(x)^2-\frac{3}{4}ab^2\left(\frac{2\sin(x)}{\sin(x)^2-1}+\log(\sin(x)+1)-\log(\sin(x)-1)\right)-\frac{1}{4}a^3\left(\frac{2\sin(x)}{\sin(x)^2-1}-\log(\sin(x)+1)+\log(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tan(x)^2 - 3/4*a*b^2*(2*sin(x)/(sin(x)^2 - 1) + log(sin(x) + 1) -
 log(sin(x) - 1)) - 1/4*a^3*(2*sin(x)/(sin(x)^2 - 1) - log(sin(x) + 1) + lo
g(sin(x) - 1)) - 1/2*b^3*(1/(sin(x)^2 - 1) - log(sin(x)^2 - 1))
```

Fricas [A] time = 2.32701, size = 219, normalized size = 2.92

$$\frac{\left(a^3-3\,ab^2+2\,b^3\right)\cos\left(x\right)^2\log\left(\sin\left(x\right)+1\right)-\left(a^3-3\,ab^2-2\,b^3\right)\cos\left(x\right)^2\log\left(-\sin\left(x\right)+1\right)+6\,a^2b+2\,b^3+2\left(a^3+3\,ab^2\right)}{4\,\cos\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(x)^2*log(sin(x) + 1) - (a^3 - 3*a*b^2 - 2*
b^3)*cos(x)^2*log(-sin(x) + 1) + 6*a^2*b + 2*b^3 + 2*(a^3 + 3*a*b^2)*sin(x)
)/cos(x)^2
```

Sympy [A] time = 5.11831, size = 122, normalized size = 1.63

$$-\frac{a^3\log\left(\sin\left(x\right)-1\right)}{4}+\frac{a^3\log\left(\sin\left(x\right)+1\right)}{4}-\frac{a^3\sin\left(x\right)}{2\sin^2\left(x\right)-2}+\frac{3a^2b\sec^2\left(x\right)}{2}+\frac{3ab^2\log\left(\sin\left(x\right)-1\right)}{4}-\frac{3ab^2\log\left(\sin\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**3,x)
```

```
[Out] -a**3*log(sin(x) - 1)/4 + a**3*log(sin(x) + 1)/4 - a**3*sin(x)/(2*sin(x)**2
 - 2) + 3*a**2*b*sec(x)**2/2 + 3*a*b**2*log(sin(x) - 1)/4 - 3*a*b**2*log(si
n(x) + 1)/4 - 3*a*b**2*sin(x)/(2*sin(x)**2 - 2) - b**3*log(sec(x)**2)/2 + b
**3*sec(x)**2/2
```

Giac [A] time = 1.14916, size = 116, normalized size = 1.55

$$\frac{1}{4}\left(a^3-3\,ab^2+2\,b^3\right)\log\left(\sin\left(x\right)+1\right)-\frac{1}{4}\left(a^3-3\,ab^2-2\,b^3\right)\log\left(-\sin\left(x\right)+1\right)-\frac{b^3\sin\left(x\right)^2+a^3\sin\left(x\right)+3\,ab^2\sin\left(x\right)+}{2\left(\sin\left(x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^3 - 3*a*b^2 + 2*b^3)*log(sin(x) + 1) - 1/4*(a^3 - 3*a*b^2 - 2*b^3)*l
og(-sin(x) + 1) - 1/2*(b^3*sin(x)^2 + a^3*sin(x) + 3*a*b^2*sin(x) + 3*a^2*b
)/(sin(x)^2 - 1)
```

3.266 $\int (a\sec(x)+b\tan(x))^2\,dx$

Optimal. Leaf size=27

$$ab\cos(x)+\sec(x)(a\sin(x)+b)(a+b\sin(x))+b^2(-x)$$

```
[Out] -(b^2*x) + a*b*Cos[x] + Sec[x]*(b + a*Sin[x])*(a + b*Sin[x])
```

Rubi [A] time = 0.0538816, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2691, 2638}

$$ab\cos(x)+\sec(x)(a\sin(x)+b)(a+b\sin(x))+b^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^2,x]
```

```
[Out] -(b^2*x) + a*b*Cos[x] + Sec[x]*(b + a*Sin[x])*(a + b*Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a\sec(x)+b\tan(x))^2\,dx = \int \sec^2(x)(a+b\sin(x))^2\,dx$$
$$= \sec(x)(b+a\sin(x))(a+b\sin(x)) - \int \left(b^2+ab\sin(x)\right)\,dx$$
$$= -b^2x+\sec(x)(b+a\sin(x))(a+b\sin(x)) - (ab)\int \sin(x)\,dx$$
$$= -b^2x+ab\cos(x)+\sec(x)(b+a\sin(x))(a+b\sin(x))$$

Mathematica [A] time = 0.0425597, size = 25, normalized size = 0.93

$$\left(a^2+b^2\right)\tan(x)+2ab\sec(x)+b^2\left(-\tan^{-1}(\tan(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^2,x]
```

```
[Out] -(b^2*ArcTan[Tan[x]]) + 2*a*b*Sec[x] + (a^2 + b^2)*Tan[x]
```

Maple [A] time = 0.044, size = 26, normalized size = 1.

$$a^2\tan(x)+2\,\frac{ab}{\cos(x)}+b^2(\tan(x)-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^2,x)
```

```
[Out] a^2*tan(x)+2*a*b/cos(x)+b^2*(tan(x)-x)
```

Maxima [A] time = 1.56571, size = 35, normalized size = 1.3

$$-b^2(x-\tan(x))+a^2\tan(x)+\frac{2\,ab}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="maxima")
```

```
[Out] -b^2*(x - tan(x)) + a^2*tan(x) + 2*a*b/cos(x)
```

Fricas [A] time = 2.08211, size = 72, normalized size = 2.67

$$-\frac{b^2 x \cos(x) - 2\,ab - (a^2 + b^2)\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
```

```
[Out] -(b^2*x*cos(x) - 2*a*b - (a^2 + b^2)*sin(x))/cos(x)
```

Sympy [A] time = 1.48019, size = 22, normalized size = 0.81

$$a^2 \tan(x) + 2ab\sec(x) + b^2(-x + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**2,x)
```

```
[Out] a**2*tan(x) + 2*a*b*sec(x) + b**2*(-x + tan(x))
```

Giac [A] time = 1.14645, size = 54, normalized size = 2.

$$-b^2 x - \frac{2\left(a^2 \tan\left(\frac{1}{2}\,x\right) + b^2 \tan\left(\frac{1}{2}\,x\right) + 2\,ab\right)}{\tan\left(\frac{1}{2}\,x\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="giac")
```

```
[Out] -b^2*x - 2*(a^2*tan(1/2*x) + b^2*tan(1/2*x) + 2*a*b)/(tan(1/2*x)^2 - 1)
```

3.267 $\int (a\sec(x)+b\tan(x))\,dx$

Optimal. Leaf size=12

$$a\tanh^{-1}(\sin(x))-b\log(\cos(x))$$

```
[Out] a*ArcTanh[Sin[x]] - b*Log[Cos[x]]
```

Rubi [A] time = 0.0074644, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$a\tanh^{-1}(\sin(x))-b\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Sec[x] + b*Tan[x],x]
```

```
[Out] a*ArcTanh[Sin[x]] - b*Log[Cos[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\sec(x)+b\tan(x))\,dx &= a\int \sec(x)\,dx+b\int \tan(x)\,dx\\ &= a\tanh^{-1}(\sin(x))-b\log(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.0053123, size = 42, normalized size = 3.5

$$-a\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+a\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-b\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sec[x] + b*Tan[x],x]
```

```
[Out] -(b*Log[Cos[x]]) - a*Log[Cos[x/2] - Sin[x/2]] + a*Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.002, size = 16, normalized size = 1.3

$$a\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)-b\ln\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sec(x)+b*tan(x),x)
```

```
[Out] a*ln(sec(x)+tan(x))-b*ln(cos(x))
```

Maxima [A] time = 1.00249, size = 19, normalized size = 1.58

$$a\log\left(\sec\left(x\right)+\tan\left(x\right)\right)+b\log\left(\sec\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x, algorithm="maxima")
```

```
[Out] a*log(sec(x) + tan(x)) + b*log(sec(x))
```

Fricas [B] time = 2.23704, size = 81, normalized size = 6.75

$$\frac{1}{2}\left(a-b\right)\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\left(a+b\right)\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(a - b)*log(sin(x) + 1) - 1/2*(a + b)*log(-sin(x) + 1)
```

Sympy [A] time = 0.107771, size = 24, normalized size = 2.

$$a\left(-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}\right)-b\log\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x)
```

```
[Out] a*(-log(sin(x) - 1)/2 + log(sin(x) + 1)/2) - b*log(cos(x))
```

Giac [B] time = 1.14809, size = 46, normalized size = 3.83

$$\frac{1}{4}a\left(\log\left(\left|\frac{1}{\sin\left(x\right)}+\sin\left(x\right)+2\right|\right)-\log\left(\left|\frac{1}{\sin\left(x\right)}+\sin\left(x\right)-2\right|\right)\right)-b\log\left(\left|\cos\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x, algorithm="giac")
```

```
[Out] 1/4*a*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) - 2))) -
 b*log(abs(cos(x)))
```

3.268 $\int \frac{1}{a\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sin(x))}{b}$$

```
[Out] Log[a + b*Sin[x]]/b
```

Rubi [A] time = 0.0342235, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2668, 31}

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx = \int \frac{\cos(x)}{a+b\sin(x)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}$$
$$= \frac{\log(a+b\sin(x))}{b}$$

Mathematica [A] time = 0.0064317, size = 11, normalized size = 1.

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Maple [A] time = 0.048, size = 12, normalized size = 1.1

$$\frac{\ln(a+b\sin(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x)),x)
```

```
[Out] ln(a+b*sin(x))/b
```

Maxima [B] time = 1.53328, size = 68, normalized size = 6.18

$$\frac{\log\left(a+\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b}-\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b - log(sin(x)
^2/(cos(x) + 1)^2 + 1)/b
```

Fricas [A] time = 2.28446, size = 28, normalized size = 2.55

$$\frac{\log\left(b\sin\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] log(b*sin(x) + a)/b
```

Sympy [A] time = 0.59975, size = 32, normalized size = 2.91

$$\begin{cases} \frac{\log\left(\frac{a\sec\left(x\right)}{b}+\tan\left(x\right)\right)}{b}-\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2b} & \text{for } b\neq 0 \\ \frac{\tan\left(x\right)}{a\sec\left(x\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x)
```

```
[Out] Piecewise((log(a*sec(x)/b + tan(x))/b - log(tan(x)**2 + 1)/(2*b), Ne(b, 0))
, (tan(x)/(a*sec(x)), True))
```

Giac [A] time = 1.13482, size = 16, normalized size = 1.45

$$\frac{\log\left(|b\sin\left(x\right)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(x) + a))/b
```

3.269 $\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$

Optimal. Leaf size=66

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}-\frac{x}{b^2}$$

```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b
^2]) - Cos[x]/(b*(a + b*Sin[x]))
```

Rubi [A] time = 0.133339, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4391, 2693, 2735, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-2),x]
```

```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b
^2]) - Cos[x]/(b*(a + b*Sin[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx = \int \frac{\cos^2(x)}{(a+b\sin(x))^2}\,dx$$
$$= -\frac{\cos(x)}{b(a+b\sin(x))} - \frac{\int \frac{\sin(x)}{a+b\sin(x)}\,dx}{b}$$
$$= -\frac{x}{b^2} - \frac{\cos(x)}{b(a+b\sin(x))} + \frac{a\int \frac{1}{a+b\sin(x)}\,dx}{b^2}$$
$$= -\frac{x}{b^2} - \frac{\cos(x)}{b(a+b\sin(x))} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^2}$$
$$= -\frac{x}{b^2} - \frac{\cos(x)}{b(a+b\sin(x))} - \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^2}$$
$$= -\frac{x}{b^2} + \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}} - \frac{\cos(x)}{b(a+b\sin(x))}$$

Mathematica [B] time = 3.75459, size = 422, normalized size = 6.39

$$\frac{\cos(x)\left(\sqrt{a+b}\left(\sqrt{a-b}\sqrt{1-\sin(x)}\left(b\left(b^2-a^2\right)\sqrt{\frac{b(\sin(x)+1)}{b-a}}\sqrt{\frac{b-b\sin(x)}{a+b}}+2a\left(a\sqrt{-b^2}+\sqrt{-b}b^{3/2}\sin(x)\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\frac{b(\sin(x)+1)}{b-a}}}{\sqrt{-b}\sqrt{\frac{b-b\sin(x)}{a+b}}}\right.\right.\right.\right.}{b^2(a-b)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-2),x]
```

```
[Out] (Cos[x]*(-2*a*(a - b)*b*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[x]))/(a - b
))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[x]))/(a + b))])]*Sqrt[1 - Sin[x]]*(a
+ b*Sin[x]) + Sqrt[a + b]*(2*ArcSinh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[x]))/(
a - b))])/(Sqrt[2]*Sqrt[b])]*Sqrt[(b - b*Sin[x])/(a + b)]*(a*b^(5/2) + a^2*
Sqrt[-b]*Sqrt[-b^2] - a*b^(5/2)*Sin[x] + (-b)^(3/2)*b*Sqrt[-b^2]*Sin[x]) +
Sqrt[a - b]*Sqrt[1 - Sin[x]]*(b*(-a^2 + b^2)*Sqrt[(b*(1 + Sin[x]))/(-a + b)
]*Sqrt[(b - b*Sin[x])/(a + b)] + 2*a*ArcTan[(Sqrt[b]*Sqrt[(b*(1 + Sin[x]))/
(-a + b)])/(Sqrt[-b]*Sqrt[(b - b*Sin[x])/(a + b)])]*(a*Sqrt[-b^2] + Sqrt[-b
]*b^(3/2)*Sin[x])))))/((a - b)^(3/2)*b^2*(a + b)^(3/2)*Sqrt[1 - Sin[x]]*Sqr
t[-((b*(1 + Sin[x]))/(a - b))]*Sqrt[(b - b*Sin[x])/(a + b)]*(a + b*Sin[x]))
```

Maple [A] time = 0.079, size = 106, normalized size = 1.6

$$-2\,\frac{\tan(x/2)}{\left(a\left(\tan(x/2)\right)^2+2\,b\tan(x/2)+a\right)a}-2\,\frac{1}{b\left(a\left(\tan(x/2)\right)^2+2\,b\tan(x/2)+a\right)}+2\,\frac{a}{b^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/\ldots}{\sqrt{a^2-\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^2,x)
```

```
[Out] -2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)/a*tan(1/2*x)-2/b/(a*tan(1/2*x)^2+2*b*t
an(1/2*x)+a)+2/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b
^2)^(1/2))-2/b^2*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39972, size = 687, normalized size = 10.41

$$\left[-\frac{2\left(a^2b-b^3\right)x\sin(x)+\left(ab\sin(x)+a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)+2\ldots}{2\left(a^3b^2-ab^4+\left(a^2b^3-b^5\right)\sin(x)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(-a^2 + b^2)*log((
(2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*
cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a
^3 - a*b^2)*x + 2*(a^2*b - b^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*
sin(x)), -((a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(a^2 - b^2)*arct
```

```
an(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^3 - a*b^2)*x + (a^2*b - b
^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(x) + b\tan(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**2,x)
```

```
[Out] Integral((a*sec(x) + b*tan(x))**(-2), x)
```

Giac [A] time = 1.14163, size = 127, normalized size = 1.92

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}b^2}-\frac{x}{b^2}-\frac{2\left(b\tan\left(\frac{1}{2}x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2+2\,b\tan\left(\frac{1}{2}x\right)+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a/(sqrt(a^2 - b^2)*b^2) - x/b^2 - 2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)
^2 + 2*b*tan(1/2*x) + a)*a*b)
```

3.270 $\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx$

Optimal. Leaf size=51

$$\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}-\frac{\log(a+b\sin(x))}{b^3}$$

```
[Out] -(Log[a + b*Sin[x]]/b^3) + (a^2 - b^2)/(2*b^3*(a + b*Sin[x])^2) - (2*a)/(b^
3*(a + b*Sin[x]))
```

Rubi [A] time = 0.0751398, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}-\frac{\log(a+b\sin(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-3),x]
```

```
[Out] -(Log[a + b*Sin[x]]/b^3) + (a^2 - b^2)/(2*b^3*(a + b*Sin[x])^2) - (2*a)/(b^
3*(a + b*Sin[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx &= \int \frac{\cos^3(x)}{(a+b\sin(x))^3}\,dx\\ &= \frac{\operatorname{Subst}\left(\int \frac{b^2-x^2}{(a+x)^3}\,dx,x,b\sin(x)\right)}{b^3}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2+b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)dx,x,b\sin(x)\right)}{b^3}\\ &= -\frac{\log(a+b\sin(x))}{b^3}+\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}\end{aligned}$$

Mathematica [A] time = 0.165935, size = 40, normalized size = 0.78

$$-\frac{\frac{3a^2+4ab\sin(x)+b^2}{2(a+b\sin(x))^2}+\log(a+b\sin(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-3),x]
```

```
[Out] -((Log[a + b*Sin[x]] + (3*a^2 + b^2 + 4*a*b*Sin[x])/(2*(a + b*Sin[x])^2))/b
^3)
```

Maple [A] time = 0.085, size = 57, normalized size = 1.1

$$-\frac{\ln(a+b\sin(x))}{b^3}-2\,\frac{a}{b^3\,(a+b\sin(x))}+\frac{a^2}{2\,b^3\,(a+b\sin(x))^2}-\frac{1}{2\,b\,(a+b\sin(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^3,x)
```

```
[Out] -ln(a+b*sin(x))/b^3-2*a/b^3/(a+b*sin(x))+1/2/b^3/(a+b*sin(x))^2*a^2-1/2/b/(
a+b*sin(x))^2
```

Maxima [B] time = 1.56461, size = 271, normalized size = 5.31

$$\frac{2\left(\frac{\left(a^3+ab^2\right)\sin(x)}{\cos(x)+1}+\frac{\left(3a^2b+b^3\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{\left(a^3+ab^2\right)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^4b^2+\frac{4a^3b^3\sin(x)}{\cos(x)+1}+\frac{4a^3b^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^4b^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{2\left(a^4b^2+2a^2b^4\right)\sin(x)^2}{(\cos(x)+1)^2}}-\frac{\log\left(a+\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^3}+\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

```
[Out] 2*((a^3 + a*b^2)*sin(x)/(cos(x) + 1) + (3*a^2*b + b^3)*sin(x)^2/(cos(x) + 1
)^2 + (a^3 + a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(x)/(c
os(x) + 1) + 4*a^3*b^3*sin(x)^3/(cos(x) + 1)^3 + a^4*b^2*sin(x)^4/(cos(x) +
 1)^4 + 2*(a^4*b^2 + 2*a^2*b^4)*sin(x)^2/(cos(x) + 1)^2) - log(a + 2*b*sin(
x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b^3 + log(sin(x)^2/(cos(x) + 1
)^2 + 1)/b^3
```

Fricas [A] time = 2.44208, size = 197, normalized size = 3.86

$$\frac{4\,ab\sin(x)+3\,a^2+b^2-2\left(b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2\right)\log(b\sin(x)+a)}{2\left(b^5\cos(x)^2-2\,ab^4\sin(x)-a^2b^3-b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*sin(x) + 3*a^2 + b^2 - 2*(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^
2)*log(b*sin(x) + a))/(b^5*cos(x)^2 - 2*a*b^4*sin(x) - a^2*b^3 - b^5)
```

Sympy [A] time = 4.61103, size = 508, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**3,x)
```

```
[Out] Piecewise((-2*a**2*log(a*sec(x)/b + tan(x))*sec(x)**2/(2*a**2*b**3*sec(x)**
2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + a**2*log(tan(x)**2 + 1)*se
c(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2)
 - 4*a*b*log(a*sec(x)/b + tan(x))*tan(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*
a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + 2*a*b*log(tan(x)**2 + 1)*tan(x)*
sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2)
+ 2*a*b*tan(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b
**5*tan(x)**2) - 2*b**2*log(a*sec(x)/b + tan(x))*tan(x)**2/(2*a**2*b**3*sec
(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + b**2*log(tan(x)**2 +
1)*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x
)**2) + 2*b**2*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) +
2*b**5*tan(x)**2) - b**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) +
2*b**5*tan(x)**2), Ne(b, 0)), ((2*tan(x)**3/(3*sec(x)**3) + tan(x)/sec(x)**
3)/a**3, True))
```

Giac [A] time = 1.15079, size = 58, normalized size = 1.14

$$-\frac{\log\left(\left|b\sin\left(x\right)+a\right|\right)}{b^3}+\frac{3\,b\sin\left(x\right)^2+2\,a\sin\left(x\right)-b}{2\,\left(b\sin\left(x\right)+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*sin(x) + a))/b^3 + 1/2*(3*b*sin(x)^2 + 2*a*sin(x) - b)/((b*sin(x
) + a)^2*b^2)
```

3.271 $\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

Optimal. Leaf size=156

$$-\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}+$$

```
[Out] x/b^4 - (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^4*(
a^2 - b^2)^(3/2)) - Cos[x]^3/(3*b*(a + b*Sin[x])^3) + (a*Cos[x]^3)/(2*b*(a^
2 - b^2)*(a + b*Sin[x])^2) + (Cos[x]*(2*(a^2 - b^2) + a*b*Sin[x]))/(2*b^3*(
a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.336986, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4391, 2693, 2864, 2863, 2735, 2660, 618, 204}

$$-\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-4),x]
```

```
[Out] x/b^4 - (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^4*(
a^2 - b^2)^(3/2)) - Cos[x]^3/(3*b*(a + b*Sin[x])^3) + (a*Cos[x]^3)/(2*b*(a^
2 - b^2)*(a + b*Sin[x])^2) + (Cos[x]*(2*(a^2 - b^2) + a*b*Sin[x]))/(2*b^3*(
a^2 - b^2)*(a + b*Sin[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx &= \int \frac{\cos^4(x)}{(a+b\sin(x))^4}\,dx \\
&= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} - \frac{\int \frac{\cos^2(x)\sin(x)}{(a+b\sin(x))^3}\,dx}{b} \\
&= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{\cos^2(x)(2b+a\sin(x))}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)} \\
&= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int}{} \\
&= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))} - \\
&= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))} - \\
&= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))} + \\
&= \frac{x}{b^4} - \frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} -
\end{aligned}
$$

Mathematica [B] time = 6.36915, size = 2677, normalized size = 17.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-4),x]
```

```
[Out] (Sec[x]*(a + b*Sin[x])^4*(-(b*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*(b/
(a + b) - (b*Sin[x])/(a + b))^(5/2))/(3*((a*b)/(a - b) - b^2/(a - b))*((a*b
)/(a + b) + b^2/(a + b))*(a + b*Sin[x])^3) - ((a*b^3*(-(b/(a - b)) - (b*Sin
[x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(2*(a^2 - b^2)*
((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x])^
2) - (-((((-3*a^2*b^5)/((a - b)^2*(a + b)^2) + (2*b^5*(3*a^2 - 2*b^2))/((a
- b)^2*(a + b)^2))*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*(b/(a + b) - (
b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b
^2/(a + b))*(a + b*Sin[x]))) - ((16*Sqrt[2]*b^6*(3*a^2 - 4*b^2)*(-(b/(a - b
)) - (b*Sin[x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[x])/(a + b)]*(1 + ((
a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)*((5*(1/(2*(1 + ((a
 - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(
a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*(((a - b)*(-(b/(a
- b)) - (b*Sin[x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[x])/(a -
 b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b
)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[x])
/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/
(2*b)])))/(32*(a - b)^3*(-(b/(a - b)) - (b*Sin[x])/(a - b))^3*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2)))/(5*(a - b)^2*(a + b)^4*Sq
rt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b]) + ((-((a*b^7*(6*a^2 - 7*b
^2))/((a - b)^3*(a + b)^3)) + (4*a*b^7*(3*a^2 - 4*b^2))/((a - b)^3*(a + b)^
3))*((4*Sqrt[2]*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(3/2)*Sqrt[b/(a + b) -
(b*Sin[x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b
))^(5/2)*((3/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2
) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1))/2 + (3*
b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]
*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*Sqr
t[b])]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*
(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b)])))/(8*(a - b)^2*(-(b/(a - b)) -
 (b*Sin[x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(
2*b))^2)))/(3*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b]) -
 ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a - b)) + b^2/(a - b))*(-(
((-((a*b)/(a + b)) - b^2/(a + b))*((-2*(-((a*b)/(a + b)) - b^2/(a + b))*Arc
Tan[(Sqrt[(a*b)/(a + b) + b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a -
b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[x])/(a +
 b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[(a*b)/(a + b) + b^2/(a
 + b)]) + (2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x]
)/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[x])/(a + b)])])/(b*Sqrt[a
+ b])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]*Sqr
t[b/(a + b) - (b*Sin[x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/
(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (
b*Sin[x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a -
b)) - (b*Sin[x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b))
)/(2*b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(
2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b])))/b
```

```
) + (4*Sqrt[2]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]*Sqrt[b/(a + b) - (b*
Sin[x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^
(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a -
b)])/(Sqrt[2]*Sqrt[b])]])/(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[
x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2
)) + (3/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1))/4))/((a + b)
*Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b])))/b))/b)/(((a*b)/(a -
b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))))/(2*((a*b)/(a - b) - b^2/(
a - b))*((a*b)/(a + b) + b^2/(a + b))))/(3*((a*b)/(a - b) - b^2/(a - b))*((
a*b)/(a + b) + b^2/(a + b)))))/((1 - (a + b*Sin[x])/(a - b))^(3/2)*(1 - (a
+ b*Sin[x])/(a + b))^(3/2)*(a*Sec[x] + b*Tan[x])^4)
```

Maple [B] time = 0.113, size = 967, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^4,x)
```

```
[Out] 1/b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a^3/(a^2-b^2)*tan(1/2*x)^5-2/(a*t
an(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a/(a^2-b^2)*tan(1/2*x)^5+2*b^2/(a*tan(1/2*x
)^2+2*b*tan(1/2*x)+a)^3/a/(a^2-b^2)*tan(1/2*x)^5+2/b^3/(a*tan(1/2*x)^2+2*b*
tan(1/2*x)+a)^3/(a^2-b^2)*a^4*tan(1/2*x)^4+3/b/(a*tan(1/2*x)^2+2*b*tan(1/2*
x)+a)^3/(a^2-b^2)*a^2*tan(1/2*x)^4-4*b/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/
(a^2-b^2)*tan(1/2*x)^4+4*b^3/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)/
a^2*tan(1/2*x)^4+12/b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a^3/(a^2-b^2)*t
an(1/2*x)^3-2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a/(a^2-b^2)*tan(1/2*x)^3-
8/3*b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/a/(a^2-b^2)*tan(1/2*x)^3+8/3*b^
4/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/a^3/(a^2-b^2)*tan(1/2*x)^3+4/b^3/(a*t
an(1/2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^4*tan(1/2*x)^2+16/b/(a*tan(1/2*
x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^2*tan(1/2*x)^2-14*b/(a*tan(1/2*x)^2+2*
b*tan(1/2*x)+a)^3/(a^2-b^2)*tan(1/2*x)^2+4*b^3/(a*tan(1/2*x)^2+2*b*tan(1/2*
x)+a)^3/(a^2-b^2)/a^2*tan(1/2*x)^2+11/b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)
^3*a^3/(a^2-b^2)*tan(1/2*x)-8/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a/(a^2-b^
2)*tan(1/2*x)+2*b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/a/(a^2-b^2)*tan(1/2
*x)+2/b^3/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^4-5/3/b/(a*tan(1/
2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^2+2/3*b/(a*tan(1/2*x)^2+2*b*tan(1/2*
x)+a)^3/(a^2-b^2)-2/b^4*a^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)
/(a^2-b^2)^(1/2))+3/b^2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(
a^2-b^2)^(1/2))+2/b^4*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96195, size = 2061, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + 2*(11*a^4*b^3 - 19*a^
2*b^5 + 8*b^7)*cos(x)^3 + 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 -
 3*a^2*b^4)*cos(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^
5)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a
*b^6)*x - 12*(a^6*b - 2*a^2*b^5 + b^7)*cos(x) + 6*(2*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3 + a^2*b^5 + b^7)*x - (5*a^5*b^2 -
 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*
b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*cos(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 +
 a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b^11)*cos(x)^2)*sin(x)), -1/6*(18*
(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + (11*a^4*b^3 - 19*a^2*b^5 + 8*b^7
)*cos(x)^3 - 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 - 3*a^2*b^4)*c
os(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^5)*cos(x)^2)*
sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) -
6*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*x - 6*(a^6*b - 2*a^2*b^5 + b^7)*cos
(x) + 3*(2*(a^4*b^3 - 2*a^2*b^5 + b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3
+ a^2*b^5 + b^7)*x - (5*a^5*b^2 - 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7
*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*co
s(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 + a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b
^11)*cos(x)^2)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(x\right)+b\tan\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**4,x)
```

```
[Out] Integral((a*sec(x) + b*tan(x))**(-4), x)
```

Giac [B] time = 1.16997, size = 498, normalized size = 3.19

$$-\frac{\left(2\,a^{3}-3\,ab^{2}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}+\frac{3\,a^{6}b\tan\left(\frac{1}{2}\,x\right)^{5}-6\,a^{4}b^{3}\tan\left(\frac{1}{2}\,x\right)^{5}+6\,a^{2}b^{5}\tan\left(\frac{1}{2}\,x\right)^{5}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

```
[Out] -(2*a^3 - 3*a*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/3*(3*a^6*b*tan
(1/2*x)^5 - 6*a^4*b^3*tan(1/2*x)^5 + 6*a^2*b^5*tan(1/2*x)^5 + 6*a^7*tan(1/2
*x)^4 + 9*a^5*b^2*tan(1/2*x)^4 - 12*a^3*b^4*tan(1/2*x)^4 + 12*a*b^6*tan(1/2
*x)^4 + 36*a^6*b*tan(1/2*x)^3 - 6*a^4*b^3*tan(1/2*x)^3 - 8*a^2*b^5*tan(1/2*
x)^3 + 8*b^7*tan(1/2*x)^3 + 12*a^7*tan(1/2*x)^2 + 48*a^5*b^2*tan(1/2*x)^2 -
 42*a^3*b^4*tan(1/2*x)^2 + 12*a*b^6*tan(1/2*x)^2 + 33*a^6*b*tan(1/2*x) - 24
*a^4*b^3*tan(1/2*x) + 6*a^2*b^5*tan(1/2*x) + 6*a^7 - 5*a^5*b^2 + 2*a^3*b^4)
/((a^5*b^3 - a^3*b^5)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^3) + x/b^4
```

3.272 $\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx$

Optimal. Leaf size=101

$$-\frac{\left(a^2-b^2\right)^2}{4b^5(a+b\sin(x))^4}+\frac{4a\left(a^2-b^2\right)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{4a}{b^5(a+b\sin(x))}+\frac{\log(a+b\sin(x))}{b^5}$$

```
[Out] Log[a + b*Sin[x]]/b^5 - (a^2 - b^2)^2/(4*b^5*(a + b*Sin[x])^4) + (4*a*(a^2
- b^2))/(3*b^5*(a + b*Sin[x])^3) - (3*a^2 - b^2)/(b^5*(a + b*Sin[x])^2) + (
4*a)/(b^5*(a + b*Sin[x]))
```

Rubi [A] time = 0.115666, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$-\frac{\left(a^2-b^2\right)^2}{4b^5(a+b\sin(x))^4}+\frac{4a\left(a^2-b^2\right)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{4a}{b^5(a+b\sin(x))}+\frac{\log(a+b\sin(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-5),x]
```

```
[Out] Log[a + b*Sin[x]]/b^5 - (a^2 - b^2)^2/(4*b^5*(a + b*Sin[x])^4) + (4*a*(a^2
- b^2))/(3*b^5*(a + b*Sin[x])^3) - (3*a^2 - b^2)/(b^5*(a + b*Sin[x])^2) + (
4*a)/(b^5*(a + b*Sin[x]))
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx &= \int \frac{\cos^5(x)}{(a+b\sin(x))^5}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^5}\,dx, x, b\sin(x)\right)}{b^5}\\ &= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(a+x)^5}-\frac{4\left(a^3-ab^2\right)}{(a+x)^4}+\frac{2\left(3a^2-b^2\right)}{(a+x)^3}-\frac{4a}{(a+x)^2}+\frac{1}{a+x}\right)dx, x, b\sin(x)\right)}{b^5}\\ &= \frac{\log(a+b\sin(x))}{b^5}-\frac{\left(a^2-b^2\right)^2}{4b^5(a+b\sin(x))^4}+\frac{4a\left(a^2-b^2\right)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{}{b^5(a+}\end{aligned}$$

Mathematica [A] time = 0.332491, size = 86, normalized size = 0.85

$$\frac{\frac{12b^2\left(9a^2+b^2\right)\sin^2(x)+8ab\left(11a^2+b^2\right)\sin(x)+2a^2b^2+25a^4+48ab^3\sin^3(x)-3b^4}{12(a+b\sin(x))^4}+\log(a+b\sin(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-5),x]
```

```
[Out] (Log[a + b*Sin[x]] + (25*a^4 + 2*a^2*b^2 - 3*b^4 + 8*a*b*(11*a^2 + b^2)*Sin
[x] + 12*b^2*(9*a^2 + b^2)*Sin[x]^2 + 48*a*b^3*Sin[x]^3)/(12*(a + b*Sin[x])
^4))/b^5
```

Maple [A] time = 0.113, size = 130, normalized size = 1.3

$$\frac{4\,a^3}{3\,b^5\,(a+b\sin(x))^3}-\frac{4\,a}{3\,b^3\,(a+b\sin(x))^3}+\frac{\ln(a+b\sin(x))}{b^5}-3\,\frac{a^2}{b^5\,(a+b\sin(x))^2}+\frac{1}{b^3\,(a+b\sin(x))^2}-\frac{a^4}{4\,b^5\,(a+b\,\text{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^5,x)
```

```
[Out] 4/3*a^3/b^5/(a+b*sin(x))^3-4/3*a/b^3/(a+b*sin(x))^3+ln(a+b*sin(x))/b^5-3/b^
5/(a+b*sin(x))^2*a^2+1/b^3/(a+b*sin(x))^2-1/4/b^5/(a+b*sin(x))^4*a^4+1/2/b^
3/(a+b*sin(x))^4*a^2-1/4/b/(a+b*sin(x))^4+4*a/b^5/(a+b*sin(x))
```

Maxima [B] time = 1.6462, size = 652, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

```
[Out] -2/3*(3*(a^7 - a^3*b^4)*sin(x)/(cos(x) + 1) + 3*(7*a^6*b - 3*a^2*b^5)*sin(x
)^2/(cos(x) + 1)^2 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^3/(co
s(x) + 1)^3 + 2*(21*a^6*b + 25*a^4*b^3 - 7*a^2*b^5 - 3*b^7)*sin(x)^4/(cos(x
) + 1)^4 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^5/(cos(x) + 1)^
5 + 3*(7*a^6*b - 3*a^2*b^5)*sin(x)^6/(cos(x) + 1)^6 + 3*(a^7 - a^3*b^4)*sin
(x)^7/(cos(x) + 1)^7)/(a^8*b^4 + 8*a^7*b^5*sin(x)/(cos(x) + 1) + 8*a^7*b^5*
sin(x)^7/(cos(x) + 1)^7 + a^8*b^4*sin(x)^8/(cos(x) + 1)^8 + 4*(a^8*b^4 + 6*
a^6*b^6)*sin(x)^2/(cos(x) + 1)^2 + 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)^3/(cos(
x) + 1)^3 + 2*(3*a^8*b^4 + 24*a^6*b^6 + 8*a^4*b^8)*sin(x)^4/(cos(x) + 1)^4
+ 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)^5/(cos(x) + 1)^5 + 4*(a^8*b^4 + 6*a^6*b^
6)*sin(x)^6/(cos(x) + 1)^6) + log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/
(cos(x) + 1)^2)/b^5 - log(sin(x)^2/(cos(x) + 1)^2 + 1)/b^5
```

Fricas [B] time = 2.56074, size = 509, normalized size = 5.04

$$\frac{25\,a^4 + 110\,a^2b^2 + 9\,b^4 - 12\left(9\,a^2b^2 + b^4\right)\cos\left(x\right)^2 + 12\left(b^4\cos\left(x\right)^4 + a^4 + 6\,a^2b^2 + b^4 - 2\left(3\,a^2b^2 + b^4\right)\cos\left(x\right)^2 - 4\left(ab^3\,\mathrm{c}\cdots\right.\right.}{12\left(b^9\cos\left(x\right)^4 + a^4b^5 + 6\,a^2b^7 + b^9 - 2\left(3\,a^2b^7 + b^9\right)\cos\left(x\right)^2 - 4\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(25*a^4 + 110*a^2*b^2 + 9*b^4 - 12*(9*a^2*b^2 + b^4)*cos(x)^2 + 12*(b^
4*cos(x)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*cos(x)^2 - 4*(a*b^
3*cos(x)^2 - a^3*b - a*b^3)*sin(x))*log(b*sin(x) + a) - 8*(6*a*b^3*cos(x)^2
```

```
 - 11*a^3*b - 7*a*b^3)*sin(x))/(b^9*cos(x)^4 + a^4*b^5 + 6*a^2*b^7 + b^9 -
2*(3*a^2*b^7 + b^9)*cos(x)^2 - 4*(a*b^8*cos(x)^2 - a^3*b^6 - a*b^8)*sin(x))
```

Sympy [A] time = 33.2854, size = 1727, normalized size = 17.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**5,x)
```

```
[Out] Piecewise((12*a**4*log(a*sec(x)/b + tan(x))*sec(x)**4/(12*a**4*b**5*sec(x)*
*4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*
a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) - 6*a**4*log(tan(x)**2 + 1)*se
c(x)**4/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b
**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) +
 16*a**4*sec(x)**4/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3
+ 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*t
an(x)**4) + 48*a**3*b*log(a*sec(x)/b + tan(x))*tan(x)*sec(x)**3/(12*a**4*b*
*5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x
)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) - 24*a**3*b*log(tan(
x)**2 + 1)*tan(x)*sec(x)**3/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*s
ec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) +
12*b**9*tan(x)**4) + 52*a**3*b*tan(x)*sec(x)**3/(12*a**4*b**5*sec(x)**4 + 4
8*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8
*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) + 72*a**2*b**2*log(a*sec(x)/b + tan(
x))*tan(x)**2*sec(x)**2/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x
)**3 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b
**9*tan(x)**4) - 36*a**2*b**2*log(tan(x)**2 + 1)*tan(x)**2*sec(x)**2/(12*a*
*4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*
sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) + 56*a**2*b**2*
tan(x)**2*sec(x)**2/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3
 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*
tan(x)**4) + 2*a**2*b**2*sec(x)**2/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*t
an(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*se
c(x) + 12*b**9*tan(x)**4) + 48*a*b**3*log(a*sec(x)/b + tan(x))*tan(x)**3*se
c(x)/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7
*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) - 24
*a*b**3*log(tan(x)**2 + 1)*tan(x)**3*sec(x)/(12*a**4*b**5*sec(x)**4 + 48*a*
*3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan
(x)**3*sec(x) + 12*b**9*tan(x)**4) + 20*a*b**3*tan(x)**3*sec(x)/(12*a**4*b*
*5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x
```

```
)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) + 8*a*b**3*tan(x)*se
c(x)/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7
*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) + 12
*b**4*log(a*sec(x)/b + tan(x))*tan(x)**4/(12*a**4*b**5*sec(x)**4 + 48*a**3*
b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)
**3*sec(x) + 12*b**9*tan(x)**4) - 6*b**4*log(tan(x)**2 + 1)*tan(x)**4/(12*a
**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7*tan(x)**2
*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) + 6*b**4*tan(x
)**2/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7
*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4) - 3*
b**4/(12*a**4*b**5*sec(x)**4 + 48*a**3*b**6*tan(x)*sec(x)**3 + 72*a**2*b**7
*tan(x)**2*sec(x)**2 + 48*a*b**8*tan(x)**3*sec(x) + 12*b**9*tan(x)**4), Ne(
b, 0)), ((8*tan(x)**5/(15*sec(x)**5) + 4*tan(x)**3/(3*sec(x)**5) + tan(x)/s
ec(x)**5)/a**5, True))
```

Giac [A] time = 1.13955, size = 123, normalized size = 1.22

$$\frac{\log\left(|b\sin\left(x\right)+a|\right)}{b^5}-\frac{25\,b^3\sin\left(x\right)^4+52\,ab^2\sin\left(x\right)^3+42\,a^2b\sin\left(x\right)^2-12\,b^3\sin\left(x\right)^2+12\,a^3\sin\left(x\right)-8\,ab^2\sin\left(x\right)-2\,a^2b}{12\left(b\sin\left(x\right)+a\right)^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(b*sin(x) + a))/b^5 - 1/12*(25*b^3*sin(x)^4 + 52*a*b^2*sin(x)^3 + 42
*a^2*b*sin(x)^2 - 12*b^3*sin(x)^2 + 12*a^3*sin(x) - 8*a*b^2*sin(x) - 2*a^2*
b + 3*b^3)/((b*sin(x) + a)^4*b^4)
```

3.273 $\int (\sec(x)+\tan(x))^5\,dx$

Optimal. Leaf size=30

$$-\frac{4}{1-\sin(x)}+\frac{2}{(1-\sin(x))^2}-\log(1-\sin(x))$$

```
[Out] -Log[1 - Sin[x]] + 2/(1 - Sin[x])^2 - 4/(1 - Sin[x])
```

Rubi [A] time = 0.0499651, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$-\frac{4}{1-\sin(x)}+\frac{2}{(1-\sin(x))^2}-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^5,x]
```

```
[Out] -Log[1 - Sin[x]] + 2/(1 - Sin[x])^2 - 4/(1 - Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (\sec(x)+\tan(x))^5\,dx = \int \sec^5(x)(1+\sin(x))^5\,dx$$
$$= \text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, \sin(x)\right)$$
$$= -\log(1-\sin(x))+\frac{2}{(1-\sin(x))^2}-\frac{4}{1-\sin(x)}$$

Mathematica [A] time = 0.111897, size = 54, normalized size = 1.8

$$\frac{11\tan^4(x)}{4}-\frac{\tan^2(x)}{2}+\frac{5\sec^4(x)}{4}+\tanh^{-1}(\sin(x))-\log(\cos(x))-\tan(x)\sec^3(x)+5\tan^3(x)\sec(x)+\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^5,x]
```

```
[Out] ArcTanh[Sin[x]] - Log[Cos[x]] + (5*Sec[x]^4)/4 + Sec[x]*Tan[x] - Sec[x]^3*T
an[x] - Tan[x]^2/2 + 5*Sec[x]*Tan[x]^3 + (11*Tan[x]^4)/4
```

Maple [B] time = 0.064, size = 106, normalized size = 3.5

$$-\left(-\frac{(\sec(x))^3}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\ln(\sec(x)+\tan(x))+\frac{5}{4(\cos(x))^4}+\frac{5(\sin(x))^3}{2(\cos(x))^4}+\frac{5(\sin(x))^3}{4(\cos(x))^2}-\frac{5\sin(x)}{8}+\frac{5(}{2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^5,x)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+ln(sec(x)+tan(x))+5/4/cos(x)^4+5/2*sin(x
)^3/cos(x)^4+5/4*sin(x)^3/cos(x)^2-5/8*sin(x)+5/2*sin(x)^4/cos(x)^4+5/4*sin
(x)^5/cos(x)^4-5/8*sin(x)^5/cos(x)^2-5/8*sin(x)^3+1/4*tan(x)^4-1/2*tan(x)^2
-ln(cos(x))
```

Maxima [B] time = 0.995535, size = 190, normalized size = 6.33

$$\frac{5}{2}\tan(x)^4 + \frac{5\left(5\sin(x)^3 - 3\sin(x)\right)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} - \frac{3\sin(x)^3 - 5\sin(x)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{5\left(\sin(x)^3 + \sin(x)\right)}{4\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{4\sin(}{4\left(\sin(x)^4 - 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*tan(x)^4 + 5/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/
8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 5/4*(sin(x)^3 + sin
(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x
)^2 + 1) + 5/4/(sin(x)^2 - 1)^2 - 1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x) +
1) - 1/2*log(sin(x) - 1)
```

Fricas [A] time = 2.077, size = 119, normalized size = 3.97

$$-\frac{\left(\cos(x)^2 + 2\sin(x) - 2\right)\log(-\sin(x) + 1) + 4\sin(x) - 2}{\cos(x)^2 + 2\sin(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="fricas")
```

```
[Out] -((cos(x)^2 + 2*sin(x) - 2)*log(-sin(x) + 1) + 4*sin(x) - 2)/(cos(x)^2 + 2*
sin(x) - 2)
```

Sympy [B] time = 8.26483, size = 68, normalized size = 2.27

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} + \frac{\log\left(\sec^2(x)\right)}{2} + \frac{5\tan^4(x)}{2} + \frac{3\sec^4(x)}{2} - \sec^2(x) + \frac{32\sin^3(x)}{8\sin^4(x) - 16\sin^2(x) + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**5,x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 + log(sec(x)**2)/2 + 5*tan(x)**4/2 +
 3*sec(x)**4/2 - sec(x)**2 + 32*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8)
```

Giac [B] time = 1.12623, size = 84, normalized size = 2.8

$$\frac{25 \tan\left(\frac{1}{2}x\right)^4 - 100 \tan\left(\frac{1}{2}x\right)^3 + 198 \tan\left(\frac{1}{2}x\right)^2 - 100 \tan\left(\frac{1}{2}x\right) + 25}{6\left(\tan\left(\frac{1}{2}x\right) - 1\right)^4} + \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 2\log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="giac")
```

```
[Out] 1/6*(25*tan(1/2*x)^4 - 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 - 100*tan(1/2*x)
 + 25)/(tan(1/2*x) - 1)^4 + log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) -
1))
```

3.274 $\int (\sec(x)+\tan(x))^4\,dx$

Optimal. Leaf size=30

$$x+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}$$

```
[Out] x + (2*Cos[x]^3)/(3*(1 - Sin[x])^3) - (2*Cos[x])/(1 - Sin[x])
```

Rubi [A] time = 0.10202, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4391, 2670, 2680, 8}

$$x+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^4,x]
```

```
[Out] x + (2*Cos[x]^3)/(3*(1 - Sin[x])^3) - (2*Cos[x])/(1 - Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^4\,dx &= \int \sec^4(x)(1+\sin(x))^4\,dx\\ &= \int \frac{\cos^4(x)}{(1-\sin(x))^4}\,dx\\ &= \frac{2\cos^3(x)}{3(1-\sin(x))^3}-\int \frac{\cos^2(x)}{(1-\sin(x))^2}\,dx\\ &= \frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}+\int 1\,dx\\ &= x+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.135676, size = 64, normalized size = 2.13

$$-\frac{-3(3x+8)\cos\left(\frac{x}{2}\right)+(3x+16)\cos\left(\frac{3x}{2}\right)+6\sin\left(\frac{x}{2}\right)(2x+x\cos(x)+4)}{6\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^4,x]
```

```
[Out] -(-3*(8 + 3*x)*Cos[x/2] + (16 + 3*x)*Cos[(3*x)/2] + 6*(4 + 2*x + x*Cos[x])*
Sin[x/2])/(6*(Cos[x/2] - Sin[x/2])^3)
```

Maple [B] time = 0.048, size = 71, normalized size = 2.4

$$-\left(-\frac{2}{3}-\frac{(\sec(x))^2}{3}\right)\tan(x)+\frac{4}{3\,(\cos(x))^3}+2\,\frac{(\sin(x))^3}{(\cos(x))^3}+\frac{4\,(\sin(x))^4}{3\,(\cos(x))^3}-\frac{4\,(\sin(x))^4}{3\,\cos(x)}-\frac{\left(8+4\,(\sin(x))^2\right)\cos(x)}{3}+\frac{(\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^4,x)
```

```
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)+4/3/cos(x)^3+2*sin(x)^3/cos(x)^3+4/3*sin(x)^4/c
os(x)^3-4/3*sin(x)^4/cos(x)-4/3*(2+sin(x)^2)*cos(x)+1/3*tan(x)^3-tan(x)+x
```

Maxima [A] time = 1.48583, size = 38, normalized size = 1.27

$$\frac{8}{3}\tan(x)^3 + x - \frac{4\left(3\cos(x)^2 - 1\right)}{3\cos(x)^3} + \frac{4}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 8/3*tan(x)^3 + x - 4/3*(3*cos(x)^2 - 1)/cos(x)^3 + 4/3/cos(x)^3
```

Fricas [B] time = 2.07358, size = 188, normalized size = 6.27

$$\frac{(3x+8)\cos(x)^2 - (3x-4)\cos(x) + ((3x-8)\cos(x) + 6x - 4)\sin(x) - 6x - 4}{3\left(\cos(x)^2 + (\cos(x)+2)\sin(x) - \cos(x) - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x + 8)*cos(x)^2 - (3*x - 4)*cos(x) + ((3*x - 8)*cos(x) + 6*x - 4)*s
in(x) - 6*x - 4)/(cos(x)^2 + (cos(x) + 2)*sin(x) - cos(x) - 2)
```

Sympy [A] time = 4.46487, size = 44, normalized size = 1.47

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)} + \frac{7\tan^3(x)}{3} + \tan(x) + \frac{8\sec^3(x)}{3} - 4\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x) + 7*tan(x)**3/3 + tan(x) + 8*se
c(x)**3/3 - 4*sec(x)
```

Giac [A] time = 1.15657, size = 27, normalized size = 0.9

$$x - \frac{8\left(3\tan\left(\frac{1}{2}x\right) - 1\right)}{3\left(\tan\left(\frac{1}{2}x\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(3*tan(1/2*x) - 1)/(tan(1/2*x) - 1)^3
```

3.275 $\int (\sec(x) + \tan(x))^3 \, dx$

Optimal. Leaf size=18

$$\frac{2}{1-\sin(x)} + \log(1-\sin(x))$$

```
[Out] Log[1 - Sin[x]] + 2/(1 - Sin[x])
```

Rubi [A] time = 0.0438103, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$\frac{2}{1-\sin(x)} + \log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^3,x]

[Out] Log[1 - Sin[x]] + 2/(1 - Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^3\,dx &= \int \sec^3(x)(1+\sin(x))^3\,dx\\ &= \text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, \sin(x)\right)\\ &= \log(1-\sin(x))+\frac{2}{1-\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.0259234, size = 31, normalized size = 1.72

$$\frac{\tan^2(x)}{2}+\frac{3\sec^2(x)}{2}-\tanh^{-1}(\sin(x))+\log(\cos(x))+2\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^3,x]
```

```
[Out] -ArcTanh[Sin[x]] + Log[Cos[x]] + (3*Sec[x]^2)/2 + 2*Sec[x]*Tan[x] + Tan[x]^
2/2
```

Maple [B] time = 0.051, size = 45, normalized size = 2.5

$$\frac{\sec(x)\tan(x)}{2}-\ln(\sec(x)+\tan(x))+\frac{3}{2(\cos(x))^2}+\frac{3(\sin(x))^3}{2(\cos(x))^2}+\frac{3\sin(x)}{2}+\frac{(\tan(x))^2}{2}+\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^3,x)
```

```
[Out] 1/2*sec(x)*tan(x)-ln(sec(x)+tan(x))+3/2/cos(x)^2+3/2*sin(x)^3/cos(x)^2+3/2*
sin(x)+1/2*tan(x)^2+ln(cos(x))
```

Maxima [B] time = 0.982467, size = 70, normalized size = 3.89

$$\frac{3}{2}\tan(x)^2-\frac{2\sin(x)}{\sin(x)^2-1}-\frac{1}{2\left(\sin(x)^2-1\right)}+\frac{1}{2}\log\left(\sin(x)^2-1\right)-\frac{1}{2}\log(\sin(x)+1)+\frac{1}{2}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*tan(x)^2 - 2*sin(x)/(sin(x)^2 - 1) - 1/2/(sin(x)^2 - 1) + 1/2*log(sin(x
)^2 - 1) - 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1)
```

Fricas [A] time = 2.03005, size = 68, normalized size = 3.78

$$\frac{(\sin(x)-1)\log(-\sin(x)+1)-2}{\sin(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] ((sin(x) - 1)*log(-sin(x) + 1) - 2)/(sin(x) - 1)
```

Sympy [B] time = 5.04155, size = 44, normalized size = 2.44

$$\frac{\log(\sin(x)-1)}{2}-\frac{\log(\sin(x)+1)}{2}-\frac{\log\left(\sec^2(x)\right)}{2}+2\sec^2(x)-\frac{4\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**3,x)
```

```
[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/2 - log(sec(x)**2)/2 + 2*sec(x)**2 - 4*
sin(x)/(2*sin(x)**2 - 2)
```

Giac [B] time = 1.18844, size = 65, normalized size = 3.61

$$-\frac{3\tan\left(\frac{1}{2}x\right)^2-10\tan\left(\frac{1}{2}x\right)+3}{\left(\tan\left(\frac{1}{2}x\right)-1\right)^2}-\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)+2\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] -(3*tan(1/2*x)^2 - 10*tan(1/2*x) + 3)/(tan(1/2*x) - 1)^2 - log(tan(1/2*x)^2
 + 1) + 2*log(abs(tan(1/2*x) - 1))
```

3.276 $\int (\sec(x) + \tan(x))^2 \, dx$

Optimal. Leaf size=16

$$\frac{2\cos(x)}{1-\sin(x)} - x$$

```
[Out] -x + (2*Cos[x])/(1 - Sin[x])
```

Rubi [A] time = 0.0698457, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4391, 2670, 2680, 8}

$$\frac{2\cos(x)}{1-\sin(x)} - x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^2,x]
```

```
[Out] -x + (2*Cos[x])/(1 - Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^2\,dx &= \int \sec^2(x)(1+\sin(x))^2\,dx\\ &= \int \frac{\cos^2(x)}{(1-\sin(x))^2}\,dx\\ &= \frac{2\cos(x)}{1-\sin(x)}-\int 1\,dx\\ &= -x+\frac{2\cos(x)}{1-\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.0118779, size = 14, normalized size = 0.88

$$-\tan^{-1}(\tan(x))+2\tan(x)+2\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^2,x]
```

```
[Out] -ArcTan[Tan[x]] + 2*Sec[x] + 2*Tan[x]
```

Maple [A] time = 0.017, size = 15, normalized size = 0.9

$$2\tan(x)+2(\cos(x))^{-1}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^2,x)
```

```
[Out] 2*tan(x)+2/cos(x)-x
```

Maxima [A] time = 1.45766, size = 19, normalized size = 1.19

$$-x+\frac{2}{\cos\left(x\right)}+2\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -x + 2/cos(x) + 2*tan(x)
```

Fricas [A] time = 2.05034, size = 89, normalized size = 5.56

$$-\frac{(x-2)\cos\left(x\right)-(x+2)\sin\left(x\right)+x-2}{\cos\left(x\right)-\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -((x - 2)*cos(x) - (x + 2)*sin(x) + x - 2)/(cos(x) - sin(x) + 1)
```

Sympy [A] time = 1.23515, size = 10, normalized size = 0.62

$$-x+2\tan\left(x\right)+2\sec\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**2,x)
```

```
[Out] -x + 2*tan(x) + 2*sec(x)
```

Giac [A] time = 1.13136, size = 19, normalized size = 1.19

$$-x-\frac{4}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] -x - 4/(tan(1/2*x) - 1)
```

3.277 $\int (\sec(x) + \tan(x))\, dx$

Optimal. Leaf size=13

$$-2\log\left(\cos\left(\frac{1}{4}(2x+\pi)\right)\right)$$

```
[Out] -2*Log[Cos[(Pi + 2*x)/4]]
```

Rubi [A] time = 0.0056346, antiderivative size = 9, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3770, 3475}

$$\tanh^{-1}(\sin(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x] + Tan[x],x]
```

```
[Out] ArcTanh[Sin[x]] - Log[Cos[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))\, dx &= \int \sec(x)\, dx + \int \tan(x)\, dx\\ &= \tanh^{-1}(\sin(x)) - \log(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.0052144, size = 38, normalized size = 2.92

$$-\log(\cos(x)) - \log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x] + Tan[x],x]
```

```
[Out] -Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.001, size = 13, normalized size = 1.

$$\ln(\sec(x) + \tan(x)) - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)+tan(x),x)
```

```
[Out] ln(sec(x)+tan(x))-ln(cos(x))
```

Maxima [A] time = 0.967093, size = 14, normalized size = 1.08

$$\log(\sec(x) + \tan(x)) + \log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="maxima")
```

```
[Out] log(sec(x) + tan(x)) + log(sec(x))
```

Fricas [A] time = 2.05595, size = 26, normalized size = 2.

$$-\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="fricas")
```

```
[Out] -log(-sin(x) + 1)
```

Sympy [A] time = 0.112976, size = 20, normalized size = 1.54

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(cos(x))
```

Giac [B] time = 1.17835, size = 42, normalized size = 3.23

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2)) -
 log(abs(cos(x)))
```

3.278 $\int \frac{1}{\sec(x)+\tan(x)} \, dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

```
[Out] Log[1 + Sin[x]]
```

Rubi [A] time = 0.0244933, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3159, 2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos(x)}{1+\sin(x)}\,dx \\
&= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right) \\
&= \log(1+\sin(x))
\end{aligned}
$$

Mathematica [B] time = 0.0138217, size = 16, normalized size = 3.2

$$2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] 2*Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.051, size = 6, normalized size = 1.2

$$\ln\left(1+\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x)),x)
```

```
[Out] ln(1+sin(x))
```

Maxima [B] time = 0.979022, size = 42, normalized size = 8.4

$$2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)-\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 2.1763, size = 23, normalized size = 4.6

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [B] time = 0.165543, size = 17, normalized size = 3.4

$$\log(\tan(x) + \sec(x)) - \frac{\log\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x)
```

```
[Out] log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

Giac [B] time = 1.12336, size = 30, normalized size = 6.

$$-\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 2\log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

3.279 $\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx$

Optimal. Leaf size=14

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

[Out] -x - (2*Cos[x])/(1 + Sin[x])

Rubi [A] time = 0.0414318, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2680, 8}

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^(-2),x]

[Out] -x - (2*Cos[x])/(1 + Sin[x])

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = \int \frac{\cos^2(x)}{(1+\sin(x))^2}\,dx$$
$$= -\frac{2\cos(x)}{1+\sin(x)} - \int 1\,dx$$
$$= -x - \frac{2\cos(x)}{1+\sin(x)}$$

Mathematica [A] time = 0.0255073, size = 27, normalized size = 1.93

$$\frac{4\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)} - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-2),x]
```

```
[Out] -x + (4*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.059, size = 15, normalized size = 1.1

$$-4\,(1+\tan(x/2))^{-1} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^2,x)
```

```
[Out] -4/(1+tan(1/2*x))-x
```

Maxima [A] time = 1.49414, size = 38, normalized size = 2.71

$$-\frac{4}{\frac{\sin(x)}{\cos(x)+1}+1} - 2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -4/(sin(x)/(cos(x) + 1) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.06679, size = 89, normalized size = 6.36

$$-\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(\tan(x)+\sec(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**2,x)
```

```
[Out] Integral((tan(x) + sec(x))**(-2), x)
```

Giac [A] time = 1.09812, size = 19, normalized size = 1.36

$$-x-\frac{4}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] -x - 4/(tan(1/2*x) + 1)
```

3.280 $\int \frac{1}{(\sec(x)+\tan(x))^3} dx$

Optimal. Leaf size=16

$$-\frac{2}{\sin(x)+1} - \log(\sin(x)+1)$$

```
[Out] -Log[1 + Sin[x]] - 2/(1 + Sin[x])
```

Rubi [A] time = 0.0462508, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$-\frac{2}{\sin(x)+1} - \log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-3),x]
```

```
[Out] -Log[1 + Sin[x]] - 2/(1 + Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sec(x)+\tan(x))^3}\,dx &= \int \frac{\cos^3(x)}{(1+\sin(x))^3}\,dx\\ &= \text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, \sin(x)\right)\\ &= -\log(1+\sin(x))-\frac{2}{1+\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.0205385, size = 34, normalized size = 2.12

$$-\frac{2}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}-2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-3),x]

[Out] -2*Log[Cos[x/2] + Sin[x/2]] - 2/(Cos[x/2] + Sin[x/2])^2
```

Maple [A] time = 0.085, size = 17, normalized size = 1.1

$$-\ln\left(1+\sin\left(x\right)\right)-2\,\left(1+\sin\left(x\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^3,x)

[Out] -ln(1+sin(x))-2/(1+sin(x))
```

Maxima [B] time = 1.50015, size = 86, normalized size = 5.38

$$\frac{4\,\sin\left(x\right)}{\left(\frac{2\,\sin(x)}{\cos(x)+1}+\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)\left(\cos\left(x\right)+1\right)}-2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)+\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 4*sin(x)/((2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1
)) - 2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 2.07822, size = 68, normalized size = 4.25

$$-\frac{\left(\sin\left(x\right)+1\right)\log\left(\sin\left(x\right)+1\right)+2}{\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] -((sin(x) + 1)*log(sin(x) + 1) + 2)/(sin(x) + 1)
```

Sympy [B] time = 1.34542, size = 306, normalized size = 19.12

$$-\frac{2\log\left(\tan\left(x\right)+\sec\left(x\right)\right)\tan^2\left(x\right)}{2\tan^2\left(x\right)+4\tan\left(x\right)\sec\left(x\right)+2\sec^2\left(x\right)}-\frac{4\log\left(\tan\left(x\right)+\sec\left(x\right)\right)\tan\left(x\right)\sec\left(x\right)}{2\tan^2\left(x\right)+4\tan\left(x\right)\sec\left(x\right)+2\sec^2\left(x\right)}-\frac{2\log\left(\tan\left(x\right)+\sec\left(x\right)\right)\sec}{2\tan^2\left(x\right)+4\tan\left(x\right)\sec\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**3,x)
```

```
[Out] -2*log(tan(x) + sec(x))*tan(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)
**2) - 4*log(tan(x) + sec(x))*tan(x)*sec(x)/(2*tan(x)**2 + 4*tan(x)*sec(x)
+ 2*sec(x)**2) - 2*log(tan(x) + sec(x))*sec(x)**2/(2*tan(x)**2 + 4*tan(x)*s
ec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*tan(x)**2/(2*tan(x)**2 + 4*tan(x)
*sec(x) + 2*sec(x)**2) + 2*log(tan(x)**2 + 1)*tan(x)*sec(x)/(2*tan(x)**2 +
4*tan(x)*sec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*sec(x)**2/(2*tan(x)**2
```

```
+ 4*tan(x)*sec(x) + 2*sec(x)**2) + 2*tan(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(
x) + 2*sec(x)**2) + 2*tan(x)*sec(x)/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(
x)**2) - 1/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)**2)
```

Giac [B] time = 1.12815, size = 61, normalized size = 3.81

$$\frac{3\,\tan\left(\frac{1}{2}\,x\right)^{2}+10\,\tan\left(\frac{1}{2}\,x\right)+3}{\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^{2}}+\log\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)-2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] (3*tan(1/2*x)^2 + 10*tan(1/2*x) + 3)/(tan(1/2*x) + 1)^2 + log(tan(1/2*x)^2
+ 1) - 2*log(abs(tan(1/2*x) + 1))
```

3.281 $\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx$

Optimal. Leaf size=26

$$x-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}+\frac{2\cos(x)}{\sin(x)+1}$$

```
[Out] x - (2*Cos[x]^3)/(3*(1 + Sin[x])^3) + (2*Cos[x])/(1 + Sin[x])
```

Rubi [A] time = 0.0689964, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2680, 8}

$$x-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}+\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-4),x]
```

```
[Out] x - (2*Cos[x]^3)/(3*(1 + Sin[x])^3) + (2*Cos[x])/(1 + Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx &= \int \frac{\cos^4(x)}{(1+\sin(x))^4}\,dx\\ &= -\frac{2\cos^3(x)}{3(1+\sin(x))^3} - \int \frac{\cos^2(x)}{(1+\sin(x))^2}\,dx\\ &= -\frac{2\cos^3(x)}{3(1+\sin(x))^3} + \frac{2\cos(x)}{1+\sin(x)} + \int 1\,dx\\ &= x - \frac{2\cos^3(x)}{3(1+\sin(x))^3} + \frac{2\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.0747828, size = 62, normalized size = 2.38

$$\frac{3(3x-8)\cos\left(\frac{x}{2}\right)+(16-3x)\cos\left(\frac{3x}{2}\right)+6\sin\left(\frac{x}{2}\right)(2x+x\cos(x)-4)}{6\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-4),x]
```

```
[Out] (3*(-8 + 3*x)*Cos[x/2] + (16 - 3*x)*Cos[(3*x)/2] + 6*(-4 + 2*x + x*Cos[x])*
Sin[x/2])/(6*(Cos[x/2] + Sin[x/2])^3)
```

Maple [A] time = 0.072, size = 23, normalized size = 0.9

$$-\frac{16}{3}\left(1+\tan\left(\frac{x}{2}\right)\right)^{-3}+8\,\left(1+\tan\left(x/2\right)\right)^{-2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^4,x)
```

```
[Out] -16/3/(1+tan(1/2*x))^3+8/(1+tan(1/2*x))^2+x
```

Maxima [B] time = 1.51288, size = 86, normalized size = 3.31

$$\frac{8\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 8/3*(3*sin(x)/(cos(x) + 1) + 1)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x)
 + 1)^2 + sin(x)^3/(cos(x) + 1)^3 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 2.149, size = 188, normalized size = 7.23

$$\frac{(3x-8)\cos(x)^2-(3x+4)\cos(x)-((3x+8)\cos(x)+6x+4)\sin(x)-6x+4}{3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x - 8)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 8)*cos(x) + 6*x + 4)*s
in(x) - 6*x + 4)/(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\tan(x)+\sec(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**4,x)
```

```
[Out] Integral((tan(x) + sec(x))**(-4), x)
```

Giac [A] time = 1.17076, size = 27, normalized size = 1.04

$$x + \frac{8\left(3\,\tan\left(\frac{1}{2}\,x\right) + 1\right)}{3\left(\tan\left(\frac{1}{2}\,x\right) + 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*tan(1/2*x) + 1)/(tan(1/2*x) + 1)^3
```

3.282 $\int \frac{1}{(\sec(x)+\tan(x))^5} \, dx$

Optimal. Leaf size=22

$$\frac{4}{\sin(x)+1} - \frac{2}{(\sin(x)+1)^2} + \log(\sin(x)+1)$$

```
[Out] Log[1 + Sin[x]] - 2/(1 + Sin[x])^2 + 4/(1 + Sin[x])
```

Rubi [A] time = 0.0479451, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$\frac{4}{\sin(x)+1} - \frac{2}{(\sin(x)+1)^2} + \log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-5),x]
```

```
[Out] Log[1 + Sin[x]] - 2/(1 + Sin[x])^2 + 4/(1 + Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx &= \int \frac{\cos^5(x)}{(1+\sin(x))^5}\,dx\\ &= \text{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, \sin(x)\right)\\ &= \log(1+\sin(x))-\frac{2}{(1+\sin(x))^2}+\frac{4}{1+\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.0465485, size = 39, normalized size = 1.77

$$\frac{4\sin(x)+2}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4}+2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-5),x]

[Out] 2*Log[Cos[x/2] + Sin[x/2]] + (2 + 4*Sin[x])/(Cos[x/2] + Sin[x/2])^4
```

Maple [A] time = 0.109, size = 23, normalized size = 1.1

$$\ln(1+\sin(x))-2\,(1+\sin(x))^{-2}+4\,(1+\sin(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^5,x)

[Out] ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))
```

Maxima [B] time = 1.5119, size = 124, normalized size = 5.64

$$-\frac{8\,\sin\left(x\right)^{2}}{\left(\frac{4\,\sin(x)}{\cos(x)+1}+\frac{6\,\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{4\,\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{\sin(x)^{4}}{(\cos(x)+1)^{4}}+1\right)\left(\cos\left(x\right)+1\right)^{2}}+2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)-\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="maxima")
```

```
[Out] -8*sin(x)^2/((4*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 4*sin(x)^
3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1)*(cos(x) + 1)^2) + 2*log(sin
(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 2.09642, size = 116, normalized size = 5.27

$$\frac{\left(\cos\left(x\right)^{2}-2\,\sin\left(x\right)-2\right)\log\left(\sin\left(x\right)+1\right)-4\,\sin\left(x\right)-2}{\cos\left(x\right)^{2}-2\,\sin\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="fricas")
```

```
[Out] ((cos(x)^2 - 2*sin(x) - 2)*log(sin(x) + 1) - 4*sin(x) - 2)/(cos(x)^2 - 2*si
n(x) - 2)
```

Sympy [B] time = 5.48686, size = 1064, normalized size = 48.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**5,x)
```

```
[Out] 12*log(tan(x) + sec(x))*tan(x)**4/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*
tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) + 48*log(tan(x) +
 sec(x))*tan(x)**3*sec(x)/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**
2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) + 72*log(tan(x) + sec(x))
*tan(x)**2*sec(x)**2/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec
```

```
(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) + 48*log(tan(x) + sec(x))*tan(
x)*sec(x)**3/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec(x)**2 +
 48*tan(x)*sec(x)**3 + 12*sec(x)**4) + 12*log(tan(x) + sec(x))*sec(x)**4/(1
2*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(
x)**3 + 12*sec(x)**4) - 6*log(tan(x)**2 + 1)*tan(x)**4/(12*tan(x)**4 + 48*t
an(x)**3*sec(x) + 72*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)*
*4) - 24*log(tan(x)**2 + 1)*tan(x)**3*sec(x)/(12*tan(x)**4 + 48*tan(x)**3*s
ec(x) + 72*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) - 36*l
og(tan(x)**2 + 1)*tan(x)**2*sec(x)**2/(12*tan(x)**4 + 48*tan(x)**3*sec(x) +
 72*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) - 24*log(tan(
x)**2 + 1)*tan(x)*sec(x)**3/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)
**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) - 6*log(tan(x)**2 + 1)*
sec(x)**4/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec(x)**2 + 48
*tan(x)*sec(x)**3 + 12*sec(x)**4) - 16*tan(x)**4/(12*tan(x)**4 + 48*tan(x)*
*3*sec(x) + 72*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) -
44*tan(x)**3*sec(x)/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec(
x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) - 40*tan(x)**2*sec(x)**2/(12*ta
n(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**
3 + 12*sec(x)**4) + 6*tan(x)**2/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*ta
n(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) - 12*tan(x)*sec(x)*
*3/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec(x)**2 + 48*tan(x)
*sec(x)**3 + 12*sec(x)**4) + 8*tan(x)*sec(x)/(12*tan(x)**4 + 48*tan(x)**3*s
ec(x) + 72*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4) + 2*se
c(x)**2/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 72*tan(x)**2*sec(x)**2 + 48*t
an(x)*sec(x)**3 + 12*sec(x)**4) - 3/(12*tan(x)**4 + 48*tan(x)**3*sec(x) + 7
2*tan(x)**2*sec(x)**2 + 48*tan(x)*sec(x)**3 + 12*sec(x)**4)
```

Giac [B] time = 1.12812, size = 86, normalized size = 3.91

$$-\frac{25\tan\left(\frac{1}{2}x\right)^4+100\tan\left(\frac{1}{2}x\right)^3+198\tan\left(\frac{1}{2}x\right)^2+100\tan\left(\frac{1}{2}x\right)+25}{6\left(\tan\left(\frac{1}{2}x\right)+1\right)^4}-\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)+2\log\left(\left|\tan\left(\frac{1}{2}x\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="giac")
```

```
[Out] -1/6*(25*tan(1/2*x)^4 + 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 + 100*tan(1/2*x
) + 25)/(tan(1/2*x) + 1)^4 - log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) +
 1))
```

3.283 $\int (a\cot(x)+b\csc(x))^5\,dx$

Optimal. Leaf size=152

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{16}(a+b)^3\left(8a^2-9ab+3b^2\right)\log(1-\cos(x))+\frac{1}{16}(a-b)^3\left(8a^2+9ab+3b^2\right)\log(\cos(x)+1)+$$

```
[Out] (a^2*b*(7*a^2 - 3*b^2)*Cos[x])/8 + ((b + a*Cos[x])^2*(2*a*(2*a^2 - b^2) + b
*(5*a^2 - 3*b^2)*Cos[x])*Csc[x]^2)/8 - ((b + a*Cos[x])^4*(a + b*Cos[x])*Csc
[x]^4)/4 + ((a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*Log[1 - Cos[x]])/16 + ((a - b
)^3*(8*a^2 + 9*a*b + 3*b^2)*Log[1 + Cos[x]])/16
```

Rubi [A] time = 0.219501, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2668, 739, 819, 774, 633, 31}

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{16}(a+b)^3\left(8a^2-9ab+3b^2\right)\log(1-\cos(x))+\frac{1}{16}(a-b)^3\left(8a^2+9ab+3b^2\right)\log(\cos(x)+1)+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^5,x]
```

```
[Out] (a^2*b*(7*a^2 - 3*b^2)*Cos[x])/8 + ((b + a*Cos[x])^2*(2*a*(2*a^2 - b^2) + b
*(5*a^2 - 3*b^2)*Cos[x])*Csc[x]^2)/8 - ((b + a*Cos[x])^4*(a + b*Cos[x])*Csc
[x]^4)/4 + ((a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*Log[1 - Cos[x]])/16 + ((a - b
)^3*(8*a^2 + 9*a*b + 3*b^2)*Log[1 + Cos[x]])/16
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^5\,dx = \int (b+a\cos(x))^5\csc^5(x)\,dx$$
$$= -\left(a^5\,\text{Subst}\left(\int \frac{(b+x)^5}{\left(a^2-x^2\right)^3}\,dx, x, a\cos(x)\right)\right)$$
$$= -\frac{1}{4}(b+a\cos(x))^4(a+b\cos(x))\csc^4(x)+\frac{1}{4}a^3\,\text{Subst}\left(\int \frac{(b+x)^3\left(4a^2-3b^2+bx\right)}{\left(a^2-x^2\right)^2}\,dx, x, a\,\text{c}\right.$$
$$= \frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)\csc^2(x)-\frac{1}{4}(b+a\cos(x))^4(a+b\cos$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)\csc^2(x)-$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)\csc^2(x)-$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)\csc^2(x)-$$

Mathematica [A] time = 0.73585, size = 143, normalized size = 0.94

$$\frac{1}{64}\left(8(a+b)^3\left(8a^2-9ab+3b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right)+8\left(8a^2+9ab+3b^2\right)(a-b)^3\log\left(\cos\left(\frac{x}{2}\right)\right)-(a+b)^5\csc^4\left(\frac{x}{2}\right)+2(7a-3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^5,x]
```

```
[Out] (2*(7*a - 3*b)*(a + b)^4*Csc[x/2]^2 - (a + b)^5*Csc[x/2]^4 + 8*(a - b)^3*(8
*a^2 + 9*a*b + 3*b^2)*Log[Cos[x/2]] + 8*(a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*L
og[Sin[x/2]] + 2*(a - b)^4*(7*a + 3*b)*Sec[x/2]^2 - (a - b)^5*Sec[x/2]^4)/6
4
```

Maple [A] time = 0.059, size = 204, normalized size = 1.3

$$-\frac{a^5(\cot(x))^4}{4}+\frac{a^5(\cot(x))^2}{2}+a^5\ln(\sin(x))-\frac{5a^4b(\cos(x))^5}{4(\sin(x))^4}+\frac{5a^4b(\cos(x))^5}{8(\sin(x))^2}+\frac{5(\cos(x))^3a^4b}{8}+\frac{15a^4b\cos(x)}{8}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^5,x)
```

```
[Out] -1/4*a^5*cot(x)^4+1/2*a^5*cot(x)^2+a^5*ln(sin(x))-5/4*a^4*b/sin(x)^4*cos(x)
^5+5/8*a^4*b/sin(x)^2*cos(x)^5+5/8*cos(x)^3*a^4*b+15/8*a^4*b*cos(x)+15/8*a^
4*b*ln(csc(x)-cot(x))-5/2*a^3*b^2/sin(x)^4*cos(x)^4-5/2*a^2*b^3/sin(x)^4*co
s(x)^3-5/4*a^2*b^3/sin(x)^2*cos(x)^3-5/4*cos(x)*a^2*b^3-5/4*a^2*b^3*ln(csc(
x)-cot(x))-5/4*a*b^4/sin(x)^4-1/4*b^5*cot(x)*csc(x)^3-3/8*b^5*csc(x)*cot(x)
+3/8*b^5*ln(csc(x)-cot(x))
```

Maxima [A] time = 1.00049, size = 254, normalized size = 1.67

$$-\frac{5}{2}\,a^3b^2\cot\left(x\right)^4-\frac{5}{16}\,a^4b\left(\frac{2\left(5\,\cos\left(x\right)^3-3\,\cos\left(x\right)\right)}{\cos\left(x\right)^4-2\,\cos\left(x\right)^2+1}+3\,\log\left(\cos\left(x\right)+1\right)-3\,\log\left(\cos\left(x\right)-1\right)\right)+\frac{1}{16}\,b^5\left(\frac{2\left(3\,\cos\left(x\right)^3-\cdots\right.}{\cos\left(x\right)^4-2\,\cos\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*a^3*b^2*cot(x)^4 - 5/16*a^4*b*(2*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2
*cos(x)^2 + 1) + 3*log(cos(x) + 1) - 3*log(cos(x) - 1)) + 1/16*b^5*(2*(3*co
s(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3*log(cos(x) + 1) + 3*log(
cos(x) - 1)) - 5/8*a^2*b^3*(2*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 +
1) - log(cos(x) + 1) + log(cos(x) - 1)) + 1/4*a^5*((4*sin(x)^2 - 1)/sin(x)^
4 + 2*log(sin(x)^2)) - 5/4*a*b^4/sin(x)^4
```

Fricas [B] time = 2.17755, size = 703, normalized size = 4.62

$$\frac{12\,a^5+40\,a^3b^2-20\,ab^4-2\left(25\,a^4b+10\,a^2b^3-3\,b^5\right)\cos\left(x\right)^3-16\left(a^5+5\,a^3b^2\right)\cos\left(x\right)^2+10\left(3\,a^4b-2\,a^2b^3-b^5\right)\cos\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^5 + 40*a^3*b^2 - 20*a*b^4 - 2*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*co
s(x)^3 - 16*(a^5 + 5*a^3*b^2)*cos(x)^2 + 10*(3*a^4*b - 2*a^2*b^3 - b^5)*cos
(x) + (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 + (8*a^5 - 15*a^4*b + 10*a^2*b
^3 - 3*b^5)*cos(x)^4 - 2*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)^2)*
```

```
log(1/2*cos(x) + 1/2) + (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 + (8*a^5 + 1
5*a^4*b - 10*a^2*b^3 + 3*b^5)*cos(x)^4 - 2*(8*a^5 + 15*a^4*b - 10*a^2*b^3 +
 3*b^5)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^4 - 2*cos(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11573, size = 228, normalized size = 1.5

$$\frac{1}{16}\left(8\,a^5-15\,a^4b+10\,a^2b^3-3\,b^5\right)\log\left(\cos\left(x\right)+1\right)+\frac{1}{16}\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\log\left(-\cos\left(x\right)+1\right)+\frac{6\,a^5+20\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(cos(x) + 1) + 1/16*(8*a^5
+ 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(-cos(x) + 1) + 1/8*(6*a^5 + 20*a^3*b^2
 - 10*a*b^4 - (25*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)^3 - 8*(a^5 + 5*a^3*b^2
)*cos(x)^2 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cos(x))/((cos(x) + 1)^2*(cos(x)
- 1)^2)
```

3.284 $\int (a\cot(x)+b\csc(x))^4\,dx$

Optimal. Leaf size=101

$$\frac{4}{3}ab\left(2a^2-b^2\right)\sin(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sin(x)\cos(x)+\frac{1}{3}\csc(x)(a\cos(x)+b)^2\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)+a^4x-\frac{1}{3}\csc$$

```
[Out] a^4*x + ((b + a*Cos[x])^2*(a*b + (3*a^2 - 2*b^2)*Cos[x])*Csc[x])/3 - ((b +
a*Cos[x])^3*(a + b*Cos[x])*Csc[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sin[x])/3 + (
a^2*(3*a^2 - 2*b^2)*Cos[x]*Sin[x])/3
```

Rubi [A] time = 0.215116, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4392, 2691, 2861, 2734}

$$\frac{4}{3}ab\left(2a^2-b^2\right)\sin(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sin(x)\cos(x)+\frac{1}{3}\csc(x)(a\cos(x)+b)^2\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)+a^4x-\frac{1}{3}\csc$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^4,x]
```

```
[Out] a^4*x + ((b + a*Cos[x])^2*(a*b + (3*a^2 - 2*b^2)*Cos[x])*Csc[x])/3 - ((b +
a*Cos[x])^3*(a + b*Cos[x])*Csc[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sin[x])/3 + (
a^2*(3*a^2 - 2*b^2)*Cos[x]*Sin[x])/3
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cot(x)+b\csc(x))^4\,dx &= \int (b+a\cos(x))^4\csc^4(x)\,dx\\ &= -\frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))\csc^3(x)-\frac{1}{3}\int (b+a\cos(x))^2\left(3a^2-2b^2+ab\cos(x)\right)\csc^2(x\\ &= \frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x)-\frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))\csc^3(x)+\\ &= a^4x+\frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x)-\frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))\csc\end{aligned}$$

Mathematica [A] time = 0.256534, size = 95, normalized size = 0.94

$$-\frac{1}{12}\csc^3(x)\left(6a^2b^2\cos(3x)+6b^2\left(3a^2+b^2\right)\cos(x)+24a^3b\cos(2x)-8a^3b-9a^4x\sin(x)+3a^4x\sin(3x)+4a^4\cos(3x)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^4,x]
```

```
[Out] -(Csc[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cos[x] + 24*a^3*b*Cos
[2*x] + 4*a^4*Cos[3*x] + 6*a^2*b^2*Cos[3*x] - 2*b^4*Cos[3*x] - 9*a^4*x*Sin[
x] + 3*a^4*x*Sin[3*x]))/12
```

Maple [A] time = 0.049, size = 93, normalized size = 0.9

$$a^4\left(-\frac{(\cot(x))^3}{3}+\cot(x)+x\right)+4a^3b\left(-1/3\frac{(\cos(x))^4}{(\sin(x))^3}+1/3\frac{(\cos(x))^4}{\sin(x)}+1/3\left(2+(\cos(x))^2\right)\sin(x)\right)-2\frac{a^2b^2(\cos(x))^3}{(\sin(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^4,x)
```

```
[Out] a^4*(-1/3*cot(x)^3+cot(x)+x)+4*a^3*b*(-1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos
(x)^4+1/3*(2+cos(x)^2)*sin(x))-2*a^2*b^2/sin(x)^3*cos(x)^3-4/3*a*b^3/sin(x)
^3+b^4*(-2/3-1/3*csc(x)^2)*cot(x)
```

Maxima [A] time = 1.50357, size = 108, normalized size = 1.07

$$-2a^2b^2\cot(x)^3+\frac{1}{3}a^4\left(3x+\frac{3\tan(x)^2-1}{\tan(x)^3}\right)+\frac{4\left(3\sin(x)^2-1\right)a^3b}{3\sin(x)^3}-\frac{\left(3\tan(x)^2+1\right)b^4}{3\tan(x)^3}-\frac{4ab^3}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*a^2*b^2*cot(x)^3 + 1/3*a^4*(3*x + (3*tan(x)^2 - 1)/tan(x)^3) + 4/3*(3*si
n(x)^2 - 1)*a^3*b/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)*b^4/tan(x)^3 - 4/3*a*b^3/
sin(x)^3
```

Fricas [A] time = 1.92686, size = 225, normalized size = 2.23

$$\frac{12a^3b\cos(x)^2-8a^3b+4ab^3+2\left(2a^4+3a^2b^2-b^4\right)\cos(x)^3-3\left(a^4-b^4\right)\cos(x)+3\left(a^4x\cos(x)^2-a^4x\right)\sin(x)}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(12*a^3*b*cos(x)^2 - 8*a^3*b + 4*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)*co
s(x)^3 - 3*(a^4 - b^4)*cos(x) + 3*(a^4*x*cos(x)^2 - a^4*x)*sin(x))/((cos(x)
```

```
^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11127, size = 290, normalized size = 2.87

$$\frac{1}{24}\,a^4\tan\left(\frac{1}{2}\,x\right)^3 - \frac{1}{6}\,a^3b\tan\left(\frac{1}{2}\,x\right)^3 + \frac{1}{4}\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3 - \frac{1}{6}\,ab^3\tan\left(\frac{1}{2}\,x\right)^3 + \frac{1}{24}\,b^4\tan\left(\frac{1}{2}\,x\right)^3 + a^4x - \frac{5}{8}\,a^4\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

```
[Out] 1/24*a^4*tan(1/2*x)^3 - 1/6*a^3*b*tan(1/2*x)^3 + 1/4*a^2*b^2*tan(1/2*x)^3 -
 1/6*a*b^3*tan(1/2*x)^3 + 1/24*b^4*tan(1/2*x)^3 + a^4*x - 5/8*a^4*tan(1/2*x
) + 3/2*a^3*b*tan(1/2*x) - 3/4*a^2*b^2*tan(1/2*x) - 1/2*a*b^3*tan(1/2*x) +
3/8*b^4*tan(1/2*x) + 1/24*(15*a^4*tan(1/2*x)^2 + 36*a^3*b*tan(1/2*x)^2 + 18
*a^2*b^2*tan(1/2*x)^2 - 12*a*b^3*tan(1/2*x)^2 - 9*b^4*tan(1/2*x)^2 - a^4 -
4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4)/tan(1/2*x)^3
```

3.285 $\int (a\cot(x)+b\csc(x))^3\,dx$

Optimal. Leaf size=77

$$-\frac{1}{2}a^2b\cos(x)-\frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x))-\frac{1}{4}(a-b)^2(2a+b)\log(\cos(x)+1)-\frac{1}{2}\csc^2(x)(a\cos(x)+b)^2(a+b\cos(x))$$

```
[Out] -(a^2*b*Cos[x])/2 - ((b + a*Cos[x])^2*(a + b*Cos[x])*Csc[x]^2)/2 - ((2*a -
b)*(a + b)^2*Log[1 - Cos[x]])/4 - ((a - b)^2*(2*a + b)*Log[1 + Cos[x]])/4
```

Rubi [A] time = 0.135766, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4392, 2668, 739, 774, 633, 31}

$$-\frac{1}{2}a^2b\cos(x)-\frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x))-\frac{1}{4}(a-b)^2(2a+b)\log(\cos(x)+1)-\frac{1}{2}\csc^2(x)(a\cos(x)+b)^2(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^3,x]
```

```
[Out] -(a^2*b*Cos[x])/2 - ((b + a*Cos[x])^2*(a + b*Cos[x])*Csc[x]^2)/2 - ((2*a -
b)*(a + b)^2*Log[1 - Cos[x]])/4 - ((a - b)^2*(2*a + b)*Log[1 + Cos[x]])/4
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
```

```
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^3\,dx = \int (b+a\cos(x))^3\csc^3(x)\,dx$$
$$= -\left(a^3\,\text{Subst}\left(\int \frac{(b+x)^3}{\left(a^2-x^2\right)^2}\,dx, x, a\cos(x)\right)\right)$$
$$= -\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{(b+x)\left(2a^2-b^2+bx\right)}{a^2-x^2}\,dx, x, a\cos(\right.$$
$$= -\frac{1}{2}a^2b\cos(x) - \frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) - \frac{1}{2}a\,\text{Subst}\left(\int \frac{-a^2b-b\left(2a^2-b^2\right)}{a^2-x^2}\right.$$
$$= -\frac{1}{2}a^2b\cos(x) - \frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) + \frac{1}{4}\left((2a-b)(a+b)^2\right)\text{Subst}\left(\int \frac{1}{a-}\right.$$
$$= -\frac{1}{2}a^2b\cos(x) - \frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) - \frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x)) -$$

Mathematica [A] time = 0.273392, size = 79, normalized size = 1.03

$$\frac{1}{8}\left(-(a+b)^3\csc^2\left(\frac{x}{2}\right)+(a-b)^3\left(-\sec^2\left(\frac{x}{2}\right)\right)-4(2a-b)(a+b)^2\log\left(\sin\left(\frac{x}{2}\right)\right)-4(2a+b)(a-b)^2\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^3,x]
```

```
[Out] (-((a + b)^3*Csc[x/2]^2) - 4*(a - b)^2*(2*a + b)*Log[Cos[x/2]] - 4*(2*a - b
)*(a + b)^2*Log[Sin[x/2]] - (a - b)^3*Sec[x/2]^2)/8
```

Maple [A] time = 0.051, size = 87, normalized size = 1.1

$$-\frac{a^3 (\cot (x))^2}{2} - a^3 \ln (\sin (x)) - \frac{3 a^2 b (\cos (x))^3}{2 (\sin (x))^2} - \frac{3 a^2 b \cos (x)}{2} - \frac{3 a^2 b \ln (\csc (x) - \cot (x))}{2} - \frac{3 a b^2}{2 (\sin (x))^2} - \frac{b^3 \csc (x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^3,x)
```

```
[Out] -1/2*a^3*cot(x)^2-a^3*ln(sin(x))-3/2*a^2*b/sin(x)^2*cos(x)^3-3/2*a^2*b*cos(
x)-3/2*a^2*b*ln(csc(x)-cot(x))-3/2*a*b^2/sin(x)^2-1/2*b^3*csc(x)*cot(x)+1/2
*b^3*ln(csc(x)-cot(x))
```

Maxima [A] time = 0.988473, size = 117, normalized size = 1.52

$$-\frac{3}{2} a b^2 \cot (x)^2 + \frac{3}{4} a^2 b \left(\frac{2 \cos (x)}{\cos (x)^2 - 1} + \log (\cos (x) + 1) - \log (\cos (x) - 1)\right) + \frac{1}{4} b^3 \left(\frac{2 \cos (x)}{\cos (x)^2 - 1} - \log (\cos (x) + 1) + \log (\cos (x) - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*cot(x)^2 + 3/4*a^2*b*(2*cos(x)/(cos(x)^2 - 1) + log(cos(x) + 1)
- log(cos(x) - 1)) + 1/4*b^3*(2*cos(x)/(cos(x)^2 - 1) - log(cos(x) + 1) + l
og(cos(x) - 1)) - 1/2*a^3*(1/sin(x)^2 + log(sin(x)^2))
```

Fricas [A] time = 2.10249, size = 313, normalized size = 4.06

$$\frac{2 a^3 + 6 a b^2 + 2 \left(3 a^2 b + b^3\right) \cos (x) + \left(2 a^3 - 3 a^2 b + b^3 - \left(2 a^3 - 3 a^2 b + b^3\right) \cos (x)^2\right) \log \left(\frac{1}{2} \cos (x) + \frac{1}{2}\right) + \left(2 a^3 + 3 a^2 \cdots\right.}{4 \left(\cos (x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3 + 6*a*b^2 + 2*(3*a^2*b + b^3)*cos(x) + (2*a^3 - 3*a^2*b + b^3 -
(2*a^3 - 3*a^2*b + b^3)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^3 + 3*a^2*b
- b^3 - (2*a^3 + 3*a^2*b - b^3)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^2
 - 1)
```

Sympy [A] time = 46.8825, size = 122, normalized size = 1.58

$$\frac{a^3\log\left(\csc^2(x)\right)}{2}-\frac{a^3\csc^2(x)}{2}-\frac{3a^2b\log(\cos(x)-1)}{4}+\frac{3a^2b\log(\cos(x)+1)}{4}+\frac{3a^2b\cos(x)}{2\cos^2(x)-2}-\frac{3ab^2\csc^2(x)}{2}+\frac{b^3\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**3,x)
```

```
[Out] a**3*log(csc(x)**2)/2 - a**3*csc(x)**2/2 - 3*a**2*b*log(cos(x) - 1)/4 + 3*a
**2*b*log(cos(x) + 1)/4 + 3*a**2*b*cos(x)/(2*cos(x)**2 - 2) - 3*a*b**2*csc(
x)**2/2 + b**3*log(cos(x) - 1)/4 - b**3*log(cos(x) + 1)/4 + b**3*cos(x)/(2*
cos(x)**2 - 2)
```

Giac [A] time = 1.18298, size = 116, normalized size = 1.51

$$-\frac{1}{4}\left(2a^3-3a^2b+b^3\right)\log(\cos(x)+1)-\frac{1}{4}\left(2a^3+3a^2b-b^3\right)\log(-\cos(x)+1)+\frac{a^3+3ab^2+\left(3a^2b+b^3\right)\cos(x)}{2(\cos(x)+1)(\cos(x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^3 - 3*a^2*b + b^3)*log(cos(x) + 1) - 1/4*(2*a^3 + 3*a^2*b - b^3)*
log(-cos(x) + 1) + 1/2*(a^3 + 3*a*b^2 + (3*a^2*b + b^3)*cos(x))/((cos(x) +
1)*(cos(x) - 1))
```

3.286 $\int (a\cot(x)+b\csc(x))^2\,dx$

Optimal. Leaf size=29

$$a^2(-x)-ab\sin(x)-\csc(x)(a\cos(x)+b)(a+b\cos(x))$$

```
[Out] -(a^2*x) - (b + a*Cos[x])*(a + b*Cos[x])*Csc[x] - a*b*Sin[x]
```

Rubi [A] time = 0.056306, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2691, 2637}

$$a^2(-x)-ab\sin(x)-\csc(x)(a\cos(x)+b)(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^2,x]
```

```
[Out] -(a^2*x) - (b + a*Cos[x])*(a + b*Cos[x])*Csc[x] - a*b*Sin[x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cot(x)+b\csc(x))^2\,dx &= \int (b+a\cos(x))^2\csc^2(x)\,dx\\ &= -(b+a\cos(x))(a+b\cos(x))\csc(x)-\int \left(a^2+ab\cos(x)\right)\,dx\\ &= -a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-(ab)\int \cos(x)\,dx\\ &= -a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-ab\sin(x)\end{aligned}$$

Mathematica [A] time = 0.129501, size = 24, normalized size = 0.83

$$-\left(a^2+b^2\right)\cot(x)-a(ax+2b\csc(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^2,x]
```

```
[Out] -((a^2 + b^2)*Cot[x]) - a*(a*x + 2*b*Csc[x])
```

Maple [A] time = 0.014, size = 29, normalized size = 1.

$$a^2\left(-\cot(x)-x\right)-2\,\frac{ab}{\sin(x)}-b^2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^2,x)
```

```
[Out] a^2*(-cot(x)-x)-2*a*b/sin(x)-b^2*cot(x)
```

Maxima [A] time = 1.48353, size = 39, normalized size = 1.34

$$-a^2\left(x+\frac{1}{\tan(x)}\right)-\frac{2\,ab}{\sin(x)}-\frac{b^2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

```
[Out] -a^2*(x + 1/tan(x)) - 2*a*b/sin(x) - b^2/tan(x)
```

Fricas [A] time = 1.95012, size = 72, normalized size = 2.48

$$-\frac{a^2 x \sin(x) + 2\,ab + \left(a^2 + b^2\right)\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*x*sin(x) + 2*a*b + (a^2 + b^2)*cos(x))/sin(x)
```

Sympy [A] time = 7.40745, size = 31, normalized size = 1.07

$$-a^2 x - \frac{a^2 \cos(x)}{\sin(x)} - 2ab\csc(x) - b^2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**2,x)
```

```
[Out] -a**2*x - a**2*cos(x)/sin(x) - 2*a*b*csc(x) - b**2*cot(x)
```

Giac [A] time = 1.14155, size = 70, normalized size = 2.41

$$-a^2 x + \frac{1}{2}\,a^2 \tan\left(\frac{1}{2}\,x\right) - ab\tan\left(\frac{1}{2}\,x\right) + \frac{1}{2}\,b^2\tan\left(\frac{1}{2}\,x\right) - \frac{a^2 + 2\,ab + b^2}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

```
[Out] -a^2*x + 1/2*a^2*tan(1/2*x) - a*b*tan(1/2*x) + 1/2*b^2*tan(1/2*x) - 1/2*(a^
2 + 2*a*b + b^2)/tan(1/2*x)
```

3.287 $\int (a\cot(x)+b\csc(x))\,dx$

Optimal. Leaf size=12

$$a\log(\sin(x))-b\tanh^{-1}(\cos(x))$$

```
[Out] -(b*ArcTanh[Cos[x]]) + a*Log[Sin[x]]
```

Rubi [A] time = 0.0074097, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3475, 3770}

$$a\log(\sin(x))-b\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Cot[x] + b*Csc[x],x]
```

```
[Out] -(b*ArcTanh[Cos[x]]) + a*Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cot(x)+b\csc(x))\,dx &= a\int \cot(x)\,dx+b\int \csc(x)\,dx\\ &= -b\tanh^{-1}(\cos(x))+a\log(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0085695, size = 25, normalized size = 2.08

$$a\log(\sin(x))+b\log\left(\sin\left(\frac{x}{2}\right)\right)-b\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cot[x] + b*Csc[x],x]
```

```
[Out] -(b*Log[Cos[x/2]]) + b*Log[Sin[x/2]] + a*Log[Sin[x]]
```

Maple [A] time = 0.003, size = 16, normalized size = 1.3

$$a\ln(\sin(x)) - b\ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cot(x)+b*csc(x),x)
```

```
[Out] a*ln(sin(x))-b*ln(cot(x)+csc(x))
```

Maxima [A] time = 0.988407, size = 20, normalized size = 1.67

$$-b\log(\cot(x) + \csc(x)) + a\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="maxima")
```

```
[Out] -b*log(cot(x) + csc(x)) + a*log(sin(x))
```

Fricas [B] time = 2.01705, size = 97, normalized size = 8.08

$$\frac{1}{2}(a-b)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}(a+b)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="fricas")
```

```
[Out] 1/2*(a - b)*log(1/2*cos(x) + 1/2) + 1/2*(a + b)*log(-1/2*cos(x) + 1/2)
```

Sympy [A] time = 0.124325, size = 24, normalized size = 2.

$$a\log(\sin(x)) + b\left(\frac{\log(\cos(x)-1)}{2} - \frac{\log(\cos(x)+1)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x)
```

```
[Out] a*log(sin(x)) + b*(log(cos(x) - 1)/2 - log(cos(x) + 1)/2)
```

Giac [A] time = 1.14671, size = 28, normalized size = 2.33

$$\frac{1}{2}\,a\log\left(-\cos(x)^2+1\right) + b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="giac")
```

```
[Out] 1/2*a*log(-cos(x)^2 + 1) + b*log(abs(tan(1/2*x)))
```

3.288 $\int \frac{1}{a\cot(x)+b\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a\cos(x)+b)}{a}$$

```
[Out] -(Log[b + a*Cos[x]]/a)
```

Rubi [A] time = 0.0350371, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3160, 2668, 31}

$$-\frac{\log(a\cos(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-1),x]
```

```
[Out] -(Log[b + a*Cos[x]]/a)
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a\cot(x)+b\csc(x)}\,dx=\int \frac{\sin(x)}{b+a\cos(x)}\,dx$$
$$=-\frac{\text{Subst}\left(\int \frac{1}{b+x}\,dx,x,a\cos(x)\right)}{a}$$
$$=-\frac{\log(b+a\cos(x))}{a}$$

Mathematica [A] time = 0.0161616, size = 12, normalized size = 1.

$$-\frac{\log(a\cos(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-1),x]
```

```
[Out] -(Log[b + a*Cos[x]]/a)
```

Maple [A] time = 0.038, size = 13, normalized size = 1.1

$$-\frac{\ln(b+a\cos(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x)),x)
```

```
[Out] -ln(b+a*cos(x))/a
```

Maxima [B] time = 1.4615, size = 61, normalized size = 5.08

$$-\frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a}+\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="maxima")
```

```
[Out] -log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(x)^2/(cos(x) + 1)
^2 + 1)/a
```

Fricas [A] time = 2.06224, size = 30, normalized size = 2.5

$$-\frac{\log\left(a\cos\left(x\right)+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="fricas")
```

```
[Out] -log(a*cos(x) + b)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{a\cot\left(x\right)+b\csc\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x)
```

```
[Out] Integral(1/(a*cot(x) + b*csc(x)), x)
```

Giac [A] time = 1.13109, size = 18, normalized size = 1.5

$$-\frac{\log\left(\left|a\cos\left(x\right)+b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="giac")
```

```
[Out] -log(abs(a*cos(x) + b))/a
```

3.289 $\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\sin(x)}{a(a\cos(x)+b)}$$

```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]) + Sin[x]/(a*(b + a*Cos[x]))
```

Rubi [A] time = 0.117185, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4392, 2693, 2735, 2659, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\sin(x)}{a(a\cos(x)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-2),x]
```


```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]) + Sin[x]/(a*(b + a*Cos[x]))
```


Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx &= \int \frac{\sin^2(x)}{(b+a\cos(x))^2}\,dx \\
&= \frac{\sin(x)}{a(b+a\cos(x))}-\frac{\int \frac{\cos(x)}{b+a\cos(x)}\,dx}{a} \\
&= -\frac{x}{a^2}+\frac{\sin(x)}{a(b+a\cos(x))}+\frac{b\int \frac{1}{b+a\cos(x)}\,dx}{a^2} \\
&= -\frac{x}{a^2}+\frac{\sin(x)}{a(b+a\cos(x))}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2} \\
&= -\frac{x}{a^2}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{\sin(x)}{a(b+a\cos(x))}
\end{aligned}
$$

Mathematica [A] time = 0.255074, size = 71, normalized size = 1.06

$$
-\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{-a\sin(x)+ax\cos(x)+bx}{a\cos(x)+b}}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-2),x]
```

```
[Out] -(((2*b*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (b*
x + a*x*Cos[x] - a*Sin[x])/(b + a*Cos[x]))/a^2)
```

Maple [A] time = 0.051, size = 86, normalized size = 1.3

$$-2\,\frac{\arctan\left(\tan\left(x/2\right)\right)}{a^{2}}-2\,\frac{\tan\left(x/2\right)}{a\left(a\left(\tan\left(x/2\right)\right)^{2}-b\left(\tan\left(x/2\right)\right)^{2}-a-b\right)}+2\,\frac{b}{a^{2}\sqrt{\left(a-b\right)\left(a+b\right)}}\mathrm{Artanh}\left(\frac{\left(a-b\right)\tan\left(x/2\right)}{\sqrt{\left(a-b\right)\left(a+b\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^2,x)
```

```
[Out] -2/a^2*arctan(tan(1/2*x))-2/a*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b
)+2/a^2*b/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39142, size = 689, normalized size = 10.28

$$\left[-\frac{2\left(a^{3}-ab^{2}\right)x\cos\left(x\right)-\left(ab\cos\left(x\right)+b^{2}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(x)-\left(a^{2}-2\,b^{2}\right)\cos(x)^{2}+2\,\sqrt{a^{2}-b^{2}}(b\cos(x)+a)\sin(x)+2\,a^{2}-b^{2}}{a^{2}\cos(x)^{2}+2\,ab\cos(x)+b^{2}}\right)+2\left(a^{2}b-\right.}{2\left(a^{4}b-a^{2}b^{3}+\left(a^{5}-a^{3}b^{2}\right)\cos\left(x\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(a^2 - b^2)*log((2
*a*b*cos(x) - (a^2 - 2*b^2)*cos(x)^2 + 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin
(x) + 2*a^2 - b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 2*(a^2*b - b^3)*x
 - 2*(a^3 - a*b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x)), -((a
^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + (a^2*b - b^3)*x - (a^3 - a*
b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cot(x) + b\csc(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**2,x)
```

```
[Out] Integral((a*cot(x) + b*csc(x))**(-2), x)
```

Giac [A] time = 1.15058, size = 144, normalized size = 2.15

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}a^2} - \frac{x}{a^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2 - b\tan\left(\frac{1}{2}x\right)^2 - a - b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a^2) - x/a^2 - 2*tan(1/2*x)
/((a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)*a)
```

3.290 $\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx$

Optimal. Leaf size=50

$$\frac{a^2-b^2}{2a^3(a\cos(x)+b)^2}+\frac{2b}{a^3(a\cos(x)+b)}+\frac{\log(a\cos(x)+b)}{a^3}$$

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cos[x])^2) + (2*b)/(a^3*(b + a*Cos[x])) + Log[b +
 a*Cos[x]]/a^3
```

Rubi [A] time = 0.0787822, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$\frac{a^2-b^2}{2a^3(a\cos(x)+b)^2}+\frac{2b}{a^3(a\cos(x)+b)}+\frac{\log(a\cos(x)+b)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-3),x]
```

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cos[x])^2) + (2*b)/(a^3*(b + a*Cos[x])) + Log[b +
 a*Cos[x]]/a^3
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx &= \int \frac{\sin^3(x)}{(b+a\cos(x))^3}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{a^2-x^2}{(b+x)^3}\,dx, x, a\cos(x)\right)}{a^3}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{-b-x}+\frac{a^2-b^2}{(b+x)^3}+\frac{2b}{(b+x)^2}\right)dx, x, a\cos(x)\right)}{a^3}\\ &= \frac{a^2-b^2}{2a^3(b+a\cos(x))^2}+\frac{2b}{a^3(b+a\cos(x))}+\frac{\log(b+a\cos(x))}{a^3}\end{aligned}$$

Mathematica [A] time = 0.111372, size = 77, normalized size = 1.54

$$\frac{a^2\cos(2x)\log(a\cos(x)+b)+a^2\log(a\cos(x)+b)+a^2+2b^2\log(a\cos(x)+b)+4ab\cos(x)(\log(a\cos(x)+b)+1)+3b}{2a^3(a\cos(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-3),x]
```

```
[Out] (a^2 + 3*b^2 + a^2*Log[b + a*Cos[x]] + 2*b^2*Log[b + a*Cos[x]] + a^2*Cos[2*
x]*Log[b + a*Cos[x]] + 4*a*b*Cos[x]*(1 + Log[b + a*Cos[x]]))/(2*a^3*(b + a*
Cos[x])^2)
```

Maple [A] time = 0.048, size = 56, normalized size = 1.1

$$\frac{\ln(b+a\cos(x))}{a^3}+2\frac{b}{a^3(b+a\cos(x))}+\frac{1}{2a(b+a\cos(x))^2}-\frac{b^2}{2a^3(b+a\cos(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^3,x)
```

```
[Out] ln(b+a*cos(x))/a^3+2*b/a^3/(b+a*cos(x))+1/2/a/(b+a*cos(x))^2-1/2/a^3/(b+a*c
os(x))^2*b^2
```

Maxima [B] time = 1.51703, size = 239, normalized size = 4.78

$$\frac{2\left(ab+b^2+\frac{(a^2-2\,ab+b^2)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5+a^4b-a^3b^2-a^2b^3-\frac{2\,(a^5-a^4b-a^3b^2+a^2b^3)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sin(x)^4}{(\cos(x)+1)^4}}+\frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}-\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(a*b + b^2 + (a^2 - 2*a*b + b^2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + a^4*b -
a^3*b^2 - a^2*b^3 - 2*(a^5 - a^4*b - a^3*b^2 + a^2*b^3)*sin(x)^2/(cos(x) +
1)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(x)^4/(cos(x) + 1)^4) + log
(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^3 - log(sin(x)^2/(cos(x) + 1)^2
 + 1)/a^3
```

Fricas [A] time = 2.31057, size = 181, normalized size = 3.62

$$\frac{4\,ab\cos(x)+a^2+3\,b^2+2\left(a^2\cos(x)^2+2\,ab\cos(x)+b^2\right)\log(a\cos(x)+b)}{2\left(a^5\cos(x)^2+2\,a^4b\cos(x)+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*cos(x) + a^2 + 3*b^2 + 2*(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)*log
(a*cos(x) + b))/(a^5*cos(x)^2 + 2*a^4*b*cos(x) + a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11597, size = 61, normalized size = 1.22

$$\frac{\log\left(|a\cos\left(x\right)+b|\right)}{a^3}+\frac{4\,b\cos\left(x\right)+\frac{a^2+3\,b^2}{a}}{2\left(a\cos\left(x\right)+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(a*cos(x) + b))/a^3 + 1/2*(4*b*cos(x) + (a^2 + 3*b^2)/a)/((a*cos(x)
+ b)^2*a^2)
```

3.291 $\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$

Optimal. Leaf size=159

$$\frac{b\sin^3(x)}{2a(a^2-b^2)(a\cos(x)+b)^2}-\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{2a^3(a^2-b^2)(a\cos(x)+b)}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}+\frac{x}{a^4}+\frac{\sin^3(x)}{3a(a\cos(x)+$$

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cos[x])*Sin[x])/(2*a
^3*(a^2 - b^2)*(b + a*Cos[x])) + Sin[x]^3/(3*a*(b + a*Cos[x])^3) + (b*Sin[x
]^3)/(2*a*(a^2 - b^2)*(b + a*Cos[x])^2)
```

Rubi [A] time = 0.338455, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2693, 2864, 2863, 2735, 2659, 208}

$$\frac{b\sin^3(x)}{2a(a^2-b^2)(a\cos(x)+b)^2}-\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{2a^3(a^2-b^2)(a\cos(x)+b)}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}+\frac{x}{a^4}+\frac{\sin^3(x)}{3a(a\cos(x)+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-4),x]
```

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cos[x])*Sin[x])/(2*a
^3*(a^2 - b^2)*(b + a*Cos[x])) + Sin[x]^3/(3*a*(b + a*Cos[x])^3) + (b*Sin[x
]^3)/(2*a*(a^2 - b^2)*(b + a*Cos[x])^2)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx = \int \frac{\sin^4(x)}{(b+a\cos(x))^4}\,dx$$
$$= \frac{\sin^3(x)}{3a(b+a\cos(x))^3} - \frac{\int \frac{\cos(x)\sin^2(x)}{(b+a\cos(x))^3}\,dx}{a}$$
$$= \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos(x))^2} - \frac{\int \frac{(2a+b\cos(x))\sin^2(x)}{(b+a\cos(x))^2}\,dx}{2a\left(a^2-b^2\right)}$$
$$= -\frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos(x))^2} + \frac{\int -}{}$$
$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos(x))^2} -$$
$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos(x))^2} -$$
$$= \frac{x}{a^4} - \frac{b\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(}$$

Mathematica [A] time = 0.466912, size = 150, normalized size = 0.94

$$\frac{\sin(x)\left(-\frac{a\left(8a^2-11b^2\right)(a\cos(x)+b)^2}{(a-b)(a+b)} - \frac{6b\left(2b^2-3a^2\right)\csc(x)(a\cos(x)+b)^3\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 2a\left(a^2-b^2\right) + 7ab(a\cos(x)+b) + 6x\csc(x)(\right.}{6a^4(a\cos(x)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-4),x]
```

```
[Out] ((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cos[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cos[x
])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cos[x])^3*Csc[x] - (6*b*(-3*a^2 + 2*b^
2)*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[x])^3*Csc[x])/(a
^2 - b^2)^(3/2))*Sin[x])/(6*a^4*(b + a*Cos[x])^3)
```

Maple [B] time = 0.062, size = 534, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^4,x)
```

```
[Out] 2/a^4*arctan(tan(1/2*x))+2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(
1/2*x)^5-1/a/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b-3/a
^2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b^2+2/a^3/(a*ta
n(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b^3-20/3/a/(a*tan(1/2*x
)^2-b*tan(1/2*x)^2-a-b)^3*tan(1/2*x)^3+4/a^3/(a*tan(1/2*x)^2-b*tan(1/2*x)^2
-a-b)^3*tan(1/2*x)^3*b^2+2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(
1/2*x)+1/a/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)*b-3/a^2/(
a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)*b^2-2/a^3/(a*tan(1/2*
x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)*b^3-3/a^2*b/(a^2-b^2)/((a-b)*(a
+b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))+2/a^4*b^3/(a^2-b^2
)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.93663, size = 1945, normalized size = 12.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 36*(a^6*b - 2*a^4*b^3 +
a^2*b^5)*x*cos(x)^2 + 36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x) + 3*(3*a^2*
```

```
b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b^4)*co
s(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(a^2 - b^2)*log((2*a*b*cos(x)
- (a^2 - 2*b^2)*cos(x)^2 - 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin(x) + 2*a^2
- b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 12*(a^4*b^3 - 2*a^2*b^5 + b^7
)*x + 2*(2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2 + 1
1*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*cos(x))*sin(x))/(
a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cos(x)^3 + 3*(
a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*
cos(x)), 1/6*(6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 18*(a^6*b - 2*a^4*
b^3 + a^2*b^5)*x*cos(x)^2 + 18*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x) - 3*(
3*a^2*b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b
^4)*cos(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + 6*(a^4*b^3 - 2*a^2*b^5
 + b^7)*x + (2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2
 + 11*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*cos(x))*sin(x
))/(a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cos(x)^3 +
 3*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b
^6)*cos(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14053, size = 381, normalized size = 2.4

$$-\frac{\left(3\,a^2b-2\,b^3\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}+\frac{6\,a^4\tan\left(\frac{1}{2}\,x\right)^5-9\,a^3b\tan\left(\frac{1}{2}\,x\right)^5-6\,a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - 2*b^3)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*x) - b*tan(1/2*x))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b
^2)) + 1/3*(6*a^4*tan(1/2*x)^5 - 9*a^3*b*tan(1/2*x)^5 - 6*a^2*b^2*tan(1/2*x
)^5 + 15*a*b^3*tan(1/2*x)^5 - 6*b^4*tan(1/2*x)^5 - 20*a^4*tan(1/2*x)^3 + 32
*a^2*b^2*tan(1/2*x)^3 - 12*b^4*tan(1/2*x)^3 + 6*a^4*tan(1/2*x) + 9*a^3*b*ta
n(1/2*x) - 6*a^2*b^2*tan(1/2*x) - 15*a*b^3*tan(1/2*x) - 6*b^4*tan(1/2*x))/(
(a^5 - a^3*b^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)^3) + x/a^4
```

3.292 $\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx$

Optimal. Leaf size=100

$$\frac{\left(a^2-b^2\right)^2}{4a^5(a\cos(x)+b)^4}+\frac{4b\left(a^2-b^2\right)}{3a^5(a\cos(x)+b)^3}-\frac{a^2-3b^2}{a^5(a\cos(x)+b)^2}-\frac{4b}{a^5(a\cos(x)+b)}-\frac{\log(a\cos(x)+b)}{a^5}$$

```
[Out] (a^2 - b^2)^2/(4*a^5*(b + a*Cos[x])^4) + (4*b*(a^2 - b^2))/(3*a^5*(b + a*Co
s[x])^3) - (a^2 - 3*b^2)/(a^5*(b + a*Cos[x])^2) - (4*b)/(a^5*(b + a*Cos[x])
) - Log[b + a*Cos[x]]/a^5
```

Rubi [A] time = 0.123181, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$\frac{\left(a^2-b^2\right)^2}{4a^5(a\cos(x)+b)^4}+\frac{4b\left(a^2-b^2\right)}{3a^5(a\cos(x)+b)^3}-\frac{a^2-3b^2}{a^5(a\cos(x)+b)^2}-\frac{4b}{a^5(a\cos(x)+b)}-\frac{\log(a\cos(x)+b)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-5),x]
```

```
[Out] (a^2 - b^2)^2/(4*a^5*(b + a*Cos[x])^4) + (4*b*(a^2 - b^2))/(3*a^5*(b + a*Co
s[x])^3) - (a^2 - 3*b^2)/(a^5*(b + a*Cos[x])^2) - (4*b)/(a^5*(b + a*Cos[x])
) - Log[b + a*Cos[x]]/a^5
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = \int \frac{\sin^5(x)}{(b+a\cos(x))^5}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(a^2-x^2\right)^2}{(b+x)^5}\,dx, x, a\cos(x)\right)}{a^5}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(b+x)^5}-\frac{4b\left(-a^2+b^2\right)}{(b+x)^4}-\frac{2\left(a^2-3b^2\right)}{(b+x)^3}-\frac{4b}{(b+x)^2}+\frac{1}{b+x}\right)dx, x, a\cos(x)\right)}{a^5}$$
$$= \frac{\left(a^2-b^2\right)^2}{4a^5(b+a\cos(x))^4}+\frac{4b\left(a^2-b^2\right)}{3a^5(b+a\cos(x))^3}-\frac{a^2-3b^2}{a^5(b+a\cos(x))^2}-\frac{4b}{a^5(b+a\cos(x))}-\frac{\log(b+$$

Mathematica [A] time = 0.3392, size = 138, normalized size = 1.38

$$-\frac{12a^2\cos^2(x)\left(a^2+6b^2\log(a\cos(x)+b)+9b^2\right)+8ab\cos(x)\left(a^2+6b^2\log(a\cos(x)+b)+11b^2\right)+2a^2b^2+12a^4\cos^4(x}{12a^5(a\cos(x)+b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-5),x]
```

```
[Out] -(-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cos[x]] + 12*a^4*Cos[x]^4*
Log[b + a*Cos[x]] + 48*a^3*b*Cos[x]^3*(1 + Log[b + a*Cos[x]]) + 12*a^2*Cos[
x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cos[x]]) + 8*a*b*Cos[x]*(a^2 + 11*b^2 +
 6*b^2*Log[b + a*Cos[x]]))/(12*a^5*(b + a*Cos[x])^4)
```

Maple [A] time = 0.056, size = 132, normalized size = 1.3

$$\frac{1}{4\,a\left(b+a\cos(x)\right)^4}-\frac{b^2}{2\,a^3\left(b+a\cos(x)\right)^4}+\frac{b^4}{4\,a^5\left(b+a\cos(x)\right)^4}-4\,\frac{b}{a^5\left(b+a\cos(x)\right)}-\frac{\ln\left(b+a\cos(x)\right)}{a^5}+\frac{4\,b}{3\,a^3\left(b+a\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^5,x)
```

```
[Out] 1/4/a/(b+a*cos(x))^4-1/2/a^3/(b+a*cos(x))^4*b^2+1/4/a^5/(b+a*cos(x))^4*b^4-
4*b/a^5/(b+a*cos(x))-ln(b+a*cos(x))/a^5+4/3*b/a^3/(b+a*cos(x))^3-4/3*b^3/a^
5/(b+a*cos(x))^3-1/a^3/(b+a*cos(x))^2+3/a^5/(b+a*cos(x))^2*b^2
```

Maxima [B] time = 1.69827, size = 671, normalized size = 6.71

$$-\frac{2\left(5\,a^4b+10\,a^3b^2+2\,a^2b^3-6\,ab^4-3\,b^5+\frac{\left(3\,a^5-17\,a^4b-6\,a^3b^2+26\,a^2b^3+3\,ab^4-9\,b^5\right)\sin(x)^2}{(\cos(x)+1)^2}-\frac{3\left(4\,a^5-13\,a\right.}{}\right.}{3\left(a^{10}+2\,a^9b-a^8b^2-4\,a^7b^3-a^6b^4+2\,a^5b^5+a^4b^6-\frac{4\left(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6\left(a^{10}-2\,a^9b-a^8b^2+4\,a^7b^3-a^6b^4-2\,a^5b^5+a^4b^6\right.}{(\cos(x)+1)^4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

```
[Out] -2/3*(5*a^4*b + 10*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 - 3*b^5 + (3*a^5 - 17*a^4*
b - 6*a^3*b^2 + 26*a^2*b^3 + 3*a*b^4 - 9*b^5)*sin(x)^2/(cos(x) + 1)^2 - 3*(
4*a^5 - 13*a^4*b + 12*a^3*b^2 + 2*a^2*b^3 - 8*a*b^4 + 3*b^5)*sin(x)^4/(cos(
x) + 1)^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin
(x)^6/(cos(x) + 1)^6)/(a^10 + 2*a^9*b - a^8*b^2 - 4*a^7*b^3 - a^6*b^4 + 2*a
^5*b^5 + a^4*b^6 - 4*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sin(x)^2/(cos
(x) + 1)^2 + 6*(a^10 - 2*a^9*b - a^8*b^2 + 4*a^7*b^3 - a^6*b^4 - 2*a^5*b^5
+ a^4*b^6)*sin(x)^4/(cos(x) + 1)^4 - 4*(a^10 - 4*a^9*b + 5*a^8*b^2 - 5*a^6*
b^4 + 4*a^5*b^5 - a^4*b^6)*sin(x)^6/(cos(x) + 1)^6 + (a^10 - 6*a^9*b + 15*a
^8*b^2 - 20*a^7*b^3 + 15*a^6*b^4 - 6*a^5*b^5 + a^4*b^6)*sin(x)^8/(cos(x) +
1)^8) - log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^5 + log(sin(x)^2/(co
s(x) + 1)^2 + 1)/a^5
```

Fricas [A] time = 2.53773, size = 409, normalized size = 4.09

$$-\frac{48\,a^3b\cos(x)^3-3\,a^4+2\,a^2b^2+25\,b^4+12\left(a^4+9\,a^2b^2\right)\cos(x)^2+8\left(a^3b+11\,ab^3\right)\cos(x)+12\left(a^4\cos(x)^4+4\,a^3b\cos\right.}{12\left(a^9\cos(x)^4+4\,a^8b\cos(x)^3+6\,a^7b^2\cos(x)^2+4\,a^6b^3\cos(x)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

```
[Out] -1/12*(48*a^3*b*cos(x)^3 - 3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*(a^4 + 9*a^2*b^2
)*cos(x)^2 + 8*(a^3*b + 11*a*b^3)*cos(x) + 12*(a^4*cos(x)^4 + 4*a^3*b*cos(x
)^3 + 6*a^2*b^2*cos(x)^2 + 4*a*b^3*cos(x) + b^4)*log(a*cos(x) + b))/(a^9*co
s(x)^4 + 4*a^8*b*cos(x)^3 + 6*a^7*b^2*cos(x)^2 + 4*a^6*b^3*cos(x) + a^5*b^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14898, size = 126, normalized size = 1.26

$$-\frac{\log\left(|a\cos\left(x\right)+b|\right)}{a^5}-\frac{48\,a^2b\cos\left(x\right)^3+12\left(a^3+9\,ab^2\right)\cos\left(x\right)^2+8\left(a^2b+11\,b^3\right)\cos\left(x\right)-\frac{3\,a^4-2\,a^2b^2-25\,b^4}{a}}{12\left(a\cos\left(x\right)+b\right)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

```
[Out] -log(abs(a*cos(x) + b))/a^5 - 1/12*(48*a^2*b*cos(x)^3 + 12*(a^3 + 9*a*b^2)*
cos(x)^2 + 8*(a^2*b + 11*b^3)*cos(x) - (3*a^4 - 2*a^2*b^2 - 25*b^4)/a)/((a*
cos(x) + b)^4*a^4)
```

3.293 $\int (\cot(x)+\csc(x))^5\,dx$

Optimal. Leaf size=28

$$\frac{4}{1-\cos(x)}-\frac{2}{(1-\cos(x))^2}+\log(1-\cos(x))$$

```
[Out] -2/(1 - Cos[x])^2 + 4/(1 - Cos[x]) + Log[1 - Cos[x]]
```

Rubi [A] time = 0.0510641, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{4}{1-\cos(x)}-\frac{2}{(1-\cos(x))^2}+\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^5,x]
```

```
[Out] -2/(1 - Cos[x])^2 + 4/(1 - Cos[x]) + Log[1 - Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (\cot(x)+\csc(x))^5\,dx &= \int (1+\cos(x))^5\csc^5(x)\,dx\\
&= -\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, \cos(x)\right)\\
&= -\frac{2}{(1-\cos(x))^2}+\frac{4}{1-\cos(x)}+\log(1-\cos(x))
\end{aligned}
$$

Mathematica [A] time = 0.0734023, size = 32, normalized size = 1.14

$$
-\frac{1}{2}\csc^4\left(\frac{x}{2}\right)+2\csc^2\left(\frac{x}{2}\right)+2\log\left(\sin\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^5,x]
```

```
[Out] 2*Csc[x/2]^2 - Csc[x/2]^4/2 + 2*Log[Sin[x/2]]
```

Maple [B] time = 0.048, size = 105, normalized size = 3.8

$$
-\frac{(\cot(x))^4}{4}+\frac{(\cot(x))^2}{2}+\ln(\sin(x))-\frac{5\,(\cos(x))^5}{4\,(\sin(x))^4}+\frac{5\,(\cos(x))^5}{8\,(\sin(x))^2}+\frac{5\,(\cos(x))^3}{8}+\frac{5\,\cos(x)}{8}+\ln(\csc(x)-\cot(x))-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^5,x)
```

```
[Out] -1/4*cot(x)^4+1/2*cot(x)^2+ln(sin(x))-5/4/sin(x)^4*cos(x)^5+5/8/sin(x)^2*co
s(x)^5+5/8*cos(x)^3+5/8*cos(x)+ln(csc(x)-cot(x))-5/2/sin(x)^4*cos(x)^4-5/2/
sin(x)^4*cos(x)^3-5/4/sin(x)^2*cos(x)^3-5/4/sin(x)^4+(-1/4*csc(x)^3-3/8*csc
(x))*cot(x)
```

Maxima [B] time = 1.01307, size = 169, normalized size = 6.04

$$-\frac{5}{2}\cot(x)^4-\frac{5\left(5\cos(x)^3-3\cos(x)\right)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{3\cos(x)^3-5\cos(x)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}-\frac{5\left(\cos(x)^3+\cos(x)\right)}{4\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{4\sin(x)^2-1}{4\sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*cot(x)^4 - 5/8*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1
/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 5/4*(cos(x)^3 + co
s(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 - 5/4/sin
(x)^4 + 1/2*log(sin(x)^2) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [A] time = 2.03381, size = 126, normalized size = 4.5

$$\frac{\left(\cos(x)^2-2\cos(x)+1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-4\cos(x)+2}{\cos(x)^2-2\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="fricas")
```

```
[Out] ((cos(x)^2 - 2*cos(x) + 1)*log(-1/2*cos(x) + 1/2) - 4*cos(x) + 2)/(cos(x)^2
 - 2*cos(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14025, size = 30, normalized size = 1.07

$$-\frac{2\,(2\,\cos(x)-1)}{(\cos(x)-1)^2}+\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="giac")
```

```
[Out] -2*(2*cos(x) - 1)/(cos(x) - 1)^2 + log(-cos(x) + 1)
```

3.294 $\int (\cot(x)+\csc(x))^4\,dx$

Optimal. Leaf size=30

$$x-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\frac{2\sin(x)}{1-\cos(x)}$$

```
[Out] x + (2*Sin[x])/(1 - Cos[x]) - (2*Sin[x]^3)/(3*(1 - Cos[x])^3)
```

Rubi [A] time = 0.101051, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$x-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\frac{2\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^4,x]
```

```
[Out] x + (2*Sin[x])/(1 - Cos[x]) - (2*Sin[x]^3)/(3*(1 - Cos[x])^3)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int(\cot(x)+\csc(x))^4\,dx&=\int(1+\cos(x))^4\csc^4(x)\,dx\\&=\int\frac{\sin^4(x)}{(1-\cos(x))^4}\,dx\\&=-\frac{2\sin^3(x)}{3(1-\cos(x))^3}-\int\frac{\sin^2(x)}{(1-\cos(x))^2}\,dx\\&=\frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\int 1\,dx\\&=x+\frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}\end{aligned}$$

Mathematica [A] time = 0.0451802, size = 30, normalized size = 1.

$$x+\frac{8}{3}\cot\left(\frac{x}{2}\right)-\frac{2}{3}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^4,x]

[Out] x + (8*Cot[x/2])/3 - (2*Cot[x/2]*Csc[x/2]^2)/3
```

Maple [B] time = 0.039, size = 68, normalized size = 2.3

$$-\frac{(\cot(x))^3}{3}+\cot(x)+x-\frac{4\,(\cos(x))^4}{3\,(\sin(x))^3}+\frac{4\,(\cos(x))^4}{3\sin(x)}+\frac{\left(8+4\,(\cos(x))^2\right)\sin(x)}{3}-2\,\frac{(\cos(x))^3}{(\sin(x))^3}-\frac{4}{3\,(\sin(x))^3}+\left(-\frac{2}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^4,x)
```

```
[Out] -1/3*cot(x)^3+cot(x)+x-4/3/sin(x)^3*cos(x)^4+4/3/sin(x)*cos(x)^4+4/3*(2+cos
(x)^2)*sin(x)-2/sin(x)^3*cos(x)^3-4/3/sin(x)^3+(-2/3-1/3*csc(x)^2)*cot(x)
```

Maxima [B] time = 1.47857, size = 76, normalized size = 2.53

$$-2\cot(x)^3 + x + \frac{4\left(3\sin(x)^2 - 1\right)}{3\sin(x)^3} - \frac{3\tan(x)^2 + 1}{3\tan(x)^3} + \frac{3\tan(x)^2 - 1}{3\tan(x)^3} - \frac{4}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*cot(x)^3 + x + 4/3*(3*sin(x)^2 - 1)/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)/tan(
x)^3 + 1/3*(3*tan(x)^2 - 1)/tan(x)^3 - 4/3/sin(x)^3
```

Fricas [A] time = 2.09382, size = 109, normalized size = 3.63

$$\frac{8\cos(x)^2 + 3(x\cos(x) - x)\sin(x) + 4\cos(x) - 4}{3(\cos(x) - 1)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(8*cos(x)^2 + 3*(x*cos(x) - x)*sin(x) + 4*cos(x) - 4)/((cos(x) - 1)*sin
(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13628, size = 27, normalized size = 0.9

$$x + \frac{2\left(3\tan\left(\frac{1}{2}x\right)^{2} - 1\right)}{3\tan\left(\frac{1}{2}x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^4,x, algorithm="giac")
```

```
[Out] x + 2/3*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3
```

3.295 $\int (\cot(x)+\csc(x))^3\,dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

```
[Out] -2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

Rubi [A] time = 0.0480181, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^3,x]
```

```
[Out] -2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))^3\,dx &= \int (1+\cos(x))^3\csc^3(x)\,dx\\ &= -\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx,x,\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx,x,\cos(x)\right)\\ &= -\frac{2}{1-\cos(x)}-\log(1-\cos(x))\end{aligned}$$

Mathematica [A] time = 0.0394034, size = 20, normalized size = 1.

$$-\csc^2\left(\frac{x}{2}\right)-2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^3,x]
```

```
[Out] -Csc[x/2]^2 - 2*Log[Sin[x/2]]
```

Maple [B] time = 0.037, size = 49, normalized size = 2.5

$$-\frac{(\cot(x))^2}{2}-\ln(\sin(x))-\frac{3\,(\cos(x))^3}{2\,(\sin(x))^2}-\frac{3\,\cos(x)}{2}-\ln(\csc(x)-\cot(x))-\frac{3}{2\,(\sin(x))^2}-\frac{\cot(x)\csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^3,x)
```

```
[Out] -1/2*cot(x)^2-ln(sin(x))-3/2/sin(x)^2*cos(x)^3-3/2*cos(x)-ln(csc(x)-cot(x))
-3/2/sin(x)^2-1/2*cot(x)*csc(x)
```

Maxima [B] time = 0.95826, size = 62, normalized size = 3.1

$$-\frac{3}{2}\cot(x)^2+\frac{2\,\cos(x)}{\cos(x)^2-1}-\frac{1}{2\,\sin(x)^2}-\frac{1}{2}\log\left(\sin(x)^2\right)+\frac{1}{2}\log(\cos(x)+1)-\frac{1}{2}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*cot(x)^2 + 2*cos(x)/(cos(x)^2 - 1) - 1/2/sin(x)^2 - 1/2*log(sin(x)^2)
+ 1/2*log(cos(x) + 1) - 1/2*log(cos(x) - 1)
```

Fricas [A] time = 2.05848, size = 77, normalized size = 3.85

$$-\frac{(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="fricas")
```

```
[Out] -((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

Sympy [B] time = 82.1944, size = 44, normalized size = 2.2

$$-\frac{\log(\cos(x)-1)}{2}+\frac{\log(\cos(x)+1)}{2}+\frac{\log\left(\csc^2(x)\right)}{2}-2\csc^2(x)+\frac{4\cos(x)}{2\cos^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**3,x)
```

```
[Out] -log(cos(x) - 1)/2 + log(cos(x) + 1)/2 + log(csc(x)**2)/2 - 2*csc(x)**2 + 4
*cos(x)/(2*cos(x)**2 - 2)
```

Giac [A] time = 1.1422, size = 24, normalized size = 1.2

$$\frac{2}{\cos(x)-1}-\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="giac")
```

```
[Out] 2/(cos(x) - 1) - log(-cos(x) + 1)
```

3.296 $\int (\cot(x) + \csc(x))^2 \, dx$

Optimal. Leaf size=16

$$-x - \frac{2\sin(x)}{1 - \cos(x)}$$

```
[Out] -x - (2*Sin[x])/(1 - Cos[x])
```

Rubi [A] time = 0.0688719, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$-x - \frac{2\sin(x)}{1 - \cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^2,x]
```

```
[Out] -x - (2*Sin[x])/(1 - Cos[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int (\cot(x)+\csc(x))^2\,dx &= \int (1+\cos(x))^2\csc^2(x)\,dx \\
&= \int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx \\
&= -\frac{2\sin(x)}{1-\cos(x)} - \int 1\,dx \\
&= -x-\frac{2\sin(x)}{1-\cos(x)}
\end{aligned}$$

Mathematica [A] time = 0.0236012, size = 12, normalized size = 0.75

$$-x-2\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^2,x]
```

```
[Out] -x - 2*Cot[x/2]
```

Maple [A] time = 0.012, size = 15, normalized size = 0.9

$$-2\cot(x)-x-2(\sin(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^2,x)
```

```
[Out] -2*cot(x)-x-2/sin(x)
```

Maxima [A] time = 1.49362, size = 22, normalized size = 1.38

$$-x - \frac{2}{\sin(x)} - \frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 2/sin(x) - 2/tan(x)
```

Fricas [A] time = 1.95338, size = 47, normalized size = 2.94

$$-\frac{x\sin(x) + 2\cos(x) + 2}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

Sympy [A] time = 8.48301, size = 17, normalized size = 1.06

$$-x - \cot(x) - 2\csc(x) - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**2,x)
```

```
[Out] -x - cot(x) - 2*csc(x) - cos(x)/sin(x)
```

Giac [A] time = 1.15252, size = 16, normalized size = 1.

$$-x - \frac{2}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="giac")
```

```
[Out] -x - 2/tan(1/2*x)
```

3.297 $\int (\cot(x)+\csc(x))\,dx$

Optimal. Leaf size=9

$$\log(\sin(x))-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]] + Log[Sin[x]]
```

Rubi [A] time = 0.0053846, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3475, 3770}

$$\log(\sin(x))-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x] + Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))\,dx &= \int \cot(x)\,dx+\int \csc(x)\,dx\\ &= -\tanh^{-1}(\cos(x))+\log(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.004463, size = 20, normalized size = 2.22

$$\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x] + Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

Maple [A] time = 0.003, size = 13, normalized size = 1.4

$$\ln(\sin(x)) - \ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)+csc(x),x)
```

```
[Out] ln(sin(x))-ln(cot(x)+csc(x))
```

Maxima [A] time = 1.0108, size = 16, normalized size = 1.78

$$-\log(\cot(x) + \csc(x)) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="maxima")
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

Fricas [A] time = 2.07307, size = 32, normalized size = 3.56

$$\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 0.106556, size = 20, normalized size = 2.22

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}+\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + log(sin(x))
```

Giac [A] time = 1.17512, size = 24, normalized size = 2.67

$$\frac{1}{2}\,\log\left(-\cos\left(x\right)^{2}+1\right)+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1) + log(abs(tan(1/2*x)))
```

3.298 $\int \frac{1}{\cot(x)+\csc(x)} dx$

Optimal. Leaf size=7

$$-\log(\cos(x)+1)$$

```
[Out] -Log[1 + Cos[x]]
```

Rubi [A] time = 0.0269082, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3160, 2667, 31}

$$-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-1),x]

[Out] -Log[1 + Cos[x]]
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx=\int \frac{\sin(x)}{1+\cos(x)}\,dx$$
$$=-\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\cos(x)\right)$$
$$=-\log(1+\cos(x))$$

Mathematica [A] time = 0.0119092, size = 9, normalized size = 1.29

$$-2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-1),x]
```

```
[Out] -2*Log[Cos[x/2]]
```

Maple [A] time = 0.047, size = 8, normalized size = 1.1

$$-\ln(1+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x)),x)
```

```
[Out] -ln(1+cos(x))
```

Maxima [B] time = 1.45563, size = 19, normalized size = 2.71

$$\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 2.13351, size = 32, normalized size = 4.57

$$-\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -log(1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cot\left(x\right)+\csc\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x)
```

```
[Out] Integral(1/(cot(x) + csc(x)), x)
```

Giac [A] time = 1.1338, size = 9, normalized size = 1.29

$$-\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] -log(cos(x) + 1)
```

3.299 $\int \frac{1}{(\cot(x)+\csc(x))^2} dx$

Optimal. Leaf size=14

$$\frac{2\sin(x)}{\cos(x)+1} - x$$

```
[Out] -x + (2*Sin[x])/(1 + Cos[x])
```

Rubi [A] time = 0.0413406, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$\frac{2\sin(x)}{\cos(x)+1} - x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-2),x]
```

```
[Out] -x + (2*Sin[x])/(1 + Cos[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx &= \int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx\\ &= \frac{2\sin(x)}{1+\cos(x)}-\int 1\,dx\\ &= -x+\frac{2\sin(x)}{1+\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.0127581, size = 12, normalized size = 0.86

$$2\tan\left(\frac{x}{2}\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-2),x]
```

```
[Out] -x + 2*Tan[x/2]
```

Maple [A] time = 0.044, size = 11, normalized size = 0.8

$$2\,\tan\left(x/2\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^2,x)
```

```
[Out] 2*tan(1/2*x)-x
```

Maxima [A] time = 1.48777, size = 31, normalized size = 2.21

$$\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1}-2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="maxima")
```

```
[Out] 2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.01411, size = 55, normalized size = 3.93

$$-\frac{x\cos\left(x\right) + x - 2\,\sin\left(x\right)}{\cos\left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\cot\left(x\right) + \csc\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**2,x)
```

```
[Out] Integral((cot(x) + csc(x))**(-2), x)
```

Giac [A] time = 1.15347, size = 14, normalized size = 1.

$$-x + 2\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="giac")
```

```
[Out] -x + 2*tan(1/2*x)
```

3.300 $\int \frac{1}{(\cot(x)+\csc(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

```
[Out] 2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

Rubi [A] time = 0.0465222, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-3),x]
```

```
[Out] 2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx &= \int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx\\ &= -\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, \cos(x)\right)\\ &= \frac{2}{1+\cos(x)}+\log(1+\cos(x))\end{aligned}$$

Mathematica [A] time = 0.0139392, size = 18, normalized size = 1.29

$$\sec^2\left(\frac{x}{2}\right)+2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-3),x]

[Out] 2*Log[Cos[x/2]] + Sec[x/2]^2
```

Maple [A] time = 0.073, size = 15, normalized size = 1.1

$$2\,(1+\cos(x))^{-1}+\ln(1+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^3,x)

[Out] 2/(1+cos(x))+ln(1+cos(x))
```

Maxima [A] time = 1.51203, size = 38, normalized size = 2.71

$$\frac{\sin(x)^2}{(\cos(x)+1)^2}-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="maxima")
```

```
[Out] sin(x)^2/(cos(x) + 1)^2 - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 1.93883, size = 74, normalized size = 5.29

$$\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="fricas")
```

```
[Out] ((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13826, size = 19, normalized size = 1.36

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="giac")
```

```
[Out] 2/(cos(x) + 1) + log(cos(x) + 1)
```

3.301 $\int \frac{1}{(\cot(x)+\csc(x))^4} dx$

Optimal. Leaf size=26

$$x + \frac{2\sin^3(x)}{3(\cos(x)+1)^3} - \frac{2\sin(x)}{\cos(x)+1}$$

```
[Out] x - (2*Sin[x])/(1 + Cos[x]) + (2*Sin[x]^3)/(3*(1 + Cos[x])^3)
```

Rubi [A] time = 0.0711771, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$x + \frac{2\sin^3(x)}{3(\cos(x)+1)^3} - \frac{2\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-4),x]
```

```
[Out] x - (2*Sin[x])/(1 + Cos[x]) + (2*Sin[x]^3)/(3*(1 + Cos[x])^3)
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx &= \int \frac{\sin^4(x)}{(1+\cos(x))^4}\,dx\\
&= \frac{2\sin^3(x)}{3(1+\cos(x))^3} - \int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx\\
&= -\frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3} + \int 1\,dx\\
&= x - \frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3}
\end{aligned}
$$

Mathematica [A] time = 0.0140729, size = 30, normalized size = 1.15

$$x - \frac{8}{3}\tan\left(\frac{x}{2}\right) + \frac{2}{3}\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-4),x]
```

```
[Out] x - (8*Tan[x/2])/3 + (2*Sec[x/2]^2*Tan[x/2])/3
```

Maple [A] time = 0.073, size = 17, normalized size = 0.7

$$\frac{2}{3}\left(\tan\left(\frac{x}{2}\right)\right)^3 - 2\tan(x/2) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^4,x)
```

```
[Out] 2/3*tan(1/2*x)^3-2*tan(1/2*x)+x
```

Maxima [A] time = 1.47989, size = 47, normalized size = 1.81

$$-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^3}{3(\cos(x)+1)^3} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*sin(x)/(cos(x) + 1) + 2/3*sin(x)^3/(cos(x) + 1)^3 + 2*arctan(sin(x)/(cos
(x) + 1))
```

Fricas [A] time = 1.93125, size = 123, normalized size = 4.73

$$\frac{3\,x\cos\left(x\right)^{2}+6\,x\cos\left(x\right)-4\,\left(2\,\cos\left(x\right)+1\right)\sin\left(x\right)+3\,x}{3\,\left(\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cos(x)^2 + 6*x*cos(x) - 4*(2*cos(x) + 1)*sin(x) + 3*x)/(cos(x)^2 +
 2*cos(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21487, size = 22, normalized size = 0.85

$$\frac{2}{3}\,\tan\left(\frac{1}{2}\,x\right)^{3}+x-2\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="giac")
```

```
[Out] 2/3*tan(1/2*x)^3 + x - 2*tan(1/2*x)
```

3.302 $\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx$

Optimal. Leaf size=24

$$-\frac{4}{\cos(x)+1}+\frac{2}{(\cos(x)+1)^2}-\log(\cos(x)+1)$$

```
[Out] 2/(1 + Cos[x])^2 - 4/(1 + Cos[x]) - Log[1 + Cos[x]]
```

Rubi [A] time = 0.0502716, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$-\frac{4}{\cos(x)+1}+\frac{2}{(\cos(x)+1)^2}-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-5),x]

[Out] 2/(1 + Cos[x])^2 - 4/(1 + Cos[x]) - Log[1 + Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx &= \int \frac{\sin^5(x)}{(1+\cos(x))^5}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, \cos(x)\right) \\
&= -\operatorname{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, \cos(x)\right) \\
&= \frac{2}{(1+\cos(x))^2}-\frac{4}{1+\cos(x)}-\log(1+\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.0134744, size = 32, normalized size = 1.33

$$\frac{1}{2}\sec^4\left(\frac{x}{2}\right)-2\sec^2\left(\frac{x}{2}\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-5),x]
```

```
[Out] -2*Log[Cos[x/2]] - 2*Sec[x/2]^2 + Sec[x/2]^4/2
```

Maple [A] time = 0.08, size = 25, normalized size = 1.

$$2\,(1+\cos(x))^{-2}-4\,(1+\cos(x))^{-1}-\ln(1+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^5,x)
```

```
[Out] 2/(1+cos(x))^2-4/(1+cos(x))-ln(1+cos(x))
```

Maxima [A] time = 1.56519, size = 53, normalized size = 2.21

$$-\frac{\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^4}{2\,(\cos(x)+1)^4}+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="maxima")
```

```
[Out] -sin(x)^2/(cos(x) + 1)^2 + 1/2*sin(x)^4/(cos(x) + 1)^4 + log(sin(x)^2/(cos(
x) + 1)^2 + 1)
```

Fricas [A] time = 2.1921, size = 126, normalized size = 5.25

$$-\frac{\left(\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+4\,\cos\left(x\right)+2}{\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="fricas")
```

```
[Out] -((cos(x)^2 + 2*cos(x) + 1)*log(1/2*cos(x) + 1/2) + 4*cos(x) + 2)/(cos(x)^2
 + 2*cos(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13402, size = 30, normalized size = 1.25

$$-\frac{2\left(2\,\cos\left(x\right)+1\right)}{\left(\cos\left(x\right)+1\right)^{2}}-\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="giac")
```

```
[Out] -2*(2*cos(x) + 1)/(cos(x) + 1)^2 - log(cos(x) + 1)
```

3.303 $\int (\csc(x) - \sin(x))^4 \, dx$

Optimal. Leaf size=44

$$\frac{35x}{8} - \frac{35\cot^3(x)}{24} + \frac{35\cot(x)}{8} + \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{7}{8}\cos^2(x)\cot^3(x)$$

```
[Out] (35*x)/8 + (35*Cot[x])/8 - (35*Cot[x]^3)/24 + (7*Cos[x]^2*Cot[x]^3)/8 + (Co
s[x]^4*Cot[x]^3)/4
```

Rubi [A] time = 0.0336686, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {290, 325, 203}

$$\frac{35x}{8} - \frac{35\cot^3(x)}{24} + \frac{35\cot(x)}{8} + \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{7}{8}\cos^2(x)\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^4,x]
```

```
[Out] (35*x)/8 + (35*Cot[x])/8 - (35*Cot[x]^3)/24 + (7*Cos[x]^2*Cot[x]^3)/8 + (Co
s[x]^4*Cot[x]^3)/4
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (\csc(x)-\sin(x))^4\,dx &= \text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^3}\,dx,x,\tan(x)\right)\\ &= \frac{1}{4}\cos^4(x)\cot^3(x)+\frac{7}{4}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= \frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)+\frac{35}{8}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\ &= -\frac{35}{24}\cot^3(x)+\frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)-\frac{35}{8}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx,x,\tan(x\right.\\ &= \frac{35\cot(x)}{8}-\frac{35\cot^3(x)}{24}+\frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)+\frac{35}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\right.\\ &= \frac{35x}{8}+\frac{35\cot(x)}{8}-\frac{35\cot^3(x)}{24}+\frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)\end{aligned}$$

Mathematica [A] time = 0.0295829, size = 38, normalized size = 0.86

$$\frac{35x}{8}+\frac{3}{4}\sin(2x)+\frac{1}{32}\sin(4x)+\frac{10\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^4,x]
```

```
[Out] (35*x)/8 + (10*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3 + (3*Sin[2*x])/4 + Sin[4*x]/
32
```

Maple [A] time = 0.022, size = 39, normalized size = 0.9

$$-\frac{\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)+\frac{35x}{8}+2\cos(x)\sin(x)+4\cot(x)+\left(-\frac{2}{3}-\frac{(\csc(x))^2}{3}\right)\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+35/8*x+2*cos(x)*sin(x)+4*cot(x)+(-2/3-1/3
*csc(x)^2)*cot(x)
```

Maxima [A] time = 1.00614, size = 49, normalized size = 1.11

$$\frac{35}{8}\,x+\frac{4}{\tan\left(x\right)}-\frac{3\,\tan\left(x\right)^{2}+1}{3\,\tan\left(x\right)^{3}}+\frac{1}{32}\,\sin\left(4\,x\right)+\frac{3}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="maxima")
```

```
[Out] 35/8*x + 4/tan(x) - 1/3*(3*tan(x)^2 + 1)/tan(x)^3 + 1/32*sin(4*x) + 3/4*sin
(2*x)
```

Fricas [A] time = 2.12102, size = 157, normalized size = 3.57

$$-\frac{6\,\cos\left(x\right)^{7}+21\,\cos\left(x\right)^{5}-140\,\cos\left(x\right)^{3}-105\,\left(x\cos\left(x\right)^{2}-x\right)\sin\left(x\right)+105\,\cos\left(x\right)}{24\,\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*cos(x)^7 + 21*cos(x)^5 - 140*cos(x)^3 - 105*(x*cos(x)^2 - x)*sin(x
) + 105*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 18.1013, size = 44, normalized size = 1.

$$\frac{35x}{8}+2\sin\left(x\right)\cos\left(x\right)-\frac{\sin\left(2x\right)}{4}+\frac{\sin\left(4x\right)}{32}-\frac{\cot^{3}\left(x\right)}{3}-\cot\left(x\right)+\frac{4\cos\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**4,x)
```

```
[Out] 35*x/8 + 2*sin(x)*cos(x) - sin(2*x)/4 + sin(4*x)/32 - cot(x)**3/3 - cot(x)
+ 4*cos(x)/sin(x)
```

Giac [A] time = 1.16528, size = 53, normalized size = 1.2

$$\frac{35}{8}x+\frac{11\tan(x)^3+13\tan(x)}{8\left(\tan(x)^2+1\right)^2}+\frac{9\tan(x)^2-1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="giac")
```

```
[Out] 35/8*x + 1/8*(11*tan(x)^3 + 13*tan(x))/(tan(x)^2 + 1)^2 + 1/3*(9*tan(x)^2 -
 1)/tan(x)^3
```

3.304 $\int (\csc(x) - \sin(x))^3 \, dx$

Optimal. Leaf size=34

$$-\frac{5\cos^3(x)}{6} - \frac{5\cos(x)}{2} - \frac{1}{2}\cos^3(x)\cot^2(x) + \frac{5}{2}\tanh^{-1}(\cos(x))$$

```
[Out] (5*ArcTanh[Cos[x]])/2 - (5*Cos[x])/2 - (5*Cos[x]^3)/6 - (Cos[x]^3*Cot[x]^2)
/2
```

Rubi [A] time = 0.0458312, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 2592, 288, 302, 206}

$$-\frac{5\cos^3(x)}{6} - \frac{5\cos(x)}{2} - \frac{1}{2}\cos^3(x)\cot^2(x) + \frac{5}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^3,x]
```

```
[Out] (5*ArcTanh[Cos[x]])/2 - (5*Cos[x])/2 - (5*Cos[x]^3)/6 - (Cos[x]^3*Cot[x]^2)
/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (\csc(x)-\sin(x))^3\,dx &= \int \cos^3(x)\cot^3(x)\,dx\\
&= -\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \cos(x)\right)\\
&= -\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(x)\right)\\
&= -\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(x)\right)\\
&= -\frac{5\cos(x)}{2}-\frac{5\cos^3(x)}{6}-\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)\\
&= \frac{5}{2}\tanh^{-1}(\cos(x))-\frac{5\cos(x)}{2}-\frac{5\cos^3(x)}{6}-\frac{1}{2}\cos^3(x)\cot^2(x)
\end{aligned}$$

Mathematica [A] time = 0.0204385, size = 61, normalized size = 1.79

$$-\frac{9\cos(x)}{4}-\frac{1}{12}\cos(3x)-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)-\frac{5}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{5}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^3,x]
```

```
[Out] (-9*Cos[x])/4 - Cos[3*x]/12 - Csc[x/2]^2/8 + (5*Log[Cos[x/2]])/2 - (5*Log[S
in[x/2]])/2 + Sec[x/2]^2/8
```

Maple [A] time = 0.019, size = 32, normalized size = 0.9

$$\frac{\left(2+\left(\sin\left(x\right)\right)^{2}\right)\cos\left(x\right)}{3}-3\cos\left(x\right)-\frac{5\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)}{2}-\frac{\cot\left(x\right)\csc\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^3,x)
```

```
[Out] 1/3*(2+sin(x)^2)*cos(x)-3*cos(x)-5/2*ln(csc(x)-cot(x))-1/2*cot(x)*csc(x)
```

Maxima [A] time = 0.986781, size = 50, normalized size = 1.47

$$-\frac{1}{3}\cos\left(x\right)^{3}+\frac{\cos\left(x\right)}{2\left(\cos\left(x\right)^{2}-1\right)}-2\cos\left(x\right)+\frac{5}{4}\log\left(\cos\left(x\right)+1\right)-\frac{5}{4}\log\left(\cos\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/3*cos(x)^3 + 1/2*cos(x)/(cos(x)^2 - 1) - 2*cos(x) + 5/4*log(cos(x) + 1)
- 5/4*log(cos(x) - 1)
```

Fricas [B] time = 2.12217, size = 197, normalized size = 5.79

$$-\frac{4\cos\left(x\right)^{5}+20\cos\left(x\right)^{3}-15\left(\cos\left(x\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+15\left(\cos\left(x\right)^{2}-1\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-30\cos\left(x\right)}{12\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(x)^5 + 20*cos(x)^3 - 15*(cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) +
 15*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2) - 30*cos(x))/(cos(x)^2 - 1)
```

Sympy [A] time = 5.87116, size = 42, normalized size = 1.24

$$-\frac{5\log\left(\cos\left(x\right)-1\right)}{4}+\frac{5\log\left(\cos\left(x\right)+1\right)}{4}-\frac{\cos^{3}\left(x\right)}{3}-2\cos\left(x\right)+\frac{\cos\left(x\right)}{2\cos^{2}\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**3,x)
```

```
[Out] -5*log(cos(x) - 1)/4 + 5*log(cos(x) + 1)/4 - cos(x)**3/3 - 2*cos(x) + cos(x
)/(2*cos(x)**2 - 2)
```

Giac [B] time = 1.16047, size = 134, normalized size = 3.94

$$\frac{\left(\frac{10\left(\cos(x)-1\right)}{\cos(x)+1}+1\right)\left(\cos\left(x\right)+1\right)}{8\left(\cos\left(x\right)-1\right)}-\frac{\cos\left(x\right)-1}{8\left(\cos\left(x\right)+1\right)}-\frac{2\left(\frac{12\left(\cos(x)-1\right)}{\cos(x)+1}-\frac{9\left(\cos(x)-1\right)^{2}}{\left(\cos(x)+1\right)^{2}}-7\right)}{3\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)^{3}}-\frac{5}{4}\log\left(-\frac{\cos\left(x\right)-1}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/8*(10*(cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(cos
(x) - 1)/(cos(x) + 1) - 2/3*(12*(cos(x) - 1)/(cos(x) + 1) - 9*(cos(x) - 1)^
2/(cos(x) + 1)^2 - 7)/((cos(x) - 1)/(cos(x) + 1) - 1)^3 - 5/4*log(-(cos(x)
- 1)/(cos(x) + 1))
```

3.305 $\int (\csc(x)-\sin(x))^2\,dx$

Optimal. Leaf size=22

$$-\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)$$

```
[Out] (-3*x)/2 - (3*Cot[x])/2 + (Cos[x]^2*Cot[x])/2
```

Rubi [A] time = 0.0238772, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {290, 325, 203}

$$-\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^2,x]
```

```
[Out] (-3*x)/2 - (3*Cot[x])/2 + (Cos[x]^2*Cot[x])/2
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (\csc(x)-\sin(x))^2\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\cos^2(x)\cot(x)+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\ &= -\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\ &= -\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)\end{aligned}$$

Mathematica [A] time = 0.0157032, size = 18, normalized size = 0.82

$$-\frac{3x}{2}-\frac{1}{4}\sin(2x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^2,x]
```

```
[Out] (-3*x)/2 - Cot[x] - Sin[2*x]/4
```

Maple [A] time = 0.016, size = 15, normalized size = 0.7

$$-\frac{\cos(x)\sin(x)}{2}-\frac{3\,x}{2}-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^2,x)
```

```
[Out] -1/2*cos(x)*sin(x)-3/2*x-cot(x)
```

Maxima [A] time = 0.994337, size = 22, normalized size = 1.

$$-\frac{3}{2}\,x - \frac{1}{\tan\left(x\right)} - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3/2*x - 1/tan(x) - 1/4*sin(2*x)
```

Fricas [A] time = 2.11598, size = 63, normalized size = 2.86

$$\frac{\cos\left(x\right)^{3} - 3\,x\sin\left(x\right) - 3\,\cos\left(x\right)}{2\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)^3 - 3*x*sin(x) - 3*cos(x))/sin(x)
```

Sympy [A] time = 1.9787, size = 15, normalized size = 0.68

$$-\frac{3x}{2} - \frac{\sin\left(2x\right)}{4} - \cot\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**2,x)
```

```
[Out] -3*x/2 - sin(2*x)/4 - cot(x)
```

Giac [A] time = 1.14499, size = 31, normalized size = 1.41

$$-\frac{3}{2}\,x - \frac{3\,\tan\left(x\right)^{2} + 2}{2\,\left(\tan\left(x\right)^{3} + \tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="giac")
```

```
[Out] -3/2*x - 1/2*(3*tan(x)^2 + 2)/(tan(x)^3 + tan(x))
```

3.306 $\int (\csc(x)-\sin(x))\,dx$

Optimal. Leaf size=8

$$\cos(x)-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rubi [A] time = 0.0054189, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3770, 2638}

$$\cos(x)-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x] - Sin[x],x]

[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\csc(x)-\sin(x))\,dx &= \int \csc(x)\,dx-\int \sin(x)\,dx\\ &= -\tanh^{-1}(\cos(x))+\cos(x)\end{aligned}$$

Mathematica [B] time = 0.003509, size = 19, normalized size = 2.38

$$\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x] - Sin[x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.5

$$\cos(x) - \ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)-sin(x),x)
```

```
[Out] cos(x)-ln(cot(x)+csc(x))
```

Maxima [A] time = 0.985122, size = 15, normalized size = 1.88

$$\cos(x) - \log(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x, algorithm="maxima")
```

```
[Out] cos(x) - log(cot(x) + csc(x))
```

Fricas [B] time = 2.15031, size = 88, normalized size = 11.

$$\cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 0.103721, size = 19, normalized size = 2.38

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

Giac [A] time = 1.14856, size = 12, normalized size = 1.5

$$\cos\left(x\right)+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x, algorithm="giac")
```

```
[Out] cos(x) + log(abs(tan(1/2*x)))
```

3.307 $\int \frac{1}{\csc(x)-\sin(x)}\,dx$

Optimal. Leaf size=2

$$\sec(x)$$

```
[Out] Sec[x]
```

Rubi [A] time = 0.0187821, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 8}

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-1),x]
```

```
[Out] Sec[x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\csc(x) - \sin(x)}\, dx = \int \sec(x)\tan(x)\, dx$$
$$= \text{Subst}(\int 1\, dx, x, \sec(x))$$
$$= \sec(x)$$

Mathematica [A] time = 0.0038522, size = 2, normalized size = 1.

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-1),x]
```

```
[Out] Sec[x]
```

Maple [A] time = 0.03, size = 5, normalized size = 2.5

$$(\cos(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x)),x)
```

```
[Out] 1/cos(x)
```

Maxima [B] time = 0.963639, size = 23, normalized size = 11.5

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 - 1)
```

Fricas [A] time = 1.97604, size = 14, normalized size = 7.

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-\sin(x) + \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x)
```

```
[Out] Integral(1/(-sin(x) + csc(x)), x)
```

Giac [B] time = 1.15938, size = 23, normalized size = 11.5

$$\frac{2}{\frac{\cos(x)-1}{\cos(x)+1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="giac")
```

```
[Out] 2/((cos(x) - 1)/(cos(x) + 1) + 1)
```

3.308 $\int \frac{1}{(\csc(x)-\sin(x))^2} dx$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

```
[Out] Tan[x]^3/3
```

Rubi [A] time = 0.014444, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30}

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-2),x]
```

```
[Out] Tan[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(\csc(x)-\sin(x))^2} dx &= \text{Subst}\left(\int x^2 \, dx, x, \tan(x)\right) \\ &= \frac{\tan^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.0029058, size = 8, normalized size = 1.

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-2),x]
```

```
[Out] Tan[x]^3/3
```

Maple [A] time = 0.036, size = 7, normalized size = 0.9

$$\frac{(\tan(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^2,x)
```

```
[Out] 1/3*tan(x)^3
```

Maxima [A] time = 1.0229, size = 8, normalized size = 1.

$$\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3
```

Fricas [B] time = 1.97841, size = 50, normalized size = 6.25

$$-\frac{\left(\cos(x)^2 - 1\right)\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\sin(x) + \csc(x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**2,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-2), x)
```

Giac [A] time = 1.13383, size = 8, normalized size = 1.

$$\frac{1}{3} \tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3
```

3.309 $\int \frac{1}{(\csc(x)-\sin(x))^3} dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0380005, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 14}

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-3),x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx &= \int \sec^3(x)\tan^3(x)\,dx\\ &= \text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \sec(x)\right)\\ &= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0201522, size = 17, normalized size = 1.

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-3),x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Maple [A] time = 0.046, size = 14, normalized size = 0.8

$$-\frac{1}{3\left(\cos\left(x\right)\right)^{3}}+\frac{1}{5\left(\cos\left(x\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^3,x)
```

```
[Out] -1/3/cos(x)^3+1/5/cos(x)^5
```

Maxima [B] time = 0.999292, size = 139, normalized size = 8.18

$$-\frac{4\left(\frac{5\sin(x)^2}{(\cos(x)+1)^2}+\frac{5\sin(x)^4}{(\cos(x)+1)^4}+\frac{15\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{15\left(\frac{5\sin(x)^2}{(\cos(x)+1)^2}-\frac{10\sin(x)^4}{(\cos(x)+1)^4}+\frac{10\sin(x)^6}{(\cos(x)+1)^6}-\frac{5\sin(x)^8}{(\cos(x)+1)^8}+\frac{\sin(x)^{10}}{(\cos(x)+1)^{10}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="maxima")
```

```
[Out] -4/15*(5*sin(x)^2/(cos(x) + 1)^2 + 5*sin(x)^4/(cos(x) + 1)^4 + 15*sin(x)^6/
(cos(x) + 1)^6 - 1)/(5*sin(x)^2/(cos(x) + 1)^2 - 10*sin(x)^4/(cos(x) + 1)^4
 + 10*sin(x)^6/(cos(x) + 1)^6 - 5*sin(x)^8/(cos(x) + 1)^8 + sin(x)^10/(cos(
x) + 1)^10 - 1)
```

Fricas [A] time = 1.93792, size = 45, normalized size = 2.65

$$-\frac{5 \cos\left(x\right)^{2} - 3}{15 \cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\sin\left(x\right) + \csc\left(x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**3,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-3), x)
```

Giac [B] time = 1.17964, size = 80, normalized size = 4.71

$$-\frac{4 \left(\frac{5 (\cos(x)-1)}{\cos(x)+1} - \frac{5 (\cos(x)-1)^{2}}{(\cos(x)+1)^{2}} + \frac{15 (\cos(x)-1)^{3}}{(\cos(x)+1)^{3}} + 1\right)}{15 \left(\frac{\cos(x)-1}{\cos(x)+1} + 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="giac")
```

```
[Out] -4/15*(5*(cos(x) - 1)/(cos(x) + 1) - 5*(cos(x) - 1)^2/(cos(x) + 1)^2 + 15*(
cos(x) - 1)^3/(cos(x) + 1)^3 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1)^5
```

3.310 $\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx$

Optimal. Leaf size=17

$$\frac{\tan^7(x)}{7}+\frac{\tan^5(x)}{5}$$

```
[Out] Tan[x]^5/5 + Tan[x]^7/7
```

Rubi [A] time = 0.0177155, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{\tan^7(x)}{7}+\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-4),x]
```

```
[Out] Tan[x]^5/5 + Tan[x]^7/7
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx &= \text{Subst}\left(\int \left(x^4+x^6\right)\,dx, x, \tan(x)\right)\\ &= \frac{\tan^5(x)}{5}+\frac{\tan^7(x)}{7}\end{aligned}$$

Mathematica [B] time = 0.0173619, size = 37, normalized size = 2.18

$$\frac{2\tan(x)}{35}+\frac{1}{7}\tan(x)\sec^6(x)-\frac{8}{35}\tan(x)\sec^4(x)+\frac{1}{35}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-4),x]
```

```
[Out] (2*Tan[x])/35 + (Sec[x]^2*Tan[x])/35 - (8*Sec[x]^4*Tan[x])/35 + (Sec[x]^6*T
an[x])/7
```

Maple [A] time = 0.043, size = 14, normalized size = 0.8

$$\frac{(\tan(x))^5}{5} + \frac{(\tan(x))^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^4,x)
```

```
[Out] 1/5*tan(x)^5+1/7*tan(x)^7
```

Maxima [A] time = 0.988165, size = 18, normalized size = 1.06

$$\frac{1}{7}\tan(x)^7 + \frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="maxima")
```

```
[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5
```

Fricas [A] time = 2.13188, size = 85, normalized size = 5.

$$\frac{\left(2\cos(x)^6 + \cos(x)^4 - 8\cos(x)^2 + 5\right)\sin(x)}{35\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(x)^6 + cos(x)^4 - 8*cos(x)^2 + 5)*sin(x)/cos(x)^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**4,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-4), x)
```

Giac [A] time = 1.14984, size = 18, normalized size = 1.06

$$\frac{1}{7}\,\tan\left(x\right)^{7}+\frac{1}{5}\,\tan\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="giac")
```

```
[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5
```

3.311 $\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx$

Optimal. Leaf size=25

$$\frac{\sec^9(x)}{9}-\frac{2\sec^7(x)}{7}+\frac{\sec^5(x)}{5}$$

```
[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

Rubi [A] time = 0.0398882, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$\frac{\sec^9(x)}{9}-\frac{2\sec^7(x)}{7}+\frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-5),x]
```

```
[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx &= \int \sec^5(x)\tan^5(x)\,dx\\ &= \text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \sec(x)\right)\\ &= \frac{\sec^5(x)}{5}-\frac{2\sec^7(x)}{7}+\frac{\sec^9(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.0160423, size = 25, normalized size = 1.

$$\frac{\sec^9(x)}{9}-\frac{2\sec^7(x)}{7}+\frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-5),x]
```

```
[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

Maple [A] time = 0.05, size = 20, normalized size = 0.8

$$\frac{1}{9\left(\cos\left(x\right)\right)^{9}}-\frac{2}{7\left(\cos\left(x\right)\right)^{7}}+\frac{1}{5\left(\cos\left(x\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^5,x)
```

```
[Out] 1/9/cos(x)^9-2/7/cos(x)^7+1/5/cos(x)^5
```

Maxima [B] time = 1.04796, size = 252, normalized size = 10.08

$$\frac{16\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}-\frac{126\sin(x)^6}{(\cos(x)+1)^6}-\frac{441\sin(x)^8}{(\cos(x)+1)^8}-\frac{315\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{210\sin(x)^{12}}{(\cos(x)+1)^{12}}-1\right)}{315\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}+\frac{84\sin(x)^6}{(\cos(x)+1)^6}-\frac{126\sin(x)^8}{(\cos(x)+1)^8}+\frac{126\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{84\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{36\sin(x)^{14}}{(\cos(x)+1)^{14}}-\frac{9\sin(x)^{16}}{(\cos(x)+1)^{16}}+\frac{\sin(x)^{18}}{(\cos(x)+1)^{18}}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="maxima")
```

```
[Out] 16/315*(9*sin(x)^2/(cos(x) + 1)^2 - 36*sin(x)^4/(cos(x) + 1)^4 - 126*sin(x)
^6/(cos(x) + 1)^6 - 441*sin(x)^8/(cos(x) + 1)^8 - 315*sin(x)^10/(cos(x) + 1
)^10 - 210*sin(x)^12/(cos(x) + 1)^12 - 1)/(9*sin(x)^2/(cos(x) + 1)^2 - 36*s
in(x)^4/(cos(x) + 1)^4 + 84*sin(x)^6/(cos(x) + 1)^6 - 126*sin(x)^8/(cos(x)
+ 1)^8 + 126*sin(x)^10/(cos(x) + 1)^10 - 84*sin(x)^12/(cos(x) + 1)^12 + 36*
sin(x)^14/(cos(x) + 1)^14 - 9*sin(x)^16/(cos(x) + 1)^16 + sin(x)^18/(cos(x)
 + 1)^18 - 1)
```

Fricas [A] time = 1.99351, size = 66, normalized size = 2.64

$$\frac{63 \cos(x)^4 - 90 \cos(x)^2 + 35}{315 \cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(63*cos(x)^4 - 90*cos(x)^2 + 35)/cos(x)^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13144, size = 136, normalized size = 5.44

$$\frac{16\left(\frac{9(\cos(x)-1)}{\cos(x)+1} + \frac{36(\cos(x)-1)^2}{(\cos(x)+1)^2} - \frac{126(\cos(x)-1)^3}{(\cos(x)+1)^3} + \frac{441(\cos(x)-1)^4}{(\cos(x)+1)^4} - \frac{315(\cos(x)-1)^5}{(\cos(x)+1)^5} + \frac{210(\cos(x)-1)^6}{(\cos(x)+1)^6} + 1\right)}{315\left(\frac{\cos(x)-1}{\cos(x)+1} + 1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="giac")
```

```
[Out] 16/315*(9*(cos(x) - 1)/(cos(x) + 1) + 36*(cos(x) - 1)^2/(cos(x) + 1)^2 - 12
6*(cos(x) - 1)^3/(cos(x) + 1)^3 + 441*(cos(x) - 1)^4/(cos(x) + 1)^4 - 315*(
cos(x) - 1)^5/(cos(x) + 1)^5 + 210*(cos(x) - 1)^6/(cos(x) + 1)^6 + 1)/((cos
(x) - 1)/(cos(x) + 1) + 1)^9
```

3.312 $\int \frac{1}{(\csc(x)-\sin(x))^6} \, dx$

Optimal. Leaf size=25

$$\frac{\tan^{11}(x)}{11}+\frac{2\tan^9(x)}{9}+\frac{\tan^7(x)}{7}$$

```
[Out] Tan[x]^7/7 + (2*Tan[x]^9)/9 + Tan[x]^11/11
```

Rubi [A] time = 0.0219173, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {270}

$$\frac{\tan^{11}(x)}{11}+\frac{2\tan^9(x)}{9}+\frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-6),x]
```

```
[Out] Tan[x]^7/7 + (2*Tan[x]^9)/9 + Tan[x]^11/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^6} \, dx &= \text{Subst}\left(\int x^6 \left(1+x^2\right)^2 \, dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^6+2x^8+x^{10}\right) \, dx, x, \tan(x)\right)\\ &= \frac{\tan^7(x)}{7}+\frac{2\tan^9(x)}{9}+\frac{\tan^{11}(x)}{11}\end{aligned}$$

Mathematica [B] time = 0.017864, size = 57, normalized size = 2.28

$$-\frac{8\tan(x)}{693}+\frac{1}{11}\tan(x)\sec^{10}(x)-\frac{23}{99}\tan(x)\sec^8(x)+\frac{113}{693}\tan(x)\sec^6(x)-\frac{1}{231}\tan(x)\sec^4(x)-\frac{4}{693}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-6),x]
```

```
[Out] (-8*Tan[x])/693 - (4*Sec[x]^2*Tan[x])/693 - (Sec[x]^4*Tan[x])/231 + (113*Se
c[x]^6*Tan[x])/693 - (23*Sec[x]^8*Tan[x])/99 + (Sec[x]^10*Tan[x])/11
```

Maple [A] time = 0.053, size = 20, normalized size = 0.8

$$\frac{(\tan(x))^7}{7} + \frac{2\,(\tan(x))^9}{9} + \frac{(\tan(x))^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^6,x)
```

```
[Out] 1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11
```

Maxima [A] time = 0.984245, size = 26, normalized size = 1.04

$$\frac{1}{11}\,\tan(x)^{11} + \frac{2}{9}\,\tan(x)^9 + \frac{1}{7}\,\tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="maxima")
```

```
[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

Fricas [B] time = 2.07, size = 135, normalized size = 5.4

$$-\frac{\left(8\,\cos(x)^{10} + 4\,\cos(x)^8 + 3\,\cos(x)^6 - 113\,\cos(x)^4 + 161\,\cos(x)^2 - 63\right)\sin(x)}{693\,\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="fricas")
```

```
[Out] -1/693*(8*cos(x)^10 + 4*cos(x)^8 + 3*cos(x)^6 - 113*cos(x)^4 + 161*cos(x)^2
 - 63)*sin(x)/cos(x)^11
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15698, size = 26, normalized size = 1.04

$$\frac{1}{11}\tan(x)^{11} + \frac{2}{9}\tan(x)^{9} + \frac{1}{7}\tan(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="giac")
```

```
[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

3.313 $\int \frac{1}{(\csc(x)-\sin(x))^7} dx$

Optimal. Leaf size=33

$$\frac{\sec^{13}(x)}{13}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^9(x)}{3}-\frac{\sec^7(x)}{7}$$

```
[Out] -Sec[x]^7/7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

Rubi [A] time = 0.0426652, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$\frac{\sec^{13}(x)}{13}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^9(x)}{3}-\frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-7),x]
```

```
[Out] -Sec[x]^7/7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx = \int \sec^7(x)\tan^7(x)\,dx$$
$$= \text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^6+3x^8-3x^{10}+x^{12}\right)dx, x, \sec(x)\right)$$
$$= -\frac{1}{7}\sec^7(x)+\frac{\sec^9(x)}{3}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^{13}(x)}{13}$$

Mathematica [A] time = 0.0173609, size = 33, normalized size = 1.

$$\frac{\sec^{13}(x)}{13}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^9(x)}{3}-\frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-7),x]
```

```
[Out] -Sec[x]^7/7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

Maple [A] time = 0.058, size = 26, normalized size = 0.8

$$-\frac{1}{7\left(\cos(x)\right)^{7}}+\frac{1}{3\left(\cos(x)\right)^{9}}+\frac{1}{13\left(\cos(x)\right)^{13}}-\frac{3}{11\left(\cos(x)\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^7,x)
```

```
[Out] -1/7/cos(x)^7+1/3/cos(x)^9+1/13/cos(x)^13-3/11/cos(x)^11
```

Maxima [B] time = 1.05678, size = 366, normalized size = 11.09

$$-\frac{32\left(\frac{13\sin(x)^2}{(\cos(x)+1)^2}-\frac{78\sin(x)^4}{(\cos(x)+1)^4}+\frac{286\sin(x)^6}{(\cos(x)+1)^6}+\frac{2288\sin(x)^8}{(\cos(x)+1)^8}+\frac{10296\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{16302\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{18018\sin(x)^{14}}{(\cos(x)+1)^{14}}\right.}{3003\left(\frac{13\sin(x)^2}{(\cos(x)+1)^2}-\frac{78\sin(x)^4}{(\cos(x)+1)^4}+\frac{286\sin(x)^6}{(\cos(x)+1)^6}-\frac{715\sin(x)^8}{(\cos(x)+1)^8}+\frac{1287\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{1716\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{1716\sin(x)^{14}}{(\cos(x)+1)^{14}}-\frac{1287\sin(x)^{16}}{(\cos(x)+1)^{16}}+\frac{715\sin(x)^{18}}{(\cos(x)+1)^{18}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="maxima")
```

```
[Out] -32/3003*(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*sin
(x)^6/(cos(x) + 1)^6 + 2288*sin(x)^8/(cos(x) + 1)^8 + 10296*sin(x)^10/(cos(
x) + 1)^10 + 16302*sin(x)^12/(cos(x) + 1)^12 + 18018*sin(x)^14/(cos(x) + 1)
^14 + 9009*sin(x)^16/(cos(x) + 1)^16 + 3003*sin(x)^18/(cos(x) + 1)^18 - 1)/
(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*sin(x)^6/(co
s(x) + 1)^6 - 715*sin(x)^8/(cos(x) + 1)^8 + 1287*sin(x)^10/(cos(x) + 1)^10
- 1716*sin(x)^12/(cos(x) + 1)^12 + 1716*sin(x)^14/(cos(x) + 1)^14 - 1287*si
n(x)^16/(cos(x) + 1)^16 + 715*sin(x)^18/(cos(x) + 1)^18 - 286*sin(x)^20/(co
s(x) + 1)^20 + 78*sin(x)^22/(cos(x) + 1)^22 - 13*sin(x)^24/(cos(x) + 1)^24
+ sin(x)^26/(cos(x) + 1)^26 - 1)
```

Fricas [A] time = 2.21782, size = 96, normalized size = 2.91

$$-\frac{429 \cos(x)^6 - 1001 \cos(x)^4 + 819 \cos(x)^2 - 231}{3003 \cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*cos(x)^6 - 1001*cos(x)^4 + 819*cos(x)^2 - 231)/cos(x)^13
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19071, size = 193, normalized size = 5.85

$$-\frac{32\left(\frac{13(\cos(x)-1)}{\cos(x)+1}+\frac{78(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{286(\cos(x)-1)^3}{(\cos(x)+1)^3}-\frac{2288(\cos(x)-1)^4}{(\cos(x)+1)^4}+\frac{10296(\cos(x)-1)^5}{(\cos(x)+1)^5}-\frac{16302(\cos(x)-1)^6}{(\cos(x)+1)^6}+\frac{18018(\cos(x)-1)^7}{(\cos(x)+1)^7}-\frac{9009(c}{(\cos}\right.}{3003\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="giac")
```

```
[Out] -32/3003*(13*(cos(x) - 1)/(cos(x) + 1) + 78*(cos(x) - 1)^2/(cos(x) + 1)^2 +
 286*(cos(x) - 1)^3/(cos(x) + 1)^3 - 2288*(cos(x) - 1)^4/(cos(x) + 1)^4 + 1
0296*(cos(x) - 1)^5/(cos(x) + 1)^5 - 16302*(cos(x) - 1)^6/(cos(x) + 1)^6 +
18018*(cos(x) - 1)^7/(cos(x) + 1)^7 - 9009*(cos(x) - 1)^8/(cos(x) + 1)^8 +
3003*(cos(x) - 1)^9/(cos(x) + 1)^9 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1)^13
```

3.314 $\int (\csc(x)-\sin(x))^{7/2}\,dx$

Optimal. Leaf size=73

$$\frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}+\frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{35}\cot(x)\csc(x)\sqrt{\cos(x)\cot(x)}+\frac{256}{35}\sec(x)\sqrt{\mathrm{co}}$$

```
[Out] (8*Cos[x]*Cot[x]^2*Sqrt[Cos[x]*Cot[x]])/7 + (2*Cos[x]^3*Cot[x]^2*Sqrt[Cos[x
]*Cot[x]])/7 - (64*Cot[x]*Sqrt[Cos[x]*Cot[x]]*Csc[x])/35 + (256*Sqrt[Cos[x]
*Cot[x]]*Sec[x])/35
```

Rubi [A] time = 0.148391, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}+\frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{35}\cot(x)\csc(x)\sqrt{\cos(x)\cot(x)}+\frac{256}{35}\sec(x)\sqrt{\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(7/2),x]
```

```
[Out] (8*Cos[x]*Cot[x]^2*Sqrt[Cos[x]*Cot[x]])/7 + (2*Cos[x]^3*Cot[x]^2*Sqrt[Cos[x
]*Cot[x]])/7 - (64*Cot[x]*Sqrt[Cos[x]*Cot[x]]*Csc[x])/35 + (256*Sqrt[Cos[x]
*Cot[x]]*Sec[x])/35
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (\csc(x)-\sin(x))^{7/2}\,dx = \int (\cos(x)\cot(x))^{7/2}\,dx$$
$$= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(12\sqrt{\cos(x)\cot(x)}\right)\int \cos^{\frac{3}{2}}(x)\cot^{\frac{7}{2}}(x)\,dx}{7\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(32\sqrt{\cos(x)\cot(x)}\right)\int \frac{c}{\sqrt{}}}{7\sqrt{\cos(x)}\sqrt{\cot(}}$$
$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{35}\cot(x)\sqrt{\cos(x)\cot(}$$
$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{35}\cot(x)\sqrt{\cos(x)\cot(}$$

Mathematica [A] time = 0.0795734, size = 37, normalized size = 0.51

$$-\frac{1}{70}\sec(x)\sqrt{\cos(x)\cot(x)}\left(115\cos^2(x)+5\cos(3x)\cos(x)+28\cot^2(x)-512\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(7/2),x]
```

```
[Out] -(Sqrt[Cos[x]*Cot[x]]*(-512 + 115*Cos[x]^2 + 5*Cos[x]*Cos[3*x] + 28*Cot[x]^
2)*Sec[x])/70
```

Maple [A] time = 0.157, size = 40, normalized size = 0.6

$$\frac{\left(10\left(\cos\left(x\right)\right)^{6}+40\left(\cos\left(x\right)\right)^{4}-320\left(\cos\left(x\right)\right)^{2}+256\right)\sin\left(x\right)}{35\left(\cos\left(x\right)\right)^{7}}\left(\frac{\left(\cos\left(x\right)\right)^{2}}{\sin\left(x\right)}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(7/2),x)
```

```
[Out] 2/35*(5*cos(x)^6+20*cos(x)^4-160*cos(x)^2+128)*sin(x)*(cos(x)^2/sin(x))^(7/
2)/cos(x)^7
```

Maxima [B] time = 1.97216, size = 780, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/280*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*
cos(x) + 1)^(1/4)*(((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13/2*x) + 5
817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x) + 2275*cos
(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) + 2275*sin(13/2*x) - 5817*sin(9/2
*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105*sin(3/2*x) - 2275*sin(1/2*x))*co
s(7/2*arctan2(sin(x), cos(x) - 1)) + (5*cos(21/2*x) + 105*cos(17/2*x) - 227
5*cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(
3/2*x) + 2275*cos(1/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*sin(13/2*
x) + 5817*sin(9/2*x) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/2*x) + 22
75*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*cos(7/2*arctan2(sin(x)
, cos(x) + 1)) + ((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13/2*x) + 581
7*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x) + 2275*cos(1
/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*sin(13/2*x) + 5817*sin(9/2*x
```

```
) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/2*x) + 2275*sin(1/2*x))*cos(
7/2*arctan2(sin(x), cos(x) - 1)) - (5*cos(21/2*x) + 105*cos(17/2*x) - 2275*
cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/
2*x) + 2275*cos(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) + 2275*sin(13/2*x)
 - 5817*sin(9/2*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105*sin(3/2*x) - 2275
*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*sin(7/2*arctan2(sin(x),
cos(x) + 1)))/(cos(x)^8 + sin(x)^8 + 4*(cos(x)^2 + 1)*sin(x)^6 - 4*cos(x)^6
 + 2*(3*cos(x)^4 + 2*cos(x)^2 + 3)*sin(x)^4 + 6*cos(x)^4 + 4*(cos(x)^6 - co
s(x)^4 - cos(x)^2 + 1)*sin(x)^2 - 4*cos(x)^2 + 1)
```

Fricas [A] time = 2.33403, size = 131, normalized size = 1.79

$$-\frac{2\left(5\cos\left(x\right)^{6}+20\cos\left(x\right)^{4}-160\cos\left(x\right)^{2}+128\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{35\left(\cos\left(x\right)^{3}-\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*cos(x)^6 + 20*cos(x)^4 - 160*cos(x)^2 + 128)*sqrt(cos(x)^2/sin(x))
/(cos(x)^3 - cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\csc\left(x\right)-\sin\left(x\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(7/2), x)
```

3.315 $\int (\csc(x) - \sin(x))^{5/2}\, dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rubi [A] time = 0.111701, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(5/2),x]
```

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}
\int (\csc(x)-\sin(x))^{5/2}\,dx &= \int (\cos(x)\cot(x))^{5/2}\,dx\\
&= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}+\frac{\left(8\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)\,dx}{5\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}+\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{\left(32\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(}}{15\sqrt{\cos(x)}\sqrt{\cot(}}\\
&= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}+\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.0781942, size = 29, normalized size = 0.58

$$-\frac{2}{15}\tan(x)\sqrt{\cos(x)\cot(x)}\left(3\cos^2(x)+5\cot^2(x)+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

Maple [A] time = 0.118, size = 34, normalized size = 0.7

$$\frac{\left(6\, (\cos(x))^4+48\, (\cos(x))^2-64\right)\sin(x)}{15\, (\cos(x))^5}\left(\frac{(\cos(x))^2}{\sin(x)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(5/2),x)
```

```
[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5
```

Maxima [B] time = 1.87634, size = 576, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*sin
(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan2(s
in(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3
*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/
2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*sin
(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) + 1)) - (
(3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) + 410*cos(
3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*sin(7/2*x)
- 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2(sin(x), c
os(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2
*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 4
10*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*sin(5/2*arc
tan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^4
 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))
```

Fricas [A] time = 2.26392, size = 103, normalized size = 2.06

$$\frac{2\left(3\cos(x)^{4}+24\cos(x)^{2}-32\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(\csc(x)-\sin(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(5/2), x)
```

3.316 $\int (\csc(x) - \sin(x))^{3/2} \, dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Rubi [A] time = 0.0812269, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4397, 4400, 2598, 2589}

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(3/2),x]
```

```
[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int (\csc(x)-\sin(x))^{3/2}\,dx &= \int (\cos(x)\cot(x))^{3/2}\,dx\\ &= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\ &= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}+\frac{\left(4\sqrt{\cos(x)\cot(x)}\right)\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}\,dx}{3\sqrt{\cos(x)}\sqrt{\cot(x)}}\\ &= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)\end{aligned}$$

Mathematica [A] time = 0.0402594, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cos^2(x)-4\right)\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(3/2),x]
```

```
[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Maple [A] time = 0.088, size = 26, normalized size = 0.8

$$\frac{\left(2\,(\cos(x))^2-8\right)\sin(x)}{3\,(\cos(x))^3}\left(\frac{(\cos(x))^2}{\sin(x)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(3/2),x)
```

```
[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3
```

Maxima [B] time = 1.86962, size = 424, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos
(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) -
 sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(s
in(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2
*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arct
an2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*
x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x
) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos
(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(
5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*
sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 +
1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

Fricas [A] time = 2.17473, size = 66, normalized size = 2.13

$$\frac{2 \left(\cos\left(x\right)^{2} - 4\right) \sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{3 \cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\csc(x) - \sin(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(3/2), x)
```

3.317 $\int \sqrt{\csc(x) - \sin(x)}\, dx$

Optimal. Leaf size=13

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rubi [A] time = 0.0486451, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4397, 4400, 2589}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = \int \sqrt{\cos(x)\cot(x)}\,dx$$
$$= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

Mathematica [A] time = 0.0276821, size = 13, normalized size = 1.

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Maple [A] time = 0.091, size = 20, normalized size = 1.5

$$2\,\frac{\sin(x)}{\cos(x)}\sqrt{\frac{(\cos(x))^2}{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(1/2),x)
```

```
[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)
```

Maxima [B] time = 1.8014, size = 254, normalized size = 19.54

$$\frac{\left(\left(\cos\left(\frac{3}{2}\,x\right)-\cos\left(\frac{1}{2}\,x\right)+\sin\left(\frac{3}{2}\,x\right)+\sin\left(\frac{1}{2}\,x\right)\right)\cos\left(\frac{1}{2}\,\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}\,x\right)-\cos\left(\frac{1}{2}\,x\right)-\sin\left(\frac{3}{2}\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

Fricas [A] time = 2.01188, size = 53, normalized size = 4.08

$$\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sin(x) + \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-sin(x) + csc(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(x) - \sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(x) - sin(x)), x)
```

3.318 $\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}-\frac{\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

```
[Out] (ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) - (ArcTa
nh[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]])
```

Rubi [A] time = 0.0906745, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4397, 4400, 2601, 2564, 329, 298, 203, 206}

$$\frac{\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}-\frac{\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] (ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) - (ArcTa
nh[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx = \int \frac{1}{\sqrt{\cos(x)\cot(x)}}\,dx$$
$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\,dx}{\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\cos(x)\int \sec(x)\sqrt{-\sin(x)}\,dx}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= -\frac{\cos(x)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, -\sin(x)\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= -\frac{(2\cos(x))\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= -\frac{\cos(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} + \frac{\cos(x)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= \frac{\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

Mathematica [A] time = 0.266845, size = 44, normalized size = 0.73

$$-\frac{\sin(x)\tan(x)\sqrt{\cos(x)\cot(x)}\left(\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)-\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)\right)}{\sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] -(((ArcTan[(Sin[x]^2)^(1/4)] - ArcTanh[(Sin[x]^2)^(1/4)])*Sqrt[Cos[x]*Cot[x
]]*Sin[x]*Tan[x])/(Sin[x]^2)^(3/4))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(1/2),x)
```

```
[Out] int(1/(csc(x)-sin(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csc(x) - sin(x)), x)
```

Fricas [B] time = 2.46269, size = 393, normalized size = 6.55

$$\frac{1}{2}\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x) - \cos(x)}\right) + \frac{1}{4}\log\left(\frac{\cos(x)^3 - 5\cos(x)^2 - \left(\cos(x)^2 + 6\cos(x) + 4\right)\sin(x) + 4\left(\cos(x)^2 - (\right.}{\cos(x)^3 + 3\cos(x)^2 - \left(\cos(x)^2 - 2\cos(x) - 4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x))) + 1/4*l
og((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin(x) + 4*(cos(x)^2
- (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(x) + 4)/(cos(x)^3
+ 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sin(x) + \csc(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-sin(x) + csc(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(csc(x) - sin(x)), x)
```

3.319 $\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}$$

```
[Out] Sec[x]/(2*Sqrt[Cos[x]*Cot[x]]) + (ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(4*Sqrt[Cos[x]*Cot[x]]) + (ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(4*Sqrt[Cos[x]*Cot[x]])
```

Rubi [A] time = 0.11552, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {4397, 4400, 2597, 2601, 2564, 329, 212, 206, 203}

$$\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-3/2),x]
```

```
[Out] Sec[x]/(2*Sqrt[Cos[x]*Cot[x]]) + (ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(4*Sqrt[Cos[x]*Cot[x]]) + (ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(4*Sqrt[Cos[x]*Cot[x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !InertTrigFreeQ[w])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \int \frac{1}{(\cos(x)\cot(x))^{3/2}}\,dx$$
$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} - \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{\sqrt{\cot(x)}}{\cos^{\frac{3}{2}}(x)}\,dx}{4\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} - \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\int \frac{\sec(x)}{\sqrt{-\sin(x)}}\,dx}{4\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, -\sin(x)\right)}{4\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{-\sin(x)}\right)}{2\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right.}{}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{4\sqrt{\cos(x)\cot(x)}} + \frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin}}{4\sqrt{\cos(x)\cot(x)}}$$

Mathematica [A] time = 0.154516, size = 60, normalized size = 0.75

$$\frac{2\sqrt[4]{\sin^2(x)}\sec(x)+\cos(x)\left(-\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)\right)-\cos(x)\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)}{4\sqrt[4]{\sin^2(x)}\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-3/2),x]
```

```
[Out] (-(ArcTan[(Sin[x]^2)^(1/4)]*Cos[x]) - ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] + 2*
Sec[x]*(Sin[x]^2)^(1/4))/(4*Sqrt[Cos[x]*Cot[x]]*(Sin[x]^2)^(1/4))
```

Maple [C] time = 0.209, size = 450, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(3/2),x)
```

```
[Out] -1/8*2^(1/2)*(-1+cos(x))*(2*I*sin(x)*cos(x)^2*((-I*cos(x)+sin(x)+I)/sin(x))
^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*Ell
ipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-I*sin(x)*cos(x)^2*((
-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-
1+cos(x))/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1
/2*I,1/2*2^(1/2))-I*sin(x)*cos(x)^2*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticPi(((
I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+sin(x)*cos(x)^2*((-
I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1
+cos(x))/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/
2*I,1/2*2^(1/2))-sin(x)*cos(x)^2*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*co
s(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticPi(((I*c
os(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*cos(x)*2^(1/2)+2*2^(
1/2))*cos(x)*(1+cos(x))^2/sin(x)^5/(cos(x)^2/sin(x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(-3/2), x)
```

Fricas [B] time = 2.40963, size = 475, normalized size = 5.94

$$\frac{2\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^3+\cos(x)^3\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)-4\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\sqrt{\frac{\cos(x)}{\sin(x)}}}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4}\right.}{16\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*cos
(x)^3 + cos(x)^3*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin
(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(
x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x
) - 4)) + 8*sqrt(cos(x)^2/sin(x))*sin(x))/cos(x)^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\sin(x) + \csc(x))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(3/2),x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(-3/2), x)
```

3.320 $\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}-\frac{3\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}+\frac{3\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}+\frac{\tan(x)\sec^2(x)}{4\sqrt{\cos(x)\cot(x)}}$$

```
[Out] (-3*ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) +
(3*ArcTanh[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) -
(3*Tan[x])/(16*Sqrt[Cos[x]*Cot[x]]) + (Sec[x]^2*Tan[x])/(4*Sqrt[Cos[x]*Cot[
x]])
```

Rubi [A] time = 0.151284, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2564, 329, 298, 203, 206}

$$-\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}-\frac{3\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}+\frac{3\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}+\frac{\tan(x)\sec^2(x)}{4\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-5/2),x]
```


```
[Out] (-3*ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) +
(3*ArcTanh[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) -
(3*Tan[x])/(16*Sqrt[Cos[x]*Cot[x]]) + (Sec[x]^2*Tan[x])/(4*Sqrt[Cos[x]*Cot[
x]])
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx &= \int \frac{1}{(\cos(x)\cot(x))^{5/2}}\,dx \\
&= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}} \\
&= \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} - \frac{\left(3\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)\sqrt{\cot(x)}}\,dx}{8\sqrt{\cos(x)\cot(x)}} \\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} - \frac{\left(3\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\,dx}{32\sqrt{\cos(x)\cot(x)}} \\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} - \frac{(3\cos(x))\int \sec(x)\sqrt{-\sin(x)}\,dx}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} \\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} + \frac{(3\cos(x))\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, -\sin(x)\right)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} \\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} + \frac{(3\cos(x))\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{-\sin(x)}\right)}{16\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} \\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} + \frac{(3\cos(x))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{(3\cos
\end{aligned}
$$

$$
= -\frac{3\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} + \frac{3\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x}{4\sqrt{\cos(}
$$

Mathematica [A] time = 0.532499, size = 69, normalized size = 0.7

$$-\frac{\sin(x)\tan(x)\sqrt{\cos(x)\cot(x)}\left(-3\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)+3\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)+\sin^2(x)^{3/4}(3\cos(2x)-5)\sec^4(x)\right)}{32\sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-5/2),x]
```

```
[Out] -(Sqrt[Cos[x]*Cot[x]]*Sin[x]*(-3*ArcTan[(Sin[x]^2)^(1/4)] + 3*ArcTanh[(Sin[
x]^2)^(1/4)] + (-5 + 3*Cos[2*x])*Sec[x]^4*(Sin[x]^2)^(3/4))*Tan[x])/(32*(Si
n[x]^2)^(3/4))
```

Maple [C] time = 0.191, size = 382, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(5/2),x)
```

```
[Out] 1/64*2^(1/2)*(-1+cos(x))*(3*I*cos(x)^4*EllipticPi(((I*cos(x)+sin(x)-I)/sin(
x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*co
s(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)-3*I*cos(x)^4*Ell
ipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-I*cos(
x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x
))/sin(x))^(1/2)+3*cos(x)^4*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)
-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)+3*cos(x)^4*EllipticPi(((I*c
os(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/
sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1
/2)-6*cos(x)^3*2^(1/2)+6*cos(x)^2*2^(1/2)+8*cos(x)*2^(1/2)-8*2^(1/2))*cos(x
)*(1+cos(x))^2/sin(x)^5/(cos(x)^2/sin(x))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(-5/2), x)
```

Fricas [B] time = 2.67216, size = 512, normalized size = 5.17

$$-\frac{6\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos\left(x\right)^5-3\cos\left(x\right)^5\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)-4\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}-2\cos(x)+4}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4}\right)-8\left(3\cos(x)^4-7\cos(x)^2+4\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{128\cos\left(x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/128*(6*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*c
os(x)^5 - 3*cos(x)^5*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)
*sin(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*
cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*c
os(x) - 4)) - 8*(3*cos(x)^4 - 7*cos(x)^2 + 4)*sqrt(cos(x)^2/sin(x)))/cos(x)
^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\csc\left(x\right)-\sin\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(-5/2), x)
```

3.321 $\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}}+\frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}$$

```
[Out] (5*Sec[x])/(192*Sqrt[Cos[x]*Cot[x]]) - (5*Sec[x]^3)/(48*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]
]) + (Sec[x]^3*Tan[x]^2)/(6*Sqrt[Cos[x]*Cot[x]])
```

Rubi [A] time = 0.179569, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2564, 329, 212, 206, 203}

$$-\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}}+\frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-7/2),x]
```

```
[Out] (5*Sec[x])/(192*Sqrt[Cos[x]*Cot[x]]) - (5*Sec[x]^3)/(48*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]
]) + (Sec[x]^3*Tan[x]^2)/(6*Sqrt[Cos[x]*Cot[x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx = \int \frac{1}{(\cos(x)\cot(x))^{7/2}}\,dx$$

$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{3}{2}}(x)}\,dx}{12\sqrt{\cos(x)\cot(x)}}$$

$$= -\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{\sqrt{\cot(x)}}{\cos^{\frac{7}{2}}(x)}\,dx}{96\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{\sqrt{\cot(x}}{\cos^{\frac{3}{2}}(x}}{128\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right)\int \frac{\sec(x}{\sqrt{-\sin}}}{128\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\Big(}{128\sqrt{\cos(x}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\Big(}{64\sqrt{\cos(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\Big(}{128\sqrt{\cos(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} - \frac{5\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{128\sqrt{\cos(x)\cot(x)}} - \frac{5\tanh^{-}}{}$$

<u>Mathematica</u> [A] time = 0.267902, size = 74, normalized size = 0.63

$$\frac{2\sqrt[4]{\sin^2(x)}\sec(x)\left(32\sec^4(x)-52\sec^2(x)+5\right)+15\cos(x)\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)+15\cos(x)\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)}{384\sqrt[4]{\sin^2(x)}\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-7/2),x]
```

```
[Out] (15*ArcTan[(Sin[x]^2)^(1/4)]*Cos[x] + 15*ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] +
 2*Sec[x]*(5 - 52*Sec[x]^2 + 32*Sec[x]^4)*(Sin[x]^2)^(1/4))/(384*Sqrt[Cos[x
]*Cot[x]]*(Sin[x]^2)^(1/4))
```

Maple [C] time = 0.227, size = 487, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(7/2),x)
```

```
[Out] -1/768*2^(1/2)*(-1+cos(x))*(15*I*sin(x)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)
*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*cos(x)^6*
EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*I*s
in(x)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2
)*(-I*(-1+cos(x))/sin(x))^(1/2)*cos(x)^6*EllipticPi(((I*cos(x)+sin(x)-I)/si
n(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-30*I*sin(x)*((-I*cos(x)+sin(x)+I)/sin(x)
)^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*co
s(x)^6*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+15*sin(x)*
cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(
1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^
(1/2),1/2+1/2*I,1/2*2^(1/2))-15*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x)
)^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*E
llipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*2^(1
/2)*cos(x)^5+10*2^(1/2)*cos(x)^4+104*cos(x)^3*2^(1/2)-104*cos(x)^2*2^(1/2)-
64*cos(x)*2^(1/2)+64*2^(1/2))*cos(x)*(1+cos(x))^2/sin(x)^7/(cos(x)^2/sin(x)
)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(-7/2), x)
```

Fricas [A] time = 2.60421, size = 528, normalized size = 4.47

$$-\frac{30 \arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^7 - 15\cos(x)^7\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)+4\left(\cos(x)^2-(\cos(x)+1)\sin(x)-\right.}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)}\right.}{1536\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/1536*(30*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))
*cos(x)^7 - 15*cos(x)^7*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) +
 4)*sin(x) + 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) -
 2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) -
2*cos(x) - 4)) - 8*(5*cos(x)^4 - 52*cos(x)^2 + 32)*sqrt(cos(x)^2/sin(x))*si
n(x))/cos(x)^7
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\csc(x)-\sin(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(-7/2), x)
```

3.322 $\int (-\cos(x)+\sec(x))^4\,dx$

Optimal. Leaf size=44

$$\frac{35x}{8}+\frac{35\tan^3(x)}{24}-\frac{35\tan(x)}{8}-\frac{1}{4}\sin^4(x)\tan^3(x)-\frac{7}{8}\sin^2(x)\tan^3(x)$$

```
[Out] (35*x)/8 - (35*Tan[x])/8 + (35*Tan[x]^3)/24 - (7*Sin[x]^2*Tan[x]^3)/8 - (Si
n[x]^4*Tan[x]^3)/4
```

Rubi [A] time = 0.0307354, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {288, 302, 203}

$$\frac{35x}{8}+\frac{35\tan^3(x)}{24}-\frac{35\tan(x)}{8}-\frac{1}{4}\sin^4(x)\tan^3(x)-\frac{7}{8}\sin^2(x)\tan^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^4,x]
```

```
[Out] (35*x)/8 - (35*Tan[x])/8 + (35*Tan[x]^3)/24 - (7*Sin[x]^2*Tan[x]^3)/8 - (Si
n[x]^4*Tan[x]^3)/4
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^4\,dx &= \text{Subst}\left(\int \frac{x^8}{\left(1+x^2\right)^3}\,dx, x, \tan(x)\right)\\
&= -\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{7}{4}\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= -\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{35}{8}\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{35}{8}\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \tan(x)\right)\\
&= -\frac{35\tan(x)}{8}+\frac{35\tan^3(x)}{24}-\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{35}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.\\
&= \frac{35x}{8}-\frac{35\tan(x)}{8}+\frac{35\tan^3(x)}{24}-\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0323595, size = 38, normalized size = 0.86

$$\frac{35x}{8}-\frac{3}{4}\sin(2x)+\frac{1}{32}\sin(4x)-\frac{10\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^4,x]
```

```
[Out] (35*x)/8 - (3*Sin[2*x])/4 + Sin[4*x]/32 - (10*Tan[x])/3 + (Sec[x]^2*Tan[x])
/3
```

Maple [A] time = 0.023, size = 40, normalized size = 0.9

$$-\left(-\frac{2}{3}-\frac{(\sec(x))^2}{3}\right)\tan(x)-4\tan(x)+\frac{35x}{8}-2\cos(x)\sin(x)+\frac{\sin(x)}{4}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^4,x)
```

```
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)-4*tan(x)+35/8*x-2*cos(x)*sin(x)+1/4*(cos(x)^3+3
/2*cos(x))*sin(x)
```

Maxima [A] time = 0.978038, size = 35, normalized size = 0.8

$$\frac{1}{3}\tan(x)^3+\frac{35}{8}x+\frac{1}{32}\sin(4x)-\frac{3}{4}\sin(2x)-3\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + 35/8*x + 1/32*sin(4*x) - 3/4*sin(2*x) - 3*tan(x)
```

Fricas [A] time = 2.10209, size = 116, normalized size = 2.64

$$\frac{105\,x\cos(x)^3+\left(6\cos(x)^6-39\cos(x)^4-80\cos(x)^2+8\right)\sin(x)}{24\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*x*cos(x)^3 + (6*cos(x)^6 - 39*cos(x)^4 - 80*cos(x)^2 + 8)*sin(x))
/cos(x)^3
```

Sympy [A] time = 32.1363, size = 44, normalized size = 1.

$$\frac{35x}{8}-2\sin(x)\cos(x)-\frac{4\sin(x)}{\cos(x)}+\frac{\sin(2x)}{4}+\frac{\sin(4x)}{32}+\frac{\tan^3(x)}{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**4,x)
```

```
[Out] 35*x/8 - 2*sin(x)*cos(x) - 4*sin(x)/cos(x) + sin(2*x)/4 + sin(4*x)/32 + tan
(x)**3/3 + tan(x)
```

Giac [A] time = 1.14325, size = 47, normalized size = 1.07

$$\frac{1}{3} \tan(x)^3 + \frac{35}{8} x - \frac{13 \tan(x)^3 + 11 \tan(x)}{8 \left(\tan(x)^2 + 1\right)^2} - 3 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + 35/8*x - 1/8*(13*tan(x)^3 + 11*tan(x))/(tan(x)^2 + 1)^2 - 3*
tan(x)
```

3.323 $\int (-\cos(x)+\sec(x))^3\,dx$

Optimal. Leaf size=34

$$\frac{5\sin^3(x)}{6}+\frac{5\sin(x)}{2}+\frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\tanh^{-1}(\sin(x))$$

```
[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sin[x])/2 + (5*Sin[x]^3)/6 + (Sin[x]^3*Tan[x]^2
)/2
```

Rubi [A] time = 0.0415915, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 2592, 288, 302, 206}

$$\frac{5\sin^3(x)}{6}+\frac{5\sin(x)}{2}+\frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^3,x]
```

```
[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sin[x])/2 + (5*Sin[x]^3)/6 + (Sin[x]^3*Tan[x]^2
)/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^3\,dx &= \int \sin^3(x)\tan^3(x)\,dx\\
&= \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(x)\right)\\
&= \frac{5\sin(x)}{2}+\frac{5\sin^3(x)}{6}+\frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{5}{2}\tanh^{-1}(\sin(x))+\frac{5\sin(x)}{2}+\frac{5\sin^3(x)}{6}+\frac{1}{2}\sin^3(x)\tan^2(x)
\end{aligned}$$

Mathematica [A] time = 0.0106327, size = 38, normalized size = 1.12

$$-\frac{1}{3}\sin^3(x)\tan^2(x)-\frac{5}{3}\sin(x)\tan^2(x)-\frac{5}{2}\tanh^{-1}(\sin(x))+\frac{5}{2}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^3,x]
```

```
[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sec[x]*Tan[x])/2 - (5*Sin[x]*Tan[x]^2)/3 - (Sin
[x]^3*Tan[x]^2)/3
```

Maple [A] time = 0.022, size = 30, normalized size = 0.9

$$\frac{\sec(x)\tan(x)}{2}-\frac{5\ln(\sec(x)+\tan(x))}{2}+3\sin(x)-\frac{\left(2+(\cos(x))^{2}\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^3,x)
```

```
[Out] 1/2*sec(x)*tan(x)-5/2*ln(sec(x)+tan(x))+3*sin(x)-1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.989054, size = 50, normalized size = 1.47

$$\frac{1}{3}\sin(x)^{3}-\frac{\sin(x)}{2\left(\sin(x)^{2}-1\right)}-\frac{5}{4}\log(\sin(x)+1)+\frac{5}{4}\log(\sin(x)-1)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(si
n(x) - 1) + 2*sin(x)
```

Fricas [A] time = 2.27556, size = 161, normalized size = 4.74

$$-\frac{15\cos(x)^{2}\log(\sin(x)+1)-15\cos(x)^{2}\log(-\sin(x)+1)+2\left(2\cos(x)^{4}-14\cos(x)^{2}-3\right)\sin(x)}{12\cos(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(15*cos(x)^2*log(sin(x) + 1) - 15*cos(x)^2*log(-sin(x) + 1) + 2*(2*co
s(x)^4 - 14*cos(x)^2 - 3)*sin(x))/cos(x)^2
```

Sympy [A] time = 8.17225, size = 42, normalized size = 1.24

$$\frac{5\log\left(\sin\left(x\right)-1\right)}{4}-\frac{5\log\left(\sin\left(x\right)+1\right)}{4}+\frac{\sin^{3}\left(x\right)}{3}+2\sin\left(x\right)-\frac{\sin\left(x\right)}{2\sin^{2}\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**3,x)
```

```
[Out] 5*log(sin(x) - 1)/4 - 5*log(sin(x) + 1)/4 + sin(x)**3/3 + 2*sin(x) - sin(x)
/(2*sin(x)**2 - 2)
```

Giac [A] time = 1.16285, size = 53, normalized size = 1.56

$$\frac{1}{3}\,\sin\left(x\right)^{3}-\frac{\sin\left(x\right)}{2\left(\sin\left(x\right)^{2}-1\right)}-\frac{5}{4}\,\log\left(\sin\left(x\right)+1\right)+\frac{5}{4}\,\log\left(-\sin\left(x\right)+1\right)+2\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(-s
in(x) + 1) + 2*sin(x)
```

3.324 $\int (-\cos(x) + \sec(x))^2 \, dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} + \frac{3\tan(x)}{2} - \frac{1}{2}\sin^2(x)\tan(x)$$

```
[Out] (-3*x)/2 + (3*Tan[x])/2 - (Sin[x]^2*Tan[x])/2
```

Rubi [A] time = 0.0205319, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {288, 321, 203}

$$-\frac{3x}{2} + \frac{3\tan(x)}{2} - \frac{1}{2}\sin^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^2,x]
```

```
[Out] (-3*x)/2 + (3*Tan[x])/2 - (Sin[x]^2*Tan[x])/2
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^2\,dx &= \text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= -\frac{1}{2}\sin^2(x)\tan(x)+\frac{3}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{3\tan(x)}{2}-\frac{1}{2}\sin^2(x)\tan(x)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{3x}{2}+\frac{3\tan(x)}{2}-\frac{1}{2}\sin^2(x)\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.0177295, size = 16, normalized size = 0.73

$$-\frac{3x}{2}+\frac{1}{4}\sin(2x)+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^2,x]
```

```
[Out] (-3*x)/2 + Sin[2*x]/4 + Tan[x]
```

Maple [A] time = 0.016, size = 13, normalized size = 0.6

$$\tan(x)-\frac{3\,x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^2,x)
```

```
[Out] tan(x)-3/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.973503, size = 16, normalized size = 0.73

$$-\frac{3}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="maxima")
```

```
[Out] -3/2*x + 1/4*sin(2*x) + tan(x)
```

Fricas [A] time = 2.06675, size = 68, normalized size = 3.09

$$-\frac{3\,x\cos\left(x\right)-\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)}{2\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

Sympy [A] time = 2.5935, size = 14, normalized size = 0.64

$$-\frac{3x}{2}+\frac{\sin\left(2x\right)}{4}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**2,x)
```

```
[Out] -3*x/2 + sin(2*x)/4 + tan(x)
```

Giac [A] time = 1.15956, size = 24, normalized size = 1.09

$$-\frac{3}{2}\,x+\frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^{2}+1\right)}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="giac")
```

```
[Out] -3/2*x + 1/2*tan(x)/(tan(x)^2 + 1) + tan(x)
```

3.325 $\int (-\cos(x)+\sec(x))\,dx$

Optimal. Leaf size=8

$$\tanh^{-1}(\sin(x))-\sin(x)$$

```
[Out] ArcTanh[Sin[x]] - Sin[x]
```

Rubi [A] time = 0.004773, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2637, 3770}

$$\tanh^{-1}(\sin(x))-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[-Cos[x] + Sec[x],x]

[Out] ArcTanh[Sin[x]] - Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (-\cos(x)+\sec(x))\,dx &= -\int \cos(x)\,dx+\int \sec(x)\,dx\\ &= \tanh^{-1}(\sin(x))-\sin(x)\end{aligned}$$

Mathematica [B] time = 0.0042199, size = 37, normalized size = 4.62

$$-\sin(x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cos[x] + Sec[x],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x]
```

Maple [A] time = 0.002, size = 12, normalized size = 1.5

$$-\sin(x) + \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)+sec(x),x)
```

```
[Out] -sin(x)+ln(sec(x)+tan(x))
```

Maxima [A] time = 0.995348, size = 15, normalized size = 1.88

$$\log(\sec(x) + \tan(x)) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="maxima")
```

```
[Out] log(sec(x) + tan(x)) - sin(x)
```

Fricas [B] time = 2.11805, size = 72, normalized size = 9.

$$\frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(-\sin(x) + 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Sympy [B] time = 0.110682, size = 19, normalized size = 2.38

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - sin(x)
```

Giac [B] time = 1.14779, size = 39, normalized size = 4.88

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2)) -
 sin(x)
```

3.326 $\int \frac{1}{-\cos(x)+\sec(x)}\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

```
[Out] -Csc[x]
```

Rubi [A] time = 0.0175251, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-1),x]
```

```
[Out] -Csc[x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{-\cos(x)+\sec(x)}\,dx = \int \cot(x)\csc(x)\,dx$$
$$= -\operatorname{Subst}\left(\int 1\,dx, x, \csc(x)\right)$$
$$= -\csc(x)$$

Mathematica [A] time = 0.0026784, size = 4, normalized size = 1.

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-1),x]
```

```
[Out] -Csc[x]
```

Maple [A] time = 0.03, size = 7, normalized size = 1.8

$$-(\sin(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x)),x)
```

```
[Out] -1/sin(x)
```

Maxima [B] time = 0.996598, size = 28, normalized size = 7.

$$-\frac{\cos(x)+1}{2\sin(x)} - \frac{\sin(x)}{2(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + 1)/sin(x) - 1/2*sin(x)/(cos(x) + 1)
```

Fricas [A] time = 1.95337, size = 15, normalized size = 3.75

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\cos(x) - \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x)
```

```
[Out] -Integral(1/(cos(x) - sec(x)), x)
```

Giac [A] time = 1.10473, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

3.327 $\int \frac{1}{(-\cos(x)+\sec(x))^2} dx$

Optimal. Leaf size=8

$$-\frac{1}{3}\cot^3(x)$$

```
[Out] -Cot[x]^3/3
```

Rubi [A] time = 0.0140351, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30}

$$-\frac{1}{3}\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-2),x]
```

```
[Out] -Cot[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x)+\sec(x))^2} dx &= \text{Subst}\left(\int \frac{1}{x^4} dx, x, \tan(x)\right) \\ &= -\frac{1}{3}\cot^3(x) \end{aligned}$$

Mathematica [A] time = 0.0025405, size = 8, normalized size = 1.

$$-\frac{1}{3}\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-2),x]
```

```
[Out] -Cot[x]^3/3
```

Maple [A] time = 0.03, size = 7, normalized size = 0.9

$$-\frac{1}{3\left(\tan\left(x\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^2,x)
```

```
[Out] -1/3/tan(x)^3
```

Maxima [A] time = 0.980036, size = 8, normalized size = 1.

$$-\frac{1}{3\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="maxima")
```

```
[Out] -1/3/tan(x)^3
```

Fricas [B] time = 2.03443, size = 51, normalized size = 6.38

$$\frac{\cos\left(x\right)^{3}}{3\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3/((cos(x)^2 - 1)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\cos\left(x\right)+\sec\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**2,x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-2), x)
```

Giac [A] time = 1.14738, size = 8, normalized size = 1.

$$-\frac{1}{3\,\tan\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="giac")
```

```
[Out] -1/3/tan(x)^3
```

3.328 $\int \frac{1}{(-\cos(x)+\sec(x))^3} dx$

Optimal. Leaf size=17

$$\frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

```
[Out] Csc[x]^3/3 - Csc[x]^5/5
```

Rubi [A] time = 0.0379673, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 14}

$$\frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-3),x]
```

```
[Out] Csc[x]^3/3 - Csc[x]^5/5
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx = \int \cot^3(x)\csc^3(x)\,dx$$
$$= -\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \csc(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \csc(x)\right)$$
$$= \frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

Mathematica [A] time = 0.0091952, size = 17, normalized size = 1.

$$\frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-3),x]
```

```
[Out] Csc[x]^3/3 - Csc[x]^5/5
```

Maple [A] time = 0.036, size = 14, normalized size = 0.8

$$\frac{1}{3\,(\sin(x))^3} - \frac{1}{5\,(\sin(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^3,x)
```

```
[Out] 1/3/sin(x)^3-1/5/sin(x)^5
```

Maxima [B] time = 0.988393, size = 99, normalized size = 5.82

$$\frac{\left(\frac{5\sin(x)^2}{(\cos(x)+1)^2} + \frac{30\sin(x)^4}{(\cos(x)+1)^4} - 3\right)(\cos(x)+1)^5}{480\sin(x)^5} + \frac{\sin(x)}{16\,(\cos(x)+1)} + \frac{\sin(x)^3}{96\,(\cos(x)+1)^3} - \frac{\sin(x)^5}{160\,(\cos(x)+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(5*sin(x)^2/(cos(x) + 1)^2 + 30*sin(x)^4/(cos(x) + 1)^4 - 3)*(cos(x)
+ 1)^5/sin(x)^5 + 1/16*sin(x)/(cos(x) + 1) + 1/96*sin(x)^3/(cos(x) + 1)^3 -
 1/160*sin(x)^5/(cos(x) + 1)^5
```

Fricas [B] time = 2.095, size = 82, normalized size = 4.82

$$-\frac{5\,\cos\left(x\right)^{2}-2}{15\,\left(\cos\left(x\right)^{4}-2\,\cos\left(x\right)^{2}+1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 2)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\cos^{3}\left(x\right)-3\cos^{2}\left(x\right)\sec\left(x\right)+3\cos\left(x\right)\sec^{2}\left(x\right)-\sec^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**3,x)
```

```
[Out] -Integral(1/(cos(x)**3 - 3*cos(x)**2*sec(x) + 3*cos(x)*sec(x)**2 - sec(x)**
3), x)
```

Giac [A] time = 1.11208, size = 19, normalized size = 1.12

$$\frac{5\,\sin\left(x\right)^{2}-3}{15\,\sin\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="giac")
```

```
[Out] 1/15*(5*sin(x)^2 - 3)/sin(x)^5
```

3.329 $\int \frac{1}{(-\cos(x)+\sec(x))^4}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

```
[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rubi [A] time = 0.0171948, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-4),x]
```

```
[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\cos(x)+\sec(x))^4}\,dx &= \text{Subst}\left(\int \left(\frac{1}{x^8}+\frac{1}{x^6}\right)dx,x,\tan(x)\right)\\ &= -\frac{1}{5}\cot^5(x)-\frac{\cot^7(x)}{7}\end{aligned}$$

Mathematica [B] time = 0.0224526, size = 37, normalized size = 2.18

$$-\frac{2\cot(x)}{35}-\frac{1}{7}\cot(x)\csc^6(x)+\frac{8}{35}\cot(x)\csc^4(x)-\frac{1}{35}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-4),x]
```

```
[Out] (-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 - (Cot[x]*Cs
c[x]^6)/7
```

Maple [A] time = 0.037, size = 14, normalized size = 0.8

$$-\frac{1}{7\left(\tan\left(x\right)\right)^{7}}-\frac{1}{5\left(\tan\left(x\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^4,x)
```

```
[Out] -1/7/tan(x)^7-1/5/tan(x)^5
```

Maxima [A] time = 0.997813, size = 19, normalized size = 1.12

$$-\frac{7\tan\left(x\right)^{2}+5}{35\tan\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

Fricas [B] time = 2.30063, size = 112, normalized size = 6.59

$$-\frac{2\cos\left(x\right)^{7}-7\cos\left(x\right)^{5}}{35\left(\cos\left(x\right)^{6}-3\cos\left(x\right)^{4}+3\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*s
in(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1702, size = 19, normalized size = 1.12

$$-\frac{7\tan(x)^2+5}{35\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="giac")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

3.330 $\int \frac{1}{(-\cos(x)+\sec(x))^5} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

```
[Out] -Csc[x]^5/5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

Rubi [A] time = 0.0410418, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-5),x]
```

```
[Out] -Csc[x]^5/5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = \int \cot^5(x)\csc^5(x)\,dx$$
$$= -\operatorname{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \csc(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \csc(x)\right)$$
$$= -\frac{1}{5}\csc^5(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^9(x)}{9}$$

Mathematica [A] time = 0.0117831, size = 25, normalized size = 1.

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-5),x]
```

```
[Out] -Csc[x]^5/5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

Maple [A] time = 0.043, size = 20, normalized size = 0.8

$$\frac{2}{7\left(\sin(x)\right)^7}-\frac{1}{9\left(\sin(x)\right)^9}-\frac{1}{5\left(\sin(x)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^5,x)
```

```
[Out] 2/7/sin(x)^7-1/9/sin(x)^9-1/5/sin(x)^5
```

Maxima [B] time = 1.0162, size = 163, normalized size = 6.52

$$\frac{\left(\frac{45\sin(x)^2}{(\cos(x)+1)^2}+\frac{252\sin(x)^4}{(\cos(x)+1)^4}-\frac{420\sin(x)^6}{(\cos(x)+1)^6}-\frac{1890\sin(x)^8}{(\cos(x)+1)^8}-35\right)(\cos(x)+1)^9}{161280\sin(x)^9}-\frac{3\sin(x)}{256(\cos(x)+1)}-\frac{\sin(x)^3}{384(\cos(x)+1)^3}+\frac{\sin}{640(\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="maxima")
```

```
[Out] 1/161280*(45*sin(x)^2/(cos(x) + 1)^2 + 252*sin(x)^4/(cos(x) + 1)^4 - 420*si
n(x)^6/(cos(x) + 1)^6 - 1890*sin(x)^8/(cos(x) + 1)^8 - 35)*(cos(x) + 1)^9/s
in(x)^9 - 3/256*sin(x)/(cos(x) + 1) - 1/384*sin(x)^3/(cos(x) + 1)^3 + 1/640
*sin(x)^5/(cos(x) + 1)^5 + 1/3584*sin(x)^7/(cos(x) + 1)^7 - 1/4608*sin(x)^9
/(cos(x) + 1)^9
```

Fricas [B] time = 2.45715, size = 139, normalized size = 5.56

$$-\frac{63 \cos\left(x\right)^{4} - 36 \cos\left(x\right)^{2} + 8}{315 \left(\cos\left(x\right)^{8} - 4 \cos\left(x\right)^{6} + 6 \cos\left(x\right)^{4} - 4 \cos\left(x\right)^{2} + 1\right) \sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(63*cos(x)^4 - 36*cos(x)^2 + 8)/((cos(x)^8 - 4*cos(x)^6 + 6*cos(x)^4
 - 4*cos(x)^2 + 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15575, size = 27, normalized size = 1.08

$$-\frac{63 \sin\left(x\right)^{4} - 90 \sin\left(x\right)^{2} + 35}{315 \sin\left(x\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="giac")
```

```
[Out] -1/315*(63*sin(x)^4 - 90*sin(x)^2 + 35)/sin(x)^9
```

3.331 $\int \frac{1}{(-\cos(x)+\sec(x))^6} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{11}\cot^{11}(x)-\frac{2\cot^9(x)}{9}-\frac{\cot^7(x)}{7}$$

```
[Out] -Cot[x]^7/7 - (2*Cot[x]^9)/9 - Cot[x]^11/11
```

Rubi [A] time = 0.0205889, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {270}

$$-\frac{1}{11}\cot^{11}(x)-\frac{2\cot^9(x)}{9}-\frac{\cot^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-6),x]
```

```
[Out] -Cot[x]^7/7 - (2*Cot[x]^9)/9 - Cot[x]^11/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\cos(x)+\sec(x))^6} \, dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^{12}} \, dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^{12}}+\frac{2}{x^{10}}+\frac{1}{x^8}\right) \, dx, x, \tan(x)\right)\\ &= -\frac{1}{7}\cot^7(x)-\frac{2\cot^9(x)}{9}-\frac{\cot^{11}(x)}{11}\end{aligned}$$

Mathematica [B] time = 0.0200594, size = 57, normalized size = 2.28

$$\frac{8\cot(x)}{693}-\frac{1}{11}\cot(x)\csc^{10}(x)+\frac{23}{99}\cot(x)\csc^{8}(x)-\frac{113}{693}\cot(x)\csc^{6}(x)+\frac{1}{231}\cot(x)\csc^{4}(x)+\frac{4}{693}\cot(x)\csc^{2}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-6),x]
```

```
[Out] (8*Cot[x])/693 + (4*Cot[x]*Csc[x]^2)/693 + (Cot[x]*Csc[x]^4)/231 - (113*Cot
[x]*Csc[x]^6)/693 + (23*Cot[x]*Csc[x]^8)/99 - (Cot[x]*Csc[x]^10)/11
```

Maple [A] time = 0.039, size = 20, normalized size = 0.8

$$-\frac{1}{7\left(\tan\left(x\right)\right)^{7}}-\frac{2}{9\left(\tan\left(x\right)\right)^{9}}-\frac{1}{11\left(\tan\left(x\right)\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^6,x)
```

```
[Out] -1/7/tan(x)^7-2/9/tan(x)^9-1/11/tan(x)^11
```

Maxima [A] time = 0.990126, size = 27, normalized size = 1.08

$$-\frac{99\,\tan\left(x\right)^{4}+154\,\tan\left(x\right)^{2}+63}{693\,\tan\left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="maxima")
```

```
[Out] -1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

Fricas [B] time = 2.45493, size = 173, normalized size = 6.92

$$\frac{8\,\cos\left(x\right)^{11}-44\,\cos\left(x\right)^{9}+99\,\cos\left(x\right)^{7}}{693\left(\cos\left(x\right)^{10}-5\,\cos\left(x\right)^{8}+10\,\cos\left(x\right)^{6}-10\,\cos\left(x\right)^{4}+5\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(8*cos(x)^11 - 44*cos(x)^9 + 99*cos(x)^7)/((cos(x)^10 - 5*cos(x)^8 +
10*cos(x)^6 - 10*cos(x)^4 + 5*cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14656, size = 27, normalized size = 1.08

$$-\frac{99 \tan(x)^4 + 154 \tan(x)^2 + 63}{693 \tan(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="giac")
```

```
[Out] -1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

3.332 $\int \frac{1}{(-\cos(x)+\sec(x))^7} dx$

Optimal. Leaf size=33

$$-\frac{1}{13}\csc^{13}(x)+\frac{3\csc^{11}(x)}{11}-\frac{\csc^9(x)}{3}+\frac{\csc^7(x)}{7}$$

```
[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

Rubi [A] time = 0.0422955, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$-\frac{1}{13}\csc^{13}(x)+\frac{3\csc^{11}(x)}{11}-\frac{\csc^9(x)}{3}+\frac{\csc^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-7),x]
```

```
[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = \int \cot^7(x)\csc^7(x)\,dx$$
$$= -\text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \csc(x)\right)$$
$$= -\text{Subst}\left(\int \left(-x^6+3x^8-3x^{10}+x^{12}\right)dx, x, \csc(x)\right)$$
$$= \frac{\csc^7(x)}{7}-\frac{\csc^9(x)}{3}+\frac{3\csc^{11}(x)}{11}-\frac{\csc^{13}(x)}{13}$$

Mathematica [A] time = 0.0133771, size = 33, normalized size = 1.

$$-\frac{1}{13}\csc^{13}(x)+\frac{3\csc^{11}(x)}{11}-\frac{\csc^9(x)}{3}+\frac{\csc^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-7),x]
```

```
[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

Maple [A] time = 0.044, size = 26, normalized size = 0.8

$$-\frac{1}{13(\sin(x))^{13}}+\frac{1}{7(\sin(x))^{7}}-\frac{1}{3(\sin(x))^{9}}+\frac{3}{11(\sin(x))^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^7,x)
```

```
[Out] -1/13/sin(x)^13+1/7/sin(x)^7-1/3/sin(x)^9+3/11/sin(x)^11
```

Maxima [B] time = 1.04915, size = 228, normalized size = 6.91

$$\frac{\left(\frac{273\sin(x)^2}{(\cos(x)+1)^2}+\frac{2002\sin(x)^4}{(\cos(x)+1)^4}-\frac{2574\sin(x)^6}{(\cos(x)+1)^6}-\frac{9009\sin(x)^8}{(\cos(x)+1)^8}+\frac{15015\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{60060\sin(x)^{12}}{(\cos(x)+1)^{12}}-231\right)(\cos(x)+1)^{13}}{24600576\sin(x)^{13}}+\frac{5\sin(x)}{2048(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="maxima")
```

```
[Out] 1/24600576*(273*sin(x)^2/(cos(x) + 1)^2 + 2002*sin(x)^4/(cos(x) + 1)^4 - 25
74*sin(x)^6/(cos(x) + 1)^6 - 9009*sin(x)^8/(cos(x) + 1)^8 + 15015*sin(x)^10
/(cos(x) + 1)^10 + 60060*sin(x)^12/(cos(x) + 1)^12 - 231)*(cos(x) + 1)^13/s
in(x)^13 + 5/2048*sin(x)/(cos(x) + 1) + 5/8192*sin(x)^3/(cos(x) + 1)^3 - 3/
8192*sin(x)^5/(cos(x) + 1)^5 - 3/28672*sin(x)^7/(cos(x) + 1)^7 + 1/12288*si
n(x)^9/(cos(x) + 1)^9 + 1/90112*sin(x)^11/(cos(x) + 1)^11 - 1/106496*sin(x)
^13/(cos(x) + 1)^13
```

Fricas [B] time = 2.30183, size = 207, normalized size = 6.27

$$-\frac{429 \cos(x)^6 - 286 \cos(x)^4 + 104 \cos(x)^2 - 16}{3003 \left(\cos(x)^{12} - 6 \cos(x)^{10} + 15 \cos(x)^8 - 20 \cos(x)^6 + 15 \cos(x)^4 - 6 \cos(x)^2 + 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*cos(x)^6 - 286*cos(x)^4 + 104*cos(x)^2 - 16)/((cos(x)^12 - 6*c
os(x)^10 + 15*cos(x)^8 - 20*cos(x)^6 + 15*cos(x)^4 - 6*cos(x)^2 + 1)*sin(x)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16044, size = 35, normalized size = 1.06

$$\frac{429 \sin(x)^6 - 1001 \sin(x)^4 + 819 \sin(x)^2 - 231}{3003 \sin(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="giac")
```

```
[Out] 1/3003*(429*sin(x)^6 - 1001*sin(x)^4 + 819*sin(x)^2 - 231)/sin(x)^13
```

3.333 $\int (-\cos(x)+\sec(x))^{7/2}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\tan(x)\sec(x)\sqrt{s}$$

```
[Out] (-256*Csc[x]*Sqrt[Sin[x]*Tan[x]])/35 + (64*Sec[x]*Tan[x]*Sqrt[Sin[x]*Tan[x]
])/35 - (8*Sin[x]*Tan[x]^2*Sqrt[Sin[x]*Tan[x]])/7 - (2*Sin[x]^3*Tan[x]^2*Sq
rt[Sin[x]*Tan[x]])/7
```

Rubi [A] time = 0.112532, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\tan(x)\sec(x)\sqrt{s}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(7/2),x]
```

```
[Out] (-256*Csc[x]*Sqrt[Sin[x]*Tan[x]])/35 + (64*Sec[x]*Tan[x]*Sqrt[Sin[x]*Tan[x]
])/35 - (8*Sin[x]*Tan[x]^2*Sqrt[Sin[x]*Tan[x]])/7 - (2*Sin[x]^3*Tan[x]^2*Sq
rt[Sin[x]*Tan[x]])/7
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \int (\sin(x)\tan(x))^{7/2}\,dx$$

$$= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$= -\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(12\sqrt{\sin(x)\tan(x)}\right)\int \sin^{\frac{3}{2}}(x)\tan^{\frac{7}{2}}(x)\,dx}{7\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$= -\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(32\sqrt{\sin(x)\tan(x)}\right)}{7\sqrt{\sin(x)}\sqrt{\mathrm{ta}}}$$

$$= \frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\mathrm{sin}}$$

$$= -\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)} + \frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\,\mathrm{t}}$$

Mathematica [A] time = 0.204435, size = 37, normalized size = 0.51

$$\frac{1}{70}\sec(x)\sqrt{\sin(x)\tan(x)}(28\tan(x) - 512\cot(x) - 5(\sin(3x) - 23\sin(x))\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(7/2),x]
```

```
[Out] (Sec[x]*Sqrt[Sin[x]*Tan[x]]*(-512*Cot[x] - 5*Cos[x]*(-23*Sin[x] + Sin[3*x])
 + 28*Tan[x]))/70
```

Maple [B] time = 0.235, size = 603, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(7/2),x)
```

```
[Out] 1/70*(-1+cos(x))^2*(-105*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos
(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^
2)^(1/2)-1)/sin(x)^2)+105*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(
x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2
)^(1/2)-1)/sin(x)^2)-315*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos
(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^
2)^(1/2)-1)/sin(x)^2)+315*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(
x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2
)^(1/2)-1)/sin(x)^2)+20*cos(x)^6-315*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*
ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)
/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+315*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)
*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/
(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-105*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln
(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(
1+cos(x))^2)^(1/2)-1)/sin(x)^2)+105*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(
-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+c
os(x))^2)^(1/2)-1)/sin(x)^2)-140*cos(x)^4-420*cos(x)^2+28)*cos(x)*(1+cos(x)
)^2*(-(-1+cos(x)^2)/cos(x))^(7/2)/sin(x)^11
```

Maxima [A] time = 1.57384, size = 111, normalized size = 1.52

$$\frac{128 \left(\frac{7 \sin(x)^4}{(\cos(x)+1)^4} - \frac{7 \sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{2 \sin(x)^{14}}{(\cos(x)+1)^{14}} - 2\right)}{35 \left(\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{7}{2}} \left(-\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{7}{2}} \left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

```
[Out] 128/35*(7*sin(x)^4/(cos(x) + 1)^4 - 7*sin(x)^10/(cos(x) + 1)^10 + 2*sin(x)^
14/(cos(x) + 1)^14 - 2)/((sin(x)/(cos(x) + 1) + 1)^(7/2)*(-sin(x)/(cos(x) +
 1) + 1)^(7/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(7/2))
```

Fricas [A] time = 2.1244, size = 134, normalized size = 1.84

$$\frac{2\left(5\cos(x)^6-35\cos(x)^4-105\cos(x)^2+7\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{35\cos(x)^2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(x)^6 - 35*cos(x)^4 - 105*cos(x)^2 + 7)*sqrt(-(cos(x)^2 - 1)/cos
(x))/(cos(x)^2*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(-\cos(x)+\sec(x))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(7/2), x)
```

3.334 $\int (-\cos(x)+\sec(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rubi [A] time = 0.085911, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(5/2),x]
```

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^{5/2}\,dx &= \int (\sin(x)\tan(x))^{5/2}\,dx \\
&= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}} \\
&= -\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{\left(8\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin(x)}\tan^{\frac{5}{2}}(x)\,dx}{5\sqrt{\sin(x)}\sqrt{\tan(x)}} \\
&= \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{\left(32\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin}}{15\sqrt{\sin(x)}\sqrt{\tan}} \\
&= \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}
\end{aligned}$$

Mathematica [A] time = 0.0763065, size = 29, normalized size = 0.58

$$\frac{2}{15}\tan(x)\sqrt{\sin(x)\tan(x)}\left(3\cos^2(x)+32\cot^2(x)+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(5/2),x]
```

```
[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

Maple [B] time = 0.146, size = 321, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(5/2),x)
```

```
[Out] -1/15*(-1+cos(x))^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln
(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-
2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)-15*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))
^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-c
os(x)/(1+cos(x))^2)^(1/2)+15*cos(x)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2
)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos
(x)/(1+cos(x))^2)^(1/2)-60*cos(x)^2-10)*cos(x)*(1+cos(x))^2*(-(-1+cos(x)^2)
/cos(x))^(5/2)/sin(x)^9
```

Maxima [B] time = 1.58506, size = 111, normalized size = 2.22

$$-\frac{32\left(\frac{5\sin(x)^4}{(\cos(x)+1)^4}-\frac{5\sin(x)^6}{(\cos(x)+1)^6}+\frac{2\sin(x)^{10}}{(\cos(x)+1)^{10}}-2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10
/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1
) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 2.24485, size = 112, normalized size = 2.24

$$-\frac{2\left(3\cos(x)^4-30\cos(x)^2-5\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*s
in(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-\cos (x) + \sec (x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(5/2), x)
```

3.335 $\int (-\cos(x)+\sec(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rubi [A] time = 0.0639886, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4397, 4400, 2598, 2589}

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(3/2),x]
```

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int(-\cos(x)+\sec(x))^{3/2}\,dx&=\int(\sin(x)\tan(x))^{3/2}\,dx\\&=\frac{\sqrt{\sin(x)\tan(x)}\int\sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\&=-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}+\frac{\left(4\sqrt{\sin(x)\tan(x)}\right)\int\frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}\,dx}{3\sqrt{\sin(x)}\sqrt{\tan(x)}}\\&=\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.03854, size = 23, normalized size = 0.74

$$\frac{2}{3}\sin(x)\left(4\csc^2(x)-1\right)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(3/2),x]
```

```
[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Maple [B] time = 0.104, size = 584, normalized size = 18.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(3/2),x)
```

```
[Out] 1/6*(-1+cos(x))^2*(3*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^
2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(
```

```
1/2)-1)/sin(x)^2)-3*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(
-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2
)-1)/sin(x)^2)+9*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2*(-
cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)
-1)/sin(x)^2)-9*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-cos
(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)
/sin(x)^2)+9*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2*(-cos(x)
/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/si
n(x)^2)-9*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-cos(x)/(1+c
os(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^
2)+3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^
(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-3*(-cos
(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)
^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+4*cos(x)^3+12*cos(x
))*(1+cos(x))^2*(-(-1+cos(x)^2)/cos(x))^(3/2)/sin(x)^7
```

Maxima [B] time = 1.57738, size = 77, normalized size = 2.48

$$-\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x
)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 2.17881, size = 76, normalized size = 2.45

$$\frac{2\left(\cos(x)^2+3\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{3\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-\cos(x) + \sec(x))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(3/2), x)
```

3.336 $\int \sqrt{-\cos(x)+\sec(x)}\,dx$

Optimal. Leaf size=13

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Rubi [A] time = 0.0410275, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4397, 4400, 2589}

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[x] + Sec[x]],x]

[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = \int \sqrt{\sin(x)\tan(x)}\,dx$$
$$= \frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= -2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Mathematica [A] time = 0.0260806, size = 13, normalized size = 1.

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Maple [B] time = 0.138, size = 174, normalized size = 13.4

$$\frac{(-1+\cos(x))\cos(x)}{2\,(\sin(x))^3}\left(4\,\cos(x)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}+4\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\ln\left(-\frac{1}{(\sin(x))^2}\left(2\,(\cos(x))^2\sqrt{-\frac{\cos(x)}{(1+\cos(x)}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(1/2),x)
```

```
[Out] 1/2*(-1+cos(x))*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+4*(-cos(x)/(1+cos(x)
)^2)^(1/2)-ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2
*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+ln(-2*(2*cos(x)^2*(-cos(x)/(1+co
s(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2
))*cos(x)*(-(-1+cos(x)^2)/cos(x))^(1/2)/sin(x)^3/(-cos(x)/(1+cos(x))^2)^(1/
2)
```

Maxima [B] time = 1.54065, size = 77, normalized size = 5.92

$$\frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)
/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))
```

Fricas [A] time = 1.98497, size = 63, normalized size = 4.85

$$-\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\cos(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(x) + sec(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\cos(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(x) + sec(x)), x)
```

3.337 $\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$

Optimal. Leaf size=52

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

```
[Out] (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (ArcTanh
[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rubi [A] time = 0.0780825, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4397, 4400, 2601, 2565, 329, 298, 203, 206}

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (ArcTanh
[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx &= \int \frac{1}{\sqrt{\sin(x)\tan(x)}}\,dx \\
&= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\,dx}{\sqrt{\sin(x)\tan(x)}} \\
&= \frac{\sin(x)\int \sqrt{\cos(x)}\csc(x)\,dx}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\sin(x)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(x)\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{(2\sin(x))\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= \frac{\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.245932, size = 43, normalized size = 0.83

$$
\frac{\cos(x)\cot(x)\sqrt{\sin(x)\tan(x)}\left(\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right)-\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)\right)}{\cos^2(x)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] ((ArcTan[(Cos[x]^2)^(1/4)] - ArcTanh[(Cos[x]^2)^(1/4)])*Cos[x]*Cot[x]*Sqrt[
Sin[x]*Tan[x]])/(Cos[x]^2)^(3/4)
```

Maple [B] time = 0.108, size = 105, normalized size = 2.

$$
-\frac{1+\cos(x)}{2\sin(x)}\left(\arctan\left(\frac{1}{2}\frac{1}{\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}\right)+\ln\left(-\frac{1}{(\sin(x))^2}\left(2\,(\cos(x))^2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-(\cos(x))^2+2\cos(x)-2\sqrt{-\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(1/2),x)
```

```
[Out] -1/2*(arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+ln(-(2*cos(x)^2*(-cos(x)/(1+
cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)
^2))*(1+cos(x))*(-cos(x)/(1+cos(x))^2)^(1/2)*((1-cos(x)^2)/cos(x))^(1/2)/si
n(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-cos(x) + sec(x)), x)
```

Fricas [A] time = 2.32496, size = 228, normalized size = 4.38

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)+\frac{1}{2}\log\left(\frac{(\cos(x)+1)\sin(x)-2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x))) +
1/2*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos
(x) - 1)*sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-cos(x) + sec(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-cos(x) + sec(x)), x)
```

3.338 $\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}}+\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

```
[Out] -Csc[x]/(2*Sqrt[Sin[x]*Tan[x]]) + (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos
[x]]*Sqrt[Sin[x]*Tan[x]]) + (ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos[x]]*
Sqrt[Sin[x]*Tan[x]])
```

Rubi [A] time = 0.093649, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {4397, 4400, 2597, 2601, 2565, 329, 212, 206, 203}

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}}+\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-3/2),x]
```

```
[Out] -Csc[x]/(2*Sqrt[Sin[x]*Tan[x]]) + (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos
[x]]*Sqrt[Sin[x]*Tan[x]]) + (ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos[x]]*
Sqrt[Sin[x]*Tan[x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx &= \int \frac{1}{(\sin(x)\tan(x))^{3/2}}\,dx \\
&= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} - \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{3}{2}}(x)}\,dx}{4\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} - \frac{\sin(x)\int \frac{\csc(x)}{\sqrt{\cos(x)}}\,dx}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \cos(x)\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{2\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.171909, size = 56, normalized size = 0.78

$$
\frac{\cot(x)\sqrt{\sin(x)\tan(x)}\left(\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right) - 2\sqrt[4]{\cos^2(x)}\csc^2(x) + \tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)\right)}{4\sqrt[4]{\cos^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-3/2),x]
```

```
[Out] (Cot[x]*(ArcTan[(Cos[x]^2)^(1/4)] + ArcTanh[(Cos[x]^2)^(1/4)] - 2*(Cos[x]^2
)^(1/4)*Csc[x]^2)*Sqrt[Sin[x]*Tan[x]])/(4*(Cos[x]^2)^(1/4))
```

Maple [B] time = 0.149, size = 265, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(3/2),x)
```

```
[Out] -1/8*(-1+cos(x))*(8*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)+16*cos(x)*(-cos(x
)/(1+cos(x))^2)^(3/2)+cos(x)^2*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-cos
(x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-co
s(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+8*(-cos(x)/(1+cos(x))^2)^(3/2)+4*cos(
x)*(-cos(x)/(1+cos(x))^2)^(1/2)-4*(-cos(x)/(1+cos(x))^2)^(1/2)-arctan(1/2/(
-cos(x)/(1+cos(x))^2)^(1/2))+ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-c
os(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2))/cos(x)/sin(x)
/(-(-1+cos(x)^2)/cos(x))^(3/2)/(-cos(x)/(1+cos(x))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\cos(x)+\sec(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-3/2), x)
```

Fricas [B] time = 2.47909, size = 371, normalized size = 5.15

$$-\frac{\left(\cos(x)^2-1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-\left(\cos(x)^2-1\right)\log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-4\sqrt{-\cdots}}{8\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x)
- 1)*sin(x)))*sin(x) - (cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) + 2*sqrt(-(c
os(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) - 4*sqrt(-(cos(x
)^2 - 1)/cos(x))*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(3/2),x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(-3/2), x)
```

3.339 $\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{3\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}}+\frac{3\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}}$$

```
[Out] (3*Cot[x])/(16*Sqrt[Sin[x]*Tan[x]]) - (Cot[x]*Csc[x]^2)/(4*Sqrt[Sin[x]*Tan[
x]]) - (3*ArcTan[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]
) + (3*ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rubi [A] time = 0.120849, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2565, 329, 298, 203, 206}

$$-\frac{3\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}}+\frac{3\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-5/2),x]
```

```
[Out] (3*Cot[x])/(16*Sqrt[Sin[x]*Tan[x]]) - (Cot[x]*Csc[x]^2)/(4*Sqrt[Sin[x]*Tan[
x]]) - (3*ArcTan[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]
) + (3*ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx &= \int \frac{1}{(\sin(x)\tan(x))^{5/2}}\,dx\\
&= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}}\\
&= -\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{\left(3\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)\sqrt{\tan(x)}}\,dx}{8\sqrt{\sin(x)\tan(x)}}\\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{\left(3\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\,dx}{32\sqrt{\sin(x)\tan(x)}}\\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{(3\sin(x))\int \sqrt{\cos(x)}\csc(x)\,dx}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}\\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(x)\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}\\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{16\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}\\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{(3\sin(\ldots}{\ldots}\\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{3\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{3\tanh^{-1}\left(\sqrt{\cos(x)}\right)\ldots}{32\sqrt{\cos(x)}\sqrt{\sin(x)\,\mathrm{t}\ldots}}
\end{aligned}$$

Mathematica [A] time = 0.647308, size = 73, normalized size = 0.8

$$-\frac{\cot(x)\sqrt{\sin(x)\tan(x)}\left(3\cos(x)\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right)-3\cos(x)\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)+\cos^2(x)^{3/4}(3\cos(2x)+5)\cot(x)\csc^3(\ldots}{32\cos^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-5/2),x]
```

```
[Out] -(Cot[x]*(3*ArcTan[(Cos[x]^2)^(1/4)]*Cos[x] - 3*ArcTanh[(Cos[x]^2)^(1/4)]*C
os[x] + (Cos[x]^2)^(3/4)*(5 + 3*Cos[2*x])*Cot[x]*Csc[x]^3)*Sqrt[Sin[x]*Tan[
x]])/(32*(Cos[x]^2)^(3/4))
```

Maple [B] time = 0.15, size = 454, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(5/2),x)
```

```
[Out] 1/64*(24*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)+40*cos(x)^2*(-cos(x)/(1+cos(
x))^2)^(3/2)-12*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)-3*cos(x)^3*ln(-(2*cos
(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^
2)^(1/2)-1)/sin(x)^2)-3*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+8
*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)+24*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/
2)+3*cos(x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x
)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)^2*arctan(1/2/(-cos(x
)/(1+cos(x))^2)^(1/2))-8*(-cos(x)/(1+cos(x))^2)^(3/2)-12*cos(x)*(-cos(x)/(1
+cos(x))^2)^(1/2)+3*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos
(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)*arctan(
1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^
(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-3*arcta
n(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)/cos(x)^2/(-(-1+cos(x)^2)/cos(x)
)^(5/2)/(-cos(x)/(1+cos(x))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-5/2), x)
```

Fricas [B] time = 2.47547, size = 452, normalized size = 4.97

$$\frac{3\left(\cos(x)^4-2\cos(x)^2+1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)+3\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{-\frac{\cos(x}{\cos}}}{(\cos(x)-1)\sin(x)}\right.}{64\left(\cos(x)^4-2\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(cos(x)^4 - 2*cos(x)^2 + 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*c
os(x)/((cos(x) - 1)*sin(x)))*sin(x) + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(((c
os(x) + 1)*sin(x) + 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*si
n(x)))*sin(x) - 4*(3*cos(x)^4 + cos(x)^2)*sqrt(-(cos(x)^2 - 1)/cos(x)))/((c
os(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\cos(x)+\sec(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(-5/2), x)
```

3.340 $\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{5\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}}-\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}}-\frac{5\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}}$$

```
[Out] (-5*Csc[x])/(192*Sqrt[Sin[x]*Tan[x]]) + (5*Csc[x]^3)/(48*Sqrt[Sin[x]*Tan[x]
]) - (Cot[x]^2*Csc[x]^3)/(6*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTan[Sqrt[Cos[x]]]*
Sin[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTanh[Sqrt[Cos[x]]]*S
in[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rubi [A] time = 0.140367, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2565, 329, 212, 206, 203}

$$-\frac{5\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}}-\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}}-\frac{5\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-7/2),x]
```

```
[Out] (-5*Csc[x])/(192*Sqrt[Sin[x]*Tan[x]]) + (5*Csc[x]^3)/(48*Sqrt[Sin[x]*Tan[x]
]) - (Cot[x]^2*Csc[x]^3)/(6*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTan[Sqrt[Cos[x]]]*
Sin[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTanh[Sqrt[Cos[x]]]*S
in[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = \int \frac{1}{(\sin(x)\tan(x))^{7/2}}\,dx$$

$$= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}}$$

$$= -\frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{\left(5\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{3}{2}}(x)}\,dx}{12\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{\left(5\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{7}{2}}(x)}\,dx}{96\sqrt{\sin(x)\tan(x)}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{\left(5\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{\sqrt{\ }}{\text{si}}}{128\sqrt{\sin(x)\tan(}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{(5\sin(x))\int \frac{\csc(x)}{\sqrt{\cos(x)}}\,dx}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x))\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1}\right.}{128\sqrt{\cos(x)}\sqrt{\sin}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x))\,\text{Subst}\left(\int \frac{1}{1-x^4}\right.}{64\sqrt{\cos(x)}\sqrt{\sin(}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x))\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{128\sqrt{\cos(x)}\sqrt{\sin(}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{5\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan}}$$

Mathematica [A] time = 0.369063, size = 74, normalized size = 0.67

$$-\frac{\cot(x)\sqrt{\sin(x)\tan(x)}\left(15\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right) + 2\sqrt[4]{\cos^2(x)}\left(32\csc^4(x) - 52\csc^2(x) + 5\right)\csc^2(x) + 15\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right.}{384\sqrt[4]{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-7/2),x]
```

```
[Out] -(Cot[x]*(15*ArcTan[(Cos[x]^2)^(1/4)] + 15*ArcTanh[(Cos[x]^2)^(1/4)] + 2*(C
os[x]^2)^(1/4)*Csc[x]^2*(5 - 52*Csc[x]^2 + 32*Csc[x]^4))*Sqrt[Sin[x]*Tan[x]
```

```
])/(384*(Cos[x]^2)^(1/4))
```

Maple [B] time = 0.167, size = 494, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(7/2),x)
```

```
[Out] 1/768*(56*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)-16*cos(x)^3*(-cos(x)/(1+cos
(x))^2)^(3/2)-15*cos(x)^4*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(
x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^4*arcta
n(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-192*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/
2)+76*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)+30*cos(x)^3*ln(-(2*cos(x)^2*(-c
os(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-
1)/sin(x)^2)-30*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+16*cos(x)
*(-cos(x)/(1+cos(x))^2)^(3/2)-148*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)+136
*(-cos(x)/(1+cos(x))^2)^(3/2)+196*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)-30*co
s(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos
(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+30*cos(x)*arctan(1/2/(-cos(x)/(1+cos(x
))^2)^(1/2))-60*(-cos(x)/(1+cos(x))^2)^(1/2)+15*ln(-(2*cos(x)^2*(-cos(x)/(1
+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x
)^2)-15*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)^3/(-1+cos(x))/cos(
x)^3/(-(-1+cos(x)^2)/cos(x))^(7/2)/(-cos(x)/(1+cos(x))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-7/2), x)
```

Fricas [B] time = 2.57107, size = 529, normalized size = 4.81

$$\frac{15\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)+15\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right.}{768\left(\cos(x)^6-3\cos(x)^4+3\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/768*(15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2
 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x)))*sin(x) + 15*(cos(x)^6 - 3*cos(x
)^4 + 3*cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos
(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) + 4*(5*cos(x)^5 + 42*cos(x)^3 -
15*cos(x))*sqrt(-(cos(x)^2 - 1)/cos(x)))/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)
^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\cos(x)+\sec(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(-7/2), x)
```

3.341 $\int (\sin(x)+\tan(x))^4\,dx$

Optimal. Leaf size=55

$$-\frac{61x}{8}-\frac{4\sin^3(x)}{3}+\frac{\tan^3(x)}{3}+5\tan(x)-2\tanh^{-1}(\sin(x))+\frac{1}{4}\sin(x)\cos^3(x)+\frac{19}{8}\sin(x)\cos(x)+2\tan(x)\sec(x)$$

```
[Out] (-61*x)/8 - 2*ArcTanh[Sin[x]] + (19*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
- (4*Sin[x]^3)/3 + 5*Tan[x] + 2*Sec[x]*Tan[x] + Tan[x]^3/3
```

Rubi [A] time = 0.109614, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {4397, 2709, 2637, 2635, 8, 2633, 3770, 3767, 3768}

$$-\frac{61x}{8}-\frac{4\sin^3(x)}{3}+\frac{\tan^3(x)}{3}+5\tan(x)-2\tanh^{-1}(\sin(x))+\frac{1}{4}\sin(x)\cos^3(x)+\frac{19}{8}\sin(x)\cos(x)+2\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^4,x]
```

```
[Out] (-61*x)/8 - 2*ArcTanh[Sin[x]] + (19*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
- (4*Sin[x]^3)/3 + 5*Tan[x] + 2*Sec[x]*Tan[x] + Tan[x]^3/3
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (\sin(x)+\tan(x))^4\,dx = \int (1+\cos(x))^4\tan^4(x)\,dx$$
$$= \int \left(-10-4\cos(x)+4\cos^2(x)+4\cos^3(x)+\cos^4(x)-4\sec(x)+4\sec^2(x)+4\sec^3(x)+\sec^4(\right.$$
$$= -10x-4\int \cos(x)\,dx+4\int \cos^2(x)\,dx+4\int \cos^3(x)\,dx-4\int \sec(x)\,dx+4\int \sec^2(x)\,dx+4$$
$$= -10x-4\tanh^{-1}(\sin(x))-4\sin(x)+2\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+2\sec(x)\tan(x)+\frac{3}{4}$$
$$= -8x-2\tanh^{-1}(\sin(x))+\frac{19}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)-\frac{4\sin^3(x)}{3}+5\tan(x)+2\sec(x$$
$$= -\frac{61x}{8}-2\tanh^{-1}(\sin(x))+\frac{19}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)-\frac{4\sin^3(x)}{3}+5\tan(x)+2\sec$$

Mathematica [B] time = 0.200432, size = 129, normalized size = 2.35

$$\frac{1}{768}\sec^3(x)\left(1395\sin(x)+672\sin(2x)+1265\sin(3x)+129\sin(5x)+32\sin(6x)+3\sin(7x)-72\cos(x)\left(61x-16\log\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^4,x]
```

```
[Out] (Sec[x]^3*(-72*Cos[x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Cos[x/2]
 + Sin[x/2]]) - 24*Cos[3*x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Co
s[x/2] + Sin[x/2]]) + 1395*Sin[x] + 672*Sin[2*x] + 1265*Sin[3*x] + 129*Sin[
5*x] + 32*Sin[6*x] + 3*Sin[7*x]))/768
```

Maple [A] time = 0.022, size = 66, normalized size = 1.2

$$\frac{23\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)-\frac{61x}{8}+\frac{2(\sin(x))^3}{3}+2\sin(x)-2\ln(\sec(x)+\tan(x))+6\frac{(\sin(x))^5}{\cos(x)}+2\frac{(\sin(x))^5}{(\cos(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^4,x)
```

```
[Out] 23/4*(sin(x)^3+3/2*sin(x))*cos(x)-61/8*x+2/3*sin(x)^3+2*sin(x)-2*ln(sec(x)+
tan(x))+6*sin(x)^5/cos(x)+2*sin(x)^5/cos(x)^2+1/3*tan(x)^3-tan(x)
```

Maxima [A] time = 1.49245, size = 92, normalized size = 1.67

$$-\frac{4}{3}\sin(x)^3+\frac{1}{3}\tan(x)^3-\frac{61}{8}x-\frac{2\sin(x)}{\sin(x)^2-1}+\frac{3\tan(x)}{\tan(x)^2+1}-\log(\sin(x)+1)+\log(\sin(x)-1)+\frac{1}{32}\sin(4x)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] -4/3*sin(x)^3 + 1/3*tan(x)^3 - 61/8*x - 2*sin(x)/(sin(x)^2 - 1) + 3*tan(x)/
(tan(x)^2 + 1) - log(sin(x) + 1) + log(sin(x) - 1) + 1/32*sin(4*x) - 1/4*si
n(2*x) + 5*tan(x)
```

Fricas [A] time = 2.2071, size = 255, normalized size = 4.64

$$-\frac{183\,x\cos(x)^3+24\cos(x)^3\log(\sin(x)+1)-24\cos(x)^3\log(-\sin(x)+1)-\left(6\cos(x)^6+32\cos(x)^5+57\cos(x)^4\right.}{24\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(183*x*cos(x)^3 + 24*cos(x)^3*log(sin(x) + 1) - 24*cos(x)^3*log(-sin(
x) + 1) - (6*cos(x)^6 + 32*cos(x)^5 + 57*cos(x)^4 - 32*cos(x)^3 + 112*cos(x
)^2 + 48*cos(x) + 8)*sin(x))/cos(x)^3
```

Sympy [A] time = 8.13968, size = 90, normalized size = 1.64

$$-\frac{61x}{8}+\log(\sin(x)-1)-\log(\sin(x)+1)-\frac{4\sin^3(x)}{3}+\frac{6\sin^3(x)}{\cos(x)}+\frac{\sin^3(x)}{3\cos^3(x)}+9\sin(x)\cos(x)-\frac{\sin(x)}{\cos(x)}-\frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**4,x)
```

```
[Out] -61*x/8 + log(sin(x) - 1) - log(sin(x) + 1) - 4*sin(x)**3/3 + 6*sin(x)**3/c
os(x) + sin(x)**3/(3*cos(x)**3) + 9*sin(x)*cos(x) - sin(x)/cos(x) - sin(2*x
```

```
)/4 + sin(4*x)/32 - 4*sin(x)/(2*sin(x)**2 - 2)
```

Giac [B] time = 8.73601, size = 1856, normalized size = 33.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] 1/24*(8*tan(1/2*x)^10*tan(x)^5 - 183*x*tan(1/2*x)^10*tan(x)^2 - 24*log(2*(t
an(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^10*tan(x)^2
+ 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)
^10*tan(x)^2 + 128*tan(1/2*x)^10*tan(x)^3 + 8*tan(1/2*x)^8*tan(x)^5 - 183*x
*tan(1/2*x)^10 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^10 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)
^2 + 1))*tan(1/2*x)^10 + 180*tan(1/2*x)^10*tan(x) - 183*x*tan(1/2*x)^8*tan(
x)^2 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1
/2*x)^8*tan(x)^2 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2
 + 1))*tan(1/2*x)^8*tan(x)^2 + 96*tan(1/2*x)^9*tan(x)^2 + 128*tan(1/2*x)^8*
tan(x)^3 - 16*tan(1/2*x)^6*tan(x)^5 - 183*x*tan(1/2*x)^8 - 24*log(2*(tan(1/
2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 24*log(2*(tan
(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 96*tan(1/2
*x)^9 + 180*tan(1/2*x)^8*tan(x) + 366*x*tan(1/2*x)^6*tan(x)^2 + 48*log(2*(t
an(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6*tan(x)^2 -
 48*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^
6*tan(x)^2 + 128*tan(1/2*x)^7*tan(x)^2 - 256*tan(1/2*x)^6*tan(x)^3 - 16*tan
(1/2*x)^4*tan(x)^5 + 366*x*tan(1/2*x)^6 + 48*log(2*(tan(1/2*x)^2 + 2*tan(1/
2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 48*log(2*(tan(1/2*x)^2 - 2*tan
(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 + 128*tan(1/2*x)^7 - 360*tan(
1/2*x)^6*tan(x) + 366*x*tan(1/2*x)^4*tan(x)^2 + 48*log(2*(tan(1/2*x)^2 + 2*
tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 - 48*log(2*(tan(1
/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 + 108
8*tan(1/2*x)^5*tan(x)^2 - 256*tan(1/2*x)^4*tan(x)^3 + 8*tan(1/2*x)^2*tan(x)
^5 + 366*x*tan(1/2*x)^4 + 48*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^4 - 48*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(ta
n(1/2*x)^2 + 1))*tan(1/2*x)^4 + 1088*tan(1/2*x)^5 - 360*tan(1/2*x)^4*tan(x)
 - 183*x*tan(1/2*x)^2*tan(x)^2 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)
/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)^2 + 24*log(2*(tan(1/2*x)^2 - 2*tan
(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)^2 + 128*tan(1/2*x)^3*t
an(x)^2 + 128*tan(1/2*x)^2*tan(x)^3 + 8*tan(x)^5 - 183*x*tan(1/2*x)^2 - 24*
log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 +
```

```
24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2
 + 128*tan(1/2*x)^3 + 180*tan(1/2*x)^2*tan(x) - 183*x*tan(x)^2 - 24*log(2*(
tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(x)^2 + 24*log(2*(t
an(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(x)^2 + 96*tan(1/2*x
)*tan(x)^2 + 128*tan(x)^3 - 183*x - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1)) + 96*tan(1/2*x) + 180*tan(x))/(tan(1/2*x)^10*tan(x)^2 + tan(1
/2*x)^10 + tan(1/2*x)^8*tan(x)^2 + tan(1/2*x)^8 - 2*tan(1/2*x)^6*tan(x)^2 -
 2*tan(1/2*x)^6 - 2*tan(1/2*x)^4*tan(x)^2 - 2*tan(1/2*x)^4 + tan(1/2*x)^2*t
an(x)^2 + tan(1/2*x)^2 + tan(x)^2 + 1) + 1/32*sin(4*x)
```

3.342 $\int (\sin(x) + \tan(x))^3 \, dx$

Optimal. Leaf size=38

$$\frac{\cos^3(x)}{3} + \frac{3\cos^2(x)}{2} + 2\cos(x) + \frac{\sec^2(x)}{2} + 3\sec(x) - 2\log(\cos(x))$$

```
[Out] 2*Cos[x] + (3*Cos[x]^2)/2 + Cos[x]^3/3 - 2*Log[Cos[x]] + 3*Sec[x] + Sec[x]^
2/2
```

Rubi [A] time = 0.049694, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4397, 2707, 75}

$$\frac{\cos^3(x)}{3} + \frac{3\cos^2(x)}{2} + 2\cos(x) + \frac{\sec^2(x)}{2} + 3\sec(x) - 2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^3,x]
```

```
[Out] 2*Cos[x] + (3*Cos[x]^2)/2 + Cos[x]^3/3 - 2*Log[Cos[x]] + 3*Sec[x] + Sec[x]^
2/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$
\begin{aligned}
\int (\sin(x)+\tan(x))^3\,dx &= \int (1+\cos(x))^3\tan^3(x)\,dx\\
&= -\operatorname{Subst}\left(\int \frac{(1-x)(1+x)^4}{x^3}\,dx, x, \cos(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(-2+\frac{1}{x^3}+\frac{3}{x^2}+\frac{2}{x}-3x-x^2\right)dx, x, \cos(x)\right)\\
&= 2\cos(x)+\frac{3\cos^2(x)}{2}+\frac{\cos^3(x)}{3}-2\log(\cos(x))+3\sec(x)+\frac{\sec^2(x)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.041343, size = 40, normalized size = 1.05

$$\frac{9\cos(x)}{4}+\frac{3}{4}\cos(2x)+\frac{1}{12}\cos(3x)+\frac{\sec^2(x)}{2}+3\sec(x)-2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^3,x]
```

```
[Out] (9*Cos[x])/4 + (3*Cos[2*x])/4 + Cos[3*x]/12 - 2*Log[Cos[x]] + 3*Sec[x] + Se
c[x]^2/2
```

Maple [A] time = 0.02, size = 39, normalized size = 1.

$$\frac{\left(16+8\,(\sin(x))^2\right)\cos(x)}{3}-\frac{3\,(\sin(x))^2}{2}-2\,\ln(\cos(x))+3\,\frac{(\sin(x))^4}{\cos(x)}+\frac{(\tan(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^3,x)
```

```
[Out] 8/3*(2+sin(x)^2)*cos(x)-3/2*sin(x)^2-2*ln(cos(x))+3*sin(x)^4/cos(x)+1/2*tan
(x)^2
```

Maxima [A] time = 0.984777, size = 57, normalized size = 1.5

$$\frac{1}{3}\,\cos(x)^3-\frac{3}{2}\,\sin(x)^2-\frac{1}{2\left(\sin(x)^2-1\right)}+\frac{3}{\cos(x)}+2\,\cos(x)-\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - 3/2*sin(x)^2 - 1/2/(sin(x)^2 - 1) + 3/cos(x) + 2*cos(x) - lo
g(sin(x)^2 - 1)
```

Fricas [A] time = 2.23008, size = 151, normalized size = 3.97

$$\frac{4\cos(x)^5 + 18\cos(x)^4 + 24\cos(x)^3 - 24\cos(x)^2\log(-\cos(x)) - 9\cos(x)^2 + 36\cos(x) + 6}{12\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(x)^5 + 18*cos(x)^4 + 24*cos(x)^3 - 24*cos(x)^2*log(-cos(x)) - 9
*cos(x)^2 + 36*cos(x) + 6)/cos(x)^2
```

Sympy [A] time = 6.66531, size = 46, normalized size = 1.21

$$-3\log(\cos(x)) - \frac{\log\left(\sec^2(x)\right)}{2} + \frac{\cos^3(x)}{3} + \frac{3\cos^2(x)}{2} + 2\cos(x) + \frac{\sec^2(x)}{2} + \frac{3}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**3,x)
```

```
[Out] -3*log(cos(x)) - log(sec(x)**2)/2 + cos(x)**3/3 + 3*cos(x)**2/2 + 2*cos(x)
+ sec(x)**2/2 + 3/cos(x)
```

Giac [B] time = 1.65285, size = 234, normalized size = 6.16

$$\frac{\tan\left(\frac{1}{2}x\right)^4\tan(x)^4 - 2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\left(\frac{1}{2}x\right)^4\tan(x)^2 - 10\tan\left(\frac{1}{2}x\right)^4\tan(x)^2 - 2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\left(\frac{1}{2}x\right)^4 - 8\tan}{2\left(\tan\left(\frac{1}{2}x\right)^4\tan(x)^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*x)^4*tan(x)^4 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4*tan(x)^2
- 10*tan(1/2*x)^4*tan(x)^2 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4 - 8*tan(1
/2*x)^4 - 3*tan(1/2*x)^2*tan(x)^2 - tan(x)^4 + 2*log(4/(tan(x)^2 + 1))*tan(
x)^2 - 3*tan(1/2*x)^2 - 11*tan(x)^2 + 2*log(4/(tan(x)^2 + 1)) - 13)/(tan(1/
2*x)^4*tan(x)^2 + tan(1/2*x)^4 - tan(x)^2 - 1) + 1/12*cos(3*x)
```

3.343 $\int (\sin(x)+\tan(x))^2\,dx$

Optimal. Leaf size=25

$$-\frac{x}{2}-2\sin(x)+\tan(x)+2\tanh^{-1}(\sin(x))-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] -x/2 + 2*ArcTanh[Sin[x]] - 2*Sin[x] - (Cos[x]*Sin[x])/2 + Tan[x]
```

Rubi [A] time = 0.0625847, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4397, 2709, 2637, 2635, 8, 3770, 3767}

$$-\frac{x}{2}-2\sin(x)+\tan(x)+2\tanh^{-1}(\sin(x))-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^2,x]
```

```
[Out] -x/2 + 2*ArcTanh[Sin[x]] - 2*Sin[x] - (Cos[x]*Sin[x])/2 + Tan[x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int (\sin(x)+\tan(x))^2\,dx &= \int (1+\cos(x))^2\tan^2(x)\,dx\\
&= \int \left(-2\cos(x)-\cos^2(x)+2\sec(x)+\sec^2(x)\right)\,dx\\
&= -(2\int \cos(x)\,dx)+2\int \sec(x)\,dx-\int \cos^2(x)\,dx+\int \sec^2(x)\,dx\\
&= 2\tanh^{-1}(\sin(x))-2\sin(x)-\frac{1}{2}\cos(x)\sin(x)-\frac{\int 1\,dx}{2}-\text{Subst}(\int 1\,dx,x,-\tan(x))\\
&= -\frac{x}{2}+2\tanh^{-1}(\sin(x))-2\sin(x)-\frac{1}{2}\cos(x)\sin(x)+\tan(x)
\end{aligned}
$$

Mathematica [B] time = 0.0956259, size = 60, normalized size = 2.4

$$-\frac{x}{2}-2\sin(x)+\frac{7\tan(x)}{8}-\frac{1}{8}\sin(3x)\sec(x)-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^2,x]
```

```
[Out] -x/2 - 2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x] -
 (Sec[x]*Sin[3*x])/8 + (7*Tan[x])/8
```

Maple [A] time = 0.016, size = 25, normalized size = 1.

$$-\frac{\cos(x)\sin(x)}{2}-\frac{x}{2}-2\sin(x)+2\ln(\sec(x)+\tan(x))+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^2,x)
```

```
[Out] -1/2*cos(x)*sin(x)-1/2*x-2*sin(x)+2*ln(sec(x)+tan(x))+tan(x)
```

Maxima [A] time = 1.47788, size = 38, normalized size = 1.52

$$-\frac{1}{2}x+\log(\sin(x)+1)-\log(\sin(x)-1)-\frac{1}{4}\sin(2x)-2\sin(x)+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + log(sin(x) + 1) - log(sin(x) - 1) - 1/4*sin(2*x) - 2*sin(x) + tan(
x)
```

Fricas [B] time = 2.11352, size = 154, normalized size = 6.16

$$-\frac{x\cos(x)-2\cos(x)\log(\sin(x)+1)+2\cos(x)\log(-\sin(x)+1)+\left(\cos(x)^2+4\cos(x)-2\right)\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x) - 2*cos(x)*log(sin(x) + 1) + 2*cos(x)*log(-sin(x) + 1) + (co
s(x)^2 + 4*cos(x) - 2)*sin(x))/cos(x)
```

Sympy [A] time = 1.84119, size = 31, normalized size = 1.24

$$-\frac{x}{2}-\log\left(\sin\left(x\right)-1\right)+\log\left(\sin\left(x\right)+1\right)-2\sin\left(x\right)-\frac{\sin\left(2x\right)}{4}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**2,x)
```

```
[Out] -x/2 - log(sin(x) - 1) + log(sin(x) + 1) - 2*sin(x) - sin(2*x)/4 + tan(x)
```

Giac [B] time = 1.24517, size = 239, normalized size = 9.56

$$\frac{1}{2}x-\frac{x\tan\left(\frac{1}{2}x\right)^{2}-\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^{2}+2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)\tan\left(\frac{1}{2}x\right)^{2}+\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^{2}-2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)\tan\left(\frac{1}{2}x\right)^{2}-\tan\left(\frac{1}{2}x\right)^{2}\tan}{\tan\left(\frac{1}{2}x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - (x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 - tan(1/2*x)^2*tan(x) + x - log(2*(tan(1/2*x)^2 + 2
*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 4*tan(1/2*x) - tan(x))/(tan(1/2*x)^2 + 1) - 1/4*s
in(2*x)
```

3.344 $\int (\sin(x) + \tan(x))\, dx$

Optimal. Leaf size=10

$$-\cos(x) - \log(\cos(x))$$

```
[Out] -Cos[x] - Log[Cos[x]]
```

Rubi [A] time = 0.0049923, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2638, 3475}

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x] + Tan[x],x]
```

```
[Out] -Cos[x] - Log[Cos[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (\sin(x) + \tan(x))\, dx &= \int \sin(x)\, dx + \int \tan(x)\, dx \\ &= -\cos(x) - \log(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.0029032, size = 10, normalized size = 1.

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x] + Tan[x],x]
```

```
[Out] -Cos[x] - Log[Cos[x]]
```

Maple [A] time = 0.002, size = 11, normalized size = 1.1

$$-\cos(x) - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)+tan(x),x)
```

```
[Out] -cos(x)-ln(cos(x))
```

Maxima [A] time = 0.982502, size = 11, normalized size = 1.1

$$-\cos(x) + \log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="maxima")
```

```
[Out] -cos(x) + log(sec(x))
```

Fricas [A] time = 2.26663, size = 32, normalized size = 3.2

$$-\cos(x) - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="fricas")
```

```
[Out] -cos(x) - log(-cos(x))
```

Sympy [A] time = 0.065387, size = 8, normalized size = 0.8

$$-\log(\cos(x)) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x)
```

```
[Out] -log(cos(x)) - cos(x)
```

Giac [A] time = 1.12834, size = 15, normalized size = 1.5

$$-\cos(x) - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="giac")
```

```
[Out] -cos(x) - log(abs(cos(x)))
```

3.345 $\int \frac{1}{\sin(x)+\tan(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rubi [A] time = 0.0583856, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\cos(x)}\,dx\\ &= -\int \cot^2(x)\csc(x)\,dx + \int \cot(x)\csc^2(x)\,dx\\ &= \frac{1}{2}\cot(x)\csc(x) + \frac{1}{2}\int \csc(x)\,dx - \text{Subst}\left(\int x\,dx, x, \csc(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x)) + \frac{1}{2}\cot(x)\csc(x) - \frac{\csc^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0150762, size = 35, normalized size = 1.46

$$-\frac{1}{4}\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

Maple [A] time = 0.037, size = 24, normalized size = 1.

$$-\frac{1}{2+2\cos(x)}-\frac{\ln(1+\cos(x))}{4}+\frac{\ln(-1+\cos(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x)),x)
```

```
[Out] -1/2/(1+cos(x))-1/4*ln(1+cos(x))+1/4*ln(-1+cos(x))
```

Maxima [A] time = 0.979031, size = 34, normalized size = 1.42

$$-\frac{\sin(x)^2}{4\,(\cos(x)+1)^2}+\frac{1}{2}\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.19473, size = 132, normalized size = 5.5

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)+2}{4\,(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) + 2)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sin(x)+\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x)
```

```
[Out] Integral(1/(sin(x) + tan(x)), x)
```

Giac [A] time = 1.12371, size = 38, normalized size = 1.58

$$\frac{\cos(x)-1}{4(\cos(x)+1)}+\frac{1}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="giac")
```

```
[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.346 $\int \frac{1}{(\sin(x)+\tan(x))^2} \, dx$

Optimal. Leaf size=33

$$-\frac{2}{5}\cot^5(x)-\frac{\cot^3(x)}{3}+\frac{2\csc^5(x)}{5}-\frac{2\csc^3(x)}{3}$$

```
[Out] -Cot[x]^3/3 - (2*Cot[x]^5)/5 - (2*Csc[x]^3)/3 + (2*Csc[x]^5)/5
```

Rubi [A] time = 0.126604, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2711, 2607, 30, 2606, 14}

$$-\frac{2}{5}\cot^5(x)-\frac{\cot^3(x)}{3}+\frac{2\csc^5(x)}{5}-\frac{2\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-2),x]
```

```
[Out] -Cot[x]^3/3 - (2*Cot[x]^5)/5 - (2*Csc[x]^3)/3 + (2*Csc[x]^5)/5
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx &= \int \frac{\cot^2(x)}{(1+\cos(x))^2}\,dx\\
&= \int \left(\cot^4(x)\csc^2(x) - 2\cot^3(x)\csc^3(x) + \cot^2(x)\csc^4(x)\right) dx\\
&= -\left(2\int \cot^3(x)\csc^3(x)\,dx\right) + \int \cot^4(x)\csc^2(x)\,dx + \int \cot^2(x)\csc^4(x)\,dx\\
&= 2\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \csc(x)\right) + \mathrm{Subst}\left(\int x^4\,dx, x, -\cot(x)\right) + \mathrm{Subst}\left(\int x^2\left(1+x^2\right.\right.\\
&= -\frac{1}{5}\cot^5(x) + 2\,\mathrm{Subst}\left(\int\left(-x^2+x^4\right)dx, x, \csc(x)\right) + \mathrm{Subst}\left(\int\left(x^2+x^4\right)dx, x, -\cot(x)\right)\\
&= -\frac{1}{3}\cot^3(x) - \frac{2\cot^5(x)}{5} - \frac{2\csc^3(x)}{3} + \frac{2\csc^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.0158117, size = 57, normalized size = 1.73

$$-\frac{7}{120}\tan\left(\frac{x}{2}\right) - \frac{1}{8}\cot\left(\frac{x}{2}\right) + \frac{1}{40}\tan\left(\frac{x}{2}\right)\sec^4\left(\frac{x}{2}\right) - \frac{11}{120}\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-2),x]
```

```
[Out] -Cot[x/2]/8 - (7*Tan[x/2])/120 - (11*Sec[x/2]^2*Tan[x/2])/120 + (Sec[x/2]^4
*Tan[x/2])/40
```

Maple [A] time = 0.041, size = 32, normalized size = 1.

$$\frac{1}{40}\left(\tan\left(\frac{x}{2}\right)\right)^{5}-\frac{1}{24}\left(\tan\left(\frac{x}{2}\right)\right)^{3}-\frac{1}{8}\tan\left(\frac{x}{2}\right)-\frac{1}{8}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^2,x)
```

```
[Out] 1/40*tan(1/2*x)^5-1/24*tan(1/2*x)^3-1/8*tan(1/2*x)-1/8/tan(1/2*x)
```

Maxima [A] time = 0.996161, size = 61, normalized size = 1.85

$$-\frac{\cos(x)+1}{8\sin(x)}-\frac{\sin(x)}{8(\cos(x)+1)}-\frac{\sin(x)^{3}}{24(\cos(x)+1)^{3}}+\frac{\sin(x)^{5}}{40(\cos(x)+1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(cos(x) + 1)/sin(x) - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x)
+ 1)^3 + 1/40*sin(x)^5/(cos(x) + 1)^5
```

Fricas [A] time = 2.13688, size = 109, normalized size = 3.3

$$-\frac{\cos(x)^{3}+2\cos(x)^{2}+8\cos(x)+4}{15\left(\cos(x)^{2}+2\cos(x)+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(x)^3 + 2*cos(x)^2 + 8*cos(x) + 4)/((cos(x)^2 + 2*cos(x) + 1)*sin
(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sin(x) + \tan(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**2,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-2), x)
```

Giac [A] time = 1.15399, size = 42, normalized size = 1.27

$$\frac{1}{40}\,\tan\left(\frac{1}{2}\,x\right)^5 - \frac{1}{24}\,\tan\left(\frac{1}{2}\,x\right)^3 - \frac{1}{8\,\tan\left(\frac{1}{2}\,x\right)} - \frac{1}{8}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] 1/40*tan(1/2*x)^5 - 1/24*tan(1/2*x)^3 - 1/8/tan(1/2*x) - 1/8*tan(1/2*x)
```

3.347 $\int \frac{1}{(\sin(x)+\tan(x))^3} dx$

Optimal. Leaf size=60

$$-\frac{1}{32(1-\cos(x))}-\frac{1}{16(\cos(x)+1)}-\frac{3}{32(\cos(x)+1)^2}+\frac{1}{6(\cos(x)+1)^3}-\frac{1}{16(\cos(x)+1)^4}+\frac{1}{32}\tanh^{-1}(\cos(x))$$

```
[Out] ArcTanh[Cos[x]]/32 - 1/(32*(1 - Cos[x])) - 1/(16*(1 + Cos[x])^4) + 1/(6*(1
+ Cos[x])^3) - 3/(32*(1 + Cos[x])^2) - 1/(16*(1 + Cos[x]))
```

Rubi [A] time = 0.0716931, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4397, 2707, 88, 207}

$$-\frac{1}{32(1-\cos(x))}-\frac{1}{16(\cos(x)+1)}-\frac{3}{32(\cos(x)+1)^2}+\frac{1}{6(\cos(x)+1)^3}-\frac{1}{16(\cos(x)+1)^4}+\frac{1}{32}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-3),x]
```

```
[Out] ArcTanh[Cos[x]]/32 - 1/(32*(1 - Cos[x])) - 1/(16*(1 + Cos[x])^4) + 1/(6*(1
+ Cos[x])^3) - 3/(32*(1 + Cos[x])^2) - 1/(16*(1 + Cos[x]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = \int \frac{\cot^3(x)}{(1+\cos(x))^3}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{x^3}{(1-x)^2(1+x)^5}\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{32(-1+x)^2} - \frac{1}{4(1+x)^5} + \frac{1}{2(1+x)^4} - \frac{3}{16(1+x)^3} - \frac{1}{16(1+x)^2} + \frac{1}{32\left(-1+x^2\right)}\right.\right.$$
$$= -\frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{1}{16(1+\cos(x))} - \frac{1}{32}$$
$$= \frac{1}{32}\tanh^{-1}(\cos(x)) - \frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} -$$

Mathematica [A] time = 0.0178364, size = 83, normalized size = 1.38

$$-\frac{1}{64}\csc^2\left(\frac{x}{2}\right) - \frac{1}{256}\sec^8\left(\frac{x}{2}\right) + \frac{1}{48}\sec^6\left(\frac{x}{2}\right) - \frac{3}{128}\sec^4\left(\frac{x}{2}\right) - \frac{1}{32}\sec^2\left(\frac{x}{2}\right) - \frac{1}{32}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{32}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-3),x]
```

```
[Out] -Csc[x/2]^2/64 + Log[Cos[x/2]]/32 - Log[Sin[x/2]]/32 - Sec[x/2]^2/32 - (3*S
ec[x/2]^4)/128 + Sec[x/2]^6/48 - Sec[x/2]^8/256
```

Maple [A] time = 0.052, size = 56, normalized size = 0.9

$$-\frac{1}{16\,(1+\cos(x))^4} + \frac{1}{6\,(1+\cos(x))^3} - \frac{3}{32\,(1+\cos(x))^2} - \frac{1}{16+16\,\cos(x)} + \frac{\ln(1+\cos(x))}{64} + \frac{1}{-32+32\,\cos(x)} - \frac{\ln(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^3,x)
```

```
[Out] -1/16/(1+cos(x))^4+1/6/(1+cos(x))^3-3/32/(1+cos(x))^2-1/16/(1+cos(x))+1/64*
ln(1+cos(x))+1/32/(-1+cos(x))-1/64*ln(-1+cos(x))
```

Maxima [A] time = 1.00228, size = 99, normalized size = 1.65

$$-\frac{(\cos(x)+1)^2}{64\sin(x)^2}-\frac{\sin(x)^2}{32(\cos(x)+1)^2}+\frac{\sin(x)^4}{64(\cos(x)+1)^4}+\frac{\sin(x)^6}{192(\cos(x)+1)^6}-\frac{\sin(x)^8}{256(\cos(x)+1)^8}-\frac{1}{32}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] -1/64*(cos(x) + 1)^2/sin(x)^2 - 1/32*sin(x)^2/(cos(x) + 1)^2 + 1/64*sin(x)^
4/(cos(x) + 1)^4 + 1/192*sin(x)^6/(cos(x) + 1)^6 - 1/256*sin(x)^8/(cos(x) +
 1)^8 - 1/32*log(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 2.24772, size = 424, normalized size = 7.07

$$-\frac{6\cos(x)^4+18\cos(x)^3-50\cos(x)^2-3\left(\cos(x)^5+3\cos(x)^4+2\cos(x)^3-2\cos(x)^2-3\cos(x)-1\right)\log\left(\frac{1}{2}\cos(x)\right.}{192\left(\cos(x)^5+3\cos(x)^4+2\cos(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(6*cos(x)^4 + 18*cos(x)^3 - 50*cos(x)^2 - 3*(cos(x)^5 + 3*cos(x)^4 +
 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^
5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(-1/2*cos(x) +
1/2) - 54*cos(x) - 16)/(cos(x)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3
*cos(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(\sin(x)+\tan(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**3,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-3), x)
```

Giac [B] time = 1.13901, size = 128, normalized size = 2.13

$$\frac{\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)(\cos(x)+1)}{64\,(\cos(x)-1)}+\frac{\cos(x)-1}{32\,(\cos(x)+1)}+\frac{(\cos(x)-1)^2}{64\,(\cos(x)+1)^2}-\frac{(\cos(x)-1)^3}{192\,(\cos(x)+1)^3}-\frac{(\cos(x)-1)^4}{256\,(\cos(x)+1)^4}-\frac{1}{64}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/64*((cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) + 1/32*(cos(
x) - 1)/(cos(x) + 1) + 1/64*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/192*(cos(x) -
 1)^3/(cos(x) + 1)^3 - 1/256*(cos(x) - 1)^4/(cos(x) + 1)^4 - 1/64*log(-(cos
(x) - 1)/(cos(x) + 1))
```

3.348 $\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx$

Optimal. Leaf size=65

$$-\frac{8}{11}\cot^{11}(x)-\frac{16\cot^9(x)}{9}-\frac{9\cot^7(x)}{7}-\frac{\cot^5(x)}{5}+\frac{8\csc^{11}(x)}{11}-\frac{20\csc^9(x)}{9}+\frac{16\csc^7(x)}{7}-\frac{4\csc^5(x)}{5}$$

```
[Out] -Cot[x]^5/5 - (9*Cot[x]^7)/7 - (16*Cot[x]^9)/9 - (8*Cot[x]^11)/11 - (4*Csc[
x]^5)/5 + (16*Csc[x]^7)/7 - (20*Csc[x]^9)/9 + (8*Csc[x]^11)/11
```

Rubi [A] time = 0.210677, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2711, 2607, 14, 2606, 270}

$$-\frac{8}{11}\cot^{11}(x)-\frac{16\cot^9(x)}{9}-\frac{9\cot^7(x)}{7}-\frac{\cot^5(x)}{5}+\frac{8\csc^{11}(x)}{11}-\frac{20\csc^9(x)}{9}+\frac{16\csc^7(x)}{7}-\frac{4\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-4),x]
```

```
[Out] -Cot[x]^5/5 - (9*Cot[x]^7)/7 - (16*Cot[x]^9)/9 - (8*Cot[x]^11)/11 - (4*Csc[
x]^5)/5 + (16*Csc[x]^7)/7 - (20*Csc[x]^9)/9 + (8*Csc[x]^11)/11
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx &= \int \frac{\cot^4(x)}{(1+\cos(x))^4}\,dx\\ &= \int \left(\cot^8(x)\csc^4(x)-4\cot^7(x)\csc^5(x)+6\cot^6(x)\csc^6(x)-4\cot^5(x)\csc^7(x)+\cot^4(x)\csc^8(\right.\\ &= -\left(4\int \cot^7(x)\csc^5(x)\,dx\right)-4\int \cot^5(x)\csc^7(x)\,dx+6\int \cot^6(x)\csc^6(x)\,dx+\int \cot^8(x)\csc\\ &= 4\,\text{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx,x,\csc(x)\right)+4\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^3\,dx,x,\csc(x)\right)+6\,\text{Subst}\\ &= 4\,\text{Subst}\left(\int \left(-x^4+3x^6-3x^8+x^{10}\right)dx,x,\csc(x)\right)+4\,\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)dx,x,\csc(\right.\\ &= -\frac{1}{5}\cot^5(x)-\frac{9\cot^7(x)}{7}-\frac{16\cot^9(x)}{9}-\frac{8\cot^{11}(x)}{11}-\frac{4\csc^5(x)}{5}+\frac{16\csc^7(x)}{7}-\frac{20\csc^9(x)}{9}+\frac{8}{}\end{aligned}$$

Mathematica [A] time = 0.0207937, size = 129, normalized size = 1.98

$$-\frac{2749\tan\left(\frac{x}{2}\right)}{110880}+\frac{1}{96}\cot\left(\frac{x}{2}\right)-\frac{1}{384}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)+\frac{\tan\left(\frac{x}{2}\right)\sec^{10}\left(\frac{x}{2}\right)}{1408}-\frac{7\tan\left(\frac{x}{2}\right)\sec^8\left(\frac{x}{2}\right)}{1584}+\frac{641\tan\left(\frac{x}{2}\right)\sec^6\left(\frac{x}{2}\right)}{88704}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-4),x]
```

```
[Out] Cot[x/2]/96 - (Cot[x/2]*Csc[x/2]^2)/384 - (2749*Tan[x/2])/110880 - (2033*Se
c[x/2]^2*Tan[x/2])/443520 + (179*Sec[x/2]^4*Tan[x/2])/73920 + (641*Sec[x/2]
^6*Tan[x/2])/88704 - (7*Sec[x/2]^8*Tan[x/2])/1584 + (Sec[x/2]^10*Tan[x/2])/
1408
```

Maple [A] time = 0.054, size = 64, normalized size = 1.

$$\frac{1}{1408}\left(\tan\left(\frac{x}{2}\right)\right)^{11}-\frac{1}{1152}\left(\tan\left(\frac{x}{2}\right)\right)^{9}-\frac{3}{896}\left(\tan\left(\frac{x}{2}\right)\right)^{7}+\frac{3}{640}\left(\tan\left(\frac{x}{2}\right)\right)^{5}+\frac{1}{128}\left(\tan\left(\frac{x}{2}\right)\right)^{3}-\frac{3}{128}\tan\left(\frac{x}{2}\right)+\frac{1}{128}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^4,x)
```

```
[Out] 1/1408*tan(1/2*x)^11-1/1152*tan(1/2*x)^9-3/896*tan(1/2*x)^7+3/640*tan(1/2*x
)^5+1/128*tan(1/2*x)^3-3/128*tan(1/2*x)+1/128/tan(1/2*x)-1/384/tan(1/2*x)^3
```

Maxima [A] time = 1.00157, size = 131, normalized size = 2.02

$$\frac{\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)^3}{384\sin(x)^3}-\frac{3\sin(x)}{128(\cos(x)+1)}+\frac{\sin(x)^3}{128(\cos(x)+1)^3}+\frac{3\sin(x)^5}{640(\cos(x)+1)^5}-\frac{3\sin(x)^7}{896(\cos(x)+1)^7}-\frac{}{1152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 1/384*(3*sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)^3/sin(x)^3 - 3/128*sin(x
)/(cos(x) + 1) + 1/128*sin(x)^3/(cos(x) + 1)^3 + 3/640*sin(x)^5/(cos(x) + 1
)^5 - 3/896*sin(x)^7/(cos(x) + 1)^7 - 1/1152*sin(x)^9/(cos(x) + 1)^9 + 1/14
08*sin(x)^11/(cos(x) + 1)^11
```

Fricas [A] time = 2.18283, size = 255, normalized size = 3.92

$$\frac{122\cos(x)^7+488\cos(x)^6+549\cos(x)^5-244\cos(x)^4-64\cos(x)^3+144\cos(x)^2+128\cos(x)+32}{3465\left(\cos(x)^6+4\cos(x)^5+5\cos(x)^4-5\cos(x)^2-4\cos(x)-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3465*(122*cos(x)^7 + 488*cos(x)^6 + 549*cos(x)^5 - 244*cos(x)^4 - 64*cos(
x)^3 + 144*cos(x)^2 + 128*cos(x) + 32)/((cos(x)^6 + 4*cos(x)^5 + 5*cos(x)^4
 - 5*cos(x)^2 - 4*cos(x) - 1)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sin(x) + \tan(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**4,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-4), x)
```

Giac [A] time = 1.14752, size = 88, normalized size = 1.35

$$\frac{1}{1408}\tan\left(\frac{1}{2}x\right)^{11} - \frac{1}{1152}\tan\left(\frac{1}{2}x\right)^{9} - \frac{3}{896}\tan\left(\frac{1}{2}x\right)^{7} + \frac{3}{640}\tan\left(\frac{1}{2}x\right)^{5} + \frac{1}{128}\tan\left(\frac{1}{2}x\right)^{3} + \frac{3\tan\left(\frac{1}{2}x\right)^{2} - 1}{384\tan\left(\frac{1}{2}x\right)^{3}} - \frac{3}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] 1/1408*tan(1/2*x)^11 - 1/1152*tan(1/2*x)^9 - 3/896*tan(1/2*x)^7 + 3/640*tan
(1/2*x)^5 + 1/128*tan(1/2*x)^3 + 1/384*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 -
3/128*tan(1/2*x)
```

3.349 $\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=74

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (c*C*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] - (b*C*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rubi [A] time = 0.0627072, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3137, 3074, 206}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*C*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] - (b*C*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx &= \frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx\\
&= \frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\,\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)\\
&= \frac{cCx}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}
\end{aligned}$$

Mathematica [A] time = 0.186413, size = 68, normalized size = 0.92

$$\frac{2A\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{C(cx-b\log(b\cos(x)+c\sin(x)))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (C*(c*x
- b*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

Maple [B] time = 0.059, size = 150, normalized size = 2.

$$-\frac{bC}{b^2+c^2}\ln\left(b\left(\tan\left(\frac{x}{2}\right)\right)^2-2c\tan(x/2)-b\right)+2\frac{Ab^2}{\left(b^2+c^2\right)^{3/2}}\text{Artanh}\left(1/2\frac{2b\tan(x/2)-2c}{\sqrt{b^2+c^2}}\right)+2\frac{Ac^2}{\left(b^2+c^2\right)^{3/2}}\text{Artanh}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] -1/(b^2+c^2)*b*C*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arct
anh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arcta
nh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*c^2+C/(b^2+c^2)*b*ln(1+tan(1
/2*x)^2)+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3812, size = 358, normalized size = 4.84

$$\frac{2\,Ccx - Cb\log\left(2\,bc\cos\left(x\right)\sin\left(x\right) + \left(b^{2} - c^{2}\right)\cos\left(x\right)^{2} + c^{2}\right) + \sqrt{b^{2} + c^{2}}A\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^{2}-c^{2}\right)\cos(x)^{2}-2\,b^{2}-c^{2}+2\,\sqrt{b^{2}\cdots}}{2\,bc\cos(x)\sin(x)+\left(b^{2}-c^{2}\right)\cos(x)^{2}\cdots}\right.}{2\left(b^{2} + c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*c*x - C*b*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2
- c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^
2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

[Out] Exception raised: AttributeError

Giac [A] time = 1.33327, size = 177, normalized size = 2.39

$$\frac{Ccx}{b^2+c^2}+\frac{Cb\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}-\frac{Cb\log\left(\left|b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right|\right)}{b^2+c^2}-\frac{A\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] C*c*x/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - C*b*log(abs(b*t
an(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) -
 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sq
rt(b^2 + c^2)
```

3.350 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=75

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

```
[Out] -((c*C*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.0601355, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3154, 3074, 206}

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((c*C*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \frac{bC-Ac\cos(x)+Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(cC)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= \frac{bC-Ac\cos(x)+Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(cC)\,\mathrm{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)}{b^2+c^2}$$

$$= -\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.297198, size = 82, normalized size = 1.09

$$\frac{A\left(b^2+c^2\right)\sin(x)+b^2C}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2cC\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*c*C*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^2
*C + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Maple [A] time = 0.087, size = 108, normalized size = 1.4

$$2\,\frac{1}{b\left(\tan(x/2)\right)^2-2\,c\tan(x/2)-b}\left(-\frac{\left(Ab^2+Ac^2+Cbc\right)\tan(x/2)}{b\left(b^2+c^2\right)}-\frac{bC}{b^2+c^2}\right)+2\,\frac{Cc}{\left(b^2+c^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{2\,b\tan(x/2)-}{\sqrt{b^2+c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*b^2+A*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)-C*b/(b^2+c^2))/(b*tan(1/2*x)
^2-2*c*tan(1/2*x)-b)+2*C*c/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)
/(b^2+c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29159, size = 487, normalized size = 6.49

$$\frac{2\,Cb^3 + 2\,Cbc^2 + \left(Cbc\cos\left(x\right) + Cc^2\sin\left(x\right)\right)\sqrt{b^2+c^2}\log\left(-\frac{2\,bc\cos(x)\sin(x) + \left(b^2-c^2\right)\cos(x)^2 - 2\,b^2 - c^2 + 2\,\sqrt{b^2+c^2}(c\cos(x) - b\sin(x))}{2\,bc\cos(x)\sin(x) + \left(b^2-c^2\right)\cos(x)^2 + c^2}\right) - 2}{2\left(\left(b^5 + 2\,b^3c^2 + bc^4\right)\cos\left(x\right) + \left(b^4c + 2\,b^2c^3 + c^5\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3 + 2*C*b*c^2 + (C*b*c*cos(x) + C*c^2*sin(x))*sqrt(b^2 + c^2)*lo
g(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 +
 c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 +
c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*
b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25145, size = 176, normalized size = 2.35

$$-\frac{Cc\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Cbc\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)+Cb^2\right)}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -C*c*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) -
 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + C*b*c*
tan(1/2*x) + A*c^2*tan(1/2*x) + C*b^2)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c
*tan(1/2*x) - b))
```

3.351 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=116

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(c^2*C*Cos[x] - b*c*C*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.110397, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3157, 3153, 3074, 206}

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```


```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(c^2*C*Cos[x] - b*c*C*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```


Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2cC+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$

$$= \frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}+\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{2\left(b^2+c^2\right)}$$

$$= \frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}-\frac{A\,\mathrm{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,d\ldots\right.}{2\left(b\ldots\right.}$$

$$= -\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c^2C\cos(x)-bcC\sin(\ldots}{\left(b^2+c^2\right)^2(b\cos(x)+c\,\mathrm{si}\ldots}$$

Mathematica [C] time = 0.363374, size = 132, normalized size = 1.14

$$\frac{\left(b^2+c^2\right)\left(Ab^2\sin(x)-Abc\cos(x)+bC(b+c\sin(2x))+2c^2C\sin^2(x)\right)+2Ab\sqrt{b^2+c^2}(b\cos(x)+c\sin(x))^2\tanh^{-1}\left(\frac{b\tan\ldots}{\sqrt{b\ldots}}\right.}{2b(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (2*A*b*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x]
 + c*Sin[x])^2 + (b^2 + c^2)*(-(A*b*c*Cos[x]) + A*b^2*Sin[x] + 2*c^2*C*Sin[
x]^2 + b*C*(b + c*Sin[2*x])))/(2*b*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Si
n[x])^2)
```

Maple [A] time = 0.104, size = 177, normalized size = 1.5

$$-2\frac{1}{\left(b\left(\tan\left(x/2\right)\right)^{2}-2c\tan\left(x/2\right)-b\right)^{2}}\left(-1/2\frac{A\left(b^{2}+2c^{2}\right)\left(\tan\left(x/2\right)\right)^{3}}{\left(b^{2}+c^{2}\right)b}-1/2\frac{\left(Ab^{2}c-2Ac^{3}+2Cb^{3}+2Cbc^{2}\right)\left(\tan\left(x/2\right.}{\left(b^{2}+c^{2}\right)b^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*A*(b^2+2*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^2*c-2*A*c^3+2*C*b^
3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*A*(b^2-2*c^2)/(b^2+c^2)/b*tan(1
/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3
/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49414, size = 659, normalized size = 5.68

$$-\frac{8Cbc^{2}\cos\left(x\right)^{2}-2Cb^{3}-6Cbc^{2}-\left(2Abc\cos\left(x\right)\sin\left(x\right)+Ac^{2}+\left(Ab^{2}-Ac^{2}\right)\cos\left(x\right)^{2}\right)\sqrt{b^{2}+c^{2}}\log\left(-\frac{2bc\cos(x)\sin(x)+(}{2b}\right.}{4\left(b^{4}c^{2}+2b^{2}c^{4}+c^{6}+\left(b^{6}+b^{4}c^{2}-b^{2}c^{4}-c^{6}\right)c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*C*b*c^2*cos(x)^2 - 2*C*b^3 - 6*C*b*c^2 - (2*A*b*c*cos(x)*sin(x) + A
*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(
x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c
^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(C*b^2*c - C*c^3)*cos(x))*sin(x))/(b^4*
c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c
 + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31973, size = 269, normalized size = 2.32

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\,\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3+2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3+2\,Cb^3\tan\left(\frac{1}{2}\,x\right)^2+Ab^2c\tan\left(\frac{1}{2}\,x\right)^2+2\,Cbc^2\tan}{\left(b^4+b^2c^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 + 2*A
*b*c^2*tan(1/2*x)^3 + 2*C*b^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*C*b*c
^2*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1
/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2)
```

3.352 $\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=73

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (b*B*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] + (B*c*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rubi [A] time = 0.0528934, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3138, 3074, 206}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (b*B*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] + (B*c*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx\\ &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\,\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)\\ &= \frac{bBx}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}\end{aligned}$$

Mathematica [A] time = 0.137927, size = 67, normalized size = 0.92

$$\frac{2A\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{B(c\log(b\cos(x)+c\sin(x))+bx)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (B*(b*x
+ c*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

Maple [B] time = 0.055, size = 150, normalized size = 2.1

$$\frac{Bc}{b^2+c^2}\ln\left(b\left(\tan\left(\frac{x}{2}\right)\right)^2-2\,c\tan(x/2)-b\right)+2\,\frac{Ab^2}{\left(b^2+c^2\right)^{3/2}}\text{Artanh}\left(1/2\,\frac{2\,b\tan(x/2)-2\,c}{\sqrt{b^2+c^2}}\right)+2\,\frac{Ac^2}{\left(b^2+c^2\right)^{3/2}}\text{Artanh}\left(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)*B*c*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arcta
nh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arctan
h(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*c^2-B/(b^2+c^2)*c*ln(1+tan(1/
2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31764, size = 358, normalized size = 4.9

$$\frac{2\,Bbx + Bc\log\left(2\,bc\cos\left(x\right)\sin\left(x\right) + \left(b^2 - c^2\right)\cos\left(x\right)^2 + c^2\right) + \sqrt{b^2 + c^2}A\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+}}{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2}\right.}{2\left(b^2 + c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*B*b*x + B*c*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2
- c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^
2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.33087, size = 177, normalized size = 2.42

$$\frac{Bbx}{b^2+c^2}-\frac{Bc\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}+\frac{Bc\log\left(\left|b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right|\right)}{b^2+c^2}-\frac{A\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] B*b*x/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + B*c*log(abs(b*t
an(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) -
 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sq
rt(b^2 + c^2)
```

3.353 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=76

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{(b^2+c^2)(b\cos(x)+c\sin(x))}-\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

```
[Out] -((b*B*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.0533076, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3155, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{(b^2+c^2)(b\cos(x)+c\sin(x))}-\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((b*B*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(bB)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB)\,\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx, x, c\cos(x)-b\sin(x)\right)}{b^2+c^2}$$

$$= -\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.225564, size = 82, normalized size = 1.08

$$\frac{A\left(b^2+c^2\right)\sin(x)-bBc}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2bB\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*b*B*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (-(b
*B*c) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Maple [A] time = 0.084, size = 109, normalized size = 1.4

$$2\frac{1}{b(\tan(x/2))^2-2c\tan(x/2)-b}\left(-\frac{\left(Ab^2+Ac^2-Bc^2\right)\tan(x/2)}{b\left(b^2+c^2\right)}+\frac{Bc}{b^2+c^2}\right)+2\frac{bB}{\left(b^2+c^2\right)^{3/2}}\text{Artanh}\left(1/2\,\frac{2\,b\tan(x/2)-}{\sqrt{b^2+c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*b^2+A*c^2-B*c^2)/b/(b^2+c^2)*tan(1/2*x)+B*c/(b^2+c^2))/(b*tan(1/2*x)
^2-2*c*tan(1/2*x)-b)+2*b*B/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)
/(b^2+c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34865, size = 489, normalized size = 6.43

$$-\frac{2\,Bb^2c+2\,Bc^3-\left(Bb^2\cos\left(x\right)+Bbc\sin\left(x\right)\right)\sqrt{b^2+c^2}\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2}\right)+}{2\left(\left(b^5+2\,b^3c^2+bc^4\right)\cos\left(x\right)+\left(b^4c+2\,b^2c^3+c^5\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*b^2*c + 2*B*c^3 - (B*b^2*cos(x) + B*b*c*sin(x))*sqrt(b^2 + c^2)*l
og(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2
+ c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 +
 c^2)) + 2*(A*b^2*c + A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2
*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33013, size = 178, normalized size = 2.34

$$-\frac{Bb\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)-Bbc\right)}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -B*b*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) -
 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + A*c^2*
tan(1/2*x) - B*c^2*tan(1/2*x) - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c
*tan(1/2*x) - b))
```

3.354 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=116

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(b*B*c*Cos[x] - b^2*B*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.10807, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3158, 3153, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(b*B*c*Cos[x] - b^2*B*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2bB+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$

$$= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}+\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}\,d\cdots}{2\left(b^2+c^2\right)}$$

$$= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}-A\,\mathrm{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\cdots\right)\cdots 2\left(\cdots\right.$$

$$= -\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(\cdots}{\left(b^2+c^2\right)^2(b\cos(x)+c\,\mathrm{si}\cdots}$$

Mathematica [C] time = 0.313424, size = 118, normalized size = 1.02

$$\frac{\left(b^2+c^2\right)(b\sin(x)(A+2B\cos(x))-Ac\cos(x)-Bc\cos(2x))+2A\sqrt{b^2+c^2}(b\cos(x)+c\sin(x))^2\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{2(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x] +
 c*Sin[x])^2 + (b^2 + c^2)*(-(A*c*Cos[x]) - B*c*Cos[2*x] + b*(A + 2*B*Cos[x
])*Sin[x]))/(2*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Sin[x])^2)
```

Maple [A] time = 0.099, size = 204, normalized size = 1.8

$$-2\,\frac{1}{\left(b\left(\tan(x/2)\right)^2-2\,c\tan(x/2)-b\right)^2}\left(-1/2\,\frac{\left(Ab^2+2\,Ac^2-2\,Bb^2-2\,Bc^2\right)\left(\tan(x/2)\right)^3}{\left(b^2+c^2\right)b}-1/2\,\frac{c\left(Ab^2-2\,Ac^2+2\,Bb^2+\right.}{\left(b^2+c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*c*(A*
b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*(A*b^2-2*A*c^2+
2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-
2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2
+c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.43362, size = 659, normalized size = 5.68

$$-\frac{8\,Bb^2c\cos(x)^2-2\,Bb^2c+2\,Bc^3-\left(2\,Abc\cos(x)\sin(x)+Ac^2+\left(Ab^2-Ac^2\right)\cos(x)^2\right)\sqrt{b^2+c^2}\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b}{2\,b}\right.}{4\left(b^4c^2+2\,b^2c^4+c^6+\left(b^6+b^4c^2-b^2c^4-c^6\right)\text{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*B*b^2*c*cos(x)^2 - 2*B*b^2*c + 2*B*c^3 - (2*A*b*c*cos(x)*sin(x) + A
*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(
x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c
^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(B*b^3 - B*b*c^2)*cos(x))*sin(x))/(b^4*
c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c
 + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39639, size = 331, normalized size = 2.85

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\,\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3-2\,Bb^3\tan\left(\frac{1}{2}\,x\right)^3+2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3-2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)^3+Ab^2c\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B
*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + A*b^2
*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*t
an(1/2*x)^2 + A*b^3*tan(1/2*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x)
```

```
+ 2*B*b*c^2*tan(1/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*ta
n(1/2*x) - b)^2)
```

3.355 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$

Optimal. Leaf size=246

$$\frac{35b\left(b^2+c^2\right)^{3/2}\sin(d+ex)}{8e}-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\right.}{4e}$$

```
[Out] (35*(b^2 + c^2)^2*x)/8 - (35*c*(b^2 + c^2)^(3/2)*Cos[d + e*x])/(8*e) + (35*
b*(b^2 + c^2)^(3/2)*Sin[d + e*x])/(8*e) - (35*(b^2 + c^2)*(c*Cos[d + e*x] -
 b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(24*e
) - (7*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] +
 b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d +
e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e)
```

Rubi [A] time = 0.169314, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3113, 2637, 2638}

$$\frac{35b\left(b^2+c^2\right)^{3/2}\sin(d+ex)}{8e}-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\right.}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```


```
[Out] (35*(b^2 + c^2)^2*x)/8 - (35*c*(b^2 + c^2)^(3/2)*Cos[d + e*x])/(8*e) + (35*
b*(b^2 + c^2)^(3/2)*Sin[d + e*x])/(8*e) - (35*(b^2 + c^2)*(c*Cos[d + e*x] -
 b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(24*e
) - (7*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] +
 b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d +
e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e)
```


Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}{4e}$$

$$= -\frac{7\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex\right.}{12e}$$

$$= -\frac{35\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+\right.}{24e}$$

$$= \frac{35}{8}\left(b^2+c^2\right)^2x-\frac{35\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2-\right.}{24e}$$

$$= \frac{35}{8}\left(b^2+c^2\right)^2x-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}+\frac{35b\left(b^2+c^2\right)^{3/2}\sin}{8e}$$

Mathematica [C] time = 1.48369, size = 238, normalized size = 0.97

$$\frac{420\left(b^2+c^2\right)^2(d+ex)+672b(b-ic)(b+ic)\sqrt{b^2+c^2}\sin(d+ex)+32b\left(b^2-3c^2\right)\sqrt{b^2+c^2}\sin(3(d+ex))+168\left(b^4-c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (420*(b^2 + c^2)^2*(d + e*x) - 672*(b - I*c)*(b + I*c)*c*Sqrt[b^2 + c^2]*Co
s[d + e*x] - 336*b*c*(b^2 + c^2)*Cos[2*(d + e*x)] + 32*c*(-3*b^2 + c^2)*Sqr
t[b^2 + c^2]*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d + e*x)] + 672*b
*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*Sin[d + e*x] + 168*(b^4 - c^4)*Sin[2*(
d + e*x)] + 32*b*(b^2 - 3*c^2)*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 3*(b^4 -
```

```
6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

Maple [B] time = 0.124, size = 514, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x)
```

```
[Out] 1/e*(2*b^2*c^2*(e*x+d)+c^4*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3
/8*e*x+3/8*d)+6*c^4*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+b^4*(1/4*(co
s(e*x+d)^3+3/2*cos(e*x+d))*sin(e*x+d)+3/8*e*x+3/8*d)+6*b^4*(1/2*sin(e*x+d)*
cos(e*x+d)+1/2*e*x+1/2*d)+b^4*(e*x+d)+c^4*(e*x+d)+4/3*(b^2+c^2)^(1/2)*b^3*(
2+cos(e*x+d)^2)*sin(e*x+d)+6*b^2*c^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2
*d)+4*(b^2+c^2)^(1/2)*b^3*sin(e*x+d)-4/3*(b^2+c^2)^(1/2)*c^3*(2+sin(e*x+d)^
2)*cos(e*x+d)+6*b^2*c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-4*(b^2+c
^2)^(1/2)*c^3*cos(e*x+d)-4*(b^2+c^2)^(1/2)*b^2*c*cos(e*x+d)^3+4*(b^2+c^2)^(
1/2)*b*c^2*sin(e*x+d)^3-cos(e*x+d)^4*b^3*c+6*b^2*c^2*(-1/4*sin(e*x+d)*cos(e
*x+d)^3+1/8*sin(e*x+d)*cos(e*x+d)+1/8*e*x+1/8*d)+b*c^3*sin(e*x+d)^4-6*cos(e
*x+d)^2*b^3*c-6*cos(e*x+d)^2*b*c^3+4*(b^2+c^2)^(1/2)*b*c^2*sin(e*x+d)-4*(b^
2+c^2)^(1/2)*b^2*c*cos(e*x+d))
```

Maxima [A] time = 1.01463, size = 478, normalized size = 1.94

$$-\frac{b^3 c \cos(ex+d)^4}{e} + \frac{bc^3 \sin(ex+d)^4}{e} + \frac{(12\,ex + 12\,d + \sin(4\,ex + 4\,d) + 8\,\sin(2\,ex + 2\,d))b^4}{32\,e} + \frac{3\,(4\,ex + 4\,d - \sin(4\,ex}{16\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="maxim
a")
```

```
[Out] -b^3*c*cos(e*x + d)^4/e + b*c^3*sin(e*x + d)^4/e + 1/32*(12*e*x + 12*d + si
n(4*e*x + 4*d) + 8*sin(2*e*x + 2*d))*b^4/e + 3/16*(4*e*x + 4*d - sin(4*e*x
+ 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x +
2*d))*c^4/e + (b^2 + c^2)^2*x - 4*(b^2 + c^2)^(3/2)*(c*cos(e*x + d)/e - b*s
in(e*x + d)/e) - 3/2*(4*b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2
*d))*b^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e)*(b^2 + c^2) - 4/3*(3*b
```

```
^2*c*cos(e*x + d)^3/e - 3*b*c^2*sin(e*x + d)^3/e + (sin(e*x + d)^3 - 3*sin(
e*x + d))*b^3/e - (cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e)*sqrt(b^2 + c^2)
```

Fricas [A] time = 2.32549, size = 509, normalized size = 2.07

$$-\frac{24\left(b^3c-bc^3\right)\cos\left(ex+d\right)^4-105\left(b^4+2\,b^2c^2+c^4\right)ex+48\left(3\,b^3c+4\,bc^3\right)\cos\left(ex+d\right)^2-3\left(2\left(b^4-6\,b^2c^2+c^4\right)\cos\left(e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="frica
s")
```

```
[Out] -1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 - 105*(b^4 + 2*b^2*c^2 + c^4)*e*x
+ 48*(3*b^3*c + 4*b*c^3)*cos(e*x + d)^2 - 3*(2*(b^4 - 6*b^2*c^2 + c^4)*cos(
e*x + d)^3 + (27*b^4 + 6*b^2*c^2 - 29*c^4)*cos(e*x + d))*sin(e*x + d) + 32*
((3*b^2*c - c^3)*cos(e*x + d)^3 + 3*(b^2*c + 2*c^3)*cos(e*x + d) - (5*b^3 +
 6*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^2))/e
```

Sympy [A] time = 4.22427, size = 882, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

```
[Out] Piecewise((3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*
x)**2/4 + 3*b**4*x*sin(d + e*x)**2 + 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*x*
cos(d + e*x)**2 + b**4*x + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5*b*
*4*sin(d + e*x)*cos(d + e*x)**3/(8*e) + 3*b**4*sin(d + e*x)*cos(d + e*x)/e
+ 6*b**3*c*sin(d + e*x)**2/e - b**3*c*cos(d + e*x)**4/e + 8*b**3*sqrt(b**2
+ c**2)*sin(d + e*x)**3/(3*e) + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)*cos(d
 + e*x)**2/e + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*b**2*c**2*x*sin(
d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/2 + 6*b**2*c*
*2*x*sin(d + e*x)**2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 6*b**2*c**2*x*cos(
d + e*x)**2 + 2*b**2*c**2*x + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e*x)/(4*e
) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) - 4*b**2*c*sqrt(b**2 + c
**2)*cos(d + e*x)**3/e - 4*b**2*c*sqrt(b**2 + c**2)*cos(d + e*x)/e - 2*b*c*
```

```
*3*sin(d + e*x)**2*cos(d + e*x)**2/e + 6*b*c**3*sin(d + e*x)**2/e - b*c**3*
cos(d + e*x)**4/e + 4*b*c**2*sqrt(b**2 + c**2)*sin(d + e*x)**3/e + 4*b*c**2
*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*c**4*x*sin(d + e*x)**4/8 + 3*c**4*x*s
in(d + e*x)**2*cos(d + e*x)**2/4 + 3*c**4*x*sin(d + e*x)**2 + 3*c**4*x*cos(
d + e*x)**4/8 + 3*c**4*x*cos(d + e*x)**2 + c**4*x - 5*c**4*sin(d + e*x)**3*
cos(d + e*x)/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) - 3*c**4*sin
(d + e*x)*cos(d + e*x)/e - 4*c**3*sqrt(b**2 + c**2)*sin(d + e*x)**2*cos(d +
 e*x)/e - 8*c**3*sqrt(b**2 + c**2)*cos(d + e*x)**3/(3*e) - 4*c**3*sqrt(b**2
 + c**2)*cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b**2 + c
**2))**4, True))
```

Giac [A] time = 1.24363, size = 387, normalized size = 1.57

$$-\frac{1}{8}\left(b^3c - bc^3\right)\cos\left(4\,xe + 4\,d\right)e^{(-1)} - \frac{1}{3}\left(3\,\sqrt{b^2 + c^2}b^2c - \sqrt{b^2 + c^2}c^3\right)\cos\left(3\,xe + 3\,d\right)e^{(-1)} - \frac{7}{2}\left(b^3c + bc^3\right)\cos\left(2\,xe + 2\,d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="giac"
)
```

```
[Out] -1/8*(b^3*c - b*c^3)*cos(4*x*e + 4*d)*e^(-1) - 1/3*(3*sqrt(b^2 + c^2)*b^2*c
 - sqrt(b^2 + c^2)*c^3)*cos(3*x*e + 3*d)*e^(-1) - 7/2*(b^3*c + b*c^3)*cos(2
*x*e + 2*d)*e^(-1) - 7*(sqrt(b^2 + c^2)*b^2*c + sqrt(b^2 + c^2)*c^3)*cos(x*
e + d)*e^(-1) + 1/32*(b^4 - 6*b^2*c^2 + c^4)*e^(-1)*sin(4*x*e + 4*d) + 1/3*
(sqrt(b^2 + c^2)*b^3 - 3*sqrt(b^2 + c^2)*b*c^2)*e^(-1)*sin(3*x*e + 3*d) + 7
/4*(b^4 - c^4)*e^(-1)*sin(2*x*e + 2*d) + 7*(sqrt(b^2 + c^2)*b^3 + sqrt(b^2
+ c^2)*b*c^2)*e^(-1)*sin(x*e + d) + 35/8*(b^4 + 2*b^2*c^2 + c^4)*x
```

3.356 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3\,dx$

Optimal. Leaf size=178

$$\frac{5b\left(b^2+c^2\right)\sin(d+ex)}{2e}-\frac{5c\left(b^2+c^2\right)\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}{3e}$$

```
[Out] (5*(b^2 + c^2)^(3/2)*x)/2 - (5*c*(b^2 + c^2)*Cos[d + e*x])/(2*e) + (5*b*(b^
2 + c^2)*Sin[d + e*x])/(2*e) - (5*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d
 + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c*C
os[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d +
 e*x])^2)/(3*e)
```

Rubi [A] time = 0.101832, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3113, 2637, 2638}

$$\frac{5b\left(b^2+c^2\right)\sin(d+ex)}{2e}-\frac{5c\left(b^2+c^2\right)\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (5*(b^2 + c^2)^(3/2)*x)/2 - (5*c*(b^2 + c^2)*Cos[d + e*x])/(2*e) + (5*b*(b^
2 + c^2)*Sin[d + e*x])/(2*e) - (5*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d
 + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c*C
os[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d +
 e*x])^2)/(3*e)
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}{3e}$$

$$= -\frac{5\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)\right.}{6e}$$

$$= \frac{5}{2}\left(b^2+c^2\right)^{3/2}x-\frac{5\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}\right.}{6e}$$

$$= \frac{5}{2}\left(b^2+c^2\right)^{3/2}x-\frac{5c\left(b^2+c^2\right)\cos(d+ex)}{2e}+\frac{5b\left(b^2+c^2\right)\sin(d+ex)}{2e}$$

Mathematica [C] time = 0.661051, size = 163, normalized size = 0.92

$$\frac{30(b-ic)(b+ic)\sqrt{b^2+c^2}(d+ex)+45b\left(b^2+c^2\right)\sin(d+ex)+9\left(b^2-c^2\right)\sqrt{b^2+c^2}\sin(2(d+ex))+b\left(b^2-3c^2\right)\sin(3(d}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (30*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*(d + e*x) - 45*c*(b^2 + c^2)*Cos[d
+ e*x] - 18*b*c*Sqrt[b^2 + c^2]*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(
d + e*x)] + 45*b*(b^2 + c^2)*Sin[d + e*x] + 9*(b^2 - c^2)*Sqrt[b^2 + c^2]*S
in[2*(d + e*x)] + b*(b^2 - 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

Maple [A] time = 0.085, size = 250, normalized size = 1.4

$$\frac{1}{e}\left(\frac{b^3\left(2+(\cos(ex+d))^2\right)\sin(ex+d)}{3}-(\cos(ex+d))^3b^2c+3\sqrt{b^2+c^2}b^2\left(1/2\sin(ex+d)\cos(ex+d)+1/2\,ex+d/2\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x)
```

```
[Out] 1/e*(1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-cos(e*x+d)^3*b^2*c+3*(b^2+c^2)^(1/
2)*b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+b*c^2*sin(e*x+d)^3-3*(b^2+
c^2)^(1/2)*b*c*cos(e*x+d)^2+3*sin(e*x+d)*b^3+3*b*c^2*sin(e*x+d)-1/3*c^3*(2+
sin(e*x+d)^2)*cos(e*x+d)+3*(b^2+c^2)^(1/2)*c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+
1/2*e*x+1/2*d)-3*cos(e*x+d)*b^2*c-3*cos(e*x+d)*c^3+(b^2+c^2)^(1/2)*b^2*(e*x
+d)+(b^2+c^2)^(1/2)*c^2*(e*x+d))
```

Maxima [A] time = 1.00097, size = 279, normalized size = 1.57

$$-\frac{b^2c\cos(ex+d)^3}{e}+\frac{bc^2\sin(ex+d)^3}{e}-\frac{\left(\sin(ex+d)^3-3\sin(ex+d)\right)b^3}{3e}+\frac{\left(\cos(ex+d)^3-3\cos(ex+d)\right)c^3}{3e}+\left(b^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="maxim
a")
```

```
[Out] -b^2*c*cos(e*x + d)^3/e + b*c^2*sin(e*x + d)^3/e - 1/3*(sin(e*x + d)^3 - 3*
sin(e*x + d))*b^3/e + 1/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e + (b^2 +
c^2)^(3/2)*x - 3*(b^2 + c^2)*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/4*(4
*b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2
*d - sin(2*e*x + 2*d))*c^2/e)*sqrt(b^2 + c^2)
```

Fricas [A] time = 2.25459, size = 342, normalized size = 1.92

$$-\frac{2\left(3b^2c-c^3\right)\cos(ex+d)^3+6\left(3b^2c+4c^3\right)\cos(ex+d)-2\left(11b^3+12bc^2+\left(b^3-3bc^2\right)\cos(ex+d)^2\right)\sin(ex+d)+}{6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="frica
s")
```

```
[Out] -1/6*(2*(3*b^2*c - c^3)*cos(e*x + d)^3 + 6*(3*b^2*c + 4*c^3)*cos(e*x + d) -
 2*(11*b^3 + 12*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 3*(6
```

```
*b*c*cos(e*x + d)^2 - 5*(b^2 + c^2)*e*x - 3*(b^2 - c^2)*cos(e*x + d)*sin(e*
x + d))*sqrt(b^2 + c^2))/e
```

Sympy [A] time = 1.881, size = 415, normalized size = 2.33

$$\begin{cases} \frac{2b^3\sin^3(d+ex)}{3e} + \frac{b^3\sin(d+ex)\cos^2(d+ex)}{e} + \frac{3b^3\sin(d+ex)}{e} - \frac{b^2c\cos^3(d+ex)}{e} - \frac{3b^2c\cos(d+ex)}{e} + \frac{3b^2x\sqrt{b^2+c^2}\sin^2(d+ex)}{2} + \frac{3b^2x\sqrt{b^2+c^2}\cos^2(d+ex)}{2} \\ x\left(b\cos(d) + c\sin(d) + \sqrt{b^2+c^2}\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

```
[Out] Piecewise((2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**2
/e + 3*b**3*sin(d + e*x)/e - b**2*c*cos(d + e*x)**3/e - 3*b**2*c*cos(d + e*
x)/e + 3*b**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*b**2*x*sqrt(b**2 +
c**2)*cos(d + e*x)**2/2 + b**2*x*sqrt(b**2 + c**2) + 3*b**2*sqrt(b**2 + c**
2)*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c**2*sin(d + e*x)**3/e + 3*b*c**2*si
n(d + e*x)/e + 3*b*c*sqrt(b**2 + c**2)*sin(d + e*x)**2/e - c**3*sin(d + e*x
)**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e) - 3*c**3*cos(d + e*x)/e
+ 3*c**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*c**2*x*sqrt(b**2 + c**2)
*cos(d + e*x)**2/2 + c**2*x*sqrt(b**2 + c**2) - 3*c**2*sqrt(b**2 + c**2)*si
n(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b*
*2 + c**2))**3, True))
```

Giac [A] time = 1.15752, size = 269, normalized size = 1.51

$$-\frac{3}{2}\sqrt{b^2+c^2}bc\cos(2xe+2d)e^{(-1)} - \frac{1}{12}\left(3b^2c - c^3\right)\cos(3xe+3d)e^{(-1)} - \frac{15}{4}\left(b^2c + c^3\right)\cos(xe+d)e^{(-1)} + \frac{1}{12}\left(b^3 - 3bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="giac"
)
```

```
[Out] -3/2*sqrt(b^2 + c^2)*b*c*cos(2*x*e + 2*d)*e^(-1) - 1/12*(3*b^2*c - c^3)*cos
(3*x*e + 3*d)*e^(-1) - 15/4*(b^2*c + c^3)*cos(x*e + d)*e^(-1) + 1/12*(b^3 -
 3*b*c^2)*e^(-1)*sin(3*x*e + 3*d) + 3/4*(sqrt(b^2 + c^2)*b^2 - sqrt(b^2 + c
^2)*c^2)*e^(-1)*sin(2*x*e + 2*d) + 15/4*(b^3 + b*c^2)*e^(-1)*sin(x*e + d) +
 (b^2 + c^2)^(3/2)*x + 3/2*(sqrt(b^2 + c^2)*b^2 + sqrt(b^2 + c^2)*c^2)*x
```

3.357 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2 dx$

Optimal. Leaf size=116

$$\frac{3b\sqrt{b^2+c^2}\sin(d+ex)}{2e}-\frac{3c\sqrt{b^2+c^2}\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}{2e}$$

```
[Out] (3*(b^2 + c^2)*x)/2 - (3*c*Sqrt[b^2 + c^2]*Cos[d + e*x])/(2*e) + (3*b*Sqrt[
b^2 + c^2]*Sin[d + e*x])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b
^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(2*e)
```

Rubi [A] time = 0.0584102, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3113, 2637, 2638}

$$\frac{3b\sqrt{b^2+c^2}\sin(d+ex)}{2e}-\frac{3c\sqrt{b^2+c^2}\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (3*(b^2 + c^2)*x)/2 - (3*c*Sqrt[b^2 + c^2]*Cos[d + e*x])/(2*e) + (3*b*Sqrt[
b^2 + c^2]*Sin[d + e*x])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b
^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(2*e)
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2 dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}{2e}$$

$$= \frac{3}{2}\left(b^2+c^2\right)x - \frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+\right.}{2e}$$

$$= \frac{3}{2}\left(b^2+c^2\right)x - \frac{3c\sqrt{b^2+c^2}\cos(d+ex)}{2e} + \frac{3b\sqrt{b^2+c^2}\sin(d+ex)}{2e} - $$

Mathematica [A] time = 0.222119, size = 111, normalized size = 0.96

$$\frac{8b\sqrt{b^2+c^2}\sin(d+ex)-8c\sqrt{b^2+c^2}\cos(d+ex)+b^2\sin(2(d+ex))+6b^2d+6b^2ex-2bc\cos(2(d+ex))-c^2\sin(2(d+e}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (6*b^2*d + 6*c^2*d + 6*b^2*e*x + 6*c^2*e*x - 8*c*Sqrt[b^2 + c^2]*Cos[d + e*
x] - 2*b*c*Cos[2*(d + e*x)] + 8*b*Sqrt[b^2 + c^2]*Sin[d + e*x] + b^2*Sin[2*
(d + e*x)] - c^2*Sin[2*(d + e*x)])/(4*e)
```

Maple [A] time = 0.061, size = 124, normalized size = 1.1

$$\frac{1}{e}\left(b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-(\cos(ex+d))^2bc+c^2\left(-\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2\sqrt{b^2+c^2}b\,\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x)
```

```
[Out] 1/e*(b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*b*c+c^2*(-1
/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+2*(b^2+c^2)^(1/2)*b*sin(e*x+d)-2*(b
```

```
^2+c^2)^(1/2)*c*cos(e*x+d)+b^2*(e*x+d)+c^2*(e*x+d))
```

Maxima [A] time = 0.986031, size = 153, normalized size = 1.32

$$b^2x + c^2x - \frac{bc\cos(ex+d)^2}{e} + \frac{(2\,ex + 2\,d + \sin(2\,ex + 2\,d))b^2}{4\,e} + \frac{(2\,ex + 2\,d - \sin(2\,ex + 2\,d))c^2}{4\,e} - 2\,\sqrt{b^2 + c^2}\left(\frac{c\cos(ex}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="maxim
a")
```

```
[Out] b^2*x + c^2*x - b*c*cos(e*x + d)^2/e + 1/4*(2*e*x + 2*d + sin(2*e*x + 2*d))
*b^2/e + 1/4*(2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 2*sqrt(b^2 + c^2)*(c*
cos(e*x + d)/e - b*sin(e*x + d)/e)
```

Fricas [A] time = 2.19603, size = 196, normalized size = 1.69

$$-\frac{2\,bc\cos(ex+d)^2 - 3\left(b^2 + c^2\right)ex - \left(b^2 - c^2\right)\cos(ex+d)\sin(ex+d) + 4\,\sqrt{b^2 + c^2}(c\cos(ex+d) - b\sin(ex+d))}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="frica
s")
```

```
[Out] -1/2*(2*b*c*cos(e*x + d)^2 - 3*(b^2 + c^2)*e*x - (b^2 - c^2)*cos(e*x + d)*s
in(e*x + d) + 4*sqrt(b^2 + c^2)*(c*cos(e*x + d) - b*sin(e*x + d)))/e
```

Sympy [A] time = 0.617558, size = 192, normalized size = 1.66

$$\begin{cases} \frac{b^2x\sin^2(d+ex)}{2} + \frac{b^2x\cos^2(d+ex)}{2} + b^2x + \frac{b^2\sin(d+ex)\cos(d+ex)}{2e} + \frac{bc\sin^2(d+ex)}{e} + \frac{2b\sqrt{b^2+c^2}\sin(d+ex)}{e} + \frac{c^2x\sin^2(d+ex)}{2} + \frac{c^2x\cos^2(d+ex)}{2} - \\ x\left(b\cos(d) + c\sin(d) + \sqrt{b^2 + c^2}\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

```
[Out] Piecewise((b**2*x*sin(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*x + b
**2*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c*sin(d + e*x)**2/e + 2*b*sqrt(b**2
 + c**2)*sin(d + e*x)/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d + e*x)**2
/2 + c**2*x - c**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*c*sqrt(b**2 + c**2)*
cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b**2 + c**2))**2,
 True))
```

Giac [A] time = 1.14036, size = 124, normalized size = 1.07

$$-\frac{1}{2}\,bc\cos\left(2\,xe+2\,d\right)e^{(-1)}-2\,\sqrt{b^2+c^2}c\cos\left(xe+d\right)e^{(-1)}+2\,\sqrt{b^2+c^2}be^{(-1)}\sin\left(xe+d\right)+\frac{1}{4}\left(b^2-c^2\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="giac"
)
```

```
[Out] -1/2*b*c*cos(2*x*e + 2*d)*e^(-1) - 2*sqrt(b^2 + c^2)*c*cos(x*e + d)*e^(-1)
+ 2*sqrt(b^2 + c^2)*b*e^(-1)*sin(x*e + d) + 1/4*(b^2 - c^2)*e^(-1)*sin(2*x*
e + 2*d) + 3/2*(b^2 + c^2)*x
```

3.358 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx$

Optimal. Leaf size=37

$$x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

```
[Out] Sqrt[b^2 + c^2]*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rubi [A] time = 0.0153018, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2637, 2638}

$$x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] Sqrt[b^2 + c^2]*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx &= \sqrt{b^2+c^2}x+b\int \cos(d+ex)\,dx+c\int \sin(d+ex)\,dx\\ &= \sqrt{b^2+c^2}x-\frac{c\cos(d+ex)}{e}+\frac{b\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0373104, size = 36, normalized size = 0.97

$$\frac{ex\sqrt{b^2+c^2}+b\sin(d+ex)-c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] (Sqrt[b^2 + c^2]*e*x - c*Cos[d + e*x] + b*Sin[d + e*x])/e
```

Maple [A] time = 0.005, size = 36, normalized size = 1.

$$-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}+x\sqrt{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x)
```

```
[Out] -c*cos(e*x+d)/e+b*sin(e*x+d)/e+x*(b^2+c^2)^(1/2)
```

Maxima [A] time = 0.984325, size = 47, normalized size = 1.27

$$\sqrt{b^2+c^2}x-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b^2 + c^2)*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

Fricas [A] time = 2.14202, size = 80, normalized size = 2.16

$$\frac{\sqrt{b^2+c^2}ex-c\cos(ex+d)+b\sin(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b^2 + c^2)*e*x - c*cos(e*x + d) + b*sin(e*x + d))/e
```

Sympy [A] time = 0.163549, size = 42, normalized size = 1.14

$$b\left(\begin{cases}\frac{\sin(d+ex)}{e} & \text{for } e\neq 0\\ x\cos(d) & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{\cos(d+ex)}{e} & \text{for } e\neq 0\\ x\sin(d) & \text{otherwise}\end{cases}\right)+x\sqrt{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2),x)
```

```
[Out] b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewise((-c
os(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + x*sqrt(b**2 + c**2)
```

Giac [A] time = 1.13188, size = 47, normalized size = 1.27

$$-c\cos(xe+d)\,e^{(-1)}+be^{(-1)}\sin(xe+d)+\sqrt{b^2+c^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="giac")
```

```
[Out] -c*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d) + sqrt(b^2 + c^2)*x
```

3.359 $\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=49

$$-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

```
[Out] -((c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])
))
```

Rubi [A] time = 0.036152, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3114}

$$-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] -((c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])
))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Mathematica [A] time = 0.0995726, size = 49, normalized size = 1.

$$\frac{\sqrt{b^2+c^2}\sin(d+ex)-c}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-c + Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Maple [A] time = 0.074, size = 50, normalized size = 1.

$$-2\,\frac{\sqrt{b^2+c^2}+b}{c^2e}\left(\tan\left(d/2+1/2\,ex\right)+\frac{\sqrt{b^2+c^2}}{c}+\frac{b}{c}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x)
```

```
[Out] -2/e*((b^2+c^2)^(1/2)+b)/c^2/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [A] time = 0.999555, size = 54, normalized size = 1.1

$$-\frac{2}{\left(c-\frac{\left(b-\sqrt{b^2+c^2}\right)\sin(ex+d)}{\cos(ex+d)+1}\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="maxim
a")
```

```
[Out] -2/((c - (b - sqrt(b^2 + c^2))*sin(e*x + d)/(cos(e*x + d) + 1))*e)
```

Fricas [A] time = 2.15614, size = 173, normalized size = 3.53

$$-\frac{b^2+c^2-\sqrt{b^2+c^2}\left(b\cos\left(ex+d\right)+c\sin\left(ex+d\right)\right)}{\left(b^2c+c^3\right)e\cos\left(ex+d\right)-\left(b^3+bc^2\right)e\sin\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="frica
s")
```

```
[Out] -(b^2 + c^2 - sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))/((b^2*c +
c^3)*e*cos(e*x + d) - (b^3 + b*c^2)*e*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1434, size = 58, normalized size = 1.18

$$-\frac{2\left(b+\sqrt{b^2+c^2}\right)e^{(-1)}}{\left(c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b+\sqrt{b^2+c^2}\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="giac"
)
```

```
[Out] -2*(b + sqrt(b^2 + c^2))*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c
^2))*c)
```

3.360 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx$

Optimal. Leaf size=129

$$-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{3ce\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}$$

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(3*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(3
*c*Sqrt[b^2 + c^2]*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rubi [A] time = 0.0852958, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3116, 3114}

$$-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{3ce\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(3*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(3
*c*Sqrt[b^2 + c^2]*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
```

```
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{3\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}+\frac{\int \frac{}{\sqrt{b^2+c^2}}}{}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{3\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{}{3c\sqrt{b^2+}}$$

Mathematica [A] time = 0.248005, size = 98, normalized size = 0.76

$$\frac{-2c\sqrt{b^2+c^2}+b^2\sin^3(d+ex)+2bc\cos^3(d+ex)+2c^2\sin(d+ex)+c^2\sin(d+ex)\cos^2(d+ex)}{3ce(c\cos(d+ex)-b\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] (-2*c*Sqrt[b^2 + c^2] + 2*b*c*Cos[d + e*x]^3 + 2*c^2*Sin[d + e*x] + c^2*Cos
[d + e*x]^2*Sin[d + e*x] + b^2*Sin[d + e*x]^3)/(3*c*e*(c*Cos[d + e*x] - b*S
in[d + e*x])^3)
```

Maple [A] time = 0.13, size = 233, normalized size = 1.8

$$2\,\frac{\sqrt{b^2+c^2}+b}{ec^2}\left(-\frac{\left(\sqrt{b^2+c^2}+b\right)(\tan(d/2+1/2\,ex))^2}{c^2}-\frac{\left(2\,b^2+c^2+2\,\sqrt{b^2+c^2}b\right)\tan(d/2+1/2\,ex)}{c^3}-2/3\,\frac{2\,\sqrt{b^2+c^2}b^2 -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x)
```

```
[Out] 2/e*((b^2+c^2)^(1/2)+b)/c^2*(-((b^2+c^2)^(1/2)+b)/c^2*tan(1/2*d+1/2*e*x)^2-
1/c^3*(2*b^2+c^2+2*(b^2+c^2)^(1/2)*b)*tan(1/2*d+1/2*e*x)-2/3*(2*(b^2+c^2)^(
1/2)*b^2+(b^2+c^2)^(1/2)*c^2+2*b^3+2*b*c^2)/c^4)/(tan(1/2*d+1/2*e*x)^2+2/c*
```

```
(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2*b/c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)
^(1/2)*b+2/c^2*b^2+1)/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.30345, size = 424, normalized size = 3.29

$$-\frac{3\,b^3\cos\left(ex+d\right)-\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^3+\left(3\,b^2c+2\,c^3-\left(3\,b^2c-c^3\right)\cos\left(ex+d\right)^2\right)\sin\left(ex+d\right)-2\left(b^2-\right.}{3\left(\left(3\,b^4c+2\,b^2c^3-c^5\right)e\cos\left(ex+d\right)^3-3\left(b^4c+b^2c^3\right)e\cos\left(ex+d\right)-\left(\left(b^5-2\,b^3c^2-3\,bc^4\right)e\cos\left(ex+d\right)^2-\left(b^5+b^3c^2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(3*b^3*cos(e*x + d) - (b^3 - 3*b*c^2)*cos(e*x + d)^3 + (3*b^2*c + 2*c^
3 - (3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - 2*(b^2 + c^2)^(3/2))/((3
*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)*e*cos(e*x
+ d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b^3*c^2)*e)*s
in(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15765, size = 216, normalized size = 1.67

$$-\frac{2\left(8\,b^{4}+10\,b^{2}c^{2}+2\,c^{4}+3\left(2\,b^{2}c^{2}+c^{4}+2\,\sqrt{b^{2}+c^{2}}bc^{2}\right)\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+3\left(4\,b^{3}c+3\,bc^{3}+\left(4\,b^{2}c+c^{3}\right)\sqrt{b^{2}+c^{2}}\right)\text{ta}}{3\left(c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b+\sqrt{b^{2}+c^{2}}\right)^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="gia
c")
```

```
[Out] -2/3*(8*b^4 + 10*b^2*c^2 + 2*c^4 + 3*(2*b^2*c^2 + c^4 + 2*sqrt(b^2 + c^2)*b
*c^2)*tan(1/2*x*e + 1/2*d)^2 + 3*(4*b^3*c + 3*b*c^3 + (4*b^2*c + c^3)*sqrt(
b^2 + c^2))*tan(1/2*x*e + 1/2*d) + 2*(4*b^3 + 3*b*c^2)*sqrt(b^2 + c^2))*e^(
-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c^2))^3*c^3)
```

3.361 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$

Optimal. Leaf size=191

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{15e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-$$

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(5*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^3) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))
/(15*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) -
 (2*(c - Sqrt[b^2 + c^2]*Sin[d + e*x]))/(15*c*(b^2 + c^2)*e*(c*Cos[d + e*x]
 - b*Sin[d + e*x]))
```

Rubi [A] time = 0.132685, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3116, 3114}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{15e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```


```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(5*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^3) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))
/(15*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) -
 (2*(c - Sqrt[b^2 + c^2]*Sin[d + e*x]))/(15*c*(b^2 + c^2)*e*(c*Cos[d + e*x]
 - b*Sin[d + e*x]))
```


Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
```

```
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}+\frac{2\int \frac{}{\left(\sqrt{b^2+\right.}}}{}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-\frac{}{15\left(b^2+\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-\frac{}{15\left(b^2+\right.}$$

Mathematica [B] time = 2.77742, size = 420, normalized size = 2.2

$$\frac{100c^4\sqrt{b^2+c^2}\sin(d+ex)+5c^4\sqrt{b^2+c^2}\sin(3(d+ex))+c^4\sqrt{b^2+c^2}\sin(5(d+ex))+110b^2c^2\sqrt{b^2+c^2}\sin(d+ex)-40b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] (-76*b^4*c - 152*b^2*c^3 - 76*c^5 + 90*b*c*(b^2 + c^2)^(3/2)*Cos[d + e*x] +
 20*c*(-b^4 + c^4)*Cos[2*(d + e*x)] + 10*b^3*c*Sqrt[b^2 + c^2]*Cos[3*(d + e
*x)] + 10*b*c^3*Sqrt[b^2 + c^2]*Cos[3*(d + e*x)] - 4*b^3*c*Sqrt[b^2 + c^2]*
Cos[5*(d + e*x)] + 4*b*c^3*Sqrt[b^2 + c^2]*Cos[5*(d + e*x)] + 10*b^4*Sqrt[b
^2 + c^2]*Sin[d + e*x] + 110*b^2*c^2*Sqrt[b^2 + c^2]*Sin[d + e*x] + 100*c^4
*Sqrt[b^2 + c^2]*Sin[d + e*x] - 40*b^3*c^2*Sin[2*(d + e*x)] - 40*b*c^4*Sin[
2*(d + e*x)] - 5*b^4*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 5*c^4*Sqrt[b^2 + c^
2]*Sin[3*(d + e*x)] + b^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)] - 6*b^2*c^2*Sqrt
[b^2 + c^2]*Sin[5*(d + e*x)] + c^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)])/(120*c
```

```
*(b^2 + c^2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x])^5)
```

Maple [B] time = 0.194, size = 496, normalized size = 2.6

$$2\frac{1}{c^4e}\left(-\frac{\left(4\sqrt{b^2+c^2}b^2+\sqrt{b^2+c^2}c^2+4b^3+3bc^2\right)(\tan(d/2+1/2ex))^4}{c^2}-2\frac{\left(8b^4+8b^2c^2+c^4+8\sqrt{b^2+c^2}b^3+4\sqrt{b^2\right.}}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x)
```

```
[Out] 2/e/c^4*(-(4*(b^2+c^2)^(1/2)*b^2+(b^2+c^2)^(1/2)*c^2+4*b^3+3*b*c^2)/c^2*tan
(1/2*d+1/2*e*x)^4-2*(8*b^4+8*b^2*c^2+c^4+8*(b^2+c^2)^(1/2)*b^3+4*(b^2+c^2)^
(1/2)*b*c^2)/c^3*tan(1/2*d+1/2*e*x)^3-4/3*(24*(b^2+c^2)^(1/2)*b^4+20*(b^2+c
^2)^(1/2)*b^2*c^2+2*(b^2+c^2)^(1/2)*c^4+24*b^5+32*b^3*c^2+9*b*c^4)/c^4*tan(
1/2*d+1/2*e*x)^2-2/3*(48*b^6+76*b^4*c^2+31*b^2*c^4+2*c^6+48*(b^2+c^2)^(1/2)
*b^5+52*(b^2+c^2)^(1/2)*b^3*c^2+11*(b^2+c^2)^(1/2)*b*c^4)/c^5*tan(1/2*d+1/2
*e*x)-1/15/c^6*(192*(b^2+c^2)^(1/2)*b^6+256*(b^2+c^2)^(1/2)*b^4*c^2+96*(b^2
+c^2)^(1/2)*b^2*c^4+7*(b^2+c^2)^(1/2)*c^6+192*b^7+352*b^5*c^2+200*b^3*c^4+3
5*b*c^6))/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2*b/
c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)^(1/2)*b+2/c^2*b^2+1)^2/(tan(1/2*d+1/2*
e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.98511, size = 1080, normalized size = 5.65

$$-\frac{7b^6+26b^4c^2+31b^2c^4+12c^6+5\left(b^6+b^4c^2-b^2c^4-c^6\right)\cos(ex+d)^2+10\left(b^5c+2b^3c^3+bc^5\right)\cos(ex+d)\sin(ex+}{15\left(\left(5b^8c-14b^4c^5-8b^2c^7+c^9\right)e\cos(ex+d)^5-10\left(b^8c+b^6c^3-b^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="fri
cas")
```

```
[Out] -1/15*(7*b^6 + 26*b^4*c^2 + 31*b^2*c^4 + 12*c^6 + 5*(b^6 + b^4*c^2 - b^2*c^
4 - c^6)*cos(e*x + d)^2 + 10*(b^5*c + 2*b^3*c^3 + b*c^5)*cos(e*x + d)*sin(e
*x + d) - (2*(b^5 - 10*b^3*c^2 + 5*b*c^4)*cos(e*x + d)^5 - 5*(b^5 - 6*b^3*c
^2 + b*c^4)*cos(e*x + d)^3 + 5*(3*b^5 + 3*b^3*c^2 + 2*b*c^4)*cos(e*x + d) +
 (15*b^4*c + 25*b^2*c^3 + 12*c^5 + 2*(5*b^4*c - 10*b^2*c^3 + c^5)*cos(e*x +
 d)^4 - (15*b^4*c - 10*b^2*c^3 - c^5)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^
2 + c^2))/((5*b^8*c - 14*b^4*c^5 - 8*b^2*c^7 + c^9)*e*cos(e*x + d)^5 - 10*(
b^8*c + b^6*c^3 - b^4*c^5 - b^2*c^7)*e*cos(e*x + d)^3 + 5*(b^8*c + 2*b^6*c^
3 + b^4*c^5)*e*cos(e*x + d) - ((b^9 - 8*b^7*c^2 - 14*b^5*c^4 + 5*b*c^8)*e*c
os(e*x + d)^4 - 2*(b^9 - 3*b^7*c^2 - 9*b^5*c^4 - 5*b^3*c^6)*e*cos(e*x + d)^
2 + (b^9 + 2*b^7*c^2 + b^5*c^4)*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14242, size = 467, normalized size = 2.45

$$-\frac{2\left(192\,b^7+352\,b^5c^2+200\,b^3c^4+35\,bc^6+15\left(4\,b^3c^4+3\,bc^6+\left(4\,b^2c^4+c^6\right)\sqrt{b^2+c^2}\right)\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^4+30\left(8\,b^4c^3+8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="gia
c")
```

```
[Out] -2/15*(192*b^7 + 352*b^5*c^2 + 200*b^3*c^4 + 35*b*c^6 + 15*(4*b^3*c^4 + 3*b
*c^6 + (4*b^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^4 + 30*(8*b^
4*c^3 + 8*b^2*c^5 + c^7 + 4*(2*b^3*c^3 + b*c^5)*sqrt(b^2 + c^2))*tan(1/2*x*
e + 1/2*d)^3 + 20*(24*b^5*c^2 + 32*b^3*c^4 + 9*b*c^6 + 2*(12*b^4*c^2 + 10*b
^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^2 + 10*(48*b^6*c + 76*b
^4*c^3 + 31*b^2*c^5 + 2*c^7 + (48*b^5*c + 52*b^3*c^3 + 11*b*c^5)*sqrt(b^2 +
 c^2))*tan(1/2*x*e + 1/2*d) + (192*b^6 + 256*b^4*c^2 + 96*b^2*c^4 + 7*c^6)*
sqrt(b^2 + c^2))*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c^2))^5*c
^5)
```

3.362 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$

Optimal. Leaf size=259

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)^{3/2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}$$

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(7*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^4) - (3*(c*Cos[d + e*x] - b*Sin[d + e*x]))
/(35*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3) -
 (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*(b^2 + c^2)^(3/2)*e*(Sqrt[b^2 +
c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(c - Sqrt[b^2 + c^2]*Sin[d
+ e*x]))/(35*c*(b^2 + c^2)^(3/2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rubi [A] time = 0.189269, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3116, 3114}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)^{3/2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(7*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^4) - (3*(c*Cos[d + e*x] - b*Sin[d + e*x]))
/(35*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3) -
 (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*(b^2 + c^2)^(3/2)*e*(Sqrt[b^2 +
c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(c - Sqrt[b^2 + c^2]*Sin[d
+ e*x]))/(35*c*(b^2 + c^2)^(3/2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
```

```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}+\frac{3\int \frac{}{(\sqrt{b}}}{}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}-\frac{}{35\left(b^2\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}-\frac{}{35\left(b^2\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}-\frac{}{35\left(b^2\right.}$$

Mathematica [B] time = 2.11432, size = 533, normalized size = 2.06

$$\frac{-1295b^4c^2\sin(d+ex)-189b^4c^2\sin(3(d+ex))+35b^4c^2\sin(5(d+ex))-15b^4c^2\sin(7(d+ex))+896b^3c^2\sqrt{b^2+c^2}\sin(2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] (832*b^4*c*Sqrt[b^2 + c^2] + 1664*b^2*c^3*Sqrt[b^2 + c^2] + 832*c^5*Sqrt[b^
2 + c^2] - 1190*b*c*(b^2 + c^2)^2*Cos[d + e*x] + 448*c*Sqrt[b^2 + c^2]*(b^4
 - c^4)*Cos[2*(d + e*x)] - 112*b^5*c*Cos[3*(d + e*x)] + 56*b^3*c^3*Cos[3*(d
```

```
 + e*x)] + 168*b*c^5*Cos[3*(d + e*x)] + 28*b^5*c*Cos[5*(d + e*x)] - 28*b*c^
5*Cos[5*(d + e*x)] - 6*b^5*c*Cos[7*(d + e*x)] + 20*b^3*c^3*Cos[7*(d + e*x)]
 - 6*b*c^5*Cos[7*(d + e*x)] - 35*b^6*Sin[d + e*x] - 1295*b^4*c^2*Sin[d + e*
x] - 2485*b^2*c^4*Sin[d + e*x] - 1225*c^6*Sin[d + e*x] + 896*b^3*c^2*Sqrt[b
^2 + c^2]*Sin[2*(d + e*x)] + 896*b*c^4*Sqrt[b^2 + c^2]*Sin[2*(d + e*x)] + 2
1*b^6*Sin[3*(d + e*x)] - 189*b^4*c^2*Sin[3*(d + e*x)] - 161*b^2*c^4*Sin[3*(
d + e*x)] + 49*c^6*Sin[3*(d + e*x)] - 7*b^6*Sin[5*(d + e*x)] + 35*b^4*c^2*S
in[5*(d + e*x)] + 35*b^2*c^4*Sin[5*(d + e*x)] - 7*c^6*Sin[5*(d + e*x)] + b^
6*Sin[7*(d + e*x)] - 15*b^4*c^2*Sin[7*(d + e*x)] + 15*b^2*c^4*Sin[7*(d + e*
x)] - c^6*Sin[7*(d + e*x)])/(1120*c*(b^2 + c^2)*e*(-(c*Cos[d + e*x]) + b*Si
n[d + e*x])^7)
```

Maple [B] time = 0.318, size = 823, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x)
```

```
[Out] -2/e/c^6*((8*b^4+8*b^2*c^2+c^4+8*(b^2+c^2)^(1/2)*b^3+4*(b^2+c^2)^(1/2)*b*c^
2)/c^2*tan(1/2*d+1/2*e*x)^6+3*(16*(b^2+c^2)^(1/2)*b^4+12*(b^2+c^2)^(1/2)*b^
2*c^2+(b^2+c^2)^(1/2)*c^4+16*b^5+20*b^3*c^2+5*b*c^4)/c^3*tan(1/2*d+1/2*e*x)
^5+2*(80*(b^2+c^2)^(1/2)*b^5+84*(b^2+c^2)^(1/2)*b^3*c^2+17*(b^2+c^2)^(1/2)*
b*c^4+80*b^6+124*b^4*c^2+49*b^2*c^4+3*c^6)/c^4*tan(1/2*d+1/2*e*x)^4+2*(160*
b^7+288*b^5*c^2+150*b^3*c^4+20*b*c^6+160*(b^2+c^2)^(1/2)*b^6+208*(b^2+c^2)^
(1/2)*b^4*c^2+66*(b^2+c^2)^(1/2)*b^2*c^4+3*(b^2+c^2)^(1/2)*c^6)/c^5*tan(1/2
*d+1/2*e*x)^3+3/5*(640*b^7*(b^2+c^2)^(1/2)+992*(b^2+c^2)^(1/2)*b^5*c^2+440*
(b^2+c^2)^(1/2)*b^3*c^4+50*(b^2+c^2)^(1/2)*b*c^6+640*b^8+1312*b^6*c^2+856*b
^4*c^4+186*b^2*c^6+7*c^8)/c^6*tan(1/2*d+1/2*e*x)^2+1/5*(1280*b^9+2944*b^7*c
^2+2288*b^5*c^4+676*b^3*c^6+57*b*c^8+1280*(b^2+c^2)^(1/2)*b^8+2304*(b^2+c^2
)^(1/2)*b^6*c^2+1296*(b^2+c^2)^(1/2)*b^4*c^4+236*(b^2+c^2)^(1/2)*b^2*c^6+7*
(b^2+c^2)^(1/2)*c^8)/c^7*tan(1/2*d+1/2*e*x)+4/35*(640*(b^2+c^2)^(1/2)*b^9+1
312*(b^2+c^2)^(1/2)*b^7*c^2+896*(b^2+c^2)^(1/2)*b^5*c^4+238*(b^2+c^2)^(1/2)
*b^3*c^6+21*(b^2+c^2)^(1/2)*b*c^8+640*b^10+1632*b^8*c^2+1472*b^6*c^4+562*b^
4*c^6+85*b^2*c^8+3*c^10)/c^8)/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan
(1/2*d+1/2*e*x)+2*b/c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)^(1/2)*b+2/c^2*b^2+
1)^3/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 5.61137, size = 1643, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="fri
cas")
```

```
[Out] 1/35*(2*(b^7 - 21*b^5*c^2 + 35*b^3*c^4 - 7*b*c^6)*cos(e*x + d)^7 - 7*(b^7 -
 15*b^5*c^2 + 15*b^3*c^4 - b*c^6)*cos(e*x + d)^5 - 14*(5*b^5*c^2 - 5*b^3*c^
4 - 2*b*c^6)*cos(e*x + d)^3 - 7*(5*b^7 + 15*b^5*c^2 + 20*b^3*c^4 + 8*b*c^6)
*cos(e*x + d) - (35*b^6*c + 105*b^4*c^3 + 112*b^2*c^5 + 40*c^7 - 2*(7*b^6*c
 - 35*b^4*c^3 + 21*b^2*c^5 - c^7)*cos(e*x + d)^6 + (35*b^6*c - 105*b^4*c^3
+ 21*b^2*c^5 + c^7)*cos(e*x + d)^4 + 2*(35*b^4*c^3 + 7*b^2*c^5 - 4*c^7)*cos
(e*x + d)^2)*sin(e*x + d) + 4*(3*b^6 + 16*b^4*c^2 + 23*b^2*c^4 + 10*c^6 + 7
*(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(e*x + d)^2 + 14*(b^5*c + 2*b^3*c^3 + b
*c^5)*cos(e*x + d)*sin(e*x + d))*sqrt(b^2 + c^2))/((7*b^10*c - 21*b^8*c^3 -
 42*b^6*c^5 + 6*b^4*c^7 + 19*b^2*c^9 - c^11)*e*cos(e*x + d)^7 - 7*(3*b^10*c
 - 4*b^8*c^3 - 14*b^6*c^5 - 4*b^4*c^7 + 3*b^2*c^9)*e*cos(e*x + d)^5 + 7*(3*
b^10*c + b^8*c^3 - 7*b^6*c^5 - 5*b^4*c^7)*e*cos(e*x + d)^3 - 7*(b^10*c + 2*
b^8*c^3 + b^6*c^5)*e*cos(e*x + d) - ((b^11 - 19*b^9*c^2 - 6*b^7*c^4 + 42*b^
5*c^6 + 21*b^3*c^8 - 7*b*c^10)*e*cos(e*x + d)^6 - (3*b^11 - 36*b^9*c^2 - 46
*b^7*c^4 + 28*b^5*c^6 + 35*b^3*c^8)*e*cos(e*x + d)^4 + 3*(b^11 - 5*b^9*c^2
- 13*b^7*c^4 - 7*b^5*c^6)*e*cos(e*x + d)^2 - (b^11 + 2*b^9*c^2 + b^7*c^4)*e
)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17106, size = 809, normalized size = 3.12

$$-\frac{2\left(2560\,b^{10}+6528\,b^{8}c^{2}+5888\,b^{6}c^{4}+2248\,b^{4}c^{6}+340\,b^{2}c^{8}+12\,c^{10}+35\left(8\,b^{4}c^{6}+8\,b^{2}c^{8}+c^{10}+4\left(2\,b^{3}c^{6}+bc^{8}\right)\sqrt{b^{2}+\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="gia
c")
```

```
[Out] -2/35*(2560*b^10 + 6528*b^8*c^2 + 5888*b^6*c^4 + 2248*b^4*c^6 + 340*b^2*c^8
 + 12*c^10 + 35*(8*b^4*c^6 + 8*b^2*c^8 + c^10 + 4*(2*b^3*c^6 + b*c^8)*sqrt(
b^2 + c^2))*tan(1/2*x*e + 1/2*d)^6 + 105*(16*b^5*c^5 + 20*b^3*c^7 + 5*b*c^9
 + (16*b^4*c^5 + 12*b^2*c^7 + c^9)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^5
+ 70*(80*b^6*c^4 + 124*b^4*c^6 + 49*b^2*c^8 + 3*c^10 + (80*b^5*c^4 + 84*b^3
*c^6 + 17*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^4 + 70*(160*b^7*c^3
+ 288*b^5*c^5 + 150*b^3*c^7 + 20*b*c^9 + (160*b^6*c^3 + 208*b^4*c^5 + 66*b^
2*c^7 + 3*c^9)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^3 + 21*(640*b^8*c^2 +
1312*b^6*c^4 + 856*b^4*c^6 + 186*b^2*c^8 + 7*c^10 + 2*(320*b^7*c^2 + 496*b^
5*c^4 + 220*b^3*c^6 + 25*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^2 + 7
*(1280*b^9*c + 2944*b^7*c^3 + 2288*b^5*c^5 + 676*b^3*c^7 + 57*b*c^9 + (1280
*b^8*c + 2304*b^6*c^3 + 1296*b^4*c^5 + 236*b^2*c^7 + 7*c^9)*sqrt(b^2 + c^2)
)*tan(1/2*x*e + 1/2*d) + 4*(640*b^9 + 1312*b^7*c^2 + 896*b^5*c^4 + 238*b^3*
c^6 + 21*b*c^8)*sqrt(b^2 + c^2))*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt
(b^2 + c^2))^7*c^7)
```

3.363 $\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))^3\,dx$

Optimal. Leaf size=157

$$\frac{4a\left(15a^2+4c^2\right)\sin(d+ex)}{3e}-\frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)(a\,\mathrm{co}}{3e}$$

```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) + (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] - a^2*Sin[d + e*
x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] - a*S
in[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```

Rubi [A] time = 0.142804, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4a\left(15a^2+4c^2\right)\sin(d+ex)}{3e}-\frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)(a\,\mathrm{co}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```


```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) + (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] - a^2*Sin[d + e*
x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] - a*S
in[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
```

```
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx = -\frac{8(c\cos(d+ex)-a\sin(d+ex))(a+a\cos(d+ex)+c\sin(d+ex))^2}{3e}+\frac{1}{3}$$

$$= -\frac{20\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)(a+a\cos(d+ex)+c\sin(d+ex))}{3e} -$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{20\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)(a+a\cos(d+ex)+}{3e}$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e} + \frac{4a\left(15a^2+4c^2\right)\sin(d+e}{3e}$$

Mathematica [A] time = 0.422082, size = 135, normalized size = 0.86

$$\frac{2\left(6a\left(5a^2+3c^2\right)(d+ex)+9a\left(5a^2+c^2\right)\sin(d+ex)+9a\left(a^2-c^2\right)\sin(2(d+ex))+a\left(a^2-3c^2\right)\sin(3(d+ex))-9c\left(5a^2\right.\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] - 18*a^2
*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] + 9*a*(5*a^2 + c^2)
*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] + a*(a^2 - 3*c^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A] time = 0.075, size = 177, normalized size = 1.1

$$8\,\frac{a^3(ex+d)+3a^3\sin(ex+d)-3a^2c\cos(ex+d)+3a^3\left(1/2\,\sin(ex+d)\cos(ex+d)+1/2\,ex+d/2\right)-3a^2c\left(\cos(ex+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(a^3*(e*x+d)+3*a^3*sin(e*x+d)-3*a^2*c*cos(e*x+d)+3*a^3*(1/2*sin(e*x+d)*
cos(e*x+d)+1/2*e*x+1/2*d)-3*a^2*c*cos(e*x+d)^2+3*a*c^2*(-1/2*sin(e*x+d)*cos
(e*x+d)+1/2*e*x+1/2*d)+1/3*a^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a^2*c*cos(e*x+d)
^3+a*c^2*sin(e*x+d)^3-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

Maxima [A] time = 1.12478, size = 258, normalized size = 1.64

$$-\frac{8\,a^2c\cos(ex+d)^3}{e}+\frac{8\,ac^2\sin(ex+d)^3}{e}+8\,a^3x-\frac{8\left(\sin(ex+d)^3-3\,\sin(ex+d)\right)a^3}{3\,e}+\frac{8\left(\cos(ex+d)^3-3\,\cos(ex+\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*c*cos(e*x + d)^3/e + 8*a*c^2*sin(e*x + d)^3/e + 8*a^3*x - 8/3*(sin(e
*x + d)^3 - 3*sin(e*x + d))*a^3/e + 8/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c
^3/e - 24*a^2*(c*cos(e*x + d)/e - a*sin(e*x + d)/e) - 6*(4*a*c*cos(e*x + d)
^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e - (2*e*x + 2*d - sin(2*e*x +
2*d))*c^2/e)*a
```

Fricas [A] time = 2.21643, size = 308, normalized size = 1.96

$$-\frac{4\left(18\,a^2c\cos(ex+d)^2+2\left(3\,a^2c-c^3\right)\cos(ex+d)^3-3\left(5\,a^3+3\,ac^2\right)ex+6\left(3\,a^2c+c^3\right)\cos(ex+d)-\left(22\,a^3+6\,ac^2+\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(18*a^2*c*cos(e*x + d)^2 + 2*(3*a^2*c - c^3)*cos(e*x + d)^3 - 3*(5*a^3
 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*(a
^3 - 3*a*c^2)*cos(e*x + d)^2 + 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d))/
e
```

Sympy [A] time = 0.883761, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x + \frac{16a^3\sin^3(d+ex)}{3e} + \frac{8a^3\sin(d+ex)\cos^2(d+ex)}{e} + \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} + \frac{24a^3\sin}{e} \\ x(2a\cos(d) + 2a + 2c\sin(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 + 16*a**3*sin(d + e*x)**3/(3*e) + 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e +
12*a**3*sin(d + e*x)*cos(d + e*x)/e + 24*a**3*sin(d + e*x)/e + 24*a**2*c*si
n(d + e*x)**2/e - 8*a**2*c*cos(d + e*x)**3/e - 24*a**2*c*cos(d + e*x)/e + 1
2*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 + 8*a*c**2*sin(d +
 e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e*x)**2
*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(2*a*cos(d)
+ 2*a + 2*c*sin(d))**3, True))
```

Giac [A] time = 1.14513, size = 204, normalized size = 1.3

$$-12\,a^2c\cos(2\,xe+2\,d)\,e^{(-1)} - \frac{2}{3}\left(3\,a^2c - c^3\right)\cos(3\,xe+3\,d)\,e^{(-1)} - 6\left(5\,a^2c + c^3\right)\cos(xe+d)\,e^{(-1)} + \frac{2}{3}\left(a^3 - 3\,ac^2\right)e^{(-1)}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -12*a^2*c*cos(2*x*e + 2*d)*e^(-1) - 2/3*(3*a^2*c - c^3)*cos(3*x*e + 3*d)*e^
(-1) - 6*(5*a^2*c + c^3)*cos(x*e + d)*e^(-1) + 2/3*(a^3 - 3*a*c^2)*e^(-1)*s
in(3*x*e + 3*d) + 6*(a^3 - a*c^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^3 + a*c^
2)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*c^2)*x
```

3.364 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2x\left(3a^2+c^2\right)+\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))}{e}$$

```
[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e + (6*a^2*Sin[d + e*x])/e - (2*(c
*Cos[d + e*x] - a*Sin[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/e
```

Rubi [A] time = 0.0495218, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2+c^2\right)+\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```


```
[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e + (6*a^2*Sin[d + e*x])/e - (2*(c
*Cos[d + e*x] - a*Sin[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/e
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a + 2a\cos(d+ex) + 2c\sin(d+ex))^2\,dx = -\frac{2(c\cos(d+ex) - a\sin(d+ex))(a + a\cos(d+ex) + c\sin(d+ex))}{e} + \frac{1}{2}\int$$
$$= 2\left(3a^2 + c^2\right)x - \frac{2(c\cos(d+ex) - a\sin(d+ex))(a + a\cos(d+ex) + c\sin($$
$$= 2\left(3a^2 + c^2\right)x - \frac{6ac\cos(d+ex)}{e} + \frac{6a^2\sin(d+ex)}{e} - \frac{2(c\cos(d+ex) - a\,\mathrm{si}}{}$$

Mathematica [A] time = 0.145591, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2 + c^2\right)(d+ex)}{2e} + \frac{\left(a^2 - c^2\right)\sin(2(d+ex))}{4e} + \frac{2a^2\sin(d+ex)}{e} - \frac{2ac\cos(d+ex)}{e} - \frac{ac\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e - (a*c*Cos[2*(d
 + e*x)])/(2*e) + (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A] time = 0.054, size = 101, normalized size = 1.3

$$4\,\frac{a^2(ex+d) + 2\,a^2\sin(ex+d) - 2\,ac\cos(ex+d) + a^2\left(1/2\,\sin(ex+d)\cos(ex+d) + 1/2\,ex + d/2\right) - ac\left(\cos(ex+d)\right)^2 +}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(e*x+d)+2*a^2*sin(e*x+d)-2*a*c*cos(e*x+d)+a^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)-a*c*cos(e*x+d)^2+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

Maxima [A] time = 1.01777, size = 134, normalized size = 1.65

$$4\,a^2x - \frac{4\,ac\cos(ex+d)^2}{e} + \frac{(2\,ex + 2\,d + \sin(2\,ex + 2\,d))a^2}{e} + \frac{(2\,ex + 2\,d - \sin(2\,ex + 2\,d))c^2}{e} - 8\,a\left(\frac{c\cos(ex+d)}{e} - \frac{a\,\mathrm{si}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x - 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e - a*sin(e*x
 + d)/e)
```

Fricas [A] time = 2.08, size = 162, normalized size = 2.

$$-\frac{2\left(2\,ac\cos\left(ex+d\right)^{2}-\left(3\,a^{2}+c^{2}\right)ex+4\,ac\cos\left(ex+d\right)-\left(4\,a^{2}+\left(a^{2}-c^{2}\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*a*c*cos(e*x + d)^2 - (3*a^2 + c^2)*e*x + 4*a*c*cos(e*x + d) - (4*a^2
+ (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.376701, size = 170, normalized size = 2.1

$$\begin{cases}2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x+\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}+\frac{8a^2\sin\left(d+ex\right)}{e}+\frac{4ac\sin^2\left(d+ex\right)}{e}-\frac{8ac\cos\left(d+ex\right)}{e}+2c^2x\\ x\left(2a\cos\left(d\right)+2a+2c\sin\left(d\right)\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x +
 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*sin(d + e*x)/e + 4*a*c*sin(d +
 e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos
(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*cos(d
) + 2*a + 2*c*sin(d))**2, True))
```

Giac [A] time = 1.17128, size = 105, normalized size = 1.3

$$-2\,ac\cos\left(2\,xe+2\,d\right)e^{(-1)}-8\,ac\cos\left(xe+d\right)e^{(-1)}+8\,a^{2}e^{(-1)}\sin\left(xe+d\right)+\left(a^{2}-c^{2}\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+2\left(3\,a^{2}+c^{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -2*a*c*cos(2*x*e + 2*d)*e^(-1) - 8*a*c*cos(x*e + d)*e^(-1) + 8*a^2*e^(-1)*s
in(x*e + d) + (a^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + c^2)*x
```

3.365 $\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx$

Optimal. Leaf size=29

$$\frac{2a\sin(d+ex)}{e} + 2ax - \frac{2c\cos(d+ex)}{e}$$

```
[Out] 2*a*x - (2*c*Cos[d + e*x])/e + (2*a*Sin[d + e*x])/e
```

Rubi [A] time = 0.015509, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$\frac{2a\sin(d+ex)}{e} + 2ax - \frac{2c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d + e*x])/e + (2*a*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a + 2a\cos(d+ex) + 2c\sin(d+ex))\,dx &= 2ax + (2a)\int \cos(d+ex)\,dx + (2c)\int \sin(d+ex)\,dx\\ &= 2ax - \frac{2c\cos(d+ex)}{e} + \frac{2a\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0156266, size = 53, normalized size = 1.83

$$\frac{2a\sin(d)\cos(ex)}{e}+\frac{2a\cos(d)\sin(ex)}{e}+2ax+\frac{2c\sin(d)\sin(ex)}{e}-\frac{2c\cos(d)\cos(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d]*Cos[e*x])/e + (2*a*Cos[e*x]*Sin[d])/e + (2*a*Cos[d]*Sin
[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

Maple [A] time = 0.001, size = 30, normalized size = 1.

$$2\,ax-2\,{\frac{c\cos\left(ex+d\right)}{e}}+2\,{\frac{a\sin\left(ex+d\right)}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x-2*c*cos(e*x+d)/e+2*a*sin(e*x+d)/e
```

Maxima [A] time = 0.985324, size = 39, normalized size = 1.34

$$2\,ax-\frac{2\,c\cos\left(ex+d\right)}{e}+\frac{2\,a\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x - 2*c*cos(e*x + d)/e + 2*a*sin(e*x + d)/e
```

Fricas [A] time = 2.14457, size = 63, normalized size = 2.17

$$\frac{2\left(aex-c\cos\left(ex+d\right)+a\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x - c*cos(e*x + d) + a*sin(e*x + d))/e
```

Sympy [A] time = 0.158888, size = 39, normalized size = 1.34

$$2ax + 2a\left(\begin{cases} \frac{\sin(d+ex)}{e} & \text{for}\ e \neq 0 \\ x\cos(d) & \text{otherwise} \end{cases}\right) + 2c\left(\begin{cases} -\frac{\cos(d+ex)}{e} & \text{for}\ e \neq 0 \\ x\sin(d) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x + 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*P
iecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

Giac [A] time = 1.1218, size = 39, normalized size = 1.34

$$-2\,c\cos(xe+d)\,e^{(-1)} + 2\,ae^{(-1)}\sin(xe+d) + 2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -2*c*cos(x*e + d)*e^(-1) + 2*a*e^(-1)*sin(x*e + d) + 2*a*x
```

3.366 $\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

Optimal. Leaf size=25

$$\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

```
[Out] Log[a + c*Tan[(d + e*x)/2]]/(2*c*e)
```

Rubi [A] time = 0.0223817, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3124, 31}

$$\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[a + c*Tan[(d + e*x)/2]]/(2*c*e)
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{4a+4cx}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e} = \frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Mathematica [B] time = 0.0533843, size = 57, normalized size = 2.28

$$\frac{1}{2}\left(\frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)}{ce}-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{ce}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-(Log[Cos[(d + e*x)/2]]/(c*e)) + Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/
2]]/(c*e))/2
```

Maple [A] time = 0.087, size = 23, normalized size = 0.9

$$\frac{1}{2\,ce}\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] 1/2*ln(a+c*tan(1/2*d+1/2*e*x))/c/e
```

Maxima [A] time = 0.997076, size = 39, normalized size = 1.56

$$\frac{\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{2\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*log(a + c*sin(e*x + d)/(cos(e*x + d) + 1))/(c*e)
```

Fricas [B] time = 2.17535, size = 157, normalized size = 6.28

$$\frac{\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2+\frac{1}{2}\left(a^2-c^2\right)\cos\left(ex+d\right)\right)-\log\left(\frac{1}{2}\,\cos\left(ex+d\right)+\frac{1}{2}\right)}{4\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d
)) - log(1/2*cos(e*x + d) + 1/2))/(c*e)
```

Sympy [A] time = 1.20441, size = 63, normalized size = 2.52

$$\begin{cases} \frac{x}{2a\cos\left(d\right)+2a} & \text{for}\ c=0\wedge e=0 \\ \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{2ae} & \text{for}\ c=0 \\ \frac{x}{2a\cos\left(d\right)+2a+2c\sin\left(d\right)} & \text{for}\ e=0 \\ \frac{\log\left(\frac{a}{c}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] Piecewise((x/(2*a*cos(d) + 2*a), Eq(c, 0) & Eq(e, 0)), (tan(d/2 + e*x/2)/(2
*a*e), Eq(c, 0)), (x/(2*a*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (log(a/c +
 tan(d/2 + e*x/2))/(2*c*e), True))
```

Giac [A] time = 1.1631, size = 31, normalized size = 1.24

$$\frac{e^{(-1)}\log\left(\left|c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*e^(-1)*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c
```

3.367 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$

```
[Out] -(a*Log[a + c*Tan[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] - a*Sin[d + e*
x])/(4*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rubi [A] time = 0.0487294, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3124, 31}

$$-\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] -(a*Log[a + c*Tan[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] - a*Sin[d + e*
x])/(4*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}+\frac{\int -\frac{2a}{2a+2a\cos(d+ex)+2c\sin(d+ex)}}{4c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}-\frac{a\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}}{2c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{4a+4cx}\,dx, x, \tan\right.}{c^2e}$$

$$= -\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}$$

Mathematica [A] time = 0.538032, size = 115, normalized size = 1.53

$$\frac{\frac{c(a^2+c^2)\sin\left(\frac{1}{2}(d+ex)\right)}{a\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)}+2a\left(\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\right)+c\tan\left(\frac{1}{2}(d+ex\right.}{8c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*a*(Log[Cos[(d + e*x)/2]] - Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]])
 + (c*(a^2 + c^2)*Sin[(d + e*x)/2])/(a*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x
)/2])) + c*Tan[(d + e*x)/2])/(8*c^3*e)
```

Maple [A] time = 0.137, size = 91, normalized size = 1.2

$$\frac{1}{8c^2e}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-\frac{a}{4c^3e}\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{a^2}{8c^3e}\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}-\frac{1}{8ce}\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] 1/8/e/c^2*tan(1/2*d+1/2*e*x)-1/4*a*ln(a+c*tan(1/2*d+1/2*e*x))/c^3/e-1/8/e/c
^3/(a+c*tan(1/2*d+1/2*e*x))*a^2-1/8/e/c/(a+c*tan(1/2*d+1/2*e*x))
```

Maxima [A] time = 1.03015, size = 122, normalized size = 1.63

$$-\frac{\frac{a^2+c^2}{ac^3+\frac{c^4\sin(ex+d)}{\cos(ex+d)+1}}+\frac{2\,a\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}-\frac{\sin(ex+d)}{c^2(\cos(ex+d)+1)}}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((a^2 + c^2)/(a*c^3 + c^4*sin(e*x + d)/(cos(e*x + d) + 1)) + 2*a*log(a
 + c*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 - sin(e*x + d)/(c^2*(cos(e*x + d)
 + 1)))/e
```

Fricas [B] time = 2.2152, size = 398, normalized size = 5.31

$$-\frac{2\,c^2\cos(ex+d)-2\,ac\sin(ex+d)+\left(a^2\cos(ex+d)+ac\sin(ex+d)+a^2\right)\log\left(ac\sin(ex+d)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2+\frac{1}{2}\,(a^2-\right.}{8\left(ac^3e\cos(ex+d)+c^4e\sin(ex+d)+ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*c^2*cos(e*x + d) - 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) + a*c*sin
(e*x + d) + a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)
```

```
*cos(e*x + d)) - (a^2*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*log(1/2*cos(e*
x + d) + 1/2))/(a*c^3*e*cos(e*x + d) + c^4*e*sin(e*x + d) + a*c^3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14521, size = 116, normalized size = 1.55

$$-\frac{1}{8}\left(\frac{2\,a\log\left(\left|c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right|\right)}{c^3}-\frac{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)}{c^2}-\frac{2\,ac\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a^2-c^2}{\left(c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right)c^3}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*a*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c^3 - tan(1/2*x*e + 1/2*d)/c
^2 - (2*a*c*tan(1/2*x*e + 1/2*d) + a^2 - c^2)/((c*tan(1/2*x*e + 1/2*d) + a)
*c^3))*e^(-1)
```

3.368 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=134

$$\frac{(3a^2+c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}+\frac{3\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{16c^4e(a\cos(d+ex)+a+c\sin(d+ex))}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex}$$

```
[Out] ((3*a^2 + c^2)*Log[a + c*Tan[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] -
a*Sin[d + e*x])/(16*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*
c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(16*c^4*e*(a + a*Cos[d + e*x] + c*Sin[d
 + e*x]))
```

Rubi [A] time = 0.111693, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3124, 31}

$$\frac{(3a^2+c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}+\frac{3\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{16c^4e(a\cos(d+ex)+a+c\sin(d+ex))}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((3*a^2 + c^2)*Log[a + c*Tan[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] -
a*Sin[d + e*x])/(16*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*
c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(16*c^4*e*(a + a*Cos[d + e*x] + c*Sin[d
 + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2} + \frac{\int \frac{-4a+2a\cos(d+ex)+2c\sin(d+e}{(2a+2a\cos(d+ex)+2c\sin(d+ex}}{8c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3\left(ac\cos(d+ex)-a^2 s\right.}{16c^4e(a+a\cos(d+ex)+}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3\left(ac\cos(d+ex)-a^2 s\right.}{16c^4e(a+a\cos(d+ex)+}$$

$$= \frac{\left(3a^2+c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e} - \frac{c\cos(d+ex)-a\sin(d+}{16c^2e(a+a\cos(d+ex)+c\sin}$$

Mathematica [A] time = 3.0798, size = 186, normalized size = 1.39

$$-\frac{4\left(3a^2+c^2\right)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)+\frac{c^2\left(a^2+c^2\right)}{\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6c\left(a^2+c^2\right)\sin\left(\frac{1}{2}(d+ex)\right)}{a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)}-4\left(3a^2+c^2\right)\log\left(a\cos\left(\right.\right.}{64c^5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] -(4*(3*a^2 + c^2)*Log[Cos[(d + e*x)/2]] - 4*(3*a^2 + c^2)*Log[a*Cos[(d + e*
x)/2] + c*Sin[(d + e*x)/2]] - c^2*Sec[(d + e*x)/2]^2 + (c^2*(a^2 + c^2))/(a
*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^2 + (6*c*(a^2 + c^2)*Sin[(d + e*x)/
2])/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]) + 6*a*c*Tan[(d + e*x)/2])/(64
*c^5*e)
```

Maple [A] time = 0.189, size = 211, normalized size = 1.6

$$\frac{1}{64\,ec^3}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2-\frac{3\,a}{32\,c^4e}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-\frac{a^4}{64\,ec^5}\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-2}-\frac{a^2}{32\,ec^3}\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-2}-\frac{1}{64\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 1/64/e/c^3*tan(1/2*d+1/2*e*x)^2-3/32/e/c^4*tan(1/2*d+1/2*e*x)*a-1/64/e/c^5/
(a+c*tan(1/2*d+1/2*e*x))^2*a^4-1/32/e/c^3/(a+c*tan(1/2*d+1/2*e*x))^2*a^2-1/
64/e/c/(a+c*tan(1/2*d+1/2*e*x))^2+3/16/e/c^5*ln(a+c*tan(1/2*d+1/2*e*x))*a^2
+1/16/e/c^3*ln(a+c*tan(1/2*d+1/2*e*x))+1/8/e*a^3/c^5/(a+c*tan(1/2*d+1/2*e*x
))+1/8/e*a/c^3/(a+c*tan(1/2*d+1/2*e*x))
```

Maxima [A] time = 1.06366, size = 257, normalized size = 1.92

$$\frac{\frac{7\,a^4+6\,a^2c^2-c^4+\frac{8\left(a^3c+ac^3\right)\sin(ex+d)}{\cos(ex+d)+1}}{a^2c^5+\frac{2\,ac^6\sin(ex+d)}{\cos(ex+d)+1}+\frac{c^7\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{\frac{6\,a\sin(ex+d)}{\cos(ex+d)+1}-\frac{c\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{c^4}+\frac{4\left(3\,a^2+c^2\right)\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}}{64\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/64*((7*a^4 + 6*a^2*c^2 - c^4 + 8*(a^3*c + a*c^3)*sin(e*x + d)/(cos(e*x +
d) + 1))/(a^2*c^5 + 2*a*c^6*sin(e*x + d)/(cos(e*x + d) + 1) + c^7*sin(e*x +
 d)^2/(cos(e*x + d) + 1)^2) - (6*a*sin(e*x + d)/(cos(e*x + d) + 1) - c*sin(
e*x + d)^2/(cos(e*x + d) + 1)^2)/c^4 + 4*(3*a^2 + c^2)*log(a + c*sin(e*x +
```

```
d)/(cos(e*x + d) + 1))/c^5)/e
```

Fricas [B] time = 2.35285, size = 986, normalized size = 7.36

$$\frac{12\,a^2c^2\cos\left(ex+d\right)^2-6\,a^2c^2+2\left(3\,a^2c^2-c^4\right)\cos\left(ex+d\right)+\left(3\,a^4+4\,a^2c^2+c^4+\left(3\,a^4-2\,a^2c^2-c^4\right)\cos\left(ex+d\right)^2+2\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 + 2*(3*a^2*c^2 - c^4)*cos(e*x +
 d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 +
 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 + (3*a^3*c + a*c^3)*
cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*
(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 -
 c^4)*cos(e*x + d)^2 + 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^
3 + (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(1/2*cos(e*x + d) + 1/
2) - 2*(3*a^3*c - a*c^3 + 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x + d))/(2*
a^2*c^5*e*cos(e*x + d) + (a^2*c^5 - c^7)*e*cos(e*x + d)^2 + (a^2*c^5 + c^7)
*e + 2*(a*c^6*e*cos(e*x + d) + a*c^6*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1766, size = 231, normalized size = 1.72

$$\frac{1}{64}\left(\frac{4\left(3\,a^2+c^2\right)\log\left(\left|c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right|\right)}{c^5}+\frac{c^3\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2-6\,ac^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)}{c^6}-\frac{18\,a^2c^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 + c^2)*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c^5 + (c^3*tan(1
/2*x*e + 1/2*d)^2 - 6*a*c^2*tan(1/2*x*e + 1/2*d))/c^6 - (18*a^2*c^2*tan(1/2
*x*e + 1/2*d)^2 + 6*c^4*tan(1/2*x*e + 1/2*d)^2 + 28*a^3*c*tan(1/2*x*e + 1/2
*d) + 4*a*c^3*tan(1/2*x*e + 1/2*d) + 11*a^4 + c^4)/((c*tan(1/2*x*e + 1/2*d)
 + a)^2*c^5))*e^(-1)
```

3.369 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=207

$$-\frac{a\left(5a^2+3c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)-a\left(15a^2+4c^2\right)\sin(d+ex)}{96c^6e(a\cos(d+ex)+a+c\sin(d+ex))}+\frac{5\left(ac\cos(d+\right.}{96c^4e(a\cos(d+}$$

```
[Out] -(a*(5*a^2 + 3*c^2)*Log[a + c*Tan[(d + e*x)/2]])/(32*c^7*e) - (c*Cos[d + e*
x] - a*Sin[d + e*x])/(48*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^3) + (
5*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(96*c^4*e*(a + a*Cos[d + e*x] + c*
Sin[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] - a*(15*a^2 + 4*c^2)*Si
n[d + e*x])/(96*c^6*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rubi [A] time = 0.248415, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3124, 31}

$$-\frac{a\left(5a^2+3c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)-a\left(15a^2+4c^2\right)\sin(d+ex)}{96c^6e(a\cos(d+ex)+a+c\sin(d+ex))}+\frac{5\left(ac\cos(d+\right.}{96c^4e(a\cos(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] -(a*(5*a^2 + 3*c^2)*Log[a + c*Tan[(d + e*x)/2]])/(32*c^7*e) - (c*Cos[d + e*
x] - a*Sin[d + e*x])/(48*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^3) + (
5*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(96*c^4*e*(a + a*Cos[d + e*x] + c*
Sin[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] - a*(15*a^2 + 4*c^2)*Si
n[d + e*x])/(96*c^6*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{\int \frac{-6a+4a\cos(d+ex)+4c\sin(d+e}{(2a+2a\cos(d+ex)+2c\sin(d+ex}}{12c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+ex)-a^2 s\right.}{96c^4e(a+a\cos(d+ex)+}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+ex)-a^2 s\right.}{96c^4e(a+a\cos(d+ex)+}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+ex)-a^2 s\right.}{96c^4e(a+a\cos(d+ex)+}$$

$$= -\frac{a\left(5a^2+3c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e} - \frac{c\cos(d+ex)-a\sin}{48c^2e(a+a\cos(d+ex)+}$$

Mathematica [B] time = 1.6272, size = 492, normalized size = 2.38

$$\frac{\cos\left(\frac{1}{2}(d+ex)\right)\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(\frac{c\left(255a^4c^2\sin(d+ex)+72a^4c^2\sin(2(d+ex))-37a^4c^2\sin(3(d+ex))+129a^2c^4\sin(d+ex)+\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (Cos[(d + e*x)/2]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])*(192*(5*a^3 + 3
*a*c^2)*Cos[(d + e*x)/2]^3*Log[Cos[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Si
n[(d + e*x)/2])^3 - 192*(5*a^3 + 3*a*c^2)*Cos[(d + e*x)/2]^3*Log[a*Cos[(d +
 e*x)/2] + c*Sin[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^3
+ (c*(150*a^5*c + 130*a^3*c^3 + 24*a*c^5 + 3*a*c*(25*a^4 + 25*a^2*c^2 - 4*c
^4)*Cos[d + e*x] - 6*(25*a^5*c + 15*a^3*c^3 + 4*a*c^5)*Cos[2*(d + e*x)] - 7
5*a^5*c*Cos[3*(d + e*x)] - 35*a^3*c^3*Cos[3*(d + e*x)] - 4*a*c^5*Cos[3*(d +
 e*x)] + 150*a^6*Sin[d + e*x] + 255*a^4*c^2*Sin[d + e*x] + 129*a^2*c^4*Sin[
d + e*x] + 12*c^6*Sin[d + e*x] + 120*a^6*Sin[2*(d + e*x)] + 72*a^4*c^2*Sin[
2*(d + e*x)] + 36*a^2*c^4*Sin[2*(d + e*x)] + 30*a^6*Sin[3*(d + e*x)] - 37*a
^4*c^2*Sin[3*(d + e*x)] - 27*a^2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*
x)]))/a))/(384*c^7*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^4)
```

Maple [A] time = 0.222, size = 378, normalized size = 1.8

$$\frac{1}{384c^4e}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^3-\frac{a}{64c^5e}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2+\frac{5a^2}{64ec^6}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{3}{128c^4e}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{3a^5}{128ec^7}\left(a+c\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x)
```

```
[Out] 1/384/e/c^4*tan(1/2*d+1/2*e*x)^3-1/64/e/c^5*tan(1/2*d+1/2*e*x)^2*a+5/64/e/c
^6*a^2*tan(1/2*d+1/2*e*x)+3/128/e/c^4*tan(1/2*d+1/2*e*x)+3/128/e*a^5/c^7/(a
+c*tan(1/2*d+1/2*e*x))^2+3/64/e*a^3/c^5/(a+c*tan(1/2*d+1/2*e*x))^2+3/128/e*
a/c^3/(a+c*tan(1/2*d+1/2*e*x))^2-1/384/e/c^7/(a+c*tan(1/2*d+1/2*e*x))^3*a^6
-1/128/e/c^5/(a+c*tan(1/2*d+1/2*e*x))^3*a^4-1/128/e/c^3/(a+c*tan(1/2*d+1/2*
e*x))^3*a^2-1/384/e/c/(a+c*tan(1/2*d+1/2*e*x))^3-5/32/e*a^3/c^7*ln(a+c*tan(
1/2*d+1/2*e*x))-3/32/e*a/c^5*ln(a+c*tan(1/2*d+1/2*e*x))-15/128/e/c^7/(a+c*t
an(1/2*d+1/2*e*x))*a^4-9/64/e/c^5/(a+c*tan(1/2*d+1/2*e*x))*a^2-3/128/e/c^3/
(a+c*tan(1/2*d+1/2*e*x))
```

Maxima [A] time = 1.14598, size = 414, normalized size = 2.

$$-\frac{\frac{37a^6+39a^4c^2+3a^2c^4+c^6+\frac{9(9a^5c+10a^3c^3+ac^5)\sin(ex+d)}{\cos(ex+d)+1}+\frac{9(5a^4c^2+6a^2c^4+c^6)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3c^7+\frac{3a^2c^8\sin(ex+d)}{\cos(ex+d)+1}+\frac{3ac^9\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{c^{10}\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}+\frac{\frac{6ac\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{c^2\sin(ex+d)^3}{(\cos(ex+d)+1)^3}-\frac{3(10a^2+3c^2)\sin(ex+d)}{\cos(ex+d)+1}}{c^6}+\frac{12(5a^3+3a\ldots}{}}{384e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*((37*a^6 + 39*a^4*c^2 + 3*a^2*c^4 + c^6 + 9*(9*a^5*c + 10*a^3*c^3 +
a*c^5)*sin(e*x + d)/(cos(e*x + d) + 1) + 9*(5*a^4*c^2 + 6*a^2*c^4 + c^6)*si
n(e*x + d)^2/(cos(e*x + d) + 1)^2)/(a^3*c^7 + 3*a^2*c^8*sin(e*x + d)/(cos(e
*x + d) + 1) + 3*a*c^9*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + c^10*sin(e*x +
 d)^3/(cos(e*x + d) + 1)^3) + (6*a*c*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 -
c^2*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 - 3*(10*a^2 + 3*c^2)*sin(e*x + d)/(
cos(e*x + d) + 1))/c^6 + 12*(5*a^3 + 3*a*c^2)*log(a + c*sin(e*x + d)/(cos(e
*x + d) + 1))/c^7)/e
```

Fricas [B] time = 2.70019, size = 1782, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*c^2 + 6*a^2*c^4 - 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(e*x
+ d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(e*x + d)^2 + 6*(5*a^4*c^2 - 2*a^2*c^
4 - 2*c^6)*cos(e*x + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 + (5*a^6 - 12*a
^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(
e*x + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*c + 1
4*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2 + 6*(
5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2
*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d)) + 3*(5*a^6 + 18*a^4*c^2 + 9*
a^2*c^4 + (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^
4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e
*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^
5)*cos(e*x + d)^2 + 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log
(1/2*cos(e*x + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5 + (15*a^5*c -
 41*a^3*c^3 - 12*a*c^5)*cos(e*x + d)^2 + 3*(10*a^5*c - 9*a^3*c^3 - a*c^5)*c
os(e*x + d))*sin(e*x + d))/((a^3*c^7 - 3*a*c^9)*e*cos(e*x + d)^3 + 3*(a^3*c
^7 - a*c^9)*e*cos(e*x + d)^2 + 3*(a^3*c^7 + a*c^9)*e*cos(e*x + d) + (a^3*c^
7 + 3*a*c^9)*e + (6*a^2*c^8*e*cos(e*x + d) + (3*a^2*c^8 - c^10)*e*cos(e*x +
 d)^2 + (3*a^2*c^8 + c^10)*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14161, size = 410, normalized size = 1.98

$$-\frac{1}{384}\left(\frac{12\left(5\,a^3+3\,ac^2\right)\log\left(\left|c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right|\right)}{c^7}-\frac{110\,a^3c^3\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3+66\,ac^5\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3+285\,a^4c^2\,\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c^7 - (11
0*a^3*c^3*tan(1/2*x*e + 1/2*d)^3 + 66*a*c^5*tan(1/2*x*e + 1/2*d)^3 + 285*a^
4*c^2*tan(1/2*x*e + 1/2*d)^2 + 144*a^2*c^4*tan(1/2*x*e + 1/2*d)^2 - 9*c^6*t
an(1/2*x*e + 1/2*d)^2 + 249*a^5*c*tan(1/2*x*e + 1/2*d) + 108*a^3*c^3*tan(1/
2*x*e + 1/2*d) - 9*a*c^5*tan(1/2*x*e + 1/2*d) + 73*a^6 + 27*a^4*c^2 - 3*a^2
*c^4 - c^6)/((c*tan(1/2*x*e + 1/2*d) + a)^3*c^7) - (c^8*tan(1/2*x*e + 1/2*d
)^3 - 6*a*c^7*tan(1/2*x*e + 1/2*d)^2 + 30*a^2*c^6*tan(1/2*x*e + 1/2*d) + 9*
c^8*tan(1/2*x*e + 1/2*d))/c^12)*e^(-1)
```

3.370 $\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(2*a*e)
```

Rubi [A] time = 0.0213659, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3124, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(2*a*e)
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{4a+4ax}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ae}$$

Mathematica [B] time = 0.0296963, size = 50, normalized size = 2.17

$$\frac{\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e}-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] (-(Log[Cos[(d + e*x)/2]]/e) + Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/e)/(
2*a)
```

Maple [A] time = 0.065, size = 21, normalized size = 0.9

$$\frac{1}{2\,ae}\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] 1/2*ln(1+tan(1/2*d+1/2*e*x))/a/e
```

Maxima [A] time = 0.987027, size = 38, normalized size = 1.65

$$\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{2\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/(a*e)
```

Fricas [A] time = 2.03023, size = 89, normalized size = 3.87

$$-\frac{\log\left(\frac{1}{2}\cos\left(ex+d\right)+\frac{1}{2}\right)-\log\left(\sin\left(ex+d\right)+1\right)}{4\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(1/2*cos(e*x + d) + 1/2) - log(sin(e*x + d) + 1))/(a*e)
```

Sympy [A] time = 0.719413, size = 36, normalized size = 1.57

$$\begin{cases} \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2ae} & \text{for } e\neq 0 \\ \frac{x}{2a\sin\left(d\right)+2a\cos\left(d\right)+2a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((log(tan(d/2 + e*x/2) + 1)/(2*a*e), Ne(e, 0)), (x/(2*a*sin(d) + 2
*a*cos(d) + 2*a), True))
```

Giac [A] time = 1.13198, size = 28, normalized size = 1.22

$$\frac{e^{(-1)}\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+1\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*e^(-1)*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a
```

3.371 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^2e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}$$

```
[Out] -Log[1 + Tan[(d + e*x)/2]]/(4*a^2*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(4
*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rubi [A] time = 0.0479758, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3124, 31}

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^2e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] -Log[1 + Tan[(d + e*x)/2]]/(4*a^2*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(4
*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}+\frac{\int -\frac{2a}{2a+2a\cos(d+ex)+2a\sin(d+ex)} \cdots}{4a^2}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}-\frac{\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\, d \cdots}{2a}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}-\frac{\text{Subst}\left(\int \frac{1}{4a+4ax}\,dx, x, \tan\left(\cdots \right.\right.}{ae}$$

$$= -\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^2e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}$$

Mathematica [A] time = 0.183596, size = 93, normalized size = 1.24

$$\frac{\tan\left(\frac{1}{2}(d+ex)\right)+2\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)+\frac{2\sin\left(\frac{1}{2}(d+ex)\right)}{\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)}-2\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)}{8a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*Log[Cos[(d + e*x)/2]] - 2*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + (2*
Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + Tan[(d + e*x)/2])
/(8*a^2*e)
```

Maple [A] time = 0.086, size = 60, normalized size = 0.8

$$\frac{1}{8a^2e}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-\frac{1}{4a^2e}\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}-\frac{1}{4a^2e}\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x)
```

```
[Out] 1/8/e/a^2*tan(1/2*d+1/2*e*x)-1/4/e/a^2/(1+tan(1/2*d+1/2*e*x))-1/4*ln(1+tan(
1/2*d+1/2*e*x))/a^2/e
```

Maxima [A] time = 1.01651, size = 108, normalized size = 1.44

$$-\frac{\frac{2}{a^2+\frac{a^2\sin(ex+d)}{\cos(ex+d)+1}}+\frac{2\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^2}-\frac{\sin(ex+d)}{a^2(\cos(ex+d)+1)}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2/(a^2 + a^2*sin(e*x + d)/(cos(e*x + d) + 1)) + 2*log(sin(e*x + d)/(c
os(e*x + d) + 1) + 1)/a^2 - sin(e*x + d)/(a^2*(cos(e*x + d) + 1)))/e
```

Fricas [A] time = 2.12157, size = 285, normalized size = 3.8

$$\frac{(\cos(ex+d)+\sin(ex+d)+1)\log\left(\frac{1}{2}\cos(ex+d)+\frac{1}{2}\right)-(\cos(ex+d)+\sin(ex+d)+1)\log(\sin(ex+d)+1)-2\cos}{8\left(a^2e\cos(ex+d)+a^2e\sin(ex+d)+a^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((cos(e*x + d) + sin(e*x + d) + 1)*log(1/2*cos(e*x + d) + 1/2) - (cos(e
*x + d) + sin(e*x + d) + 1)*log(sin(e*x + d) + 1) - 2*cos(e*x + d) + 2*sin(
```

```
e*x + d))/(a^2*e*cos(e*x + d) + a^2*e*sin(e*x + d) + a^2*e)
```

Sympy [A] time = 2.48721, size = 168, normalized size = 2.24

$$\begin{cases} -\frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} - \frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} + \frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} - \frac{3}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} & \text{for } e \neq 0 \\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((-2*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(8*a**2*e*tan(d/2
+ e*x/2) + 8*a**2*e) - 2*log(tan(d/2 + e*x/2) + 1)/(8*a**2*e*tan(d/2 + e*x/
2) + 8*a**2*e) + tan(d/2 + e*x/2)**2/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2*e)
 - 3/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*
a*cos(d) + 2*a)**2, True))
```

Giac [A] time = 1.14342, size = 92, normalized size = 1.23

$$-\frac{1}{8}\left(\frac{2\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right|\right)}{a^2} - \frac{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)}{a^2} - \frac{2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)}{a^2\left(\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^2 - tan(1/2*x*e + 1/2*d)/a^2 -
 2*tan(1/2*x*e + 1/2*d)/(a^2*(tan(1/2*x*e + 1/2*d) + 1)))*e^(-1)
```

3.372 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^3e}+\frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(4*a^3*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(16
*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) + (3*(Cos[d + e*x] - Sin[
d + e*x]))/(16*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rubi [A] time = 0.107318, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3124, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^3e}+\frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(4*a^3*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(16
*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) + (3*(Cos[d + e*x] - Sin[
d + e*x]))/(16*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
```

```
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx = -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{\int\frac{-4a+2a\cos(d+ex)+2a\sin(d+}{(2a+2a\cos(d+ex)+2a\sin(d+e}}{8a^2}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{3(\cos(d+ex)-\sin}{16e\left(a^3+a^3\cos(d+ex)+\right.}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{3(\cos(d+ex)-\sin}{16e\left(a^3+a^3\cos(d+ex)+\right.}$$

$$= \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^3e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}$$

Mathematica [A] time = 0.582234, size = 135, normalized size = 1.1

$$\frac{\sec^2\left(\frac{1}{2}(d+ex)\right)+2\left(-3\tan\left(\frac{1}{2}(d+ex)\right)-8\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-\frac{6\sin\left(\frac{1}{2}(d+ex)\right)}{\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)}-\frac{1}{\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)^2}+\right.}{64a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] (Sec[(d + e*x)/2]^2 + 2*(-8*Log[Cos[(d + e*x)/2]] + 8*Log[Cos[(d + e*x)/2]
+ Sin[(d + e*x)/2]] - (Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^(-2) - (6*Sin[(
d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) - 3*Tan[(d + e*x)/2]))/(
64*a^3*e)
```

Maple [A] time = 0.099, size = 100, normalized size = 0.8

$$\frac{1}{64\,a^3e}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2-\frac{3}{32\,a^3e}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{1}{4\,a^3e}\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}-\frac{1}{16\,a^3e}\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-2}+\frac{1}{4\,a^3e}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x)
```

```
[Out] 1/64/e/a^3*tan(1/2*d+1/2*e*x)^2-3/32/e/a^3*tan(1/2*d+1/2*e*x)+1/4/e/a^3/(1+
tan(1/2*d+1/2*e*x))-1/16/e/a^3/(1+tan(1/2*d+1/2*e*x))^2+1/4*ln(1+tan(1/2*d+
1/2*e*x))/a^3/e
```

Maxima [A] time = 1.07133, size = 197, normalized size = 1.6

$$\frac{\frac{4\left(\frac{4\,\sin(ex+d)}{\cos(ex+d)+1}+3\right)}{a^3+\frac{2\,a^3\sin(ex+d)}{\cos(ex+d)+1}+\frac{a^3\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{\frac{6\,\sin(ex+d)}{\cos(ex+d)+1}-\frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3}+\frac{16\,\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^3}}{64\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(4*sin(e*x + d)/(cos(e*x + d) + 1) + 3)/(a^3 + 2*a^3*sin(e*x + d)/(
cos(e*x + d) + 1) + a^3*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - (6*sin(e*x +
 d)/(cos(e*x + d) + 1) - sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/a^3 + 16*log(
sin(e*x + d)/(cos(e*x + d) + 1) + 1)/a^3)/e
```

Fricas [A] time = 2.13307, size = 405, normalized size = 3.29

$$\frac{6 \cos(ex+d)^2 - 4\,((\cos(ex+d)+1)\sin(ex+d) + \cos(ex+d) + 1)\log\left(\frac{1}{2}\cos(ex+d) + \frac{1}{2}\right) + 4\,((\cos(ex+d)+1)\sin(}{32\left(a^3 e\cos(ex+d) + a^3 e + \left(a^3 e\cos(ex+d) + a^3 e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(e*x + d)^2 - 4*((cos(e*x + d) + 1)*sin(e*x + d) + cos(e*x + d)
+ 1)*log(1/2*cos(e*x + d) + 1/2) + 4*((cos(e*x + d) + 1)*sin(e*x + d) + cos
(e*x + d) + 1)*log(sin(e*x + d) + 1) + 2*cos(e*x + d) - 2*sin(e*x + d) - 3)
/(a^3*e*cos(e*x + d) + a^3*e + (a^3*e*cos(e*x + d) + a^3*e)*sin(e*x + d))
```

Sympy [A] time = 9.65431, size = 423, normalized size = 3.44

$$\begin{cases} \frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{32\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{}{64a} \\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((16*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(64*a**3*e*tan(
d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 32*log(tan(d/2
 + e*x/2) + 1)*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e
*tan(d/2 + e*x/2) + 64*a**3*e) + 16*log(tan(d/2 + e*x/2) + 1)/(64*a**3*e*ta
n(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + tan(d/2 + e*
x/2)**4/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a
**3*e) - 4*tan(d/2 + e*x/2)**3/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*
tan(d/2 + e*x/2) + 64*a**3*e) + 32*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*
x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 23/(64*a**3*e*tan(d/2
+ e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e), Ne(e, 0)), (x/(2*a*
sin(d) + 2*a*cos(d) + 2*a)**3, True))
```

Giac [A] time = 1.15918, size = 144, normalized size = 1.17

$$\frac{1}{64}\left(\frac{16\,\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+1\right|\right)}{a^3}-\frac{4\left(6\,\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+8\,\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+3\right)}{a^3\left(\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+1\right)^2}+\frac{a^3\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2-6\,a^3\,\mathrm{ta}}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(16*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^3 - 4*(6*tan(1/2*x*e + 1/2*d)
^2 + 8*tan(1/2*x*e + 1/2*d) + 3)/(a^3*(tan(1/2*x*e + 1/2*d) + 1)^2) + (a^3*
tan(1/2*x*e + 1/2*d)^2 - 6*a^3*tan(1/2*x*e + 1/2*d))/a^6)*e^(-1)
```

3.373 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=168

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^4e}-\frac{19(a\cos(d+ex)-a\sin(d+ex))}{96e\left(a^5\sin(d+ex)+a^5\cos(d+ex)+a^5\right)}+\frac{5(\cos(d+ex)-\sin(d+ex))}{96e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}-$$

```
[Out] -Log[1 + Tan[(d + e*x)/2]]/(4*a^4*e) - (Cos[d + e*x] - Sin[d + e*x])/(48*a*
e*(a + a*Cos[d + e*x] + a*Sin[d + e*x])^3) + (5*(Cos[d + e*x] - Sin[d + e*x
]))/(96*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) - (19*(a*Cos[d + e
*x] - a*Sin[d + e*x]))/(96*e*(a^5 + a^5*Cos[d + e*x] + a^5*Sin[d + e*x]))
```

Rubi [A] time = 0.186493, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3124, 31}

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^4e}-\frac{19(a\cos(d+ex)-a\sin(d+ex))}{96e\left(a^5\sin(d+ex)+a^5\cos(d+ex)+a^5\right)}+\frac{5(\cos(d+ex)-\sin(d+ex))}{96e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] -Log[1 + Tan[(d + e*x)/2]]/(4*a^4*e) - (Cos[d + e*x] - Sin[d + e*x])/(48*a*
e*(a + a*Cos[d + e*x] + a*Sin[d + e*x])^3) + (5*(Cos[d + e*x] - Sin[d + e*x
]))/(96*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) - (19*(a*Cos[d + e
*x] - a*Sin[d + e*x]))/(96*e*(a^5 + a^5*Cos[d + e*x] + a^5*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a + 2a\cos(d+ex) + 2a\sin(d+ex))^4} dx = -\frac{\cos(d+ex) - \sin(d+ex)}{48ae(a + a\cos(d+ex) + a\sin(d+ex))^3} + \frac{\int \frac{-6a+4a\cos(d+ex)+4a\sin(d+ex)}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}}{12a^2}$$

$$= -\frac{\cos(d+ex) - \sin(d+ex)}{48ae(a + a\cos(d+ex) + a\sin(d+ex))^3} + \frac{5(\cos(d+ex) - \sin(}{96e\left(a^2 + a^2\cos(d+ex) + a\right.}$$

$$= -\frac{\cos(d+ex) - \sin(d+ex)}{48ae(a + a\cos(d+ex) + a\sin(d+ex))^3} + \frac{5(\cos(d+ex) - \sin(}{96e\left(a^2 + a^2\cos(d+ex) + a\right.}$$

$$= -\frac{\cos(d+ex) - \sin(d+ex)}{48ae(a + a\cos(d+ex) + a\sin(d+ex))^3} + \frac{5(\cos(d+ex) - \sin(}{96e\left(a^2 + a^2\cos(d+ex) + a\right.}$$

$$= -\frac{\log\left(1 + \tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\cos(d+ex) - \sin(d+ex)}{48ae(a + a\cos(d+ex) + a\sin(d+ex))^3} +$$

Mathematica [A] time = 0.973845, size = 247, normalized size = 1.47

$$\frac{19\tan\left(\frac{1}{2}(d+ex)\right)}{192a^4e} - \frac{\sec^2\left(\frac{1}{2}(d+ex)\right)}{64a^4e} + \frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} + \frac{19\sin\left(\frac{1}{2}(d+ex)\right)}{96a^4e\left(\sin\left(\frac{1}{2}(d+ex)\right) + \cos\left(\frac{1}{2}(d+ex)\right)\right)} + \frac{}{192a^4e\left(\mathrm{si}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] Log[Cos[(d + e*x)/2]]/(4*a^4*e) - Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/
(4*a^4*e) - Sec[(d + e*x)/2]^2/(64*a^4*e) + Sin[(d + e*x)/2]/(96*a^4*e*(Cos
[(d + e*x)/2] + Sin[(d + e*x)/2])^3) + 5/(192*a^4*e*(Cos[(d + e*x)/2] + Sin
[(d + e*x)/2])^2) + (19*Sin[(d + e*x)/2])/(96*a^4*e*(Cos[(d + e*x)/2] + Sin
[(d + e*x)/2])) + (19*Tan[(d + e*x)/2])/(192*a^4*e) + (Sec[(d + e*x)/2]^2*T
an[(d + e*x)/2])/(384*a^4*e)
```

Maple [A] time = 0.112, size = 140, normalized size = 0.8

$$\frac{1}{384\,ea^4}\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^3 - \frac{1}{64\,ea^4}\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2 + \frac{13}{128\,ea^4}\tan\left(\frac{d}{2} + \frac{ex}{2}\right) - \frac{9}{32\,ea^4}\left(1 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^{-1} + \frac{3}{32\,ea^4}\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x)
```

```
[Out] 1/384/e/a^4*tan(1/2*d+1/2*e*x)^3-1/64/e/a^4*tan(1/2*d+1/2*e*x)^2+13/128/e/a
^4*tan(1/2*d+1/2*e*x)-9/32/e/a^4/(1+tan(1/2*d+1/2*e*x))+3/32/e/a^4/(1+tan(1
/2*d+1/2*e*x))^2-1/4*ln(1+tan(1/2*d+1/2*e*x))/a^4/e-1/48/e/a^4/(1+tan(1/2*d
+1/2*e*x))^3
```

Maxima [A] time = 1.12096, size = 281, normalized size = 1.67

$$-\frac{\frac{4\left(\frac{45\sin(ex+d)}{\cos(ex+d)+1}+\frac{27\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+20\right)}{a^4+\frac{3a^4\sin(ex+d)}{\cos(ex+d)+1}+\frac{3a^4\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{a^4\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}-\frac{\frac{39\sin(ex+d)}{\cos(ex+d)+1}-\frac{6\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{a^4}+\frac{96\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^4}}{384e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*(4*(45*sin(e*x + d)/(cos(e*x + d) + 1) + 27*sin(e*x + d)^2/(cos(e*x
+ d) + 1)^2 + 20)/(a^4 + 3*a^4*sin(e*x + d)/(cos(e*x + d) + 1) + 3*a^4*sin(
e*x + d)^2/(cos(e*x + d) + 1)^2 + a^4*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)
- (39*sin(e*x + d)/(cos(e*x + d) + 1) - 6*sin(e*x + d)^2/(cos(e*x + d) + 1)
^2 + sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/a^4 + 96*log(sin(e*x + d)/(cos(e*
x + d) + 1) + 1)/a^4)/e
```

Fricas [A] time = 2.14268, size = 645, normalized size = 3.84

$$\frac{38\cos(ex+d)^3+66\cos(ex+d)^2+24\left(\cos(ex+d)^3-\left(\cos(ex+d)^2+3\cos(ex+d)+2\right)\sin(ex+d)-3\cos(ex+\right.}{192\left(a^4e\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(38*cos(e*x + d)^3 + 66*cos(e*x + d)^2 + 24*(cos(e*x + d)^3 - (cos(e*
x + d)^2 + 3*cos(e*x + d) + 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(1/2*c
```

```
os(e*x + d) + 1/2) - 24*(cos(e*x + d)^3 - (cos(e*x + d)^2 + 3*cos(e*x + d)
+ 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(sin(e*x + d) + 1) + (38*cos(e*x
 + d)^2 - 35)*sin(e*x + d) - 3*cos(e*x + d) - 33)/(a^4*e*cos(e*x + d)^3 - 3
*a^4*e*cos(e*x + d) - 2*a^4*e - (a^4*e*cos(e*x + d)^2 + 3*a^4*e*cos(e*x + d
) + 2*a^4*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1494, size = 188, normalized size = 1.12

$$-\frac{1}{384}\left(\frac{96\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right|\right)}{a^4}-\frac{4\left(44\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^3+105\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2+87\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+24\right)}{a^4\left(\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)^3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(96*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^4 - 4*(44*tan(1/2*x*e + 1/2
*d)^3 + 105*tan(1/2*x*e + 1/2*d)^2 + 87*tan(1/2*x*e + 1/2*d) + 24)/(a^4*(ta
n(1/2*x*e + 1/2*d) + 1)^3) - (a^8*tan(1/2*x*e + 1/2*d)^3 - 6*a^8*tan(1/2*x*
e + 1/2*d)^2 + 39*a^8*tan(1/2*x*e + 1/2*d))/a^12)*e^(-1)
```

3.374 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))^3 \, dx$

Optimal. Leaf size=157

$$-\frac{4a(15a^2+4c^2)\sin(d+ex)}{3e} - \frac{4c(15a^2+4c^2)\cos(d+ex)}{3e} + 4ax(5a^2+3c^2) - \frac{20(a^2\sin(d+ex)+ac\cos(d+ex))(a(}{3e}$$

```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) - (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] + a^2*Sin[d + e*
x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] + a*S
in[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```

Rubi [A] time = 0.133825, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$-\frac{4a(15a^2+4c^2)\sin(d+ex)}{3e} - \frac{4c(15a^2+4c^2)\cos(d+ex)}{3e} + 4ax(5a^2+3c^2) - \frac{20(a^2\sin(d+ex)+ac\cos(d+ex))(a(}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) - (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] + a^2*Sin[d + e*
x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] + a*S
in[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
```

```
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx = -\frac{8(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin(d+ex))^2}{3e}+\frac{1}{3}$$

$$= -\frac{20\left(ac\cos(d+ex)+a^2\sin(d+ex)\right)(a-a\cos(d+ex)+c\sin(d+ex))}{3e}-$$

$$= 4a\left(5a^2+3c^2\right)x-\frac{20\left(ac\cos(d+ex)+a^2\sin(d+ex)\right)(a-a\cos(d+ex)+$$

$$= 4a\left(5a^2+3c^2\right)x-\frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e}-\frac{4a\left(15a^2+4c^2\right)\sin(d+e$$

Mathematica [A] time = 0.446984, size = 136, normalized size = 0.87

$$\frac{2\left(6a\left(5a^2+3c^2\right)(d+ex)-9a\left(5a^2+c^2\right)\sin(d+ex)+9a\left(a^2-c^2\right)\sin(2(d+ex))-a\left(a^2-3c^2\right)\sin(3(d+ex))-9c\left(5a^2\right.\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] + 18*a^2
*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] - 9*a*(5*a^2 + c^2)
*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] - a*(a^2 - 3*c^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A] time = 0.075, size = 178, normalized size = 1.1

$$8\,\frac{-1/3\,a^3\left(2+(\cos(ex+d))^2\right)\sin(ex+d)-a^2c\,(\cos(ex+d))^3+3\,a^3\,(1/2\,\sin(ex+d)\cos(ex+d)+1/2\,ex+d/2)-ac^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(-1/3*a^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a^2*c*cos(e*x+d)^3+3*a^3*(1/2*sin
(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-a*c^2*sin(e*x+d)^3+3*a^2*c*cos(e*x+d)^2-3
*a^3*sin(e*x+d)-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)+3*a*c^2*(-1/2*sin(e*x+d
)*cos(e*x+d)+1/2*e*x+1/2*d)-3*a^2*c*cos(e*x+d)+a^3*(e*x+d))
```

Maxima [A] time = 0.99652, size = 254, normalized size = 1.62

$$-\frac{8\,a^2c\cos(ex+d)^3}{e}-\frac{8\,ac^2\sin(ex+d)^3}{e}+8\,a^3x+\frac{8\left(\sin(ex+d)^3-3\,\sin(ex+d)\right)a^3}{3\,e}+\frac{8\left(\cos(ex+d)^3-3\,\cos(ex+\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*c*cos(e*x + d)^3/e - 8*a*c^2*sin(e*x + d)^3/e + 8*a^3*x + 8/3*(sin(e
*x + d)^3 - 3*sin(e*x + d))*a^3/e + 8/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c
^3/e - 24*a^2*(c*cos(e*x + d)/e + a*sin(e*x + d)/e) + 6*(4*a*c*cos(e*x + d)
^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d - sin(2*e*x +
2*d))*c^2/e)*a
```

Fricas [A] time = 2.26303, size = 306, normalized size = 1.95

$$\frac{4\left(18\,a^2c\cos(ex+d)^2-2\left(3\,a^2c-c^3\right)\cos(ex+d)^3+3\left(5\,a^3+3\,ac^2\right)ex-6\left(3\,a^2c+c^3\right)\cos(ex+d)-\left(22\,a^3+6\,ac^2+2\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(18*a^2*c*cos(e*x + d)^2 - 2*(3*a^2*c - c^3)*cos(e*x + d)^3 + 3*(5*a^3
+ 3*a*c^2)*e*x - 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*(a^
3 - 3*a*c^2)*cos(e*x + d)^2 - 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.906979, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x - \frac{16a^3\sin^3(d+ex)}{3e} - \frac{8a^3\sin(d+ex)\cos^2(d+ex)}{e} + \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} - \frac{24a^3\sin(}{e} \\ x(-2a\cos(d) + 2a + 2c\sin(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 - 16*a**3*sin(d + e*x)**3/(3*e) - 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e +
12*a**3*sin(d + e*x)*cos(d + e*x)/e - 24*a**3*sin(d + e*x)/e - 24*a**2*c*si
n(d + e*x)**2/e - 8*a**2*c*cos(d + e*x)**3/e - 24*a**2*c*cos(d + e*x)/e + 1
2*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 - 8*a*c**2*sin(d +
 e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e*x)**2
*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(-2*a*cos(d)
 + 2*a + 2*c*sin(d))**3, True))
```

Giac [A] time = 1.13794, size = 204, normalized size = 1.3

$$12\,a^2c\cos(2\,xe + 2\,d)\,e^{(-1)} - \frac{2}{3}\left(3\,a^2c - c^3\right)\cos(3\,xe + 3\,d)\,e^{(-1)} - 6\left(5\,a^2c + c^3\right)\cos(xe + d)\,e^{(-1)} - \frac{2}{3}\left(a^3 - 3\,ac^2\right)e^{(-1)}\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 12*a^2*c*cos(2*x*e + 2*d)*e^(-1) - 2/3*(3*a^2*c - c^3)*cos(3*x*e + 3*d)*e^(
-1) - 6*(5*a^2*c + c^3)*cos(x*e + d)*e^(-1) - 2/3*(a^3 - 3*a*c^2)*e^(-1)*si
n(3*x*e + 3*d) + 6*(a^3 - a*c^2)*e^(-1)*sin(2*x*e + 2*d) - 6*(5*a^3 + a*c^2
)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*c^2)*x
```

3.375 $\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2x\left(3a^2+c^2\right)-\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex}{e}$$

```
[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e - (6*a^2*Sin[d + e*x])/e - (2*(c
*Cos[d + e*x] + a*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/e
```

Rubi [A] time = 0.0469545, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2+c^2\right)-\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e - (6*a^2*Sin[d + e*x])/e - (2*(c
*Cos[d + e*x] + a*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/e
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = -\frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin(d+ex))}{e}+\frac{1}{2}\int$$
$$= 2\left(3a^2+c^2\right)x-\frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin($$
$$= 2\left(3a^2+c^2\right)x-\frac{6ac\cos(d+ex)}{e}-\frac{6a^2\sin(d+ex)}{e}-\frac{2(c\cos(d+ex)+a\,\mathrm{si}}{}$$

Mathematica [A] time = 0.154857, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2+c^2\right)(d+ex)}{2e}+\frac{\left(a^2-c^2\right)\sin(2(d+ex))}{4e}-\frac{2a^2\sin(d+ex)}{e}-\frac{2ac\cos(d+ex)}{e}+\frac{ac\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e + (a*c*Cos[2*(d
 + e*x)])/(2*e) - (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A] time = 0.054, size = 100, normalized size = 1.2

$$4\,\frac{a^2\left(1/2\,\sin(ex+d)\cos(ex+d)+1/2\,ex+d/2\right)+ac\left(\cos(ex+d)\right)^2-2\,a^2\sin(ex+d)+c^2\left(-1/2\,\sin(ex+d)\cos(ex+d\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+a*c*cos(e*x+d)^2-2*a^2*s
in(e*x+d)+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-2*a*c*cos(e*x+d)+a
^2*(e*x+d))
```

Maxima [A] time = 0.986615, size = 132, normalized size = 1.63

$$4\,a^2x+\frac{4\,ac\cos(ex+d)^2}{e}+\frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))a^2}{e}+\frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))c^2}{e}-8\,a\left(\frac{c\cos(ex+d)}{e}+\frac{a\,\mathrm{si}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x + 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e + a*sin(e*x
 + d)/e)
```

Fricas [A] time = 2.29069, size = 161, normalized size = 1.99

$$\frac{2\left(2\,ac\cos\left(ex+d\right)^{2}+\left(3\,a^{2}+c^{2}\right)ex-4\,ac\cos\left(ex+d\right)-\left(4\,a^{2}-\left(a^{2}-c^{2}\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 2*(2*a*c*cos(e*x + d)^2 + (3*a^2 + c^2)*e*x - 4*a*c*cos(e*x + d) - (4*a^2 -
 (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.381613, size = 170, normalized size = 2.1

$$\begin{cases} 2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x+\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{8a^2\sin\left(d+ex\right)}{e}-\frac{4ac\sin^2\left(d+ex\right)}{e}-\frac{8ac\cos\left(d+ex\right)}{e}+2c^2x \\ x\left(-2a\cos\left(d\right)+2a+2c\sin\left(d\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x +
 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*sin(d + e*x)/e - 4*a*c*sin(d +
 e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos
(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*cos(
d) + 2*a + 2*c*sin(d))**2, True))
```

Giac [A] time = 1.13912, size = 105, normalized size = 1.3

$$2\,ac\cos\left(2\,xe+2\,d\right)e^{(-1)}-8\,ac\cos\left(xe+d\right)e^{(-1)}-8\,a^{2}e^{(-1)}\sin\left(xe+d\right)+\left(a^{2}-c^{2}\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+2\left(3\,a^{2}+c^{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*a*c*cos(2*x*e + 2*d)*e^(-1) - 8*a*c*cos(x*e + d)*e^(-1) - 8*a^2*e^(-1)*si
n(x*e + d) + (a^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + c^2)*x
```

3.376 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx$

Optimal. Leaf size=29

$$-\frac{2a\sin(d+ex)}{e} + 2ax - \frac{2c\cos(d+ex)}{e}$$

```
[Out] 2*a*x - (2*c*Cos[d + e*x])/e - (2*a*Sin[d + e*x])/e
```

Rubi [A] time = 0.014309, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$-\frac{2a\sin(d+ex)}{e} + 2ax - \frac{2c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d + e*x])/e - (2*a*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx &= 2ax - (2a)\int \cos(d+ex)\,dx + (2c)\int \sin(d+ex)\,dx\\ &= 2ax - \frac{2c\cos(d+ex)}{e} - \frac{2a\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0148812, size = 53, normalized size = 1.83

$$-\frac{2a\sin(d)\cos(ex)}{e}-\frac{2a\cos(d)\sin(ex)}{e}+2ax+\frac{2c\sin(d)\sin(ex)}{e}-\frac{2c\cos(d)\cos(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d]*Cos[e*x])/e - (2*a*Cos[e*x]*Sin[d])/e - (2*a*Cos[d]*Sin
[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

Maple [A] time = 0.001, size = 30, normalized size = 1.

$$2\,ax-2\,\frac{c\cos\left(ex+d\right)}{e}-2\,\frac{a\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e
```

Maxima [A] time = 0.979215, size = 39, normalized size = 1.34

$$2\,ax-\frac{2\,c\cos\left(ex+d\right)}{e}-\frac{2\,a\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x - 2*c*cos(e*x + d)/e - 2*a*sin(e*x + d)/e
```

Fricas [A] time = 2.00739, size = 63, normalized size = 2.17

$$\frac{2\left(aex-c\cos\left(ex+d\right)-a\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x - c*cos(e*x + d) - a*sin(e*x + d))/e
```

Sympy [A] time = 0.15735, size = 39, normalized size = 1.34

$$2ax - 2a\left(\begin{cases} \frac{\sin(d+ex)}{e} & \text{for } e \neq 0 \\ x\cos(d) & \text{otherwise} \end{cases}\right) + 2c\left(\begin{cases} -\frac{\cos(d+ex)}{e} & \text{for } e \neq 0 \\ x\sin(d) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x - 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*P
iecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

Giac [A] time = 1.13433, size = 39, normalized size = 1.34

$$-2\,c\cos\left(xe + d\right)e^{(-1)} - 2\,ae^{(-1)}\sin\left(xe + d\right) + 2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -2*c*cos(x*e + d)*e^(-1) - 2*a*e^(-1)*sin(x*e + d) + 2*a*x
```

3.377 $\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

Optimal. Leaf size=25

$$-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

```
[Out] -Log[a + c*Cot[(d + e*x)/2]]/(2*c*e)
```

Rubi [A] time = 0.0210431, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3121, 31}

$$-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] -Log[a + c*Cot[(d + e*x)/2]]/(2*c*e)
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{2a+2cx}\,dx, x, \cot\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= -\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Mathematica [A] time = 0.151511, size = 50, normalized size = 2.

$$\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)-\log\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] (Log[Sin[(d + e*x)/2]] - Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])/(2*c
*e)
```

Maple [A] time = 0.1, size = 42, normalized size = 1.7

$$-\frac{1}{2\,ce}\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{1}{2\,ce}\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] -1/2/e/c*ln(c+a*tan(1/2*d+1/2*e*x))+1/2/e/c*ln(tan(1/2*d+1/2*e*x))
```

Maxima [B] time = 0.986877, size = 73, normalized size = 2.92

$$-\frac{\frac{\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c}-\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*(log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c - log(sin(e*x + d)/(cos(
e*x + d) + 1))/c)/e
```

Fricas [B] time = 2.13042, size = 159, normalized size = 6.36

$$-\frac{\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}a^2+\frac{1}{2}c^2-\frac{1}{2}\left(a^2-c^2\right)\cos\left(ex+d\right)\right)-\log\left(-\frac{1}{2}\cos\left(ex+d\right)+\frac{1}{2}\right)}{4\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x +
d)) - log(-1/2*cos(e*x + d) + 1/2))/(c*e)
```

Sympy [A] time = 1.46485, size = 95, normalized size = 3.8

$$\begin{cases} \frac{\tilde{\infty}x}{\sin\left(d\right)} & \text{for}\: a = 0 \wedge c = 0 \wedge e = 0 \\ -\frac{1}{2ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)} & \text{for}\: c = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{for}\: a = 0 \\ \frac{x}{-2a\cos\left(d\right)+2a+2c\sin\left(d\right)} & \text{for}\: e = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}\right)}{2ce}+\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/sin(d), Eq(a, 0) & Eq(c, 0) & Eq(e, 0)), (-1/(2*a*e*tan(d/
2 + e*x/2)), Eq(c, 0)), (log(tan(d/2 + e*x/2))/(2*c*e), Eq(a, 0)), (x/(-2*a
*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (-log(tan(d/2 + e*x/2) + c/a)/(2*c*
e) + log(tan(d/2 + e*x/2))/(2*c*e), True))
```

Giac [A] time = 1.14731, size = 57, normalized size = 2.28

$$-\frac{1}{2}\left(\frac{\log\left(\left|a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c\right|\right)}{c}-\frac{\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)\right|\right)}{c}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(a*tan(1/2*x*e + 1/2*d) + c))/c - log(abs(tan(1/2*x*e + 1/2*d)
))/c)*e^(-1)
```

3.378 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$\frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

```
[Out] (a*Log[a + c*Cot[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] + a*Sin[d + e*x
])/(4*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rubi [A] time = 0.0531722, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3121, 31}

$$\frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[a + c*Cot[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] + a*Sin[d + e*x
])/(4*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a - 2a\cos(d+ex) + 2c\sin(d+ex))^2}\,dx = -\frac{c\cos(d+ex) + a\sin(d+ex)}{4c^2e(a - a\cos(d+ex) + c\sin(d+ex))} + \frac{\int -\frac{2a}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,d}{4c^2}$$

$$= -\frac{c\cos(d+ex) + a\sin(d+ex)}{4c^2e(a - a\cos(d+ex) + c\sin(d+ex))} - \frac{a\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,d}{2c^2}$$

$$= -\frac{c\cos(d+ex) + a\sin(d+ex)}{4c^2e(a - a\cos(d+ex) + c\sin(d+ex))} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2a+2cx}\,dx, x, \cot\right.}{2c^2e}$$

$$= \frac{a\log\left(a + c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e} - \frac{c\cos(d+ex) + a\sin(d+ex)}{4c^2e(a - a\cos(d+ex) + c\sin(d+ex))}$$

Mathematica [B] time = 0.413908, size = 229, normalized size = 3.05

$$-\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right) + c\cos\left(\frac{1}{2}(d+ex)\right)\right)\left(\cos(d+ex)\left(2a^2\log\left(a\sin\left(\frac{1}{2}(d+ex)\right) + c\cos\left(\frac{1}{2}(d+ex)\right)\right) - 2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] -(Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(Cos[d + e*x]*
(a^2 + 2*c^2 - 2*a^2*Log[Sin[(d + e*x)/2]] + 2*a^2*Log[c*Cos[(d + e*x)/2] +
 a*Sin[(d + e*x)/2]]) + a*(a*(-1 + 2*Log[Sin[(d + e*x)/2]] - 2*Log[c*Cos[(d
 + e*x)/2] + a*Sin[(d + e*x)/2]]) + c*(1 + 2*Log[Sin[(d + e*x)/2]] - 2*Log[
c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])*Sin[d + e*x])))/(4*c^3*e*(a - a*C
os[d + e*x] + c*Sin[d + e*x])^2)
```

Maple [A] time = 0.151, size = 110, normalized size = 1.5

$$-\frac{a}{8\,c^2e}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}-\frac{1}{8\,ae}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}+\frac{a}{4\,c^3e}\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{1}{8\,c^2e}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] -1/8/e/c^2*a/(c+a*tan(1/2*d+1/2*e*x))-1/8/e/a/(c+a*tan(1/2*d+1/2*e*x))+1/4/
e/c^3*a*ln(c+a*tan(1/2*d+1/2*e*x))-1/8/e/c^2/tan(1/2*d+1/2*e*x)-1/4/e/c^3*a
*ln(tan(1/2*d+1/2*e*x))
```

Maxima [A] time = 1.01679, size = 185, normalized size = 2.47

$$-\frac{\frac{ac+\frac{(2\,a^2+c^2)\sin(ex+d)}{\cos(ex+d)+1}}{\frac{ac^3\sin(ex+d)}{\cos(ex+d)+1}+\frac{a^2c^2\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{2\,a\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}+\frac{2\,a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((a*c + (2*a^2 + c^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a*c^3*sin(e*x
+ d)/(cos(e*x + d) + 1) + a^2*c^2*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - 2*
a*log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 + 2*a*log(sin(e*x + d)/(co
s(e*x + d) + 1))/c^3)/e
```

Fricas [B] time = 2.34212, size = 398, normalized size = 5.31

$$\frac{2\,c^2\cos(ex+d)+2\,ac\sin(ex+d)+\left(a^2\cos(ex+d)-ac\sin(ex+d)-a^2\right)\log\left(ac\sin(ex+d)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2-\frac{1}{2}\left(a^2-c^2\right.\right.}{8\left(ac^3e\cos(ex+d)-c^4e\sin(ex+d)-ac^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*c^2*cos(e*x + d) + 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) - a*c*sin(
e*x + d) - a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*
cos(e*x + d)) - (a^2*cos(e*x + d) - a*c*sin(e*x + d) - a^2)*log(-1/2*cos(e*
x + d) + 1/2))/(a*c^3*e*cos(e*x + d) - c^4*e*sin(e*x + d) - a*c^3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15568, size = 155, normalized size = 2.07

$$\frac{1}{8}\left(\frac{2\,a\log\left(\left|a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c\right|\right)}{c^3}-\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right|\right)}{c^3}-\frac{2\,a^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+ac}{\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right)ac^2}\right)e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*a*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/c^3 - 2*a*log(abs(tan(1/2*x*e
 + 1/2*d)))/c^3 - (2*a^2*tan(1/2*x*e + 1/2*d) + c^2*tan(1/2*x*e + 1/2*d) +
a*c)/((a*tan(1/2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))*a*c^2))*e^(-1)
```

3.379 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=134

$$\frac{3\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{16c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))}-\frac{\left(3a^2+c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{a\sin(d+ex)+c\cos(d+ex}{16c^2e(a(-\cos(d+ex))+a+c\sin(}$$

```
[Out] -((3*a^2 + c^2)*Log[a + c*Cot[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] +
 a*Sin[d + e*x])/(16*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a
*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(16*c^4*e*(a - a*Cos[d + e*x] + c*Sin[
d + e*x]))
```

Rubi [A] time = 0.112631, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3121, 31}

$$\frac{3\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{16c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))}-\frac{\left(3a^2+c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{a\sin(d+ex)+c\cos(d+ex}{16c^2e(a(-\cos(d+ex))+a+c\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] -((3*a^2 + c^2)*Log[a + c*Cot[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] +
 a*Sin[d + e*x])/(16*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a
*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(16*c^4*e*(a - a*Cos[d + e*x] + c*Sin[
d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2} + \frac{\int \frac{-4a-2a\cos(d+ex)+2c\sin(d+ex)}{(2a-2a\cos(d+ex)+2c\sin(d+ex)}}{8c^2}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3\left(ac\cos(d+ex)+a^2\sin\right.}{16c^4e(a-a\cos(d+ex)+}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3\left(ac\cos(d+ex)+a^2\sin\right.}{16c^4e(a-a\cos(d+ex)+}$$

$$= -\frac{\left(3a^2+c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e} - \frac{c\cos(d+ex)+a\sin(d}{16c^2e(a-a\cos(d+ex)+c\sin}$$

Mathematica [C] time = 0.610723, size = 350, normalized size = 2.61

$$\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+ex)\right)\right)\left(-6a\left(a^2+c^2\right)\sin^3\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(c^2*((-I)*a +
c)*(I*a + c)*Sin[(d + e*x)/2]^2 - 6*a*(a^2 + c^2)*Sin[(d + e*x)/2]^3*(c*Cos
[(d + e*x)/2] + a*Sin[(d + e*x)/2]) - c^2*(c*Cos[(d + e*x)/2] + a*Sin[(d +
e*x)/2])^2 + 4*(3*a^2 + c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Co
s[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2 - 4*(3*a^2 + c^2)*Log[c*Cos[(d + e*x
)/2] + a*Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Cos[(d + e*x)/2] + a*Sin[(
d + e*x)/2])^2 + 3*a*c*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2*Sin[d +
e*x]))/(8*c^5*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3)
```

Maple [B] time = 0.196, size = 272, normalized size = 2.

$$\frac{3\,a^2}{32\,c^4 e}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}+\frac{1}{16\,c^2 e}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}-\frac{1}{32\,a^2 e}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}+\frac{a^2}{64\,ec^3}\left(c+a\tan\left(\frac{d}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 3/32/e*a^2/c^4/(c+a*tan(1/2*d+1/2*e*x))+1/16/e/c^2/(c+a*tan(1/2*d+1/2*e*x))
-1/32/e/a^2/(c+a*tan(1/2*d+1/2*e*x))+1/64/e*a^2/c^3/(c+a*tan(1/2*d+1/2*e*x)
)^2+1/32/e/c/(c+a*tan(1/2*d+1/2*e*x))^2+1/64/e/a^2*c/(c+a*tan(1/2*d+1/2*e*x
))^2-3/16/e/c^5*ln(c+a*tan(1/2*d+1/2*e*x))*a^2-1/16/e/c^3*ln(c+a*tan(1/2*d+
1/2*e*x))-1/64/e/c^3/tan(1/2*d+1/2*e*x)^2+3/16/e/c^5*ln(tan(1/2*d+1/2*e*x))
*a^2+1/16/e/c^3*ln(tan(1/2*d+1/2*e*x))+3/32/e/c^4*a/tan(1/2*d+1/2*e*x)
```

Maxima [B] time = 1.06342, size = 358, normalized size = 2.67

$$-\frac{\frac{a^2c^3-\frac{4\,a^3c^2\sin(ex+d)}{\cos(ex+d)+1}-\frac{\left(18\,a^4c+6\,a^2c^3-c^5\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{2\left(6\,a^5+2\,a^3c^2-ac^4\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{\frac{a^2c^6\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{2\,a^3c^5\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{a^4c^4\sin(ex+d)^4}{(\cos(ex+d)+1)^4}}+\frac{4\left(3\,a^2+c^2\right)\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}-\frac{4\left(3\,a^2+c^2\right)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}}{64\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -1/64*((a^2*c^3 - 4*a^3*c^2*sin(e*x + d)/(cos(e*x + d) + 1) - (18*a^4*c + 6
*a^2*c^3 - c^5)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 2*(6*a^5 + 2*a^3*c^2
- a*c^4)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(a^2*c^6*sin(e*x + d)^2/(cos(
e*x + d) + 1)^2 + 2*a^3*c^5*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + a^4*c^4*s
in(e*x + d)^4/(cos(e*x + d) + 1)^4) + 4*(3*a^2 + c^2)*log(c + a*sin(e*x + d
)/(cos(e*x + d) + 1))/c^5 - 4*(3*a^2 + c^2)*log(sin(e*x + d)/(cos(e*x + d)
+ 1))/c^5)/e
```

Fricas [B] time = 2.46851, size = 987, normalized size = 7.37

$$\frac{12\,a^2c^2\cos\left(ex+d\right)^2-6\,a^2c^2-2\left(3\,a^2c^2-c^4\right)\cos\left(ex+d\right)+\left(3\,a^4+4\,a^2c^2+c^4+\left(3\,a^4-2\,a^2c^2-c^4\right)\cos\left(ex+d\right)^2-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 - 2*(3*a^2*c^2 - c^4)*cos(e*x +
 d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 -
 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 - (3*a^3*c + a*c^3)*
cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*
(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 -
 c^4)*cos(e*x + d)^2 - 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^
3 - (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(-1/2*cos(e*x + d) + 1
/2) - 2*(3*a^3*c - a*c^3 - 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x + d))/(2
*a^2*c^5*e*cos(e*x + d) - (a^2*c^5 - c^7)*e*cos(e*x + d)^2 - (a^2*c^5 + c^7
)*e + 2*(a*c^6*e*cos(e*x + d) - a*c^6*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29775, size = 323, normalized size = 2.41

$$\frac{1}{64}\left(\frac{4\left(3a^2+c^2\right)\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)\right|\right)}{c^5}-\frac{4\left(3a^3+ac^2\right)\log\left(\left|a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c\right|\right)}{ac^5}+\frac{12a^5\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^3+4a^3c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 + c^2)*log(abs(tan(1/2*x*e + 1/2*d)))/c^5 - 4*(3*a^3 + a*c^2
)*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/(a*c^5) + (12*a^5*tan(1/2*x*e + 1/2*
d)^3 + 4*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 2*a*c^4*tan(1/2*x*e + 1/2*d)^3 +
18*a^4*c*tan(1/2*x*e + 1/2*d)^2 + 6*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - c^5*ta
n(1/2*x*e + 1/2*d)^2 + 4*a^3*c^2*tan(1/2*x*e + 1/2*d) - a^2*c^3)/((a*tan(1/
2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))^2*a^2*c^4))*e^(-1)
```

3.380 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4} dx$

Optimal. Leaf size=207

$$-\frac{a\left(15a^2+4c^2\right)\sin(d+ex)+c\left(15a^2+4c^2\right)\cos(d+ex)}{96c^6e(a(-\cos(d+ex))+a+c\sin(d+ex))}+\frac{5\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{96c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}+\frac{a\left(5a^2+3c^2\right)}{}$$

```
[Out] (a*(5*a^2 + 3*c^2)*Log[a + c*Cot[(d + e*x)/2]])/(32*c^7*e) - (c*Cos[d + e*x
] + a*Sin[d + e*x])/(48*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5
*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(96*c^4*e*(a - a*Cos[d + e*x] + c*S
in[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] + a*(15*a^2 + 4*c^2)*Sin
[d + e*x])/(96*c^6*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rubi [A] time = 0.24045, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3121, 31}

$$-\frac{a\left(15a^2+4c^2\right)\sin(d+ex)+c\left(15a^2+4c^2\right)\cos(d+ex)}{96c^6e(a(-\cos(d+ex))+a+c\sin(d+ex))}+\frac{5\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{96c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}+\frac{a\left(5a^2+3c^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```


```
[Out] (a*(5*a^2 + 3*c^2)*Log[a + c*Cot[(d + e*x)/2]])/(32*c^7*e) - (c*Cos[d + e*x
] + a*Sin[d + e*x])/(48*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5
*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(96*c^4*e*(a - a*Cos[d + e*x] + c*S
in[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] + a*(15*a^2 + 4*c^2)*Sin
[d + e*x])/(96*c^6*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```


Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{\int \frac{-6a-4a\cos(d+ex)+4c\sin(d+e}{(2a-2a\cos(d+ex)+2c\sin(d+ex)}}{12c^2}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+ex)+a^2\,s\right.}{96c^4e(a-a\cos(d+ex)+}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+ex)+a^2\,s\right.}{96c^4e(a-a\cos(d+ex)+}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+ex)+a^2\,s\right.}{96c^4e(a-a\cos(d+ex)+}$$

$$= \frac{a\left(5a^2+3c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e} - \frac{c\cos(d+ex)+a\sin(}{48c^2e(a-a\cos(d+ex)+c\,s}$$

Mathematica [B] time = 1.08889, size = 494, normalized size = 2.39

$$\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+ex)\right)\right)\left(75a^3c^3\sin(d+ex)-156a^3c^3\sin(2(d+ex))+79a^3c^3\sin(3(d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(150*a^6 + 130*
a^4*c^2 + 24*a^2*c^4 - 225*a^6*Cos[d + e*x] - 255*a^4*c^2*Cos[d + e*x] - 42
*a^2*c^4*Cos[d + e*x] - 24*c^6*Cos[d + e*x] + 90*a^6*Cos[2*(d + e*x)] + 174
*a^4*c^2*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] - 49*a^4*c^2*Cos[3*(d +
 e*x)] + 18*a^2*c^4*Cos[3*(d + e*x)] + 8*c^6*Cos[3*(d + e*x)] - 192*(5*a^3
+ 3*a*c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a
*Sin[(d + e*x)/2])^3 + 192*(5*a^3 + 3*a*c^2)*Log[c*Cos[(d + e*x)/2] + a*Sin
[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])
^3 + 75*a^5*c*Sin[d + e*x] + 75*a^3*c^3*Sin[d + e*x] - 12*a*c^5*Sin[d + e*x
] - 60*a^5*c*Sin[2*(d + e*x)] - 156*a^3*c^3*Sin[2*(d + e*x)] - 12*a*c^5*Sin
[2*(d + e*x)] + 15*a^5*c*Sin[3*(d + e*x)] + 79*a^3*c^3*Sin[3*(d + e*x)] + 2
0*a*c^5*Sin[3*(d + e*x)]))/(384*c^7*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])
^4)
```

Maple [B] time = 0.229, size = 416, normalized size = 2.

$$-\frac{a^3}{64\,c^5e}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-2}-\frac{3\,a}{128\,c^3e}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-2}+\frac{c}{128\,a^3e}\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-2}-\frac{5\,a^3}{64\,ec^6}\left(c+a\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x)
```

```
[Out] -1/64/e*a^3/c^5/(c+a*tan(1/2*d+1/2*e*x))^2-3/128/e*a/c^3/(c+a*tan(1/2*d+1/2
*e*x))^2+1/128/e/a^3*c/(c+a*tan(1/2*d+1/2*e*x))^2-5/64/e/c^6*a^3/(c+a*tan(1
/2*d+1/2*e*x))-9/128/e/c^4*a/(c+a*tan(1/2*d+1/2*e*x))-1/128/e/a^3/(c+a*tan(
1/2*d+1/2*e*x))-1/384/e*a^3/c^4/(c+a*tan(1/2*d+1/2*e*x))^3-1/128/e*a/c^2/(c
+a*tan(1/2*d+1/2*e*x))^3-1/128/e/a/(c+a*tan(1/2*d+1/2*e*x))^3-1/384/e/a^3*c
^2/(c+a*tan(1/2*d+1/2*e*x))^3+5/32/e*a^3/c^7*ln(c+a*tan(1/2*d+1/2*e*x))+3/3
2/e*a/c^5*ln(c+a*tan(1/2*d+1/2*e*x))-1/384/e/c^4/tan(1/2*d+1/2*e*x)^3-5/64/
e/c^6/tan(1/2*d+1/2*e*x)*a^2-3/128/e/c^4/tan(1/2*d+1/2*e*x)+1/64/e/c^5*a/ta
n(1/2*d+1/2*e*x)^2-5/32/e*a^3/c^7*ln(tan(1/2*d+1/2*e*x))-3/32/e*a/c^5*ln(ta
n(1/2*d+1/2*e*x))
```

Maxima [A] time = 1.12117, size = 516, normalized size = 2.49

$$-\frac{\frac{a^3c^5-\frac{3\,a^4c^4\sin(ex+d)}{\cos(ex+d)+1}+\frac{3\left(5\,a^5c^3+3\,a^3c^5\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\left(110\,a^6c^2+66\,a^4c^4+3\,a^2c^6+c^8\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{3\left(50\,a^7c+30\,a^5c^3+ac^7\right)\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{3\left(20\,a^8+12\,a^6c^2+a^2c^6\right)\sin(ex+d)^5}{(\cos(ex+d)+1)^5}}{\frac{a^3c^9\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{3\,a^4c^8\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{3\,a^5c^7\sin(ex+d)^5}{(\cos(ex+d)+1)^5}+\frac{a^6c^6\sin(ex+d)^6}{(\cos(ex+d)+1)^6}}-\frac{12(}{}}{384\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*((a^3*c^5 - 3*a^4*c^4*sin(e*x + d)/(cos(e*x + d) + 1) + 3*(5*a^5*c^3
 + 3*a^3*c^5)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (110*a^6*c^2 + 66*a^4*c
^4 + 3*a^2*c^6 + c^8)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + 3*(50*a^7*c + 3
0*a^5*c^3 + a*c^7)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 3*(20*a^8 + 12*a^6
*c^2 + a^2*c^6)*sin(e*x + d)^5/(cos(e*x + d) + 1)^5)/(a^3*c^9*sin(e*x + d)^
3/(cos(e*x + d) + 1)^3 + 3*a^4*c^8*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 3*
a^5*c^7*sin(e*x + d)^5/(cos(e*x + d) + 1)^5 + a^6*c^6*sin(e*x + d)^6/(cos(e
*x + d) + 1)^6) - 12*(5*a^3 + 3*a*c^2)*log(c + a*sin(e*x + d)/(cos(e*x + d)
 + 1))/c^7 + 12*(5*a^3 + 3*a*c^2)*log(sin(e*x + d)/(cos(e*x + d) + 1))/c^7)
/e
```

Fricas [B] time = 2.64159, size = 1783, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*c^2 + 6*a^2*c^4 + 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(e*x
+ d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(e*x + d)^2 - 6*(5*a^4*c^2 - 2*a^2*c^
4 - 2*c^6)*cos(e*x + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 - (5*a^6 - 12*a
^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(
e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*c + 1
4*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2 - 6*(
5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2
*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x + d)) + 3*(5*a^6 + 18*a^4*c^2 + 9*
a^2*c^4 - (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^
4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e
*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^
5)*cos(e*x + d)^2 - 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log
(-1/2*cos(e*x + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5 + (15*a^5*c
- 41*a^3*c^3 - 12*a*c^5)*cos(e*x + d)^2 - 3*(10*a^5*c - 9*a^3*c^3 - a*c^5)*
cos(e*x + d))*sin(e*x + d))/((a^3*c^7 - 3*a*c^9)*e*cos(e*x + d)^3 - 3*(a^3*
c^7 - a*c^9)*e*cos(e*x + d)^2 + 3*(a^3*c^7 + a*c^9)*e*cos(e*x + d) - (a^3*c
^7 + 3*a*c^9)*e + (6*a^2*c^8*e*cos(e*x + d) - (3*a^2*c^8 - c^10)*e*cos(e*x
+ d)^2 - (3*a^2*c^8 + c^10)*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20794, size = 490, normalized size = 2.37

$$-\frac{1}{384}\left(\frac{12\left(5a^3+3ac^2\right)\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)\right|\right)}{c^7}-\frac{12\left(5a^4+3a^2c^2\right)\log\left(\left|a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c\right|\right)}{ac^7}+\frac{60a^8\tan\left(\frac{1}{2}xe+\frac{1}{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(tan(1/2*x*e + 1/2*d)))/c^7 - 12*(5*a^4
 + 3*a^2*c^2)*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/(a*c^7) + (60*a^8*tan(1/
2*x*e + 1/2*d)^5 + 36*a^6*c^2*tan(1/2*x*e + 1/2*d)^5 + 3*a^2*c^6*tan(1/2*x*
e + 1/2*d)^5 + 150*a^7*c*tan(1/2*x*e + 1/2*d)^4 + 90*a^5*c^3*tan(1/2*x*e +
1/2*d)^4 + 3*a*c^7*tan(1/2*x*e + 1/2*d)^4 + 110*a^6*c^2*tan(1/2*x*e + 1/2*d
)^3 + 66*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 3*a^2*c^6*tan(1/2*x*e + 1/2*d)^3
+ c^8*tan(1/2*x*e + 1/2*d)^3 + 15*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 9*a^3*c^
5*tan(1/2*x*e + 1/2*d)^2 - 3*a^4*c^4*tan(1/2*x*e + 1/2*d) + a^3*c^5)/((a*ta
n(1/2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))^3*a^3*c^6))*e^(-1)
```

3.381 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx$

Optimal. Leaf size=157

$$\frac{4b\left(15a^2+4b^2\right)\sin(d+ex)}{3e}-\frac{4a\left(15a^2+4b^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3b^2\right)-\frac{20\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)(a\,\mathrm{si}}{3e}$$

```
[Out] 4*a*(5*a^2 + 3*b^2)*x - (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) - (8*(a + b*Cos[d + e*x] + a*Sin[d + e*x
])^2*(a*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e) - (20*(a + b*Cos[d + e*x] + a
*Sin[d + e*x])*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(3*e)
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4b\left(15a^2+4b^2\right)\sin(d+ex)}{3e}-\frac{4a\left(15a^2+4b^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3b^2\right)-\frac{20\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)(a\,\mathrm{si}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```


```
[Out] 4*a*(5*a^2 + 3*b^2)*x - (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) - (8*(a + b*Cos[d + e*x] + a*Sin[d + e*x
])^2*(a*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e) - (20*(a + b*Cos[d + e*x] + a
*Sin[d + e*x])*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(3*e)
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
```

```
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx = -\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e}+\frac{1}{3}$$

$$= -\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{20}{}$$

$$= 4a\left(5a^2+3b^2\right)x-\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b}{3e}$$

$$= 4a\left(5a^2+3b^2\right)x-\frac{4a\left(15a^2+4b^2\right)\cos(d+ex)}{3e}+\frac{4b\left(15a^2+4b^2\right)\sin(d+}{3e}$$

Mathematica [A] time = 0.464093, size = 135, normalized size = 0.86

$$\frac{2\left(6a\left(5a^2+3b^2\right)(d+ex)-9a\left(a^2-b^2\right)\sin(2(d+ex))+9b\left(5a^2+b^2\right)\sin(d+ex)+b\left(b^2-3a^2\right)\sin(3(d+ex))-9a\left(5a^2\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) - 9*a*(5*a^2 + b^2)*Cos[d + e*x] - 18*a^2
*b*Cos[2*(d + e*x)] + a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*
Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A] time = 0.069, size = 177, normalized size = 1.1

$$8\frac{a^3(ex+d)+3\sin(ex+d)a^2b-3a^3\cos(ex+d)+3ab^2(1/2\sin(ex+d)\cos(ex+d)+1/2ex+d/2)-3(\cos(ex+d)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(a^3*(e*x+d)+3*sin(e*x+d)*a^2*b-3*a^3*cos(e*x+d)+3*a*b^2*(1/2*sin(e*x+d
)*cos(e*x+d)+1/2*e*x+1/2*d)-3*cos(e*x+d)^2*a^2*b+3*a^3*(-1/2*sin(e*x+d)*cos
(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-cos(e*x+d)^3*a*b
^2+a^2*b*sin(e*x+d)^3-1/3*a^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

Maxima [A] time = 0.996859, size = 258, normalized size = 1.64

$$-\frac{8\,ab^2\cos(ex+d)^3}{e}+\frac{8\,a^2b\sin(ex+d)^3}{e}+8\,a^3x+\frac{8\left(\cos(ex+d)^3-3\,\cos(ex+d)\right)a^3}{3\,e}-\frac{8\left(\sin(ex+d)^3-3\,\sin(ex+\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a*b^2*cos(e*x + d)^3/e + 8*a^2*b*sin(e*x + d)^3/e + 8*a^3*x + 8/3*(cos(e
*x + d)^3 - 3*cos(e*x + d))*a^3/e - 8/3*(sin(e*x + d)^3 - 3*sin(e*x + d))*b
^3/e - 24*a^2*(a*cos(e*x + d)/e - b*sin(e*x + d)/e) - 6*(4*a*b*cos(e*x + d)
^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e - (2*e*x + 2*d + sin(2*e*x +
2*d))*b^2/e)*a
```

Fricas [A] time = 2.30833, size = 293, normalized size = 1.87

$$-\frac{4\left(18\,a^2b\cos(ex+d)^2+24\,a^3\cos(ex+d)-2\left(a^3-3\,ab^2\right)\cos(ex+d)^3-3\left(5\,a^3+3\,ab^2\right)ex-\left(24\,a^2b+4\,b^3-2\left(3\,a^2\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*cos
(e*x + d)^3 - 3*(5*a^3 + 3*a*b^2)*e*x - (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^
3)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.935438, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x - \frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} - \frac{16a^3\cos^3(d+ex)}{3e} - \frac{24a^3\cos}{e} \\ x(2a\sin(d) + 2a + 2b\cos(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 - 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x
)/e - 16*a**3*cos(d + e*x)**3/(3*e) - 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin
(d + e*x)**3/e + 24*a**2*b*sin(d + e*x)**2/e + 24*a**2*b*sin(d + e*x)/e + 1
2*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d
+ e*x)*cos(d + e*x)/e - 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**
3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(2*a*sin(d)
+ 2*a + 2*b*cos(d))**3, True))
```

Giac [A] time = 1.10934, size = 204, normalized size = 1.3

$$-12a^2b\cos(2xe+2d)e^{(-1)} + \frac{2}{3}\left(a^3 - 3ab^2\right)\cos(3xe+3d)e^{(-1)} - 6\left(5a^3 + ab^2\right)\cos(xe+d)e^{(-1)} - \frac{2}{3}\left(3a^2b - b^3\right)e^{(-1)}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -12*a^2*b*cos(2*x*e + 2*d)*e^(-1) + 2/3*(a^3 - 3*a*b^2)*cos(3*x*e + 3*d)*e^
(-1) - 6*(5*a^3 + a*b^2)*cos(x*e + d)*e^(-1) - 2/3*(3*a^2*b - b^3)*e^(-1)*s
in(3*x*e + 3*d) - 6*(a^3 - a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^2*b + b^
3)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*b^2)*x
```

3.382 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2x\left(3a^2+b^2\right)-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}-\frac{2(a\sin(d+ex)+a+b\cos(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}$$

```
[Out] 2*(3*a^2 + b^2)*x - (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e - (2*(a
 + b*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - b*Sin[d + e*x]))/e
```

Rubi [A] time = 0.0466635, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2+b^2\right)-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}-\frac{2(a\sin(d+ex)+a+b\cos(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + b^2)*x - (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e - (2*(a
 + b*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - b*Sin[d + e*x]))/e
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = -\frac{2(a+b\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}+\frac{1}{2}\int$$
$$= 2\left(3a^2+b^2\right)x-\frac{2(a+b\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-b\sin($$
$$= 2\left(3a^2+b^2\right)x-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}-\frac{2(a+b\cos(d+ex)}{}$$

Mathematica [A] time = 0.151361, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2+b^2\right)(d+ex)}{2e}-\frac{\left(a^2-b^2\right)\sin(2(d+ex))}{4e}-\frac{2a^2\cos(d+ex)}{e}+\frac{2ab\sin(d+ex)}{e}-\frac{ab\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + b^2)*(d + e*x))/(2*e) - (2*a^2*Cos[d + e*x])/e - (a*b*Cos[2*(d
 + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A] time = 0.057, size = 101, normalized size = 1.3

$$4\,\frac{a^2(ex+d)+2\,ab\sin(ex+d)-2\,a^2\cos(ex+d)+b^2\left(1/2\,\sin(ex+d)\cos(ex+d)+1/2\,ex+d/2\right)-(\cos(ex+d))^2ab+}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)-2*a^2*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*a*b+a^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

Maxima [A] time = 0.992526, size = 134, normalized size = 1.65

$$4\,a^2x-\frac{4\,ab\cos(ex+d)^2}{e}+\frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))a^2}{e}+\frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))b^2}{e}-8\,a\left(\frac{a\cos(ex+d)}{e}-\frac{b\sin}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x - 4*a*b*cos(e*x + d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - 8*a*(a*cos(e*x + d)/e - b*sin(e*x
 + d)/e)
```

Fricas [A] time = 2.06676, size = 162, normalized size = 2.

$$-\frac{2\left(2\,ab\cos\left(ex+d\right)^{2}-\left(3\,a^{2}+b^{2}\right)ex+4\,a^{2}\cos\left(ex+d\right)-\left(4\,ab-\left(a^{2}-b^{2}\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*a*b*cos(e*x + d)^2 - (3*a^2 + b^2)*e*x + 4*a^2*cos(e*x + d) - (4*a*b
- (a^2 - b^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.369924, size = 170, normalized size = 2.1

$$\begin{cases} 2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x-\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{8a^2\cos\left(d+ex\right)}{e}+\frac{4ab\sin^2\left(d+ex\right)}{e}+\frac{8ab\sin\left(d+ex\right)}{e}+2b^2x \\ x\left(2a\sin\left(d\right)+2a+2b\cos\left(d\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x -
 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*cos(d + e*x)/e + 4*a*b*sin(d +
 e*x)**2/e + 8*a*b*sin(d + e*x)/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos
(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*sin(d
) + 2*a + 2*b*cos(d))**2, True))
```

Giac [A] time = 1.12001, size = 107, normalized size = 1.32

$$-2\,ab\cos\left(2\,xe+2\,d\right)e^{(-1)}-8\,a^{2}\cos\left(xe+d\right)e^{(-1)}+8\,abe^{(-1)}\sin\left(xe+d\right)-\left(a^{2}-b^{2}\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+2\left(3\,a^{2}+b^{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*cos(2*x*e + 2*d)*e^(-1) - 8*a^2*cos(x*e + d)*e^(-1) + 8*a*b*e^(-1)*s
in(x*e + d) - (a^2 - b^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + b^2)*x
```

3.383 $\int (2a + 2b\cos(d + ex) + 2a\sin(d + ex))\,dx$

Optimal. Leaf size=29

$$-\frac{2a\cos(d+ex)}{e} + 2ax + \frac{2b\sin(d+ex)}{e}$$

```
[Out] 2*a*x - (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rubi [A] time = 0.0162816, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$-\frac{2a\cos(d+ex)}{e} + 2ax + \frac{2b\sin(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a+2b\cos(d+ex)+2a\sin(d+ex))\,dx &= 2ax+(2a)\int \sin(d+ex)\,dx+(2b)\int \cos(d+ex)\,dx\\ &= 2ax-\frac{2a\cos(d+ex)}{e}+\frac{2b\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0175271, size = 53, normalized size = 1.83

$$\frac{2a\sin(d)\sin(ex)}{e} - \frac{2a\cos(d)\cos(ex)}{e} + 2ax + \frac{2b\sin(d)\cos(ex)}{e} + \frac{2b\cos(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Sin
[e*x])/e + (2*a*Sin[d]*Sin[e*x])/e
```

Maple [A] time = 0.001, size = 30, normalized size = 1.

$$2\,ax - 2\,\frac{a\cos(ex+d)}{e} + 2\,\frac{b\sin(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x-2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Maxima [A] time = 0.968986, size = 39, normalized size = 1.34

$$2\,ax - \frac{2\,a\cos(ex+d)}{e} + \frac{2\,b\sin(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x - 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

Fricas [A] time = 2.00037, size = 63, normalized size = 2.17

$$\frac{2\,(aex - a\cos(ex+d) + b\sin(ex+d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x - a*cos(e*x + d) + b*sin(e*x + d))/e
```

Sympy [A] time = 0.155403, size = 39, normalized size = 1.34

$$2ax + 2a\left(\begin{cases} -\frac{\cos(d+ex)}{e} & \text{for}\ e \neq 0 \\ x\sin(d) & \text{otherwise} \end{cases}\right) + 2b\left(\begin{cases} \frac{\sin(d+ex)}{e} & \text{for}\ e \neq 0 \\ x\cos(d) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x + 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b*
Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

Giac [A] time = 1.12217, size = 39, normalized size = 1.34

$$-2\,a\cos(xe+d)\,e^{(-1)} + 2\,be^{(-1)}\sin(xe+d) + 2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -2*a*cos(x*e + d)*e^(-1) + 2*b*e^(-1)*sin(x*e + d) + 2*a*x
```

3.384 $\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx$

Optimal. Leaf size=33

$$-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

```
[Out] -Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)
```

Rubi [A] time = 0.0217124, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3123, 31}

$$-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] -Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{2a+2bx}\,dx, x, \cot\left(\frac{\pi}{4}+\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= -\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}$$

Mathematica [B] time = 0.0701459, size = 93, normalized size = 2.82

$$\frac{1}{2}\left(\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)}{be}-\frac{\log\left(a\sin\left(\frac{1}{2}(d+ex)\right)+a\cos\left(\frac{1}{2}(d+ex)\right)-b\sin\left(\frac{1}{2}(d+ex)\right)+b\cos\left(\frac{1}{2}(d\right.\right.}{be}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] (Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/(b*e) - Log[a*Cos[(d + e*x)/2] +
b*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2] - b*Sin[(d + e*x)/2]]/(b*e))/2
```

Maple [B] time = 0.093, size = 104, normalized size = 3.2

$$\frac{1}{2\,be}\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{a}{2\,be\,(a-b)}\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)+\frac{1}{2\,e\,(a-b)}\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] 1/2/e/b*ln(1+tan(1/2*d+1/2*e*x))-1/2/e/b/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*ta
n(1/2*d+1/2*e*x)+a+b)*a+1/2/e/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2
*e*x)+a+b)
```

Maxima [B] time = 0.988727, size = 89, normalized size = 2.7

$$-\frac{\frac{\log\left(-a-b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b}-\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{b}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*(log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*x
 + d)/(cos(e*x + d) + 1) + 1)/b)/e
```

Fricas [B] time = 2.11244, size = 136, normalized size = 4.12

$$-\frac{\log\left(2\,ab\cos\left(ex+d\right)+a^2+b^2+\left(a^2-b^2\right)\sin\left(ex+d\right)\right)-\log\left(\sin\left(ex+d\right)+1\right)}{4\,be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x + d)) - log(
sin(e*x + d) + 1))/(b*e)
```

Sympy [A] time = 61.5814, size = 107, normalized size = 3.24

$$\begin{cases} \frac{\tilde{\infty}x}{\cos\left(d\right)} & \text{for}\: a=0 \wedge b=0 \wedge e=0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2be} & \text{for}\: a=b \\ -\frac{1}{ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ae} & \text{for}\: b=0 \\ \frac{x}{2a\sin\left(d\right)+2a+2b\cos\left(d\right)} & \text{for}\: e=0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2be}-\frac{\log\left(\frac{a}{a-b}+\frac{b}{a-b}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (log(tan(d/2 + e*
x/2) + 1)/(2*b*e), Eq(a, b)), (-1/(a*e*tan(d/2 + e*x/2) + a*e), Eq(b, 0)),
(x/(2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0)), (log(tan(d/2 + e*x/2) + 1)/(
2*b*e) - log(a/(a - b) + b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))
```

Giac [B] time = 1.15237, size = 96, normalized size = 2.91

$$-\frac{1}{2}\left(\frac{(a-b)\log\left(\left|a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+a+b\right|\right)}{ab-b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right|\right)}{b}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] -1/2*((a - b)*log(abs(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + a +
 b))/(a*b - b^2) - log(abs(tan(1/2*x*e + 1/2*d) + 1))/b)*e^(-1)
```

3.385 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=83

$$\frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$

```
[Out] (a*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) - (a*Cos[d + e*x] - b*Si
n[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rubi [A] time = 0.0495658, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3123, 31}

$$\frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) - (a*Cos[d + e*x] - b*Si
n[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}+\frac{\int -\frac{2a}{2a+2b\cos(d+ex)+2a\sin(d+ex)}}{4b^2}\\ &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}-\frac{a\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}}{2b^2}\\ &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{2a+2bx}\,dx,x,\cot\right.}{2b^2e}\\ &= \frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.506601, size = 162, normalized size = 1.95

$$\frac{\frac{b(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)}+a\log\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)-a\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right.}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (-(a*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]) + a*Log[(a + b)*Cos[(d + e*x
)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] +
 Sin[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Cos
[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

Maple [B] time = 0.162, size = 166, normalized size = 2.

$$-\frac{1}{4\,b^{2}e}\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^{-1}-\frac{a}{4\,b^{3}e}\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{a^{2}}{4\,b^{2}e\,(a-b)}\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x)
```

```
[Out] -1/4/e/b^2/(1+tan(1/2*d+1/2*e*x))-1/4/e*a/b^3*ln(1+tan(1/2*d+1/2*e*x))-1/4/
e/b^2/(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^2-1/4/e/(a-b)
/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)+1/4/e*a/b^3*ln(a*tan(1/2*d
+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)
```

Maxima [B] time = 1.0459, size = 250, normalized size = 3.01

$$-\frac{\frac{2\left(a^{2}+\frac{\left(a^{2}-ab+b^{2}\right)\sin(ex+d)}{\cos(ex+d)+1}\right)}{a^{2}b^{2}-b^{4}+\frac{2\left(a^{2}b^{2}-ab^{3}\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(a^{2}b^{2}-2\,ab^{3}+b^{4}\right)\sin(ex+d)^{2}}{(\cos(ex+d)+1)^{2}}}-\frac{a\log\left(-a-b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b^{3}}+\frac{a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{b^{3}}}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 + (a^2 - a*b + b^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a^2*b^2
- b^4 + 2*(a^2*b^2 - a*b^3)*sin(e*x + d)/(cos(e*x + d) + 1) + (a^2*b^2 - 2*
a*b^3 + b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - a*log(-a - b - (a - b)*
sin(e*x + d)/(cos(e*x + d) + 1))/b^3 + a*log(sin(e*x + d)/(cos(e*x + d) + 1
) + 1)/b^3)/e
```

Fricas [B] time = 2.36147, size = 377, normalized size = 4.54

$$-\frac{2\,ab\cos(ex+d)-2\,b^{2}\sin(ex+d)-\left(ab\cos(ex+d)+a^{2}\sin(ex+d)+a^{2}\right)\log\left(2\,ab\cos(ex+d)+a^{2}+b^{2}+\left(a^{2}-b^{2}\right)\right.}{8\left(b^{4}e\cos(ex+d)+ab^{3}e\sin(ex+d)+ab^{3}e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*cos(e*x + d) - 2*b^2*sin(e*x + d) - (a*b*cos(e*x + d) + a^2*sin
(e*x + d) + a^2)*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x +
 d)) + (a*b*cos(e*x + d) + a^2*sin(e*x + d) + a^2)*log(sin(e*x + d) + 1))/(
b^4*e*cos(e*x + d) + a*b^3*e*sin(e*x + d) + a*b^3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20223, size = 250, normalized size = 3.01

$$\frac{1}{4}\left(\frac{\left(a^2 - ab\right)\log\left(\left|a\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - b\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a + b\right|\right)}{ab^3 - b^4} - \frac{2\left(a^2\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - ab\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right.}{\left(ab^2 - b^3\right)\left(a\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - b\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/4*((a^2 - a*b)*log(abs(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) +
a + b))/(a*b^3 - b^4) - 2*(a^2*tan(1/2*x*e + 1/2*d) - a*b*tan(1/2*x*e + 1/2
*d) + b^2*tan(1/2*x*e + 1/2*d) + a^2)/((a*b^2 - b^3)*(a*tan(1/2*x*e + 1/2*d
)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + a + b)) - a*log
(abs(tan(1/2*x*e + 1/2*d) + 1))/b^3)*e^(-1)
```

3.386 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=142

$$\frac{3\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{16b^4e(a\sin(d+ex)+a+b\cos(d+ex))}-\frac{\left(3a^2+b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+}$$

```
[Out] -((3*a^2 + b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) - (a*Cos[d
 + e*x] - b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x])^2
) + (3*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*x]
 + a*Sin[d + e*x]))
```

Rubi [A] time = 0.112195, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3123, 31}

$$\frac{3\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{16b^4e(a\sin(d+ex)+a+b\cos(d+ex))}-\frac{\left(3a^2+b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -((3*a^2 + b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) - (a*Cos[d
 + e*x] - b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x])^2
) + (3*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*x]
 + a*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx = -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2}+\frac{\int \frac{-4a+2b\cos(d+ex)+2a\sin(d+}{(2a+2b\cos(d+ex)+2a\sin(d+ex}}{8b^2}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2}+\frac{3\left(a^2\cos(d+ex)-ab\,s\right.}{16b^4e(a+b\cos(d+ex)+}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2}+\frac{3\left(a^2\cos(d+ex)-ab\,s\right.}{16b^4e(a+b\cos(d+ex)+}$$

$$= -\frac{\left(3a^2+b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e}-\frac{a\cos(d+ex)-b\sin(}{16b^2e(a+b\cos(d+ex)+a}$$

Mathematica [A] time = 2.41739, size = 255, normalized size = 1.8

$$-\frac{-\frac{b^2\left(a^2+b^2\right)}{\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6ab\left(a^2+b^2\right)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)}-2\left(3a^2+b^2\right)\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -(-2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 2*(3*a^2 + b^
2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + b^2/(Cos[(d +
 e*x)/2] + Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2]
 + Sin[(d + e*x)/2]) - (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (a - b
)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a +
 b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]))))/(32*b^5*e)
```

Maple [B] time = 0.178, size = 639, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)
```

```
[Out] -1/16/e/b^3/(1+tan(1/2*d+1/2*e*x))^2+3/16/e/b^4/(1+tan(1/2*d+1/2*e*x))*a+1/
16/e/b^3/(1+tan(1/2*d+1/2*e*x))+3/16/e/b^5*ln(1+tan(1/2*d+1/2*e*x))*a^2+1/1
6/e/b^3*ln(1+tan(1/2*d+1/2*e*x))-3/16/e/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b
*tan(1/2*d+1/2*e*x)+a+b)*a^3+3/16/e/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan
(1/2*d+1/2*e*x)+a+b)*a^2-1/16/e/b^3/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2
*d+1/2*e*x)+a+b)*a+1/16/e/b^2/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2
*e*x)+a+b)+1/16/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+
b)^2*a^4+1/8/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*
a^2+1/16/e*b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2+3/16
/e/b^4/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^4-1/4/e/b^
3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^3+1/8/e/b^2/(a-
b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^2-1/4/e/b/(a-b)^2/(a
*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a-1/16/e/(a-b)^2/(a*tan(1/2*d
+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)
```

Maxima [B] time = 1.12918, size = 666, normalized size = 4.69

$$\frac{\frac{2\left(3a^5-4a^3b^2-ab^4+\frac{\left(9a^5-9a^4b-2a^3b^2+2a^2b^3-5ab^4+b^5\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(9a^5-18a^4b+12a^3b^2-6a^2b^3+ab^4\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\left(3a^5-9a^4b+10a^3b^2-6a^2b^3+ab^4+b^5\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}\right)}{a^4b^4-2a^2b^6+b^8+\frac{4\left(a^4b^4-a^3b^5-a^2b^6+ab^7\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{2\left(3a^4b^4-6a^3b^5+2a^2b^6+2ab^7-b^8\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{4\left(a^4b^4-3a^3b^5+3a^2b^6-ab^7\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{\left(a^4b^4-4a^3b^5+6a^2b^6-4ab^7+b^8\right)\sin}{(\cos(ex+d)+1)^4}}}{16e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 + (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b + 1
2*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (3*a^5
 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos(e*x
+ d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 + 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 +
a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*b
^6 + 2*a*b^7 - b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 4*(a^4*b^4 - 3*a^
3*b^5 + 3*a^2*b^6 - a*b^7)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + (a^4*b^4 -
 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4
) - (3*a^2 + b^2)*log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^5
 + (3*a^2 + b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/b^5)/e
```

Fricas [B] time = 2.4519, size = 940, normalized size = 6.62

$$\frac{12\,a^2b^2\cos\left(ex+d\right)^2-6\,a^2b^2+2\left(3\,a^3b-ab^3\right)\cos\left(ex+d\right)-\left(6\,a^4+2\,a^2b^2-\left(3\,a^4-2\,a^2b^2-b^4\right)\cos\left(ex+d\right)^2+2\left(3\,a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x +
 d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*(3*
a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e*
x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(
e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 +
 2*(3*a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*
cos(e*x + d))*sin(e*x + d))*log(sin(e*x + d) + 1) - 2*(3*a^2*b^2 - b^4 - 3*
(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d))/(2*a*b^6*e*cos(e*x + d) + 2*a^2
*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 + 2*(a*b^6*e*cos(e*x + d) + a^2*b
^5*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19224, size = 653, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -1/16*((3*a^3 - 3*a^2*b + a*b^2 - b^3)*log(abs(a*tan(1/2*x*e + 1/2*d) - b*t
an(1/2*x*e + 1/2*d) + a + b))/(a*b^5 - b^6) - 2*(3*a^5*tan(1/2*x*e + 1/2*d)
^3 - 9*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 10*a^3*b^2*tan(1/2*x*e + 1/2*d)^3 - 6
*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + a*b^4*tan(1/2*x*e + 1/2*d)^3 + b^5*tan(1/
2*x*e + 1/2*d)^3 + 9*a^5*tan(1/2*x*e + 1/2*d)^2 - 18*a^4*b*tan(1/2*x*e + 1/
2*d)^2 + 12*a^3*b^2*tan(1/2*x*e + 1/2*d)^2 - 6*a^2*b^3*tan(1/2*x*e + 1/2*d)
^2 + a*b^4*tan(1/2*x*e + 1/2*d)^2 + 9*a^5*tan(1/2*x*e + 1/2*d) - 9*a^4*b*ta
n(1/2*x*e + 1/2*d) - 2*a^3*b^2*tan(1/2*x*e + 1/2*d) + 2*a^2*b^3*tan(1/2*x*e
 + 1/2*d) - 5*a*b^4*tan(1/2*x*e + 1/2*d) + b^5*tan(1/2*x*e + 1/2*d) + 3*a^5
 - 4*a^3*b^2 - a*b^4)/((a^2*b^4 - 2*a*b^5 + b^6)*(a*tan(1/2*x*e + 1/2*d)^2
- b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + a + b)^2) - (3*a^2
+ b^2)*log(abs(tan(1/2*x*e + 1/2*d) + 1))/b^5)*e^(-1)
```

3.387 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=215

$$\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{96b^4e(a\sin(d+ex)+a+b\cos(d+ex))^2}-\frac{a\left(15a^2+4b^2\right)\cos(d+ex)-b\left(15a^2+4b^2\right)\sin(d+ex)}{96b^6e(a\sin(d+ex)+a+b\cos(d+ex))}+\frac{a\left(5a^2+3b^2\right)\log}{}$$

```
[Out] (a*(5*a^2 + 3*b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) - (a*Co
s[d + e*x] - b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]
)^3) + (5*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d + e
*x] + a*Sin[d + e*x])^2) - (a*(15*a^2 + 4*b^2)*Cos[d + e*x] - b*(15*a^2 + 4
*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rubi [A] time = 0.244099, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3123, 31}

$$\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{96b^4e(a\sin(d+ex)+a+b\cos(d+ex))^2}-\frac{a\left(15a^2+4b^2\right)\cos(d+ex)-b\left(15a^2+4b^2\right)\sin(d+ex)}{96b^6e(a\sin(d+ex)+a+b\cos(d+ex))}+\frac{a\left(5a^2+3b^2\right)\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (a*(5*a^2 + 3*b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) - (a*Co
s[d + e*x] - b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]
)^3) + (5*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d + e
*x] + a*Sin[d + e*x])^2) - (a*(15*a^2 + 4*b^2)*Cos[d + e*x] - b*(15*a^2 + 4
*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx = -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{\int \frac{-6a+4b\cos(d+ex)+4a\sin(d+}{(2a+2b\cos(d+ex)+2a\sin(d+ex}}{12b^2}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5\left(a^2\cos(d+ex)-ab\right.}{96b^4e(a+b\cos(d+ex)+}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5\left(a^2\cos(d+ex)-ab\right.}{96b^4e(a+b\cos(d+ex)+}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5\left(a^2\cos(d+ex)-ab\right.}{96b^4e(a+b\cos(d+ex)+}$$

$$= \frac{a\left(5a^2+3b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e} - \frac{a\cos(d+ex)-b\sin}{48b^2e(a+b\cos(d+ex)+}$$

Mathematica [B] time = 2.9166, size = 632, normalized size = 2.94

$$\frac{b\left(180a^4b^2\sin(d+ex)+54a^4b^2\sin(2(d+ex))-4a^4b^2\sin(3(d+ex))+15a^3b^3\sin(d+ex)+102a^3b^3\sin(2(d+ex))+3a^3b^3\sin(3(d+ex))+27a^2b^4\sin(d+ex)+6a^2b^4\sin(2(d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (-12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 12*a*(5*a
^2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*(
150*a^6 + 130*a^4*b^2 + 24*a^2*b^4 - 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2 -
 30*a*b^3 + 4*b^4)*Cos[d + e*x] - 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2*
a*b^3 - 2*b^4)*Cos[2*(d + e*x)] + 15*a^6*Cos[3*(d + e*x)] - 30*a^5*b*Cos[3*
(d + e*x)] - 41*a^4*b^2*Cos[3*(d + e*x)] - 38*a^3*b^3*Cos[3*(d + e*x)] - 12
*a^2*b^4*Cos[3*(d + e*x)] - 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x]
 + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e*
x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x]
- 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*(
d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*a
*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)]
 - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Sin
[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)]))/((a + b
)*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3*((a + b)*Cos[(d + e*x)/2] + (a -
b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

Maple [B] time = 0.223, size = 1069, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x)
```

```
[Out] 3/32/e/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a-1/16/e/(
a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^3*a^2+1/16/e/b^5/(1+
tan(1/2*d+1/2*e*x))^2*a-5/32/e/b^6/(1+tan(1/2*d+1/2*e*x))*a^2-1/16/e/b^5/(1
+tan(1/2*d+1/2*e*x))*a-5/32/e*a^3/b^7*ln(1+tan(1/2*d+1/2*e*x))-3/32/e*a/b^5
*ln(1+tan(1/2*d+1/2*e*x))-1/48/e*b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/
2*d+1/2*e*x)+a+b)^3+1/32/e*b/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*
e*x)+a+b)^2-1/16/e/b^4/(1+tan(1/2*d+1/2*e*x))+3/16/e/b^2/(a-b)^3/(a*tan(1/2
*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^3-5/32/e/b^6/(a-b)^3/(a*tan(1/2*d
+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^6+3/8/e/b^5/(a-b)^3/(a*tan(1/2*d+1/2*
e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^5-3/8/e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-
b*tan(1/2*d+1/2*e*x)+a+b)*a^4+3/8/e/b^3/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan
(1/2*d+1/2*e*x)+a+b)*a^3-9/32/e/b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2
*d+1/2*e*x)+a+b)*a^2+5/32/e*a^4/b^7/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2
*d+1/2*e*x)+a+b)-5/32/e*a^3/b^6/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1
/2*e*x)+a+b)+3/32/e*a^2/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e
*x)+a+b)-3/32/e/b^3/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)
^2*a^4-3/32/e*a/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)
-1/48/e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^3*a^6-1
/16/e/b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^3*a^4-1/1
6/e/b^5/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^6+3/32/
e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^5-1/16/e/
(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)-1/48/e/b^4/(1+tan(1
/2*d+1/2*e*x))^3+1/32/e/b^4/(1+tan(1/2*d+1/2*e*x))^2
```

Maxima [B] time = 1.25344, size = 1300, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 + 3*(25*a^8 - 25*a^7
*b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b
^7 + 2*b^8)*sin(e*x + d)/(cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6
*b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin(e*x
 + d)^2/(cos(e*x + d) + 1)^2 + 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^
5*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(e*x + d
)^3/(cos(e*x + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^
3 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(e*x + d)^4/(cos(e*
x + d) + 1)^4 + 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4
- 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(e*x + d)^5/(cos(e*x + d) +
 1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 + 6*(a^6*b^6 - a^5*b^7 - 2*
a^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(e*x + d)/(cos(e*x + d) + 1) +
3*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 - 2*a*b^11 +
b^12)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*
a^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(e*x + d)^3/(cos(e*x + d) +
 1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^10 + 4*
a*b^11 - b^12)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 6*(a^6*b^6 - 5*a^5*b^7
 + 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(e*x + d)^5/(cos(e*x +
 d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*b^9 + 15*a^2*b^10 -
 6*a*b^11 + b^12)*sin(e*x + d)^6/(cos(e*x + d) + 1)^6) - 3*(5*a^3 + 3*a*b^2
)*log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^7 + 3*(5*a^3 + 3*
a*b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/b^7)/e
```

Fricas [B] time = 2.90969, size = 1632, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(e*
x + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(e*x + d)^2 - 6*(10*a^5*b - 9*a^3*b
^3 - 2*a*b^5)*cos(e*x + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3
 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)
^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 -
 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x +
 d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x
 + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(e*x
+ d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*
a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2*
b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin(e*x + d))*l
```

```
og(sin(e*x + d) + 1) + 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*a^4*b^2 - 3*
a^2*b^4 - 4*b^6)*cos(e*x + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*b^5)*cos(e*x
+ d))*sin(e*x + d))/(6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (3*a^2*b^8 -
b^10)*e*cos(e*x + d)^3 - 3*(a^3*b^7 - a*b^9)*e*cos(e*x + d)^2 + (6*a^2*b^8*
e*cos(e*x + d) + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*e*cos(e*x + d)^2)*sin(e*
x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**4,x)

[Out] Timed out
```

Giac [B] time = 1.18962, size = 1368, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")

[Out] 1/96*(3*(5*a^4 - 5*a^3*b + 3*a^2*b^2 - 3*a*b^3)*log(abs(a*tan(1/2*x*e + 1/2
*d) - b*tan(1/2*x*e + 1/2*d) + a + b))/(a*b^7 - b^8) - 2*(15*a^8*tan(1/2*x*
e + 1/2*d)^5 - 75*a^7*b*tan(1/2*x*e + 1/2*d)^5 + 159*a^6*b^2*tan(1/2*x*e +
1/2*d)^5 - 195*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 + 165*a^4*b^4*tan(1/2*x*e + 1
/2*d)^5 - 105*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 + 51*a^2*b^6*tan(1/2*x*e + 1/2
*d)^5 - 21*a*b^7*tan(1/2*x*e + 1/2*d)^5 + 6*b^8*tan(1/2*x*e + 1/2*d)^5 + 75
*a^8*tan(1/2*x*e + 1/2*d)^4 - 300*a^7*b*tan(1/2*x*e + 1/2*d)^4 + 495*a^6*b^
2*tan(1/2*x*e + 1/2*d)^4 - 480*a^5*b^3*tan(1/2*x*e + 1/2*d)^4 + 345*a^4*b^4
*tan(1/2*x*e + 1/2*d)^4 - 180*a^3*b^5*tan(1/2*x*e + 1/2*d)^4 + 57*a^2*b^6*t
an(1/2*x*e + 1/2*d)^4 - 12*a*b^7*tan(1/2*x*e + 1/2*d)^4 + 150*a^8*tan(1/2*x
*e + 1/2*d)^3 - 450*a^7*b*tan(1/2*x*e + 1/2*d)^3 + 500*a^6*b^2*tan(1/2*x*e
+ 1/2*d)^3 - 300*a^5*b^3*tan(1/2*x*e + 1/2*d)^3 + 126*a^4*b^4*tan(1/2*x*e +
 1/2*d)^3 + 22*a^3*b^5*tan(1/2*x*e + 1/2*d)^3 - 48*a^2*b^6*tan(1/2*x*e + 1/
2*d)^3 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^3 - 4*b^8*tan(1/2*x*e + 1/2*d)^3 + 1
```

```
50*a^8*tan(1/2*x*e + 1/2*d)^2 - 300*a^7*b*tan(1/2*x*e + 1/2*d)^2 + 120*a^6*
b^2*tan(1/2*x*e + 1/2*d)^2 + 60*a^5*b^3*tan(1/2*x*e + 1/2*d)^2 - 102*a^4*b^
4*tan(1/2*x*e + 1/2*d)^2 + 144*a^3*b^5*tan(1/2*x*e + 1/2*d)^2 - 60*a^2*b^6*
tan(1/2*x*e + 1/2*d)^2 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^2 + 75*a^8*tan(1/2*x
*e + 1/2*d) - 75*a^7*b*tan(1/2*x*e + 1/2*d) - 75*a^6*b^2*tan(1/2*x*e + 1/2*
d) + 75*a^5*b^3*tan(1/2*x*e + 1/2*d) - 39*a^4*b^4*tan(1/2*x*e + 1/2*d) + 39
*a^3*b^5*tan(1/2*x*e + 1/2*d) + 33*a^2*b^6*tan(1/2*x*e + 1/2*d) - 15*a*b^7*
tan(1/2*x*e + 1/2*d) + 6*b^8*tan(1/2*x*e + 1/2*d) + 15*a^8 - 31*a^6*b^2 + 9
*a^4*b^4 + 15*a^2*b^6)/((a^3*b^6 - 3*a^2*b^7 + 3*a*b^8 - b^9)*(a*tan(1/2*x*
e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + a + b)
^3) - 3*(5*a^3 + 3*a*b^2)*log(abs(tan(1/2*x*e + 1/2*d) + 1))/b^7)*e^(-1)
```

3.388 $\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))^3\,dx$

Optimal. Leaf size=157

$$\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e} + \frac{4a(15a^2+4b^2)\cos(d+ex)}{3e} + 4ax(5a^2+3b^2) + \frac{20(a^2\cos(d+ex)+ab\sin(d+ex))(a(-\mathrm{s}}{3e}$$

```
[Out] 4*a*(5*a^2 + 3*b^2)*x + (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) + (8*(a + b*Cos[d + e*x] - a*Sin[d + e*x
])^2*(a*Cos[d + e*x] + b*Sin[d + e*x]))/(3*e) + (20*(a + b*Cos[d + e*x] - a
*Sin[d + e*x])*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(3*e)
```

Rubi [A] time = 0.135751, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e} + \frac{4a(15a^2+4b^2)\cos(d+ex)}{3e} + 4ax(5a^2+3b^2) + \frac{20(a^2\cos(d+ex)+ab\sin(d+ex))(a(-\mathrm{s}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```


```
[Out] 4*a*(5*a^2 + 3*b^2)*x + (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) + (8*(a + b*Cos[d + e*x] - a*Sin[d + e*x
])^2*(a*Cos[d + e*x] + b*Sin[d + e*x]))/(3*e) + (20*(a + b*Cos[d + e*x] - a
*Sin[d + e*x])*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(3*e)
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
```

```
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a + 2b\cos(d+ex) - 2a\sin(d+ex))^3\,dx = \frac{8(a+b\cos(d+ex)-a\sin(d+ex))^2(a\cos(d+ex)+b\sin(d+ex))}{3e} + \frac{1}{3}$$

$$= \frac{8(a+b\cos(d+ex)-a\sin(d+ex))^2(a\cos(d+ex)+b\sin(d+ex))}{3e} + \frac{20}{}$$

$$= 4a\left(5a^2+3b^2\right)x + \frac{8(a+b\cos(d+ex)-a\sin(d+ex))^2(a\cos(d+ex)+}{3e}$$

$$= 4a\left(5a^2+3b^2\right)x + \frac{4a\left(15a^2+4b^2\right)\cos(d+ex)}{3e} + \frac{4b\left(15a^2+4b^2\right)\sin(d}{3e}$$

Mathematica [A] time = 0.442698, size = 136, normalized size = 0.87

$$\frac{2\left(6a\left(5a^2+3b^2\right)(d+ex)-9a\left(a^2-b^2\right)\sin(2(d+ex))+9b\left(5a^2+b^2\right)\sin(d+ex)+b\left(b^2-3a^2\right)\sin(3(d+ex))+9a\left(5a\right.\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) + 9*a*(5*a^2 + b^2)*Cos[d + e*x] + 18*a^2
*b*Cos[2*(d + e*x)] - a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*
Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A] time = 0.07, size = 176, normalized size = 1.1

$$8\,\frac{a^{3}\left(ex+d\right)+3\,\sin\left(ex+d\right)a^{2}b+3\,a^{3}\cos\left(ex+d\right)+3\,ab^{2}\left(1/2\,\sin\left(ex+d\right)\cos\left(ex+d\right)+1/2\,ex+d/2\right)+3\,\left(\cos\left(ex+d\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(a^3*(e*x+d)+3*sin(e*x+d)*a^2*b+3*a^3*cos(e*x+d)+3*a*b^2*(1/2*sin(e*x+d
)*cos(e*x+d)+1/2*e*x+1/2*d)+3*cos(e*x+d)^2*a^2*b+3*a^3*(-1/2*sin(e*x+d)*cos
(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)+cos(e*x+d)^3*a*b
^2+a^2*b*sin(e*x+d)^3+1/3*a^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

Maxima [A] time = 0.991341, size = 254, normalized size = 1.62

$$\frac{8\,ab^{2}\cos\left(ex+d\right)^{3}}{e}+\frac{8\,a^{2}b\sin\left(ex+d\right)^{3}}{e}+8\,a^{3}x-\frac{8\left(\cos\left(ex+d\right)^{3}-3\,\cos\left(ex+d\right)\right)a^{3}}{3\,e}-\frac{8\left(\sin\left(ex+d\right)^{3}-3\,\sin\left(ex+d\right)\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 8*a*b^2*cos(e*x + d)^3/e + 8*a^2*b*sin(e*x + d)^3/e + 8*a^3*x - 8/3*(cos(e*
x + d)^3 - 3*cos(e*x + d))*a^3/e - 8/3*(sin(e*x + d)^3 - 3*sin(e*x + d))*b^
3/e + 24*a^2*(a*cos(e*x + d)/e + b*sin(e*x + d)/e) + 6*(4*a*b*cos(e*x + d)^
2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d + sin(2*e*x + 2
*d))*b^2/e)*a
```

Fricas [A] time = 2.21198, size = 292, normalized size = 1.86

$$\frac{4\left(18\,a^{2}b\cos\left(ex+d\right)^{2}+24\,a^{3}\cos\left(ex+d\right)-2\left(a^{3}-3\,ab^{2}\right)\cos\left(ex+d\right)^{3}+3\left(5\,a^{3}+3\,ab^{2}\right)ex+\left(24\,a^{2}b+4\,b^{3}-2\left(3\,a^{2}b-\right.\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*cos(
e*x + d)^3 + 3*(5*a^3 + 3*a*b^2)*e*x + (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^3
)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.974717, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x + \frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} + \frac{16a^3\cos^3(d+ex)}{3e} + \frac{24a^3\cos \\ x(-2a\sin(d) + 2a + 2b\cos(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 + 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x
)/e + 16*a**3*cos(d + e*x)**3/(3*e) + 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin
(d + e*x)**3/e - 24*a**2*b*sin(d + e*x)**2/e + 24*a**2*b*sin(d + e*x)/e + 1
2*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d
+ e*x)*cos(d + e*x)/e + 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**
3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(-2*a*sin(d)
 + 2*a + 2*b*cos(d))**3, True))
```

Giac [A] time = 1.11012, size = 204, normalized size = 1.3

$$12\,a^2b\cos(2\,xe + 2\,d)\,e^{(-1)} - \frac{2}{3}\left(a^3 - 3\,ab^2\right)\cos(3\,xe + 3\,d)\,e^{(-1)} + 6\left(5\,a^3 + ab^2\right)\cos(xe + d)\,e^{(-1)} - \frac{2}{3}\left(3\,a^2b - b^3\right)e^{(-1)}\,\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 12*a^2*b*cos(2*x*e + 2*d)*e^(-1) - 2/3*(a^3 - 3*a*b^2)*cos(3*x*e + 3*d)*e^(
-1) + 6*(5*a^3 + a*b^2)*cos(x*e + d)*e^(-1) - 2/3*(3*a^2*b - b^3)*e^(-1)*si
n(3*x*e + 3*d) - 6*(a^3 - a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^2*b + b^3
)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*b^2)*x
```

3.389 $\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2x\left(3a^2+b^2\right)+\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}+\frac{2(a(-\sin(d+ex))+a+b\cos(d+ex))(a\cos(d+ex)+b\sin(d+ex)}{e}$$

```
[Out] 2*(3*a^2 + b^2)*x + (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e + (2*(a
 + b*Cos[d + e*x] - a*Sin[d + e*x])*(a*Cos[d + e*x] + b*Sin[d + e*x]))/e
```

Rubi [A] time = 0.0469208, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2+b^2\right)+\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}+\frac{2(a(-\sin(d+ex))+a+b\cos(d+ex))(a\cos(d+ex)+b\sin(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + b^2)*x + (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e + (2*(a
 + b*Cos[d + e*x] - a*Sin[d + e*x])*(a*Cos[d + e*x] + b*Sin[d + e*x]))/e
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx = \frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b\sin(d+ex))}{e}+\frac{1}{2}\int$$
$$= 2\left(3a^2+b^2\right)x+\frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b\,\text{si}}{e}$$
$$= 2\left(3a^2+b^2\right)x+\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}+\frac{2(a+b\cos(d+ex}{}$$

Mathematica [A] time = 0.1516, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2+b^2\right)(d+ex)}{2e}-\frac{\left(a^2-b^2\right)\sin(2(d+ex))}{4e}+\frac{2a^2\cos(d+ex)}{e}+\frac{2ab\sin(d+ex)}{e}+\frac{ab\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + b^2)*(d + e*x))/(2*e) + (2*a^2*Cos[d + e*x])/e + (a*b*Cos[2*(d
 + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A] time = 0.055, size = 100, normalized size = 1.2

$$4\,\frac{a^2(ex+d)+2\,ab\sin(ex+d)+2\,a^2\cos(ex+d)+b^2(1/2\,\sin(ex+d)\cos(ex+d)+1/2\,ex+d/2)+(\cos(ex+d))^2ab}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)+2*a^2*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)+cos(e*x+d)^2*a*b+a^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

Maxima [A] time = 0.980257, size = 132, normalized size = 1.63

$$4\,a^2x+\frac{4\,ab\cos(ex+d)^2}{e}+\frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))a^2}{e}+\frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))b^2}{e}+8\,a\left(\frac{a\cos(ex+d)}{e}+\frac{b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x + 4*a*b*cos(e*x + d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e + 8*a*(a*cos(e*x + d)/e + b*sin(e*x
 + d)/e)
```

Fricas [A] time = 2.14909, size = 161, normalized size = 1.99

$$\frac{2\left(2\,ab\cos\left(ex+d\right)^{2}+\left(3\,a^{2}+b^{2}\right)ex+4\,a^{2}\cos\left(ex+d\right)+\left(4\,ab-\left(a^{2}-b^{2}\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 2*(2*a*b*cos(e*x + d)^2 + (3*a^2 + b^2)*e*x + 4*a^2*cos(e*x + d) + (4*a*b -
 (a^2 - b^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.367994, size = 170, normalized size = 2.1

$$\begin{cases} 2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x-\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}+\frac{8a^2\cos\left(d+ex\right)}{e}-\frac{4ab\sin^2\left(d+ex\right)}{e}+\frac{8ab\sin\left(d+ex\right)}{e}+2b^2x\,\mathrm{s} \\ x\left(-2a\sin\left(d\right)+2a+2b\cos\left(d\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x -
 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*cos(d + e*x)/e - 4*a*b*sin(d +
 e*x)**2/e + 8*a*b*sin(d + e*x)/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos
(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*sin(
d) + 2*a + 2*b*cos(d))**2, True))
```

Giac [A] time = 1.12162, size = 107, normalized size = 1.32

$$2\,ab\cos\left(2\,xe+2\,d\right)e^{(-1)}+8\,a^{2}\cos\left(xe+d\right)e^{(-1)}+8\,abe^{(-1)}\sin\left(xe+d\right)-\left(a^{2}-b^{2}\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+2\left(3\,a^{2}+b^{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*cos(2*x*e + 2*d)*e^(-1) + 8*a^2*cos(x*e + d)*e^(-1) + 8*a*b*e^(-1)*si
n(x*e + d) - (a^2 - b^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + b^2)*x
```

3.390 $\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))\,dx$

Optimal. Leaf size=29

$$\frac{2a\cos(d + ex)}{e} + 2ax + \frac{2b\sin(d + ex)}{e}$$

```
[Out] 2*a*x + (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rubi [A] time = 0.0144019, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$\frac{2a\cos(d + ex)}{e} + 2ax + \frac{2b\sin(d + ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x + (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))\,dx &= 2ax - (2a)\int \sin(d + ex)\,dx + (2b)\int \cos(d + ex)\,dx \\ &= 2ax + \frac{2a\cos(d + ex)}{e} + \frac{2b\sin(d + ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0119253, size = 53, normalized size = 1.83

$$-\frac{2a\sin(d)\sin(ex)}{e}+\frac{2a\cos(d)\cos(ex)}{e}+2ax+\frac{2b\sin(d)\cos(ex)}{e}+\frac{2b\cos(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x + (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Sin
[e*x])/e - (2*a*Sin[d]*Sin[e*x])/e
```

Maple [A] time = 0., size = 30, normalized size = 1.

$$2\,ax+2\,\frac{a\cos(ex+d)}{e}+2\,\frac{b\sin(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x+2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Maxima [A] time = 0.990877, size = 39, normalized size = 1.34

$$2\,ax+\frac{2\,a\cos(ex+d)}{e}+\frac{2\,b\sin(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x + 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

Fricas [A] time = 2.14906, size = 63, normalized size = 2.17

$$\frac{2\,(aex+a\cos(ex+d)+b\sin(ex+d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x + a*cos(e*x + d) + b*sin(e*x + d))/e
```

Sympy [A] time = 0.153007, size = 39, normalized size = 1.34

$$2ax-2a\left(\begin{cases}-\frac{\cos(d+ex)}{e} & \text{for } e\neq 0\\ x\sin(d) & \text{otherwise}\end{cases}\right)+2b\left(\begin{cases}\frac{\sin(d+ex)}{e} & \text{for } e\neq 0\\ x\cos(d) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x - 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b*
Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

Giac [A] time = 1.12482, size = 39, normalized size = 1.34

$$2\,a\cos\left(xe+d\right)e^{(-1)}+2\,be^{(-1)}\sin\left(xe+d\right)+2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="giac")
```

```
[Out] 2*a*cos(x*e + d)*e^(-1) + 2*b*e^(-1)*sin(x*e + d) + 2*a*x
```

3.391 $\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx$

Optimal. Leaf size=33

$$\frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

[Out] Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)

Rubi [A] time = 0.0217288, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3122, 31}

$$\frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]

[Out] Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{2a+2bx}\,dx, x, \tan\left(\frac{\pi}{4}+\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= \frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}$$

Mathematica [B] time = 0.101587, size = 96, normalized size = 2.91

$$\frac{\log\left(-a\sin\left(\frac{1}{2}(d+ex)\right)+a\cos\left(\frac{1}{2}(d+ex)\right)+b\sin\left(\frac{1}{2}(d+ex)\right)+b\cos\left(\frac{1}{2}(d+ex)\right)\right)}{2be} - \frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+e\right.\right.}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] -Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]]/(2*b*e) + Log[a*Cos[(d + e*x)/2]
+ b*Cos[(d + e*x)/2] - a*Sin[(d + e*x)/2] + b*Sin[(d + e*x)/2]]/(2*b*e)
```

Maple [B] time = 0.1, size = 61, normalized size = 1.9

$$-\frac{1}{2\,be}\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)+\frac{1}{2\,be}\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)
```

```
[Out] -1/2/e/b*ln(tan(1/2*d+1/2*e*x)-1)+1/2/e/b*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2
*d+1/2*e*x)-a-b)
```

Maxima [B] time = 0.995999, size = 84, normalized size = 2.55

$$\frac{\frac{\log\left(a+b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b}-\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)}{b}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*x +
 d)/(cos(e*x + d) + 1) - 1)/b)/e
```

Fricas [B] time = 2.18151, size = 136, normalized size = 4.12

$$\frac{\log\left(2\,ab\cos\left(ex+d\right)+a^2+b^2-\left(a^2-b^2\right)\sin\left(ex+d\right)\right)-\log\left(-\sin\left(ex+d\right)+1\right)}{4\,be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x + d)) - log(-
sin(e*x + d) + 1))/(b*e)
```

Sympy [A] time = 61.2379, size = 109, normalized size = 3.3

$$\begin{cases} \frac{\tilde{\infty}x}{\cos\left(d\right)} & \text{for}\: a=0 \wedge b=0 \wedge e=0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be} & \text{for}\: a=b \\ -\frac{1}{ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ae} & \text{for}\: b=0 \\ \frac{x}{-2a\sin\left(d\right)+2a+2b\cos\left(d\right)} & \text{for}\: e=0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be}+\frac{\log\left(-\frac{a}{a-b}-\frac{b}{a-b}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-log(tan(d/2 + e
*x/2) - 1)/(2*b*e), Eq(a, b)), (-1/(a*e*tan(d/2 + e*x/2) - a*e), Eq(b, 0)),
 (x/(-2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0)), (-log(tan(d/2 + e*x/2) - 1
)/(2*b*e) + log(-a/(a - b) - b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))
```

Giac [B] time = 1.19495, size = 101, normalized size = 3.06

$$\frac{1}{2}\left(\frac{(a-b)\log\left(\left|a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-a-b\right|\right)}{ab-b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-1\right|\right)}{b}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*((a - b)*log(abs(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) - a -
b))/(a*b - b^2) - log(abs(tan(1/2*x*e + 1/2*d) - 1))/b)*e^(-1)
```

3.392 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=83

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}$$

```
[Out] -(a*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) + (a*Cos[d + e*x] + b*S
in[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

Rubi [A] time = 0.0529481, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3122, 31}

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] -(a*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) + (a*Cos[d + e*x] + b*S
in[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} + \frac{\int -\frac{2a}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx}{4b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} - \frac{a\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx}{2b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2a+2bx}\,dx, x, \tan\left(\frac{\pi}{4}\right.\right.}{2b^2e}$$

$$= -\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e} + \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex)}$$

Mathematica [A] time = 0.556618, size = 166, normalized size = 2.

$$\frac{\frac{b\left(a^2+b^2\right)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} - a\log\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right) + a\log\left(\cos\left(\frac{1}{2}(d+ex)\right) -\right.}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - a*Log[(a + b)*Cos[(d + e*x)/2
] + (-a + b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] - S
in[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Cos[(
d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

Maple [B] time = 0.148, size = 178, normalized size = 2.1

$$-\frac{1}{4b^2e}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)^{-1}+\frac{a}{4b^3e}\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)-\frac{a^2}{4b^2e(a-b)}\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)
```

```
[Out] -1/4/e/b^2/(tan(1/2*d+1/2*e*x)-1)+1/4/e*a/b^3*ln(tan(1/2*d+1/2*e*x)-1)-1/4/
e/b^2/(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2-1/4/e/(a-b)
/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/4/e*a/b^3*ln(a*tan(1/2*d
+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)
```

Maxima [B] time = 1.00558, size = 246, normalized size = 2.96

$$\frac{\frac{2\left(a^2-\frac{\left(a^2-ab+b^2\right)\sin(ex+d)}{\cos(ex+d)+1}\right)}{a^2b^2-b^4-\frac{2\left(a^2b^2-ab^3\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(a^2b^2-2ab^3+b^4\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{a\log\left(a+b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b^3}+\frac{a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)}{b^3}}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2 - (a^2 - a*b + b^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a^2*b^2 -
 b^4 - 2*(a^2*b^2 - a*b^3)*sin(e*x + d)/(cos(e*x + d) + 1) + (a^2*b^2 - 2*a
*b^3 + b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - a*log(a + b - (a - b)*si
n(e*x + d)/(cos(e*x + d) + 1))/b^3 + a*log(sin(e*x + d)/(cos(e*x + d) + 1)
- 1)/b^3)/e
```

Fricas [B] time = 2.27783, size = 377, normalized size = 4.54

$$\frac{2ab\cos(ex+d)+2b^2\sin(ex+d)-\left(ab\cos(ex+d)-a^2\sin(ex+d)+a^2\right)\log\left(2ab\cos(ex+d)+a^2+b^2-\left(a^2-b^2\right)\right.}{8\left(b^4e\cos(ex+d)-ab^3e\sin(ex+d)+ab^3e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*b*cos(e*x + d) + 2*b^2*sin(e*x + d) - (a*b*cos(e*x + d) - a^2*sin(
e*x + d) + a^2)*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x +
d)) + (a*b*cos(e*x + d) - a^2*sin(e*x + d) + a^2)*log(-sin(e*x + d) + 1))/(
b^4*e*cos(e*x + d) - a*b^3*e*sin(e*x + d) + a*b^3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16489, size = 258, normalized size = 3.11

$$-\frac{1}{4}\left(\frac{\left(a^2-ab\right)\log\left(\left|a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-a-b\right|\right)}{ab^3-b^4}+\frac{2\left(a^2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-ab\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)\right.}{\left(ab^2-b^3\right)\left(a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2 - a*b)*log(abs(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) -
 a - b))/(a*b^3 - b^4) + 2*(a^2*tan(1/2*x*e + 1/2*d) - a*b*tan(1/2*x*e + 1/
2*d) + b^2*tan(1/2*x*e + 1/2*d) - a^2)/((a*b^2 - b^3)*(a*tan(1/2*x*e + 1/2*
d)^2 - b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) + a + b)) - a*lo
g(abs(tan(1/2*x*e + 1/2*d) - 1))/b^3)*e^(-1)
```

3.393 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3} dx$

Optimal. Leaf size=142

$$\frac{(3a^2+b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{3\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{16b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))}+\frac{a\cos(d+ex)+b\sin(d}{16b^2e(a(-\sin(d+ex))+a+b}$$

```
[Out] ((3*a^2 + b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) + (a*Cos[d
+ e*x] + b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x])^2)
 - (3*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*x]
- a*Sin[d + e*x]))
```

Rubi [A] time = 0.107658, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3122, 31}

$$\frac{(3a^2+b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{3\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{16b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))}+\frac{a\cos(d+ex)+b\sin(d}{16b^2e(a(-\sin(d+ex))+a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```


```
[Out] ((3*a^2 + b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) + (a*Cos[d
+ e*x] + b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x])^2)
 - (3*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*x]
- a*Sin[d + e*x]))
```


Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} + \frac{\int \frac{-4a+2b\cos(d+ex)-2a\sin(d+ex)}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,d}{8b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} - \frac{3\left(a^2\cos(d+ex)+ab\sin(\right.}{16b^4e(a+b\cos(d+ex)-a\,\mathrm{si}}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} - \frac{3\left(a^2\cos(d+ex)+ab\sin(\right.}{16b^4e(a+b\cos(d+ex)-a\,\mathrm{si}}$$

$$= \frac{\left(3a^2+b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e} + \frac{a\cos(d+ex)+b\sin(d-}{16b^2e(a+b\cos(d+ex)-a\sin}$$

Mathematica [A] time = 2.62173, size = 261, normalized size = 1.84

$$-\frac{\frac{b^2\left(a^2+b^2\right)}{\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)^2} + \frac{6ab\left(a^2+b^2\right)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} + 2\left(3a^2+b^2\right)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -(2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 2*(3*a^2 + b^2
)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] - b^2/(Cos[(d +
 e*x)/2] - Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2]
 - Sin[(d + e*x)/2]) + (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (-a +
b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a
+ b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(32*b^5*e)
```

Maple [B] time = 0.17, size = 687, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x)
```

```
[Out] 1/16/e/b^3/(tan(1/2*d+1/2*e*x)-1)^2+3/16/e/b^4/(tan(1/2*d+1/2*e*x)-1)*a+1/1
6/e/b^3/(tan(1/2*d+1/2*e*x)-1)-3/16/e/b^5*ln(tan(1/2*d+1/2*e*x)-1)*a^2-1/16
/e/b^3*ln(tan(1/2*d+1/2*e*x)-1)+3/16/e/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*
tan(1/2*d+1/2*e*x)-a-b)*a^3-3/16/e/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(
1/2*d+1/2*e*x)-a-b)*a^2+1/16/e/b^3/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*
d+1/2*e*x)-a-b)*a-1/16/e/b^2/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*
e*x)-a-b)-1/16/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b
)^2*a^4-1/8/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a
^2-1/16/e*b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2+3/16/
e/b^4/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^4-1/4/e/b^3
/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^3+1/8/e/b^2/(a-b
)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2-1/4/e/b/(a-b)^2/(a*
tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a-1/16/e/(a-b)^2/(a*tan(1/2*d+
1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)
```

Maxima [B] time = 1.10031, size = 663, normalized size = 4.67

$$-\frac{\frac{2\left(3\,a^5-4\,a^3b^2-ab^4-\frac{\left(9\,a^5-9\,a^4b-2\,a^3b^2+2\,a^2b^3-5\,ab^4+b^5\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(9\,a^5-18\,a^4b+12\,a^3b^2-6\,a^2b^3+ab^4\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{\left(3\,a^5-9\,a^4b+10\,a^3b^2-6\,a^2b^3+ab^4+b^5\right)\sin(ex+d)}{(\cos(ex+d)+1)^3}}{a^4b^4-2\,a^2b^6+b^8-\frac{4\left(a^4b^4-a^3b^5-a^2b^6+ab^7\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{2\left(3\,a^4b^4-6\,a^3b^5+2\,a^2b^6+2\,ab^7-b^8\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{4\left(a^4b^4-3\,a^3b^5+3\,a^2b^6-ab^7\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{\left(a^4b^4-4\,a^3b^5+6\,a^2b^6-4\,ab^7+b^8\right.}{(\cos(ex+d)+1)^4}}}{16\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 - (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*
b^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b +
12*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - (3*a^
5 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos(e*x
 + d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 - 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 +
 a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*
b^6 + 2*a*b^7 - b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 4*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + (a^4*b^4
- 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(e*x + d)^4/(cos(e*x + d) + 1)^
4) - (3*a^2 + b^2)*log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^5
 + (3*a^2 + b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1)/b^5)/e
```

Fricas [B] time = 2.4585, size = 942, normalized size = 6.63

$$-\frac{12\,a^2b^2\cos(ex+d)^2 - 6\,a^2b^2 + 2\left(3\,a^3b - ab^3\right)\cos(ex+d) - \left(6\,a^4 + 2\,a^2b^2 - \left(3\,a^4 - 2\,a^2b^2 - b^4\right)\cos(ex+d)^2 + 2\left(3\,a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x
+ d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*(3
*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e
*x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin
(e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2
+ 2*(3*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)
*cos(e*x + d))*sin(e*x + d))*log(-sin(e*x + d) + 1) + 2*(3*a^2*b^2 - b^4 -
3*(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d))/(2*a*b^6*e*cos(e*x + d) + 2*a
^2*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 - 2*(a*b^6*e*cos(e*x + d) + a^2
*b^5*e)*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20773, size = 659, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/16*((3*a^3 - 3*a^2*b + a*b^2 - b^3)*log(abs(a*tan(1/2*x*e + 1/2*d) - b*ta
n(1/2*x*e + 1/2*d) - a - b))/(a*b^5 - b^6) + 2*(3*a^5*tan(1/2*x*e + 1/2*d)^
3 - 9*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 10*a^3*b^2*tan(1/2*x*e + 1/2*d)^3 - 6*
a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + a*b^4*tan(1/2*x*e + 1/2*d)^3 + b^5*tan(1/2
*x*e + 1/2*d)^3 - 9*a^5*tan(1/2*x*e + 1/2*d)^2 + 18*a^4*b*tan(1/2*x*e + 1/2
*d)^2 - 12*a^3*b^2*tan(1/2*x*e + 1/2*d)^2 + 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^
2 - a*b^4*tan(1/2*x*e + 1/2*d)^2 + 9*a^5*tan(1/2*x*e + 1/2*d) - 9*a^4*b*tan
(1/2*x*e + 1/2*d) - 2*a^3*b^2*tan(1/2*x*e + 1/2*d) + 2*a^2*b^3*tan(1/2*x*e
+ 1/2*d) - 5*a*b^4*tan(1/2*x*e + 1/2*d) + b^5*tan(1/2*x*e + 1/2*d) - 3*a^5
+ 4*a^3*b^2 + a*b^4)/((a^2*b^4 - 2*a*b^5 + b^6)*(a*tan(1/2*x*e + 1/2*d)^2 -
 b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) + a + b)^2) - (3*a^2 +
 b^2)*log(abs(tan(1/2*x*e + 1/2*d) - 1))/b^5)*e^(-1)
```

3.394 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=215

$$-\frac{a\left(5a^2+3b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{32b^7e}-\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{96b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}+\frac{b\left(15a^2+4b^2\right)\sin(d+ex)}{96b^6e(a(-\sin(d+ex$$

```
[Out] -(a*(5*a^2 + 3*b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) + (a*C
os[d + e*x] + b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x
])^3) - (5*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d +
e*x] - a*Sin[d + e*x])^2) + (a*(15*a^2 + 4*b^2)*Cos[d + e*x] + b*(15*a^2 +
4*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

Rubi [A] time = 0.235165, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3122, 31}

$$-\frac{a\left(5a^2+3b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{32b^7e}-\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{96b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}+\frac{b\left(15a^2+4b^2\right)\sin(d+ex)}{96b^6e(a(-\sin(d+ex$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```


```
[Out] -(a*(5*a^2 + 3*b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) + (a*C
os[d + e*x] + b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x
])^3) - (5*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d +
e*x] - a*Sin[d + e*x])^2) + (a*(15*a^2 + 4*b^2)*Cos[d + e*x] + b*(15*a^2 +
4*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```


Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} + \frac{\int \frac{-6a+4b\cos(d+ex)-4a\sin(d+ex)}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}}{12b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5\left(a^2\cos(d+ex)+ab\sin\right.}{96b^4e(a+b\cos(d+ex)-a\sin}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5\left(a^2\cos(d+ex)+ab\sin\right.}{96b^4e(a+b\cos(d+ex)-a\sin}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5\left(a^2\cos(d+ex)+ab\sin\right.}{96b^4e(a+b\cos(d+ex)-a\sin}$$

$$= -\frac{a\left(5a^2+3b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e} + \frac{a\cos(d+ex)+b\sin}{48b^2e(a+b\cos(d+ex)-}$$

Mathematica [B] time = 1.85416, size = 636, normalized size = 2.96

$$\frac{b\left(180a^4b^2\sin(d+ex)+54a^4b^2\sin(2(d+ex))-4a^4b^2\sin(3(d+ex))+15a^3b^3\sin(d+ex)+102a^3b^3\sin(2(d+ex))+3a^3b^3\sin(3(d+ex))+27a^2b^4\sin(d+ex)+6a^2b^4\sin(2(d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 12*a*(5*a^
2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] + (b*(
-150*a^6 - 130*a^4*b^2 - 24*a^2*b^4 + 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2
- 30*a*b^3 + 4*b^4)*Cos[d + e*x] + 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2
*a*b^3 - 2*b^4)*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] + 30*a^5*b*Cos[3
*(d + e*x)] + 41*a^4*b^2*Cos[3*(d + e*x)] + 38*a^3*b^3*Cos[3*(d + e*x)] + 1
2*a^2*b^4*Cos[3*(d + e*x)] + 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x
] + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e
*x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x]
 - 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*
(d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*
a*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)
] - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Si
n[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)]))/((a +
b)*(Cos[(d + e*x)/2] - Sin[(d + e*x)/2])^3*((a + b)*Cos[(d + e*x)/2] + (-a
+ b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

Maple [B] time = 0.224, size = 1149, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x)
```

```
[Out] -1/16/e/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/32/e/b^4/
(tan(1/2*d+1/2*e*x)-1)^2-1/16/e/b^4/(tan(1/2*d+1/2*e*x)-1)-1/48/e/b^4/(tan(
1/2*d+1/2*e*x)-1)^3+3/32/e*a/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+
1/2*e*x)-a-b)-1/48/e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)
-a-b)^3*a^6-1/16/e/b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a
-b)^3*a^4+1/16/e/b^5/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b
)^2*a^6-3/32/e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^
2*a^5+3/32/e/b^3/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*
a^4-3/16/e/b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^
3-5/32/e/b^6/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^6+3/
8/e/b^5/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^5-3/8/e/b
^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^4+3/8/e/b^3/(a
-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^3-9/32/e/b^2/(a-b)^
3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2-5/32/e*a^4/b^7/(a-b)*
ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)+5/32/e*a^3/b^6/(a-b)*ln(a
*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-3/32/e*a^2/b^5/(a-b)*ln(a*tan
(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/16/e/b^5/(tan(1/2*d+1/2*e*x)-1)
^2*a-5/32/e/b^6/(tan(1/2*d+1/2*e*x)-1)*a^2-1/16/e/b^5/(tan(1/2*d+1/2*e*x)-1
)*a+5/32/e*a^3/b^7*ln(tan(1/2*d+1/2*e*x)-1)+3/32/e*a/b^5*ln(tan(1/2*d+1/2*e
*x)-1)-1/48/e*b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^3
-1/32/e*b/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2-1/16/e/
(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^3*a^2-3/32/e/(a-b)^
3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a
```

Maxima [B] time = 1.25758, size = 1295, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 - 3*(25*a^8 - 25*a^7*
b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b^
7 + 2*b^8)*sin(e*x + d)/(cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6*
b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin(e*x
+ d)^2/(cos(e*x + d) + 1)^2 - 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^5
*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(e*x + d)
^3/(cos(e*x + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^3
 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(e*x + d)^4/(cos(e*x
 + d) + 1)^4 - 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4 -
 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(e*x + d)^5/(cos(e*x + d) +
1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 - 6*(a^6*b^6 - a^5*b^7 - 2*a
^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(e*x + d)/(cos(e*x + d) + 1) + 3
*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 - 2*a*b^11 + b
^12)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*a
^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(e*x + d)^3/(cos(e*x + d) +
1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^10 + 4*a
*b^11 - b^12)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 - 6*(a^6*b^6 - 5*a^5*b^7
+ 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(e*x + d)^5/(cos(e*x +
d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*b^9 + 15*a^2*b^10 -
6*a*b^11 + b^12)*sin(e*x + d)^6/(cos(e*x + d) + 1)^6) - 3*(5*a^3 + 3*a*b^2)
*log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^7 + 3*(5*a^3 + 3*a*
b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1)/b^7)/e
```

Fricas [B] time = 2.82511, size = 1635, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] -1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(e
*x + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(e*x + d)^2 - 6*(10*a^5*b - 9*a^3*
b^3 - 2*a*b^5)*cos(e*x + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^
3 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d
)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) - (20*a^6 + 12*a^4*b^2 - (5*a^6
- 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x
+ d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*
x + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(e*x
 + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3
*a^3*b^3)*cos(e*x + d) - (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2
*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin(e*x + d))*
```

```
log(-sin(e*x + d) + 1) - 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*a^4*b^2 -
3*a^2*b^4 - 4*b^6)*cos(e*x + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*b^5)*cos(e*
x + d))*sin(e*x + d))/(6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (3*a^2*b^8
- b^10)*e*cos(e*x + d)^3 - 3*(a^3*b^7 - a*b^9)*e*cos(e*x + d)^2 - (6*a^2*b^
8*e*cos(e*x + d) + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*e*cos(e*x + d)^2)*sin(
e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1823, size = 1373, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/96*(3*(5*a^4 - 5*a^3*b + 3*a^2*b^2 - 3*a*b^3)*log(abs(a*tan(1/2*x*e + 1/
2*d) - b*tan(1/2*x*e + 1/2*d) - a - b))/(a*b^7 - b^8) + 2*(15*a^8*tan(1/2*x
*e + 1/2*d)^5 - 75*a^7*b*tan(1/2*x*e + 1/2*d)^5 + 159*a^6*b^2*tan(1/2*x*e +
 1/2*d)^5 - 195*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 + 165*a^4*b^4*tan(1/2*x*e +
1/2*d)^5 - 105*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 + 51*a^2*b^6*tan(1/2*x*e + 1/
2*d)^5 - 21*a*b^7*tan(1/2*x*e + 1/2*d)^5 + 6*b^8*tan(1/2*x*e + 1/2*d)^5 - 7
5*a^8*tan(1/2*x*e + 1/2*d)^4 + 300*a^7*b*tan(1/2*x*e + 1/2*d)^4 - 495*a^6*b
^2*tan(1/2*x*e + 1/2*d)^4 + 480*a^5*b^3*tan(1/2*x*e + 1/2*d)^4 - 345*a^4*b^
4*tan(1/2*x*e + 1/2*d)^4 + 180*a^3*b^5*tan(1/2*x*e + 1/2*d)^4 - 57*a^2*b^6*
tan(1/2*x*e + 1/2*d)^4 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^4 + 150*a^8*tan(1/2*
x*e + 1/2*d)^3 - 450*a^7*b*tan(1/2*x*e + 1/2*d)^3 + 500*a^6*b^2*tan(1/2*x*e
 + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*x*e + 1/2*d)^3 + 126*a^4*b^4*tan(1/2*x*e
+ 1/2*d)^3 + 22*a^3*b^5*tan(1/2*x*e + 1/2*d)^3 - 48*a^2*b^6*tan(1/2*x*e + 1
/2*d)^3 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^3 - 4*b^8*tan(1/2*x*e + 1/2*d)^3 -
```

```
150*a^8*tan(1/2*x*e + 1/2*d)^2 + 300*a^7*b*tan(1/2*x*e + 1/2*d)^2 - 120*a^6
*b^2*tan(1/2*x*e + 1/2*d)^2 - 60*a^5*b^3*tan(1/2*x*e + 1/2*d)^2 + 102*a^4*b
^4*tan(1/2*x*e + 1/2*d)^2 - 144*a^3*b^5*tan(1/2*x*e + 1/2*d)^2 + 60*a^2*b^6
*tan(1/2*x*e + 1/2*d)^2 - 12*a*b^7*tan(1/2*x*e + 1/2*d)^2 + 75*a^8*tan(1/2*
x*e + 1/2*d) - 75*a^7*b*tan(1/2*x*e + 1/2*d) - 75*a^6*b^2*tan(1/2*x*e + 1/2
*d) + 75*a^5*b^3*tan(1/2*x*e + 1/2*d) - 39*a^4*b^4*tan(1/2*x*e + 1/2*d) + 3
9*a^3*b^5*tan(1/2*x*e + 1/2*d) + 33*a^2*b^6*tan(1/2*x*e + 1/2*d) - 15*a*b^7
*tan(1/2*x*e + 1/2*d) + 6*b^8*tan(1/2*x*e + 1/2*d) - 15*a^8 + 31*a^6*b^2 -
9*a^4*b^4 - 15*a^2*b^6)/((a^3*b^6 - 3*a^2*b^7 + 3*a*b^8 - b^9)*(a*tan(1/2*x
*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) + a + b
)^3) - 3*(5*a^3 + 3*a*b^2)*log(abs(tan(1/2*x*e + 1/2*d) - 1))/b^7)*e^(-1)
```

3.395 $\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx$

Optimal. Leaf size=260

$$\frac{5ab\left(10a^2+11\left(b^2+c^2\right)\right)\sin(d+ex)}{24e}-\frac{5ac\left(10a^2+11\left(b^2+c^2\right)\right)\cos(d+ex)}{24e}-\frac{\left(c\left(26a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(\right.\right.}{}$$

```
[Out] ((8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*x)/8 - (5*a*c*(10*a^2 + 11*
(b^2 + c^2))*Cos[d + e*x])/(24*e) + (5*a*b*(10*a^2 + 11*(b^2 + c^2))*Sin[d
+ e*x])/(24*e) - (7*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*
x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e) - ((a + b*Cos[d + e*x] + c*Sin[d +
e*x])*(c*(26*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(26*a^2 + 9*(b^2 + c^2))
*Sin[d + e*x]))/(24*e)
```

Rubi [A] time = 0.399687, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3120, 3146, 2637, 2638}

$$\frac{5ab\left(10a^2+11\left(b^2+c^2\right)\right)\sin(d+ex)}{24e}-\frac{5ac\left(10a^2+11\left(b^2+c^2\right)\right)\cos(d+ex)}{24e}-\frac{\left(c\left(26a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*x)/8 - (5*a*c*(10*a^2 + 11*
(b^2 + c^2))*Cos[d + e*x])/(24*e) + (5*a*b*(10*a^2 + 11*(b^2 + c^2))*Sin[d
+ e*x])/(24*e) - (7*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*
x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e) - ((a + b*Cos[d + e*x] + c*Sin[d +
e*x])*(c*(26*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(26*a^2 + 9*(b^2 + c^2))
*Sin[d + e*x]))/(24*e)
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}+\frac{1}{4}\int (a$$

$$= -\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{12e}-\frac{(c\cos}{}$$

$$= -\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{12e}-\frac{(c\cos}{}$$

$$= \frac{1}{8}\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)x-\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(}{12}$$

$$= \frac{1}{8}\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)x-\frac{5ac\left(10a^2+11\left(b^2+c^2\right)\right)\cos(d+}{24e}$$

Mathematica [A] time = 1.08928, size = 237, normalized size = 0.91

$$\frac{12\left(24a^2\left(b^2+c^2\right)+8a^4+3\left(b^2+c^2\right)^2\right)(d+ex)+96ab\left(4a^2+3\left(b^2+c^2\right)\right)\sin(d+ex)+24\left(b^2-c^2\right)\left(6a^2+b^2+c^2\right)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*(d + e*x) - 96*a*c*(4*a^
2 + 3*(b^2 + c^2))*Cos[d + e*x] - 48*b*c*(6*a^2 + b^2 + c^2)*Cos[2*(d + e*x
)] + 32*a*c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d +
 e*x)] + 96*a*b*(4*a^2 + 3*(b^2 + c^2))*Sin[d + e*x] + 24*(b^2 - c^2)*(6*a^
2 + b^2 + c^2)*Sin[2*(d + e*x)] + 32*a*b*(b^2 - 3*c^2)*Sin[3*(d + e*x)] + 3
*(b^4 - 6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

Maple [A] time = 0.072, size = 335, normalized size = 1.3

$$\frac{1}{e}\left(b^4\left(\frac{\sin(ex+d)}{4}\left((\cos(ex+d))^3+\frac{3\cos(ex+d)}{2}\right)+\frac{3ex}{8}+\frac{3d}{8}\right)+c^4\left(-\frac{\cos(ex+d)}{4}\left((\sin(ex+d))^3+\frac{3\sin(ex+d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x)
```

```
[Out] 1/e*(b^4*(1/4*(cos(e*x+d)^3+3/2*cos(e*x+d))*sin(e*x+d)+3/8*e*x+3/8*d)+c^4*(
-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)+a^4*(e*x+d)-6*
a^2*b*c*cos(e*x+d)^2-4*a*b^2*c*cos(e*x+d)^3+4*a*b*c^2*sin(e*x+d)^3+4*a^3*b*
sin(e*x+d)-4*cos(e*x+d)*a^3*c+6*a^2*b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+
1/2*d)+6*a^2*c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+4/3*a*b^3*(2+co
s(e*x+d)^2)*sin(e*x+d)-4/3*a*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)-cos(e*x+d)^4*b
^3*c+6*b^2*c^2*(-1/4*sin(e*x+d)*cos(e*x+d)^3+1/8*sin(e*x+d)*cos(e*x+d)+1/8*
e*x+1/8*d)+b*c^3*sin(e*x+d)^4)
```

Maxima [A] time = 1.02385, size = 446, normalized size = 1.72

$$-\frac{b^3c\cos(ex+d)^4}{e}+\frac{bc^3\sin(ex+d)^4}{e}+a^4x+\frac{(12\,ex+12\,d+\sin(4\,ex+4\,d)+8\,\sin(2\,ex+2\,d))b^4}{32\,e}+\frac{3\,(4\,ex+4\,d-\mathrm{s}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -b^3*c*cos(e*x + d)^4/e + b*c^3*sin(e*x + d)^4/e + a^4*x + 1/32*(12*e*x + 1
2*d + sin(4*e*x + 4*d) + 8*sin(2*e*x + 2*d))*b^4/e + 3/16*(4*e*x + 4*d - si
n(4*e*x + 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(
2*e*x + 2*d))*c^4/e - 4*a^3*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/2*(4*
b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2*
d - sin(2*e*x + 2*d))*c^2/e)*a^2 - 4/3*(3*b^2*c*cos(e*x + d)^3/e - 3*b*c^2*
sin(e*x + d)^3/e + (sin(e*x + d)^3 - 3*sin(e*x + d))*b^3/e - (cos(e*x + d)^
3 - 3*cos(e*x + d))*c^3/e)*a
```

Fricas [A] time = 2.25867, size = 576, normalized size = 2.22

$$-\frac{24\left(b^{3}c-bc^{3}\right)\cos\left(ex+d\right)^{4}+32\left(3\,ab^{2}c-ac^{3}\right)\cos\left(ex+d\right)^{3}-3\left(8\,a^{4}+24\,a^{2}b^{2}+3\,b^{4}+3\,c^{4}+6\left(4\,a^{2}+b^{2}\right)c^{2}\right)ex+48\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 + 32*(3*a*b^2*c - a*c^3)*cos(e*x +
 d)^3 - 3*(8*a^4 + 24*a^2*b^2 + 3*b^4 + 3*c^4 + 6*(4*a^2 + b^2)*c^2)*e*x +
48*(3*a^2*b*c + b*c^3)*cos(e*x + d)^2 + 96*(a^3*c + a*c^3)*cos(e*x + d) - (
96*a^3*b + 64*a*b^3 + 96*a*b*c^2 + 6*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^3
 + 32*(a*b^3 - 3*a*b*c^2)*cos(e*x + d)^2 + 3*(24*a^2*b^2 + 3*b^4 - 5*c^4 -
6*(4*a^2 - b^2)*c^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 2.27997, size = 707, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sin(d + e*x)/e - 4*a**3*c*cos(d + e*x)/e + 3*a
**2*b**2*x*sin(d + e*x)**2 + 3*a**2*b**2*x*cos(d + e*x)**2 + 3*a**2*b**2*si
n(d + e*x)*cos(d + e*x)/e + 6*a**2*b*c*sin(d + e*x)**2/e + 3*a**2*c**2*x*si
n(d + e*x)**2 + 3*a**2*c**2*x*cos(d + e*x)**2 - 3*a**2*c**2*sin(d + e*x)*co
s(d + e*x)/e + 8*a*b**3*sin(d + e*x)**3/(3*e) + 4*a*b**3*sin(d + e*x)*cos(d
 + e*x)**2/e - 4*a*b**2*c*cos(d + e*x)**3/e + 4*a*b*c**2*sin(d + e*x)**3/e
```

```
- 4*a*c**3*sin(d + e*x)**2*cos(d + e*x)/e - 8*a*c**3*cos(d + e*x)**3/(3*e)
+ 3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 +
 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5
*b**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) - b**3*c*cos(d + e*x)**4/e + 3*b**
2*c**2*x*sin(d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/
2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e
*x)/(4*e) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) - 2*b*c**3*sin(d
 + e*x)**2*cos(d + e*x)**2/e - b*c**3*cos(d + e*x)**4/e + 3*c**4*x*sin(d +
e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*c**4*x*cos(d + e
*x)**4/8 - 5*c**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*c**4*sin(d + e*x)*
cos(d + e*x)**3/(8*e), Ne(e, 0)), (x*(a + b*cos(d) + c*sin(d))**4, True))
```

Giac [A] time = 1.12332, size = 386, normalized size = 1.48

$$-\frac{1}{8}\left(b^3c - bc^3\right)\cos\left(4\,xe + 4\,d\right)e^{(-1)} - \frac{1}{3}\left(3\,ab^2c - ac^3\right)\cos\left(3\,xe + 3\,d\right)e^{(-1)} - \frac{1}{2}\left(6\,a^2bc + b^3c + bc^3\right)\cos\left(2\,xe + 2\,d\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/8*(b^3*c - b*c^3)*cos(4*x*e + 4*d)*e^(-1) - 1/3*(3*a*b^2*c - a*c^3)*cos(
3*x*e + 3*d)*e^(-1) - 1/2*(6*a^2*b*c + b^3*c + b*c^3)*cos(2*x*e + 2*d)*e^(-
1) - (4*a^3*c + 3*a*b^2*c + 3*a*c^3)*cos(x*e + d)*e^(-1) + 1/32*(b^4 - 6*b^
2*c^2 + c^4)*e^(-1)*sin(4*x*e + 4*d) + 1/3*(a*b^3 - 3*a*b*c^2)*e^(-1)*sin(3
*x*e + 3*d) + 1/4*(6*a^2*b^2 + b^4 - 6*a^2*c^2 - c^4)*e^(-1)*sin(2*x*e + 2*
d) + (4*a^3*b + 3*a*b^3 + 3*a*b*c^2)*e^(-1)*sin(x*e + d) + 1/8*(8*a^4 + 24*
a^2*b^2 + 3*b^4 + 24*a^2*c^2 + 6*b^2*c^2 + 3*c^4)*x
```

3.396 $\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$

Optimal. Leaf size=170

$$\frac{b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{6e}-\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(2a^2+3\left(b^2+c^2\right)\right)-\frac{(c\cos(d+ex)-b\sin($$

```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*x)/2 - (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x])
/(6*e) + (b*(11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*e) - (5*(a*c*Cos[d +
e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c
*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3
*e)
```

Rubi [A] time = 0.186289, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3120, 3146, 2637, 2638}

$$\frac{b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{6e}-\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(2a^2+3\left(b^2+c^2\right)\right)-\frac{(c\cos(d+ex)-b\sin($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*x)/2 - (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x])
/(6*e) + (b*(11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*e) - (5*(a*c*Cos[d +
e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c
*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3
*e)
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}+\frac{1}{3}\int (a$$

$$= -\frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{6e} - \frac{(c\,c$$

$$= \frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x - \frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+e}{6e}$$

$$= \frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x - \frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)}{6e} + \frac{b\left(11a^2+4\left(\right.\right.}{}$$

Mathematica [A] time = 0.437597, size = 144, normalized size = 0.85

$$\frac{6a\left(2a^2+3\left(b^2+c^2\right)\right)(d+ex)+9b\left(4a^2+b^2+c^2\right)\sin(d+ex)-9c\left(4a^2+b^2+c^2\right)\cos(d+ex)+9a\left(b^2-c^2\right)\sin(2(d+}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*(b^2 + c^2))*(d + e*x) - 9*c*(4*a^2 + b^2 + c^2)*Cos[d + e*
x] - 18*a*b*c*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] + 9*b*(4
```

```
*a^2 + b^2 + c^2)*Sin[d + e*x] + 9*a*(b^2 - c^2)*Sin[2*(d + e*x)] + b*(b^2
- 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

Maple [A] time = 0.06, size = 177, normalized size = 1.

$$\frac{1}{e}\left(a^3(ex+d)+3\sin(ex+d)a^2b-3a^2c\cos(ex+d)+3ab^2(1/2\sin(ex+d)\cos(ex+d)+1/2ex+d/2)-3abc(\cos(ex\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x)
```

```
[Out] 1/e*(a^3*(e*x+d)+3*sin(e*x+d)*a^2*b-3*a^2*c*cos(e*x+d)+3*a*b^2*(1/2*sin(e*x
+d)*cos(e*x+d)+1/2*e*x+1/2*d)-3*a*b*c*cos(e*x+d)^2+3*a*c^2*(-1/2*sin(e*x+d)
*cos(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-cos(e*x+d)^3
*b^2*c+b*c^2*sin(e*x+d)^3-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

Maxima [A] time = 0.998739, size = 255, normalized size = 1.5

$$-\frac{b^2c\cos(ex+d)^3}{e}+\frac{bc^2\sin(ex+d)^3}{e}+a^3x-\frac{\left(\sin(ex+d)^3-3\sin(ex+d)\right)b^3}{3e}+\frac{\left(\cos(ex+d)^3-3\cos(ex+d)\right)c^3}{3e}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -b^2*c*cos(e*x + d)^3/e + b*c^2*sin(e*x + d)^3/e + a^3*x - 1/3*(sin(e*x + d
)^3 - 3*sin(e*x + d))*b^3/e + 1/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e -
 3*a^2*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/4*(4*b*c*cos(e*x + d)^2/e
- (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))
*c^2/e)*a
```

Fricas [A] time = 2.22173, size = 338, normalized size = 1.99

$$-\frac{18abc\cos(ex+d)^2+2\left(3b^2c-c^3\right)\cos(ex+d)^3-3\left(2a^3+3ab^2+3ac^2\right)ex+6\left(3a^2c+c^3\right)\cos(ex+d)-\left(18a^2b+4\right.}{6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a*b*c*cos(e*x + d)^2 + 2*(3*b^2*c - c^3)*cos(e*x + d)^3 - 3*(2*a^3
 + 3*a*b^2 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (18*a^2*b + 4*
b^3 + 6*b*c^2 + 2*(b^3 - 3*b*c^2)*cos(e*x + d)^2 + 9*(a*b^2 - a*c^2)*cos(e*
x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.90262, size = 294, normalized size = 1.73

$$\begin{cases} a^3x + \frac{3a^2b\sin(d+ex)}{e} - \frac{3a^2c\cos(d+ex)}{e} + \frac{3ab^2x\sin^2(d+ex)}{2} + \frac{3ab^2x\cos^2(d+ex)}{2} + \frac{3ab^2\sin(d+ex)\cos(d+ex)}{2e} + \frac{3abc\sin^2(d+ex)}{e} + \frac{3ac^2x\sin^2(d\cdots}{2} \\ x\left(a+b\cos(d)+c\sin(d)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sin(d + e*x)/e - 3*a**2*c*cos(d + e*x)/e + 3*a
*b**2*x*sin(d + e*x)**2/2 + 3*a*b**2*x*cos(d + e*x)**2/2 + 3*a*b**2*sin(d +
 e*x)*cos(d + e*x)/(2*e) + 3*a*b*c*sin(d + e*x)**2/e + 3*a*c**2*x*sin(d + e
*x)**2/2 + 3*a*c**2*x*cos(d + e*x)**2/2 - 3*a*c**2*sin(d + e*x)*cos(d + e*x
)/(2*e) + 2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**2/
e - b**2*c*cos(d + e*x)**3/e + b*c**2*sin(d + e*x)**3/e - c**3*sin(d + e*x)
**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(a + b*cos
(d) + c*sin(d))**3, True))
```

Giac [A] time = 1.1238, size = 225, normalized size = 1.32

$$-\frac{3}{2}\,abc\cos\left(2\,xe+2\,d\right)e^{(-1)} - \frac{1}{12}\left(3\,b^2c-c^3\right)\cos\left(3\,xe+3\,d\right)e^{(-1)} - \frac{3}{4}\left(4\,a^2c+b^2c+c^3\right)\cos\left(xe+d\right)e^{(-1)} + \frac{1}{12}\left(b^3-3\,b\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -3/2*a*b*c*cos(2*x*e + 2*d)*e^(-1) - 1/12*(3*b^2*c - c^3)*cos(3*x*e + 3*d)*
e^(-1) - 3/4*(4*a^2*c + b^2*c + c^3)*cos(x*e + d)*e^(-1) + 1/12*(b^3 - 3*b*
c^2)*e^(-1)*sin(3*x*e + 3*d) + 3/4*(a*b^2 - a*c^2)*e^(-1)*sin(2*x*e + 2*d)
```

```
+ 3/4*(4*a^2*b + b^3 + b*c^2)*e^(-1)*sin(x*e + d) + 1/2*(2*a^3 + 3*a*b^2 +
3*a*c^2)*x
```

3.397 $\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}x\left(2a^2+b^2+c^2\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}+\frac{3ab\sin(d+ex)}{2e}-\frac{3ac\cos(d+}{2e}$$

```
[Out] ((2*a^2 + b^2 + c^2)*x)/2 - (3*a*c*Cos[d + e*x])/(2*e) + (3*a*b*Sin[d + e*x
])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d
 + e*x]))/(2*e)
```

Rubi [A] time = 0.0463345, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3120, 2637, 2638}

$$\frac{1}{2}x\left(2a^2+b^2+c^2\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}+\frac{3ab\sin(d+ex)}{2e}-\frac{3ac\cos(d+}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] ((2*a^2 + b^2 + c^2)*x)/2 - (3*a*c*Cos[d + e*x])/(2*e) + (3*a*b*Sin[d + e*x
])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d
 + e*x]))/(2*e)
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}+\frac{1}{2}\int \Big(2a^2$$
$$=\frac{1}{2}\left(2a^2+b^2+c^2\right)x-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(}{2e}$$
$$=\frac{1}{2}\left(2a^2+b^2+c^2\right)x-\frac{3ac\cos(d+ex)}{2e}+\frac{3ab\sin(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\,\mathrm{s}}{}$$

Mathematica [A] time = 0.170644, size = 77, normalized size = 0.85

$$\frac{2\left(2a^2+b^2+c^2\right)(d+ex)+8ab\sin(d+ex)-8ac\cos(d+ex)+\left(b^2-c^2\right)\sin(2(d+ex))-2bc\cos(2(d+ex))}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2 + c^2)*(d + e*x) - 8*a*c*Cos[d + e*x] - 2*b*c*Cos[2*(d + e*
x)] + 8*a*b*Sin[d + e*x] + (b^2 - c^2)*Sin[2*(d + e*x)])/(4*e)
```

Maple [A] time = 0.051, size = 99, normalized size = 1.1

$$\frac{1}{e}\left(a^2(ex+d)+2\,ab\sin(ex+d)-2\,ac\cos(ex+d)+b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-(\cos(ex+d))^2\,bc+c^2\Big(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x)
```

```
[Out] 1/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)-2*a*c*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*b*c+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

Maxima [A] time = 0.981224, size = 135, normalized size = 1.48

$$a^2x - \frac{bc\cos(ex+d)^2}{e} + \frac{(2\,ex + 2\,d + \sin(2\,ex + 2\,d))b^2}{4\,e} + \frac{(2\,ex + 2\,d - \sin(2\,ex + 2\,d))c^2}{4\,e} - 2\,a\left(\frac{c\cos(ex+d)}{e} - \frac{b\sin(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - b*c*cos(e*x + d)^2/e + 1/4*(2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e +
 1/4*(2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 2*a*(c*cos(e*x + d)/e - b*sin
(e*x + d)/e)
```

Fricas [A] time = 2.23692, size = 173, normalized size = 1.9

$$-\frac{2\,bc\cos(ex+d)^2 - \left(2\,a^2 + b^2 + c^2\right)ex + 4\,ac\cos(ex+d) - \left(4\,ab + \left(b^2 - c^2\right)\cos(ex+d)\right)\sin(ex+d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*cos(e*x + d)^2 - (2*a^2 + b^2 + c^2)*e*x + 4*a*c*cos(e*x + d) -
 (4*a*b + (b^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 0.387578, size = 162, normalized size = 1.78

$$\begin{cases} a^2x + \frac{2ab\sin(d+ex)}{e} - \frac{2ac\cos(d+ex)}{e} + \frac{b^2x\sin^2(d+ex)}{2} + \frac{b^2x\cos^2(d+ex)}{2} + \frac{b^2\sin(d+ex)\cos(d+ex)}{2e} + \frac{bc\sin^2(d+ex)}{e} + \frac{c^2x\sin^2(d+ex)}{2} + \frac{c^2x\,\mathrm{co}}{} \\ x\left(a + b\cos(d) + c\sin(d)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sin(d + e*x)/e - 2*a*c*cos(d + e*x)/e + b**2*x*si
n(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*sin(d + e*x)*cos(d + e*x)
/(2*e) + b*c*sin(d + e*x)**2/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d +
e*x)**2/2 - c**2*sin(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(a + b*cos(
```

```
d) + c*sin(d))**2, True))
```

Giac [A] time = 1.12857, size = 109, normalized size = 1.2

$$-\frac{1}{2}\,bc\cos\left(2\,xe+2\,d\right)e^{(-1)}-2\,ac\cos\left(xe+d\right)e^{(-1)}+2\,abe^{(-1)}\sin\left(xe+d\right)+\frac{1}{4}\left(b^2-c^2\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+\frac{1}{2}\left(2\,a^2+b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*cos(2*x*e + 2*d)*e^(-1) - 2*a*c*cos(x*e + d)*e^(-1) + 2*a*b*e^(-1)
*sin(x*e + d) + 1/4*(b^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 1/2*(2*a^2 + b^2
+ c^2)*x
```

3.398 $\int (a+b\cos(d+ex)+c\sin(d+ex))\,dx$

Optimal. Leaf size=27

$$ax+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

```
[Out] a*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rubi [A] time = 0.0157105, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2637, 2638}

$$ax+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] a*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(d+ex)+c\sin(d+ex))\,dx &= ax+b\int \cos(d+ex)\,dx+c\int \sin(d+ex)\,dx\\ &= ax-\frac{c\cos(d+ex)}{e}+\frac{b\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0143011, size = 49, normalized size = 1.81

$$ax + \frac{b \sin(d) \cos(ex)}{e} + \frac{b \cos(d) \sin(ex)}{e} + \frac{c \sin(d) \sin(ex)}{e} - \frac{c \cos(d) \cos(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] a*x - (c*Cos[d]*Cos[e*x])/e + (b*Cos[e*x]*Sin[d])/e + (b*Cos[d]*Sin[e*x])/e
 + (c*Sin[d]*Sin[e*x])/e
```

Maple [A] time = 0.002, size = 28, normalized size = 1.

$$ax - \frac{c \cos(ex + d)}{e} + \frac{b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(e*x+d)+c*sin(e*x+d),x)
```

```
[Out] a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e
```

Maxima [A] time = 0.978144, size = 36, normalized size = 1.33

$$ax - \frac{c \cos(ex + d)}{e} + \frac{b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] a*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

Fricas [A] time = 2.01996, size = 61, normalized size = 2.26

$$\frac{aex - c \cos(ex + d) + b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] (a*e*x - c*cos(e*x + d) + b*sin(e*x + d))/e
```

Sympy [A] time = 0.154906, size = 34, normalized size = 1.26

$$ax + b\left(\begin{cases}\frac{\sin{(d+ex)}}{e} & \text{for } e \neq 0\\ x\cos{(d)} & \text{otherwise}\end{cases}\right) + c\left(\begin{cases}-\frac{\cos{(d+ex)}}{e} & \text{for } e \neq 0\\ x\sin{(d)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x)
```

```
[Out] a*x + b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewi
se((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

Giac [A] time = 1.11897, size = 36, normalized size = 1.33

$$-c\cos{(xe+d)}\,e^{(-1)} + be^{(-1)}\sin{(xe+d)} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -c*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d) + a*x
```

3.399 $\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=61

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

```
[Out] (2*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2
- b^2 - c^2]*e)
```

Rubi [A] time = 0.0837496, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (2*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2
- b^2 - c^2]*e)
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx, x, 2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= \frac{2\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,e}$$

Mathematica [A] time = 0.117129, size = 57, normalized size = 0.93

$$-\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{e\sqrt{-a^2+b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(Sqrt[-
a^2 + b^2 + c^2]*e)
```

Maple [A] time = 0.079, size = 61, normalized size = 1.

$$2\,\frac{1}{e\sqrt{a^2-b^2-c^2}}\arctan\left(1/2\,\frac{2\,(a-b)\tan(d/2+1/2\,ex)+2\,c}{\sqrt{a^2-b^2-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x)
```

```
[Out] 2/e/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^
2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33889, size = 953, normalized size = 15.62

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\log\left(-\frac{a^2b^2-2\,b^4-c^4-\left(a^2+3\,b^2\right)c^2-\left(2\,a^2b^2-b^4-2\,a^2c^2+c^4\right)\cos(ex+d)^2-2\left(ab^3+abc^2\right)\cos(ex+d)-2\left(ab^2c+ac^3-\left(bc^3-\left(2\,a^2b-b^3\right)c\right)\cos(ex}{2\,ab\cos(ex+d)+\left(b^2-c^2\right)\cos(ex+d)^2+a^2+c}\right.}{2\left(a^2-b^2-c^2\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^
2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2
)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x +
 d))*sin(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e
*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-
a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 +
c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d)))/((a^2 - b^2 - c^2)*e), arct
an(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)
/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d)))
/(sqrt(a^2 - b^2 - c^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11689, size = 123, normalized size = 2.02

$$-\frac{2\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)e^{(-1)}}{\sqrt{a^2-b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x
*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*e^(-1)/sq
rt(a^2 - b^2 - c^2)
```

3.400 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=121

$$\frac{2a\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{3/2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] (2*a*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/((a^2 -
b^2 - c^2)^(3/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/((a^2 - b^2 - c^2)*
e*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rubi [A] time = 0.107953, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3129, 12, 3124, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{3/2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/((a^2 -
b^2 - c^2)^(3/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/((a^2 - b^2 - c^2)*
e*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{\int \frac{a}{a+b\cos(d+ex)+c\sin(d+ex)}}{-a^2+b^2+c^2}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} + \frac{a\int \frac{1}{a+b\cos(d+ex)+c\sin(d+}}{a^2-b^2-c^2}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx+}\right.}{\left(a^2\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-\right.}\right.}{}$$

$$= \frac{2a\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}e} + \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d}$$

Mathematica [A] time = 0.341416, size = 116, normalized size = 0.96

$$\frac{\frac{ac+\left(b^2+c^2\right)\sin(d+ex)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] ((2*a*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2
 + b^2 + c^2)^(3/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2
)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/e
```

Maple [B] time = 0.142, size = 424, normalized size = 3.5

$$-2\frac{a\tan(d/2+1/2\,ex)\,b}{e\left(a\left(\tan(d/2+1/2\,ex)\right)^2-b\left(\tan(d/2+1/2\,ex)\right)^2+2\,c\tan(d/2+1/2\,ex)+a+b\right)\left(a^3-a^2b-ab^2-ac^2+b^3+bc^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x)
```

```
[Out] -2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+
a+b)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)*a*b+2/e/(a*tan(1/
2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)/(a^3-a^2*
b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)*b^2+2/e/(a*tan(1/2*d+1/2*e*x)^2
-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)/(a^3-a^2*b-a*b^2-a*c^2+
b^3+b*c^2)*tan(1/2*d+1/2*e*x)*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1
/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)*a*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)
+2/e*a/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2
-b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.98829, size = 1796, normalized size = 14.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(-a^2 + b^2 + c^2)*lo
g((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2
 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*x + d) - 2*(a*b^2*c + a*
c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin(e*x + d) - 2*(2*a*b*c*c
os(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2 + (a*b^2
- a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x
 + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)
*sin(e*x + d))) - 2*(c^3 - (a^2 - b^2)*c)*cos(e*x + d) - 2*(a^2*b - b^3 - b
*c^2)*sin(e*x + d))/((a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2
)*e*cos(e*x + d) + (c^5 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*
sin(e*x + d) + (a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e),
((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(a^2 - b^2 - c^2)*arctan(-
(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c
^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - (
c^3 - (a^2 - b^2)*c)*cos(e*x + d) - (a^2*b - b^3 - b*c^2)*sin(e*x + d))/((a
^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2)*e*cos(e*x + d) + (c^5
 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*sin(e*x + d) + (a^5 - 2
*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14243, size = 300, normalized size = 2.48

$$-2\left(\frac{\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)a}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{ab\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)}{\left(a^3-a^2b-ab^2+b^3-ac^2+bc^2\right)\left(a\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*a/(a^2 -
 b^2 - c^2)^(3/2) + (a*b*tan(1/2*x*e + 1/2*d) - b^2*tan(1/2*x*e + 1/2*d) -
c^2*tan(1/2*x*e + 1/2*d) - a*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b*c^2
)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e +
1/2*d) + a + b)))*e^(-1)
```

3.401 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=197

$$\frac{\left(2a^2+b^2+c^2\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{5/2}}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{2e\left(a^2-b^2-c^2\right)^2(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{c\cos(d}{2e\left(a^2-b^2-c^2\right)(a}$$

```
[Out] ((2*a^2 + b^2 + c^2)*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 -
 c^2]])/((a^2 - b^2 - c^2)^(5/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*
(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*c*Cos[
d + e*x] - a*b*Sin[d + e*x]))/(2*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x]
+ c*Sin[d + e*x]))
```

Rubi [A] time = 0.197937, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3129, 3153, 3124, 618, 204}

$$\frac{\left(2a^2+b^2+c^2\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{5/2}}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{2e\left(a^2-b^2-c^2\right)^2(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{c\cos(d}{2e\left(a^2-b^2-c^2\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2 + c^2)*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 -
 c^2]])/((a^2 - b^2 - c^2)^(5/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*
(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*c*Cos[
d + e*x] - a*b*Sin[d + e*x]))/(2*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x]
+ c*Sin[d + e*x]))
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
```

```
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} - \frac{\int \frac{-2a+b\cos(d+ex)+c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^2}}{2\left(a^2-b^2-c^2\right)}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac\cos(d}{2\left(a^2-b^2-c^2\right)^2 e(a}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac\cos(d}{2\left(a^2-b^2-c^2\right)^2 e(a}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac\cos(d}{2\left(a^2-b^2-c^2\right)^2 e(a}$$

$$= \frac{\left(2a^2+b^2+c^2\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}e} + \frac{c\cos(d+ex)-b\,\mathrm{s}}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d-}$$

Mathematica [A] time = 0.908594, size = 200, normalized size = 1.02

$$\frac{\frac{ac+\left(b^2+c^2\right)\sin(d+ex)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2} - \frac{c\left(2a^2+b^2+c^2\right)+3a\left(b^2+c^2\right)\sin(d+ex)}{b\left(-a^2+b^2+c^2\right)^2(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{2\left(2a^2+b^2+c^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{5/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2 + c^2)*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 +
 b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(
b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c*(2*a^2 +
 b^2 + c^2) + 3*a*(b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2)^2*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])))/(2*e)
```

Maple [B] time = 0.191, size = 3933, normalized size = 20.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x)
```

```
[Out] 4/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a
+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*a^4-1/e
/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)
^2*c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*a^2-1/e/
(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^
2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*b^4-1/e/(a*
tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c
^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*b^2-1/e/(a*t
an(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a
-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*b^4-2/
e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b
)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*
c^4-2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*
x)+a+b)^2*c^5/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*t
an(1/2*d+1/2*e*x)^2+1/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*
tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-
2*a*b+b^2)*tan(1/2*d+1/2*e*x)*b^5+1/e/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^
2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2
-b^2-c^2)^(1/2))*b^2+1/e/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b
^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1/
2))*c^2+13/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1
/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2
)*tan(1/2*d+1/2*e*x)^2*a^2*b^2-2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*
e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b
^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*a*b*c^2-6/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(
1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b
^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*a*b^3-12/e/(a*tan(1/
2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-
2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2
*a^3*b-6/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2
*e*x)+a+b)^2*c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2
)*tan(1/2*d+1/2*e*x)^2*a*b-3/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)
^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4
)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a^2*b*c^2-7/e/(a*tan(1/2*d+1/2*e*x)^2-
b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c
^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a*b^2*c^2+1/e/(a*t
an(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/
(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e
*x)^2*b^4-1/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+
1/2*e*x)+a+b)^2*c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+
```

```
b^2)*tan(1/2*d+1/2*e*x)^2*b^2-4/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e
*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+
c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a^4*b+5/e/(a*tan(1/2*d+1/2*e*x)^2-b
*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^
2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a^3*b^2+11/e/(a*tan
(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4
-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*
a^3*c^2+3/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/
2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*t
an(1/2*d+1/2*e*x)*a^2*b^3-5/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^
2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)
/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a*b^4-2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan
(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^
4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a*c^4-1/e/(a*tan(1/2*d+
1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*
b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*b^3*c^2
-2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+
a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*
d+1/2*e*x)*b*c^4-3/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan
(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*a^2*b^2-4/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*ta
n(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*t
an(1/2*d+1/2*e*x)^3*a^3*b+7/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^
2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^
2+c^4)*tan(1/2*d+1/2*e*x)^3*a^2*b^2+5/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d
+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^
4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*a^2*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2-b
*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*
a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*a*b^3-3/e/(a*tan(1/2*d+1/2*
e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^
2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*b^2*c^2+2/e/(a^4-2*
a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b
)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a^2+4/e/(a*tan(1/2*d+1/2*e*x
)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2
*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*a^4+7/e/(a
*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*
c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1
/2*e*x)^2*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.45739, size = 4077, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*a*b*c^3 - 12*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(e*x + d)^2 - (2*a^4
+ a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2*a^2*c^2 - c^4)*c
os(e*x + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(e*x + d) + 2*(a*c^3 + (2*
a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(e*x + d))*sin(e*x + d))*sq
rt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a
^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*
x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin
(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d)
- (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^
2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*
(b*c*cos(e*x + d) + a*c)*sin(e*x + d))) - 6*(a^3*b - a*b^3)*c + 2*(c^5 - (5
*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(e*x + d) - 2*(4*a^4*b
- 5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^
3*c^2 + a*c^4)*cos(e*x + d))*sin(e*x + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*
b^4 + 2*b^6)*c^2)*e*cos(e*x + d)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*e
*cos(e*x + d) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b
^2)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*
c^7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*c)*e*cos(e*x + d) + (a*c^7 - 3*(a^3 - a*b^2)*c^5 +
 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)
*e)*sin(e*x + d)), 1/2*(3*a*b*c^3 - 6*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(e*x
 + d)^2 + (2*a^4 + a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2
*a^2*c^2 - c^4)*cos(e*x + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(e*x + d)
 + 2*(a*c^3 + (2*a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(e*x + d))
*sin(e*x + d))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(e*x + d) + a*c*sin(e*
x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x +
```

```
d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - 3*(a^3*b - a*b^3)*c + (c^5 - (5
*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(e*x + d) - (4*a^4*b -
5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^3*
c^2 + a*c^4)*cos(e*x + d))*sin(e*x + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*b^
4 + 2*b^6)*c^2)*e*cos(e*x + d)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*e*c
os(e*x + d) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b^2
)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*c^
7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*c)*e*cos(e*x + d) + (a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3
*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*e
)*sin(e*x + d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17864, size = 1204, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*
e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*(2*a^2 + b
^2 + c^2)/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 -
 b^2 - c^2)) + (4*a^4*b*tan(1/2*x*e + 1/2*d)^3 - 11*a^3*b^2*tan(1/2*x*e + 1
/2*d)^3 + 9*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 - a*b^4*tan(1/2*x*e + 1/2*d)^3 -
 b^5*tan(1/2*x*e + 1/2*d)^3 - 5*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 + 7*a^2*b*c^
2*tan(1/2*x*e + 1/2*d)^3 + a*b^2*c^2*tan(1/2*x*e + 1/2*d)^3 - 3*b^3*c^2*tan
```

```
(1/2*x*e + 1/2*d)^3 + 2*a*c^4*tan(1/2*x*e + 1/2*d)^3 - 2*b*c^4*tan(1/2*x*e
+ 1/2*d)^3 - 4*a^4*c*tan(1/2*x*e + 1/2*d)^2 + 12*a^3*b*c*tan(1/2*x*e + 1/2*
d)^2 - 13*a^2*b^2*c*tan(1/2*x*e + 1/2*d)^2 + 6*a*b^3*c*tan(1/2*x*e + 1/2*d)
^2 - b^4*c*tan(1/2*x*e + 1/2*d)^2 - 7*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 + 6*a*
b*c^3*tan(1/2*x*e + 1/2*d)^2 + b^2*c^3*tan(1/2*x*e + 1/2*d)^2 + 2*c^5*tan(1
/2*x*e + 1/2*d)^2 + 4*a^4*b*tan(1/2*x*e + 1/2*d) - 5*a^3*b^2*tan(1/2*x*e +
1/2*d) - 3*a^2*b^3*tan(1/2*x*e + 1/2*d) + 5*a*b^4*tan(1/2*x*e + 1/2*d) - b^
5*tan(1/2*x*e + 1/2*d) - 11*a^3*c^2*tan(1/2*x*e + 1/2*d) + 3*a^2*b*c^2*tan(
1/2*x*e + 1/2*d) + 7*a*b^2*c^2*tan(1/2*x*e + 1/2*d) + b^3*c^2*tan(1/2*x*e +
 1/2*d) + 2*a*c^4*tan(1/2*x*e + 1/2*d) + 2*b*c^4*tan(1/2*x*e + 1/2*d) - 4*a
^4*c + 3*a^2*b^2*c + b^4*c + a^2*c^3 + b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 +
 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^
2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x*e + 1/2*d)^2 -
b*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a + b)^2))*e^(-1)
```

3.402 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=292

$$\frac{a\left(2a^2+3\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{7/2}}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{6e\left(a^2-b^2-c^2\right)^3(a+b\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 -
 b^2 - c^2]])/((a^2 - b^2 - c^2)^(7/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x
])/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5*(a*
c*Cos[d + e*x] - a*b*Sin[d + e*x]))/(6*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^2) + (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x] - b*(
11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*(a^2 - b^2 - c^2)^3*e*(a + b*Cos[d
 + e*x] + c*Sin[d + e*x]))
```

Rubi [A] time = 0.376103, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3129, 3156, 3153, 3124, 618, 204}

$$\frac{a\left(2a^2+3\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{7/2}}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{6e\left(a^2-b^2-c^2\right)^3(a+b\cos(d+ex)+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```


```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 -
 b^2 - c^2]])/((a^2 - b^2 - c^2)^(7/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x
])/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5*(a*
c*Cos[d + e*x] - a*b*Sin[d + e*x]))/(6*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^2) + (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x] - b*(
11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*(a^2 - b^2 - c^2)^3*e*(a + b*Cos[d
 + e*x] + c*Sin[d + e*x]))
```


Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
```

```
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} - \frac{\int \frac{-3a+2b\cos(d+ex)+2c\,\text{s}}{(a+b\cos(d+ex)+c\sin(}}{3\left(a^2-b^2-\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(ac\cos(d}{6\left(a^2-b^2-c^2\right)^2 e(a}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(ac\cos(d}{6\left(a^2-b^2-c^2\right)^2 e(a}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(ac\cos(d}{6\left(a^2-b^2-c^2\right)^2 e(a}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(ac\cos(d}{6\left(a^2-b^2-c^2\right)^2 e(a}$$

$$= \frac{a\left(2a^2+3\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{7/2}e} + \frac{c\cos(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\,\text{c}}$$

Mathematica [B] time = 2.08475, size = 606, normalized size = 2.08

$$\frac{72a^2b^2c^2\sin(d+ex)+30a^2bc\left(2a^2+3\left(b^2+c^2\right)\right)\cos(d+ex)-6ac\left(a^2\left(7b^2+11c^2\right)+2b^2c^2-2b^4+4c^4\right)\cos(2(d+ex))-22a^2b^3c\cos(3(d+ex))+82a^3b^2c-9a^2b^4\sin(d+ex)+1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] ((24*a*(2*a^2 + 3*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(7/2) + (44*a^5*c + 82*a^3*b^2*c + 2
4*a*b^4*c + 82*a^3*c^3 + 48*a*b^2*c^3 + 24*a*c^5 + 30*a^2*b*c*(2*a^2 + 3*(b
^2 + c^2))*Cos[d + e*x] - 6*a*c*(-2*b^4 + 2*b^2*c^2 + 4*c^4 + a^2*(7*b^2 +
```

```
11*c^2))*Cos[2*(d + e*x)] - 22*a^2*b^3*c*Cos[3*(d + e*x)] - 8*b^5*c*Cos[3*(
d + e*x)] - 22*a^2*b*c^3*Cos[3*(d + e*x)] - 16*b^3*c^3*Cos[3*(d + e*x)] - 8
*b*c^5*Cos[3*(d + e*x)] + 72*a^4*b^2*Sin[d + e*x] - 9*a^2*b^4*Sin[d + e*x]
+ 12*b^6*Sin[d + e*x] + 132*a^4*c^2*Sin[d + e*x] + 72*a^2*b^2*c^2*Sin[d + e
*x] + 36*b^4*c^2*Sin[d + e*x] + 81*a^2*c^4*Sin[d + e*x] + 36*b^2*c^4*Sin[d
+ e*x] + 12*c^6*Sin[d + e*x] + 54*a^3*b^3*Sin[2*(d + e*x)] + 6*a*b^5*Sin[2*
(d + e*x)] + 78*a^3*b*c^2*Sin[2*(d + e*x)] + 48*a*b^3*c^2*Sin[2*(d + e*x)]
+ 42*a*b*c^4*Sin[2*(d + e*x)] + 11*a^2*b^4*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(
d + e*x)] + 4*b^4*c^2*Sin[3*(d + e*x)] - 11*a^2*c^4*Sin[3*(d + e*x)] - 4*b^
2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*x)])/(b*(-a^2 + b^2 + c^2)^3*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^3))/(24*e)
```

Maple [B] time = 0.291, size = 16909, normalized size = 57.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x)

[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 5.09825, size = 8541, normalized size = 29.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*b*c^5 + 12*(4*a^3*b + a*b^3)*c^3 + 2*(4*c^7 + (7*a^2 - 4*b^2)*c^
5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 - 7*a^2*b^4 - 4*b^6)
*c)*cos(e*x + d)^3 - 12*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3 - (9*a^5*b - 8*a
^3*b^3 - a*b^5)*c)*cos(e*x + d)^2 + 3*(2*a^6 + 3*a^4*b^2 + 9*a^2*c^4 + (2*a
^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(e*x + d)^3 + 9*(a
^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e
*x + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b + 3*a*b^3)*c^2)*c
os(e*x + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5 + 2*(a^3 - 3*a*b
^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(e*x + d)^2 + 3*(2*a^5 + 3*a^3*b^2)
*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(e*x + d))*sin(e*x + d))*
sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*
a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e
*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*si
n(e*x + d) - 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d)
 - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b
^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2
*(b*c*cos(e*x + d) + a*c)*sin(e*x + d))) - 6*(9*a^5*b - 8*a^3*b^3 - a*b^5)*
c - 6*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^3 - (6*a^6 - 15*a^
4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(e*x + d) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a
^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - 7*a^2*b^3 + 10*b^5)*c^2
 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 12*b*c^6 + (21*a^2*b + 20*b^3)*c^4 - (
33*a^4*b - 14*a^2*b^3 - 4*b^5)*c^2)*cos(e*x + d)^2 + 3*(9*a^5*b^2 - 8*a^3*b
^4 - a*b^6 + a*c^6 + (8*a^3 + a*b^2)*c^4 - (9*a^5 + a*b^4)*c^2)*cos(e*x + d
))*sin(e*x + d))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 - 3*b
*c^10 + (12*a^2*b - 11*b^3)*c^8 - 2*(9*a^4*b - 16*a^2*b^3 + 7*b^5)*c^6 + 6*
(2*a^6*b - 5*a^4*b^3 + 4*a^2*b^5 - b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*
b^5 - b^9)*c^2)*e*cos(e*x + d)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10 - a*c^10 + (4*a^3 - 3*a*b^2)*c^8 - 2*(3*a^5 - 4*a^3*b^2 + a
*b^4)*c^6 + 2*(2*a^7 - 3*a^5*b^2 + a*b^6)*c^4 - (a^9 - 6*a^5*b^4 + 8*a^3*b^
6 - 3*a*b^8)*c^2)*e*cos(e*x + d)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*
a^4*b^7 + a^2*b^9 + b*c^10 - (3*a^2*b - 4*b^3)*c^8 + 2*(a^4*b - 4*a^2*b^3 +
 3*b^5)*c^6 + 2*(a^6*b - 3*a^2*b^5 + 2*b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*
a^4*b^5 - b^9)*c^2)*e*cos(e*x + d) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*
b^6 + a^3*b^8 + 3*a*c^10 - (11*a^3 - 12*a*b^2)*c^8 + 2*(7*a^5 - 16*a^3*b^2
+ 9*a*b^4)*c^6 - 6*(a^7 - 4*a^5*b^2 + 5*a^3*b^4 - 2*a*b^6)*c^4 - (a^9 - 6*a
^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*c^2)*e - ((c^11 - (4*a^2 - b^2)*c^9 + 6*(a^4
- b^4)*c^7 - 2*(2*a^6 + 3*a^4*b^2 - 12*a^2*b^4 + 7*b^6)*c^5 + (a^8 + 8*a^6*
b^2 - 30*a^4*b^4 + 32*a^2*b^6 - 11*b^8)*c^3 - 3*(a^8*b^2 - 4*a^6*b^4 + 6*a^
4*b^6 - 4*a^2*b^8 + b^10)*c)*e*cos(e*x + d)^2 - 6*(a*b*c^9 - 4*(a^3*b - a*b
^3)*c^7 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^5 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*c^3 + (a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*c)*e
*cos(e*x + d) - (c^11 - (a^2 - 4*b^2)*c^9 - 6*(a^4 - b^4)*c^7 + 2*(7*a^6 -
```

12*a^4*b^2 + 3*a^2*b^4 + 2*b^6)*c^5 - (11*a^8 - 32*a^6*b^2 + 30*a^4*b^4 - 8
*a^2*b^6 - b^8)*c^3 + 3*(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8
)*c)*e)*sin(e*x + d)), 1/6*(3*a*b*c^5 + 6*(4*a^3*b + a*b^3)*c^3 + (4*c^7 +
(7*a^2 - 4*b^2)*c^5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 -
7*a^2*b^4 - 4*b^6)*c)*cos(e*x + d)^3 - 6*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3
 - (9*a^5*b - 8*a^3*b^3 - a*b^5)*c)*cos(e*x + d)^2 + 3*(2*a^6 + 3*a^4*b^2 +
 9*a^2*c^4 + (2*a^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(
e*x + d)^3 + 9*(a^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 -
 3*a^2*c^4)*cos(e*x + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b
+ 3*a*b^3)*c^2)*cos(e*x + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5
 + 2*(a^3 - 3*a*b^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(e*x + d)^2 + 3*(2
*a^5 + 3*a^3*b^2)*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(e*x + d
))*sin(e*x + d))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(e*x + d) + a*c*sin(
e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x
+ d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - 3*(9*a^5*b - 8*a^3*b^3 - a*b^
5)*c - 3*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^3 - (6*a^6 - 15
*a^4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(e*x + d) - (18*a^6*b - 23*a^4*b^3 + 7*
a^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - 7*a^2*b^3 + 10*b^5)*c^
2 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 12*b*c^6 + (21*a^2*b + 20*b^3)*c^4 -
(33*a^4*b - 14*a^2*b^3 - 4*b^5)*c^2)*cos(e*x + d)^2 + 3*(9*a^5*b^2 - 8*a^3*
b^4 - a*b^6 + a*c^6 + (8*a^3 + a*b^2)*c^4 - (9*a^5 + a*b^4)*c^2)*cos(e*x +
d))*sin(e*x + d))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 - 3*
b*c^10 + (12*a^2*b - 11*b^3)*c^8 - 2*(9*a^4*b - 16*a^2*b^3 + 7*b^5)*c^6 + 6
*(2*a^6*b - 5*a^4*b^3 + 4*a^2*b^5 - b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4
*b^5 - b^9)*c^2)*e*cos(e*x + d)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*
a^3*b^8 + a*b^10 - a*c^10 + (4*a^3 - 3*a*b^2)*c^8 - 2*(3*a^5 - 4*a^3*b^2 +
a*b^4)*c^6 + 2*(2*a^7 - 3*a^5*b^2 + a*b^6)*c^4 - (a^9 - 6*a^5*b^4 + 8*a^3*b
^6 - 3*a*b^8)*c^2)*e*cos(e*x + d)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4
*a^4*b^7 + a^2*b^9 + b*c^10 - (3*a^2*b - 4*b^3)*c^8 + 2*(a^4*b - 4*a^2*b^3
+ 3*b^5)*c^6 + 2*(a^6*b - 3*a^2*b^5 + 2*b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6
*a^4*b^5 - b^9)*c^2)*e*cos(e*x + d) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5
*b^6 + a^3*b^8 + 3*a*c^10 - (11*a^3 - 12*a*b^2)*c^8 + 2*(7*a^5 - 16*a^3*b^2
 + 9*a*b^4)*c^6 - 6*(a^7 - 4*a^5*b^2 + 5*a^3*b^4 - 2*a*b^6)*c^4 - (a^9 - 6*
a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*c^2)*e - ((c^11 - (4*a^2 - b^2)*c^9 + 6*(a^4
 - b^4)*c^7 - 2*(2*a^6 + 3*a^4*b^2 - 12*a^2*b^4 + 7*b^6)*c^5 + (a^8 + 8*a^6
*b^2 - 30*a^4*b^4 + 32*a^2*b^6 - 11*b^8)*c^3 - 3*(a^8*b^2 - 4*a^6*b^4 + 6*a
^4*b^6 - 4*a^2*b^8 + b^10)*c)*e*cos(e*x + d)^2 - 6*(a*b*c^9 - 4*(a^3*b - a*
b^3)*c^7 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^5 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3
*b^5 - a*b^7)*c^3 + (a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*c)*
e*cos(e*x + d) - (c^11 - (a^2 - 4*b^2)*c^9 - 6*(a^4 - b^4)*c^7 + 2*(7*a^6 -
 12*a^4*b^2 + 3*a^2*b^4 + 2*b^6)*c^5 - (11*a^8 - 32*a^6*b^2 + 30*a^4*b^4 -
8*a^2*b^6 - b^8)*c^3 + 3*(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^
8)*c)*e)*sin(e*x + d))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49059, size = 3606, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2 + 3*a*c^2)*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/
sqrt(a^2 - b^2 - c^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 3*a^4*c^2 + 6
*a^2*b^2*c^2 - 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 - c^6)*sqrt(a^2 - b^2 - c^
2)) + (18*a^7*b*tan(1/2*x*e + 1/2*d)^5 - 81*a^6*b^2*tan(1/2*x*e + 1/2*d)^5
+ 141*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 - 120*a^4*b^4*tan(1/2*x*e + 1/2*d)^5 +
 60*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 - 33*a^2*b^6*tan(1/2*x*e + 1/2*d)^5 + 21
*a*b^7*tan(1/2*x*e + 1/2*d)^5 - 6*b^8*tan(1/2*x*e + 1/2*d)^5 - 27*a^6*c^2*t
an(1/2*x*e + 1/2*d)^5 + 81*a^5*b*c^2*tan(1/2*x*e + 1/2*d)^5 - 72*a^4*b^2*c^
2*tan(1/2*x*e + 1/2*d)^5 + 18*a^3*b^3*c^2*tan(1/2*x*e + 1/2*d)^5 - 27*a^2*b
^4*c^2*tan(1/2*x*e + 1/2*d)^5 + 45*a*b^5*c^2*tan(1/2*x*e + 1/2*d)^5 - 18*b^
6*c^2*tan(1/2*x*e + 1/2*d)^5 + 18*a^4*c^4*tan(1/2*x*e + 1/2*d)^5 - 36*a^3*b
*c^4*tan(1/2*x*e + 1/2*d)^5 + 36*a*b^3*c^4*tan(1/2*x*e + 1/2*d)^5 - 18*b^4*
c^4*tan(1/2*x*e + 1/2*d)^5 - 6*a^2*c^6*tan(1/2*x*e + 1/2*d)^5 + 12*a*b*c^6*
tan(1/2*x*e + 1/2*d)^5 - 6*b^2*c^6*tan(1/2*x*e + 1/2*d)^5 - 18*a^7*c*tan(1/
2*x*e + 1/2*d)^4 + 108*a^6*b*c*tan(1/2*x*e + 1/2*d)^4 - 261*a^5*b^2*c*tan(1
/2*x*e + 1/2*d)^4 + 336*a^4*b^3*c*tan(1/2*x*e + 1/2*d)^4 - 264*a^3*b^4*c*ta
n(1/2*x*e + 1/2*d)^4 + 144*a^2*b^5*c*tan(1/2*x*e + 1/2*d)^4 - 57*a*b^6*c*ta
n(1/2*x*e + 1/2*d)^4 + 12*b^7*c*tan(1/2*x*e + 1/2*d)^4 - 81*a^5*c^3*tan(1/2
*x*e + 1/2*d)^4 + 216*a^4*b*c^3*tan(1/2*x*e + 1/2*d)^4 - 198*a^3*b^2*c^3*ta
n(1/2*x*e + 1/2*d)^4 + 108*a^2*b^3*c^3*tan(1/2*x*e + 1/2*d)^4 - 81*a*b^4*c^
3*tan(1/2*x*e + 1/2*d)^4 + 36*b^5*c^3*tan(1/2*x*e + 1/2*d)^4 + 36*a^3*c^5*t
```

an(1/2*x*e + 1/2*d)^4 - 36*a^2*b*c^5*tan(1/2*x*e + 1/2*d)^4 - 36*a*b^2*c^5*tan(1/2*x*e + 1/2*d)^4 + 36*b^3*c^5*tan(1/2*x*e + 1/2*d)^4 - 12*a*c^7*tan(1/2*x*e + 1/2*d)^4 + 12*b*c^7*tan(1/2*x*e + 1/2*d)^4 + 36*a^7*b*tan(1/2*x*e + 1/2*d)^3 - 108*a^6*b^2*tan(1/2*x*e + 1/2*d)^3 + 76*a^5*b^3*tan(1/2*x*e + 1/2*d)^3 + 60*a^4*b^4*tan(1/2*x*e + 1/2*d)^3 - 100*a^3*b^5*tan(1/2*x*e + 1/2*d)^3 + 44*a^2*b^6*tan(1/2*x*e + 1/2*d)^3 - 12*a*b^7*tan(1/2*x*e + 1/2*d)^3 + 4*b^8*tan(1/2*x*e + 1/2*d)^3 - 108*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 + 240*a^5*b*c^2*tan(1/2*x*e + 1/2*d)^3 - 162*a^4*b^2*c^2*tan(1/2*x*e + 1/2*d)^3 + 122*a^3*b^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 174*a^2*b^4*c^2*tan(1/2*x*e + 1/2*d)^3 + 78*a*b^5*c^2*tan(1/2*x*e + 1/2*d)^3 + 4*b^6*c^2*tan(1/2*x*e + 1/2*d)^3 - 42*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 162*a^3*b*c^4*tan(1/2*x*e + 1/2*d)^3 - 210*a^2*b^2*c^4*tan(1/2*x*e + 1/2*d)^3 + 102*a*b^3*c^4*tan(1/2*x*e + 1/2*d)^3 - 12*b^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 8*a^2*c^6*tan(1/2*x*e + 1/2*d)^3 + 12*a*b*c^6*tan(1/2*x*e + 1/2*d)^3 - 20*b^2*c^6*tan(1/2*x*e + 1/2*d)^3 - 8*c^8*tan(1/2*x*e + 1/2*d)^3 - 36*a^7*c*tan(1/2*x*e + 1/2*d)^2 + 108*a^6*b*c*tan(1/2*x*e + 1/2*d)^2 - 108*a^5*b^2*c*tan(1/2*x*e + 1/2*d)^2 + 12*a^4*b^3*c*tan(1/2*x*e + 1/2*d)^2 + 84*a^3*b^4*c*tan(1/2*x*e + 1/2*d)^2 - 108*a^2*b^5*c*tan(1/2*x*e + 1/2*d)^2 + 60*a*b^6*c*tan(1/2*x*e + 1/2*d)^2 - 12*b^7*c*tan(1/2*x*e + 1/2*d)^2 - 120*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 132*a^4*b*c^3*tan(1/2*x*e + 1/2*d)^2 + 42*a^3*b^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 36*a^2*b^3*c^3*tan(1/2*x*e + 1/2*d)^2 + 18*a*b^4*c^3*tan(1/2*x*e + 1/2*d)^2 - 36*b^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 18*a^3*c^5*tan(1/2*x*e + 1/2*d)^2 + 72*a^2*b*c^5*tan(1/2*x*e + 1/2*d)^2 - 54*a*b^2*c^5*tan(1/2*x*e + 1/2*d)^2 - 36*b^3*c^5*tan(1/2*x*e + 1/2*d)^2 - 12*a*c^7*tan(1/2*x*e + 1/2*d)^2 - 12*b*c^7*tan(1/2*x*e + 1/2*d)^2 + 18*a^7*b*tan(1/2*x*e + 1/2*d) - 27*a^6*b^2*tan(1/2*x*e + 1/2*d) - 21*a^5*b^3*tan(1/2*x*e + 1/2*d) + 48*a^4*b^4*tan(1/2*x*e + 1/2*d) - 12*a^3*b^5*tan(1/2*x*e + 1/2*d) - 15*a^2*b^6*tan(1/2*x*e + 1/2*d) + 15*a*b^7*tan(1/2*x*e + 1/2*d) - 6*b^8*tan(1/2*x*e + 1/2*d) - 81*a^6*c^2*tan(1/2*x*e + 1/2*d) + 27*a^5*b*c^2*tan(1/2*x*e + 1/2*d) + 90*a^4*b^2*c^2*tan(1/2*x*e + 1/2*d) + 9*a^2*b^4*c^2*tan(1/2*x*e + 1/2*d) - 27*a*b^5*c^2*tan(1/2*x*e + 1/2*d) - 18*b^6*c^2*tan(1/2*x*e + 1/2*d) + 12*a^4*c^4*tan(1/2*x*e + 1/2*d) + 42*a^3*b*c^4*tan(1/2*x*e + 1/2*d) + 18*a^2*b^2*c^4*tan(1/2*x*e + 1/2*d) - 54*a*b^3*c^4*tan(1/2*x*e + 1/2*d) - 18*b^4*c^4*tan(1/2*x*e + 1/2*d) - 6*a^2*c^6*tan(1/2*x*e + 1/2*d) - 12*a*b*c^6*tan(1/2*x*e + 1/2*d) - 6*b^2*c^6*tan(1/2*x*e + 1/2*d) - 18*a^7*c + 21*a^5*b^2*c + 12*a^3*b^4*c - 15*a*b^6*c + 5*a^5*c^3 + 16*a^3*b^2*c^3 - 21*a*b^4*c^3 - 2*a^3*c^5 - 6*a*b^2*c^5)/((a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9 - 3*a^7*c^2 + 9*a^6*b*c^2 - 3*a^5*b^2*c^2 - 15*a^4*b^3*c^2 + 15*a^3*b^4*c^2 + 3*a^2*b^5*c^2 - 9*a*b^6*c^2 + 3*b^7*c^2 + 3*a^5*c^4 - 9*a^4*b*c^4 + 6*a^3*b^2*c^4 + 6*a^2*b^3*c^4 - 9*a*b^4*c^4 + 3*b^5*c^4 - a^3*c^6 + 3*a^2*b*c^6 - 3*a*b^2*c^6 + b^3*c^6)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a + b)^3))*e^(-1)

3.403 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=185

$$\frac{64\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}-\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+$$

```
[Out] (796*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(15*e) + (64*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (32*(5*Cos[d + e*x] - 3*Sin[d + e*x])*S
qrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(15*e) - (2*(5*Cos[d + e*x] - 3*S
in[d + e*x])*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2))/(5*e)
```

Rubi [A] time = 0.267904, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3120, 3146, 3149, 3118, 2653, 3126, 2661}

$$-\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}-\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex}}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```



```
[Out] (796*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(15*e) + (64*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (32*(5*Cos[d + e*x] - 3*Sin[d + e*x])*S
qrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(15*e) - (2*(5*Cos[d + e*x] - 3*S
in[d + e*x])*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2))/(5*e)
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx = -\frac{2(5\cos(d+ex)-3\sin(d+ex))(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}{5e} + \frac{2}{5}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{15e} - \frac{2(}{}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{15e} - \frac{2(}{}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{15e} - \frac{2(}{}$$

$$= \frac{796\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} + \frac{64F\left(\frac{1}{2}\left(d+ex\right.\right.}{}$$

Mathematica [C] time = 6.05264, size = 399, normalized size = 2.16

$$1276\sqrt{\frac{10}{3}}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{17\sin(}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-2388*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 2*Sqrt[2 + 3*Cos[d +
 e*x] + 5*Sin[d + e*x]]*(550*Cos[d + e*x] + 3*(-398 + 75*Cos[2*(d + e*x)] -
 110*Sin[d + e*x] + 40*Sin[2*(d + e*x)])) + 1276*Sqrt[10/3]*AppellF1[1/2, 1
/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]),
(Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*
x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*
x + ArcTan[3/5]]] + (1990*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e
*x - ArcTan[5/3]]/Sqrt[34]] - (1990*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2
, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] +
17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*
```

```
Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[
5/3]]])/(75*e)
```

Maple [C] time = 2.494, size = 464, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x)
```

```
[Out] (-732/17*34^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1
/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e
*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3
/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2)
)-64*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin
(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3
/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2)
)/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+796/17*(-(1
7*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+ar
ctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-3
4^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1
/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)+68/5*34^(1/
2)*sin(e*x+d+arctan(3/5))^4-116/15*34^(1/2)*sin(e*x+d+arctan(3/5))^2+1904/1
5*sin(e*x+d+arctan(3/5))^3-1904/15*sin(e*x+d+arctan(3/5))-88/15*34^(1/2))/c
os(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (3 \cos(ex+d) + 5 \sin(ex+d) + 2)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(16\cos(ex+d)^2-10(3\cos(ex+d)+2)\sin(ex+d)-12\cos(ex+d)-29\right)\sqrt{3\cos(ex+d)+5\sin(ex+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(16*cos(e*x + d)^2 - 10*(3*cos(e*x + d) + 2)*sin(e*x + d) - 12*co
s(e*x + d) - 29)*sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.404 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=139

$$\frac{20\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+}}{3e}$$

```
[Out] (16*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[3
4]))/15])/(3*e) + (20*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (2*(5*Cos[d + e*x] - 3*Sin[d + e*x])*Sqrt
[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(3*e)
```

Rubi [A] time = 0.13866, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3120, 3149, 3118, 2653, 3126, 2661}

$$-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}+\frac{20F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}+\frac{16}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```


```
[Out] (16*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[3
4]))/15])/(3*e) + (20*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (2*(5*Cos[d + e*x] - 3*Sin[d + e*x])*Sqrt
[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(3*e)
```


Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{3e}+\frac{2}{3}\int$$

$$= -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{3e}+\frac{8}{3}\int$$

$$= -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{3e}+\frac{8}{3}\int$$

$$= \frac{16\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3e}+\frac{20F\left(\frac{1}{2}\left(d+ex-\right.\right.}{}$$

Mathematica [C] time = 3.5708, size = 349, normalized size = 2.51

$$\frac{2\left(\sqrt{\sin^2\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\left(23\sqrt{30}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sqrt{\sqrt{34}\cos\left(d+e\right.}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```

```
[Out] (2*(-60*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x
 - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]
])/(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-15*(30*Cos[d + e*x] + 15
*Cos[2*(d + e*x)] - 18*Sin[d + e*x] + 8*Sin[2*(d + e*x)]) + 23*Sqrt[30]*App
ellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 +
 Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqr
t[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]
]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]]
)*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2]))/(45*e*Sqrt[2 + Sqrt[34]*Cos[d + e*x
- ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

Maple [C] time = 2.675, size = 449, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x)
```

```
[Out] (-60/17*34^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/
2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*
x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/
5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))
-20*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(
e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/
5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))
/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+68/3*sin(e*x
+d+arctan(3/5))^3-68/3*sin(e*x+d+arctan(3/5))+4/3*34^(1/2)*sin(e*x+d+arctan
(3/5))^2-4/3*34^(1/2)+80/17*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/
2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)
*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+
d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+
17))^(1/2))*34^(1/2))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5
))+2)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (3\cos(ex+d) + 5\sin(ex+d) + 2)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((3\cos(ex+d) + 5\sin(ex+d) + 2)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (3\cos(ex + d) + 5\sin(ex + d) + 2)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

3.405 $\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

```
[Out] (2*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/e
```

Rubi [A] time = 0.0305556, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3118, 2653}

$$\frac{2\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/e
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx = \int \sqrt{2+\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\,dx$$
$$= \frac{2\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

Mathematica [C] time = 2.30288, size = 326, normalized size = 7.24

$$\frac{\sqrt{\sin^2\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\left(2\sqrt{30}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sqrt{\sqrt{34}\cos\left(d+ex-\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (-15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x -
ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/
(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-75*Cos[d + e*x] + 45*Sin[d
+ e*x] + 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x
 + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]
])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Co
s[d + e*x - ArcTan[5/3]]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[
d + e*x + ArcTan[3/5]]])*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/(15*e*Sqrt[2 +
 Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

Maple [C] time = 2.867, size = 316, normalized size = 7.

$$-\frac{\left(2\sqrt{34}-34\right)\sqrt{17}}{17\cos\left(ex+d+\arctan\left(3/5\right)\right)e}\sqrt{-\frac{17\sin\left(ex+d+\arctan\left(3/5\right)\right)+\sqrt{34}}{-\sqrt{34}+17}}\sqrt{-17\frac{\sin\left(ex+d+\arctan\left(3/5\right)\right)-1}{\sqrt{34}+17}}\sqrt{\frac{1+\mathrm{s}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x)
```

```
[Out] -2/17*(34^(1/2)-17)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^
(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin
(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(EllipticE((-(17*sin(e*x+d+arcta
n(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1
/2))*34^(1/2)-34^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-3
4^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+2*EllipticE((-(1
7*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/
(34^(1/2)+17))^(1/2))-2*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-
34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2)))/cos(e*x+d+arct
an(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3\cos(ex+d)+5\sin(ex+d)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{3\cos(ex+d)+5\sin(ex+d)+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{5\sin(d+ex)+3\cos(d+ex)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

3.406 $\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx$

Optimal. Leaf size=45

$$\frac{2\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}$$

```
[Out] (2*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 +
Sqrt[34]]*e)
```

Rubi [A] time = 0.0372017, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3126, 2661}

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```



```
[Out] (2*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 +
Sqrt[34]]*e)
```


Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{2+\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}}\,dx$$
$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)\middle|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}$$

Mathematica [C] time = 0.261814, size = 128, normalized size = 2.84

$$\frac{\sqrt{\frac{2}{15}}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{17\sin\left(d+ex+\text{ta}\right.}{-17+}\right.}{e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2/15]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTa
n[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 +
 Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*S
qrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]])/e
```

Maple [C] time = 2.134, size = 158, normalized size = 3.5

$$\frac{\left(2\sqrt{34}-34\right)\sqrt{17}}{17\cos\left(ex+d+\arctan\left(3/5\right)\right)e}\sqrt{-\frac{17\sin\left(ex+d+\arctan\left(3/5\right)\right)+\sqrt{34}}{-\sqrt{34}+17}}\sqrt{-17\frac{\sin\left(ex+d+\arctan\left(3/5\right)\right)-1}{\sqrt{34}+17}}\sqrt{\frac{1+\sin}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x)
```

```
[Out] 2/17*(34^(1/2)-17)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(
1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(
e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(
3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2
```

```
))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos(ex+d)+5\sin(ex+d)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\cos(ex+d)+5\sin(ex+d)+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)

[Out] Integral(1/sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3 \cos(ex+d) + 5 \sin(ex+d) + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

3.407 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}$$

```
[Out] -(Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]
))/15])/(15*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(15*e*Sqrt[2 + 3*Cos[d +
 e*x] + 5*Sin[d + e*x]])
```

Rubi [A] time = 0.0535315, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3128, 3118, 2653}

$$-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

```
[Out] -(Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]
))/15])/(15*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(15*e*Sqrt[2 + 3*Cos[d +
 e*x] + 5*Sin[d + e*x]])
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
```

```
+ c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}} - \frac{1}{30}\int \sqrt{2+3\cos(d+ex)+5\sin}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}} - \frac{1}{30}\int \sqrt{2+\sqrt{34}\cos\left(d+ex-\text{t}\right.}$$

$$= -\frac{\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} - \frac{5\cos(d+ex)}{15e\sqrt{2+3\cos(d}}$$

Mathematica [C] time = 6.14109, size = 528, normalized size = 5.62

$$-\frac{17\left(-\frac{5\sqrt{\frac{1}{34}\left(17+\sqrt{34}\right)}\sin\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};-\frac{\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)+2}{\sqrt{34}\left(1-\sqrt{\frac{2}{17}}\right)},-\frac{\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)+2}{\sqrt{34}\left(-1-\sqrt{\frac{2}{17}}\right)}\right)}{17\sqrt{1-\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\sqrt{-\frac{\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)+1}{\sqrt{34}-17}}\sqrt{\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)+2}} - \frac{\frac{3}{17}\left(\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)+2\right)}{\sqrt{\sqrt{34}\cos\left(d+ex-\tan\right.}}\right.}{75e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]]*(-34/225 + (2*(5 + 17*Sin[d + e*
x]))/(45*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]))))/e - (Sqrt[34/(17 + Sqrt[3
4])]*AppellF1[1/2, 1/2, 1/2, 3/2, -((2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]
])/(Sqrt[34]*(1 - Sqrt[2/17]))), -((2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]
)/(Sqrt[34]*(-1 - Sqrt[2/17])))]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[1 - Sin[d
+ e*x + ArcTan[3/5]]]*Sqrt[-((1 + Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[3
4]))]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]])/(15*e) - (17*((-5*Sqrt
[(17 + Sqrt[34])/34]*AppellF1[-1/2, -1/2, -1/2, 1/2, -((2 + Sqrt[34]*Cos[d
+ e*x - ArcTan[5/3]])/(Sqrt[34]*(1 - Sqrt[2/17]))), -((2 + Sqrt[34]*Cos[d +
```

```
 e*x - ArcTan[5/3]])/(Sqrt[34]*(-1 - Sqrt[2/17]))]*Sin[d + e*x - ArcTan[5/
3]])/(17*Sqrt[1 - Cos[d + e*x - ArcTan[5/3]]]*Sqrt[-((1 + Cos[d + e*x - Arc
Tan[5/3]])/(-17 + Sqrt[34]))]*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]
) - ((3*(2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]))/17 - (5*Sin[d + e*x - Ar
cTan[5/3]])/Sqrt[34])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]))/(75*e
)
```

Maple [C] time = 3.376, size = 425, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x)
```

```
[Out] 1/4335*34^(1/2)*(255*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x
+d+arctan(3/5))^2)^(1/2)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17
))^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+
arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+
34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-25
5*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^
(1/2)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*((1+sin(e*
x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34
^(1/2)+17))^(1/2)*EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)
+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))+289*((34^(1/2)*sin(e*
x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*sin(e*x+d+arctan(3/5))^
2-289*((34^(1/2)*sin(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2))
*17^(1/2)/((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3
/5))^2)^(1/2)/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1
/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2}}{16\cos\left(ex+d\right)^{2}-10\left(3\cos\left(ex+d\right)+2\right)\sin\left(ex+d\right)-12\cos\left(ex+d\right)-29},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(16*cos(e*x + d)^2 - 10
*(3*cos(e*x + d) + 2)*sin(e*x + d) - 12*cos(e*x + d) - 29), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(5\sin\left(d+ex\right)+3\cos\left(d+ex\right)+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)
```

3.408 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{45\sqrt{2+\sqrt{34}}e}+\frac{4(5\cos(d+ex)-3\sin(d+ex))}{675e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{5\cos(d+ex)-3}{45e(5\sin(d+ex)+3\text{c}}$$

```
[Out] (4*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(675*e) + EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))
/15]/(45*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(45*e*(2
 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) + (4*(5*Cos[d + e*x] - 3*Sin[d +
 e*x]))/(675*e*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])
```

Rubi [A] time = 0.199658, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3118, 2653, 3126, 2661}

$$\frac{4(5\cos(d+ex)-3\sin(d+ex))}{675e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}+\frac{F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17}{45\sqrt{2+\sqrt{34}}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```



```
[Out] (4*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(675*e) + EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))
/15]/(45*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(45*e*(2
 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) + (4*(5*Cos[d + e*x] - 3*Sin[d +
 e*x]))/(675*e*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])
```


Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{1}{45}\int \frac{-3+\frac{3}{2}\cos(d+ex)+}{(2+3\cos(d+ex)+5}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d+ex)-3\sin(}{675e\sqrt{2+3\cos(d+ex)+5}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d+ex)-3\sin(}{675e\sqrt{2+3\cos(d+ex)+5}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d+ex)-3\sin(}{675e\sqrt{2+3\cos(d+ex)+5}}$$

$$= \frac{4\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{675e} + \frac{F\left(\frac{1}{2}\left(d+ex-\mathrm{t}\right.\right.}{45}$$

Mathematica [C] time = 3.25928, size = 430, normalized size = 2.3

$$\frac{23\sqrt{\frac{10}{3}}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{17\sin(d+}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-24*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (272*Sqrt[2 + 3*Cos[d
+ e*x] + 5*Sin[d + e*x]])/3 + (100*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*
x] + 5*Sin[d + e*x])^(3/2) - (10*(115 + 136*Sin[d + e*x]))/(3*Sqrt[2 + 3*Co
s[d + e*x] + 5*Sin[d + e*x]]) + 23*Sqrt[10/3]*AppellF1[1/2, 1/2, 1/2, 3/2,
(Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17
*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5
]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5
]]] + (20*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]
```

```
]/Sqrt[34]] - (20*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*C
os[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - A
rcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x -
 ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(6750*e)
```

Maple [C] time = 6.476, size = 524, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x)
```

```
[Out] (-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*(-1/
1530*34^(1/2)*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))
^2)^(1/2)/(sin(e*x+d+arctan(3/5))+1/17*34^(1/2))^2+68/675*34^(1/2)*cos(e*x+
d+arctan(3/5))^2/(-(-289*sin(e*x+d+arctan(3/5))-17*34^(1/2))*34^(1/2)*cos(e
*x+d+arctan(3/5))^2)^(1/2)+23/675*(1/17*34^(1/2)+1)*((17*sin(e*x+d+arctan(3
/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1
/2)+17))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)/(-(-34
^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*EllipticF(
((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+
17)*(34^(1/2)+17))^(1/2))+4/675*34^(1/2)*(1/17*34^(1/2)+1)*((17*sin(e*x+d+a
rctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/
(-34^(1/2)+17))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)
/(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*((-
1/17*34^(1/2)+1)*EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+
17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-EllipticF(((17*sin(e*x
+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2
)+17))^(1/2))))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^
(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\cos(ex+d) + 5\sin(ex+d) + 2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\cos(ex+d)+5\sin(ex+d)+2}}{198\cos(ex+d)^3+96\cos(ex+d)^2-5\left(2\cos(ex+d)^2+36\cos(ex+d)+37\right)\sin(ex+d)-261\cos(ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(198*cos(e*x + d)^3 + 9
6*cos(e*x + d)^2 - 5*(2*cos(e*x + d)^2 + 36*cos(e*x + d) + 37)*sin(e*x + d)
 - 261*cos(e*x + d) - 158), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)
```

3.409 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{8\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3375\sqrt{2+\sqrt{34}}e}-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{101250e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}+\frac{8(5\cos(d+e}{3375e(5\sin(d+ex}$$

```
[Out] (-199*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt
[34]))/15])/(101250*e) - (8*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - S
qrt[34]))/15])/(3375*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*
x])/(75*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2)) + (8*(5*Cos[d + e*x]
 - 3*Sin[d + e*x]))/(3375*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) -
(199*(5*Cos[d + e*x] - 3*Sin[d + e*x]))/(101250*e*Sqrt[2 + 3*Cos[d + e*x] +
 5*Sin[d + e*x]])
```

Rubi [A] time = 0.259548, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3118, 2653, 3126, 2661}

$$-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{101250e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}+\frac{8(5\cos(d+ex)-3\sin(d+ex))}{3375e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}-\frac{5\cos(d+ex)-3\sin(}{75e(5\sin(d+ex)+3\cos(d}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```


```
[Out] (-199*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt
[34]))/15])/(101250*e) - (8*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - S
qrt[34]))/15])/(3375*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*
x])/(75*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2)) + (8*(5*Cos[d + e*x]
 - 3*Sin[d + e*x]))/(3375*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) -
(199*(5*Cos[d + e*x] - 3*Sin[d + e*x]))/(101250*e*Sqrt[2 + 3*Cos[d + e*x] +
 5*Sin[d + e*x]])
```


Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
```

```
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
```

```
b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{1}{75}\int \frac{-5+\frac{9}{2}\cos(d+ex)+\ldots}{(2+3\cos(d+ex)+5\,\mathrm{s}\ldots}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)-3\sin(\ldots}{3375e(2+3\cos(d+ex)+5\,\mathrm{s}\ldots}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)-3\sin(\ldots}{3375e(2+3\cos(d+ex)+5\,\mathrm{s}\ldots}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)-3\sin(\ldots}{3375e(2+3\cos(d+ex)+5\,\mathrm{s}\ldots}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)-3\sin(\ldots}{3375e(2+3\cos(d+ex)+5\,\mathrm{s}\ldots}$$

$$= -\frac{199\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{101250e} - \frac{8F\left(\frac{1}{2}\left(d+ex\ldots\right.\right.}{3\ldots}$$

Mathematica [C] time = 3.92931, size = 436, normalized size = 1.87

$$\frac{-638\sqrt{30}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{17\sin(d\ldots}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (-13532*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] + (597*(12 + 43*Cos[d + e
*x] + 15*Sin[d + e*x]))/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (27
000*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2) - (3
00*(305 + 272*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2) +
(20*(1595 + 3383*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] -
 638*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + Arc
Tan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17
 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]
*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] + (2985*Sqrt[30]*AppellF1[-1
/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt
[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d +
e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Co
s[d + e*x - ArcTan[5/3]]])/(3037500*e)
```

Maple [C] time = 5.834, size = 589, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x)
```

```
[Out] (-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*(-1/
2550*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)
/(sin(e*x+d+arctan(3/5))+1/17*34^(1/2))^3+4/57375*34^(1/2)*(-(-34^(1/2)*sin
(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)/(sin(e*x+d+arctan(3/
5))+1/17*34^(1/2))^2-3383/101250*34^(1/2)*cos(e*x+d+arctan(3/5))^2/(-(-289*
sin(e*x+d+arctan(3/5))-17*34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2
)-319/50625*(-1+1/17*34^(1/2))*((-17*sin(e*x+d+arctan(3/5))-34^(1/2))/(-34^
(1/2)+17))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)*((17
*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))^(1/2)/(-(-34^(1/2)*sin(e*x+d+ar
ctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*EllipticF(((-17*sin(e*x+d+arc
tan(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^
(1/2))-199/101250*34^(1/2)*(-1+1/17*34^(1/2))*((-17*sin(e*x+d+arctan(3/5))-
34^(1/2))/(-34^(1/2)+17))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+
17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))^(1/2)/(-(-34^(1/
2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*((-1/17*34^(1/
2)-1)*EllipticE(((-17*sin(e*x+d+arctan(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2
),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+EllipticF(((-17*sin(e*x+d+arctan(
3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2
))))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2}}{644\,\cos\left(ex+d\right)^{4}+1584\,\cos\left(ex+d\right)^{3}+284\,\cos\left(ex+d\right)^{2}+20\left(48\,\cos\left(ex+d\right)^{3}-4\,\cos\left(ex+d\right)^{2}-111\,\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(644*cos(e*x + d)^4 + 1
584*cos(e*x + d)^3 + 284*cos(e*x + d)^2 + 20*(48*cos(e*x + d)^3 - 4*cos(e*x
 + d)^2 - 111*cos(e*x + d) - 58)*sin(e*x + d) - 1896*cos(e*x + d) - 1241),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)
```

3.410 $\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=347

$$-\frac{16a\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(b,c)+d+ex\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a-}}{}$$

```
[Out] (-16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[
d + e*x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x
] + c*Sin[d + e*x])^(3/2))/(5*e) + (2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d
 + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]])/(15*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin
[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (16*a*(a^2 - b^2 - c^2)*EllipticF[(d +
 e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(15*e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rubi [A] time = 0.531804, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3120, 3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{16a\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+}}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[
d + e*x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x
] + c*Sin[d + e*x])^(3/2))/(5*e) + (2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d
 + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]])/(15*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin
[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (16*a*(a^2 - b^2 - c^2)*EllipticF[(d +
 e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(15*e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}+\frac{2}{5}\int$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e}-\frac{2(}{}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e}-\frac{2(}{}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e}-\frac{2(}{}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e}-\frac{2(}{}$$

Mathematica [C] time = 6.59523, size = 3767, normalized size = 10.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((2*b*(23*a^2 + 9*b^2 + 9*c^2))/
(15*c) - (22*a*c*Cos[d + e*x])/15 - (2*b*c*Cos[2*(d + e*x)])/5 + (22*a*b*Si
n[d + e*x])/15 + ((b^2 - c^2)*Sin[2*(d + e*x)])/5))/e + (2*a^3*AppellF1[1/2
, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sq
rt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]
*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2
]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqr
t[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2]
)]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqr
t[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-
a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (34*a*b^2*AppellF1
[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])
/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/
c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2
/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c
*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/
c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c
*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]
)/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*c*e) + (34*a*c*App
ellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b
/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 +
 b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1
+ b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2
] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 +
c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sq
rt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[
b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*e) + (23*a^2*
b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))),
 -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2
]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
 c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e
*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^
2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[
(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])
])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^
2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[
1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*c*e) + (3*b^4*(-((c*AppellF1
[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[
1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*
Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])
/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*
x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*C
```

```
os[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b
*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*
x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d
 + e*x - ArcTan[c/b]]]))/(5*c*e) + (23*a^2*c*(-((c*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))
]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)
/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^
2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]
]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - Arc
Tan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^
2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]]
)/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[
c/b]]]))/(15*e) + (6*b^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*S
qrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b
*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - Ar
cTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^
2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sq
rt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^
2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b
*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - A
rcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/
b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5*e) + (
3*c^3*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[
d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))
), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b
^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d +
 e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 +
c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqr
t[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2
])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 +
c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqr
t[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5*e)
```

Maple [B] time = 10.443, size = 2303, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*((b^2+c^2)^(3/2)*(-2/5
/(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))*(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(e*x+d-arc
tan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c
^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^
(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/
2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-ar
ctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2
)^(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2
+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e
*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*
x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-a
rctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2
+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+
EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)
))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+(3*a*b^2+3*a*c^
2)*(-2/3/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(
-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-
b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))
^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2
)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/
2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))
^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(
1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/
2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^
(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arct
an(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^
(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+
(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+
6*a^2*(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-
arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*
(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^
2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*Ellip
ticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
```

```
/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)
^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)
^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*a^3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+
c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*
x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x
+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(e
*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d
-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b
^2+c^2)^(1/2)))^(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c
))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,ab\cos(ex+d)+\left(b^2-c^2\right)\cos(ex+d)^2+a^2+c^2+2\,(bc\cos(ex+d)+ac)\sin(ex+d)\right)\sqrt{b\cos(ex+d)+c\,\text{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(
b*c*cos(e*x + d) + a*c)*sin(e*x + d))*sqrt(b*cos(e*x + d) + c*sin(e*x + d)
+ a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.411 $\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=283

$$-\frac{2\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(b,c)+d+ex\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin}}{3e\sqrt{}}$$

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(3*e) + (8*a*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2]
)/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e*Sq
rt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2
- b^2 - c^2)*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a +
 Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2
 + c^2])])/(3*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rubi [A] time = 0.281805, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3120, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\right.}{3e\sqrt{\frac{a+b\cos(d+ex)}{a+\sqrt{b}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(3*e) + (8*a*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2]
)/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e*Sq
rt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2
- b^2 - c^2)*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a +
 Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2
 + c^2])])/(3*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
```

e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}+\frac{2}{3}\int$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}+\frac{1}{3}(4a)$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}+\frac{(4a\sqrt{}}{}$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}+\frac{8aE\Big(}{}$$

Mathematica [C] time = 6.28603, size = 2190, normalized size = 7.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*b*Sin[d + e*x])/3)*Sqrt[a + b*C
os[d + e*x] + c*Sin[d + e*x]])/e + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a
 + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcT
an[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*
x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Si
n[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b
^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] +
c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2
)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((
a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
 a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Arc
Tan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e
*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*S
in[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(
b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] +
```

```
 c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^
2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1
- a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Ar
cTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d +
e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*
Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
+ c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c
^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*e) + (4*a*b^2*(-((c*AppellF1[-1/2, -1/2, -
1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
+ c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[
d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^
2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(
b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b
]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - A
rcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/
b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTa
n[c/b]]]))/(3*c*e) + (4*a*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*
Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(
b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/
b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b
^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*S
qrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b
^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a +
b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x -
ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2
/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(3*e)
```

Maple [B] time = 7.295, size = 1516, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*((b^2+c^2)*(-2/3/(b^2+
```

```
c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arct
an(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^
2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b
^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^
2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(
e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e
*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-
arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^
2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(
-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))
+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+4*a*(b^2+c^2)^
(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)
/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/
(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a
+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2
)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d
-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+
c^2)^(1/2)))^(1/2)))+2*a^2*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e
*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c)
)+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c))
)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(e*x+d-arctan(-b,c
))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d-arctan(-b,c))^2
/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c)
)-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^
(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d
-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(ex+d) + c\sin(ex+d) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

3.412 $\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(a + b*Cos[d +
e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
```

Rubi [A] time = 0.0708033, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3119, 2653}

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(a + b*Cos[d +
e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}\,dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

$$= \frac{2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.27412, size = 1408, normalized size = 13.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*b*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(c*e) + (2*a*AppellF1[1/2,
1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt
[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c
*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*
c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]
*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[
(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a
+ c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (b^2*(-((c*AppellF1[
-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]
])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 +
c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1
 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x
 - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Co
s[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*
Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x
```

```
 - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]]]))/(c*e) + (c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(
1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Ar
cTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d +
e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*
Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*
Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])
/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d +
 e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[
1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/e
```

Maple [B] time = 3.265, size = 720, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x)
```

```
[Out] -2/(b^2+c^2)^(1/2)*(-a+(b^2+c^2)^(1/2))*(-((b^2+c^2)^(1/2)*sin(e*x+d-arctan
(-b,c))+a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c
^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)
^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))
*cos(e*x+d-arctan(-b,c))^2+cos(e*x+d-arctan(-b,c))^2*a)^(1/2)*((b^2+c^2)^(1
/2)*EllipticF((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2))*sin(e*x+d-arctan(-b,c)
)-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))
^(1/2))-(b^2+c^2)^(1/2)*EllipticE((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2))*si
n(e*x+d-arctan(-b,c))-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/2))/
(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)
)*sin(e*x+d-arctan(-b,c))-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/
2))/(a+(b^2+c^2)^(1/2)))^(1/2))*a-EllipticE((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)
^(1/2))*sin(e*x+d-arctan(-b,c))-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^
2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))*a)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*s
in(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+
c^2)^(1/2))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(ex+d) + c\sin(ex+d) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(ex+d) + c\sin(ex+d) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\cos(d+ex) + c\sin(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(ex+d) + c\sin(ex+d) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

3.413 $\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(b,c)+d+ex\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] (2*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rubi [A] time = 0.0702036, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3127, 2661}

$$\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx=\frac{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}}\,dx}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$
$$=\frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] time = 0.647812, size = 285, normalized size = 2.64

$$\frac{2\sec\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)\sqrt{-\frac{c\sqrt{\frac{b^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)-1\right)}{a+c\sqrt{\frac{b^2}{c^2}+1}}}\sqrt{\frac{c\sqrt{\frac{b^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)+1\right)}{c\sqrt{\frac{b^2}{c^2}+1}-a}}\sqrt{a+c\sqrt{\frac{b^2}{c^2}+1}\sin\left(\tan^{-1}\left(\frac{b}{c}\right)+\cdots\right.}}{ce\sqrt{\frac{b^2}{c^2}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcT
an[b/c]])/(a - Sqrt[1 + b^2/c^2]*c), (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x +
 ArcTan[b/c]])/(a + Sqrt[1 + b^2/c^2]*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[-
((Sqrt[1 + b^2/c^2]*c*(-1 + Sin[d + e*x + ArcTan[b/c]]))/(a + Sqrt[1 + b^2/
c^2]*c))]*Sqrt[(Sqrt[1 + b^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[b/c]]))/(-a +
 Sqrt[1 + b^2/c^2]*c)]*Sqrt[a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/
c]]])/(Sqrt[1 + b^2/c^2]*c*e)
```

Maple [B] time = 2.115, size = 303, normalized size = 2.8

$$-2\,\frac{-a+\sqrt{b^2+c^2}}{\sqrt{b^2+c^2}\cos\left(ex+d-\arctan\left(-b,c\right)\right)e}\sqrt{-\frac{\sqrt{b^2+c^2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)+a}{-a+\sqrt{b^2+c^2}}}\sqrt{-\frac{\left(\sin\left(ex+d-\arctan\left(-b,c\right)\cdots\right.\right.}{a+\sqrt{b^2+c^2}\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x)
```

```
[Out] -2*(-a+(b^2+c^2)^(1/2))*(-((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(-a+(
b^2+c^2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)/(a+(b^
2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+
c^2)^(1/2)))^(1/2)*EllipticF((-((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/
(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/
2))/(b^2+c^2)^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c
^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cos\left(d+ex\right)+c\sin\left(d+ex\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(ex+d)+c\sin(ex+d)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

3.414 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(a}}$$

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/((a^2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d
 + e*x] + c*Sin[d + e*x]]) + (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sq
rt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/((a^2 - b^2 - c^2)*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
Sqrt[b^2 + c^2])])
```

Rubi [A] time = 0.103961, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3128, 3119, 2653}

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/((a^2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d
 + e*x] + c*Sin[d + e*x]]) + (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sq
rt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/((a^2 - b^2 - c^2)*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
Sqrt[b^2 + c^2])])
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
```

```
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{\left(a^2-b^2-c^2\right)e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{\int \sqrt{a+b\cos(d+ex)+}}{a^2-b^2-}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{\left(a^2-b^2-c^2\right)e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{\sqrt{a+b\cos(d+ex)+c}}{\left(a^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{\left(a^2-b^2-c^2\right)e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\right.\right.}{\left(a^2-\right.}$$

Mathematica [C] time = 6.37446, size = 1540, normalized size = 8.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2))/(b*c*(-a^2 + b
^2 + c^2)) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(b*(-a^2 + b^2
```

```
 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e - (2*a*AppellF1[1/2, 1/2
, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1
+ b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Si
n[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))
*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^
2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^
2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c
*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)*e) - (b^2
*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e
*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(
(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(
-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^
2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x
- ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/
b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^
2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]))
 - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2)
- (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)*e) - (c*(-((c
*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b
*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a
/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]
*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcT
an[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*C
os[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^
2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2
*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*S
in[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b
^2]*Cos[d + e*x - ArcTan[c/b]]]))/((-a^2 + b^2 + c^2)*e)
```

Maple [B] time = 8.075, size = 2388, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*((b^2+
```

c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)+1/(a^2-b^2-c^2)*(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+1/(a^2-b^2-c^2)*(b^2+c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))-(1/2*b^2+1/2*c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)/a*EllipticPi(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b^2+c^2))^(1/2)+a*(b^2+c^2)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b^2+c^2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(-(b^2+c^2)^(3/2)+2*(b^2+c^2)^(1/2)*b^2+2*(b^2+c^2)^(1/2)*c^2)/(2*a^2-2*b^2-2*c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b^2+c^2))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+1/2*(b^2+c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c))

```
)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d
-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b^2+c^2))^(1/2)/a*EllipticPi((
(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1
/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+
c^2)^(1/2)))^(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+
c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(ex+d)+c\sin(ex+d)+a}}{2\,ab\cos(ex+d)+\left(b^2-c^2\right)\cos(ex+d)^2+a^2+c^2+2\,(bc\cos(ex+d)+ac)\sin(ex+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/(2*a*b*cos(e*x + d) + (b
^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x +
 d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)
```

3.415 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=382

$$-\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(b,c)+d+ex\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\Big(\frac{1}{2}}{3e\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\,\text{co}}{}}}$$

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(3/2)) + (8*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/
(3*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (8*a*
EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c
^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*(a^2 - b^2 - c^2)^2*e*S
qrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*Elli
pticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])
]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(3*(a^
2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rubi [A] time = 0.362805, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\Big(\frac{1}{2}\big(d+ex-\text{t}}{3e\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(d+ex)+c\sin(}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(3/2)) + (8*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/
(3*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (8*a*
EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c
^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*(a^2 - b^2 - c^2)^2*e*S
qrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*Elli
pticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])
]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(3*(a^
```

```
2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cos(d+e\ldots}{(a+b\cos(d+ex)+\ldots}}{3\left(a^2-b\ldots\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8(ac\cos\ldots}{3\left(a^2-b^2-c^2\right)^2\ldots}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8(ac\cos\ldots}{3\left(a^2-b^2-c^2\right)^2\ldots}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8(ac\cos\ldots}{3\left(a^2-b^2-c^2\right)^2\ldots}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8(ac\cos\ldots}{3\left(a^2-b^2-c^2\right)^2\ldots}$$

Mathematica [C] time = 6.3863, size = 2408, normalized size = 6.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((8*a*(b^2 + c^2))/(3*b*c*(a^2 -
 b^2 - c^2)^2) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(3*b*(-a^2
 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(3*a^2*c + b^2*
c + c^3 + 4*a*b^2*Sin[d + e*x] + 4*a*c^2*Sin[d + e*x]))/(3*b*(-a^2 + b^2 +
c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e + (2*a^2*AppellF1[1/2, 1/
2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1
 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*S
in[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c)
)*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b
^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*S
qrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b
^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a +
c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^2*e) + (
2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x +
 ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a +
 Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a
/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 +
 c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqr
t[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[
b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x
+ ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c*(-a
^2 + b^2 + c^2)^2*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^
2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^
2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqr
t[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c
]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + Ar
cTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 +
c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(3*S
qrt[1 + b^2/c^2]*(-a^2 + b^2 + c^2)^2*e) + (4*a*b^2*(-((c*AppellF1[-1/2, -1
/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sq
rt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]
*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b
^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2
 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*S
qrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTa
```

```
n[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*
x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTa
n[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x -
ArcTan[c/b]]]))/(3*c*(-a^2 + b^2 + c^2)^2*e) + (4*a*c*(-((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*
Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^
2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2
/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b
^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b
*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - Arc
Tan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d +
e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1
 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x
- ArcTan[c/b]]]))/(3*(-a^2 + b^2 + c^2)^2*e)
```

Maple [B] time = 29.057, size = 2967, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*(1/4*(b^2+c^2)/(a^2-b^
2-c^2)/a*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c)
)+a))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2
+c^2)^(1/2))+1/3/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+
1/(b^2+c^2)^(1/2)*a)^2+4/3*(b^2+c^2)^(1/2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b
^2-c^2)^2*a/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b
,c))+a))^(1/2)+2*(-7/24/(a^2-b^2-c^2)+2/3*a^2/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)
^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)
))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))
^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+
a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+
c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(1/8/
a/(a^2-b^2-c^2)*(b^2+c^2)^(1/2)+2/3*a*(b^2+c^2)^(1/2)/(a^2-b^2-c^2)^2)*(1/(
b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+
```

c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^
2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)
^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan
(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*si
n(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b
,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)
))^(1/2)))+1/8*(5*a^2-b^2-c^2)/a^2/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((
-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-
sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+s
in(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*
x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*Elli
pticPi(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^
(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/
(a+(b^2+c^2)^(1/2)))^(1/2))-1/4/a/(a^2-b^2-c^2)*(b^2+c^2)^(1/2)*(cos(e*x+d-
arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2
)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2
))+1/3/(a^2-b^2-c^2)/(b^2+c^2)*(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/(b^2
+c^2)^(1/2)*a)^2-4/3*(-b^2-c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)^2*a
/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))+a))^(1/2)+2*(-1/24/(a^2-b^2-c^2)*(b^2+c^2)^(1/2)+2/3*a^2*(b^2+c^2)^(1/2
)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arc
tan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^
2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c
^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*
((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(
1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(
1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(13*a^2*b^2+13*a^2*c^2+3*b^4+6*b^2*c^2+
3*c^4)/(24*a^5-48*a^3*b^2-48*a^3*c^2+24*a*b^4+48*a*b^2*c^2+24*a*c^4)*(1/(b^
2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^
2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)
^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+
d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^
(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^
(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-a
rctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^
2)^(1/2)))^(1/2)))-1/8*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)/a^2/(a^2-b^2
-c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-
arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*
(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^
2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^
2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticPi(((-(b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b

```
^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2
)))^(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e
*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos{(ex+d)} + c\sin{(ex+d)} + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos{(ex+d)} + c\sin{(ex+d)} + a}}{\left(b^3 - 3\,bc^2\right)\cos{(ex+d)}^3 + a^3 + 3\,ac^2 + 3\left(ab^2 - ac^2\right)\cos{(ex+d)}^2 + 3\left(a^2b + bc^2\right)\cos{(ex+d)} + \left(6\,abc\cos{(e}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/((b^3 - 3*b*c^2)*cos(e*x
 + d)^3 + a^3 + 3*a*c^2 + 3*(a*b^2 - a*c^2)*cos(e*x + d)^2 + 3*(a^2*b + b*c
^2)*cos(e*x + d) + (6*a*b*c*cos(e*x + d) + 3*a^2*c + c^3 + (3*b^2*c - c^3)*
cos(e*x + d)^2)*sin(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

3.416 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

Optimal. Leaf size=490

$$-\frac{16a\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(b,c)+d+ex\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\left(a^2-b^2-c^2\right)^2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+e}}{15e\left(a^2-b^2-\right.}$$

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(5*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(5/2)) + (16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))
/(15*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (
2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
Sqrt[b^2 + c^2])]) - (16*a*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])
/(a + Sqrt[b^2 + c^2])])/(15*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x]
+ c*Sin[d + e*x]]) + (2*(c*(23*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(23*a^
2 + 9*(b^2 + c^2))*Sin[d + e*x]))/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[a + b*Cos[
d + e*x] + c*Sin[d + e*x]])
```

Rubi [A] time = 0.619259, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$-\frac{16a\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\left(a^2-b^2-c^2\right)^2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(}}{15e\left(a^2-b^2-c^2\right)^3\sqrt{\frac{a+b}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(5*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(5/2)) + (16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))
/(15*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (
2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
```

```
Sqrt[b^2 + c^2])]) - (16*a*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])
/(a + Sqrt[b^2 + c^2])])/(15*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x]
+ c*Sin[d + e*x]]) + (2*(c*(23*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(23*a^
2 + 9*(b^2 + c^2))*Sin[d + e*x]))/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[a + b*Cos[
d + e*x] + c*Sin[d + e*x]])
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cos(d+ex)}{(a+b\cos(d+ex)+c}}{5\left(a^2-b^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{16(ac\cos}{15\left(a^2-b^2-c^2\right)^2 e}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{16(ac\cos}{15\left(a^2-b^2-c^2\right)^2 e}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{16(ac\cos}{15\left(a^2-b^2-c^2\right)^2 e}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{16(ac\cos}{15\left(a^2-b^2-c^2\right)^2 e}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{16(ac\cos}{15\left(a^2-b^2-c^2\right)^2 e}$$

Mathematica [C] time = 6.62076, size = 4116, normalized size = 8.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2)*(23*a^2 + 9*b^2
 + 9*c^2))/(15*b*c*(-a^2 + b^2 + c^2)^3) + (2*(a*c + b^2*Sin[d + e*x] + c^2
*Sin[d + e*x]))/(5*b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x
])^3) - (2*(5*a^2*c + 3*b^2*c + 3*c^3 + 8*a*b^2*Sin[d + e*x] + 8*a*c^2*Sin[
d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x])
^2) + (2*(15*a^3*c + 17*a*b^2*c + 17*a*c^3 + 23*a^2*b^2*Sin[d + e*x] + 9*b^
4*Sin[d + e*x] + 23*a^2*c^2*Sin[d + e*x] + 18*b^2*c^2*Sin[d + e*x] + 9*c^4*
Sin[d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^3*(a + b*Cos[d + e*x] + c*Sin[d + e
```

*x]))))/e - (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(15*Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(15*Sqrt[1 + b^2/c^2]*(-a^2 + b^2 + c^2)^3*e) - (23*a^2*b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*c*(-a^2 + b^2 + c^2)^3*e) - (3*b^4*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Ar

```
cTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b
^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5*c*(-a^2
 + b^2 + c^2)^3*e) - (23*a^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a +
 b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 -
a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan
[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x
- ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt
[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])
]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt
[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a
 + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x
 - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*(
-a^2 + b^2 + c^2)^3*e) - (6*b^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((
a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1
 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e
*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*S
qrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^
2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d +
e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5
*(-a^2 + b^2 + c^2)^3*e) - (3*c^3*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((
a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1
 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e
*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*S
qrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^
2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d +
e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5
*(-a^2 + b^2 + c^2)^3*e)
```

Maple [B] time = 94., size = 3876, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(1/8/a
*(b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1
/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(
e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))^2+1/5/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*
(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/
2)/(sin(e*x+d-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^3-3/32*(b^2+c^2)^(1/2)*(5*
a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)/(a^2-b^2-c^2)^2/a^2*(cos(e*x+d-arctan(
-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(b^2*sin(e*x+d-
arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))+8/15/(a^2-b^2-
c^2)^2*a*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c)
)+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2+1/15*(b^2+c^2)*
cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)^3*(23*a^2+9*b^2+9*c^2)/(cos(e*x+d-a
rctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+2*(1/64*(
11*a^2+b^2+c^2)*(b^2+c^2)^(1/2)/a/(a^2-b^2-c^2)^2-4/15*a*(b^2+c^2)^(1/2)/(a
^2-b^2-c^2)^2+1/30*a*(b^2+c^2)^(1/2)*(23*a^2+9*b^2+9*c^2)/(a^2-b^2-c^2)^3)*
(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(
b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^
2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+
c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-ar
ctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-
a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1
/2))+2*(-3/64*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)/(a^2-b^2-c^2)^2/a^2+1
/30*(b^2+c^2)*(23*a^2+9*b^2+9*c^2)/(a^2-b^2-c^2)^3)*(1/(b^2+c^2)^(1/2)*a-1)
*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*
((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((
1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos
(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*(
(-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c
))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))
^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^
2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))-1/64*(43
*a^4*b^2+43*a^4*c^2+2*a^2*b^4+4*a^2*b^2*c^2+2*a^2*c^4+3*b^6+9*b^4*c^2+9*b^2
*c^4+3*c^6)/(a^2-b^2-c^2)^2/a^3/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-
(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-s
in(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+si
n(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x
+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*Ellip
ticPi(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(
1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))-1/8/a/(a^2-b^2-c^2)*(b^2+c^2)^(3/2)*(cos(e*x+d-a
rctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)
```

```
/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2)
)^2+1/5/(a^2-b^2-c^2)/(b^2+c^2)*(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+
c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/(b^
2+c^2)^(1/2)*a)^3+3/32*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)/(a^2-b^2-c^2
)^2/a^2*(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arc
tan(-b,c))+a))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c
))-a*(b^2+c^2)^(1/2))+8/15/(a^2-b^2-c^2)^2*a/(b^2+c^2)^(1/2)*(cos(e*x+d-arc
tan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(
sin(e*x+d-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2-1/15*(b^2+c^2)^(1/2)*(-b^2-c
^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)^3*(23*a^2+9*b^2+9*c^2)/(cos(e*x
+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(
1/2)+2*(-1/64*(11*a^2*b^2+11*a^2*c^2+b^4+2*b^2*c^2+c^4)/a/(a^2-b^2-c^2)^2-4
/15*a*(b^2+c^2)/(a^2-b^2-c^2)^2+1/30*a*(b^2+c^2)*(23*a^2+9*b^2+9*c^2)/(a^2-
b^2-c^2)^3)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^
(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/
2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*si
n(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/64*(b^2+c^2)^(1/2)*(5*a^2*b^2+5*a^2*c^2-b^4
-2*b^2*c^2-c^4)/(a^2-b^2-c^2)^2/a^2-1/30*(b^2+c^2)^(3/2)*(23*a^2+9*b^2+9*c^
2)/(a^2-b^2-c^2)^3+1/30*(b^2+c^2)^(1/2)*(2*b^2+2*c^2)/(a^2-b^2-c^2)^3*(23*a
^2+9*b^2+9*c^2))*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arcta
n(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+
c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2
)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((
b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*
EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)
))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2
+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2
+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+1/64*(43*a^4*b^2+43*a^4*c^2+2*a^2
*b^4+4*a^2*b^2*c^2+2*a^2*c^4+3*b^6+9*b^4*c^2+9*b^2*c^4+3*c^6)/(a^2-b^2-c^2)
^2/a^3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a
)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)
/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-
a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(
1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticPi(((-(b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a
+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos
(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,
c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos{(ex+d)} + c\sin{(ex+d)} + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{\left(b^4 - 6\,b^2c^2 + c^4\right)\cos{(ex+d)}^4 + a^4 + 6\,a^2c^2 + c^4 + 4\left(ab^3 - 3\,abc^2\right)\cos{(ex+d)}^3 + 2\left(3\,a^2b^2 - c^4 - 3\left(a^2 - b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/((b^4 - 6*b^2*c^2 + c^4)
*cos(e*x + d)^4 + a^4 + 6*a^2*c^2 + c^4 + 4*(a*b^3 - 3*a*b*c^2)*cos(e*x + d
)^3 + 2*(3*a^2*b^2 - c^4 - 3*(a^2 - b^2)*c^2)*cos(e*x + d)^2 + 4*(a^3*b + 3
*a*b*c^2)*cos(e*x + d) + 4*(a^3*c + a*c^3 + (b^3*c - b*c^3)*cos(e*x + d)^3
+ (3*a*b^2*c - a*c^3)*cos(e*x + d)^2 + (3*a^2*b*c + b*c^3)*cos(e*x + d))*si
n(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)
```

3.417 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}-\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex}}{3e}$$

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*S
in[d + e*x]]) - (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*
x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rubi [A] time = 0.0648625, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}-\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*S
in[d + e*x]]) - (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*
x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
```

2 - b^2 - c^2, 0]

Rubi steps

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}{5e}+8\int$$
$$= -\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}{3e}-\frac{2(3}{}$$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{}}{3e}$$

Mathematica [A] time = 0.627387, size = 130, normalized size = 0.94

$$-\frac{(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}\left(3750\cos\left(\frac{1}{2}(d+ex)\right)+1625\cos\left(\frac{3}{2}(d+ex)\right)+3\left(-3750\sin\left(\frac{1}{2}(d+ex)\right)-375\sin\right.\right.}{30e\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] -((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(3750*Cos[(d + e*x)/2] + 1625
*Cos[(3*(d + e*x))/2] + 3*(79*Cos[(5*(d + e*x))/2] - 3750*Sin[(d + e*x)/2]
- 375*Sin[(3*(d + e*x))/2] + 3*Sin[(5*(d + e*x))/2])))/(30*e*(3*Cos[(d + e*
x)/2] + Sin[(d + e*x)/2])^5)
```

Maple [A] time = 1.355, size = 74, normalized size = 0.5

$$\frac{(50+50\sin(ex+d+\arctan(4/3)))\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(3(\sin(ex+d+\arctan(4/3)))^2+14\sin(ex+d+\right.}{3\cos(ex+d+\arctan(4/3))e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] 50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(3*sin(e*x+d+arc
tan(4/3))^2+14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(5+5*sin(e
```

```
*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (4 \cos(ex+d) + 3 \sin(ex+d) + 5)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)
```

Fricas [A] time = 1.82639, size = 293, normalized size = 2.11

$$-\frac{2\left(237 \cos(ex+d)^3 + 931 \cos(ex+d)^2 + 9\left(\cos(ex+d)^2 - 62 \cos(ex+d) - 344\right)\sin(ex+d) + 1166 \cos(ex+d) + \cdots\right.}{15\,(3\,e\cos(ex+d) + e\sin(ex+d) + 3\,e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(237*cos(e*x + d)^3 + 931*cos(e*x + d)^2 + 9*(cos(e*x + d)^2 - 62*cos
(e*x + d) - 344)*sin(e*x + d) + 1166*cos(e*x + d) + 472)*sqrt(4*cos(e*x + d
) + 3*sin(e*x + d) + 5)/(3*e*cos(e*x + d) + e*sin(e*x + d) + 3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.418 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=93

$$-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}(3\cos(d+ex)-4\sin(d+ex))}{3e}-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

```
[Out] (-40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*x]
 + 3*Sin[d + e*x]])/(3*e)
```

Rubi [A] time = 0.0399338, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}(3\cos(d+ex)-4\sin(d+ex))}{3e}-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*x]
 + 3*Sin[d + e*x]])/(3*e)
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}{3e}+\frac{20}{3}\int$$
$$= -\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{5}}{3e}$$

Mathematica [A] time = 0.34001, size = 104, normalized size = 1.12

$$\frac{(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}\left(9\left(15\sin\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{3}{2}(d+ex)\right)\right)-45\cos\left(\frac{1}{2}(d+ex)\right)-13\cos\left(\frac{3}{2}(d+ex)\right)\right)}{3e\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] ((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(-45*Cos[(d + e*x)/2] - 13*Cos
[(3*(d + e*x))/2] + 9*(15*Sin[(d + e*x)/2] + Sin[(3*(d + e*x))/2])))/(3*e*(
3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3)
```

Maple [A] time = 1.44, size = 60, normalized size = 0.7

$$\frac{(50+50\sin(ex+d+\arctan(4/3)))\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5\right)}{3\cos(ex+d+\arctan(4/3))\,e}\frac{1}{\sqrt{5+5\sin(ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] 50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(sin(e*x+d+arcta
n(4/3))+5)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (4\cos(ex+d)+3\sin(ex+d)+5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)
```

Fricas [A] time = 1.7542, size = 228, normalized size = 2.45

$$-\frac{2\left(13\cos\left(ex+d\right)^{2}-9\left(\cos\left(ex+d\right)+8\right)\sin\left(ex+d\right)+29\cos\left(ex+d\right)+16\right)\sqrt{4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+5}}{3\left(3e\cos\left(ex+d\right)+e\sin\left(ex+d\right)+3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(13*cos(e*x + d)^2 - 9*(cos(e*x + d) + 8)*sin(e*x + d) + 29*cos(e*x +
d) + 16)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)/(3*e*cos(e*x + d) + e*si
n(e*x + d) + 3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+5\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)
```

3.419 $\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx$

Optimal. Leaf size=44

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d
 + e*x]])
```

Rubi [A] time = 0.0182359, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d
 + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx=-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}$$

Mathematica [A] time = 0.0383478, size = 75, normalized size = 1.7

$$-\frac{2\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}{e\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin
[d + e*x]])/(e*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))
```

Maple [A] time = 0.89, size = 50, normalized size = 1.1

$$10\,\frac{(\sin(ex+d+\arctan(4/3))-1)\,(1+\sin(ex+d+\arctan(4/3)))}{\cos(ex+d+\arctan(4/3))\,\sqrt{5+5\,\sin(ex+d+\arctan(4/3))}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4
/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{4\,\cos(ex+d)+3\,\sin(ex+d)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

Fricas [A] time = 1.74993, size = 167, normalized size = 3.8

$$-\frac{2\,\sqrt{4\,\cos(ex+d)+3\,\sin(ex+d)+5}(\cos(ex+d)-3\,\sin(ex+d)+1)}{3\,e\cos(ex+d)+e\sin(ex+d)+3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)*(cos(e*x + d) - 3*sin(e*x + d)
 + 1)/(3*e*cos(e*x + d) + e*sin(e*x + d) + 3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3\sin(d+ex)+4\cos(d+ex)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos(ex+d)+3\sin(ex+d)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

3.420 $\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{e}$$

```
[Out] (Sqrt[2/5]*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x
 - ArcTan[3/4]]])])/e
```

Rubi [A] time = 0.0648061, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2/5]*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x
 - ArcTan[3/4]]])])/e
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{10-x^2}\,dx, x, -\frac{5\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

$$= \frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] time = 0.105108, size = 101, normalized size = 2.1

$$-\frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\tan^{-1}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\left(3\tan\left(\frac{1}{4}(d+ex)\right)-1\right)\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] ((-2/5 - (6*I)/5)*Sqrt[4/5 + (3*I)/5]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (
3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))
/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

Maple [A] time = 0.89, size = 77, normalized size = 1.6

$$-\frac{\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\sqrt{10}}{5\cos\left(ex+d+\arctan(4/3)\right)e}\sqrt{-5\sin\left(ex+d+\arctan(4/3)\right)+5}\,\text{Artanh}\left(\frac{\sqrt{10}}{10}\sqrt{-5\sin\left(ex+d+\arctan(4/3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] -1/5*(1+sin(e*x+d+arctan(4/3)))*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2
)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))/cos(e*x+d+arct
an(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

Fricas [B] time = 1.77431, size = 428, normalized size = 8.92

$$\frac{\sqrt{5}\sqrt{2}\log\left(-\frac{9\,\cos\left(ex+d\right)^{2}+\left(13\,\cos\left(ex+d\right)-6\right)\sin\left(ex+d\right)+2\left(\sqrt{5}\sqrt{2}\cos\left(ex+d\right)-3\,\sqrt{5}\sqrt{2}\sin\left(ex+d\right)+\sqrt{5}\sqrt{2}\right)\sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}-33\,\cos\left(ex+d\right)-42}{9\,\cos\left(ex+d\right)^{2}+\left(13\,\cos\left(ex+d\right)+14\right)\sin\left(ex+d\right)+27\,\cos\left(ex+d\right)+18}\right)}{10\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*log(-(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 6)*sin(e*x
 + d) + 2*(sqrt(5)*sqrt(2)*cos(e*x + d) - 3*sqrt(5)*sqrt(2)*sin(e*x + d) +
sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*cos(e*x + d
) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*cos(e*
x + d) + 18))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\sin\left(d+ex\right)+4\cos\left(d+ex\right)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4 \cos (ex+d)+3 \sin (ex+d)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

3.421 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{10\sqrt{10}e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

```
[Out] ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(5 + 4*Co
s[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.0532055, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{10\sqrt{10}e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(5 + 4*Co
s[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}+\frac{1}{20}\int \frac{1}{\sqrt{5+4\cos(d+ex)+}}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}+\frac{1}{20}\int \frac{1}{\sqrt{5+5\cos\left(d+ex-\right.}}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{10-x^2}\,dx, x, -\frac{5\sin}{\sqrt{5+5}}\right.}{10e}$$

$$= \frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex)}$$

Mathematica [C] time = 0.298685, size = 154, normalized size = 1.6

$$-\frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)-(1-i)\sqrt{20+15i}\,\text{ta}\right.}{e(3\sin(d+ex)+4\cos(d+ex)+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ((-1/250 + I/125)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((5 + 10*I)*(Cos[
(d + e*x)/2] - 3*Sin[(d + e*x)/2]) - (1 - I)*Sqrt[20 + 15*I]*ArcTan[(1/10 +
 (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/
2] + Sin[(d + e*x)/2])^2))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Maple [A] time = 1.286, size = 117, normalized size = 1.2

$$-\frac{1}{100\cos\left(ex+d+\arctan\left(4/3\right)\right)e}\left(\sqrt{10}\text{Artanh}\left(\frac{\sqrt{10}}{10}\sqrt{-5\sin\left(ex+d+\arctan\left(4/3\right)\right)+5}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] -1/100*(10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))
*sin(e*x+d+arctan(4/3))+10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)
^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-5*sin(e*x+d+arcta
n(4/3))+5)^(1/2)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/
e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-3/2), x)
```

Fricas [B] time = 1.95377, size = 782, normalized size = 8.15

$$\frac{\left(9\sqrt{10}\cos\left(ex+d\right)^{2}+\left(13\sqrt{10}\cos\left(ex+d\right)+14\sqrt{10}\right)\sin\left(ex+d\right)+27\sqrt{10}\cos\left(ex+d\right)+18\sqrt{10}\right)\log\left(-\frac{9\cos(ex+d)^{2}+(13\ldots}{\ldots}\right.}{200\left(9e\cos\left(ex+d\right)^{2}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/200*((9*sqrt(10)*cos(e*x + d)^2 + (13*sqrt(10)*cos(e*x + d) + 14*sqrt(10)
)*sin(e*x + d) + 27*sqrt(10)*cos(e*x + d) + 18*sqrt(10))*log(-(9*cos(e*x +
d)^2 + (13*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(10)*cos(e*x + d) - 3*sq
rt(10)*sin(e*x + d) + sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) -
 33*cos(e*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x +
 d) + 27*cos(e*x + d) + 18)) - 20*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)
*(cos(e*x + d) - 3*sin(e*x + d) + 1))/(9*e*cos(e*x + d)^2 + 27*e*cos(e*x +
d) + (13*e*cos(e*x + d) + 14*e)*sin(e*x + d) + 18*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\sin\left(d+ex\right)+4\cos\left(d+ex\right)+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-3/2), x)
```

Giac [B] time = 1.95805, size = 383, normalized size = 3.99

$$\frac{1}{100}\left(\frac{\sqrt{10}\log\left(\frac{\left|-2\sqrt{10}+2\sqrt{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+1}-2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-6\right|}{\left|2\sqrt{10}+2\sqrt{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+1}-2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-6\right|}\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+3\right)}-\frac{20\left(19\left(\sqrt{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+1}-\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right)^{3}-51\right.}{\left(\left(\sqrt{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+1}-\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] 1/100*(sqrt(10)*log(abs(-2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) -
2*tan(1/2*x*e + 1/2*d) - 6)/abs(2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2
+ 1) - 2*tan(1/2*x*e + 1/2*d) - 6))/sgn(tan(1/2*x*e + 1/2*d) + 3) - 20*(19*
(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 51*(sqrt(tan(
1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 17*sqrt(tan(1/2*x*e + 1
/2*d)^2 + 1) + 17*tan(1/2*x*e + 1/2*d) - 3)/(((sqrt(tan(1/2*x*e + 1/2*d)^2
+ 1) - tan(1/2*x*e + 1/2*d))^2 - 6*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 6*tan
(1/2*x*e + 1/2*d) - 1)^2*sgn(tan(1/2*x*e + 1/2*d) + 3)))*e^(-1)
```

3.422 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}+\frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{400\sqrt{10}e}$$

```
[Out] (3*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTa
n[3/4]]])])/(400*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(5 +
 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d + e
*x]))/(400*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.0767014, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 206}

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}+\frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{400\sqrt{10}e}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```


```
[Out] (3*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTa
n[3/4]]])])/(400*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(5 +
 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d + e
*x]))/(400*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```


Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}+\frac{3}{40}\int \frac{1}{(5+4\cos(d+ex)+3\mathrm{s}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{3(3\cos(d+ex)-4\sin(d}{400e(5+4\cos(d+ex)+3\sin}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{3(3\cos(d+ex)-4\sin(d}{400e(5+4\cos(d+ex)+3\sin}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{3(3\cos(d+ex)-4\sin(d}{400e(5+4\cos(d+ex)+3\sin}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex)}$$

Mathematica [C] time = 0.403138, size = 180, normalized size = 1.27

$$-\frac{\left(\frac{1}{20000}-\frac{i}{10000}\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(-165\sin\left(\frac{1}{2}(d+ex)\right)-27\sin\left(\frac{3}{2}(d+ex)\right)+55\cos\left(\frac{1}{2}(d\right.\right.\right.}{e(3\sin(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] ((-1/20000 + I/10000)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((-6 + 6*I)*S
qrt[20 + 15*I]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d
+ e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^4 + (5 + 10*I)*(55*Cos[
(d + e*x)/2] + 39*Cos[(3*(d + e*x))/2] - 165*Sin[(d + e*x)/2] - 27*Sin[(3*(
d + e*x))/2])))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

Maple [A] time = 1.46, size = 190, normalized size = 1.3

$$-\frac{1}{(4000+4000\sin(ex+d+\arctan(4/3)))\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)e}\left(3\sqrt{10}\mathrm{Artanh}\left(1/10\sqrt{-5\sin(ex+d+\arctan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] -1/4000*(3*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/
2))*sin(e*x+d+arctan(4/3))^2+6*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4
/3))+5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctanh(1/10*(-5*
sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))+6*(-5*sin(e*x+d+arctan(4/3))+5)^(
1/2)*sin(e*x+d+arctan(4/3))+14*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-5*sin
(e*x+d+arctan(4/3))+5)^(1/2)/(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4/
3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4\cos(ex+d)+3\sin(ex+d)+5)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-5/2), x)
```

Fricas [B] time = 1.97268, size = 998, normalized size = 7.03

$$\frac{3\left(3\sqrt{10}\cos(ex+d)^3 - 111\sqrt{10}\cos(ex+d)^2 - \left(79\sqrt{10}\cos(ex+d)^2 + 202\sqrt{10}\cos(ex+d) + 124\sqrt{10}\right)\sin(ex+d) - 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8000*(3*(3*sqrt(10)*cos(e*x + d)^3 - 111*sqrt(10)*cos(e*x + d)^2 - (79*sq
rt(10)*cos(e*x + d)^2 + 202*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*sin(e*x +
 d) - 246*sqrt(10)*cos(e*x + d) - 132*sqrt(10))*log(-(9*cos(e*x + d)^2 + (1
3*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(10)*cos(e*x + d) - 3*sqrt(10)*si
n(e*x + d) + sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*cos(e
*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*
cos(e*x + d) + 18)) + 20*(39*cos(e*x + d)^2 - 3*(9*cos(e*x + d) + 32)*sin(e
*x + d) + 47*cos(e*x + d) + 8)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5))/(
3*e*cos(e*x + d)^3 - 111*e*cos(e*x + d)^2 - 246*e*cos(e*x + d) - (79*e*cos(
e*x + d)^2 + 202*e*cos(e*x + d) + 124*e)*sin(e*x + d) - 132*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.97177, size = 563, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] 1/4000*(3*sqrt(10)*log(abs(-2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1)
 - 2*tan(1/2*x*e + 1/2*d) - 6)/abs(2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)
^2 + 1) - 2*tan(1/2*x*e + 1/2*d) - 6))/sgn(tan(1/2*x*e + 1/2*d) + 3) - 20*(
797*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^7 - 7137*(sqr
t(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^6 + 27543*(sqrt(tan(1
/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^5 - 30015*(sqrt(tan(1/2*x*e
+ 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^4 - 27105*(sqrt(tan(1/2*x*e + 1/2*d
)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 7491*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1)
 - tan(1/2*x*e + 1/2*d))^2 - 859*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 859*tan
(1/2*x*e + 1/2*d) - 69)/(((sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e +
 1/2*d))^2 - 6*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 6*tan(1/2*x*e + 1/2*d) -
1)^4*sgn(tan(1/2*x*e + 1/2*d) + 3)))*e^(-1)
```

3.423 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx$

Optimal. Leaf size=185

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}+\frac{24(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+}{7e}$$

```
[Out] (6400*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(7*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) - (320*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
 e*x] + 3*Sin[d + e*x]])/(7*e) + (24*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5
+ 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(7*e) - (2*(3*Cos[d + e*x] - 4*Si
n[d + e*x])*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))/(7*e)
```

Rubi [A] time = 0.0935227, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}+\frac{24(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```


```
[Out] (6400*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(7*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) - (320*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
 e*x] + 3*Sin[d + e*x]])/(7*e) + (24*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5
+ 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(7*e) - (2*(3*Cos[d + e*x] - 4*Si
n[d + e*x])*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))/(7*e)
```


Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
```

```
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}{7e}$$

$$= \frac{24(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}{7e}$$

$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{7e}$$

$$= \frac{6400(3\cos(d+ex)-4\sin(d+ex))}{7e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{320(3\cos(d+ex)-4\sin(d+e}{}$$

Mathematica [A] time = 1.92178, size = 151, normalized size = 0.82

$$\frac{(3\sin(d+ex)+4\cos(d+ex)-5)^{7/2}\left(30625\sin\left(\frac{1}{2}(d+ex)\right)-15925\sin\left(\frac{3}{2}(d+ex)\right)+3871\sin\left(\frac{5}{2}(d+ex)\right)-307\sin\left(\right.\right.}{28e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2)*(91875*Cos[(d + e*x)/2] - 110
25*Cos[(3*(d + e*x))/2] - 147*Cos[(5*(d + e*x))/2] + 249*Cos[(7*(d + e*x))/
2] + 30625*Sin[(d + e*x)/2] - 15925*Sin[(3*(d + e*x))/2] + 3871*Sin[(5*(d +
 e*x))/2] - 307*Sin[(7*(d + e*x))/2]))/(28*e*(Cos[(d + e*x)/2] - 3*Sin[(d +
 e*x)/2])^7)
```

Maple [A] time = 1.25, size = 86, normalized size = 0.5

$$\frac{(250\sin(ex+d+\arctan(4/3))-250)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(5(\sin(ex+d+\arctan(4/3)))^3-27(\sin(ex\right.}{7\cos(ex+d+\arctan(4/3))e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x)
```

```
[Out] 250/7*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(5*sin(e*x+d+ar
ctan(4/3))^3-27*sin(e*x+d+arctan(4/3))^2+71*sin(e*x+d+arctan(4/3))-177)/cos
(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (4 \cos (ex + d) + 3 \sin (ex + d) - 5)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)
```

Fricas [A] time = 1.76515, size = 358, normalized size = 1.94

$$-\frac{2 \left(249 \cos (ex + d)^{4} + 51 \cos (ex + d)^{3} - 3042 \cos (ex + d)^{2} - \left(307 \cos (ex + d)^{3} - 1782 \cos (ex + d)^{2} + 2860 \cos (ex -}{7 (e \cos (ex + d) - 3 e \sin (ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/7*(249*cos(e*x + d)^4 + 51*cos(e*x + d)^3 - 3042*cos(e*x + d)^2 - (307*c
os(e*x + d)^3 - 1782*cos(e*x + d)^2 + 2860*cos(e*x + d) - 1392)*sin(e*x + d
) + 10068*cos(e*x + d) + 12912)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(
e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.424 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}+\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)\cdots}}{3e}$$

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) + (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
e*x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5 +
4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rubi [A] time = 0.0739644, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}+\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)\cdots}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) + (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
e*x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5 +
4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
```

```
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}{5e}$$
$$= \frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{3e} - $$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} + \frac{16(3\cos(d+ex)-4\sin(d+e}{}$$

Mathematica [A] time = 0.523038, size = 127, normalized size = 0.91

$$\frac{(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}\left(3750\sin\left(\frac{1}{2}(d+ex)\right)-1625\sin\left(\frac{3}{2}(d+ex)\right)+237\sin\left(\frac{5}{2}(d+ex)\right)+11250\cos\left(\frac{1}{2}\right.\right.}{30e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(11250*Cos[(d + e*x)/2] - 112
5*Cos[(3*(d + e*x))/2] - 9*Cos[(5*(d + e*x))/2] + 3750*Sin[(d + e*x)/2] - 1
625*Sin[(3*(d + e*x))/2] + 237*Sin[(5*(d + e*x))/2]))/(30*e*(Cos[(d + e*x)/
2] - 3*Sin[(d + e*x)/2])^5)
```

Maple [A] time = 1.248, size = 74, normalized size = 0.5

$$\frac{(50\sin(ex+d+\arctan(4/3))-50)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(3(\sin(ex+d+\arctan(4/3)))^2-14\sin(ex+d\right.}{3\cos(ex+d+\arctan(4/3))e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] 50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(3*sin(e*x+d+arc
tan(4/3))^2-14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(-5+5*sin(
```

```
e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (4\cos(ex+d) + 3\sin(ex+d) - 5)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)
```

Fricas [A] time = 1.66577, size = 293, normalized size = 2.11

$$-\frac{2\left(9\cos(ex+d)^3 + 567\cos(ex+d)^2 - \left(237\cos(ex+d)^2 - 694\cos(ex+d) + 472\right)\sin(ex+d) - 2538\cos(ex+d) - \right.}{15\left(e\cos(ex+d) - 3e\sin(ex+d) + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(9*cos(e*x + d)^3 + 567*cos(e*x + d)^2 - (237*cos(e*x + d)^2 - 694*co
s(e*x + d) + 472)*sin(e*x + d) - 2538*cos(e*x + d) - 3096)*sqrt(4*cos(e*x +
 d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.425 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=93

$$\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

```
[Out] (40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d + e*x
] + 3*Sin[d + e*x]])/(3*e)
```

Rubi [A] time = 0.0383577, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```


```
[Out] (40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d + e*x
] + 3*Sin[d + e*x]])/(3*e)
```


Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{3e} - $$
$$= \frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{2(3\cos(d+ex)-4\sin(d+ex))}{}$$

Mathematica [A] time = 0.236961, size = 103, normalized size = 1.11

$$\frac{(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}\left(45\sin\left(\frac{1}{2}(d+ex)\right)-13\sin\left(\frac{3}{2}(d+ex)\right)+135\cos\left(\frac{1}{2}(d+ex)\right)-9\cos\left(\frac{3}{2}(d+ex)\right)\right)}{3e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(135*Cos[(d + e*x)/2] - 9*Cos
[(3*(d + e*x))/2] + 45*Sin[(d + e*x)/2] - 13*Sin[(3*(d + e*x))/2]))/(3*e*(C
os[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^3)
```

Maple [A] time = 1.193, size = 60, normalized size = 0.7

$$\frac{(50\sin(ex+d+\arctan(4/3))-50)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5\right)}{3\cos(ex+d+\arctan(4/3))\,e}\frac{}{\sqrt{-5+5\sin(ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] 50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arcta
n(4/3))-5)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)
```

Fricas [A] time = 1.70999, size = 225, normalized size = 2.42

$$\frac{2\left(9\cos\left(ex+d\right)^{2}+\left(13\cos\left(ex+d\right)-16\right)\sin\left(ex+d\right)-63\cos\left(ex+d\right)-72\right)\sqrt{4\cos\left(ex+d\right)+3\sin\left(ex+d\right)-5}}{3\left(e\cos\left(ex+d\right)-3\,e\sin\left(ex+d\right)+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 16)*sin(e*x + d) - 63*cos(e*x +
d) - 72)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*si
n(e*x + d) + e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)-5\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)
```

3.426 $\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx$

Optimal. Leaf size=44

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[
d + e*x]])
```

Rubi [A] time = 0.0173385, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[
d + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx=-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}$$

Mathematica [A] time = 0.0417613, size = 75, normalized size = 1.7

$$\frac{2\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (2*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin
[d + e*x]])/(e*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))
```

Maple [A] time = 1.077, size = 50, normalized size = 1.1

$$10\,\frac{(\sin(ex+d+\arctan(4/3))-1)\,(1+\sin(ex+d+\arctan(4/3)))}{\cos(ex+d+\arctan(4/3))\,\sqrt{-5+5\,\sin(ex+d+\arctan(4/3))}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4
/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{4\,\cos(ex+d)+3\,\sin(ex+d)-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

Fricas [A] time = 1.76369, size = 163, normalized size = 3.7

$$\frac{2\,\sqrt{4\,\cos(ex+d)+3\,\sin(ex+d)-5}(3\,\cos(ex+d)+\sin(ex+d)+3)}{e\cos(ex+d)-3\,e\sin(ex+d)+e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)*(3*cos(e*x + d) + sin(e*x + d)
+ 3)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3\sin(d+ex)+4\cos(d+ex)-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos(ex+d)+3\sin(ex+d)-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

3.427 $\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{e}$$

```
[Out] -((Sqrt[2/5]*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e
*x - ArcTan[3/4]]])])/e)
```

Rubi [A] time = 0.0608601, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3115, 2649, 204}

$$-\frac{\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] -((Sqrt[2/5]*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e
*x - ArcTan[3/4]]])])/e)
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{-5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-10-x^2}\,dx, x, -\frac{5\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{-5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

$$= -\frac{\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] time = 0.0891382, size = 99, normalized size = 2.02

$$\frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\tanh^{-1}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(\tan\left(\frac{1}{4}(d+ex)\right)+3\right)\right)}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] ((2/5 + (6*I)/5)*Sqrt[-4/5 - (3*I)/5]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 -
 (3*I)/5]*(3 + Tan[(d + e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))/
(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

Maple [A] time = 1.171, size = 77, normalized size = 1.6

$$\frac{\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\sqrt{10}}{5\,\cos\left(ex+d+\arctan\left(4/3\right)\right)e}\sqrt{-5\,\sin\left(ex+d+\arctan\left(4/3\right)\right)-5}\arctan\left(\frac{\sqrt{10}}{10}\sqrt{-5\,\sin\left(ex+d+\arctan\left(4/3\right)\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] 1/5*(sin(e*x+d+arctan(4/3))-1)*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2)
*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))/cos(e*x+d+arctan
(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

Fricas [B] time = 1.85431, size = 269, normalized size = 5.49

$$\frac{\sqrt{5}\sqrt{2}\arctan\left(-\frac{\left(3\sqrt{5}\sqrt{2}\cos(ex+d)+\sqrt{5}\sqrt{2}\sin(ex+d)+3\sqrt{5}\sqrt{2}\right)\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}}{10\left(\cos(ex+d)-3\sin(ex+d)+1\right)}\right)}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(2)*arctan(-1/10*(3*sqrt(5)*sqrt(2)*cos(e*x + d) + sqrt(5)*
sqrt(2)*sin(e*x + d) + 3*sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x +
 d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

3.428 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{10\sqrt{10}e}$$

```
[Out] ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(-5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.0523508, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 204}

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{10\sqrt{10}e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```


```
[Out] ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(-5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))
```


Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} - \frac{1}{20}\int \frac{1}{\sqrt{-5+4\cos(d+ex)+\cdots}}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} - \frac{1}{20}\int \frac{1}{\sqrt{-5+5\cos\left(d+ex\cdots\right.}}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{-10-x^2}\,dx, x, -\frac{5\cdots}{\sqrt{-5\cdots}}\right.}{10e}$$

$$= \frac{\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}e} + \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+e\cdots}$$

Mathematica [C] time = 0.29477, size = 152, normalized size = 1.58

$$\frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)-(1-i)\sqrt{-20-15i}\left(\text{co}\cdots\right.\right.}{e(3\sin(d+ex)+4\cos(d+ex)-5)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ((1/250 - I/125)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((-1 + I)*Sqrt[-20
 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d + e*x)/
4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^2 + (5 + 10*I)*(3*Cos[(d + e*x
)/2] + Sin[(d + e*x)/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Maple [A] time = 1.116, size = 118, normalized size = 1.2

$$\frac{1}{100\,\cos\left(ex+d+\arctan\left(4/3\right)\right)e}\left(-\sqrt{10}\arctan\left(\frac{\sqrt{10}}{10}\sqrt{-5\,\sin\left(ex+d+\arctan\left(4/3\right)\right)-5}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] 1/100*(-10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))*
sin(e*x+d+arctan(4/3))+10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(
1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e*x+d+arctan(
4/3))-5)^(1/2)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)-5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-3/2), x)
```

Fricas [B] time = 1.78628, size = 614, normalized size = 6.4

$$-\frac{\left(13\sqrt{10}\cos\left(ex+d\right)^{2}-9\left(\sqrt{10}\cos\left(ex+d\right)-2\sqrt{10}\right)\sin\left(ex+d\right)-\sqrt{10}\cos\left(ex+d\right)-14\sqrt{10}\right)\arctan\left(-\frac{\left(3\sqrt{10}\cos(ex+d\right.}{}\right.}{100\left(13\,e\cos\left(ex+d\right)^{2}-e\cos\left(ex+d\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/100*((13*sqrt(10)*cos(e*x + d)^2 - 9*(sqrt(10)*cos(e*x + d) - 2*sqrt(10)
)*sin(e*x + d) - sqrt(10)*cos(e*x + d) - 14*sqrt(10))*arctan(-1/10*(3*sqrt(
10)*cos(e*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x + d)
+ 3*sin(e*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*sqrt(4*cos(
e*x + d) + 3*sin(e*x + d) - 5)*(3*cos(e*x + d) + sin(e*x + d) + 3))/(13*e*c
os(e*x + d)^2 - e*cos(e*x + d) - 9*(e*cos(e*x + d) - 2*e)*sin(e*x + d) - 14
*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-3/2), x)
```

Giac [C] time = 1.52085, size = 336, normalized size = 3.5

$$-\frac{1}{450}\left(\frac{9\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\left(-3i\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}+3i\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-i\right)\right)}{\operatorname{sgn}\left(-3\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)}+\frac{10\left(33i\left(\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}\right.\right.}{\left(-3i\left(\sqrt{\tan\left(\frac{1}{2}xe+\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] -1/450*(9*sqrt(10)*arctan(1/10*sqrt(10)*(-3*I*sqrt(tan(1/2*x*e + 1/2*d)^2 +
 1) + 3*I*tan(1/2*x*e + 1/2*d) - I))/sgn(-3*tan(1/2*x*e + 1/2*d) + 1) + 10*
(33*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 7*I*(sq
```

```
rt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 21*I*sqrt(tan(1/
2*x*e + 1/2*d)^2 + 1) - 21*I*tan(1/2*x*e + 1/2*d) + 9*I)/((-3*I*(sqrt(tan(1
/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 2*I*sqrt(tan(1/2*x*e + 1
/2*d)^2 + 1) + 2*I*tan(1/2*x*e + 1/2*d) + 3*I)^2*sgn(-3*tan(1/2*x*e + 1/2*d
) + 1)))*e^(-1)
```

3.429 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-}}\right.}{400\sqrt{10}e}$$

```
[Out] (-3*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcT
an[3/4]]])])/(400*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(-5
 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d +
 e*x]))/(400*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.0751394, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 204}

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-}}\right.}{400\sqrt{10}e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcT
an[3/4]]])])/(400*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(-5
 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d +
 e*x]))/(400*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3}{40}\int \frac{1}{(-5+4\cos(d+ex)}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex)-4\,\mathrm{s}}{400e(-5+4\cos(d+ex)+}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex)-4\,\mathrm{s}}{400e(-5+4\cos(d+ex)+}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex)-4\,\mathrm{s}}{400e(-5+4\cos(d+ex)+}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}e} + \frac{3\cos(d+ex)-4\sin(d+}{20e(-5+4\cos(d+ex)+3\sin(}$$

Mathematica [C] time = 0.394032, size = 178, normalized size = 1.25

$$\frac{\left(\frac{1}{10000}+\frac{i}{20000}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((10-5i)\left(55\sin\left(\frac{1}{2}(d+ex)\right)-39\sin\left(\frac{3}{2}(d+ex)\right)+165\cos\left(\frac{1}{2}(d\right.\right.\right.}{e(3\sin(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] ((1/10000 + I/20000)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((6 + 6*I)*Sqr
t[-20 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d +
e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^4 + (10 - 5*I)*(165*Cos[(
d + e*x)/2] - 27*Cos[(3*(d + e*x))/2] + 55*Sin[(d + e*x)/2] - 39*Sin[(3*(d
+ e*x))/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

Maple [A] time = 1.611, size = 190, normalized size = 1.3

$$-\frac{1}{\left(4000\sin\left(ex+d+\arctan\left(4/3\right)\right)-4000\right)\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)e}\left(-3\sqrt{10}\arctan\left(1/10\sqrt{-5\sin\left(ex+d+\arctan\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] -1/4000*(-3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/
2))*sin(e*x+d+arctan(4/3))^2+6*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/
3))-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+6*(-5*sin(e*x+d+arctan(4/3))-
5)^(1/2)*sin(e*x+d+arctan(4/3))-3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan
(4/3))-5)^(1/2)*10^(1/2))-14*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e
*x+d+arctan(4/3))-5)^(1/2)/(sin(e*x+d+arctan(4/3))-1)/cos(e*x+d+arctan(4/3)
)/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)-5\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-5/2), x)
```

Fricas [B] time = 1.87787, size = 830, normalized size = 5.85

$$\frac{3\left(79\sqrt{10}\cos\left(ex+d\right)^{3}-123\sqrt{10}\cos\left(ex+d\right)^{2}+3\left(\sqrt{10}\cos\left(ex+d\right)^{2}+38\sqrt{10}\cos\left(ex+d\right)-44\sqrt{10}\right)\sin\left(ex+d\right)-7}{4000\left(79\,e\cos\left(e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4000*(3*(79*sqrt(10)*cos(e*x + d)^3 - 123*sqrt(10)*cos(e*x + d)^2 + 3*(sq
rt(10)*cos(e*x + d)^2 + 38*sqrt(10)*cos(e*x + d) - 44*sqrt(10))*sin(e*x + d
) - 78*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*arctan(-1/10*(3*sqrt(10)*cos(e
*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e
*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*(27*cos(e*x + d)^2 +
 (39*cos(e*x + d) - 8)*sin(e*x + d) - 69*cos(e*x + d) - 96)*sqrt(4*cos(e*x
+ d) + 3*sin(e*x + d) - 5))/(79*e*cos(e*x + d)^3 - 123*e*cos(e*x + d)^2 - 7
8*e*cos(e*x + d) + 3*(e*cos(e*x + d)^2 + 38*e*cos(e*x + d) - 44*e)*sin(e*x
+ d) + 124*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.57696, size = 514, normalized size = 3.62

$$-\frac{1}{162000}\left(\frac{243\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\left(3i\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}-3i\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+i\right)\right)}{\operatorname{sgn}\left(-3\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)}+\frac{10\left(15039i\left(\sqrt{\tan\left(\frac{1}{2}xe+\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] -1/162000*(243*sqrt(10)*arctan(1/10*sqrt(10)*(3*I*sqrt(tan(1/2*x*e + 1/2*d)
^2 + 1) - 3*I*tan(1/2*x*e + 1/2*d) + I))/sgn(-3*tan(1/2*x*e + 1/2*d) + 1) +
 10*(15039*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^7 +
6291*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^6 - 579*I*
(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^5 + 1645*I*(sqrt(
tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^4 + 25365*I*(sqrt(tan(1
/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 11367*I*(sqrt(tan(1/2*x*
e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 4887*I*sqrt(tan(1/2*x*e + 1/2
*d)^2 + 1) - 4887*I*tan(1/2*x*e + 1/2*d) + 3807*I)/((3*I*(sqrt(tan(1/2*x*e
+ 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 2*I*sqrt(tan(1/2*x*e + 1/2*d)^2
 + 1) - 2*I*tan(1/2*x*e + 1/2*d) - 3*I)^4*sgn(-3*tan(1/2*x*e + 1/2*d) + 1))
)*e^(-1)
```

3.430 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}\,dx$

Optimal. Leaf size=258

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}-\frac{24\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+e$$

```
[Out] (-256*(b^2 + c^2)^(3/2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*e*Sqrt[Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (64*(b^2 + c^2)*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(35*e) - (24*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))/(35*e) - (2*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])
^(5/2))/(7*e)
```

Rubi [A] time = 0.178605, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3113, 3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}-\frac{24\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+e$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]
```


```
[Out] (-256*(b^2 + c^2)^(3/2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*e*Sqrt[Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (64*(b^2 + c^2)*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(35*e) - (24*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))/(35*e) - (2*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])
^(5/2))/(7*e)
```


Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
```

```
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(\right.}{7e}$$

$$= -\frac{24\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+e\right.}{35e}$$

$$= -\frac{64\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+}}{35e}$$

$$= -\frac{256\left(b^2+c^2\right)^{3/2}(c\cos(d+ex)-b\sin(d+ex))}{35e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{64\left(b^2+c^2\right)(c\cos}{}$$

Mathematica [C] time = 32.745, size = 11888, normalized size = 46.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]

[Out] Result too large to show
```

Maple [A] time = 1.937, size = 306, normalized size = 1.2

$$\frac{(2+2\sin(ex+d-\arctan(-b,c)))(\sin(ex+d-\arctan(-b,c))-1)\left(5b^4(\sin(ex+d-\arctan(-b,c)))^3+10b^2c^2(\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x)
```

```
[Out] 2/35*(1+sin(e*x+d-arctan(-b,c)))*(sin(e*x+d-arctan(-b,c))-1)*(5*b^4*sin(e*x
+d-arctan(-b,c))^3+10*b^2*c^2*sin(e*x+d-arctan(-b,c))^3+5*c^4*sin(e*x+d-arc
tan(-b,c))^3+27*b^4*sin(e*x+d-arctan(-b,c))^2+54*b^2*c^2*sin(e*x+d-arctan(-
b,c))^2+27*c^4*sin(e*x+d-arctan(-b,c))^2+71*b^4*sin(e*x+d-arctan(-b,c))+142
*b^2*c^2*sin(e*x+d-arctan(-b,c))+71*c^4*sin(e*x+d-arctan(-b,c))+177*b^4+354
*b^2*c^2+177*c^4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2
*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.00733, size = 645, normalized size = 2.5

$$\frac{2\left(5\left(b^4-6b^2c^2+c^4\right)\cos(ex+d)^4-177b^4-310b^2c^2-128c^4+2\left(22b^4+15b^2c^2-27c^4\right)\cos(ex+d)^2+4\left(5\left(b^3c-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="f
ricas")
```

```
[Out] 2/35*(5*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^4 - 177*b^4 - 310*b^2*c^2 - 12
8*c^4 + 2*(22*b^4 + 15*b^2*c^2 - 27*c^4)*cos(e*x + d)^2 + 4*(5*(b^3*c - b*c
^3)*cos(e*x + d)^3 + (22*b^3*c + 27*b*c^3)*cos(e*x + d))*sin(e*x + d) + 2*(
11*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (53*b^3 + 86*b*c^2)*cos(e*x + d) + (53*
b^2*c + 64*c^3 + 11*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2
+ c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*
x + d) - b*e*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.431 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx$

Optimal. Leaf size=190

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}-\frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+e$$

```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[Sqrt[b^2 + c
^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (16*Sqrt[b^2 + c^2]*(c*Cos[d + e*
x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]
])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```

Rubi [A] time = 0.122111, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3113, 3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}-\frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+e$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```


```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[Sqrt[b^2 + c
^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (16*Sqrt[b^2 + c^2]*(c*Cos[d + e*
x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]
])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```


Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(\right.}{5e}$$

$$= -\frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+e}}{15e}$$

$$= -\frac{64\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{16\sqrt{b^2+c^2}(c\cos}{}$$

Mathematica [C] time = 33.0913, size = 11771, normalized size = 61.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 1.45, size = 200, normalized size = 1.1

$$\frac{(2+2\sin(ex+d-\arctan(-b,c)))(\sin(ex+d-\arctan(-b,c))-1)\left(3(\sin(ex+d-\arctan(-b,c)))^2 b^2+3c^2(\sin(e\right.}{15\cos(ex+d-\mathrm{arc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] 2/15*(1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1
)*(3*sin(e*x+d-arctan(-b,c))^2*b^2+3*c^2*sin(e*x+d-arctan(-b,c))^2+14*b^2*s
in(e*x+d-arctan(-b,c))+14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos(e*
x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))
+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.95729, size = 463, normalized size = 2.44

$$\frac{2\left(3\left(b^{3}-3\,bc^{2}\right)\cos\left(ex+d\right)^{3}+\left(29\,b^{3}+38\,bc^{2}\right)\cos\left(ex+d\right)+\left(29\,b^{2}c+32\,c^{3}+3\left(3\,b^{2}c-c^{3}\right)\cos\left(ex+d\right)^{2}\right)\sin\left(ex+d\right.}{15\left(ce\cos\left(e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d) +
 (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) + (22*
b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 - 32
*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^
2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.432 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+}}$$

```
[Out] (-8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rubi [A] time = 0.0747549, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3113, 3112}

$$-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
```

```
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin}}{3e}$$

$$= -\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{2(c\cos(d+ex)-}{}$$

Mathematica [C] time = 21.9587, size = 11679, normalized size = 92.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 1.641, size = 126, normalized size = 1.

$$\frac{(2+2\sin(ex+d-\arctan(-b,c)))\left(b^2+c^2\right)(\sin(ex+d-\arctan(-b,c))-1)(\sin(ex+d-\arctan(-b,c))+5)}{3\cos(ex+d-\arctan(-b,c))e}\frac{}{\sqrt{(b^2 s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] 2/3*(1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)*(sin(e*x+d-arctan(-b,c))-1)*(sin(
e*x+d-arctan(-b,c))+5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c)
)+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.70591, size = 313, normalized size = 2.48

$$\frac{2\left(2\,bc\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(b^2-c^2\right)\cos\left(ex+d\right)^2-5\,b^2-4\,c^2+4\,\sqrt{b^2+c^2}\left(b\cos\left(ex+d\right)+c\sin\left(ex+d\right)\right)\right)\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+\sqrt{b^2+c^2}}}{3\left(ce\cos\left(ex+d\right)-be\sin\left(ex+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2 -
 4*c^2 + 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(e*
x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x
+ d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.433 $\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=55

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x]])
```

Rubi [A] time = 0.0332414, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] time = 21.9791, size = 11586, normalized size = 210.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.319, size = 113, normalized size = 2.1

$$2\,\frac{\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)\sqrt{b^2+c^2}\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)}{\cos\left(ex+d-\arctan\left(-b,c\right)\right)e}\,\frac{1}{\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-}{\sqrt{b^2+c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] 2*(1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1)/c
os(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-
b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.64086, size = 201, normalized size = 3.65

$$\frac{2\sqrt{b\cos(ex+d)+c\sin(ex+d)+\sqrt{b^2+c^2}}\left(b\cos(ex+d)+c\sin(ex+d)-\sqrt{b^2+c^2}\right)}{ce\cos(ex+d)-be\sin(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*(b*cos(e*x + d) +
 c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.434 $\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

```
[Out] (Sqrt[2]*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*S
qrt[Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b^2
 + c^2)^(1/4)*e)
```

Rubi [A] time = 0.119567, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*S
qrt[Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b^2
 + c^2)^(1/4)*e)
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\,dx$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2\sqrt{b^2+c^2}-x^2}\,dx, x, -\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{e}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{\sqrt[4]{b^2+c^2}\,e}$$

Mathematica [C] time = 33.8649, size = 63264, normalized size = 718.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.432, size = 172, normalized size = 2.

$$-\frac{(1+\sin(ex+d-\arctan(-b,c)))\sqrt{2}}{\cos(ex+d-\arctan(-b,c))\,e}\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] -(1+sin(e*x+d-arctan(-b,c)))*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))
^(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctanh(1/2*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arc
tan(-b,c))-1))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^2
*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/
2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(d+ex) + c\sin(d+ex) + \sqrt{b^2+c^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.435 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e\left(b^2+c^2\right)^{3/4}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

```
[Out] ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.132945, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3116, 3115, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e\left(b^2+c^2\right)^{3/4}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}+\frac{\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}+\frac{\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\mathrm{Sub}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e}-\frac{c\,\mathrm{co}}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2-\right.}}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 1.644, size = 350, normalized size = 2.2

$$-\frac{1}{4\cos\left(ex+d-\arctan\left(-b,c\right)\right)e}\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-\sqrt{b^2+c^2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] -1/4/(b^2+c^2)^(7/4)*(sin(e*x+d-arctan(-b,c))*arctanh(1/2*(-(b^2+c^2)^(1/2)
*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*2^
(1/2)*(b^2+c^2)+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c
))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctanh(1/2*
(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b
^2+c^2)^(1/4))*c^2+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1
/2))^(1/2)*(b^2+c^2)^(3/4))*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^
(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-a
rctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.436 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

Optimal. Leaf size=226

$$\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{}{4e\sqrt{b^2+}}$$

```
[Out] (3*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sq
rt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2]
*(b^2 + c^2)^(5/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^2
]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Cos[
d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.185823, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3116, 3115, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{}{4e\sqrt{b^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sq
rt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2]
*(b^2 + c^2)^(5/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^2
]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Cos[
d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
```

```
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} + \frac{3\int \frac{}{(\sqrt{}}}{}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{}{16\left(b^2\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{}{16\left(b^2\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{}{16\left(b^2\right.}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{16\sqrt{2}\left(b^2+c^2\right)^{5/4}e} - \frac{c\,\mathrm{co}}{4\sqrt{b^2+c^2}e\left(\sqrt{b^2\right.}}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 1.552, size = 350, normalized size = 1.6

$$\frac{1}{4\cos\left(ex+d-\arctan\left(-b,c\right)\right)e}\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-\sqrt{b^2+c^2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/4*(sin(e*x+d-arctan(-b,c))*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan
(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*2^(1/2)*(b^2+c^2)+2
^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2
))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)
*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*c^
2+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*(b^2+c
^2)^(3/4))*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^(1/2)/(b^2+c^2)^(
5/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-ar
ctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.437 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx$

Optimal. Leaf size=196

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}+\frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+$$

```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[-Sqrt[b^2 +
c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (16*Sqrt[b^2 + c^2]*(c*Cos[d + e
*x] - b*Sin[d + e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*
x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(-Sqrt[b^2 + c^2] + b*Co
s[d + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```

Rubi [A] time = 0.133782, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3113, 3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}+\frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```


```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[-Sqrt[b^2 +
c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (16*Sqrt[b^2 + c^2]*(c*Cos[d + e
*x] - b*Sin[d + e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*
x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(-Sqrt[b^2 + c^2] + b*Co
s[d + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```


Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}dx=-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}{5e}$$

$$=\frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+}}{15e}$$

$$=-\frac{64\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}+\frac{16\sqrt{b^2+c^2}(c}{}$$

Mathematica [C] time = 34.2309, size = 11602, normalized size = 59.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 1.753, size = 204, normalized size = 1.

$$\frac{(2\sin(ex+d-\arctan(-b,c))-2)(1+\sin(ex+d-\arctan(-b,c)))\left(3(\sin(ex+d-\arctan(-b,c)))^2b^2+3c^2(\sin(e\right.}{15\cos(ex+d-\mathrm{arc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] 2/15*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))
)*(3*sin(e*x+d-arctan(-b,c))^2*b^2+3*c^2*sin(e*x+d-arctan(-b,c))^2-14*b^2*s
in(e*x+d-arctan(-b,c))-14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos(e*
x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))
-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.89513, size = 463, normalized size = 2.36

$$\frac{2\left(3\left(b^{3}-3\,bc^{2}\right)\cos\left(ex+d\right)^{3}+\left(29\,b^{3}+38\,bc^{2}\right)\cos\left(ex+d\right)+\left(29\,b^{2}c+32\,c^{3}+3\left(3\,b^{2}c-c^{3}\right)\cos\left(ex+d\right)^{2}\right)\sin\left(ex+d\right.}{15\left(ce\cos\left(e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d) +
 (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - (22*
b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 - 32
*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^
2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.438 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

```
[Out] (8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[-Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rubi [A] time = 0.0807832, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3113, 3112}

$$\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[-Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
```

```
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$
$$= \frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{2(c\cos(d+ex)-}{}$$

Mathematica [C] time = 21.2547, size = 11512, normalized size = 88.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 1.591, size = 130, normalized size = 1.

$$\frac{(2\sin(ex+d-\arctan(-b,c))-2)\left(b^2+c^2\right)(1+\sin(ex+d-\arctan(-b,c)))(\sin(ex+d-\arctan(-b,c))-5)}{3\cos(ex+d-\arctan(-b,c))e}\frac{}{\sqrt{(b^2\,\mathrm{s}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] 2/3*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)*(1+sin(e*x+d-arctan(-b,c)))*(sin(
e*x+d-arctan(-b,c))-5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c)
)+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.88337, size = 313, normalized size = 2.41

$$\frac{2\left(2\,bc\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(b^2-c^2\right)\cos\left(ex+d\right)^2-5\,b^2-4\,c^2-4\,\sqrt{b^2+c^2}(b\cos\left(ex+d\right)+c\sin\left(ex+d\right))\right)\sqrt{b\cos}}{3\left(ce\cos\left(ex+d\right)-be\sin\left(ex+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2 -
 4*c^2 - 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(e*
x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x
+ d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.439 $\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=57

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d +
 e*x] + c*Sin[d + e*x]])
```

Rubi [A] time = 0.0381648, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d +
 e*x] + c*Sin[d + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] time = 21.5246, size = 11415, normalized size = 200.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.706, size = 117, normalized size = 2.1

$$2\,\frac{\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)\,(1+\sin(ex+d-\arctan(-b,c)))}{\cos(ex+d-\arctan(-b,c))\,e}\,\frac{1}{\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-}{\sqrt{b^2+c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] 2*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c)))/c
os(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-
b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.86122, size = 201, normalized size = 3.53

$$\frac{2\left(b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+\sqrt{b^{2}+c^{2}}\right)\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)-\sqrt{b^{2}+c^{2}}}}{ce\cos\left(ex+d\right)-be\sin\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) +
 c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^{2}+c^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.440 $\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

```
[Out] -((Sqrt[2]*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*
Sqrt[-Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b
^2 + c^2)^(1/4)*e))
```

Rubi [A] time = 0.0978455, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3115, 2649, 204}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*
Sqrt[-Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b
^2 + c^2)^(1/4)*e))
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx=\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\,dx$$

$$=-\frac{2\,\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2+c^2}-x^2}\,dx,x,-\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{e}$$

$$=-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{\sqrt[4]{b^2+c^2}e}$$

Mathematica [C] time = 33.8815, size = 61904, normalized size = 680.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.314, size = 175, normalized size = 1.9

$$\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{2}}{\cos(ex+d-\arctan(-b,c))\,e}\sqrt{-\sqrt{b^2+c^2}\,(1+\sin(ex+d-\arctan(-b,c)))}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-\sqrt{b^2+c^2}\,(1+\sin(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] (sin(e*x+d-arctan(-b,c))-1)*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^
(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctan(1/2*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arc
tan(-b,c))))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^2*s
in(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2)
)^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.441 $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=164

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e\left(b^2+c^2\right)^{3/4}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

```
[Out] ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.125628, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3116, 3115, 2649, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e\left(b^2+c^2\right)^{3/4}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} - \frac{\int}{}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} - \frac{\int}{}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} + \frac{\text{Su}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e} + \frac{c}{2\sqrt{b^2+c^2}e\left(-\sqrt{}\right.}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 1.934, size = 363, normalized size = 2.2

$$\frac{1}{4\cos\left(ex+d-\arctan\left(-b,c\right)\right)e}\left(-\sin\left(ex+d-\arctan\left(-b,c\right)\right)\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-\sqrt{b^2+c^2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] 1/4/(b^2+c^2)^(7/4)*(-sin(e*x+d-arctan(-b,c))*arctan(1/2*(-(b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*2^(
1/2)*(b^2+c^2)+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))
-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan(1/2*(-(
b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+
c^2)^(1/4))*c^2+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2)
)^(1/2)*(b^2+c^2)^(3/4))*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/
2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arct
an(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.442 $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

Optimal. Leaf size=232

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}+\frac{\quad}{4e\sqrt{b^2 \cdots}}$$

```
[Out] (-3*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-S
qrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2
]*(b^2 + c^2)^(5/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^
2]*e*(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Co
s[d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(-Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))
```

Rubi [A] time = 0.170474, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3116, 3115, 2649, 204}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}+\frac{\quad}{4e\sqrt{b^2 \cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-S
qrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2
]*(b^2 + c^2)^(5/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^
2]*e*(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Co
s[d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(-Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
```

```
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{3\int \cdots}{\cdots}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{\cdots}{16\left(b\cdots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{\cdots}{16\left(b\cdots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{\cdots}{16\left(b\cdots\right.}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{16\sqrt{2}\left(b^2+c^2\right)^{5/4}e} + \frac{\cdots}{4\sqrt{b^2+c^2}e\left(-\cdots\right.}$$

Mathematica [F] time = 180.014, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 2.082, size = 363, normalized size = 1.6

$$-\frac{1}{4\cos\left(ex+d-\arctan\left(-b,c\right)\right)e}\left(-\sin\left(ex+d-\arctan\left(-b,c\right)\right)\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-\sqrt{b^2+c^2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] -1/4*(-sin(e*x+d-arctan(-b,c))*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arcta
n(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*2^(1/2)*(b^2+c^2)+
2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2
))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*c^2
+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*(b^2+c^
2)^(3/4))*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/2)/(b^2+c^2)^(5
/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arc
tan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.443 $\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=101

$$-\frac{2ac\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

```
[Out] (c*x)/(b^2 + c^2) - (2*a*c*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c
^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[a + b*Cos[x] + c*Sin[x]]
)/(b^2 + c^2)
```

Rubi [A] time = 0.0964729, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3137, 3124, 618, 204}

$$-\frac{2ac\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*x)/(b^2 + c^2) - (2*a*c*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c
^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[a + b*Cos[x] + c*Sin[x]]
)/(b^2 + c^2)
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx &= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(ac)\int\frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\
&= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(2ac)\operatorname{Subst}\left(\int\frac{1}{a+b+2cx+(a-b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2+c^2}\\
&= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{(4ac)\operatorname{Subst}\left(\int\frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx,x,2c+2(a-\right.}{b^2+c^2}\\
&= \frac{cx}{b^2+c^2}-\frac{2ac\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}
\end{aligned}$$

Mathematica [A] time = 0.224024, size = 80, normalized size = 0.79

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}-b\log(a+b\cos(x)+c\sin(x))+cx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*x + (2*a*c*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[
-a^2 + b^2 + c^2] - b*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [B] time = 0.048, size = 438, normalized size = 4.3

$$-2\frac{\ln\left(a(\tan(x/2))^2-b(\tan(x/2))^2+2c\tan(x/2)+a+b\right)ab}{(2b^2+2c^2)(a-b)}+2\frac{\ln\left(a(\tan(x/2))^2-b(\tan(x/2))^2+2c\tan(x/2)+a\right.}{(2b^2+2c^2)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] -2/(2*b^2+2*c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)
*a*b+2/(2*b^2+2*c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+
a+b)*b^2-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*a*c-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1
/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c*b+4/(2*b^2+2*c^2)/(a^2-b
^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a
-b)*a*b-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+
2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2+2/(2*b^2+2*c^2)*b*ln(1+tan(1/2*x)^2)+
4/(2*b^2+2*c^2)*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14762, size = 1291, normalized size = 12.78

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\,ac\log\left(\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(x)^2-2(ab^3+abc^2)\cos(x)-2(ab^2c+ac^3-(bc^3-(2a^2b-b^3)c)\cos(x))\sin}{2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2 + c^2)*a*c*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2
)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*
cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) -
2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2
- a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2
)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(c^3 - (a^2 - b^
2)*c)*x + (a^2*b - b^3 - b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a
^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^
2)*c^2), -1/2*(2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-(a*b*cos(x) + a*c*sin(x)
 + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b
- b^3 - b*c^2)*sin(x))) + 2*(c^3 - (a^2 - b^2)*c)*x + (a^2*b - b^3 - b*c^2)
*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)
*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15365, size = 216, normalized size = 2.14

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)ac}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}+\frac{cx}{b^2+c^2}-\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\right.}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*a*c/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2)
) + c*x/(b^2 + c^2) - b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) + b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2)
```

3.444 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=22

$$\frac{x}{2}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

```
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

Rubi [A] time = 0.0306626, antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3137, 3124, 31}

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]

[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\int \frac{1}{1+\cos(x)+\sin(x)}\,dx$$
$$= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\text{Subst}\left(\int \frac{1}{2+2x}\,dx,x,\tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

Mathematica [A] time = 0.047012, size = 22, normalized size = 1.

$$\frac{x}{2}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.043, size = 25, normalized size = 1.1

$$-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{1}{2}\ln\left(1+\left(\tan\left(\frac{x}{2}\right)\right)^2\right)+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] -ln(1+tan(1/2*x))+1/2*ln(1+tan(1/2*x)^2)+1/2*x
```

Maxima [B] time = 1.48185, size = 55, normalized size = 2.5

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)
^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 1.86841, size = 39, normalized size = 1.77

$$\frac{1}{2}x - \frac{1}{2}\log\left(\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

Sympy [A] time = 0.31174, size = 22, normalized size = 1.

$$\frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right) + 1\right) + \frac{\log\left(\tan^{2}\left(\frac{x}{2}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

Giac [A] time = 1.1655, size = 34, normalized size = 1.55

$$\frac{1}{2}x + \frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^{2} + 1\right) - \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

3.445 $\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=97

$$\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) + (b*Log[c + a*Cos[x] + b*Sin[x]
])/(a^2 + b^2)
```

Rubi [A] time = 0.127024, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3159, 3138, 3124, 618, 206}

$$\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```


```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) + (b*Log[c + a*Cos[x] + b*Sin[x]
])/(a^2 + b^2)
```


Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
```

```
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx &= \int \frac{\cos(x)}{c+a\cos(x)+b\sin(x)}\,dx\\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{(ac)\int \frac{1}{c+a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{(2ac)\,\mathrm{Subst}\left(\int \frac{1}{a+c+2bx+(-a+c)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}+\frac{(4ac)\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx,x,2b+2(-a-\right.}{a^2+b^2}\\
&= \frac{ax}{a^2+b^2}+\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.192824, size = 79, normalized size = 0.81

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{(c-a)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}+b\log(a\cos(x)+b\sin(x)+c)+ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] (a*x + (2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[
a^2 + b^2 - c^2] + b*Log[c + a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

Maple [B] time = 0.064, size = 414, normalized size = 4.3

$$\frac{ab}{\left(a^2+b^2\right)(a-c)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2-\left(\tan\left(\frac{x}{2}\right)\right)^2c-2b\tan(x/2)-a-c\right)-\frac{cb}{\left(a^2+b^2\right)(a-c)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2-\left(\tan\left(\frac{x}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] 1/(a^2+b^2)/(a-c)*ln(a*tan(1/2*x)^2-tan(1/2*x)^2*c-2*b*tan(1/2*x)-a-c)*a*b-
1/(a^2+b^2)/(a-c)*ln(a*tan(1/2*x)^2-tan(1/2*x)^2*c-2*b*tan(1/2*x)-a-c)*c*b+
2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-
b^2+c^2)^(1/2))*a*c-2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*ta
n(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2+2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*ar
ctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2/(a-c)*a-2/(a^2+
b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2
)^(1/2))*b^2/(a-c)*c-1/(a^2+b^2)*b*ln(1+tan(1/2*x)^2)+2/(a^2+b^2)*a*arctan(
tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56773, size = 1277, normalized size = 13.16

$$\left[\frac{\sqrt{a^2+b^2-c^2}ac\log\left(\frac{2\,a^4+3\,a^2b^2+b^4-\left(a^2-b^2\right)c^2+2\left(a^3+ab^2\right)c\cos(x)-\left(a^4-b^4-2\left(a^2-b^2\right)c^2\right)\cos(x)^2+2\left(\left(a^2b+b^3\right)c-\left(a^3b+ab^3-2\,abc^2\right)\cos(x)\right)\sin(x)+2\,(2}{2\,ac\cos(x)+\left(a^2-b^2\right)\cos(x)^2+b^2+c^2+2\,(ab\cos(x)+bc)\,\mathrm{si}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*
c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 +
 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b
*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*c
os(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2
+ b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)*x + (
a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 +
2*(a*b*cos(x) + b*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1
/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) + b*c*sin(x) + a^2 + b^
2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(x) - (a^3 + a*b^2 - a*
c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x - (a^2*b + b^3 - b*c^2)*log(2*a*c
*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)))/
(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\tan{(x)} + c\sec{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(1/(a + b*tan(x) + c*sec(x)), x)
```

Giac [A] time = 1.17449, size = 213, normalized size = 2.2

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2c\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\left(a^2+b^2\right)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2}+\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+\right.}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*ta
n(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c
^2)) + a*x/(a^2 + b^2) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1
/2*x) + a + c)/(a^2 + b^2) - b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

3.446 $\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

```
[Out] (-2*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rubi [A] time = 0.0760878, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3165, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx &= \int \frac{1}{c+a\cos(x)+b\sin(x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{a+c+2bx+(-a+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx, x, 2b+2(-a+c)\tan\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}\end{aligned}$$

Mathematica [A] time = 0.0450145, size = 50, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(c-a)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
- c^2]
```

Maple [A] time = 0.059, size = 53, normalized size = 1.

$$-2\,\frac{1}{\sqrt{-a^2-b^2+c^2}}\arctan\left(1/2\,\frac{2\,(a-c)\tan(x/2)-2\,b}{\sqrt{-a^2-b^2+c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] -2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14003, size = 799, normalized size = 15.67

$$\left[\frac{\log\left(-\frac{2a^4+3a^2b^2+b^4-\left(a^2-b^2\right)c^2+2\left(a^3+ab^2\right)c\cos(x)-\left(a^4-b^4-2\left(a^2-b^2\right)c^2\right)\cos(x)^2+2\left(\left(a^2b+b^3\right)c-\left(a^3b+ab^3-2abc^2\right)\cos(x)\right)\sin(x)-2\left(2abc\cos(x)^2-abc+(a \cdots\right.}{2ac\cos(x)+\left(a^2-b^2\right)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)}\right)}{2\sqrt{a^2+b^2-c^2}} \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*c^2 + 2*(a^3 + a*b^2)*c*co
s(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^2*b + b^3)*c - (a^3
*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^2
*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 +
b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x
) + b*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((a*c
*cos(x) + b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*
c^2)*cos(x) - (a^3 + a*b^2 - a*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)}{a+b\tan(x)+c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(sec(x)/(a + b*tan(x) + c*sec(x)), x)
```

Giac [A] time = 1.16505, size = 99, normalized size = 1.94

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2c)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2*c) + arctan((a*tan(1/2*x) - c*tan(
1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

3.447 $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=142

$$-\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log\left(-(a-c)\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)+c\right)}{b^2-c^2}-\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{b-c}$$

```
[Out] (-2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) - Log[1 - Tan[x/2]]/(b + c) - Log[1 + Tan[x/2]]/(b
- c) + (b*Log[a + c + 2*b*Tan[x/2] - (a - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rubi [A] time = 0.514689, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4397, 1075, 634, 618, 206, 628, 633, 31}

$$-\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log\left(-(a-c)\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)+c\right)}{b^2-c^2}-\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) - Log[1 - Tan[x/2]]/(b + c) - Log[1 + Tan[x/2]]/(b
- c) + (b*Log[a + c + 2*b*Tan[x/2] - (a - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx = \int \frac{\sec(x)}{c+a\cos(x)+b\sin(x)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)\left(a+c+2bx-(a-c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{4c-4bx}{1-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{2\left(b^2-c^2\right)} - \frac{\text{Subst}\left(\int \frac{-4b^2+(-a+c)^2-(a+c)^2-4b(-a+c)x}{a+c+2bx+(-a+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{2\left(b^2-c^2\right)}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b+c} + \frac{b\,\text{Subst}\left(\int \frac{2b+2(-a+c)x}{a+c+2bx+(-a+c}\right.}{b^2-c^2}$$
$$= -\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{b\log\left(a+c+2b\tan\left(\frac{x}{2}\right)-(a-c)\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2} -$$
$$= -\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}} - \frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{b\log\left(a+c+2b\right.}{}$$

Mathematica [A] time = 0.289371, size = 120, normalized size = 0.85

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{(c-a)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} - b\log(a\cos(x)+b\sin(x)+c) + (b-c)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + (b+c)\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{(c-b)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] ((2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
b^2 - c^2] + (b - c)*Log[Cos[x/2] - Sin[x/2]] + (b + c)*Log[Cos[x/2] + Sin[
x/2]] - b*Log[c + a*Cos[x] + b*Sin[x]])/((-b + c)*(b + c))
```

Maple [B] time = 0.059, size = 430, normalized size = 3.

$$\frac{ab}{(b-c)(b+c)(a-c)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2-\left(\tan\left(\frac{x}{2}\right)\right)^2c-2b\tan(x/2)-a-c\right)-\frac{cb}{(b-c)(b+c)(a-c)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2-\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] 1/(b-c)/(b+c)/(a-c)*ln(a*tan(1/2*x)^2-tan(1/2*x)^2*c-2*b*tan(1/2*x)-a-c)*a*
b-1/(b-c)/(b+c)/(a-c)*ln(a*tan(1/2*x)^2-tan(1/2*x)^2*c-2*b*tan(1/2*x)-a-c)*
c*b-2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/
(-a^2-b^2+c^2)^(1/2))*a*c-2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*
(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2+2/(b-c)/(b+c)/(-a^2-b^2+c^2
)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2/(a-c)
*a-2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(
-a^2-b^2+c^2)^(1/2))*b^2/(a-c)*c-2/(-2*c+2*b)*ln(1+tan(1/2*x))-2/(2*b+2*c)*
ln(tan(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 21.9246, size = 1539, normalized size = 10.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)
*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
+ 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*
cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2
 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) - (a^2*b + b^3 - b*c^2)*log(2*
a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)
) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) + (a^2*b +
b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 -
```

```
(a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) +
 b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(
x) - (a^3 + a*b^2 - a*c^2)*sin(x))) + (a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x
) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)) - (a^2*
b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) - (a^2*b + b^3 - b*c
^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 - (a^2 + 2
*b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{a + b\tan(x) + c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(a + b*tan(x) + c*sec(x)), x)
```

Giac [A] time = 1.23609, size = 217, normalized size = 1.53

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2c\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}\left(b^2-c^2\right)}+\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2\,b\tan\left(\frac{1}{2}x\right)\right.}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*tan
(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 - c^
2)) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a + c)/(b^2
 - c^2) - log(abs(tan(1/2*x) + 1))/(b - c) - log(abs(tan(1/2*x) - 1))/(b +
c)
```

3.448 $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,c)+d+ex\right),\frac{2\sqrt{a^2}}{\sqrt{a^2+}}\right.}{3e\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2}$$

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])
^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/
2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d + e*x
- ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*C
os[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] +
c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rubi [A] time = 0.446565, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3167, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2}+\frac{8b}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])
^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/
2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d + e*x
- ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*C
```

```
os[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] +
c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}dx = \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\int(b+a\cos(d+ex)+c\sin(d+ex))}{\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))} +$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))} +$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))} +$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))} +$$

Mathematica [C] time = 6.45211, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x]
```

```
[Out] (((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*a*Sin[d + e*x])/3)*(a + b*Sec[d
 + e*x] + c*Tan[d + e*x])^(3/2))/(e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x]
+ c*Sin[d + e*x])) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^
2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^
2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqr
t[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c
]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + Ar
cTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 +
c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*
x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (2*b^2*AppellF1[1/2
, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sq
rt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]
*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2
]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqr
t[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2]
)]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqr
t[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-
b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/
(Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e
*x])^(3/2)) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*
Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)
)*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(
c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]
])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d +
e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d +
 e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*e*Sec[d + e*x]^(3/2)*(b
 + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*a^2*b*(-((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*
Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^
2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2
/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a
^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a
*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - Arc
Tan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d +
e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1
 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x
- ArcTan[c/a]]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*c*e*Sec[d
```

```
+ e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*b*c*(-((c*Ap
pellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcT
an[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sq
rt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a
*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sq
rt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[
c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[
d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/
a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*
(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[
d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]
*Cos[d + e*x - ArcTan[c/a]]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/
(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Maple [C] time = 2.027, size = 21265, normalized size = 57.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}}{\sec(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}}{\sec(ex+d)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2)/sec(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}}{\sec(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x
)
```

3.449 $\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rubi [A] time = 0.14404, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3167, 3119, 2653}

$$\frac{2\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```


```
[Out] (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```


Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx &= \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\,dx}{\sqrt{\sec(d+ex)}\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}\\
&= \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}\,dx}{\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}\\
&= \frac{2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{e\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}
\end{aligned}$$

Mathematica [C] time = 6.25379, size = 1580, normalized size = 13.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```

```
[Out] (2*a*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(c*e*Sqrt[Sec[d + e*x]]) +
(2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b +
Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/
(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 +
c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt
```

```
[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a
/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x +
 ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[a + b*Sec[d + e*x] + c*
Tan[d + e*x]])/(Sqrt[1 + a^2/c^2]*c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Cos[d +
 e*x] + c*Sin[d + e*x]]) + (a^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b
+ a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 -
 b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTa
n[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x
 - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqr
t[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2]
)]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqr
t[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-
b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*
x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
 c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]])*Sqrt[
a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Co
s[d + e*x] + c*Sin[d + e*x]]) + (c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(
1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - Ar
cTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d +
e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*
Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a
^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*
Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])
/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d +
 e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[
1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]])*Sq
rt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*C
os[d + e*x] + c*Sin[d + e*x]])
```

Maple [C] time = 0.666, size = 12462, normalized size = 105.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}{\sqrt{\sec(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}{\sqrt{\sec(ex+d)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2)/sec(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))/sqrt(sec(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}{\sqrt{\sec(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

3.450 $\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,c)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right)}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[Sec[d + e*x]]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Rubi [A] time = 0.166285, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3167, 3127, 2661}

$$\frac{2\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```


```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[Sec[d + e*x]]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```


Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
```

```
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{\left(\sqrt{\sec(d+ex)}\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\right)\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$= \frac{\left(\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right)\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}}\,dx}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Mathematica [C] time = 0.91945, size = 339, normalized size = 2.87

$$\frac{2\sqrt{\sec(d+ex)}\sec\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)\sqrt{-\frac{c\sqrt{\frac{a^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)-1\right)}{c\sqrt{\frac{a^2}{c^2}+1}+b}}\sqrt{\frac{c\sqrt{\frac{a^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)+1\right)}{c\sqrt{\frac{a^2}{c^2}+1}-b}}\sqrt{c\sqrt{\frac{a^2}{c^2}+1}\sin\left(\tan\right.}}{ce\sqrt{\frac{a^2}{c^2}+1}\sqrt{a+b\sec}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcT
an[a/c]])/(b - Sqrt[1 + a^2/c^2]*c), (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
 ArcTan[a/c]])/(b + Sqrt[1 + a^2/c^2]*c)]*Sqrt[Sec[d + e*x]]*Sec[d + e*x +
ArcTan[a/c]]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]*Sqrt[-((Sqrt[1 + a^2
/c^2]*c*(-1 + Sin[d + e*x + ArcTan[a/c]]))/(b + Sqrt[1 + a^2/c^2]*c))]*Sqrt
[(Sqrt[1 + a^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[a/c]]))/(-b + Sqrt[1 + a^2/
```

```
c^2]*c)]*Sqrt[b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]]])/(Sqrt[1
+ a^2/c^2]*c*e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Maple [C] time = 0.596, size = 722, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x)
```

```
[Out] -4*I/e/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)*EllipticF(((I*a-I*b-(a^2-b^2+c^2)^(1
/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(I*sin(e*x+d)+cos(e*x+d)))^(1/2),((I
*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2
-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2))*(1/cos(e*x+d))^(
1/2)*((b+a*cos(e*x+d)+c*sin(e*x+d))/cos(e*x+d))^(1/2)*((I*a-I*b-(a^2-b^2+c^
2)^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(I*sin(e*x+d)+cos(e*x+d)))^(1/2
)*(-I/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-c)*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-a*sin
(e*x+d)+b*sin(e*x+d)+c*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)+c)/(I*cos(e*x+d)+I+si
n(e*x+d)))^(1/2)*(I/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(a*sin(e*x+d)-b*sin(e*x
+d)+cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)-c)/(I*c
os(e*x+d)+I+sin(e*x+d)))^(1/2)*(cos(e*x+d)+1)^2*cos(e*x+d)*(cos(e*x+d)-1)^2
*(I*a*cos(e*x+d)-I*cos(e*x+d)*b-I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*c*sin(e*
x+d)+cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+a*sin(e*x+d)-b*sin(e*x+d))
/sin(e*x+d)^4/(b+a*cos(e*x+d)+c*sin(e*x+d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(ex+d\right)}}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec(ex+d)}}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(d + e*x))/sqrt(a + b*sec(d + e*x) + c*tan(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sec(ex+d)}}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

3.451 $\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-}{e\left(a^2-b^2+c^2\right)(a}$$

```
[Out] (-2*Sec[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e
*x])^2)/((a^2 - b^2 + c^2)*e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Rubi [A] time = 0.216318, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3167, 3128, 3119, 2653}

$$-\frac{2\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-}{e\left(a^2-b^2+c^2\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```


```
[Out] (-2*Sec[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e
*x])^2)/((a^2 - b^2 + c^2)*e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```


Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
```

```
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \frac{\left(\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}\right)\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex}}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+}{\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+}{\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+}{\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

Mathematica [C] time = 6.39843, size = 1732, normalized size = 7.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] (Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*((-2*(a^2 + c^2
))/(a*c*(a^2 - b^2 + c^2)) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d + e*x])
)/(a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1
- b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Ar
cTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d +
e*x]^(3/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
^(3/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x
+ ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/
c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^
2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])/
(Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*
x])^(3/2)) - (a^2*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^
(3/2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[
d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))
), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a
```

```
^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2])))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1
 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d +
 e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 +
c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqr
t[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2
])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 +
c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqr
t[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/(c*(a^2 - b^2 + c^2)*e*(a + b*
Sec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (c*Sec[d + e*x]^(3/2)*(b + a*Cos[d
+ e*x] + c*Sin[d + e*x])^(3/2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b +
 a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 -
b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan
[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2])))))]*Sin[d + e*x
- ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt
[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])
]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt
[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b
 + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x
 - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/((a^2
 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Maple [C] time = 0.589, size = 12572, normalized size = 52.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(ex+d)^{\frac{3}{2}}}{(b\sec(ex+d) + c\tan(ex+d) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(e*x + d)^(3/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\sec\left(ex+d\right)^{\frac{3}{2}}}{b^{2}\sec\left(ex+d\right)^{2}+c^{2}\tan\left(ex+d\right)^{2}+2\,ab\sec\left(ex+d\right)+a^{2}+2\,\left(bc\sec\left(ex+d\right)+ac\right)\tan\left(ex+d\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sec(e*x + d)^(3/2)/(b^2*
sec(e*x + d)^2 + c^2*tan(e*x + d)^2 + 2*a*b*sec(e*x + d) + a^2 + 2*(b*c*sec
(e*x + d) + a*c)*tan(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(ex+d\right)^{\frac{3}{2}}}{\left(b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(e*x + d)^(3/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2), x
)
```

3.452 $\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$\frac{2\sec^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a\cos(d+ex)+b+c\sin(d+ex))^2\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,c)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

```
[Out] (-2*Sec[d + e*x]^(5/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*Sec[d + e*x]^(5/2)*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])
*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a
*Cos[d + e*x] + c*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Sqrt[(b + a*Cos
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*
Sin[d + e*x])^2*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 +
c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Rubi [A] time = 0.519064, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3167, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sec^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8b\sec^{\frac{5}{2}}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x]
```

```
[Out] (-2*Sec[d + e*x]^(5/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*Sec[d + e*x]^(5/2)*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])
*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
```

```
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a
*Cos[d + e*x] + c*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Sqrt[(b + a*Cos
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*
Sin[d + e*x])^2*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 +
c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \frac{\left(\sec^{\frac{5}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}\right)\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+}}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

Mathematica [C] time = 6.51367, size = 2708, normalized size = 5.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x]
```

```
[Out] (Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^3*((8*b*(a^2 + c^
2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d +
 e*x]))/(3*a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2) - (
2*(a^2*c + 3*b^2*c + c^3 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3
*a*(a^2 - b^2 + c^2)^2*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(
1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d
```

```
+ e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x
])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*
x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2
)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(
a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]
)/(3*Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[
d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/
c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/
c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[
1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x]^(5/2)*Sec[d +
 e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqr
t[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b
 + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x +
 ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[
d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])/(Sqrt[1 + a^2/c^2]
*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*
c*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Arc
Tan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqr
t[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sq
rt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*S
qrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^
2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*S
qrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/
(-b + c*Sqrt[(a^2 + c^2)/c^2])])/(3*Sqrt[1 + a^2/c^2]*(a^2 - b^2 + c^2)^2*e
*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (4*a^2*b*Sec[d + e*x]^(5/2)
*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*(-((c*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d
+ e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))
]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)
/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^
2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]
]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - Arc
Tan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^
2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]]
)/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[
c/a]]]))/(3*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(
5/2)) + (4*b*c*Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/
2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d +
 e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))),
-((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]
*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*
x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2
```

```
)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(
a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])]
)) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2
) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1
 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/(3*(a^2 - b^2 + c^2)^2*e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Maple [C] time = 1.75, size = 63949, normalized size = 130.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(ex+d)^{\frac{5}{2}}}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(e*x + d)^(5/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sec(ex}{b^3\sec(ex+d)^3+c^3\tan(ex+d)^3+3\,ab^2\sec(ex+d)^2+3\,a^2b\sec(ex+d)+a^3+3\left(bc^2\sec(ex+d)+ac^2\right)\mathrm{t}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sec(e*x + d)^(5/2)/(b^3*
sec(e*x + d)^3 + c^3*tan(e*x + d)^3 + 3*a*b^2*sec(e*x + d)^2 + 3*a^2*b*sec(
e*x + d) + a^3 + 3*(b*c^2*sec(e*x + d) + a*c^2)*tan(e*x + d)^2 + 3*(b^2*c*s
ec(e*x + d)^2 + 2*a*b*c*sec(e*x + d) + a^2*c)*tan(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.453 $\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,c)+c\right.\right.}{3e(a\cos(d+ex)+b+c\sin(d+ex))^2}$$

```
[Out] (-2*Cos[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x
] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*Cos[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3
*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*Cos[d + e*x]^(3/2)*
EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c
^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a
+ b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rubi [A] time = 0.389821, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3163, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{}}{b+}\right.}{3e(a\cos(d+ex)+b+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```


```
[Out] (-2*Cos[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x
] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*Cos[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3
*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*Cos[d + e*x]^(3/2)*
EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c
^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a
```

```
+ b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx = \frac{\left(\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\right)\int(b+a\,\text{c}}{(b+a\cos(d+ex)+c\sin(d+ex}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+}{3e(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+}{3e(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+}{3e(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+}{3e(b+a\cos(d+ex)+c\sin(d+ex))}$$

Mathematica [F] time = 151.125, size = 0, normalized size = 0.

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2), x
]
```

Maple [C] time = 0.709, size = 21015, normalized size = 56.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(ex+d) + c\tan(ex+d) + a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left((b\cos(ex+d)\sec(ex+d) + c\cos(ex+d)\tan(ex+d) + a\cos(ex+d))\sqrt{b\sec(ex+d) + c\tan(ex+d) + a}\sqrt{\mathrm{c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((b*cos(e*x + d)*sec(e*x + d) + c*cos(e*x + d)*tan(e*x + d) + a*cos
(e*x + d))*sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)**(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(ex+d) + c\tan(ex+d) + a)^{\frac{3}{2}} \cos(ex+d)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2), x
)
```

3.454 $\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] (2*Sqrt[Cos[d + e*x]]*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rubi [A] time = 0.146648, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3163, 3119, 2653}

$$\frac{2\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*Sqrt[Cos[d + e*x]]*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \frac{\left(\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\right)\int \sqrt{b+a\cos \ldots}}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}$$

$$= \frac{\left(\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\right)\int \sqrt{\frac{b}{b+\sqrt{a^2+c \ldots}}}}{\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

$$= \frac{2\sqrt{\cos(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec \ldots}}{e\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [F] time = 21.2875, size = 0, normalized size = 0.

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]], x]
```

Maple [C] time = 0.444, size = 12460, normalized size = 105.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\sqrt{\cos(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)**(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

3.455 $\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,c)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right)}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Rubi [A] time = 0.151947, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3163, 3127, 2661}

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
```

```
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}\,dx}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$= \frac{\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}}\,dx}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Mathematica [C] time = 2.94404, size = 506, normalized size = 4.29

$$\frac{4\left(\sqrt{a^2-b^2+c^2}+ia-ib+c\right)(\cos(d+ex)+i\sin(d+ex))\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)-c\right)}{\left(\sqrt{a^2-b^2+c^2}-ia+ib-c\right)\left(\tan\left(\frac{1}{2}(d+ex)\right)-i\right)}}\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}+(b-a)\tan\left(\right.\right.}{\left(\sqrt{a^2-b^2+c^2}+ia-ib+c\right)\left(\tan\right.}}}{e\left(a+i\left(\sqrt{a^2-b^2+c^2}+ib+c\right)\right)\sqrt{\cos}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),
x]
```

```
[Out] (4*(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[(((-I)*a +
 I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(I*a -
I*b + c + Sqrt[a^2 - b^2 + c^2])]], (b + I*Sqrt[a^2 - b^2 + c^2])/(b - I*Sq
rt[a^2 - b^2 + c^2])]*Sqrt[(((-I)*a + I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Co
s[d + e*x] + I*Sin[d + e*x]))/(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])]*(Cos
```

```
[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(-c + Sqrt[a^2 - b^2 + c^2] + (a - b
)*Tan[(d + e*x)/2]))/(((-I)*a + I*b - c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[
(d + e*x)/2]))]*Sqrt[((-I)*(c + Sqrt[a^2 - b^2 + c^2] + (-a + b)*Tan[(d + e
*x)/2]))/((I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))]
)/((a + I*(I*b + c + Sqrt[a^2 - b^2 + c^2]))*e*Sqrt[Cos[d + e*x]]*Sqrt[a +
b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Maple [C] time = 0.415, size = 714, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x)
```

```
[Out] 4*I/e/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)*((b+a*cos(e*x+d)+c*sin(e*x+d))/cos(e*
x+d))^(1/2)*((I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c
)*(I*sin(e*x+d)+cos(e*x+d)))^(1/2)*(-I/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-c)*(cos
(e*x+d)*(a^2-b^2+c^2)^(1/2)-a*sin(e*x+d)+b*sin(e*x+d)+c*cos(e*x+d)+(a^2-b^2
+c^2)^(1/2)+c)/(I*cos(e*x+d)+I+sin(e*x+d)))^(1/2)*(I/(I*a-I*b+(a^2-b^2+c^2)
^(1/2)-c)*(a*sin(e*x+d)-b*sin(e*x+d)+cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e
*x+d)+(a^2-b^2+c^2)^(1/2)-c)/(I*cos(e*x+d)+I+sin(e*x+d)))^(1/2)*(cos(e*x+d)
+1)^2*EllipticF(((I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/
2)-c)*(I*sin(e*x+d)+cos(e*x+d)))^(1/2),((I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(I*
a-I*b+(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2-
b^2+c^2)^(1/2)-c))^(1/2))*cos(e*x+d)^(1/2)*(cos(e*x+d)-1)^2*(I*(a^2-b^2+c^2
)^(1/2)*sin(e*x+d)-I*a*cos(e*x+d)+I*cos(e*x+d)*b-I*c*sin(e*x+d)-cos(e*x+d)*
(a^2-b^2+c^2)^(1/2)+c*cos(e*x+d)-a*sin(e*x+d)+b*sin(e*x+d))/sin(e*x+d)^4/(b
+a*cos(e*x+d)+c*sin(e*x+d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\sqrt{\cos\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\sqrt{\cos\left(ex+d\right)}}{b\cos\left(ex+d\right)\sec\left(ex+d\right)+c\cos\left(ex+d\right)\tan\left(ex+d\right)+a\cos\left(ex+d\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b*co
s(e*x + d)*sec(e*x + d) + c*cos(e*x + d)*tan(e*x + d) + a*cos(e*x + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+b\sec\left(d+ex\right)+c\tan\left(d+ex\right)}\sqrt{\cos\left(d+ex\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x))),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\sqrt{\cos\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))),
 x)
```

3.456 $\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2(a\cos(d+ex)+b+c\sin(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2(c\cos(d+ex)-a\sin(d}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+}$$

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/((a^2 - b^2 + c^2)*e*Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/((a^2 - b^2
 + c^2)*e*Cos[d + e*x]^(3/2)*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Rubi [A] time = 0.211226, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3163, 3128, 3119, 2653}

$$-\frac{2(a\cos(d+ex)+b+c\sin(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2(c\cos(d+ex)-a\sin(d}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)),x]
```


```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/((a^2 - b^2 + c^2)*e*Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/((a^2 - b^2
 + c^2)*e*Cos[d + e*x]^(3/2)*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```


Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
```

```
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex)}}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{\left(a^2-b^2+c^2\right)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{\left(a^2-b^2+c^2\right)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d}{\left(a^2-b^2+c^2\right)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+}$$

Mathematica [F] time = 24.5685, size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)
),x]
```

```
[Out] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)
), x]
```

Maple [C] time = 0.44, size = 12562, normalized size = 52.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}}{b^2\cos(ex+d)^2\sec(ex+d)^2+c^2\cos(ex+d)^2\tan(ex+d)^2+2\,ab\cos(ex+d)^2\sec(ex+d)+a^2\cos(ex+d)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b^2*
cos(e*x + d)^2*sec(e*x + d)^2 + c^2*cos(e*x + d)^2*tan(e*x + d)^2 + 2*a*b*c
os(e*x + d)^2*sec(e*x + d) + a^2*cos(e*x + d)^2 + 2*(b*c*cos(e*x + d)^2*sec
(e*x + d) + a*c*cos(e*x + d)^2)*tan(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2)
), x)
```

3.457 $\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a\cos(d+ex)+b+c\sin(d+ex))^2\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(a,c)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right)}{3e\left(a^2-b^2+c^2\right)\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8\ldots}{3e\left(a^2-b\ldots\right.}$$

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/(3*(a^2 - b^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*S
in[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*Sqrt[(b + a*Cos
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*Sqr
t[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b
^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Rubi [A] time = 0.494709, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3163, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8b(a\cos(d+\ldots}{3e\left(a^2-b^2+c^2\right)^2\cos\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/(3*(a^2 - b^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
```

```
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*S
in[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*Sqrt[(b + a*Cos
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*Sqr
t[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b
^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex)}}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d}$$

Mathematica [F] time = 28.2006, size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)
),x]
```

```
[Out] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)
), x]
```

Maple [C] time = 1.151, size = 63939, normalized size = 130.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{5}{2}}\cos(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2)*cos(e*x + d)^(5/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{b^3\cos(ex+d)^3\sec(ex+d)^3+c^3\cos(ex+d)^3\tan(ex+d)^3+3\,ab^2\cos(ex+d)^3\sec(ex+d)^2+3\,a^2b\cos(ex+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b^3*
cos(e*x + d)^3*sec(e*x + d)^3 + c^3*cos(e*x + d)^3*tan(e*x + d)^3 + 3*a*b^2
*cos(e*x + d)^3*sec(e*x + d)^2 + 3*a^2*b*cos(e*x + d)^3*sec(e*x + d) + a^3*
cos(e*x + d)^3 + 3*(b*c^2*cos(e*x + d)^3*sec(e*x + d) + a*c^2*cos(e*x + d)^
3)*tan(e*x + d)^2 + 3*(b^2*c*cos(e*x + d)^3*sec(e*x + d)^2 + 2*a*b*c*cos(e*
x + d)^3*sec(e*x + d) + a^2*c*cos(e*x + d)^3)*tan(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{5}{2}}\cos(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2)*cos(e*x + d)^(5/2)
), x)
```

3.458 $\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=98

$$\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) - (b*Log[c + b*Cos[x] + a*Sin[x]
])/(a^2 + b^2)
```

Rubi [A] time = 0.102696, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3160, 3137, 3124, 618, 206}

$$\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) - (b*Log[c + b*Cos[x] + a*Sin[x]
])/(a^2 + b^2)
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
```

```
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx &= \int \frac{\sin(x)}{c+b\cos(x)+a\sin(x)}\,dx\\
&= \frac{ax}{a^2+b^2}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}-\frac{(ac)\int \frac{1}{c+b\cos(x)+a\sin(x)}\,dx}{a^2+b^2}\\
&= \frac{ax}{a^2+b^2}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}-\frac{(2ac)\,\mathrm{Subst}\left(\int \frac{1}{b+c+2ax+(-b+c)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{ax}{a^2+b^2}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}+\frac{(4ac)\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx,x,2a+2(-b
\end{aligned}$$

$$= \frac{ax}{a^2+b^2}+\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A] time = 0.224312, size = 80, normalized size = 0.82

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{a+(c-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}-b\log(a\sin(x)+b\cos(x)+c)+ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

```
[Out] (a*x + (2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[
a^2 + b^2 - c^2] - b*Log[c + b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

Maple [B] time = 0.051, size = 446, normalized size = 4.6

$$-2\,\frac{\ln\left(b\left(\tan\left(x/2\right)\right)^{2}-\left(\tan\left(x/2\right)\right)^{2}c-2\,a\tan\left(x/2\right)-b-c\right)b^{2}}{\left(2\,a^{2}+2\,b^{2}\right)\left(b-c\right)}+2\,\frac{\ln\left(b\left(\tan\left(x/2\right)\right)^{2}-\left(\tan\left(x/2\right)\right)^{2}c-2\,a\tan\left(x/2\right)-b-\right.}{\left(2\,a^{2}+2\,b^{2}\right)\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] -2/(2*a^2+2*b^2)/(b-c)*ln(b*tan(1/2*x)^2-tan(1/2*x)^2*c-2*a*tan(1/2*x)-b-c)
*b^2+2/(2*a^2+2*b^2)/(b-c)*ln(b*tan(1/2*x)^2-tan(1/2*x)^2*c-2*a*tan(1/2*x)-
b-c)*c*b+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x
)-2*a)/(-a^2-b^2+c^2)^(1/2))*a*b+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arcta
n(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a*c-4/(2*a^2+2*b^2)/(-
a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2)
)*a/(b-c)*b^2+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(
1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a/(b-c)*c*b+2/(2*a^2+2*b^2)*b*ln(1+tan(1/
2*x)^2)+4/(2*a^2+2*b^2)*a*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17959, size = 1277, normalized size = 13.03

$$\left[\frac{\sqrt{a^2+b^2-c^2}ac\log\left(\frac{a^4+3a^2b^2+2b^4+\left(a^2-b^2\right)c^2+2\left(a^2b+b^3\right)c\cos(x)+\left(a^4-b^4-2\left(a^2-b^2\right)c^2\right)\cos(x)^2+2\left(\left(a^3+ab^2\right)c-\left(a^3b+ab^3-2abc^2\right)\cos(x)\right)\sin(x)+2}{2bc\cos(x)-\left(a^2-b^2\right)\cos(x)^2+a^2+c^2+2(ab\cos(x)+ac}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*
c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 +
 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b
*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*c
os(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2
+ a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)*x - (
a^2*b + b^3 - b*c^2)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 +
2*(a*b*cos(x) + a*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1
/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((b*c*cos(x) + a*c*sin(x) + a^2 + b^
2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*c^2)*cos(x) - (a^2*b + b^3 - b*
c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x + (a^2*b + b^3 - b*c^2)*log(2*b*c
*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)))/
(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{a+b\cot(x)+c\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(1/(a + b*cot(x) + c*csc(x)), x)
```

Giac [A] time = 1.16176, size = 213, normalized size = 2.17

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2b+2c)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\left(a^2+b^2\right)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2}-\frac{b\log\left(-b\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2\right.}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*ta
n(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c
^2)) + a*x/(a^2 + b^2) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1
/2*x) + b + c)/(a^2 + b^2) + b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

3.459 $\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

```
[Out] (-2*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rubi [A] time = 0.0717676, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx &= \int \frac{1}{c+b\cos(x)+a\sin(x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{b+c+2ax+(-b+c)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx,x,2a+2(-b+c)\tan\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}\end{aligned}$$

Mathematica [A] time = 0.0459554, size = 50, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{a+(c-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
- c^2]
```

Maple [A] time = 0.042, size = 53, normalized size = 1.

$$-2\,\frac{1}{\sqrt{-a^2-b^2+c^2}}\arctan\left(1/2\,\frac{2\,(b-c)\tan(x/2)-2\,a}{\sqrt{-a^2-b^2+c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] -2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1668, size = 799, normalized size = 15.67

$$\left[\frac{\log\left(-\frac{a^4+3a^2b^2+2b^4+\left(a^2-b^2\right)c^2+2\left(a^2b+b^3\right)c\cos(x)+\left(a^4-b^4-2\left(a^2-b^2\right)c^2\right)\cos(x)^2+2\left(\left(a^3+ab^2\right)c-\left(a^3b+ab^3-2abc^2\right)\cos(x)\right)\sin(x)-2\left(2abc\cos(x)^2-abc+\right.}{2bc\cos(x)-\left(a^2-b^2\right)\cos(x)^2+a^2+c^2+2\left(ab\cos(x)+ac\right)\sin(x)}\right.}{2\sqrt{a^2+b^2-c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*c^2 + 2*(a^2*b + b^3)*c*co
s(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^3 + a*b^2)*c - (a^3
*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^3
 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 +
b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x
) + a*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((b*c
*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*
c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*cot(x) + c*csc(x)), x)
```

Giac [A] time = 1.17391, size = 99, normalized size = 1.94

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2b-2c)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*b - 2*c) + arctan((b*tan(1/2*x) - c*tan(
1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

3.460 $\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=120

$$-\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log\left(2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

```
[Out] (-2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) + Log[Tan[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tan[x/
2] - (b - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rubi [A] time = 0.533174, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4397, 12, 1628, 634, 618, 206, 628}

$$-\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log\left(2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) + Log[Tan[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tan[x/
2] - (b - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx = \int \frac{\csc(x)}{c+b\cos(x)+a\sin(x)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1+x^2}{2x\left(b+c+2ax+(-b+c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \text{Subst}\left(\int \frac{1+x^2}{x\left(b+c+2ax+(-b+c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{(b+c)x} + \frac{2(-a+bx)}{(b+c)\left(b+c+2ax-(b-c)x^2\right)}\right)dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c} + \frac{2\,\text{Subst}\left(\int \frac{-a+bx}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b+c}$$
$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\,\text{Subst}\left(\int \frac{2a+2(-b+c)x}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^2-c^2} + \frac{(2ac)\,\text{Subst}\left(\int \frac{1}{b+c+2ax+(-\cdots}\right.}{b^2-c^{\cdots}}$$
$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2} - \frac{(4ac)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c\cdots\right.}\right.}{\cdots}$$
$$= -\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}} + \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\cdots\right.\right.}{b^2-c^2}$$

Mathematica [A] time = 0.286086, size = 104, normalized size = 0.87

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{a+(c-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} + b\log(a\sin(x)+b\cos(x)+c) + (c-b)\log\left(\sin\left(\frac{x}{2}\right)\right) - (b+c)\log\left(\cos\left(\frac{x}{2}\right)\right)}{(c-b)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] ((2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
b^2 - c^2] - (b + c)*Log[Cos[x/2]] + (-b + c)*Log[Sin[x/2]] + b*Log[c + b*C
os[x] + a*Sin[x]])/((-b + c)*(b + c))
```

Maple [A] time = 0.045, size = 184, normalized size = 1.5

$$-\frac{b}{(b+c)(b-c)}\ln\left(b\left(\tan\left(\frac{x}{2}\right)\right)^2-\left(\tan\left(\frac{x}{2}\right)\right)^2c-2a\tan(x/2)-b-c\right)+2\frac{a}{(b+c)\sqrt{-a^2-b^2+c^2}}\arctan\left(1/2\frac{2(b-c)\mathrm{t}}{\sqrt{-a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] -1/(b+c)*b/(b-c)*ln(b*tan(1/2*x)^2-tan(1/2*x)^2*c-2*a*tan(1/2*x)-b-c)+2/(b+
c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^
(1/2))*a-2/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(
-a^2-b^2+c^2)^(1/2))*b*a/(b-c)+ln(tan(1/2*x))/(b+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 23.5262, size = 1571, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)
*c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
+ 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*
cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2
 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + (a^2*b + b^3 - b*c^2)*log(2*
b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)
```

```
) - (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*cos(x) + 1/2) - (a^
2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x) + 1/2))/(a^2*b^2 +
 b^4 + c^4 - (a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((
b*c*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 -
 a*c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x))) - (a^2*b + b^3 - b*c^2)*log
(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin
(x)) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*cos(x) + 1/2) +
(a^2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x) + 1/2))/(a^2*b^
2 + b^4 + c^4 - (a^2 + 2*b^2)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{a + b\cot(x) + c\csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cot(x) + c*csc(x)), x)
```

Giac [A] time = 1.17926, size = 192, normalized size = 1.6

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2b+2c)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}\left(b^2-c^2\right)}-\frac{b\log\left(-b\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2a\tan\left(\frac{1}{2}x\right.\right.}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*tan
(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 - c^
2)) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b + c)/(b^2
 - c^2) + log(abs(tan(1/2*x)))/(b + c)
```

3.461 $\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx$

Optimal. Leaf size=21

$$x+2\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)+2}\right)$$

```
[Out] x + 2*ArcTan[(Cos[x] - Sin[x])/(2 + Cos[x] + Sin[x])]
```

Rubi [A] time = 0.0478138, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 204}

$$x+2\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
```

```
[Out] x + 2*ArcTan[(Cos[x] - Sin[x])/(2 + Cos[x] + Sin[x])]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx &= \int \frac{1}{3+2\cos(x)+2\sin(x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{5+4x+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, 4+2\tan\left(\frac{x}{2}\right)\right)\right)\\ &= x+2\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{2+\cos(x)+\sin(x)}\right)\end{aligned}$$

Mathematica [B] time = 0.0248894, size = 51, normalized size = 2.43

$$\tan^{-1}\left(\sec\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)\right)\right)-\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
```

```
[Out] -ArcTan[Cos[x/2]/(2*Cos[x/2] + Sin[x/2])] + ArcTan[Sec[x/2]*(2*Cos[x/2] + S
in[x/2])]
```

Maple [A] time = 0.039, size = 10, normalized size = 0.5

$$2\ \arctan\left(2+\tan\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(2+2*cot(x)+3*csc(x)),x)
```

```
[Out] 2*arctan(2+tan(1/2*x))
```

Maxima [A] time = 1.48324, size = 19, normalized size = 0.9

$$2\arctan\left(\frac{\sin(x)}{\cos(x)+1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1) + 2)
```

Fricas [A] time = 2.19288, size = 74, normalized size = 3.52

$$-\arctan\left(-\frac{3\cos(x)+3\sin(x)+4}{\cos(x)-\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="fricas")
```

```
[Out] -arctan(-(3*cos(x) + 3*sin(x) + 4)/(cos(x) - sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc(x)}{2\cot(x)+3\csc(x)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x)
```

```
[Out] Integral(csc(x)/(2*cot(x) + 3*csc(x) + 2), x)
```

<u>Giac</u> [A] time = 1.14316, size = 30, normalized size = 1.43

$$2\,\pi \left\lfloor \frac{x}{2\,\pi} + \frac{1}{2} \right\rfloor + 2\,\arctan\left(\tan\left(\frac{1}{2}\,x\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="giac")
```

```
[Out] 2*pi*floor(1/2*x/pi + 1/2) + 2*arctan(tan(1/2*x) + 2)
```

3.462 $\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(c,a)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}\ldots}\right.}{3e\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2}$$

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(
3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Si
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(a*Cos[d +
e*x] - c*Sin[d + e*x]))/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin
[d + e*x]))
```

Rubi [A] time = 0.432118, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3168, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2}+\frac{8b(a\ldots}{3e\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x]
```

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(
3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Si
```

```
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(a*Cos[d +
e*x] - c*Sin[d + e*x]))/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin
[d + e*x]))
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx = \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\int(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin(d+ex))}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))} + \frac{(2}{}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin(d+ex))}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))} + \frac{(4}{}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin(d+ex))}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))} + \frac{(4}{}$$

$$= \frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right.}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [C] time = 6.43539, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x]
```

```
[Out] ((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*((8*b*c)/(3*a) - (2*a*Cos[d +
e*x])/3 + (2*c*Sin[d + e*x])/3))/(e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x]
+ a*Sin[d + e*x])) + (4*a*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(-(
(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x -
 ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b +
 Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b
/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]
*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Ar
cTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]
*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 +
c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - (
(2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a
*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c
^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d +
e*x] + a*Sin[d + e*x])^(3/2)) + (4*b*c^2*(a + c*Cot[d + e*x] + b*Csc[d + e*
x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*
Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)
)*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt
[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos
[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^
2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c
*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)
/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^
2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + S
qrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*a*e*Csc[d + e*x]^(3/2)*
(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)) + (2*a*AppellF1[1/2, 1/2, 1/2,
 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^
2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d +
e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*
(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sec[d + e*x + ArcTan[c/a]]*Sqrt
[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/
a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d
+ e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^
2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*Sqrt[1 +
 c^2/a^2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2))
 + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e
*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -(
(b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(
-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2
)*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 +
c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
```

```
 + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 +
c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqr
t[(a^2 + c^2)/a^2])])/(a*Sqrt[1 + c^2/a^2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[
d + e*x] + a*Sin[d + e*x])^(3/2)) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((
b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1
 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + Arc
Tan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*(a + c*Co
t[d + e*x] + b*Csc[d + e*x])^(3/2)*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[
(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b +
 a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + A
rcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d
+ e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*a*Sqrt[1 + c^2/a^
2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2))
```

Maple [C] time = 1.618, size = 20627, normalized size = 55.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\cot(ex+d) + b\csc(ex+d) + a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/csc(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x
)
```

3.463 $\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[Csc[d + e*x]]*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rubi [A] time = 0.14369, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3168, 3119, 2653}

$$\frac{2\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```



```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[Csc[d + e*x]]*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```


Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx = \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\,dx}{\sqrt{\csc(d+ex)}\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}$$

$$= \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}\,dx}{\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

$$= \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [C] time = 6.2559, size = 1580, normalized size = 13.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```

```
[Out] (2*c*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])/(a*e*Sqrt[Csc[d + e*x]]) +
(a*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^
2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d
+ e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))
]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)
```

```
/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^
2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]
]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Arc
Tan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]
*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]]
)/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[
a/c]]]))/(e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]) +
 (c^2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2,
-1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 +
 a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos
[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*
c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c
^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[
(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/
c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x -
ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c
^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/
c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcT
an[a/c]]]))/(a*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x
]]) + (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d +
e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -
((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*
(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]
*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c
^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
+ a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c
^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt
[(a^2 + c^2)/a^2])])/(a*Sqrt[1 + c^2/a^2]*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*C
os[d + e*x] + a*Sin[d + e*x]])
```

Maple [C] time = 0.55, size = 12367, normalized size = 104.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\cot(ex+d) + b\csc(ex+d) + a}}{\sqrt{\csc(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\cot(ex+d) + b\csc(ex+d) + a}}{\sqrt{\csc(ex+d)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\csc(d+ex) + c\cot(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/csc(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))/sqrt(csc(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}{\sqrt{\csc(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

3.464 $\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(c,a)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right)}{e\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

```
[Out] (2*Sqrt[Csc[d + e*x]]*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

Rubi [A] time = 0.165704, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3168, 3127, 2661}

$$\frac{2\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```


```
[Out] (2*Sqrt[Csc[d + e*x]]*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```


Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
```

```
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx = \frac{\left(\sqrt{\csc(d+ex)}\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\right)\int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}\,dx}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

$$= \frac{\left(\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right)\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}}\,dx}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

$$= \frac{2\sqrt{\csc(d+ex)}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

Mathematica [C] time = 0.908103, size = 339, normalized size = 2.87

$$\frac{2\sqrt{\csc(d+ex)}\sec\left(\tan^{-1}\left(\frac{c}{a}\right)+d+ex\right)\sqrt{-\frac{a\sqrt{\frac{c^2}{a^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{c}{a}\right)+d+ex\right)-1\right)}{a\sqrt{\frac{c^2}{a^2}+1}+b}}\sqrt{\frac{a\sqrt{\frac{c^2}{a^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{c}{a}\right)+d+ex\right)+1\right)}{a\sqrt{\frac{c^2}{a^2}+1}-b}}\sqrt{a\sqrt{\frac{c^2}{a^2}+1}\sin\left(\tan\right.}}{ae\sqrt{\frac{c^2}{a^2}+1}\sqrt{a+b\csc}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcT
an[c/a]])/(b - a*Sqrt[1 + c^2/a^2]), (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x +
 ArcTan[c/a]])/(b + a*Sqrt[1 + c^2/a^2])]*Sqrt[Csc[d + e*x]]*Sec[d + e*x +
ArcTan[c/a]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]*Sqrt[-((a*Sqrt[1 + c
^2/a^2]*(-1 + Sin[d + e*x + ArcTan[c/a]]))/(b + a*Sqrt[1 + c^2/a^2]))]*Sqrt
[(a*Sqrt[1 + c^2/a^2]*(1 + Sin[d + e*x + ArcTan[c/a]]))/(-b + a*Sqrt[1 + c^
```

```
2/a^2])]*Sqrt[b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]]])/(a*Sqrt[
1 + c^2/a^2]*e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

Maple [C] time = 0.587, size = 715, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x)
```

```
[Out] 4*I/e/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)*(1/sin(e*x+d))^(1/2)*((b+c*cos(e*x+d)
+a*sin(e*x+d))/sin(e*x+d))^(1/2)*((I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+
(a^2-b^2+c^2)^(1/2)+a)*(I*sin(e*x+d)+cos(e*x+d)))^(1/2)*EllipticF(((I*b-I*c
-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*sin(e*x+d)+cos(e
*x+d)))^(1/2),((I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*b-I*c+(a^2-b^2+c^2)^(1/2)
-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(1/2))
*(-I/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a)*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-a*cos(
e*x+d)-b*sin(e*x+d)+c*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-a)/(I*cos(e*x+d)+I+sin
(e*x+d)))^(1/2)*(I/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(b*sin(e*x+d)-c*sin(e*x+
d)+cos(e*x+d)*(a^2-b^2+c^2)^(1/2)+a*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)+a)/(I*co
s(e*x+d)+I+sin(e*x+d)))^(1/2)*(cos(e*x+d)+1)^2*(cos(e*x+d)-1)^2*(I*(a^2-b^2
+c^2)^(1/2)*sin(e*x+d)-I*cos(e*x+d)*b+I*cos(e*x+d)*c+I*sin(e*x+d)*a-cos(e*x
+d)*(a^2-b^2+c^2)^(1/2)-a*cos(e*x+d)-b*sin(e*x+d)+c*sin(e*x+d))/sin(e*x+d)^
3/(b+c*cos(e*x+d)+a*sin(e*x+d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\csc\left(ex+d\right)}}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\csc\left(ex+d\right)}}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\csc\left(d+ex\right)}}{\sqrt{a+b\csc\left(d+ex\right)+c\cot\left(d+ex\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(d + e*x))/sqrt(a + b*csc(d + e*x) + c*cot(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\csc\left(ex+d\right)}}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

3.465 $\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\cos(d+ex)}{e\left(a^2-b^2+c^2\right)}$$

```
[Out] (-2*Csc[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2
 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*Csc[d + e*x]^(3/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(
(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
```

Rubi [A] time = 0.212266, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3168, 3128, 3119, 2653}

$$-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\cos(d+ex)}{e\left(a^2-b^2+c^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]
```

```
[Out] (-2*Csc[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2
 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*Csc[d + e*x]^(3/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(
(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
```

```
a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx = \frac{\left(\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}\right)\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+e}}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sqrt{\frac{b+c\cos(d+ex)+}{b+\sqrt{a^2}}}}$$

Mathematica [C] time = 6.41498, size = 1732, normalized size = 7.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]
```

```
[Out] (Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*((-2*(a^2 + c^2
))/(a*c*(a^2 - b^2 + c^2)) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d + e*x])
)/(c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (a*Csc[d + e*x]^(3/2)*(b + c*Cos[d +
 e*x] + a*Sin[d + e*x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b +
Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(
Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[
a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x -
 ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[
(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]
*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[
(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b
+ c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x
- ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/((a^2
- b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (c^2*Csc[d +
e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*(-((a*AppellF1[-1/2,
```

```
 -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(S
qrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2
]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^
2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(
a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b +
c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Ar
cTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d +
 e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1
+ a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - Ar
cTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x
 - ArcTan[a/c]]]))/(a*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e
*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*
Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2
])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c
^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(3/2)*Sec[d + e*x +
ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*Sqrt[(a*Sqrt[(a^2
+ c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sq
rt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan
[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x
 + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*(a^2 - b^2 + c^2)*Sqrt
[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
```

Maple [C] time = 0.556, size = 12477, normalized size = 52.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(ex+d)^{\frac{3}{2}}}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(csc(e*x + d)^(3/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}\csc\left(ex+d\right)^{\frac{3}{2}}}{c^2\cot\left(ex+d\right)^2+b^2\csc\left(ex+d\right)^2+2\,ac\cot\left(ex+d\right)+a^2+2\left(bc\cot\left(ex+d\right)+ab\right)\csc\left(ex+d\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*csc(e*x + d)^(3/2)/(c^2*
cot(e*x + d)^2 + b^2*csc(e*x + d)^2 + 2*a*c*cot(e*x + d) + a^2 + 2*(b*c*cot
(e*x + d) + a*b)*csc(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] sage0*x
```

3.466 $\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$\frac{2\csc^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a\sin(d+ex)+b+c\cos(d+ex))^2\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(c,a)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right.}{3e\left(a^2-b^2+c^2\right)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

```
[Out] (8*b*Csc[d + e*x]^(5/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
 c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(
a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sqrt[(b +
c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*Csc[d + e*x]^
(5/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a
^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2)) - (2*Csc[d + e*x]^(5/2)*(b + c*Cos[d +
 e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 +
c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (8*Csc[d + e*x]^(5/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*
x]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2))
```

Rubi [A] time = 0.496998, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3168, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\csc^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8b\csc}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]
```

```
[Out] (8*b*Csc[d + e*x]^(5/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
 c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(
a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sqrt[(b +
c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*Csc[d + e*x]^
(5/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a
```

```
^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2)) - (2*Csc[d + e*x]^(5/2)*(b + c*Cos[d +
 e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 +
c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (8*Csc[d + e*x]^(5/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*
x]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2))
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx = \frac{\left(\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}\right)\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex}}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= \frac{8b\csc^{\frac{5}{2}}(d+ex)E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a}{3\left(a^2-b^2+c^2\right)^2 e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sqrt{\frac{b+c\cos(d+ex)+a}{b+\sqrt{a^2+}}}}$$

Mathematica [C] time = 6.51315, size = 2708, normalized size = 5.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]
```

```
[Out] (Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3*((8*b*(a^2 + c^
2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d +
 e*x]))/(3*c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2) - (
2*(a^3 + 3*a*b^2 + a*c^2 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3
*c*(a^2 - b^2 + c^2)^2*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*a*b*Csc[d + e*x]^(5/2)*(b + c*Cos
[d + e*x] + a*Sin[d + e*x])^(5/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((
b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 -
 b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - Arc
```

Tan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*b*c^2*Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*a*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d +

```
e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 +
c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
 + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 +
c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqr
t[(a^2 + c^2)/a^2])])/(3*a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2))
```

Maple [C] time = 1.804, size = 64199, normalized size = 130.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x)
```

```
[Out] result too large to display
```

Maximima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(ex+d)^{\frac{5}{2}}}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(csc(e*x + d)^(5/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\,\csc(ex+}{c^3\cot(ex+d)^3+b^3\csc(ex+d)^3+3\,ac^2\cot(ex+d)^2+3\,a^2c\cot(ex+d)+a^3+3\left(b^2c\cot(ex+d)+ab^2\right)\csc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*csc(e*x + d)^(5/2)/(c^3*
cot(e*x + d)^3 + b^3*csc(e*x + d)^3 + 3*a*c^2*cot(e*x + d)^2 + 3*a^2*c*cot(
e*x + d) + a^3 + 3*(b^2*c*cot(e*x + d) + a*b^2)*csc(e*x + d)^2 + 3*(b*c^2*c
ot(e*x + d)^2 + 2*a*b*c*cot(e*x + d) + a^2*b)*csc(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.467 $\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(c,a)+d+\right.\right.}{3e(a\sin(d+ex)+b+c\cos(d+ex))^2}$$

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2))/
(3*e*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos[d + e*x
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*e*(b + c*Cos[d + e*x] + a*Si
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x
]^(3/2)*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*e*(b + c*Cos[d + e*x] + a*Sin
[d + e*x]))
```

Rubi [A] time = 0.383481, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3164, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2}}{b+\sqrt{a^2}}\right.}{3e(a\sin(d+ex)+b+c\cos(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]
```

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2))/
(3*e*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos[d + e*x
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*e*(b + c*Cos[d + e*x] + a*Si
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x
```

]^(3/2)*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*e*(b + c*Cos[d + e*x] + a*Sin [d + e*x]))

Rule 3164

Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rule 3120

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx = \frac{\left((a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\right)\int(b+c\cos}{(b+c\cos(d+ex)+a\sin(d+ex))^{3}}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)(a\cos(d+}{3e(b+c\cos(d+ex)+a\sin(d+ex))}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)(a\cos(d+}{3e(b+c\cos(d+ex)+a\sin(d+ex))}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)(a\cos(d+}{3e(b+c\cos(d+ex)+a\sin(d+ex))}$$

$$= \frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a\right.\right.}{3e(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos(d+e}{b+\sqrt{}}}}$$

Mathematica [F] time = 53.4074, size = 0, normalized size = 0.

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]
```

```
[Out] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2), x
]
```

Maple [C] time = 0.677, size = 20858, normalized size = 56.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(ex+d) + b\csc(ex+d) + a)^{\frac{3}{2}} \sin(ex+d)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((c\cot(ex+d) + b\csc(ex+d) + a)^{\frac{3}{2}} \sin(ex+d)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)*sin(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x
)
```

3.468 $\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[Sin[d + e*x]])/(e*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rubi [A] time = 0.140733, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3164, 3119, 2653}

$$\frac{2\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```


```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[Sin[d + e*x]])/(e*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```


Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] && !IntegerQ[n]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx = \frac{\left(\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\right)\int \sqrt{b+c\cos(d}}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}$$

$$= \frac{\left(\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\right)\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+}}{\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

$$= \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2}{b}\right.}{e\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [F] time = 12.6705, size = 0, normalized size = 0.

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```

```
[Out] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]], x]
```

Maple [C] time = 0.435, size = 12362, normalized size = 104.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\sqrt{\sin(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)*sin(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

3.469 $\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(c,a)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right)}{e\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

```
[Out] (2*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])
```

Rubi [A] time = 0.148861, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3164, 3127, 2661}

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
```

```
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx &= \frac{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}\,dx}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\\
&= \frac{\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}}\,dx}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\\
&= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}
\end{aligned}$$

Mathematica [C] time = 2.88962, size = 519, normalized size = 4.4

$$\frac{4\left(i\sqrt{a^2-b^2+c^2}-ia-b+c\right)(\cos(d+ex)+i\sin(d+ex))\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}+a+(b-c)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{\left(\sqrt{a^2-b^2+c^2}+a-ib+ic\right)\left(\tan\left(\frac{1}{2}(d+ex)\right)-i\right)}}\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}-a+(c-b)\tan\left(\cdots\right.\right.}{\left(\sqrt{a^2-b^2+c^2}-a+ib-ic\right)\left(\tan\left(\cdots\right.\right.}}\cdots}{e\left(-\sqrt{a^2-b^2+c^2}+a+ib-ic\right)\sqrt{\sin(\cdots}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),
x]
```

```
[Out] (4*((-I)*a - b + c + I*Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[((-a -
I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(-a +
I*b - I*c + Sqrt[a^2 - b^2 + c^2])]], (I*b + Sqrt[a^2 - b^2 + c^2])/(I*b -
Sqrt[a^2 - b^2 + c^2])]*Sqrt[((-a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Co
s[d + e*x] + I*Sin[d + e*x]))/(-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])]*(Co
```

```
s[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(a + Sqrt[a^2 - b^2 + c^2] + (b - c
)*Tan[(d + e*x)/2]))/((a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d
+ e*x)/2]))]*Sqrt[((-I)*(-a + Sqrt[a^2 - b^2 + c^2] + (-b + c)*Tan[(d + e*x
)/2]))/((-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))])
/((a + I*b - I*c - Sqrt[a^2 - b^2 + c^2])*e*Sqrt[a + c*Cot[d + e*x] + b*Csc
[d + e*x]]*Sqrt[Sin[d + e*x]])
```

Maple [C] time = 0.403, size = 705, normalized size = 6.

$$\frac{4\,i\,(\cos(ex+d)+1)^2\,(\cos(ex+d)-1)^2}{e\,(b+c\cos(ex+d)+a\sin(ex+d))}\mathrm{EllipticF}\left(\sqrt{(i\sin(ex+d)+\cos(ex+d))\left(ib-ic-\sqrt{a^2-b^2+c^2}-a\right)\left(ib-ic\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x)
```

```
[Out] 4*I/e/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)*EllipticF(((I*b-I*c-(a^2-b^2+c^2)^(1/
2)-a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*sin(e*x+d)+cos(e*x+d)))^(1/2),((I*
b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*b-I*c+(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-
b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(1/2))*((b+c*cos(e*x+d)+
a*sin(e*x+d))/sin(e*x+d))^(1/2)*((I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(
a^2-b^2+c^2)^(1/2)+a)*(I*sin(e*x+d)+cos(e*x+d)))^(1/2)*(-I/(I*b-I*c-(a^2-b^
2+c^2)^(1/2)+a)*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-a*cos(e*x+d)-b*sin(e*x+d)+c
*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-a)/(I*cos(e*x+d)+I+sin(e*x+d)))^(1/2)*(I/(I
*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(b*sin(e*x+d)-c*sin(e*x+d)+cos(e*x+d)*(a^2-b^
2+c^2)^(1/2)+a*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)+a)/(I*cos(e*x+d)+I+sin(e*x+d)
))^(1/2)*(cos(e*x+d)+1)^2*(cos(e*x+d)-1)^2*(I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d
)-I*cos(e*x+d)*b+I*cos(e*x+d)*c+I*sin(e*x+d)*a-cos(e*x+d)*(a^2-b^2+c^2)^(1/
2)-a*cos(e*x+d)-b*sin(e*x+d)+c*sin(e*x+d))/sin(e*x+d)^(7/2)/(b+c*cos(e*x+d)
+a*sin(e*x+d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}\sqrt{\sin\left(ex+d\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+b\csc\left(d+ex\right)+c\cot\left(d+ex\right)}\sqrt{\sin\left(d+ex\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/sin(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x))),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}\sqrt{\sin\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))),
 x)
```

3.470 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=240

$$-\frac{2(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2(a\cos(d+ex)-c\sin(d+e}{e\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)}$$

```
[Out] (-2*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2 - b^2 + c^2)*e*(a
+ c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e
*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2))
```

Rubi [A] time = 0.205311, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3164, 3128, 3119, 2653}

$$-\frac{2(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2(a\cos(d+ex)-c\sin(d+e}{e\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)),x]
```


```
[Out] (-2*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2 - b^2 + c^2)*e*(a
+ c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e
*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2))
```


Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
```

```
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \frac{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/}}}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d-}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d-}$$

$$= -\frac{2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+}$$

Mathematica [F] time = 21.461, size = 0, normalized size = 0.

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)
),x]
```

```
[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)
), x]
```

Maple [C] time = 0.433, size = 12467, normalized size = 52.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a\right)^{\frac{3}{2}}\sin\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex\right.}}{a^2\cos\left(ex+d\right)^2+\left(c^2\cos\left(ex+d\right)^2-c^2\right)\cot\left(ex+d\right)^2+\left(b^2\cos\left(ex+d\right)^2-b^2\right)\csc\left(ex+d\right)^2-a^2+2\left(ac\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))/(a^2
*cos(e*x + d)^2 + (c^2*cos(e*x + d)^2 - c^2)*cot(e*x + d)^2 + (b^2*cos(e*x
+ d)^2 - b^2)*csc(e*x + d)^2 - a^2 + 2*(a*c*cos(e*x + d)^2 - a*c)*cot(e*x +
 d) + 2*(a*b*cos(e*x + d)^2 - a*b + (b*c*cos(e*x + d)^2 - b*c)*cot(e*x + d)
)*csc(e*x + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/sin(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2)
), x)
```

3.471 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

Optimal. Leaf size=492

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a\sin(d+ex)+b+c\cos(d+ex))^2\text{EllipticF}\left(\frac{1}{2}\left(-\tan^{-1}(c,a)+d+ex\right),\frac{2\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}+b}\right)}{3e\left(a^2-b^2+c^2\right)\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{}{3e\left(a^2-\right.}$$

```
[Out] (8*b*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^
2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*
e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)*Sqrt[(b +
c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*EllipticF[(d
+ e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*
Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc
[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) - (2*(b + c*Cos[d + e*x] + a*Sin[d + e
*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d
 + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) + (8*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*x]))/(3*(a^2 - b^2
+ c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2))
```

Rubi [A] time = 0.491416, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3164, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8b(a\sin(d}{3e\left(a^2-b^2+c^2\right)^2 s}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)),x]
```


```
[Out] (8*b*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^
2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*
e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)*Sqrt[(b +
c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*EllipticF[(d
+ e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*
```

```
Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc
[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) - (2*(b + c*Cos[d + e*x] + a*Sin[d + e
*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d
 + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) + (8*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*x]))/(3*(a^2 - b^2
+ c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2))
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \frac{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{5/}}}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d}$$

$$= \frac{8bE\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+}{3\left(a^2-b^2+c^2\right)^2 e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d}$$

Mathematica [F] time = 25.9738, size = 0, normalized size = 0.

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)
),x]
```

```
[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)
), x]
```

Maple [C] time = 1.202, size = 64189, normalized size = 130.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{5}{2}}\sin(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2)*sin(e*x + d)^(5/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{}{\left(a^3\cos(ex+d)^2+\left(c^3\cos(ex+d)^2-c^3\right)\cot(ex+d)^3+\left(b^3\cos(ex+d)^2-b^3\right)\csc(ex+d)^3-a^3+3\left(ac^2\cos\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/((a^3*cos(e*x + d)^2 +
(c^3*cos(e*x + d)^2 - c^3)*cot(e*x + d)^3 + (b^3*cos(e*x + d)^2 - b^3)*csc(
e*x + d)^3 - a^3 + 3*(a*c^2*cos(e*x + d)^2 - a*c^2)*cot(e*x + d)^2 + 3*(a*b
^2*cos(e*x + d)^2 - a*b^2 + (b^2*c*cos(e*x + d)^2 - b^2*c)*cot(e*x + d))*cs
c(e*x + d)^2 + 3*(a^2*c*cos(e*x + d)^2 - a^2*c)*cot(e*x + d) + 3*(a^2*b*cos
(e*x + d)^2 - a^2*b + (b*c^2*cos(e*x + d)^2 - b*c^2)*cot(e*x + d)^2 + 2*(a*
b*c*cos(e*x + d)^2 - a*b*c)*cot(e*x + d))*csc(e*x + d))*sqrt(sin(e*x + d)))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2)/sin(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{5}{2}}\sin(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2)*sin(e*x + d)^(5/2)
), x)
```

3.472 $\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0092337, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + Sin[x]^2)^(-1),x]

[Out] x
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0004585, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-1),x]
```

```
[Out] x
```

Maple [A] time = 0.014, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+sin(x)^2),x)
```

```
[Out] x
```

Maxima [A] time = 1.47255, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.5574, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] time = 0.423272, size = 10, normalized size = 10.

$$\frac{x}{\sin^2(x) + \cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2),x)
```

```
[Out] x/(sin(x)**2 + cos(x)**2)
```

Giac [A] time = 1.14716, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="giac")
```

```
[Out] x
```

3.473 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0093517, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + Sin[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0002989, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-2),x]
```

```
[Out] x
```

Maple [A] time = 0.016, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+sin(x)^2)^2,x)
```

```
[Out] x
```

Maxima [A] time = 1.49555, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.87529, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="fricas")
```

[Out] x

Sympy [B] time = 1.08813, size = 22, normalized size = 22.

$$\frac{x}{\sin^4(x) + 2\sin^2(x)\cos^2(x) + \cos^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2)**2,x)
```

```
[Out] x/(sin(x)**4 + 2*sin(x)**2*cos(x)**2 + cos(x)**4)
```

Giac [A] time = 1.1291, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

3.474 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0087911, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + Sin[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0003298, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-3),x]
```

```
[Out] x
```

Maple [A] time = 0.016, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+sin(x)^2)^3,x)
```

```
[Out] x
```

Maxima [A] time = 1.45669, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.63574, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] time = 3.3206, size = 34, normalized size = 34.

$$\frac{x}{\sin^6(x) + 3\sin^4(x)\cos^2(x) + 3\sin^2(x)\cos^4(x) + \cos^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2)**3,x)
```

```
[Out] x/(sin(x)**6 + 3*sin(x)**4*cos(x)**2 + 3*sin(x)**2*cos(x)**4 + cos(x)**6)
```

Giac [A] time = 1.09798, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] x
```

3.475 $\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rubi [A] time = 0.0154245, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {206}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0048526, size = 23, normalized size = 2.09

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2}\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

```
[Out] -Log[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x]]/2
```

Maple [A] time = 0.025, size = 4, normalized size = 0.4

$$\mathrm{Artanh}\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2),x)
```

```
[Out] arctanh(tan(x))
```

Maxima [A] time = 1.03051, size = 20, normalized size = 1.82

$$\frac{1}{2}\log\left(\tan\left(x\right)+1\right)-\frac{1}{2}\log\left(\tan\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

Fricas [B] time = 1.85709, size = 84, normalized size = 7.64

$$\frac{1}{4}\log\left(2\cos\left(x\right)\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)
```

Sympy [B] time = 0.468747, size = 36, normalized size = 3.27

$$\frac{\log\left(\tan^{2}\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\log\left(\tan^{2}\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2),x)
```

```
[Out] log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 - log(tan(x/2)**2 + 2*tan(x/2) - 1)/2
```

Giac [B] time = 1.12207, size = 45, normalized size = 4.09

$$\frac{1}{8}\,\log\left(\left|\frac{1}{\sin\left(2\,x\right)}+\sin\left(2\,x\right)+2\right|\right)-\frac{1}{8}\,\log\left(\left|\frac{1}{\sin\left(2\,x\right)}+\sin\left(2\,x\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(1/sin(2*x) + sin(2*x) + 2)) - 1/8*log(abs(1/sin(2*x) + sin(2*x)
 - 2))
```

3.476 $\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{1-\tan^2(x)}$$

```
[Out] Tan[x]/(1 - Tan[x]^2)
```

Rubi [A] time = 0.0229803, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {383}

$$\frac{\tan(x)}{1-\tan^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

```
[Out] Tan[x]/(1 - Tan[x]^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{\left(1-x^2\right)^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{1-\tan^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0032264, size = 8, normalized size = 0.62

$$\frac{1}{2}\tan(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

```
[Out] Tan[2*x]/2
```

Maple [A] time = 0.031, size = 18, normalized size = 1.4

$$-\frac{1}{2+2\tan(x)}-\frac{1}{2\tan(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2)^2,x)
```

```
[Out] -1/2/(1+tan(x))-1/2/(tan(x)-1)
```

Maxima [A] time = 0.983312, size = 16, normalized size = 1.23

$$-\frac{\tan(x)}{\tan(x)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -tan(x)/(tan(x)^2 - 1)
```

Fricas [A] time = 1.64872, size = 43, normalized size = 3.31

$$\frac{\cos(x)\sin(x)}{2\cos(x)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] cos(x)*sin(x)/(2*cos(x)^2 - 1)
```

Sympy [B] time = 2.39591, size = 48, normalized size = 3.69

$$-\frac{2\tan^3\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-6\tan^2\left(\frac{x}{2}\right)+1}+\frac{2\tan\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-6\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2)**2,x)
```

```
[Out] -2*tan(x/2)**3/(tan(x/2)**4 - 6*tan(x/2)**2 + 1) + 2*tan(x/2)/(tan(x/2)**4
- 6*tan(x/2)**2 + 1)
```

Giac [A] time = 1.11712, size = 8, normalized size = 0.62

$$\frac{1}{2}\tan\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*tan(2*x)
```

3.477 $\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^3} dx$

Optimal. Leaf size=32

$$\frac{\tan(x)\sec^2(x)}{2\left(1-\tan^2(x)\right)^2}+\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/4 + (Sec[x]^2*Tan[x])/(2*(1 - Tan[x]^2)^2)
```

Rubi [A] time = 0.0273345, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {413, 21, 206}

$$\frac{\tan(x)\sec^2(x)}{2\left(1-\tan^2(x)\right)^2}+\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/4 + (Sec[x]^2*Tan[x])/(2*(1 - Tan[x]^2)^2)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tan(x)\right)\\
&= \frac{\sec^2(x)\tan(x)}{2\left(1-\tan^2(x)\right)^2}-\frac{1}{4}\,\text{Subst}\left(\int \frac{-2+2x^2}{\left(1-x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \frac{\sec^2(x)\tan(x)}{2\left(1-\tan^2(x)\right)^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{4}\tanh^{-1}(2\cos(x)\sin(x))+\frac{\sec^2(x)\tan(x)}{2\left(1-\tan^2(x)\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0066755, size = 22, normalized size = 0.69

$$\frac{1}{4}\tanh^{-1}(\sin(2x))+\frac{1}{4}\tan(2x)\sec(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

```
[Out] ArcTanh[Sin[2*x]]/4 + (Sec[2*x]*Tan[2*x])/4
```

Maple [A] time = 0.037, size = 48, normalized size = 1.5

$$-\frac{1}{4\,(1+\tan(x))^2}+\frac{1}{4+4\,\tan(x)}+\frac{\ln(1+\tan(x))}{4}+\frac{1}{4\,(\tan(x)-1)^2}+\frac{1}{4\,\tan(x)-4}-\frac{\ln(\tan(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2)^3,x)
```

```
[Out] -1/4/(1+tan(x))^2+1/4/(1+tan(x))+1/4*ln(1+tan(x))+1/4/(tan(x)-1)^2+1/4/(tan
(x)-1)-1/4*ln(tan(x)-1)
```

Maxima [A] time = 1.00548, size = 51, normalized size = 1.59

$$\frac{\tan(x)^3+\tan(x)}{2\left(\tan(x)^4-2\tan(x)^2+1\right)}+\frac{1}{4}\log(\tan(x)+1)-\frac{1}{4}\log(\tan(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(x)^3 + tan(x))/(tan(x)^4 - 2*tan(x)^2 + 1) + 1/4*log(tan(x) + 1) -
 1/4*log(tan(x) - 1)
```

Fricas [B] time = 1.88887, size = 227, normalized size = 7.09

$$\frac{\left(4\cos(x)^4-4\cos(x)^2+1\right)\log(2\cos(x)\sin(x)+1)-\left(4\cos(x)^4-4\cos(x)^2+1\right)\log(-2\cos(x)\sin(x)+1)+4\cos}{8\left(4\cos(x)^4-4\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((4*cos(x)^4 - 4*cos(x)^2 + 1)*log(2*cos(x)*sin(x) + 1) - (4*cos(x)^4 -
 4*cos(x)^2 + 1)*log(-2*cos(x)*sin(x) + 1) + 4*cos(x)*sin(x))/(4*cos(x)^4 -
 4*cos(x)^2 + 1)
```

Sympy [B] time = 9.11282, size = 765, normalized size = 23.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2)**3,x)
```

```
[Out] log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan(x/2)*
*6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*log(tan(x/2)**2 - 2*tan(x/2
) - 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*t
an(x/2)**2 + 4) + 38*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(4*tan(x
/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*log(ta
n(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 1
52*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + log(tan(x/2)**2 - 2*tan(x/2) - 1)/(4
*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - log
(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan(x/2)**6
+ 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 12*log(tan(x/2)**2 + 2*tan(x/2) -
 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(
x/2)**2 + 4) - 38*log(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**4/(4*tan(x/2)
**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 12*log(tan(x
/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*
tan(x/2)**4 - 48*tan(x/2)**2 + 4) - log(tan(x/2)**2 + 2*tan(x/2) - 1)/(4*ta
n(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 4*tan(
x/2)**7/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2
+ 4) - 4*tan(x/2)**5/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48
*tan(x/2)**2 + 4) + 4*tan(x/2)**3/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan
(x/2)**4 - 48*tan(x/2)**2 + 4) + 4*tan(x/2)/(4*tan(x/2)**8 - 48*tan(x/2)**6
 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4)
```

Giac [A] time = 1.15252, size = 50, normalized size = 1.56

$$-\frac{\sin(2x)}{4\left(\sin(2x)^2 - 1\right)} + \frac{1}{8}\log(\sin(2x) + 1) - \frac{1}{8}\log(-\sin(2x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*sin(2*x)/(sin(2*x)^2 - 1) + 1/8*log(sin(2*x) + 1) - 1/8*log(-sin(2*x)
+ 1)
```

3.478 $\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\tan^{-1}(a\tan(x))}{a}$$

```
[Out] ArcTan[a*Tan[x]]/a
```

Rubi [A] time = 0.0182644, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {203}

$$\frac{\tan^{-1}(a\tan(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[a*Tan[x]]/a
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}(a\tan(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0339063, size = 9, normalized size = 1.

$$\frac{\tan^{-1}(a\tan(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[a*Tan[x]]/a
```

Maple [A] time = 0.038, size = 10, normalized size = 1.1

$$\frac{\arctan\left(a\tan\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+a^2*sin(x)^2),x)
```

```
[Out] arctan(a*tan(x))/a
```

Maxima [A] time = 1.51675, size = 12, normalized size = 1.33

$$\frac{\arctan\left(a\tan\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x))/a
```

Fricas [B] time = 1.83198, size = 88, normalized size = 9.78

$$-\frac{\arctan\left(\frac{\left(a^{2}+1\right)\cos\left(x\right)^{2}-a^{2}}{2\,a\cos\left(x\right)\sin\left(x\right)}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + 1)*cos(x)^2 - a^2)/(a*cos(x)*sin(x)))/a
```

Sympy [B] time = 29.4254, size = 2011, normalized size = 223.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((-16*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**
2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) -
 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) + 16*a**5*sqrt(-2*a**2 - 2*a*sqrt(a
**2 - 1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a*
*5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) + 16*a
**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2
 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) -
16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) - 16*a**4*sqrt(a**2 - 1)*sqrt(-2*a**
2 - 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + ta
n(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1)
 + 2*a) + 20*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2
- 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 1
6*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) - 20*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**
2 - 1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5
 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) - 4*a**3
*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*s
qrt(a**2 - 1) + 2*a) + 4*a**3*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(sq
rt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a*
*2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) - 12*a**2*sqrt(a**2 - 1)*s
qrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1
) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqr
t(a**2 - 1) + 2*a) + 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 1
6*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) + 4*a**2*sqr
t(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3
 + 8*a**2*sqrt(a**2 - 1) + 2*a) - 4*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/
(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a)
- 5*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a
**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a
```

```
**2*sqrt(a**2 - 1) + 2*a) + 5*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(
sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(
a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) + 3*a*sqrt(-2*a**2 + 2*a
*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2)
)/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a
) - 3*a*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt(
a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*
a**2*sqrt(a**2 - 1) + 2*a) + sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 -
16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) - sqrt(a**2
 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a*
*2 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a*
*2*sqrt(a**2 - 1) + 2*a) - sqrt(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1)
 + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(16*a**5 - 16
*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2*sqrt(a**2 - 1) + 2*a) + sqrt(a**2 -
 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt(a**2
 - 1) + 1) + tan(x/2))/(16*a**5 - 16*a**4*sqrt(a**2 - 1) - 16*a**3 + 8*a**2
*sqrt(a**2 - 1) + 2*a), Ne(a, 0)), (-2*tan(x/2)/(tan(x/2)**2 - 1), True))
```

Giac [B] time = 1.12955, size = 27, normalized size = 3.

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(a\tan\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)))/a
```

3.479 $\int \frac{1}{b^2 \cos^2(x)+\sin^2(x)} \, dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

[Out] ArcTan[Tan[x]/b]/b

Rubi [A] time = 0.0197121, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]

[Out] ArcTan[Tan[x]/b]/b

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2 \cos^2(x) + \sin^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{b^2 + x^2} \, dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0318574, size = 11, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[x]/b]/b
```

Maple [A] time = 0.033, size = 12, normalized size = 1.1

$$\frac{1}{b}\arctan\left(\frac{\tan(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+sin(x)^2),x)
```

```
[Out] arctan(tan(x)/b)/b
```

Maxima [A] time = 1.4628, size = 15, normalized size = 1.36

$$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(tan(x)/b)/b
```

Fricas [B] time = 1.91518, size = 85, normalized size = 7.73

$$-\frac{\arctan\left(\frac{\left(b^2+1\right)\cos(x)^2-1}{2\,b\cos(x)\sin(x)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((b^2 + 1)*cos(x)^2 - 1)/(b*cos(x)*sin(x)))/b
```

Sympy [A] time = 32.779, size = 2118, normalized size = 192.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+sin(x)**2),x)
```

```
[Out] Piecewise((b**4*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log
(-sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqr
t(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) - b**4*sqrt(1 - b
**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 - 2*sqrt(1 - b**2)
/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*
b**2*sqrt(1 - b**2) + 16*b**2) - b**4*sqrt(1 - b**2)*sqrt(1 + 2*sqrt(1 - b*
*2)/b**2 - 2/b**2)*log(-sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2)
)/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b
**2) + b**4*sqrt(1 - b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqr
t(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 -
 b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) - 5*b**4*sqrt(1 - 2*sq
rt(1 - b**2)/b**2 - 2/b**2)*log(-sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2) +
 tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**
2) + 16*b**2) + 5*b**4*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1
- 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**
2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) + 3*b**4*sqrt(1 + 2*sqrt(1
 - b**2)/b**2 - 2/b**2)*log(-sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan
(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) +
 16*b**2) - 3*b**4*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 + 2*
sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) -
 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) - 12*b**2*sqrt(1 - b**2)*sqrt(
1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(-sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2
/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt
(1 - b**2) + 16*b**2) + 12*b**2*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b*
*2 - 2/b**2)*log(sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b*
*6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) +
4*b**2*sqrt(1 - b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(-sqrt(1
+ 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**
```

```
2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) - 4*b**2*sqrt(1 - b**2)*sq
rt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 + 2*sqrt(1 - b**2)/b**2 -
 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sq
rt(1 - b**2) + 16*b**2) + 20*b**2*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*
log(-sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*
sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) - 20*b**2*sqrt
(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2
/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt
(1 - b**2) + 16*b**2) - 4*b**2*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log
(-sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqr
t(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) + 4*b**2*sqrt(1 +
 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**
2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 -
 b**2) + 16*b**2) + 16*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b*
*2)*log(-sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b
**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) - 16*sqrt(
1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 - 2*sqrt(1 -
b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4
- 16*b**2*sqrt(1 - b**2) + 16*b**2) - 16*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2
/b**2)*log(-sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**6 +
8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sqrt(1 - b**2) + 16*b**2) + 16*sq
rt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 - 2*sqrt(1 - b**2)/b**2 -
 2/b**2) + tan(x/2))/(2*b**6 + 8*b**4*sqrt(1 - b**2) - 16*b**4 - 16*b**2*sq
rt(1 - b**2) + 16*b**2), Ne(b, 0)), (tan(x/2)/2 - 1/(2*tan(x/2)), True))
```

Giac [A] time = 1.11093, size = 30, normalized size = 2.73

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(tan(x)/b))/b
```

3.480 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rubi [A] time = 0.0257435, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.0397942, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A] time = 0.04, size = 16, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{a\tan(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)
```

```
[Out] arctan(a*tan(x)/b)/a/b
```

Maxima [A] time = 1.48158, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Fricas [B] time = 1.81065, size = 99, normalized size = 6.6

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

Sympy [A] time = 49.3371, size = 2866, normalized size = 191.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-2*tan(x/
2)/(b**2*(tan(x/2)**2 - 1)), Eq(a, 0)), ((tan(x/2)/2 - 1/(2*tan(x/2)))/a**2
, Eq(b, 0)), (-16*a**5*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a*
*5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(
a**2 - b**2) + 2*a*b**6) + 16*a**5*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x
/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2
*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 16*a**4*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 -
 b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 16*a**4
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b*
*2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2
- b**2) + 2*a*b**6) + 20*a**3*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**
2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 20*a**3*b**2*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a
**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 4*a**3*b**2*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**
2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 4*a*
*3*b**2*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**
4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2
*a*b**6) - 12*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4
+ 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 12*a**2*b**2*sqrt(a**2 - b**2
```

```
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2
*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**
6) + 4*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**
2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 4*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a*
*2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 5*a
*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4
*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*
a*b**6) + 5*a*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b*
*2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2
 - b**2) + 2*a*b**6) + 3*a*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2)
)/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b*
*4*sqrt(a**2 - b**2) + 2*a*b**6) - 3*a*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4
 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + b**4*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(
a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - b
**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*l
og(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5
*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a*
*2 - b**2) + 2*a*b**6) - b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3
*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + b**4*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*s
qrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6)
, True))
```

Giac [A] time = 1.16441, size = 35, normalized size = 2.33

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

3.481 $\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sin(2x+1)\cos(2x+1)}{\cos^2(2x+1)+2\sqrt{3}+3}\right)}{4\sqrt{3}}$$

```
[Out] x/(2*Sqrt[3]) - ArcTan[(Cos[1 + 2*x]*Sin[1 + 2*x])/(3 + 2*Sqrt[3] + Cos[1 +
 2*x]^2)]/(4*Sqrt[3])
```

Rubi [A] time = 0.0373063, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {203}

$$\frac{x}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sin(2x+1)\cos(2x+1)}{\cos^2(2x+1)+2\sqrt{3}+3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) - ArcTan[(Cos[1 + 2*x]*Sin[1 + 2*x])/(3 + 2*Sqrt[3] + Cos[1 +
 2*x]^2)]/(4*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{4+3x^2}\,dx, x, \tan(1+2x)\right)\\ &= \frac{x}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\cos(1+2x)\sin(1+2x)}{3+2\sqrt{3}+\cos^2(1+2x)}\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.042783, size = 25, normalized size = 0.47

$$\frac{\tan^{-1}\left(\frac{1}{2}\sqrt{3}\tan(2x+1)\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3]*Tan[1 + 2*x])/2]/(4*Sqrt[3])
```

Maple [A] time = 0.055, size = 18, normalized size = 0.3

$$\frac{\sqrt{3}}{12}\arctan\left(\frac{\sqrt{3}\tan(1+2x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x)
```

```
[Out] 1/12*3^(1/2)*arctan(1/2*3^(1/2)*tan(1+2*x))
```

Maxima [A] time = 1.46692, size = 23, normalized size = 0.43

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}\tan(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/2*sqrt(3)*tan(2*x + 1))
```

Fricas [A] time = 1.85086, size = 128, normalized size = 2.42

$$-\frac{1}{24}\sqrt{3}\arctan\left(\frac{7\sqrt{3}\cos(2x+1)^2-3\sqrt{3}}{12\cos(2x+1)\sin(2x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(2*x + 1)^2 - 3*sqrt(3))/(cos(2*x +
 1)*sin(2*x + 1)))
```

Sympy [A] time = 1.29113, size = 87, normalized size = 1.64

$$\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(x+\frac{1}{2}\right)}{3}-\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12}+\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(x+\frac{1}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)**2+3*sin(1+2*x)**2),x)
```

```
[Out] sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 - sqrt(3)/3) + pi*floor((x - pi/2 +
1/2)/pi))/12 + sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 + sqrt(3)/3) + pi*flo
or((x - pi/2 + 1/2)/pi))/12
```

Giac [A] time = 1.14089, size = 82, normalized size = 1.55

$$\frac{1}{12}\sqrt{3}\left(2x+\arctan\left(-\frac{2\sqrt{3}\sin\left(4x+2\right)-3\sin\left(4x+2\right)}{2\sqrt{3}\cos\left(4x+2\right)+2\sqrt{3}-3\cos\left(4x+2\right)+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(2*x + arctan(-(2*sqrt(3)*sin(4*x + 2) - 3*sin(4*x + 2))/(2*sq
rt(3)*cos(4*x + 2) + 2*sqrt(3) - 3*cos(4*x + 2) + 3)) + 1)
```

3.482 $\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}-\frac{x}{a-b}$$

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/((a - b)*Sqrt[b])
```

Rubi [A] time = 0.149493, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {481, 203, 205}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}-\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/((a - b)*Sqrt[b])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\left(a+bx^2\right)}\,dx,x,\tan(x)\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)}{a-b}+\frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(x)\right)}{a-b}\\ &= -\frac{x}{a-b}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0989602, size = 36, normalized size = 0.84

$$\frac{x-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{b-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[b])/(-a + b)
```

Maple [A] time = 0.051, size = 38, normalized size = 0.9

$$-\frac{\arctan(\tan(x))}{a-b}+\frac{a}{a-b}\arctan\left(b\tan(x)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x)
```

```
[Out] -1/(a-b)*arctan(tan(x))+a/(a-b)/(a*b)^(1/2)*arctan(tan(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99184, size = 433, normalized size = 10.07

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(x)^4-2\left(3\,ab+b^2\right)\cos(x)^2+4\left(\left(ab+b^2\right)\cos(x)^3-b^2\cos(x)\right)\sqrt{-\frac{a}{b}}\sin(x)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(x)^4+2\left(ab-b^2\right)\cos(x)^2+b^2}\right)+4\,x}{4\,(a-b)}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{\left((a+b)\cos(x)^2-b\right)\sqrt{\frac{a}{b}}}{2\,a\cos(x)\sin(x)}\right)}{2\,(a-b)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a/b)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x
)^2 + 4*((a*b + b^2)*cos(x)^3 - b^2*cos(x))*sqrt(-a/b)*sin(x) + b^2)/((a^2
- 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) + 4*x)/(a - b), -1
/2*(sqrt(a/b)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(a/b)/(a*cos(x)*sin(x))
) + 2*x)/(a - b)]
```

Sympy [A] time = 2.53662, size = 241, normalized size = 5.6

$$\begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{x\sin^2(x)}{2b\sin^2(x)+2b\cos^2(x)}+\frac{x\cos^2(x)}{2b\sin^2(x)+2b\cos^2(x)}-\frac{\sin(x)\cos(x)}{2b\sin^2(x)+2b\cos^2(x)} & \text{for } a=b \\ \frac{-x+\frac{\sin(x)}{\cos(x)}}{a} & \text{for } b=0 \\ -\frac{2i\sqrt{b}x\sqrt{\frac{1}{a}}}{2ia\sqrt{b}\sqrt{\frac{1}{a}}-2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}}-2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}}-2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)**2+b*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (x*sin(x)**2/(2*b*
sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) -
sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*cos(x)**2), Eq(a, b)), ((-x + sin(x)/cos
(x))/a, Eq(b, 0)), (-2*I*sqrt(b)*x*sqrt(1/a)/(2*I*a*sqrt(b)*sqrt(1/a) - 2*I
*b**(3/2)*sqrt(1/a)) - log(-I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a*sqr
t(b)*sqrt(1/a) - 2*I*b**(3/2)*sqrt(1/a)) + log(I*sqrt(b)*sqrt(1/a)*sin(x) +
 cos(x))/(2*I*a*sqrt(b)*sqrt(1/a) - 2*I*b**(3/2)*sqrt(1/a)), True))
```

Giac [B] time = 1.21473, size = 151, normalized size = 3.51

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(x)}{\sqrt{\frac{a+b+\sqrt{(a+b)^2-4\,ab}}{b}}}\right)}{|-a+b|}-\frac{\sqrt{ab}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(x)}{\sqrt{\frac{a+b-\sqrt{(a+b)^2-4\,ab}}{b}}}\right)\right)|b|}{b^2|-a+b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(2*sqrt(1/2)*tan(x)/sqrt((a + b + sqrt((a + b
)^2 - 4*a*b))/b)))/abs(-a + b) - sqrt(a*b)*(pi*floor(x/pi + 1/2) + arctan(2
*sqrt(1/2)*tan(x)/sqrt((a + b - sqrt((a + b)^2 - 4*a*b))/b)))*abs(b)/(b^2*a
bs(-a + b))
```

3.483 $\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a - b))
```

Rubi [A] time = 0.109112, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {391, 203, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a - b))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{a-b} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(x)\right)}{a-b}$$
$$= \frac{x}{a-b} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

Mathematica [A] time = 0.0583285, size = 36, normalized size = 0.84

$$\frac{x-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] (x - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[a])/(a - b)
```

Maple [A] time = 0.044, size = 36, normalized size = 0.8

$$-\frac{b}{a-b}\arctan\left(b\tan(x)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x)
```

```
[Out] -b/(a-b)/(a*b)^(1/2)*arctan(tan(x)*b/(a*b)^(1/2))+x/(a-b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94024, size = 432, normalized size = 10.05

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(x)^4-2\left(3\,ab+b^2\right)\cos(x)^2-4\left(\left(a^2+ab\right)\cos(x)^3-ab\cos(x)\right)\sqrt{-\frac{b}{a}}\sin(x)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(x)^4+2\left(ab-b^2\right)\cos(x)^2+b^2}\right)-4\,x}{4\,(a-b)},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(x)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(x)\sin(x)}\right)+}{2\,(a-b)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x
)^2 - 4*((a^2 + a*b)*cos(x)^3 - a*b*cos(x))*sqrt(-b/a)*sin(x) + b^2)/((a^2
- 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) - 4*x)/(a - b), 1/
2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(b/a)/(b*cos(x)*sin(x)))
 + 2*x)/(a - b)]
```

Sympy [A] time = 2.5537, size = 267, normalized size = 6.21

$$\begin{cases} \tilde{\infty}\left(-x-\frac{\cos(x)}{\sin(x)}\right) & \text{for } a=0 \wedge b=0 \\ \frac{-x-\frac{\cos(x)}{\sin(x)}}{b} & \text{for } a=0 \\ \frac{x\sin^2(x)}{2b\sin^2(x)+2b\cos^2(x)}+\frac{x\cos^2(x)}{2b\sin^2(x)+2b\cos^2(x)}+\frac{\sin(x)\cos(x)}{2b\sin^2(x)+2b\cos^2(x)} & \text{for } a=b \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2ia\sqrt{b}x\sqrt{\frac{1}{a}}}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}-2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{b\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}-2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{b\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}-2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a*cos(x)**2+b*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*(-x - cos(x)/sin(x)), Eq(a, 0) & Eq(b, 0)), ((-x - cos(x)/si
n(x))/b, Eq(a, 0)), (x*sin(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)
**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*co
s(x)**2), Eq(a, b)), (x/a, Eq(b, 0)), (2*I*a*sqrt(b)*x*sqrt(1/a)/(2*I*a**2*
sqrt(b)*sqrt(1/a) - 2*I*a*b**(3/2)*sqrt(1/a)) + b*log(-I*sqrt(b)*sqrt(1/a)*
sin(x) + cos(x))/(2*I*a**2*sqrt(b)*sqrt(1/a) - 2*I*a*b**(3/2)*sqrt(1/a)) -
b*log(I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a**2*sqrt(b)*sqrt(1/a) - 2*
I*a*b**(3/2)*sqrt(1/a)), True))
```

Giac [B] time = 1.12483, size = 200, normalized size = 4.65

$$-\frac{2\sqrt{ab}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(x)}{\sqrt{\frac{a+b-\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)|b|}{(a-b)^2b-\left(ab+b^2\right)|-a+b|}-\frac{2\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan(x)}{\sqrt{\frac{a+b+\sqrt{(a+b)^2-4ab}}{b}}}\right)\right)b}{(a-b)^2+a|-a+b|+b|-a+b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a*b)*(pi*floor(x/pi + 1/2) + arctan(2*sqrt(1/2)*tan(x)/sqrt((a + b
- sqrt((a + b)^2 - 4*a*b))/b)))*abs(b)/((a - b)^2*b - (a*b + b^2)*abs(-a +
b)) - 2*(pi*floor(x/pi + 1/2) + arctan(2*sqrt(1/2)*tan(x)/sqrt((a + b + sqr
t((a + b)^2 - 4*a*b))/b)))*b/((a - b)^2 + a*abs(-a + b) + b*abs(-a + b))
```

3.484 $\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx$

Optimal. Leaf size=36

$$\sqrt{2}x - x + \sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)$$

```
[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]
```

Rubi [A] time = 0.0277595, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1093, 203}

$$\sqrt{2}x - x + \sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{1+3x^2+2x^4}\,dx, x, \tan(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \tan(x)\right)$$
$$= -x+\sqrt{2}x+\sqrt{2}\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)$$

Mathematica [A] time = 0.0465269, size = 19, normalized size = 0.53

$$\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]
```

Maple [A] time = 0.046, size = 16, normalized size = 0.4

$$\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2),x)
```

```
[Out] 2^(1/2)*arctan(tan(x)*2^(1/2))-x
```

Maxima [A] time = 1.46873, size = 20, normalized size = 0.56

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

Fricas [A] time = 1.77284, size = 107, normalized size = 2.97

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) -
 x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\tan^2(x)+\sec^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2),x)
```

```
[Out] Integral(1/(tan(x)**2 + sec(x)**2), x)
```

Giac [A] time = 1.11276, size = 20, normalized size = 0.56

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

3.485 $\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{x}{\sqrt{2}}+x+\frac{\tan(x)}{2\tan^2(x)+1}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2] +
Tan[x]/(1 + 2*Tan[x]^2)
```

Rubi [A] time = 0.0452656, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {414, 12, 481, 203}

$$-\frac{x}{\sqrt{2}}+x+\frac{\tan(x)}{2\tan^2(x)+1}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2] +
Tan[x]/(1 + 2*Tan[x]^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(1+2x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{1+2\tan^2(x)}-\frac{1}{2}\,\text{Subst}\left(\int -\frac{2x^2}{\left(1+x^2\right)\left(1+2x^2\right)}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{1+2\tan^2(x)}+\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\left(1+2x^2\right)}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{1+2\tan^2(x)}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)-\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right)\\
&= x-\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}+\frac{\tan(x)}{1+2\tan^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.136855, size = 42, normalized size = 0.86

$$\frac{-3x-\sin(2x)+x\cos(2x)}{\cos(2x)-3}-\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

```
[Out] -(ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]) + (-3*x + x*Cos[2*x] - Sin[2*x])/(-3 + Co
s[2*x])
```

Maple [A] time = 0.057, size = 27, normalized size = 0.6

$$\frac{\tan(x)}{2}\left((\tan(x))^2+\frac{1}{2}\right)^{-1}-\frac{\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2)^2,x)
```

```
[Out] 1/2*tan(x)/(tan(x)^2+1/2)-1/2*2^(1/2)*arctan(tan(x)*2^(1/2))+x
```

Maxima [A] time = 1.48363, size = 36, normalized size = 0.73

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+x+\frac{\tan(x)}{2\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

Fricas [A] time = 1.82123, size = 207, normalized size = 4.22

$$\frac{4\,x\cos(x)^2+\left(\sqrt{2}\cos(x)^2-2\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-4\cos(x)\sin(x)-8\,x}{4\left(\cos(x)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 - 2*sqrt(2))*arctan(1/4*(3*sqrt(2)*co
s(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) - 8*x)/(cos(x)^2 - 2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\tan^2(x) + \sec^2(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2)**2,x)
```

```
[Out] Integral((tan(x)**2 + sec(x)**2)**(-2), x)
```

Giac [A] time = 1.12385, size = 36, normalized size = 0.73

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + x + \frac{\tan(x)}{2\tan(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

3.486 $\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3}\,dx$

Optimal. Leaf size=74

$$\frac{7x}{4\sqrt{2}}-x-\frac{\tan(x)}{4\left(2\tan^2(x)+1\right)}+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}+\frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] -x + (7*x)/(4*Sqrt[2]) + (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)
])/(4*Sqrt[2]) + Tan[x]/(2*(1 + 2*Tan[x]^2)^2) - Tan[x]/(4*(1 + 2*Tan[x]^2)
)
```

Rubi [A] time = 0.0546415, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {414, 527, 522, 203}

$$\frac{7x}{4\sqrt{2}}-x-\frac{\tan(x)}{4\left(2\tan^2(x)+1\right)}+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}+\frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

```
[Out] -x + (7*x)/(4*Sqrt[2]) + (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)
])/(4*Sqrt[2]) + Tan[x]/(2*(1 + 2*Tan[x]^2)^2) - Tan[x]/(4*(1 + 2*Tan[x]^2)
)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(1+2x^2\right)^3}\,dx,x,\tan(x)\right)\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{1}{4}\text{Subst}\left(\int \frac{-2-6x^2}{\left(1+x^2\right)\left(1+2x^2\right)^2}\,dx,x,\tan(x)\right)\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}+\frac{1}{8}\text{Subst}\left(\int \frac{6-2x^2}{\left(1+x^2\right)\left(1+2x^2\right)}\,dx,x,\tan(x)\right)\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)-\text{Subst}\left(\int \frac{}{1}\right.\\
&= -x+\frac{7x}{4\sqrt{2}}+\frac{7\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.184788, size = 79, normalized size = 1.07

$$-\frac{(\cos(2x)-3)\sec^6(x)\left(-76x-2\sin(2x)+3\sin(4x)+48x\cos(2x)-4x\cos(4x)+7\sqrt{2}(\cos(2x)-3)^2\tan^{-1}\left(\sqrt{2}\tan(x)\right)\right.}{64\left(\tan^2(x)+\sec^2(x)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

```
[Out] -((-3 + Cos[2*x])*Sec[x]^6*(-76*x + 7*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]*(-3 +
Cos[2*x])^2 + 48*x*Cos[2*x] - 4*x*Cos[4*x] - 2*Sin[2*x] + 3*Sin[4*x]))/(64*
(Sec[x]^2 + Tan[x]^2)^3)
```

Maple [A] time = 0.054, size = 40, normalized size = 0.5

$$8\,\frac{-1/16\,(\tan(x))^3+1/32\,\tan(x)}{\left(1+2\,(\tan(x))^2\right)^2}+\frac{7\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)}{8}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2)^3,x)
```

```
[Out] 8*(-1/16*tan(x)^3+1/32*tan(x))/(1+2*tan(x)^2)^2+7/8*2^(1/2)*arctan(tan(x)*2
^(1/2))-x
```

Maxima [A] time = 1.48526, size = 61, normalized size = 0.82

$$\frac{7}{8}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x-\frac{2\,\tan(x)^3-\tan(x)}{4\left(4\,\tan(x)^4+4\,\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(4*tan(x
)^4 + 4*tan(x)^2 + 1)
```

Fricas [A] time = 1.84311, size = 305, normalized size = 4.12

$$-\frac{16\,x\cos(x)^4-64\,x\cos(x)^2+7\left(\sqrt{2}\cos(x)^4-4\sqrt{2}\cos(x)^2+4\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-4\left(3\,\cos(x)^3-2\,\cos(\right.}{16\left(\cos(x)^4-4\,\cos(x)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*x*cos(x)^4 - 64*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(
x)^2 + 4*sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x
))) - 4*(3*cos(x)^3 - 2*cos(x))*sin(x) + 64*x)/(cos(x)^4 - 4*cos(x)^2 + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\tan^2(x) + \sec^2(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2)**3,x)
```

```
[Out] Integral((tan(x)**2 + sec(x)**2)**(-3), x)
```

Giac [A] time = 1.16182, size = 53, normalized size = 0.72

$$\frac{7}{8}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x - \frac{2\tan(x)^3 - \tan(x)}{4\left(2\tan(x)^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(2*tan(x
)^2 + 1)^2
```

3.487 $\int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.012288, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0005178, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-1),x]
```

```
[Out] x
```

Maple [C] time = 0.024, size = 4, normalized size = 4.

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2),x)
```

```
[Out] arctan(tan(x))
```

Maxima [A] time = 1.48744, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.6008, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\tan(x) + \sec(x))(\tan(x) + \sec(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2),x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))*(tan(x) + sec(x))), x)
```

Giac [A] time = 1.11368, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="giac")
```

```
[Out] x
```

3.488 $\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0125699, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.0004217, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

```
[Out] x
```

Maple [C] time = 0.024, size = 4, normalized size = 4.

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2)^2,x)
```

```
[Out] arctan(tan(x))
```

Maxima [A] time = 1.49048, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.57713, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-\tan(x) + \sec(x))^2 (\tan(x) + \sec(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2)**2,x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))**2*(tan(x) + sec(x))**2), x)
```

Giac [A] time = 1.14096, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

3.489 $\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0123656, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0004323, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

```
[Out] x
```

Maple [C] time = 0.028, size = 4, normalized size = 4.

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2)^3,x)
```

```
[Out] arctan(tan(x))
```

Maxima [A] time = 1.50833, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.61466, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\tan\left(x\right) + \sec\left(x\right)\right)^{3}\left(\tan\left(x\right) + \sec\left(x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2)**3,x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))**3*(tan(x) + sec(x))**3), x)
```

Giac [A] time = 1.11735, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="giac")
```

```
[Out] x
```

3.490 $\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx$

Optimal. Leaf size=37

$$\sqrt{2}x - x - \sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)$$

```
[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]
```

Rubi [A] time = 0.0310605, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1130, 203}

$$\sqrt{2}x - x - \sqrt{2}\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = \text{Subst}\left(\int \frac{x^2}{2+3x^2+x^4}\,dx, x, \tan(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= -x + \sqrt{2}x - \sqrt{2}\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)$$

Mathematica [A] time = 0.0426427, size = 19, normalized size = 0.51

$$\sqrt{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]
```

Maple [A] time = 0.077, size = 17, normalized size = 0.5

$$\sqrt{2}\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2),x)
```

```
[Out] 2^(1/2)*arctan(1/2*tan(x)*2^(1/2))-x
```

Maxima [A] time = 1.48216, size = 22, normalized size = 0.59

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

Fricas [A] time = 1.89927, size = 104, normalized size = 2.81

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\cot^2(x)+\csc^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2),x)
```

```
[Out] Integral(1/(cot(x)**2 + csc(x)**2), x)
```

Giac [A] time = 1.13108, size = 66, normalized size = 1.78

$$\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt
(2) - cos(2*x) + 1))) - x
```

3.491 $\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan(x)}{\tan^2(x)+2}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] x - x/Sqrt[2] + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/Sqrt[2] -
Tan[x]/(2 + Tan[x]^2)
```

Rubi [A] time = 0.0400156, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 12, 391, 203}

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan(x)}{\tan^2(x)+2}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-2),x]
```

```
[Out] x - x/Sqrt[2] + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/Sqrt[2] -
Tan[x]/(2 + Tan[x]^2)
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)\left(2+x^2\right)^2}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\frac{1}{2}\,\text{Subst}\left(\int \frac{2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(2+x^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)-\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(x)\right)\\
&= x-\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}-\frac{\tan(x)}{2+\tan^2(x)}
\end{aligned}
$$

Mathematica [A] time = 0.106036, size = 64, normalized size = 1.36

$$
\frac{(\cos(2x)+3)\csc^4(x)\left(6x-2\sin(2x)+2x\cos(2x)-\sqrt{2}(\cos(2x)+3)\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)\right)}{8\left(\cot^2(x)+\csc^2(x)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-2),x]
```

```
[Out] ((3 + Cos[2*x])*Csc[x]^4*(6*x + 2*x*Cos[2*x] - Sqrt[2]*ArcTan[Tan[x]/Sqrt[2
]]*(3 + Cos[2*x]) - 2*Sin[2*x]))/(8*(Cot[x]^2 + Csc[x]^2)^2)
```

Maple [A] time = 0.103, size = 28, normalized size = 0.6

$$-\frac{\tan(x)}{2+(\tan(x))^2}-\frac{\sqrt{2}}{2}\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2)^2,x)
```

```
[Out] -tan(x)/(2+tan(x)^2)-1/2*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))+x
```

Maxima [A] time = 1.49496, size = 36, normalized size = 0.77

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)+x-\frac{\tan(x)}{\tan(x)^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) + x - tan(x)/(tan(x)^2 + 2)
```

Fricas [A] time = 1.9151, size = 201, normalized size = 4.28

$$\frac{4\,x\cos(x)^2+\left(\sqrt{2}\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-4\cos(x)\sin(x)+4\,x}{4\left(\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(
x)^2 - sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) + 4*x)/(cos(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15392, size = 81, normalized size = 1.72

$$-\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)+x-\frac{\tan(x)}{\tan(x)^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) + x - tan(x)/(tan(x)^2 + 2)
```

3.492 $\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3}\,dx$

Optimal. Leaf size=72

$$\frac{7x}{4\sqrt{2}}-x-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}+\frac{\tan(x)}{4\left(\tan^2(x)+2\right)}-\frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] -x + (7*x)/(4*Sqrt[2]) - (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)
])/(4*Sqrt[2]) - Tan[x]^3/(2*(2 + Tan[x]^2)^2) + Tan[x]/(4*(2 + Tan[x]^2))
```

Rubi [A] time = 0.0755278, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 578, 522, 203}

$$\frac{7x}{4\sqrt{2}}-x-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}+\frac{\tan(x)}{4\left(\tan^2(x)+2\right)}-\frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```


```
[Out] -x + (7*x)/(4*Sqrt[2]) - (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)
])/(4*Sqrt[2]) - Tan[x]^3/(2*(2 + Tan[x]^2)^2) + Tan[x]/(4*(2 + Tan[x]^2))
```


Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
```

```
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)\left(2+x^2\right)^3}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(6+2x^2\right)}{\left(1+x^2\right)\left(2+x^2\right)^2}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}-\frac{1}{8}\text{Subst}\left(\int \frac{2-6x^2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(x)\right)-\text{Subst}\left(\int \frac{1}{1+
\right.\\
&= -x+\frac{7x}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.16371, size = 66, normalized size = 0.92

$$\frac{-76x+2\sin(2x)+3\sin(4x)-48x\cos(2x)-4x\cos(4x)+7\sqrt{2}(\cos(2x)+3)^2\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{8(\cos(2x)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```

```
[Out] (-76*x - 48*x*Cos[2*x] + 7*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]*(3 + Cos[2*x])^2
- 4*x*Cos[4*x] + 2*Sin[2*x] + 3*Sin[4*x])/(8*(3 + Cos[2*x])^2)
```

Maple [A] time = 0.121, size = 39, normalized size = 0.5

$$2\,\frac{-1/8\,\left(\tan\left(x\right)\right)^{3}+1/4\,\tan\left(x\right)}{\left(2+\left(\tan\left(x\right)\right)^{2}\right)^{2}}+\frac{7\,\sqrt{2}}{8}\arctan\left(\frac{\tan\left(x\right)\sqrt{2}}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2)^3,x)
```

```
[Out] 2*(-1/8*tan(x)^3+1/4*tan(x))/(2+tan(x)^2)^2+7/8*2^(1/2)*arctan(1/2*tan(x)*2
^(1/2))-x
```

Maxima [A] time = 1.51483, size = 57, normalized size = 0.79

$$\frac{7}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\tan\left(x\right)\right)-x-\frac{\tan\left(x\right)^{3}-2\,\tan\left(x\right)}{4\left(\tan\left(x\right)^{4}+4\,\tan\left(x\right)^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x - 1/4*(tan(x)^3 - 2*tan(x))/(tan
(x)^4 + 4*tan(x)^2 + 4)
```

Fricas [A] time = 1.85431, size = 297, normalized size = 4.12

$$-\frac{16\,x\cos\left(x\right)^{4}+32\,x\cos\left(x\right)^{2}+7\left(\sqrt{2}\cos\left(x\right)^{4}+2\,\sqrt{2}\cos\left(x\right)^{2}+\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\cos(x)^{2}-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-4\left(3\,\cos\left(x\right)^{3}-\cos\left(x\right)\right)\text{s}}{16\left(\cos\left(x\right)^{4}+2\,\cos\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*x*cos(x)^4 + 32*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(
x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
- 4*(3*cos(x)^3 - cos(x))*sin(x) + 16*x)/(cos(x)^4 + 2*cos(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13138, size = 93, normalized size = 1.29

$$\frac{7}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-\cos\left(2x\right)+1}\right)\right)-x-\frac{\tan\left(x\right)^{3}-2\tan\left(x\right)}{4\left(\tan\left(x\right)^{2}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) - x - 1/4*(tan(x)^3 - 2*tan(x))/(tan(x)^2 + 2)^2
```

3.493 $\int \frac{1}{\cot^2(x)-\csc^2(x)} \, dx$

Optimal. Leaf size=3

$$-x$$

```
[Out] -x
```

Rubi [A] time = 0.0131306, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$-x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-1),x]

[Out] -x
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\cot^2(x)-\csc^2(x)} \, dx &= -\int 1 \, dx \\ &= -x \end{aligned}$$

Mathematica [A] time = 0.0004945, size = 3, normalized size = 1.

$$-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-1),x]
```

```
[Out] -x
```

Maple [C] time = 0.023, size = 6, normalized size = 2.

$$-\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2),x)
```

```
[Out] -arctan(tan(x))
```

Maxima [A] time = 1.48515, size = 4, normalized size = 1.33

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="maxima")
```

```
[Out] -x
```

Fricas [A] time = 1.64396, size = 5, normalized size = 1.67

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="fricas")
```

```
[Out] -x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\cot(x) - \csc(x))(\cot(x) + \csc(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2),x)
```

```
[Out] Integral(1/((cot(x) - csc(x))*(cot(x) + csc(x))), x)
```

Giac [A] time = 1.12555, size = 4, normalized size = 1.33

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="giac")
```

```
[Out] -x
```

3.494 $\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} \, dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.0129495, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 - Csc[x]^2)^(-2),x]

[Out] x

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cot^2(x) - \csc^2(x)\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0006205, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-2),x]
```

```
[Out] x
```

Maple [C] time = 0.027, size = 4, normalized size = 4.

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2)^2,x)
```

```
[Out] arctan(tan(x))
```

Maxima [A] time = 1.48911, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.76238, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14184, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

3.495 $\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx$

Optimal. Leaf size=3

$$-x$$

```
[Out] -x
```

Rubi [A] time = 0.0131694, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$-x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

```
[Out] -x
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx &= -\int 1\,dx\\ &= -x\end{aligned}$$

Mathematica [A] time = 0.0005598, size = 3, normalized size = 1.

$$-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

```
[Out] -x
```

Maple [C] time = 0.029, size = 6, normalized size = 2.

$$-\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2)^3,x)
```

```
[Out] -arctan(tan(x))
```

Maxima [A] time = 1.48669, size = 4, normalized size = 1.33

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -x
```

Fricas [A] time = 1.48543, size = 5, normalized size = 1.67

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12165, size = 4, normalized size = 1.33

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="giac")
```

```
[Out] -x
```

3.496 $\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{a+c}}$$

```
[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

Rubi [A] time = 0.0503087, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+b+(a+c)x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{a+c}}\end{aligned}$$

Mathematica [A] time = 0.0625933, size = 33, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

Maple [A] time = 0.032, size = 27, normalized size = 0.8

$$\arctan\left((a+c)\tan(x)\frac{1}{\sqrt{(a+b)(a+c)}}\right)\frac{1}{\sqrt{(a+b)(a+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2+c*sin(x)^2),x)
```

```
[Out] 1/((a+b)*(a+c))^(1/2)*arctan((a+c)*tan(x)/((a+b)*(a+c))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14726, size = 660, normalized size = 20.

$$\left[-\frac{\sqrt{-a^2-ab-(a+b)c}\log\left(\frac{\left(8a^2+8ab+b^2+2(4a+3b)c+c^2\right)\cos(x)^4-2\left(4a^2+3ab+(5a+3b)c+c^2\right)\cos(x)^2+4\left((2a+b+c)\cos(x)^3-(a+c)\cos(x)\right)\sqrt{-a^2-}}{\left(b^2-2bc+c^2\right)\cos(x)^4+2\left(ab-(a-b)c-c^2\right)\cos(x)^2+a^2+2ac+c^2}\right.}{4\left(a^2+ab+(a+b)c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b - (a + b)*c)*log(((8*a^2 + 8*a*b + b^2 + 2*(4*a + 3*b
)*c + c^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b + (5*a + 3*b)*c + c^2)*cos(x)^2 + 4*
((2*a + b + c)*cos(x)^3 - (a + c)*cos(x))*sqrt(-a^2 - a*b - (a + b)*c)*sin(
x) + a^2 + 2*a*c + c^2)/((b^2 - 2*b*c + c^2)*cos(x)^4 + 2*(a*b - (a - b)*c
- c^2)*cos(x)^2 + a^2 + 2*a*c + c^2))/(a^2 + a*b + (a + b)*c), -1/2*arctan(
1/2*((2*a + b + c)*cos(x)^2 - a - c)/(sqrt(a^2 + a*b + (a + b)*c)*cos(x)*si
n(x)))/sqrt(a^2 + a*b + (a + b)*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\cos^2(x) + c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(1/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

Giac [B] time = 1.14046, size = 82, normalized size = 2.48

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2c)+\arctan\left(\frac{a\tan(x)+c\tan(x)}{\sqrt{a^2+ab+ac+bc}}\right)}{\sqrt{a^2+ab+ac+bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*c) + arctan((a*tan(x) + c*tan(x))/sqrt(a^
2 + a*b + a*c + b*c)))/sqrt(a^2 + a*b + a*c + b*c)
```

3.497 $\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=239

$$-\frac{\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}+\frac{\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}+\frac{ix\log\left(1+\right.}{2\sqrt{a}}$$

```
[Out] ((-I/2)*x*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a
 + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x*Log[1 + ((b - c)*E^((2*I)*x))
/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) - Po
lyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a + c]))
]/(4*Sqrt[a + b]*Sqrt[a + c]) + PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b
 + c + 2*Sqrt[a + b]*Sqrt[a + c]))]/(4*Sqrt[a + b]*Sqrt[a + c])
```

Rubi [A] time = 0.491629, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4587, 3321, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}+\frac{\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}+\frac{ix\log\left(1+\right.}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-I/2)*x*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a
 + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x*Log[1 + ((b - c)*E^((2*I)*x))
/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) - Po
lyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a + c]))
]/(4*Sqrt[a + b]*Sqrt[a + c]) + PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b
 + c + 2*Sqrt[a + b]*Sqrt[a + c]))]/(4*Sqrt[a + b]*Sqrt[a + c])
```

Rule 4587

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(2*a
+ b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g},
 x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx = 2\int \frac{x}{2a+b+c+(b-c)\cos(2x)}\,dx$$

$$= 4\int \frac{e^{2ix}x}{b-c+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}}\,dx$$

$$= \frac{(2(b-c))\int \frac{e^{2ix}x}{-4\sqrt{a+b}\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2ix}}\,dx}{\sqrt{a+b}\sqrt{a+c}} - \frac{(2(b-c))\int \frac{e^{2ix}x}{4\sqrt{a+b}\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2\ldots}}}{\sqrt{a+b}\sqrt{a+c}}$$

$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{i\int\log\left(1+\frac{2\ldots}{-4\sqrt{a+b}\ldots}\right.}{2\sqrt{a+b}\ldots}$$

$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{\ldots}{-4\sqrt{\ldots}}\right.}{\ldots}\right.}{4\sqrt{a}\ldots}$$

$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\ldots}\right.}{4\sqrt{a+b}\sqrt{a+c}\ldots}$$

Mathematica [B] time = 3.1501, size = 507, normalized size = 2.12

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)\left(2x+\frac{i\left(-\text{PolyLog}\left(2,\frac{\sqrt{a+b}-i\sqrt{a+c}\tan(x)}{\sqrt{a+b}-\sqrt{a+c}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{a+b}-i\sqrt{a+c}\tan(x)}{\sqrt{a+b}+\sqrt{a+c}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{a+b}+i\sqrt{a+c}\tan(x)}{\sqrt{a+b}-\sqrt{a+c}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{a+b}+i\sqrt{a+c}\ldots}{\sqrt{a+b}+\ldots}\right.\ldots}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]*(2*x + (I*(Log[(Sqrt[a + c]*(1 +
I*Tan[x]))/(Sqrt[a + b] + Sqrt[a + c])]*Log[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt
[a + b]] - Log[(I*Sqrt[a + c]*(I + Tan[x]))/(Sqrt[a + b] - Sqrt[a + c])]*Lo
g[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] + Log[(Sqrt[a + c]*(1 - I*Tan[x])
)/(Sqrt[a + b] + Sqrt[a + c])]*Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]]
- Log[(Sqrt[a + c]*(1 + I*Tan[x]))/(-Sqrt[a + b] + Sqrt[a + c])]*Log[1 + (I
*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] - PolyLog[2, (Sqrt[a + b] - I*Sqrt[a + c]
*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])] + PolyLog[2, (Sqrt[a + b] - I*Sqrt[a
+ c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])] - PolyLog[2, (Sqrt[a + b] + I*Sqr
t[a + c]*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])] + PolyLog[2, (Sqrt[a + b] + I
*Sqrt[a + c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])]))/(Log[1 - (I*Sqrt[a + c]
```

```
*Tan[x])/Sqrt[a + b]] - Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]])))/(2*S
qrt[a + b]*Sqrt[a + c])
```

Maple [B] time = 0.105, size = 820, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(x)^2+c*sin(x)^2),x)
```

```
[Out] -I/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))
^(1/2)-2*a-b-c))*x-I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*l
n(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*a*x-1/2*I/((a+b)*(a+
c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)
*(a+c))^(1/2)-2*a-b-c))*b*x-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/
2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*c*x-1/(
-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x^2-1/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))
^(1/2)-2*a-b-c)*a*x^2-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b
-c)*b*x^2-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x^2-1/
2/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a
+c))^(1/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*a-1/4/((a+b
)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/
(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*b-1/4/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c
))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))*c-1/2*I/((a+b)*(a+c))^(1/2)*x*ln(1-(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1
/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)*x^2-1/4/((a+b)*(a+c))^(1/2)*polylog(2
,(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cos{(x)}^2 + c\sin{(x)}^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

Fricas [B] time = 4.13438, size = 7385, normalized size = 30.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-
1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*
cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) +
2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b - c)*x*sqrt((a^2 + a*b
 + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) - (4*I*
a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a
*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*c)/(
b - c)) - 4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*l
og(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b
- c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c
 + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x*sqrt((a^2 +
 a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) - (
-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*
c)/(b - c)) - 4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2
))*log(-1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*(
(b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^
2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) -
 (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*
c)/(b - c)) + 4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2
))*log(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*
((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b - c)*x*sqrt((a
```

```
^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x)
 - (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x
))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b
- 2*c)/(b - c)) + 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2
))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*
((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
 c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) + 4*(b - c)*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*co
s(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*si
n(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) +
2*b - 2*c)/(b - c) + 1) + 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b
*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*si
n(x) - 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) + 4*(b - c
)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b
 + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b -
 I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(
b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b
- c)) + 2*b - 2*c)/(b - c) + 1) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b
^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*
I*c)*sin(x) + 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a
+ b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) - 4*
(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2
*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (
I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(
(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)
/(b - c)) + 2*b - 2*c)/(b - c) + 1) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c
)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b
 - 2*I*c)*sin(x) + 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b
 + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c) + 1
) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*
((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos
(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a
- b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1))/(a^2 + a*b + (a + b)*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(x/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cos(x)^2+c\sin(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

3.498 $\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=365

$$-\frac{x\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}+\frac{x\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}-\frac{i\text{PolyLog}\left(3,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}+\frac{i\text{PolyI}}{}$$

```
[Out] ((-I/2)*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt
[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x^2*Log[1 + ((b - c)*E^((2*I)
*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c])
- (x*PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a
 + c]))])/(2*Sqrt[a + b]*Sqrt[a + c]) + (x*PolyLog[2, -(((b - c)*E^((2*I)*x
))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(2*Sqrt[a + b]*Sqrt[a + c])
 - ((I/4)*PolyLog[3, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*S
qrt[a + c]))])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/4)*PolyLog[3, -(((b - c)*E^(
(2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(Sqrt[a + b]*Sqrt[a
+ c])
```

Rubi [A] time = 0.738861, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4587, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}+\frac{x\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}-\frac{i\text{PolyLog}\left(3,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}+\frac{i\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```


```
[Out] ((-I/2)*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt
[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x^2*Log[1 + ((b - c)*E^((2*I)
*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c])
- (x*PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a
 + c]))])/(2*Sqrt[a + b]*Sqrt[a + c]) + (x*PolyLog[2, -(((b - c)*E^((2*I)*x
))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(2*Sqrt[a + b]*Sqrt[a + c])
 - ((I/4)*PolyLog[3, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*S
qrt[a + c]))])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/4)*PolyLog[3, -(((b - c)*E^(
(2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(Sqrt[a + b]*Sqrt[a
+ c])
```

Rule 4587

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(2*a
+ b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g},
 x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx = 2\int \frac{x^2}{2a+b+c+(b-c)\cos(2x)}\,dx$$

$$= 4\int \frac{e^{2ix}x^2}{b-c+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}}\,dx$$

$$= \frac{(2(b-c))\int \frac{e^{2ix}x^2}{-4\sqrt{a+b}\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2ix}}\,dx}{\sqrt{a+b}\sqrt{a+c}} - \frac{(2(b-c))\int \frac{e^{2ix}x^2}{4\sqrt{a+b}\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2ix}}}{\sqrt{a+b}\sqrt{a+c}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{i\int x\log\left(1+\frac{}{-4\sqrt{a-}}\right.}{\sqrt{a+b}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{x\text{Li}_2\left(-\frac{(b-c)e^{2i}}{2a+b+c-2\sqrt{a+}}\right.}{2\sqrt{a+b}\sqrt{a+}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{x\text{Li}_2\left(-\frac{(b-c)e^{2i}}{2a+b+c-2\sqrt{a+}}\right.}{2\sqrt{a+b}\sqrt{a+}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{x\text{Li}_2\left(-\frac{(b-c)e^{2i}}{2a+b+c-2\sqrt{a+}}\right.}{2\sqrt{a+b}\sqrt{a+}}$$

Mathematica [A] time = 4.03247, size = 258, normalized size = 0.71

$$-\frac{i\left(-2ix\text{PolyLog}\left(2,\frac{e^{2ix}(c-b)}{-2\sqrt{(a+b)(a+c)}+2a+b+c}\right)+2ix\text{PolyLog}\left(2,\frac{e^{2ix}(c-b)}{2\sqrt{(a+b)(a+c)}+2a+b+c}\right)+\text{PolyLog}\left(3,\frac{e^{2ix}(c-b)}{-2\sqrt{(a+b)(a+c)}+2a+b+c}\right)-\text{Po}\right.}{4\sqrt{(a+b)(a+c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-I/4)*(2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[(a + b)*
(a + c)])] - 2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[(a +
 b)*(a + c)])] - (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a + b + c - 2
*Sqrt[(a + b)*(a + c)])] + (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a +
 b + c + 2*Sqrt[(a + b)*(a + c)])] + PolyLog[3, ((-b + c)*E^((2*I)*x))/(2*a
 + b + c - 2*Sqrt[(a + b)*(a + c)])] - PolyLog[3, ((-b + c)*E^((2*I)*x))/(2
*a + b + c + 2*Sqrt[(a + b)*(a + c)])]))/Sqrt[(a + b)*(a + c)]
```

Maple [B] time = 0.11, size = 1161, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(x)^2+c*sin(x)^2),x)
```

```
[Out] -2/3/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x^3-1/4*I/((a+b)*(a+c))^(1/2)/(-2*((a
+b)*(a+c))^(1/2)-2*a-b-c)*c*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1
/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x*
polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/4*I/((a+b)*(
a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*polylog(3,(b-c)*exp(2*I*x)/(
-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1
/2)-2*a-b-c)*a*x*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c
))-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x^2*ln(1-(b
-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/3/((a+b)*(a+c))^(1/2)/(-
2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x^3-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(
a+c))^(1/2)-2*a-b-c)*a*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2
*a-b-c))-1/2/((a+b)*(a+c))^(1/2)*x*polylog(2,(b-c)*exp(2*I*x)/(2*((a+b)*(a+
c))^(1/2)-2*a-b-c))-1/4*I/((a+b)*(a+c))^(1/2)*polylog(3,(b-c)*exp(2*I*x)/(2
*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x*polylog
(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2*I/(-2*((a+b)*(a+c
))^(1/2)-2*a-b-c)*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x*polylog(2,
(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-I/(-2*((a+b)*(a+c))^(1/2
)-2*a-b-c)*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/3/
((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x^3-I/((a+b)*(a+c))^
(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+
b)*(a+c))^(1/2)-2*a-b-c))-1/3/((a+b)*(a+c))^(1/2)*x^3-1/2*I/((a+b)*(a+c))^(
1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b
)*(a+c))^(1/2)-2*a-b-c))-2/3/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*
a-b-c)*a*x^3-1/2*I/((a+b)*(a+c))^(1/2)*x^2*ln(1-(b-c)*exp(2*I*x)/(2*((a+b)*
(a+c))^(1/2)-2*a-b-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cos(x)^2 + c\sin(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

Fricas [C] time = 4.55783, size = 11062, normalized size = 30.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log
(-1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c
)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
 c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
+ 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b - c)*x^2*sqrt((a^2 +
 a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) - (
4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*
c)/(b - c)) - 4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2))*log(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) -
4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c
+ c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x^2*sqr
t((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*co
s(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b - I*c)*si
n(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) +
2*b - 2*c)/(b - c)) - 4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2
*b*c + c^2))*log(-1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*si
n(x) + 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
```

/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x
^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b +
c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*
c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b -
c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)
) + 2*b - 2*c)/(b - c)) + 4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2
- 2*b*c + c^2))*log(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*
c)*sin(x) + 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(
b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b
- c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a
+ b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-
I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(
(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)
/(b - c)) + 2*b - 2*c)/(b - c)) + 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b +
2*I*c)*sin(x) - 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b +
(a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)
*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c) + 1)
+ 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*
((2*(2*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(
x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
)*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a
+ b + c)/(b - c)) + 2*b - 2*c)/(b - c) + 1) + 8*(b - c)*x*sqrt((a^2 + a*b +
(a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (-4*I
*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a
*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(
b - c) + 1) + 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
dilog(-1/2((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*
((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2
*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*c)/(b - c) + 1) - 8*(b - c)*x*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*co
s(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) + (I*b - I*c)*sin
(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqr
t((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*
b + 2*c)/(b - c) + 1) - 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b
*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*si
n(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1) - 8*(b - c)
*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b
+ c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) + (-I*b

```
+ I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(
b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b
- c)) - 2*b + 2*c)/(b - c) + 1) - 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b
- 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b
)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1)
 + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*poly
log(3, -1/2*(2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((
b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)) + 2*a + b + c)/(b - c))/(b - c)) + 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) -
(4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^
2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))/(b - c))
+ 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*poly
log(3, -1/2*(2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*(
(b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)) + 2*a + b + c)/(b - c))/(b - c)) + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b +
 (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (
-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))/(b - c)) +
 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*polyl
og(3, -1/2*(2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b
 - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*
c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
) - 2*a - b - c)/(b - c))/(b - c)) + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b + (a
 + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (4*I
*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*
b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c))/(b - c)) + 4*(2
*I*b - 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3,
-1/2*(2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)
*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) -
2*a - b - c)/(b - c))/(b - c)) + 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (-4*I*a
 - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a
^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b
 + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c))/(b - c)))/(a^2 +
 a*b + (a + b)*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(x**2/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cos(x)^2+c\sin(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

3.499 $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2$

Optimal. Leaf size=195

$$-\frac{b\left(69a^2b^2+32a^4+4b^4\right)\cos(d+ex)}{10e}-\frac{\left(5a^2+4b^2\right)\cos(d+ex)(a\sin(d+ex)+b)^3}{20e}-\frac{b\left(17a^2+4b^2\right)\cos(d+ex)(a\sin(d}{20e}$$

```
[Out] (3*a*(a^4 + 12*a^2*b^2 + 8*b^4)*x)/8 - (b*(32*a^4 + 69*a^2*b^2 + 4*b^4)*Cos
[d + e*x])/(10*e) - (a*(15*a^4 + 82*a^2*b^2 + 8*b^4)*Cos[d + e*x]*Sin[d + e
*x])/(40*e) - (b*(17*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^2)/(20*
e) - ((5*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^3)/(20*e) - (b*Cos[
d + e*x]*(b + a*Sin[d + e*x])^4)/(5*e)
```

Rubi [A] time = 0.392681, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3288, 2753, 2734}

$$-\frac{b\left(69a^2b^2+32a^4+4b^4\right)\cos(d+ex)}{10e}-\frac{\left(5a^2+4b^2\right)\cos(d+ex)(a\sin(d+ex)+b)^3}{20e}-\frac{b\left(17a^2+4b^2\right)\cos(d+ex)(a\sin(d}{20e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2,
x]
```


```
[Out] (3*a*(a^4 + 12*a^2*b^2 + 8*b^4)*x)/8 - (b*(32*a^4 + 69*a^2*b^2 + 4*b^4)*Cos
[d + e*x])/(10*e) - (a*(15*a^4 + 82*a^2*b^2 + 8*b^4)*Cos[d + e*x]*Sin[d + e
*x])/(40*e) - (b*(17*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^2)/(20*
e) - ((5*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^3)/(20*e) - (b*Cos[
d + e*x]*(b + a*Sin[d + e*x])^4)/(5*e)
```


Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx=\frac{\int\left(2ab+2a^2\sin(d+ex)\right)^4(a+b\sin(d+ex))\,dx}{16a^4}$$
$$=-\frac{b\cos(d+ex)(b+a\sin(d+ex))^4}{5e}+\frac{\int\left(2ab+2a^2\right.}{}$$
$$=-\frac{\left(5a^2+4b^2\right)\cos(d+ex)(b+a\sin(d+ex))^3}{20e}-\frac{b\,\mathrm{co}}{}$$
$$=-\frac{b\left(17a^2+4b^2\right)\cos(d+ex)(b+a\sin(d+ex))^2}{20e}-$$
$$=\frac{3}{8}a\left(a^4+12a^2b^2+8b^4\right)x-\frac{b\left(32a^4+69a^2b^2+4b^4\right.}{10e}$$

Mathematica [A] time = 0.942855, size = 149, normalized size = 0.76

$$\frac{a\left(60\left(12a^2b^2+a^4+8b^4\right)(d+ex)-40\left(10a^2b^2+a^4+4b^4\right)\sin(2(d+ex))+5\left(4a^2b^2+a^4\right)\sin(4(d+ex))+10\left(7a^3b+\right.\right.}{160e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^2,x]
```

```
[Out] (-20*b*(29*a^4 + 68*a^2*b^2 + 8*b^4)*Cos[d + e*x] + a*(60*(a^4 + 12*a^2*b^2
 + 8*b^4)*(d + e*x) + 10*(7*a^3*b + 8*a*b^3)*Cos[3*(d + e*x)] - 2*a^3*b*Cos
[5*(d + e*x)] - 40*(a^4 + 10*a^2*b^2 + 4*b^4)*Sin[2*(d + e*x)] + 5*(a^4 + 4
*a^2*b^2)*Sin[4*(d + e*x)]))/(160*e)
```

Maple [A] time = 0.035, size = 255, normalized size = 1.3

$$\frac{1}{e}\left(ab^4(ex+d)-4\cos(ex+d)a^2b^3+6a^3b^2(-1/2\sin(ex+d)\cos(ex+d)+1/2ex+d/2)-\frac{4a^4b\left(2+(\sin(ex+d))^2\right)\cos}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x)
```

```
[Out] 1/e*(a*b^4*(e*x+d)-4*cos(e*x+d)*a^2*b^3+6*a^3*b^2*(-1/2*sin(e*x+d)*cos(e*x+
d)+1/2*e*x+1/2*d)-4/3*a^4*b*(2+sin(e*x+d)^2)*cos(e*x+d)+a^5*(-1/4*(sin(e*x+
d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-cos(e*x+d)*b^5+4*a*b^4*(-1/2
*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-2*a^2*b^3*(2+sin(e*x+d)^2)*cos(e*x+d)
+4*a^3*b^2*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-1/
5*a^4*b*(8/3+sin(e*x+d)^4+4/3*sin(e*x+d)^2)*cos(e*x+d))
```

Maxima [A] time = 1.03736, size = 332, normalized size = 1.7

$$\frac{15(12ex+12d+\sin(4ex+4d)-8\sin(2ex+2d))a^5-32\left(3\cos(ex+d)^5-10\cos(ex+d)^3+15\cos(ex+d)\right)a^4b+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/480*(15*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x + 2*d))*a^5 - 32*
(3*cos(e*x + d)^5 - 10*cos(e*x + d)^3 + 15*cos(e*x + d))*a^4*b + 640*(cos(e
*x + d)^3 - 3*cos(e*x + d))*a^4*b + 60*(12*e*x + 12*d + sin(4*e*x + 4*d) -
8*sin(2*e*x + 2*d))*a^3*b^2 + 720*(2*e*x + 2*d - sin(2*e*x + 2*d))*a^3*b^2
+ 960*(cos(e*x + d)^3 - 3*cos(e*x + d))*a^2*b^3 + 480*(2*e*x + 2*d - sin(2*
e*x + 2*d))*a*b^4 + 480*(e*x + d)*a*b^4 - 1920*a^2*b^3*cos(e*x + d) - 480*b
^5*cos(e*x + d))/e
```

Fricas [A] time = 1.82638, size = 348, normalized size = 1.78

$$-\frac{8a^4b\cos(ex+d)^5-80\left(a^4b+a^2b^3\right)\cos(ex+d)^3-15\left(a^5+12a^3b^2+8ab^4\right)ex+40\left(5a^4b+10a^2b^3+b^5\right)\cos(ex+d)}{40e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] -1/40*(8*a^4*b*cos(e*x + d)^5 - 80*(a^4*b + a^2*b^3)*cos(e*x + d)^3 - 15*(a
^5 + 12*a^3*b^2 + 8*a*b^4)*e*x + 40*(5*a^4*b + 10*a^2*b^3 + b^5)*cos(e*x +
d) - 5*(2*(a^5 + 4*a^3*b^2)*cos(e*x + d)^3 - (5*a^5 + 44*a^3*b^2 + 16*a*b^4
)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [A] time = 3.51857, size = 566, normalized size = 2.9

$$\begin{cases} \frac{3a^5x\sin^4(d+ex)}{8} + \frac{3a^5x\sin^2(d+ex)\cos^2(d+ex)}{4} + \frac{3a^5x\cos^4(d+ex)}{8} - \frac{5a^5\sin^3(d+ex)\cos(d+ex)}{8e} - \frac{3a^5\sin(d+ex)\cos^3(d+ex)}{8e} - \frac{a^4b\sin^4(d+ex)\cos(d+ex)}{e} \\ x(a+b\sin(d))\left(a^2\sin^2(d)+2ab\sin(d)+b^2\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)
```

```
[Out] Piecewise((3*a**5*x*sin(d + e*x)**4/8 + 3*a**5*x*sin(d + e*x)**2*cos(d + e*
x)**2/4 + 3*a**5*x*cos(d + e*x)**4/8 - 5*a**5*sin(d + e*x)**3*cos(d + e*x)/
(8*e) - 3*a**5*sin(d + e*x)*cos(d + e*x)**3/(8*e) - a**4*b*sin(d + e*x)**4*
cos(d + e*x)/e - 4*a**4*b*sin(d + e*x)**2*cos(d + e*x)**3/(3*e) - 4*a**4*b*
sin(d + e*x)**2*cos(d + e*x)/e - 8*a**4*b*cos(d + e*x)**5/(15*e) - 8*a**4*b
*cos(d + e*x)**3/(3*e) + 3*a**3*b**2*x*sin(d + e*x)**4/2 + 3*a**3*b**2*x*si
n(d + e*x)**2*cos(d + e*x)**2 + 3*a**3*b**2*x*sin(d + e*x)**2 + 3*a**3*b**2
*x*cos(d + e*x)**4/2 + 3*a**3*b**2*x*cos(d + e*x)**2 - 5*a**3*b**2*sin(d +
e*x)**3*cos(d + e*x)/(2*e) - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)**3/(2*e)
 - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)/e - 6*a**2*b**3*sin(d + e*x)**2*co
s(d + e*x)/e - 4*a**2*b**3*cos(d + e*x)**3/e - 4*a**2*b**3*cos(d + e*x)/e +
 2*a*b**4*x*sin(d + e*x)**2 + 2*a*b**4*x*cos(d + e*x)**2 + a*b**4*x - 2*a*b
**4*sin(d + e*x)*cos(d + e*x)/e - b**5*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b
*sin(d))*(a**2*sin(d)**2 + 2*a*b*sin(d) + b**2)**2, True))
```

Giac [A] time = 1.16671, size = 213, normalized size = 1.09

$$-\frac{1}{80}a^4b\cos(5xe+5d)e^{(-1)} + \frac{1}{16}\left(7a^4b+8a^2b^3\right)\cos(3xe+3d)e^{(-1)} - \frac{1}{8}\left(29a^4b+68a^2b^3+8b^5\right)\cos(xe+d)e^{(-1)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] -1/80*a^4*b*cos(5*x*e + 5*d)*e^(-1) + 1/16*(7*a^4*b + 8*a^2*b^3)*cos(3*x*e
+ 3*d)*e^(-1) - 1/8*(29*a^4*b + 68*a^2*b^3 + 8*b^5)*cos(x*e + d)*e^(-1) + 1
/32*(a^5 + 4*a^3*b^2)*e^(-1)*sin(4*x*e + 4*d) - 1/4*(a^5 + 10*a^3*b^2 + 4*a
*b^4)*e^(-1)*sin(2*x*e + 2*d) + 3/8*(a^5 + 12*a^3*b^2 + 8*a*b^4)*x
```

3.500 $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)$

Optimal. Leaf size=109

$$\frac{\left(-8a^2b^2+a^4-3b^4\right)\cos(d+ex)}{3be}+\frac{a\left(a^2-6b^2\right)\sin(d+ex)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(a^2+4b^2\right)-\frac{a^2\cos(d+ex)(a+b\sin(d+$$

```
[Out] (a*(a^2 + 4*b^2)*x)/2 + ((a^4 - 8*a^2*b^2 - 3*b^4)*Cos[d + e*x])/(3*b*e) +
(a*(a^2 - 6*b^2)*Cos[d + e*x]*Sin[d + e*x])/(6*e) - (a^2*Cos[d + e*x]*(a +
b*Sin[d + e*x])^2)/(3*b*e)
```

Rubi [A] time = 0.0987093, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {3023, 2734}

$$\frac{\left(-8a^2b^2+a^4-3b^4\right)\cos(d+ex)}{3be}+\frac{a\left(a^2-6b^2\right)\sin(d+ex)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(a^2+4b^2\right)-\frac{a^2\cos(d+ex)(a+b\sin(d+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]
```

```
[Out] (a*(a^2 + 4*b^2)*x)/2 + ((a^4 - 8*a^2*b^2 - 3*b^4)*Cos[d + e*x])/(3*b*e) +
(a*(a^2 - 6*b^2)*Cos[d + e*x]*Sin[d + e*x])/(6*e) - (a^2*Cos[d + e*x]*(a +
b*Sin[d + e*x])^2)/(3*b*e)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx = -\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}+\frac{\int(a+b\sin(d+$$

$$=\frac{1}{2}a\left(a^2+4b^2\right)x+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos(d+ex)}{3be}+\frac{a}{}$$

Mathematica [A] time = 0.296101, size = 77, normalized size = 0.71

$$\frac{a\left(6\left(a^2+4b^2\right)(d+ex)-3\left(a^2+2b^2\right)\sin(2(d+ex))+ab\cos(3(d+ex))\right)-3b\left(11a^2+4b^2\right)\cos(d+ex)}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2),x]
```

```
[Out] (-3*b*(11*a^2 + 4*b^2)*Cos[d + e*x] + a*(6*(a^2 + 4*b^2)*(d + e*x) + a*b*Co
s[3*(d + e*x)] - 3*(a^2 + 2*b^2)*Sin[2*(d + e*x)]))/(12*e)
```

Maple [A] time = 0.024, size = 115, normalized size = 1.1

$$\frac{1}{e}\left(-\frac{a^2b\left(2+(\sin(ex+d))^2\right)\cos(ex+d)}{3}+a^3\left(-\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2\,ab^2\,(-1/2\,\sin(ex+d)\cos(ex+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x)
```

```
[Out] 1/e*(-1/3*a^2*b*(2+sin(e*x+d)^2)*cos(e*x+d)+a^3*(-1/2*sin(e*x+d)*cos(e*x+d)
+1/2*e*x+1/2*d)+2*a*b^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-2*cos(e*
x+d)*a^2*b-cos(e*x+d)*b^3+a*b^2*(e*x+d))
```

Maxima [A] time = 0.991491, size = 151, normalized size = 1.39

$$\frac{3\,(2\,ex+2\,d-\sin(2\,ex+2\,d))a^3+4\left(\cos(ex+d)^3-3\,\cos(ex+d)\right)a^2b+6\,(2\,ex+2\,d-\sin(2\,ex+2\,d))ab^2+12\,(ex+d}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*(2*e*x + 2*d - sin(2*e*x + 2*d))*a^3 + 4*(cos(e*x + d)^3 - 3*cos(e*
x + d))*a^2*b + 6*(2*e*x + 2*d - sin(2*e*x + 2*d))*a*b^2 + 12*(e*x + d)*a*b
^2 - 24*a^2*b*cos(e*x + d) - 12*b^3*cos(e*x + d))/e
```

Fricas [A] time = 1.74264, size = 182, normalized size = 1.67

$$\frac{2\,a^2b\cos\left(ex+d\right)^3+3\left(a^3+4\,ab^2\right)ex-3\left(a^3+2\,ab^2\right)\cos\left(ex+d\right)\sin\left(ex+d\right)-6\left(3\,a^2b+b^3\right)\cos\left(ex+d\right)}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(2*a^2*b*cos(e*x + d)^3 + 3*(a^3 + 4*a*b^2)*e*x - 3*(a^3 + 2*a*b^2)*cos
(e*x + d)*sin(e*x + d) - 6*(3*a^2*b + b^3)*cos(e*x + d))/e
```

Sympy [A] time = 0.781657, size = 204, normalized size = 1.87

$$\begin{cases}\frac{a^3x\sin^2\left(d+ex\right)}{2}+\frac{a^3x\cos^2\left(d+ex\right)}{2}-\frac{a^3\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e}-\frac{a^2b\sin^2\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{2a^2b\cos^3\left(d+ex\right)}{3e}-\frac{2a^2b\cos\left(d+ex\right)}{e}+ab^2x\sin^2\left(d+\right.\\x\left(a+b\sin\left(d\right)\right)\left(a^2\sin^2\left(d\right)+2ab\sin\left(d\right)+b^2\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

```
[Out] Piecewise((a**3*x*sin(d + e*x)**2/2 + a**3*x*cos(d + e*x)**2/2 - a**3*sin(d
 + e*x)*cos(d + e*x)/(2*e) - a**2*b*sin(d + e*x)**2*cos(d + e*x)/e - 2*a**2
*b*cos(d + e*x)**3/(3*e) - 2*a**2*b*cos(d + e*x)/e + a*b**2*x*sin(d + e*x)*
*2 + a*b**2*x*cos(d + e*x)**2 + a*b**2*x - a*b**2*sin(d + e*x)*cos(d + e*x)
/e - b**3*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b*sin(d))*(a**2*sin(d)**2 + 2*
a*b*sin(d) + b**2), True))
```

Giac [A] time = 1.13899, size = 107, normalized size = 0.98

$$\frac{1}{12}\,a^2b\cos\left(3\,xe+3\,d\right)e^{(-1)}-\frac{1}{4}\left(11\,a^2b+4\,b^3\right)\cos\left(xe+d\right)e^{(-1)}-\frac{1}{4}\left(a^3+2\,ab^2\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+\frac{1}{2}\left(a^3+4\,ab^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] 1/12*a^2*b*cos(3*x*e + 3*d)*e^(-1) - 1/4*(11*a^2*b + 4*b^3)*cos(x*e + d)*e^
(-1) - 1/4*(a^3 + 2*a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 1/2*(a^3 + 4*a*b^2)*x
```

3.501 $\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(d+ex)}{e(a\sin(d+ex)+b)}$$

```
[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

Rubi [A] time = 0.0893867, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3288, 2754, 8}

$$-\frac{\cos(d+ex)}{e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]
```

```
[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \left(4a^2\right)\int \frac{a+b\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^2}\,dx$$
$$= -\frac{\cos(d+ex)}{e(b+a\sin(d+ex))} + \frac{\int 0\,dx}{a^2-b^2}$$
$$= -\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}$$

Mathematica [A] time = 0.0623663, size = 23, normalized size = 1.

$$-\frac{\cos(d+ex)}{e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2),x]
```

```
[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

Maple [B] time = 0.092, size = 52, normalized size = 2.3

$$2\,\frac{1}{e\left(b\left(\tan\left(d/2+1/2\,ex\right)\right)^2+2\,a\tan\left(d/2+1/2\,ex\right)+b\right)}\left(-\frac{a\tan\left(d/2+1/2\,ex\right)}{b}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x)
```

```
[Out] 2/e*(-a*tan(1/2*d+1/2*e*x)/b-1)/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e
*x)+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67657, size = 54, normalized size = 2.35

$$-\frac{\cos(ex+d)}{ae\sin(ex+d)+be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] -cos(e*x + d)/(a*e*sin(e*x + d) + b*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19627, size = 70, normalized size = 3.04

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b\right)e^{(-1)}}{\left(b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -2*(a*tan(1/2*x*e + 1/2*d) + b)*e^(-1)/((b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan
(1/2*x*e + 1/2*d) + b)*b)
```

3.502 $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=157

$$-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{3e\left(a^2-b^2\right)^2(a\sin(d+ex)+b)}+\frac{b\cos(d+ex)}{3e\left(a^2-b^2\right)(a\sin(d+ex)+b)^2}+\frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{e\left(a^2-b^2\right)^{5/2}}-\frac{\cos(d+ex)}{3e(a\sin(d+ex)$$

```
[Out] (2*a*b*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*e) - Cos[d + e*x]/(3*e*(b + a*Sin[d + e*x])^3) + (b*Cos[d + e*x])/(3*(a^2
 - b^2)*e*(b + a*Sin[d + e*x])^2) - ((2*a^2 + b^2)*Cos[d + e*x])/(3*(a^2 -
b^2)^2*e*(b + a*Sin[d + e*x]))
```

Rubi [A] time = 0.415349, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3288, 2754, 12, 2660, 618, 206}

$$-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{3e\left(a^2-b^2\right)^2(a\sin(d+ex)+b)}+\frac{b\cos(d+ex)}{3e\left(a^2-b^2\right)(a\sin(d+ex)+b)^2}+\frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{e\left(a^2-b^2\right)^{5/2}}-\frac{\cos(d+ex)}{3e(a\sin(d+ex)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2,
x]
```



```
[Out] (2*a*b*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*e) - Cos[d + e*x]/(3*e*(b + a*Sin[d + e*x])^3) + (b*Cos[d + e*x])/(3*(a^2
 - b^2)*e*(b + a*Sin[d + e*x])^2) - ((2*a^2 + b^2)*Cos[d + e*x])/(3*(a^2 -
b^2)^2*e*(b + a*Sin[d + e*x]))
```


Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx=\left(16a^4\right)\int \frac{a+b\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^4}\,dx$$

$$=-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{\left(4a^2\right)\int \frac{4a\left(a^2-b^2\right)\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^3}\,dx}{3\left(a^2-b^2\right)}$$

$$=-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{1}{3}\left(16a^3\right)\int \frac{\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^3}\,dx$$

$$=-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2}+\frac{(2a)\int \cdots}{\cdots}$$

$$=-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2}-\frac{\left(2a\cdots\right.}{3\left(a^2-\cdots\right.}$$

$$=-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2}-\frac{\left(2a\cdots\right.}{3\left(a^2-\cdots\right.}$$

$$=-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2}-\frac{\left(2a\cdots\right.}{3\left(a^2-\cdots\right.}$$

$$=-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2}-\frac{\left(2a\cdots\right.}{3\left(a^2-\cdots\right.}$$

$$=\frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}e}-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{b\cos\cdots}{3\left(a^2-b^2\right)e\cdots}$$

Mathematica [A] time = 0.973576, size = 140, normalized size = 0.89

$$-\frac{\frac{6ab\tan^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+\frac{\cos(d+ex)\left(a^2\left(2a^2+b^2\right)\sin^2(d+ex)+3ab\left(a^2+b^2\right)\sin(d+ex)-a^2b^2+a^4+3b^4\right)}{(a-b)^2(a+b)^2(a\sin(d+ex)+b)^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^2,x]
```

```
[Out] -((6*a*b*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + (Cos[d + e*x]*(a^4 - a^2*b^2 + 3*b^4 + 3*a*b*(a^2 + b^2)*Sin[d + e*x]
 + a^2*(2*a^2 + b^2)*Sin[d + e*x]^2))/((a - b)^2*(a + b)^2*(b + a*Sin[d + e
*x])^3))/(3*e)
```

Maple [B] time = 0.118, size = 1297, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x)
```

```
[Out] -2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a^5/b/(a^4-2*a^2*b
^2+b^4)*tan(1/2*d+1/2*e*x)^5+4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*
e*x)+b)^3*a^3*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^5-4/e/(b*tan(1/2*d+1
/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1
/2*e*x)^5-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/b^2/(a^4-
2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^4*a^6+6/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan
(1/2*d+1/2*e*x)+b)^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^4*a^4-10/e/(b*t
an(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*b^2/(a^4-2*a^2*b^2+b^4)*tan
(1/2*d+1/2*e*x)^4*a^2-2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)
^3*b^4/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^4-8/3/e/(b*tan(1/2*d+1/2*e*x)
^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a^7/b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*
x)^3-4/3/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a^5/b/(a^4-2
*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^3-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*
d+1/2*e*x)+b)^3*a^3*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^3-12/e/(b*tan(
1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*tan(
1/2*d+1/2*e*x)^3-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/b^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^2*a^6-12/e/(b*tan(1/2*d+1/2*e*x)^2
+2*a*tan(1/2*d+1/2*e*x)+b)^3*b^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^2*a
^2-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*b^4/(a^4-2*a^2*b
^2+b^4)*tan(1/2*d+1/2*e*x)^2-2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*
e*x)+b)^3*a^5/b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)-8/e/(b*tan(1/2*d+1/2
*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2
*e*x)-2/3/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/(a^4-2*a^2*
b^2+b^4)*a^4+2/3/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/(a^4
-2*a^2*b^2+b^4)*a^2*b^2-2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+
b)^3/(a^4-2*a^2*b^2+b^4)*b^4-2/e*a*b/(a^4-2*a^2*b^2+b^4)/(-a^2+b^2)^(1/2)*a
rctan(1/2*(2*b*tan(1/2*d+1/2*e*x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16481, size = 1705, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] [-1/6*(2*(2*a^6 - a^4*b^2 - a^2*b^4)*cos(e*x + d)^3 - 6*(a^5*b - a*b^5)*cos
(e*x + d)*sin(e*x + d) - 3*(3*a^3*b^2*cos(e*x + d)^2 - 3*a^3*b^2 - a*b^4 +
(a^4*b*cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*sin(e*x + d))*sqrt(a^2 - b^2)*lo
g(((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b*sin(e*x + d) + a^2 + b^2 + 2*(b*cos
(e*x + d)*sin(e*x + d) + a*cos(e*x + d))*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)
^2 - 2*a*b*sin(e*x + d) - a^2 - b^2)) - 6*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*c
os(e*x + d))/(3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*cos(e*x + d)^2
- (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4
 - a^3*b^6)*e*cos(e*x + d)^2 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*s
in(e*x + d)), -1/3*((2*a^6 - a^4*b^2 - a^2*b^4)*cos(e*x + d)^3 - 3*(a^5*b -
 a*b^5)*cos(e*x + d)*sin(e*x + d) - 3*(3*a^3*b^2*cos(e*x + d)^2 - 3*a^3*b^2
 - a*b^4 + (a^4*b*cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*sin(e*x + d))*sqrt(-a
^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) + a)/((a^2 - b^2)*cos(e*
x + d))) - 3*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*cos(e*x + d))/(3*(a^8*b - 3*a^
6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*cos(e*x + d)^2 - (3*a^8*b - 8*a^6*b^3 + 6*a^
4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*e*cos(e*x + d)^2
- (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*sin(e*x + d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27347, size = 613, normalized size = 3.9

$$-\frac{2}{3}\left(\frac{3\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)ab}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{3a^5b^2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^5-6a^3b^4\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^5+6ab^6\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x*e +
1/2*d) + a)/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2
)) + (3*a^5*b^2*tan(1/2*x*e + 1/2*d)^5 - 6*a^3*b^4*tan(1/2*x*e + 1/2*d)^5 +
 6*a*b^6*tan(1/2*x*e + 1/2*d)^5 + 6*a^6*b*tan(1/2*x*e + 1/2*d)^4 - 9*a^4*b^
3*tan(1/2*x*e + 1/2*d)^4 + 15*a^2*b^5*tan(1/2*x*e + 1/2*d)^4 + 3*b^7*tan(1/
2*x*e + 1/2*d)^4 + 4*a^7*tan(1/2*x*e + 1/2*d)^3 + 2*a^5*b^2*tan(1/2*x*e + 1
/2*d)^3 + 6*a^3*b^4*tan(1/2*x*e + 1/2*d)^3 + 18*a*b^6*tan(1/2*x*e + 1/2*d)^
3 + 6*a^6*b*tan(1/2*x*e + 1/2*d)^2 + 18*a^2*b^5*tan(1/2*x*e + 1/2*d)^2 + 6*
b^7*tan(1/2*x*e + 1/2*d)^2 + 3*a^5*b^2*tan(1/2*x*e + 1/2*d) + 12*a*b^6*tan(
1/2*x*e + 1/2*d) + a^4*b^3 - a^2*b^5 + 3*b^7)/((a^4*b^3 - 2*a^2*b^5 + b^7)*
(b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b)^3))*e^(-1)
```

3.503 $\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}+\frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

```
[Out] (Sqrt[2]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b - Sqrt[b^2
- 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]])
])/Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]] + (Sqrt[2]*(e - (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b + Sqrt[b^2 - 4*a*c])*Tan[x/2])/(Sqr
t[2]*Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]])])/Sqrt[b^2 - 2*c*(a + c
) + b*Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.940384, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3292, 2660, 618, 204}

$$\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}+\frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

```
[Out] (Sqrt[2]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b - Sqrt[b^2
- 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]])
])/Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]] + (Sqrt[2]*(e - (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b + Sqrt[b^2 - 4*a*c])*Tan[x/2])/(Sqr
t[2]*Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]])])/Sqrt[b^2 - 2*c*(a + c
) + b*Sqrt[b^2 - 4*a*c]]
```

Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
```

```
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\sin(x)}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b-\sqrt{b^2-4ac}+$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}+4cx+\left(b+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)$$

$$= -\left(\left(4\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{4\left(4c^2-\left(b+\sqrt{b^2-4ac}\right)^2\right)-x^2}\,dx, x, 4c+2\left(b+\sqrt{b^2\right.\right.\right.$$

$$= \frac{\sqrt{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right.}{\sqrt{2}\sqrt{b^2-2c(a+c)+}}\right.}{\sqrt{b^2-2c(a+c)+b\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.70011, size = 286, normalized size = 1.18

$$\frac{\frac{\left(e\left(\sqrt{4ac-b^2}+ib\right)-2icd\right)\tan^{-1}\left(\frac{2c+\tan\left(\frac{x}{2}\right)\left(b-i\sqrt{4ac-b^2}\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}+\frac{\left(e\left(\sqrt{4ac-b^2}-ib\right)+2icd\right)\tan^{-1}\left(\frac{2c+\tan\left(\frac{x}{2}\right)\left(b+i\sqrt{4ac-b^2}\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}}{\sqrt{2ac-\frac{b^2}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

```
[Out] ((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b - I*Sqrt[-b^
2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a
*c]])]])/Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]] + (((2*I)*c*d + ((
-I)*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b + I*Sqrt[-b^2 + 4*a*c])*Tan
[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])]])/Sqrt[b^
2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2/2 + 2*a*c]
```

Maple [B] time = 0.095, size = 832, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x)
```

```
[Out] 8*a/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*
a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^
2)^(1/2))*d*c-2/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2
)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2
)^(1/2)+4*a^2)^(1/2))*d*b^2+4*a*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2
-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^
(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*e-2*(-4*a*c+b^2)^(
1/2)/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2
*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a
^2)^(1/2))*d*b-8*a/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(
1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c
+b^2)^(1/2)+4*a^2)^(1/2))*d*c+2/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(
1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^
2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b^2+4*a*(-4*a*c+b^2)^(1/2)/(4*a*c-
```

```
b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*
x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*
e-2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^
2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4
*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e\sin(x)+d}{c\sin(x)^2+b\sin(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(x) + d)/(c*sin(x)^2 + b*sin(x) + a), x)
```

Fricas [B] time = 61.352, size = 13609, normalized size = 56.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 -
 b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b
^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b
^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*
a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*
a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*d^4 + 4*a*b
*c*e^4 - 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 12*(a*b*c + b*c^2)*d^2*e^2 - 4
*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2
*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 -
 (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*
a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4
 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2
- 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2
)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5
```

- 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*
(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)
c^2 - 2(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2
*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)
*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2
*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -
(16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)
c^2 - 4(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - ((b^4 - 4*a*b^2*c)*d^3 -
3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*
c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b -
b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e +
(2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 -
2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(
2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^
2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*
a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 -
b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4
+ a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2
- (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x)) - 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*
c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 -
b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 +
b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 -
4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*
c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*
(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*d^4 + 4*a*b*c*e^4 - 4*(b^2*c + 2*a*c^2 +
2*c^3)*d^3*e + 12*(a*b*c + b*c^2)*d^2*e^2 - 4*(2*a*c^2 + (2*a^2 + b^2)*c)*d
e^3 - 2((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2
- b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^
3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2
- 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3
e + 2(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^
2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3
- 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin
(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3
- 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)
*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a
*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(
a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*
d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^
3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*
a*b^4)*c))*cos(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*
b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*
c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b
^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (

a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((
b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d
^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)
c^2 - 2(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c
+ 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*
e^3)*sin(x)) + 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)
*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)
c^2 - 2(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e
+ 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2
- 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 -
2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2
*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-4*b
*c^2*d^4 - 4*a*b*c*e^4 + 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 12*(a*b*c + b*
c^2)*d^2*e^2 + 4*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 + 2*((4*a*c^4 + (8*a^2 -
b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b
^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*
b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^
2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c +
2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5
- (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^
6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*
a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*
a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*
b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 +
b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 +
b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + ((b^4 - 4
*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^
2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c
^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(
a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*
a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b +
b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)
) - 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 +
b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x)) - 1/4*sqrt(2)*sq
rt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*
e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)
*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2
*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -

```
(16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)
*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2
 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-4*b*c^2*d^4 - 4*a*b*c*e^4 + 4*(b
^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 12*(a*b*c + b*c^2)*d^2*e^2 + 4*(2*a*c^2 + (
2*a^2 + b^2)*c)*d*e^3 - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^
2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b
^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*
a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 -
4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*
c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2
*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 +
 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2
)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a
^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3
 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^
4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2
 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^
2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*
a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 -
 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*
cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2
 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3
*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 +
 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 -
 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*
b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^
3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) - 2*(b^2*c*d^4 + a*b^2*e^4
- (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b +
b^3 + 2*a*b*c)*d*e^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e\sin(x)+d}{c\sin(x)^2+b\sin(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(x) + d)/(c*sin(x)^2 + b*sin(x) + a), x)
```

3.504 $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)$

Optimal. Leaf size=331

$$\frac{5a^4bx\left(3a^2+4b^2\right)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}{8\left(a^2\sin(d+ex)+ab\right)^3}-\frac{a^4b\left(29a^2+6b^2\right)\sin(d+ex)\cos(d+ex)\left(a^2\sin^2(d+ex\right.}{24e\left(a^2\sin(d+ex)+ab\right)^3}$$

```
[Out] -(b*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(4*
e) - ((4*a^4 + 28*a^2*b^2 + 3*b^4)*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] +
 a^2*Sin[d + e*x]^2)^(3/2))/(6*e*(b + a*Sin[d + e*x])^3) - ((4*a^2 + 3*b^2)
*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(12*e*
(b + a*Sin[d + e*x])) + (5*a^4*b*(3*a^2 + 4*b^2)*x*(b^2 + 2*a*b*Sin[d + e*x
] + a^2*Sin[d + e*x]^2)^(3/2))/(8*(a*b + a^2*Sin[d + e*x])^3) - (a^4*b*(29*
a^2 + 6*b^2)*Cos[d + e*x]*Sin[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[
d + e*x]^2)^(3/2))/(24*e*(a*b + a^2*Sin[d + e*x])^3)
```

Rubi [A] time = 0.322718, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3290, 2753, 2734}

$$\frac{5a^4bx\left(3a^2+4b^2\right)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}{8\left(a^2\sin(d+ex)+ab\right)^3}-\frac{a^4b\left(29a^2+6b^2\right)\sin(d+ex)\cos(d+ex)\left(a^2\sin^2(d+ex\right.}{24e\left(a^2\sin(d+ex)+ab\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3
/2),x]
```

```
[Out] -(b*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(4*
e) - ((4*a^4 + 28*a^2*b^2 + 3*b^4)*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] +
 a^2*Sin[d + e*x]^2)^(3/2))/(6*e*(b + a*Sin[d + e*x])^3) - ((4*a^2 + 3*b^2)
*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(12*e*
(b + a*Sin[d + e*x])) + (5*a^4*b*(3*a^2 + 4*b^2)*x*(b^2 + 2*a*b*Sin[d + e*x
] + a^2*Sin[d + e*x]^2)^(3/2))/(8*(a*b + a^2*Sin[d + e*x])^3) - (a^4*b*(29*
a^2 + 6*b^2)*Cos[d + e*x]*Sin[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[
d + e*x]^2)^(3/2))/(24*e*(a*b + a^2*Sin[d + e*x])^3)
```

Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\frac{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\int\left(2ab+\right.}{\left(2ab+2a^2\sin(d+\right.}$$

$$=-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+e\right.}{4e}$$

$$=-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+e\right.}{4e}$$

$$=-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+e\right.}{4e}$$

Mathematica [A] time = 0.894213, size = 140, normalized size = 0.42

$$\frac{\sqrt{(a\sin(d+ex)+b)^2}\left(3ab\left(20\left(3a^2+4b^2\right)(d+ex)-8\left(4a^2+3b^2\right)\sin(2(d+ex))+a^2\sin(4(d+ex))\right)-24\left(21a^2b^2+3a^4\right.\right.}{96e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sin[d + e*x])^2]*(-24*(3*a^4 + 21*a^2*b^2 + 4*b^4)*Cos[d + e*x
] + 8*a*(a^3 + 3*a*b^2)*Cos[3*(d + e*x)] + 3*a*b*(20*(3*a^2 + 4*b^2)*(d + e
*x) - 8*(4*a^2 + 3*b^2)*Sin[2*(d + e*x)] + a^2*Sin[4*(d + e*x)])))/(96*e*(b
 + a*Sin[d + e*x]))
```

Maple [A] time = 0.306, size = 269, normalized size = 0.8

$$-\frac{6\,(\cos(ex+d))^3\sin(ex+d)\,a^3b+8\,a^4\,(\cos(ex+d))^3+24\,a^2b^2\,(\cos(ex+d))^3-51\,\sin(ex+d)\cos(ex+d)\,a^3b-36}{24\,e\left((\cos(ex+d))^2\sin(ex+d)\,a^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/24/e*(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(3/2)*(6*cos(e*x+d)^3*
sin(e*x+d)*a^3*b+8*a^4*cos(e*x+d)^3+24*a^2*b^2*cos(e*x+d)^3-51*sin(e*x+d)*c
os(e*x+d)*a^3*b-36*cos(e*x+d)*sin(e*x+d)*a*b^3-24*a^4*cos(e*x+d)-144*a^2*b^
2*cos(e*x+d)-24*cos(e*x+d)*b^4+45*(e*x+d)*a^3*b+60*(e*x+d)*a*b^3-16*a^4-120
*a^2*b^2-24*b^4)/(cos(e*x+d)^2*sin(e*x+d)*a^3+3*cos(e*x+d)^2*a^2*b-a^3*sin(
e*x+d)-3*sin(e*x+d)*a*b^2-3*a^2*b-b^3)
```

Maxima [A] time = 1.58749, size = 751, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/12*(4*(3*(3*a^2*b + 2*b^3)*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - (4*a
^3 + 18*a*b^2 + 9*a^2*b*sin(e*x + d)/(cos(e*x + d) + 1) + 18*a*b^2*sin(e*x
+ d)^4/(cos(e*x + d) + 1)^4 - 9*a^2*b*sin(e*x + d)^5/(cos(e*x + d) + 1)^5 +
 12*(a^3 + 3*a*b^2)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/(3*sin(e*x + d)^2/
```

```
(cos(e*x + d) + 1)^2 + 3*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + sin(e*x + d)
^6/(cos(e*x + d) + 1)^6 + 1))*a + 3*(3*(a^3 + 4*a*b^2)*arctan(sin(e*x + d)/
(cos(e*x + d) + 1)) - (16*a^2*b + 8*b^3 + 8*b^3*sin(e*x + d)^6/(cos(e*x + d
) + 1)^6 + 3*(a^3 + 4*a*b^2)*sin(e*x + d)/(cos(e*x + d) + 1) + 8*(8*a^2*b +
 3*b^3)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (11*a^3 + 12*a*b^2)*sin(e*x +
 d)^3/(cos(e*x + d) + 1)^3 + 24*(2*a^2*b + b^3)*sin(e*x + d)^4/(cos(e*x + d
) + 1)^4 - (11*a^3 + 12*a*b^2)*sin(e*x + d)^5/(cos(e*x + d) + 1)^5 - 3*(a^3
 + 4*a*b^2)*sin(e*x + d)^7/(cos(e*x + d) + 1)^7)/(4*sin(e*x + d)^2/(cos(e*x
 + d) + 1)^2 + 6*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 4*sin(e*x + d)^6/(co
s(e*x + d) + 1)^6 + sin(e*x + d)^8/(cos(e*x + d) + 1)^8 + 1))*b)/e
```

Fricas [A] time = 1.83632, size = 263, normalized size = 0.79

$$\frac{8\left(a^4+3\,a^2b^2\right)\cos\left(ex+d\right)^3+15\left(3\,a^3b+4\,ab^3\right)ex-24\left(a^4+6\,a^2b^2+b^4\right)\cos\left(ex+d\right)+3\left(2\,a^3b\cos\left(ex+d\right)^3-\left(17\,a^3b\right.\right.}{24\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/24*(8*(a^4 + 3*a^2*b^2)*cos(e*x + d)^3 + 15*(3*a^3*b + 4*a*b^3)*e*x - 24*
(a^4 + 6*a^2*b^2 + b^4)*cos(e*x + d) + 3*(2*a^3*b*cos(e*x + d)^3 - (17*a^3*
b + 12*a*b^3)*cos(e*x + d))*sin(e*x + d))/e
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33431, size = 323, normalized size = 0.98

$$\frac{1}{32}\,a^3be^{(-1)}\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)\sin\left(4\,xe+4\,d\right)+\frac{1}{12}\left(a^4\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)+3\,a^2b^2\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)\right)\cos\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/32*a^3*b*e^(-1)*sgn(a*sin(x*e + d) + b)*sin(4*x*e + 4*d) + 1/12*(a^4*sgn(
a*sin(x*e + d) + b) + 3*a^2*b^2*sgn(a*sin(x*e + d) + b))*cos(3*x*e + 3*d)*e
^(-1) - 1/4*(3*a^4*sgn(a*sin(x*e + d) + b) + 21*a^2*b^2*sgn(a*sin(x*e + d)
+ b) + 4*b^4*sgn(a*sin(x*e + d) + b))*cos(x*e + d)*e^(-1) - 1/4*(4*a^3*b*sg
n(a*sin(x*e + d) + b) + 3*a*b^3*sgn(a*sin(x*e + d) + b))*e^(-1)*sin(2*x*e +
 2*d) + 5/8*(3*a^3*b*sgn(a*sin(x*e + d) + b) + 4*a*b^3*sgn(a*sin(x*e + d) +
 b))*x
```

3.505 $\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,d$

Optimal. Leaf size=185

$$\frac{3a^2bx\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2\left(a^2\sin(d+ex)+ab\right)}-\frac{a^2b\sin(d+ex)\cos(d+ex)\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2e\left(a^2\sin(d+ex)+ab\right)}-\frac{(a$$

```
[Out] -(((a^2 + b^2)*Cos[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2])/(e*(b + a*Sin[d + e*x]))) + (3*a^2*b*x*Sqrt[b^2 + 2*a*b*Sin[d + e*x]
+ a^2*Sin[d + e*x]^2])/(2*(a*b + a^2*Sin[d + e*x])) - (a^2*b*Cos[d + e*x]*S
in[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2])/(2*e*(a*b
+ a^2*Sin[d + e*x]))
```

Rubi [A] time = 0.109408, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {3290, 2734}

$$\frac{3a^2bx\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2\left(a^2\sin(d+ex)+ab\right)}-\frac{a^2b\sin(d+ex)\cos(d+ex)\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2e\left(a^2\sin(d+ex)+ab\right)}-\frac{(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2
],x]
```


```
[Out] -(((a^2 + b^2)*Cos[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2])/(e*(b + a*Sin[d + e*x]))) + (3*a^2*b*x*Sqrt[b^2 + 2*a*b*Sin[d + e*x]
+ a^2*Sin[d + e*x]^2])/(2*(a*b + a^2*Sin[d + e*x])) - (a^2*b*Cos[d + e*x]*S
in[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2])/(2*e*(a*b
+ a^2*Sin[d + e*x]))
```


Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\int\left(2ab+2a^2\right.}{2ab+2a^2\sin(d+e}$$

$$= -\frac{\left(a^2+b^2\right)\cos(d+ex)\sqrt{b^2+2ab\sin(d+ex)+a^2\sin}}{e(b+a\sin(d+ex))}$$

Mathematica [A] time = 0.193099, size = 70, normalized size = 0.38

$$-\frac{\sqrt{(a\sin(d+ex)+b)^2}\left(4\left(a^2+b^2\right)\cos(d+ex)+ab(\sin(2(d+ex))-6(d+ex))\right)}{4e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2],x]
```

```
[Out] -(Sqrt[(b + a*Sin[d + e*x])^2]*(4*(a^2 + b^2)*Cos[d + e*x] + a*b*(-6*(d + e
*x) + Sin[2*(d + e*x)])))/(4*e*(b + a*Sin[d + e*x]))
```

Maple [A] time = 0.209, size = 107, normalized size = 0.6

$$-\frac{\sin(ex+d)\cos(ex+d)ab+2a^2\cos(ex+d)+2\cos(ex+d)b^2-3(ex+d)ab+2a^2+2b^2}{2e(b+a\sin(ex+d))}\sqrt{-a^2(\cos(ex+d))^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x)
```

```
[Out] -1/2/e*(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(1/2)*(sin(e*x+d)*cos(e
*x+d)*a*b+2*a^2*cos(e*x+d)+2*cos(e*x+d)*b^2-3*(e*x+d)*a*b+2*a^2+2*b^2)/(b+a
```

```
*sin(e*x+d))
```

Maxima [A] time = 1.55904, size = 252, normalized size = 1.36

$$\frac{2\left(b\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{a}{\frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+1}\right)a+\left(a\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{2\,b+\frac{a\sin(ex+d)}{\cos(ex+d)+1}+\frac{2\,b\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{a\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{\frac{2\,\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+1}\right)b}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] (2*(b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - a/(sin(e*x + d)^2/(cos(e*x
+ d) + 1)^2 + 1))*a + (a*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - (2*b + a
*sin(e*x + d)/(cos(e*x + d) + 1) + 2*b*sin(e*x + d)^2/(cos(e*x + d) + 1)^2
- a*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(2*sin(e*x + d)^2/(cos(e*x + d) +
1)^2 + sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 1))*b)/e
```

Fricas [A] time = 1.81702, size = 108, normalized size = 0.58

$$\frac{3\,abex-ab\cos\left(ex+d\right)\sin\left(ex+d\right)-2\left(a^2+b^2\right)\cos\left(ex+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(3*a*b*e*x - a*b*cos(e*x + d)*sin(e*x + d) - 2*(a^2 + b^2)*cos(e*x + d)
)/e
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22403, size = 132, normalized size = 0.71

$$-a^2\cos(xe+d)\,e^{(-1)}\mathrm{sgn}(a\sin(xe+d)+b)-b^2\cos(xe+d)\,e^{(-1)}\mathrm{sgn}(a\sin(xe+d)+b)-\frac{1}{4}abe^{(-1)}\mathrm{sgn}(a\sin(xe+d)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] -a^2*cos(x*e + d)*e^(-1)*sgn(a*sin(x*e + d) + b) - b^2*cos(x*e + d)*e^(-1)*
sgn(a*sin(x*e + d) + b) - 1/4*a*b*e^(-1)*sgn(a*sin(x*e + d) + b)*sin(2*x*e
+ 2*d) + 3/2*a*b*x*sgn(a*sin(x*e + d) + b)
```

3.506 $\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx$

Optimal. Leaf size=137

$$\frac{bx(a\sin(d+ex)+b)}{a\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}-\frac{2\sqrt{a^2-b^2}(a\sin(d+ex)+b)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{ae\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

```
[Out] (b*x*(b + a*Sin[d + e*x]))/(a*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e
*x]^2]) - (2*Sqrt[a^2 - b^2]*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^
2]]*(b + a*Sin[d + e*x]))/(a*e*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2])
```

Rubi [A] time = 0.198201, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3290, 2735, 2660, 618, 206}

$$\frac{bx(a\sin(d+ex)+b)}{a\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}-\frac{2\sqrt{a^2-b^2}(a\sin(d+ex)+b)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{ae\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2
],x]
```


```
[Out] (b*x*(b + a*Sin[d + e*x]))/(a*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e
*x]^2]) - (2*Sqrt[a^2 - b^2]*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^
2]]*(b + a*Sin[d + e*x]))/(a*e*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2])
```


Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx = \frac{\left(2ab+2a^2\sin(d+ex)\right)\int \frac{a+b\sin(d+ex)}{2ab+2a^2\sin(d+ex)}\,dx}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{\left(\left(-2a^3+2ab^2\right)\left(2ab+2a^2 s\right.\right.}{2a^2\sqrt{b^2+2ab\sin(}}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{\left(\left(-2a^3+2ab^2\right)\left(2ab+2a^2 s\right.\right.}{a^2e\sqrt{}}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} + \frac{\left(2\left(-2a^3+2ab^2\right)\left(2ab+2a\right.\right.}{}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{2\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{ae\sqrt{b^2+2ab\sin(d+}}$$

Mathematica [A] time = 0.183296, size = 85, normalized size = 0.62

$$\frac{(a\sin(d+ex)+b)\left(b(d+ex)-2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)\right)}{ae\sqrt{(a\sin(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2],x]
```

```
[Out] ((b*(d + e*x) - 2*Sqrt[-a^2 + b^2]*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^
2 + b^2]])*(b + a*Sin[d + e*x]))/(a*e*Sqrt[(b + a*Sin[d + e*x])^2])
```

Maple [A] time = 0.164, size = 176, normalized size = 1.3

$$-\frac{b+a\sin(ex+d)}{ae}\left(2\arctan\left(\frac{b\cos(ex+d)-a\sin(ex+d)-b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)a^2-2\arctan\left(\frac{b\cos(ex+d)-a\sin(ex+d)-b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)b^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x)
```

```
[Out] -1/e/a/(-a^2+b^2)^(1/2)*(2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/
(-a^2+b^2)^(1/2))*a^2-2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a
^2+b^2)^(1/2))*b^2-b*(e*x+d)*(-a^2+b^2)^(1/2))*(b+a*sin(e*x+d))/(-a^2*cos(e
*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88315, size = 462, normalized size = 3.37

$$\left[\frac{2\,bex + \sqrt{a^2 - b^2}\log\left(-\frac{\left(a^2-2\,b^2\right)\cos(ex+d)^2+2\,ab\sin(ex+d)+a^2+b^2-2\,(b\cos(ex+d)\sin(ex+d)+a\cos(ex+d))\sqrt{a^2-b^2}}{a^2\cos(ex+d)^2-2\,ab\sin(ex+d)-a^2-b^2}\right)}{2\,ae}, \frac{bex - \sqrt{-a^2 + b^2}\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/2*(2*b*e*x + sqrt(a^2 - b^2)*log(-((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b*
sin(e*x + d) + a^2 + b^2 - 2*(b*cos(e*x + d)*sin(e*x + d) + a*cos(e*x + d))
*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*x + d) - a^2 - b^2)))/(
a*e), (b*e*x - sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) +
a)/((a^2 - b^2)*cos(e*x + d))))/(a*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34954, size = 281, normalized size = 2.05

$$\left(\frac{\left(xe-2\,\pi\left\lfloor\frac{xe+d}{2\,\pi}+\frac{1}{2}\right\rfloor+d\right)b}{a\mathrm{sgn}\left(b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{4}+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{3}+2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b\right)}+\frac{}{\sqrt{-a^{2}+b^{2}}a\mathrm{sgn}\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] ((x*e - 2*pi*floor(1/2*(x*e + d)/pi + 1/2) + d)*b/(a*sgn(b*tan(1/2*x*e + 1/
2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(
1/2*x*e + 1/2*d) + b)) + 2*(a^2 - b^2)*arctan((b*tan(1/2*x*e + 1/2*d) + a)/
sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*ta
n(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d
) + b)))*e^(-1)
```

3.507 $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{b\cos(d+ex)\left(a^2\sin(d+ex)+ab\right)^3}{2e\left(a^2-b^2\right)\left(a^3b+a^4\sin(d+ex)\right)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}-\frac{\cos(d+ex)(a\sin(d+ex)+b)}{2e\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b\right.}$$

```
[Out] -(Cos[d + e*x]*(b + a*Sin[d + e*x]))/(2*e*(b^2 + 2*a*b*Sin[d + e*x] + a^2*S
in[d + e*x]^2)^(3/2)) - (ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]]*
(a*b + a^2*Sin[d + e*x])^3)/(a^2*(a^2 - b^2)^(3/2)*e*(b^2 + 2*a*b*Sin[d + e
*x] + a^2*Sin[d + e*x]^2)^(3/2)) + (b*Cos[d + e*x]*(a*b + a^2*Sin[d + e*x])
^3)/(2*(a^2 - b^2)*e*(a^3*b + a^4*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] +
 a^2*Sin[d + e*x]^2)^(3/2))
```

Rubi [A] time = 0.271869, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3290, 2754, 12, 2660, 618, 206}

$$\frac{b\cos(d+ex)\left(a^2\sin(d+ex)+ab\right)^3}{2e\left(a^2-b^2\right)\left(a^3b+a^4\sin(d+ex)\right)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}-\frac{\cos(d+ex)(a\sin(d+ex)+b)}{2e\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3
/2),x]
```



```
[Out] -(Cos[d + e*x]*(b + a*Sin[d + e*x]))/(2*e*(b^2 + 2*a*b*Sin[d + e*x] + a^2*S
in[d + e*x]^2)^(3/2)) - (ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]]*
(a*b + a^2*Sin[d + e*x])^3)/(a^2*(a^2 - b^2)^(3/2)*e*(b^2 + 2*a*b*Sin[d + e
*x] + a^2*Sin[d + e*x]^2)^(3/2)) + (b*Cos[d + e*x]*(a*b + a^2*Sin[d + e*x])
^3)/(2*(a^2 - b^2)*e*(a^3*b + a^4*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] +
 a^2*Sin[d + e*x]^2)^(3/2))
```


Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
```

```
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}dx = \frac{\left(2ab+2a^2\sin(d+ex)\right)^3\int \frac{a+b\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^3}dx}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{\left(2ab+2a^2\sin\right.}{8a^2\left(a^2-b^2\right)\left(b^2+\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{\left(2ab+2a^2\sin(d+\right.}{4a\left(b^2+2ab\sin(\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e\left(a^3b-\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e\left(a^3b-\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e\left(a^3b-\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e\left(a^3b-\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{a+b\,t}{}\right.}{a^2\left(a^2-b^2\right)^{3/2}e\left(b^2\right.}$$

Mathematica [A] time = 0.352519, size = 144, normalized size = 0.6

$$\frac{\sqrt{b^2-a^2}\cos(d+ex)\left(a^2-ab\sin(d+ex)-2b^2\right)-2a(a\sin(d+ex)+b)^2\tan^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{2e(b-a)(a+b)\sqrt{b^2-a^2}(a\sin(d+ex)+b)\sqrt{(a\sin(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^(3/2),x]
```

```
[Out] (-2*a*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(b + a*Sin[d + e*x]
)^2 + Sqrt[-a^2 + b^2]*Cos[d + e*x]*(a^2 - 2*b^2 - a*b*Sin[d + e*x]))/(2*(-
a + b)*(a + b)*Sqrt[-a^2 + b^2]*e*(b + a*Sin[d + e*x])*Sqrt[(b + a*Sin[d +
e*x])^2])
```

Maple [B] time = 0.153, size = 738, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/2/e/(-a^2+b^2)^(1/2)/(a^2-b^2)/b^2*(-2*cos(e*x+d)^2*sin(e*x+d)*arctan((b
*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^4*b^2+cos(e*x+d)
^3*(-a^2+b^2)^(1/2)*a^2*b^3-cos(e*x+d)^2*sin(e*x+d)*(-a^2+b^2)^(1/2)*a^5+2*
cos(e*x+d)^2*sin(e*x+d)*(-a^2+b^2)^(1/2)*a^3*b^2-6*cos(e*x+d)^2*arctan((b*c
os(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^3*b^3-3*cos(e*x+d)
^2*(-a^2+b^2)^(1/2)*a^4*b+6*cos(e*x+d)^2*(-a^2+b^2)^(1/2)*a^2*b^3+cos(e*x+d
)*sin(e*x+d)*(-a^2+b^2)^(1/2)*a^3*b^2-3*cos(e*x+d)*sin(e*x+d)*(-a^2+b^2)^(1
/2)*a*b^4+2*sin(e*x+d)*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^
2+b^2)^(1/2))*a^4*b^2+6*sin(e*x+d)*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin
(e*x+d)/(-a^2+b^2)^(1/2))*a^2*b^4-2*cos(e*x+d)*(-a^2+b^2)^(1/2)*b^5+sin(e*x
+d)*(-a^2+b^2)^(1/2)*a^5+sin(e*x+d)*(-a^2+b^2)^(1/2)*a^3*b^2-6*sin(e*x+d)*(
-a^2+b^2)^(1/2)*a*b^4+6*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a
^2+b^2)^(1/2))*a^3*b^3+2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-
a^2+b^2)^(1/2))*a*b^5+3*(-a^2+b^2)^(1/2)*a^4*b-5*(-a^2+b^2)^(1/2)*a^2*b^3-2
*(-a^2+b^2)^(1/2)*b^5)/(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10839, size = 1170, normalized size = 4.9

$$\left[-\frac{2\left(a^{3}b-ab^{3}\right)\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(a^{3}\cos\left(ex+d\right)^{2}-2\,a^{2}b\sin\left(ex+d\right)-a^{3}-ab^{2}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{\left(a^{2}-2\,b^{2}\right)\cos(ex+d}{}\right.}{4\left(\left(a^{6}-2\,a^{4}b^{2}+a^{2}b^{4}\right)e\cos\left(ex+d\right)^{2}-2\left(a^{5}b-2\,a^{3}b^{3}+ab^{5}\right)e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3*b - a*b^3)*cos(e*x + d)*sin(e*x + d) + (a^3*cos(e*x + d)^2 -
2*a^2*b*sin(e*x + d) - a^3 - a*b^2)*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(
e*x + d)^2 + 2*a*b*sin(e*x + d) + a^2 + b^2 + 2*(b*cos(e*x + d)*sin(e*x + d
) + a*cos(e*x + d))*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*x +
d) - a^2 - b^2)) - 2*(a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 - 2*a^4*
b^2 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(e*x +
 d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e), -1/2*((a^3*b - a*b^3)*cos(e*x + d
)*sin(e*x + d) + (a^3*cos(e*x + d)^2 - 2*a^2*b*sin(e*x + d) - a^3 - a*b^2)*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) + a)/((a^2 - b^2)
*cos(e*x + d))) - (a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 - 2*a^4*b^2
 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(e*x + d)
 - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.59743, size = 647, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] (a*arctan((b*tan(1/2*x*e + 1/2*d) + a)/sqrt(-a^2 + b^2))/((a^2*sgn(b*tan(1/
2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2
+ 2*a*tan(1/2*x*e + 1/2*d) + b) - b^2*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*ta
n(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d
) + b))*sqrt(-a^2 + b^2)) - (2*a^3*b*tan(1/2*x*e + 1/2*d)^3 - 3*a*b^3*tan(1
/2*x*e + 1/2*d)^3 + 2*a^4*tan(1/2*x*e + 1/2*d)^2 - 3*a^2*b^2*tan(1/2*x*e +
1/2*d)^2 - 2*b^4*tan(1/2*x*e + 1/2*d)^2 + 2*a^3*b*tan(1/2*x*e + 1/2*d) - 5*
a*b^3*tan(1/2*x*e + 1/2*d) + a^2*b^2 - 2*b^4)/((a^2*b^2*sgn(b*tan(1/2*x*e +
 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*t
an(1/2*x*e + 1/2*d) + b) - b^4*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x
*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b))
*(b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b)^2))*e^(-1)
```

3.508 $\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\sin(x)}{a\cos(x)+b}$$

```
[Out] Sin[x]/(b + a*Cos[x])
```

Rubi [A] time = 0.0808742, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3289, 2754, 8}

$$\frac{\sin(x)}{a\cos(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]
```

```
[Out] Sin[x]/(b + a*Cos[x])
```

Rule 3289

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx &= \left(4a^2\right)\int \frac{a+b\cos(x)}{\left(2ab+2a^2\cos(x)\right)^2}\,dx \\
&= \frac{\sin(x)}{b+a\cos(x)}+\frac{\int 0\,dx}{a^2-b^2} \\
&= \frac{\sin(x)}{b+a\cos(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0524191, size = 11, normalized size = 1.

$$\frac{\sin(x)}{a\cos(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]
```

```
[Out] Sin[x]/(b + a*Cos[x])
```

Maple [B] time = 0.037, size = 33, normalized size = 3.

$$-2\,\frac{\tan(x/2)}{a\left(\tan(x/2)\right)^2-b\left(\tan(x/2)\right)^2-a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x)
```

```
[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70045, size = 31, normalized size = 2.82

$$\frac{\sin(x)}{a\cos(x)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="fricas
")
```

```
[Out] sin(x)/(a*cos(x) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b**2+2*a*b*cos(x)+a**2*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16215, size = 43, normalized size = 3.91

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2-a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)
```

3.509 $\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx$

Optimal. Leaf size=246

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c - Sqrt[b^2 -
4*a*c]]*Tan[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[
b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tan[x/2])/Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.789015, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3293, 2659, 205}

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c - Sqrt[b^2 -
4*a*c]]*Tan[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[
b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tan[x/2])/Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]])
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
```

```
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cos(x)}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b-\sqrt{b^2-4ac}+$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}+\left(b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tan\right.$$

$$= \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right.}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4a}}}$$

Mathematica [A] time = 0.57916, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[((b - 2*c + Sqrt[
b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e)*ArcTanh[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^
2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt
[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 0.071, size = 2556, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x)
```

```
[Out] -2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*ar
ctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d-a/(-4
*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-
a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d*b-3*a/(-4*a*c
+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-
c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*e+2*a/(-4*a*c+b^2
)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*t
an(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d+3*b/(-4*a*c+b^2)^(1
/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2
*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d-3*b/(-4*a*c+b^2)^(1/2)/(a
-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/
(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d-2*c/(-4*a*c+b^2)^(1/2)/(a-b+c
)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*
a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a*e+c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*
a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)
^(1/2)+a-c)*(a-b+c))^(1/2))*b*e+2*c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^
2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/
2)-a+c)*(a-b+c))^(1/2))*a*e-c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/
2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c
)*(a-b+c))^(1/2))*b*e+a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c
)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c
))^(1/2))*d*b+3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b
+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/
2))*b*e+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*t
an(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d-a/(a-b+c)/(((-4*a*c+b
^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2
)+a-c)*(a-b+c))^(1/2))*e+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)
```

```
*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d-a/
(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x
)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-b/(a-b+c)/(((-4*a*c+b^2)^(1/2
)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(
a-b+c))^(1/2))*d+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan(
(a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e-b/(a-b+c)/((
(-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*
c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-
b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^
(1/2))*e+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*
tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d-c/(a-b+c)/(((-4*a*c+
b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/
2)+a-c)*(a-b+c))^(1/2))*e+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2
)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d-c
/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*
x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-2/(-4*a*c+b^2)^(1/2)/(a-b+c)
/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a
*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*e+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4
*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b
^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*e-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c
+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2))*d*b^2-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^
(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-
c)*(a-b+c))^(1/2))*b^2*e+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-
a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(
a-b+c))^(1/2))*d*b^2+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)
*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+
c))^(1/2))*b^2*e-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-
b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1
/2))*c^2*d+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^
(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))
*c^2*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e\cos(x) + d}{c\cos(x)^2 + b\cos(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(x) + d)/(c*cos(x)^2 + b*cos(x) + a), x)
```

Fricas [B] time = 65.6355, size = 13604, normalized size = 55.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 -
 b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b
^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b
^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*
a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*
a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b*c^2*d^4 + 2*a*b
*c*e^4 - 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*
(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^
3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8
*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2
*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 +
b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4
*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c
^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3
*a^3*b^2 + 2*a*b^4)*c))*cos(x) + 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2
*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^
2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e
)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c +
2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5
- (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4
)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + ((b^4 - 4*a*b^2*c)*d^3 -
 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3
*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b -
 b^3)*c)*e^3)*sin(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e
+ (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*
(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a
^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8
*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
- b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4
```

+ a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 -
(2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*cos(x)) - 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c
- 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 -
b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 +
b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4
*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c
^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3
*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(
2*a^3 - 3*a*b^2)*c))*log(2*b*c^2*d^4 + 2*a*b*c*e^4 - 2*(b^2*c + 2*a*c^2 + 2
*c^3)*d^3*e + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e
^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b
^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b
- 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*
(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e +
2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 -
2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2
*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x)
- 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3
- 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)
*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a
*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(
a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*
d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^
3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*
a*b^4)*c))*sin(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*
b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*
c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*sin(x))*sqrt(-((b
^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (
a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((
b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d
^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)
c^2 - 2(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c +
2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^
3)*cos(x)) + 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d
*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c
^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e +
2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 -
2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2
*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b
^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-2*b*c
^2*d^4 - 2*a*b*c*e^4 + 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 6*(a*b*c + b*c^2
)*d^2*e^2 + 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)
c^3 + 2(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 -

4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 -
 a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sq
rt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^
2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + 1/2*sqrt(2)*(((a^2*b^4 - b^6
 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a
^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a
*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b
^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 +
b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 +
b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11
*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) - ((b^4 - 4*
a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2
 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^
2 - (4*a^2*b - b^3)*c)*e^3)*sin(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a
*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a
^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b
*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/
(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b
^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*
c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))
 - (b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^
2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*cos(x)) - 1/4*sqrt(2)*sqrt(
-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2
 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sq
rt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^
2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-2*b*c^2*d^4 - 2*a*b*c*e^4 + 2*(b^2*
c + 2*a*c^2 + 2*c^3)*d^3*e - 6*(a*b*c + b*c^2)*d^2*e^2 + 2*(2*a*c^2 + (2*a^
2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2
 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^
2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 -
a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b
 + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e
^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 -
 a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b
^4)*c))*cos(x) - 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*
c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3
*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 -
 (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4
+ b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 -
 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)

```
*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 -
 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*
b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2
*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*si
n(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 +
 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a
*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4
*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4
*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3
- (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) - (b^2*c*d^4 + a*b^2*e^4 - (b
^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3
+ 2*a*b*c)*d*e^3)*cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.510 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex\right)$

Optimal. Leaf size=144

$$\frac{\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}{3e}+\frac{b\left(a^2+b^2\right)(a\tan(d+ex)+b)^2}{2e}-\frac{a\left(a^4-b^4\right)\tan(d+ex)}{e}+\frac{b\left(3a^2-b^2\right)\left(a^2+b^2\right)\log(}{e}$$

```
[Out] a*(a^2 - 3*b^2)*(a^2 + b^2)*x + (b*(3*a^2 - b^2)*(a^2 + b^2)*Log[Cos[d + e*
x]])/e - (a*(a^4 - b^4)*Tan[d + e*x])/e + (b*(a^2 + b^2)*(b + a*Tan[d + e*x
])^2)/(2*e) + ((a^2 + b^2)*(b + a*Tan[d + e*x])^3)/(3*e) + (b*(b + a*Tan[d
+ e*x])^4)/(4*e)
```

Rubi [A] time = 0.268532, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3708, 3528, 12, 3525, 3475}

$$\frac{\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}{3e}+\frac{b\left(a^2+b^2\right)(a\tan(d+ex)+b)^2}{2e}-\frac{a\left(a^4-b^4\right)\tan(d+ex)}{e}+\frac{b\left(3a^2-b^2\right)\left(a^2+b^2\right)\log(}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2,
x]
```

```
[Out] a*(a^2 - 3*b^2)*(a^2 + b^2)*x + (b*(3*a^2 - b^2)*(a^2 + b^2)*Log[Cos[d + e*
x]])/e - (a*(a^4 - b^4)*Tan[d + e*x])/e + (b*(a^2 + b^2)*(b + a*Tan[d + e*x
])^2)/(2*e) + ((a^2 + b^2)*(b + a*Tan[d + e*x])^3)/(3*e) + (b*(b + a*Tan[d
+ e*x])^4)/(4*e)
```

Rule 3708

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2\,dx &= \frac{\int \left(2ab+2a^2\tan(d+ex)\right)^4(a+b\tan(d+ex))\,dx}{16a^4}\\
&= \frac{b(b+a\tan(d+ex))^4}{4e}+\frac{\int 2a\left(a^2+b^2\right)\tan(d+ex)}{1}\\
&= \frac{b(b+a\tan(d+ex))^4}{4e}+\frac{\left(a^2+b^2\right)\int \tan(d+ex)\left(2\right.}{8a}\\
&= \frac{\left(a^2+b^2\right)(b+a\tan(d+ex))^3}{3e}+\frac{b(b+a\tan(d+ex}{4e}\\
&= \frac{b\left(a^2+b^2\right)(b+a\tan(d+ex))^2}{2e}+\frac{\left(a^2+b^2\right)(b+a\,t}{3e}\\
&= a\left(a^2-3b^2\right)\left(a^2+b^2\right)x-\frac{a\left(a^4-b^4\right)\tan(d+ex)}{e}+\\
&= a\left(a^2-3b^2\right)\left(a^2+b^2\right)x+\frac{b\left(3a^2-b^2\right)\left(a^2+b^2\right)\log}{e}\end{aligned}$$

Mathematica [C] time = 2.32042, size = 153, normalized size = 1.06

$$\frac{4a^3\left(a^2+4b^2\right)\tan^3(d+ex)+18a^2b\left(a^2+2b^2\right)\tan^2(d+ex)-12a\left(-2a^2b^2+a^4-4b^4\right)\tan(d+ex)+6\left(a^2+b^2\right)\left(i(a+\right.}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^2,x]
```

```
[Out] (6*(a^2 + b^2)*((-I)*(a - I*b)^3*Log[I - Tan[d + e*x]] + I*(a + I*b)^3*Log[
I + Tan[d + e*x]]) - 12*a*(a^4 - 2*a^2*b^2 - 4*b^4)*Tan[d + e*x] + 18*a^2*b
*(a^2 + 2*b^2)*Tan[d + e*x]^2 + 4*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3 + 3*a^4*
b*Tan[d + e*x]^4)/(12*e)
```

Maple [A] time = 0.008, size = 245, normalized size = 1.7

$$\frac{a^4b\left(\tan\left(ex+d\right)\right)^4}{4e}+\frac{\left(\tan\left(ex+d\right)\right)^3a^5}{3e}+\frac{4\left(\tan\left(ex+d\right)\right)^3a^3b^2}{3e}+\frac{3\left(\tan\left(ex+d\right)\right)^2a^4b}{2e}+3\frac{a^2\left(\tan\left(ex+d\right)\right)^2b^3}{e}-\frac{a^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x)
```

```
[Out] 1/4/e*a^4*b*tan(e*x+d)^4+1/3/e*tan(e*x+d)^3*a^5+4/3/e*tan(e*x+d)^3*a^3*b^2+
3/2/e*tan(e*x+d)^2*a^4*b+3/e*tan(e*x+d)^2*a^2*b^3-1/e*a^5*tan(e*x+d)+2/e*a^
3*b^2*tan(e*x+d)+4/e*a*b^4*tan(e*x+d)-3/2/e*ln(1+tan(e*x+d)^2)*a^4*b-1/e*ln
(1+tan(e*x+d)^2)*a^2*b^3+1/2/e*ln(1+tan(e*x+d)^2)*b^5+1/e*arctan(tan(e*x+d)
)*a^5-2/e*arctan(tan(e*x+d))*a^3*b^2-3/e*arctan(tan(e*x+d))*a*b^4
```

Maxima [A] time = 1.51236, size = 203, normalized size = 1.41

$$\frac{3\,a^4b\tan\left(ex+d\right)^4+4\left(a^5+4\,a^3b^2\right)\tan\left(ex+d\right)^3+18\left(a^4b+2\,a^2b^3\right)\tan\left(ex+d\right)^2+12\left(a^5-2\,a^3b^2-3\,ab^4\right)(ex+d)-}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/12*(3*a^4*b*tan(e*x + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x + d)^3 + 18*(a^4
*b + 2*a^2*b^3)*tan(e*x + d)^2 + 12*(a^5 - 2*a^3*b^2 - 3*a*b^4)*(e*x + d) -
 6*(3*a^4*b + 2*a^2*b^3 - b^5)*log(tan(e*x + d)^2 + 1) - 12*(a^5 - 2*a^3*b^
2 - 4*a*b^4)*tan(e*x + d))/e
```

Fricas [A] time = 1.85719, size = 342, normalized size = 2.38

$$\frac{3\,a^4b\tan\left(ex+d\right)^4+4\left(a^5+4\,a^3b^2\right)\tan\left(ex+d\right)^3+12\left(a^5-2\,a^3b^2-3\,ab^4\right)ex+18\left(a^4b+2\,a^2b^3\right)\tan\left(ex+d\right)^2+6\left(3\,a^4 \cdots\right.}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] 1/12*(3*a^4*b*tan(e*x + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x + d)^3 + 12*(a^5
 - 2*a^3*b^2 - 3*a*b^4)*e*x + 18*(a^4*b + 2*a^2*b^3)*tan(e*x + d)^2 + 6*(3*
a^4*b + 2*a^2*b^3 - b^5)*log(1/(tan(e*x + d)^2 + 1)) - 12*(a^5 - 2*a^3*b^2
- 4*a*b^4)*tan(e*x + d))/e
```

Sympy [A] time = 0.741305, size = 248, normalized size = 1.72

$$\begin{cases} a^5x+\frac{a^5\tan^3\left(d+ex\right)}{3e}-\frac{a^5\tan\left(d+ex\right)}{e}-\frac{3a^4b\log\left(\tan^2\left(d+ex\right)+1\right)}{2e}+\frac{a^4b\tan^4\left(d+ex\right)}{4e}+\frac{3a^4b\tan^2\left(d+ex\right)}{2e}-2a^3b^2x+\frac{4a^3b^2\tan^3\left(d+ex\right)}{3e}+\frac{2a^3b^2 \cdots}{} \\ x\left(a+b\tan\left(d\right)\right)\left(a^2\tan^2\left(d\right)+2ab\tan\left(d\right)+b^2\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)
```

```
[Out] Piecewise((a**5*x + a**5*tan(d + e*x)**3/(3*e) - a**5*tan(d + e*x)/e - 3*a*
*4*b*log(tan(d + e*x)**2 + 1)/(2*e) + a**4*b*tan(d + e*x)**4/(4*e) + 3*a**4
*b*tan(d + e*x)**2/(2*e) - 2*a**3*b**2*x + 4*a**3*b**2*tan(d + e*x)**3/(3*e
) + 2*a**3*b**2*tan(d + e*x)/e - a**2*b**3*log(tan(d + e*x)**2 + 1)/e + 3*a
**2*b**3*tan(d + e*x)**2/e - 3*a*b**4*x + 4*a*b**4*tan(d + e*x)/e + b**5*lo
g(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*tan(d)**2
+ 2*a*b*tan(d) + b**2)**2, True))
```

Giac [B] time = 7.67141, size = 3140, normalized size = 21.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/12*(12*a^5*x*e*tan(x*e)^4*tan(d)^4 - 24*a^3*b^2*x*e*tan(x*e)^4*tan(d)^4 -
 36*a*b^4*x*e*tan(x*e)^4*tan(d)^4 + 18*a^4*b*log(4*(tan(d)^2 + 1)/(tan(x*e)
^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*ta
n(x*e)*tan(d) + 1))*tan(x*e)^4*tan(d)^4 + 12*a^2*b^3*log(4*(tan(d)^2 + 1)/(
tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^
2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)^4*tan(d)^4 - 6*b^5*log(4*(tan(d)^2 + 1
)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*
e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)^4*tan(d)^4 - 48*a^5*x*e*tan(x*e)^3*
tan(d)^3 + 96*a^3*b^2*x*e*tan(x*e)^3*tan(d)^3 + 144*a*b^4*x*e*tan(x*e)^3*ta
n(d)^3 + 15*a^4*b*tan(x*e)^4*tan(d)^4 + 36*a^2*b^3*tan(x*e)^4*tan(d)^4 - 72
*a^4*b*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + ta
n(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)^3*tan(d)^
3 - 48*a^2*b^3*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan
(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)^3
*tan(d)^3 + 24*b^5*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3
*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*
e)^3*tan(d)^3 + 12*a^5*tan(x*e)^4*tan(d)^3 - 24*a^3*b^2*tan(x*e)^4*tan(d)^3
 - 48*a*b^4*tan(x*e)^4*tan(d)^3 + 12*a^5*tan(x*e)^3*tan(d)^4 - 24*a^3*b^2*t
an(x*e)^3*tan(d)^4 - 48*a*b^4*tan(x*e)^3*tan(d)^4 + 72*a^5*x*e*tan(x*e)^2*t
an(d)^2 - 144*a^3*b^2*x*e*tan(x*e)^2*tan(d)^2 - 216*a*b^4*x*e*tan(x*e)^2*ta
n(d)^2 + 18*a^4*b*tan(x*e)^4*tan(d)^2 + 36*a^2*b^3*tan(x*e)^4*tan(d)^2 - 24
*a^4*b*tan(x*e)^3*tan(d)^3 - 72*a^2*b^3*tan(x*e)^3*tan(d)^3 + 18*a^4*b*tan(
x*e)^2*tan(d)^4 + 36*a^2*b^3*tan(x*e)^2*tan(d)^4 - 4*a^5*tan(x*e)^4*tan(d)
- 16*a^3*b^2*tan(x*e)^4*tan(d) + 108*a^4*b*log(4*(tan(d)^2 + 1)/(tan(x*e)^4
*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(
x*e)*tan(d) + 1))*tan(x*e)^2*tan(d)^2 + 72*a^2*b^3*log(4*(tan(d)^2 + 1)/(ta
n(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2
- 2*tan(x*e)*tan(d) + 1))*tan(x*e)^2*tan(d)^2 - 36*b^5*log(4*(tan(d)^2 + 1)
/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e
)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)^2*tan(d)^2 - 48*a^5*tan(x*e)^3*tan(d
)^2 + 24*a^3*b^2*tan(x*e)^3*tan(d)^2 + 144*a*b^4*tan(x*e)^3*tan(d)^2 - 48*a
^5*tan(x*e)^2*tan(d)^3 + 24*a^3*b^2*tan(x*e)^2*tan(d)^3 + 144*a*b^4*tan(x*e
)^2*tan(d)^3 - 4*a^5*tan(x*e)*tan(d)^4 - 16*a^3*b^2*tan(x*e)*tan(d)^4 + 3*a
^4*b*tan(x*e)^4 - 48*a^5*x*e*tan(x*e)*tan(d) + 96*a^3*b^2*x*e*tan(x*e)*tan(
d) + 144*a*b^4*x*e*tan(x*e)*tan(d) - 24*a^4*b*tan(x*e)^3*tan(d) - 72*a^2*b^
```

```
3*tan(x*e)^3*tan(d) + 36*a^4*b*tan(x*e)^2*tan(d)^2 + 72*a^2*b^3*tan(x*e)^2*
tan(d)^2 - 24*a^4*b*tan(x*e)*tan(d)^3 - 72*a^2*b^3*tan(x*e)*tan(d)^3 + 3*a^
4*b*tan(d)^4 + 4*a^5*tan(x*e)^3 + 16*a^3*b^2*tan(x*e)^3 - 72*a^4*b*log(4*(t
an(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)
^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)*tan(d) - 48*a^2*b^3*log(
4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*ta
n(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)*tan(d) + 24*b^5*log(
4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*ta
n(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)*tan(d) + 48*a^5*tan(
x*e)^2*tan(d) - 24*a^3*b^2*tan(x*e)^2*tan(d) - 144*a*b^4*tan(x*e)^2*tan(d)
+ 48*a^5*tan(x*e)*tan(d)^2 - 24*a^3*b^2*tan(x*e)*tan(d)^2 - 144*a*b^4*tan(x
*e)*tan(d)^2 + 4*a^5*tan(d)^3 + 16*a^3*b^2*tan(d)^3 + 12*a^5*x*e - 24*a^3*b
^2*x*e - 36*a*b^4*x*e + 18*a^4*b*tan(x*e)^2 + 36*a^2*b^3*tan(x*e)^2 - 24*a^
4*b*tan(x*e)*tan(d) - 72*a^2*b^3*tan(x*e)*tan(d) + 18*a^4*b*tan(d)^2 + 36*a
^2*b^3*tan(d)^2 + 18*a^4*b*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*ta
n(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)
) + 12*a^2*b^3*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan
(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)) - 6*b^5*lo
g(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*
tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)) - 12*a^5*tan(x*e) + 24*a^3*
b^2*tan(x*e) + 48*a*b^4*tan(x*e) - 12*a^5*tan(d) + 24*a^3*b^2*tan(d) + 48*a
*b^4*tan(d) + 15*a^4*b + 36*a^2*b^3)/(e*tan(x*e)^4*tan(d)^4 - 4*e*tan(x*e)^
3*tan(d)^3 + 6*e*tan(x*e)^2*tan(d)^2 - 4*e*tan(x*e)*tan(d) + e)
```

3.511 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)$

Optimal. Leaf size=72

$$-\frac{b\left(a^2+b^2\right)\log(\cos(d+ex))}{e}-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

```
[Out] -(a*(a^2 + b^2)*x) - (b*(a^2 + b^2)*Log[Cos[d + e*x]])/e + (2*a*b^2*Tan[d +
 e*x])/e + (a^2*(a + b*Tan[d + e*x])^2)/(2*b*e)
```

Rubi [A] time = 0.0755105, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3630, 3525, 3475}

$$-\frac{b\left(a^2+b^2\right)\log(\cos(d+ex))}{e}-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x]
```

```
[Out] -(a*(a^2 + b^2)*x) - (b*(a^2 + b^2)*Log[Cos[d + e*x]])/e + (2*a*b^2*Tan[d +
 e*x])/e + (a^2*(a + b*Tan[d + e*x])^2)/(2*b*e)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)dx=\frac{a^2(a+b\tan(d+ex))^2}{2be}+\int (a+b\tan(d+ex))\left(-a^2\right.$$
$$=-a\left(a^2+b^2\right)x+\frac{2ab^2\tan(d+ex)}{e}+\frac{a^2(a+b\tan(d}{2be}$$
$$=-a\left(a^2+b^2\right)x-\frac{b\left(a^2+b^2\right)\log(\cos(d+ex))}{e}+\frac{2ab^2}{}$$

Mathematica [C] time = 0.333865, size = 88, normalized size = 1.22

$$\frac{2a\left(a^2+2b^2\right)\tan(d+ex)+\left(a^2+b^2\right)((b+ia)\log(-\tan(d+ex)+i)+(b-ia)\log(\tan(d+ex)+i))+a^2b\tan^2(d+ex)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2),x]
```

```
[Out] ((a^2 + b^2)*((I*a + b)*Log[I - Tan[d + e*x]] + ((-I)*a + b)*Log[I + Tan[d
+ e*x]]) + 2*a*(a^2 + 2*b^2)*Tan[d + e*x] + a^2*b*Tan[d + e*x]^2)/(2*e)
```

Maple [A] time = 0.005, size = 117, normalized size = 1.6

$$\frac{a^2b(\tan(ex+d))^2}{2e}+\frac{a^3\tan(ex+d)}{e}+2\frac{ab^2\tan(ex+d)}{e}+\frac{\ln\left(1+(\tan(ex+d))^2\right)a^2b}{2e}+\frac{\ln\left(1+(\tan(ex+d))^2\right)b^3}{2e}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x)
```

```
[Out] 1/2/e*a^2*b*tan(e*x+d)^2+1/e*a^3*tan(e*x+d)+2*a*b^2*tan(e*x+d)/e+1/2/e*ln(1
+tan(e*x+d)^2)*a^2*b+1/2/e*ln(1+tan(e*x+d)^2)*b^3-1/e*arctan(tan(e*x+d))*a^
3-1/e*arctan(tan(e*x+d))*a*b^2
```

Maxima [A] time = 1.50719, size = 100, normalized size = 1.39

$$\frac{a^2b\tan(ex+d)^2 - 2\left(a^3+ab^2\right)(ex+d) + \left(a^2b+b^3\right)\log\left(\tan(ex+d)^2+1\right) + 2\left(a^3+2\,ab^2\right)\tan(ex+d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(a^2*b*tan(e*x + d)^2 - 2*(a^3 + a*b^2)*(e*x + d) + (a^2*b + b^3)*log(t
an(e*x + d)^2 + 1) + 2*(a^3 + 2*a*b^2)*tan(e*x + d))/e
```

Fricas [A] time = 1.79282, size = 174, normalized size = 2.42

$$\frac{a^2b\tan(ex+d)^2 - 2\left(a^3+ab^2\right)ex - \left(a^2b+b^3\right)\log\left(\frac{1}{\tan(ex+d)^2+1}\right) + 2\left(a^3+2\,ab^2\right)\tan(ex+d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(a^2*b*tan(e*x + d)^2 - 2*(a^3 + a*b^2)*e*x - (a^2*b + b^3)*log(1/(tan(
e*x + d)^2 + 1)) + 2*(a^3 + 2*a*b^2)*tan(e*x + d))/e
```

Sympy [A] time = 0.325562, size = 122, normalized size = 1.69

$$\begin{cases} -a^3x + \frac{a^3\tan(d+ex)}{e} + \frac{a^2b\log\left(\tan^2(d+ex)+1\right)}{2e} + \frac{a^2b\tan^2(d+ex)}{2e} - ab^2x + \frac{2ab^2\tan(d+ex)}{e} + \frac{b^3\log\left(\tan^2(d+ex)+1\right)}{2e} & \text{for}\ e \neq 0 \\ x\left(a+b\tan(d)\right)\left(a^2\tan^2(d) + 2ab\tan(d) + b^2\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)
```

```
[Out] Piecewise((-a**3*x + a**3*tan(d + e*x)/e + a**2*b*log(tan(d + e*x)**2 + 1)/
(2*e) + a**2*b*tan(d + e*x)**2/(2*e) - a*b**2*x + 2*a*b**2*tan(d + e*x)/e +
```

```
 b**3*log(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*ta
n(d)**2 + 2*a*b*tan(d) + b**2), True))
```

Giac [B] time = 2.07487, size = 957, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*(2*a^3*x*e*tan(x*e)^2*tan(d)^2 + 2*a*b^2*x*e*tan(x*e)^2*tan(d)^2 + a^2
*b*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*
e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)^2*tan(d)^2 +
b^3*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x
*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*e)^2*tan(d)^2 -
 4*a^3*x*e*tan(x*e)*tan(d) - 4*a*b^2*x*e*tan(x*e)*tan(d) - a^2*b*tan(x*e)^2
*tan(d)^2 - 2*a^2*b*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^
3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x
*e)*tan(d) - 2*b^3*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3
*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1))*tan(x*
e)*tan(d) + 2*a^3*tan(x*e)^2*tan(d) + 4*a*b^2*tan(x*e)^2*tan(d) + 2*a^3*tan
(x*e)*tan(d)^2 + 4*a*b^2*tan(x*e)*tan(d)^2 + 2*a^3*x*e + 2*a*b^2*x*e - a^2*
b*tan(x*e)^2 - a^2*b*tan(d)^2 + a^2*b*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(
d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*
tan(d) + 1)) + b^3*log(4*(tan(d)^2 + 1)/(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3
*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)) - 2*a^
3*tan(x*e) - 4*a*b^2*tan(x*e) - 2*a^3*tan(d) - 4*a*b^2*tan(d) - a^2*b)/(e*t
an(x*e)^2*tan(d)^2 - 2*e*tan(x*e)*tan(d) + e)
```

3.512 $\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

Optimal. Leaf size=101

$$-\frac{a^2-b^2}{e\left(a^2+b^2\right)\left(a\tan(d+ex)+b\right)}+\frac{b\left(3a^2-b^2\right)\log(a\sin(d+ex)+b\cos(d+ex))}{e\left(a^2+b^2\right)^2}-\frac{ax\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] -((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^2) + (b*(3*a^2 - b^2)*Log[b*Cos[d + e*x]
+ a*Sin[d + e*x]])/((a^2 + b^2)^2*e) - (a^2 - b^2)/((a^2 + b^2)*e*(b + a*Ta
n[d + e*x]))
```

Rubi [A] time = 0.257506, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3708, 3529, 3531, 3530}

$$-\frac{a^2-b^2}{e\left(a^2+b^2\right)\left(a\tan(d+ex)+b\right)}+\frac{b\left(3a^2-b^2\right)\log(a\sin(d+ex)+b\cos(d+ex))}{e\left(a^2+b^2\right)^2}-\frac{ax\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x]
```

```
[Out] -((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^2) + (b*(3*a^2 - b^2)*Log[b*Cos[d + e*x]
+ a*Sin[d + e*x]])/((a^2 + b^2)^2*e) - (a^2 - b^2)/((a^2 + b^2)*e*(b + a*Ta
n[d + e*x]))
```

Rule 3708

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \left(4a^2\right)\int \frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^2}\,dx$$

$$= -\frac{a^2-b^2}{\left(a^2+b^2\right)e(b+a\tan(d+ex))} + \frac{\int \frac{4a^2b-2a\left(a^2-b^2\right)\tan(d+ex)}{2ab+2a^2\tan(d+ex)}\,dx}{a^2+b^2}$$

$$= -\frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^2} - \frac{a^2-b^2}{\left(a^2+b^2\right)e(b+a\tan(d+ex))} + \frac{\left(b\left(3a^2-b^2\right)\right)\int \frac{2a^2-2a}{2ab+2a}}{\left(a^2+b^2\right)^2}$$

$$= -\frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^2} + \frac{b\left(3a^2-b^2\right)\log(b\cos(d+ex)+a\sin(d+ex))}{\left(a^2+b^2\right)^2e} - \frac{}{\left(a^2+\right.}$$

Mathematica [C] time = 2.0414, size = 187, normalized size = 1.85

$$\frac{\frac{b(-(a+ib)\log(-\tan(d+ex)+i)-(a-ib)\log(\tan(d+ex)+i)+2a\log(a\tan(d+ex)+b))}{a^2+b^2} + (a-b)(a+b)\left(\frac{2a\left(2b\log(a\tan(d+ex)+b)-\frac{a^2+b^2}{a\tan(d+ex)+b}\right)}{\left(a^2+b^2\right)^2} + \frac{i\log(-\tan}{(a-}\right.}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2),x]
```

```
[Out] ((b*(-((a + I*b)*Log[I - Tan[d + e*x]]) - (a - I*b)*Log[I + Tan[d + e*x]] +
 2*a*Log[b + a*Tan[d + e*x]]))/(a^2 + b^2) + (a - b)*(a + b)*((I*Log[I - Ta
n[d + e*x]])/(a - I*b)^2 - (I*Log[I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*
b*Log[b + a*Tan[d + e*x]] - (a^2 + b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^
2))/(2*a*e)
```

Maple [B] time = 0.05, size = 222, normalized size = 2.2

$$-\frac{a^2}{e\left(a^2+b^2\right)\left(b+a\tan\left(ex+d\right)\right)}+\frac{b^2}{e\left(a^2+b^2\right)\left(b+a\tan\left(ex+d\right)\right)}+3\,\frac{b\ln\left(b+a\tan\left(ex+d\right)\right)a^2}{e\left(a^2+b^2\right)^2}-\frac{b^3\ln\left(b+a\tan\left(ex\right.\right.}{e\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x)
```

```
[Out] -1/e/(a^2+b^2)/(b+a*tan(e*x+d))*a^2+1/e/(a^2+b^2)/(b+a*tan(e*x+d))*b^2+3/e*
b/(a^2+b^2)^2*ln(b+a*tan(e*x+d))*a^2-1/e*b^3/(a^2+b^2)^2*ln(b+a*tan(e*x+d))
-3/2/e/(a^2+b^2)^2*ln(1+tan(e*x+d)^2)*a^2*b+1/2/e/(a^2+b^2)^2*ln(1+tan(e*x+
d)^2)*b^3-1/e/(a^2+b^2)^2*arctan(tan(e*x+d))*a^3+3/e/(a^2+b^2)^2*arctan(tan
(e*x+d))*a*b^2
```

Maxima [A] time = 1.50309, size = 217, normalized size = 2.15

$$-\frac{\frac{2\left(a^3-3\,ab^2\right)(ex+d)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(3\,a^2b-b^3\right)\log(a\tan(ex+d)+b)}{a^4+2\,a^2b^2+b^4}+\frac{\left(3\,a^2b-b^3\right)\log\left(\tan(ex+d)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(a^2-b^2\right)}{a^2b+b^3+\left(a^3+ab^2\right)\tan(ex+d)}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(e*x + d)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^2*b - b^
3)*log(a*tan(e*x + d) + b)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*log(ta
n(e*x + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2 - b^2)/(a^2*b + b^3 + (a
```

```
^3 + a*b^2)*tan(e*x + d)))/e
```

Fricas [A] time = 1.73982, size = 417, normalized size = 4.13

$$-\frac{2\,a^4-2\,a^2b^2+2\left(a^3b-3\,ab^3\right)ex-\left(3\,a^2b^2-b^4+\left(3\,a^3b-ab^3\right)\tan\left(ex+d\right)\right)\log\left(\frac{a^2\tan(ex+d)^2+2\,ab\tan(ex+d)+b^2}{\tan(ex+d)^2+1}\right)-2\left(a^3b-\right.}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)e\tan\left(ex+d\right)+\left(a^4b+2\,a^2b^3+b^5\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] -1/2*(2*a^4 - 2*a^2*b^2 + 2*(a^3*b - 3*a*b^3)*e*x - (3*a^2*b^2 - b^4 + (3*a
^3*b - a*b^3)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
b^2)/(tan(e*x + d)^2 + 1)) - 2*(a^3*b - a*b^3 - (a^4 - 3*a^2*b^2)*e*x)*tan(
e*x + d))/((a^5 + 2*a^3*b^2 + a*b^4)*e*tan(e*x + d) + (a^4*b + 2*a^2*b^3 +
b^5)*e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.57471, size = 275, normalized size = 2.72

$$-\frac{1}{2}\left(\frac{2\left(a^3-3\,ab^2\right)(xe+d)}{a^4+2\,a^2b^2+b^4}+\frac{\left(3\,a^2b-b^3\right)\log\left(\tan\left(xe+d\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(3\,a^3b-ab^3\right)\log\left(\left|a\tan\left(xe+d\right)+b\right|\right)}{a^5+2\,a^3b^2+ab^4}+\frac{2\left(3\,a^3b\tan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(x*e + d)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)
*log(tan(x*e + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^3*b - a*b^3)*log(
abs(a*tan(x*e + d) + b))/(a^5 + 2*a^3*b^2 + a*b^4) + 2*(3*a^3*b*tan(x*e + d
) - a*b^3*tan(x*e + d) + a^4 + 3*a^2*b^2 - 2*b^4)/((a^4 + 2*a^2*b^2 + b^4)*
(a*tan(x*e + d) + b)))*e^(-1)
```

3.513 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=197

$$-\frac{a^2-b^2}{3e\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}+\frac{-6a^2b^2+a^4+b^4}{e\left(a^2+b^2\right)^3(a\tan(d+ex)+b)}-\frac{b\left(3a^2-b^2\right)}{2e\left(a^2+b^2\right)^2(a\tan(d+ex)+b)^2}-\frac{b\left(-10a^2b^2+\right.}{}$$

```
[Out] (a*(a^4 - 10*a^2*b^2 + 5*b^4)*x)/(a^2 + b^2)^4 - (b*(5*a^4 - 10*a^2*b^2 + b
^4)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 + b^2)^4*e) - (a^2 - b^2)/(
3*(a^2 + b^2)*e*(b + a*Tan[d + e*x])^3) - (b*(3*a^2 - b^2))/(2*(a^2 + b^2)^
2*e*(b + a*Tan[d + e*x])^2) + (a^4 - 6*a^2*b^2 + b^4)/((a^2 + b^2)^3*e*(b +
 a*Tan[d + e*x]))
```

Rubi [A] time = 0.535338, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3708, 3529, 3531, 3530}

$$-\frac{a^2-b^2}{3e\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}+\frac{-6a^2b^2+a^4+b^4}{e\left(a^2+b^2\right)^3(a\tan(d+ex)+b)}-\frac{b\left(3a^2-b^2\right)}{2e\left(a^2+b^2\right)^2(a\tan(d+ex)+b)^2}-\frac{b\left(-10a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2,
x]
```


```
[Out] (a*(a^4 - 10*a^2*b^2 + 5*b^4)*x)/(a^2 + b^2)^4 - (b*(5*a^4 - 10*a^2*b^2 + b
^4)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 + b^2)^4*e) - (a^2 - b^2)/(
3*(a^2 + b^2)*e*(b + a*Tan[d + e*x])^3) - (b*(3*a^2 - b^2))/(2*(a^2 + b^2)^
2*e*(b + a*Tan[d + e*x])^2) + (a^4 - 6*a^2*b^2 + b^4)/((a^2 + b^2)^3*e*(b +
 a*Tan[d + e*x]))
```


Rule 3708

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx = \left(16a^4\right)\int \frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^4}\,dx$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} + \frac{\left(4a^2\right)\int \frac{4a^2b-2a\left(a^2-b^2\right)\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^3}\,dx}{a^2+b^2}$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} - \frac{b\left(3a^2-b^2\right)}{2\left(a^2+b^2\right)^2 e(b+a\tan(d+ex))^2}$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} - \frac{b\left(3a^2-b^2\right)}{2\left(a^2+b^2\right)^2 e(b+a\tan(d+ex))^2}$$

$$= \frac{a\left(a^4-10a^2b^2+5b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} - \frac{}{2\left(a^2+\right.}$$

$$= \frac{a\left(a^4-10a^2b^2+5b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{b\left(5a^4-10a^2b^2+b^4\right)\log(b\cos(d+ex)+a}{\left(a^2+b^2\right)^4 e}$$

Mathematica [C] time = 4.89778, size = 308, normalized size = 1.56

$$\frac{3b\left(\frac{a\left(-\frac{\left(a^2+b^2\right)\left(a^2+4ab\tan(d+ex)+5b^2\right)}{(a\tan(d+ex)+b)^2}-2\left(a^2-3b^2\right)\log(a\tan(d+ex)+b)\right)}{\left(a^2+b^2\right)^3}+\frac{\log(-\tan(d+ex)+i)}{(a-ib)^3}+\frac{\log(\tan(d+ex)+i)}{(a+ib)^3}\right)-(a-b)(a+b)\left(-\frac{6a\left(a^2-3\right.}{\left(a^2+b^2\right)^3(a\tan}\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^2,x]
```

```
[Out] (-((a - b)*(a + b)*(((3*I)*Log[I - Tan[d + e*x]])/(a - I*b)^4 - ((3*I)*Log[
I + Tan[d + e*x]])/(a + I*b)^4 + (24*a*(a - b)*b*(a + b)*Log[b + a*Tan[d +
e*x]])/(a^2 + b^2)^4 + (2*a)/((a^2 + b^2)*(b + a*Tan[d + e*x])^3) + (6*a*b)
/((a^2 + b^2)^2*(b + a*Tan[d + e*x])^2) - (6*a*(a^2 - 3*b^2))/((a^2 + b^2)^
3*(b + a*Tan[d + e*x])))) + 3*b*(Log[I - Tan[d + e*x]]/(a - I*b)^3 + Log[I
+ Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3*b^2)*Log[b + a*Tan[d + e*x]]
- ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e*x]))/(b + a*Tan[d + e*x])^2))
/(a^2 + b^2)^3))/(6*a*e)
```

Maple [B] time = 0.059, size = 458, normalized size = 2.3

$$-\frac{a^{2}}{3e\left(a^{2}+b^{2}\right)\left(b+a\tan\left(ex+d\right)\right)^{3}}+\frac{b^{2}}{3e\left(a^{2}+b^{2}\right)\left(b+a\tan\left(ex+d\right)\right)^{3}}-\frac{3a^{2}b}{2e\left(a^{2}+b^{2}\right)^{2}\left(b+a\tan\left(ex+d\right)\right)^{2}}+\frac{}{2e\left(a^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x)
```

```
[Out] -1/3/e/(a^2+b^2)/(b+a*tan(e*x+d))^3*a^2+1/3/e/(a^2+b^2)/(b+a*tan(e*x+d))^3*
b^2-3/2/e*b/(a^2+b^2)^2/(b+a*tan(e*x+d))^2*a^2+1/2/e*b^3/(a^2+b^2)^2/(b+a*t
an(e*x+d))^2+1/e/(a^2+b^2)^3/(b+a*tan(e*x+d))*a^4-6/e/(a^2+b^2)^3/(b+a*tan(
e*x+d))*a^2*b^2+1/e/(a^2+b^2)^3/(b+a*tan(e*x+d))*b^4-5/e*b/(a^2+b^2)^4*ln(b
+a*tan(e*x+d))*a^4+10/e*b^3/(a^2+b^2)^4*ln(b+a*tan(e*x+d))*a^2-1/e*b^5/(a^2
+b^2)^4*ln(b+a*tan(e*x+d))+5/2/e/(a^2+b^2)^4*ln(1+tan(e*x+d)^2)*a^4*b-5/e/(
a^2+b^2)^4*ln(1+tan(e*x+d)^2)*a^2*b^3+1/2/e/(a^2+b^2)^4*ln(1+tan(e*x+d)^2)*
b^5+1/e/(a^2+b^2)^4*arctan(tan(e*x+d))*a^5-10/e/(a^2+b^2)^4*arctan(tan(e*x+
d))*a^3*b^2+5/e/(a^2+b^2)^4*arctan(tan(e*x+d))*a*b^4
```

Maxima [B] time = 1.59177, size = 566, normalized size = 2.87

$$\frac{\frac{6\left(a^{5}-10a^{3}b^{2}+5ab^{4}\right)(ex+d)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}-\frac{6\left(5a^{4}b-10a^{2}b^{3}+b^{5}\right)\log(a\tan(ex+d)+b)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}+\frac{3\left(5a^{4}b-10a^{2}b^{3}+b^{5}\right)\log\left(\tan(ex+d)^{2}+1\right)}{a^{8}+4a^{6}b^{2}+6a^{4}b^{4}+4a^{2}b^{6}+b^{8}}-\frac{2a^{6}+5a}{a^{6}b^{3}+3a^{4}b^{5}+3a^{2}b^{7}+b^{9}+\left(a^{9}+3a\right.}}{6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(e*x + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) - 6*(5*a^4*b - 10*a^2*b^3 + b^5)*log(a*tan(e*x + d) + b)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3
+ b^5)*log(tan(e*x + d)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8) - (2*a^6 + 5*a^4*b^2 + 40*a^2*b^4 - 11*b^6 - 6*(a^6 - 6*a^4*b^2 + a^2*b
^4)*tan(e*x + d)^2 - 3*(a^5*b - 26*a^3*b^3 + 5*a*b^5)*tan(e*x + d))/(a^6*b^
3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + (a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*t
an(e*x + d)^3 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(e*x + d)^2
+ 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(e*x + d)))/e
```

Fricas [B] time = 2.07852, size = 1266, normalized size = 6.43

$$-\frac{2a^8 + 7a^6b^2 + 66a^4b^4 - 27a^2b^6 + \left(21a^7b - 56a^5b^3 + 11a^3b^5 - 6\left(a^8 - 10a^6b^2 + 5a^4b^4\right)ex\right)\tan(ex+d)^3 - 6\left(a^5b^3 - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] -1/6*(2*a^8 + 7*a^6*b^2 + 66*a^4*b^4 - 27*a^2*b^6 + (21*a^7*b - 56*a^5*b^3
+ 11*a^3*b^5 - 6*(a^8 - 10*a^6*b^2 + 5*a^4*b^4)*e*x)*tan(e*x + d)^3 - 6*(a^
5*b^3 - 10*a^3*b^5 + 5*a*b^7)*e*x - 3*(2*a^8 - 31*a^6*b^2 + 46*a^4*b^4 - 9*
a^2*b^6 + 6*(a^7*b - 10*a^5*b^3 + 5*a^3*b^5)*e*x)*tan(e*x + d)^2 + 3*(5*a^4
*b^4 - 10*a^2*b^6 + b^8 + (5*a^7*b - 10*a^5*b^3 + a^3*b^5)*tan(e*x + d)^3 +
 3*(5*a^6*b^2 - 10*a^4*b^4 + a^2*b^6)*tan(e*x + d)^2 + 3*(5*a^5*b^3 - 10*a^
3*b^5 + a*b^7)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
 b^2)/(tan(e*x + d)^2 + 1)) - 3*(a^7*b - 46*a^5*b^3 + 35*a^3*b^5 - 6*a*b^7
+ 6*(a^6*b^2 - 10*a^4*b^4 + 5*a^2*b^6)*e*x)*tan(e*x + d))/((a^11 + 4*a^9*b^
2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*e*tan(e*x + d)^3 + 3*(a^10*b + 4*a^8*b
^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*e*tan(e*x + d)^2 + 3*(a^9*b^2 + 4*a^7
*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*e*tan(e*x + d) + (a^8*b^3 + 4*a^6*b^
5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.62634, size = 609, normalized size = 3.09

$$\frac{1}{6}\left(\frac{6\left(a^5 - 10a^3b^2 + 5ab^4\right)(xe+d)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} + \frac{3\left(5a^4b - 10a^2b^3 + b^5\right)\log\left(\tan(xe+d)^2 + 1\right)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{6\left(5a^5b - 10a^3b^3 + ab^5\right)\log(|a}{a^9 + 4a^7b^2 + 6a^5b^4 + 4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(x*e + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*log(tan(x*e + d)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(5*a^5*b - 10*a^3*b^3
+ a*b^5)*log(abs(a*tan(x*e + d) + b))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8) + (55*a^7*b*tan(x*e + d)^3 - 110*a^5*b^3*tan(x*e + d)^3 + 11*a
^3*b^5*tan(x*e + d)^3 + 6*a^8*tan(x*e + d)^2 + 135*a^6*b^2*tan(x*e + d)^2 -
 360*a^4*b^4*tan(x*e + d)^2 + 39*a^2*b^6*tan(x*e + d)^2 + 3*a^7*b*tan(x*e +
 d) + 90*a^5*b^3*tan(x*e + d) - 393*a^3*b^5*tan(x*e + d) + 48*a*b^7*tan(x*e
 + d) - 2*a^8 - 7*a^6*b^2 + 10*a^4*b^4 - 139*a^2*b^6 + 22*b^8)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(a*tan(x*e + d) + b)^3))*e^(-1)
```

3.514 $\int (a+b\tan(d+ex)) \left(b^2 + 2ab\tan(d+ex) + a^2\tan^2(d+ex)\right)$

Optimal. Leaf size=284

$$-\frac{2a^4bx\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}{\left(a^2\tan(d+ex)+ab\right)^3}+\frac{a^4b\left(a^2+b^2\right)\tan(d+ex)\left(a^2\tan^2(d+ex)+2ab\tan(d+\right.}{e\left(a^2\tan(d+ex)+ab\right)^3}$$

```
[Out] (b*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(3*e) + ((a^4 - b
^4)*Log[Cos[d + e*x]]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)
)/(e*(b + a*Tan[d + e*x])^3) + ((a^2 + b^2)*(b^2 + 2*a*b*Tan[d + e*x] + a^2
*Tan[d + e*x]^2)^(3/2))/(2*e*(b + a*Tan[d + e*x])) - (2*a^4*b*(a^2 + b^2)*x
*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(a*b + a^2*Tan[d +
e*x])^3 + (a^4*b*(a^2 + b^2)*Tan[d + e*x]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2)^(3/2))/(e*(a*b + a^2*Tan[d + e*x])^3)
```

Rubi [A] time = 0.226139, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3710, 3528, 12, 3525, 3475}

$$-\frac{2a^4bx\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}{\left(a^2\tan(d+ex)+ab\right)^3}+\frac{a^4b\left(a^2+b^2\right)\tan(d+ex)\left(a^2\tan^2(d+ex)+2ab\tan(d+\right.}{e\left(a^2\tan(d+ex)+ab\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3
/2),x]
```

```
[Out] (b*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(3*e) + ((a^4 - b
^4)*Log[Cos[d + e*x]]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)
)/(e*(b + a*Tan[d + e*x])^3) + ((a^2 + b^2)*(b^2 + 2*a*b*Tan[d + e*x] + a^2
*Tan[d + e*x]^2)^(3/2))/(2*e*(b + a*Tan[d + e*x])) - (2*a^4*b*(a^2 + b^2)*x
*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(a*b + a^2*Tan[d +
e*x])^3 + (a^4*b*(a^2 + b^2)*Tan[d + e*x]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2)^(3/2))/(e*(a*b + a^2*Tan[d + e*x])^3)
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
```

```
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}\,dx = \frac{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}\int\left(2a\right.}{\left(2ab+2a^2\tan\right.}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{3e} + \frac{\left(\right.}{}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{3e} + \frac{\left(\right.}{}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{3e} + \frac{\left(\right.}{}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{3e} + \frac{\left(\right.}{}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{3e} + \frac{\left(\right.}{}$$

Mathematica [C] time = 1.36673, size = 147, normalized size = 0.52

$$\frac{\sqrt{(a\tan(d+ex)+b)^2}\left(3a^2\left(a^2+3b^2\right)\tan^2(d+ex)+6ab\left(2a^2+3b^2\right)\tan(d+ex)-3\left(a^2+b^2\right)\left((a-ib)^2\log(-\tan(d+e\right.\right.}{6e(a\tan(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-3*(a^2 + b^2)*((a - I*b)^2*Log[I - Tan[d +
e*x]] + (a + I*b)^2*Log[I + Tan[d + e*x]]) + 6*a*b*(2*a^2 + 3*b^2)*Tan[d +
e*x] + 3*a^2*(a^2 + 3*b^2)*Tan[d + e*x]^2 + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(
b + a*Tan[d + e*x]))
```

Maple [A] time = 0.088, size = 158, normalized size = 0.6

$$-\frac{-2\left(\tan(ex+d)\right)^3a^3b-3\left(\tan(ex+d)\right)^2a^4-9\left(\tan(ex+d)\right)^2a^2b^2+3\ln\left(1+\left(\tan(ex+d)\right)^2\right)a^4-3\ln\left(1+\left(\tan(ex\right.\right.}{6e\left(b+a\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/6/e*((b+a*tan(e*x+d))^2)^(3/2)*(-2*tan(e*x+d)^3*a^3*b-3*tan(e*x+d)^2*a^4
-9*tan(e*x+d)^2*a^2*b^2+3*ln(1+tan(e*x+d)^2)*a^4-3*ln(1+tan(e*x+d)^2)*b^4+1
2*arctan(tan(e*x+d))*a^3*b+12*arctan(tan(e*x+d))*a*b^3-12*tan(e*x+d)*a^3*b-
18*tan(e*x+d)*a*b^3)/(b+a*tan(e*x+d))^3
```

Maxima [A] time = 1.52516, size = 224, normalized size = 0.79

$$\frac{3\left(a^{3}\tan\left(ex+d\right)^{2}+6\,a^{2}b\tan\left(ex+d\right)-2\left(3\,a^{2}b-b^{3}\right)\left(ex+d\right)-\left(a^{3}-3\,ab^{2}\right)\log\left(\tan\left(ex+d\right)^{2}+1\right)\right)a+\left(2\,a^{3}\tan\left(ex\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/6*(3*(a^3*tan(e*x + d)^2 + 6*a^2*b*tan(e*x + d) - 2*(3*a^2*b - b^3)*(e*x
+ d) - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1))*a + (2*a^3*tan(e*x + d)^3 +
 9*a^2*b*tan(e*x + d)^2 + 6*(a^3 - 3*a*b^2)*(e*x + d) - 3*(3*a^2*b - b^3)*l
og(tan(e*x + d)^2 + 1) - 6*(a^3 - 3*a*b^2)*tan(e*x + d))*b)/e
```

Fricas [A] time = 1.75058, size = 236, normalized size = 0.83

$$\frac{2\,a^{3}b\tan\left(ex+d\right)^{3}-12\left(a^{3}b+ab^{3}\right)ex+3\left(a^{4}+3\,a^{2}b^{2}\right)\tan\left(ex+d\right)^{2}+3\left(a^{4}-b^{4}\right)\log\left(\frac{1}{\tan\left(ex+d\right)^{2}+1}\right)+6\left(2\,a^{3}b+3\,ab^{3}\right)}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*b*tan(e*x + d)^3 - 12*(a^3*b + a*b^3)*e*x + 3*(a^4 + 3*a^2*b^2)*
tan(e*x + d)^2 + 3*(a^4 - b^4)*log(1/(tan(e*x + d)^2 + 1)) + 6*(2*a^3*b + 3
*a*b^3)*tan(e*x + d))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(d + ex))\left((a\tan(d + ex) + b)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))*((a*tan(d + e*x) + b)**2)**(3/2), x)
```

Giac [A] time = 1.52229, size = 328, normalized size = 1.15

$$-2\left(a^3b\mathrm{sgn}\left(a\tan(xe + d) + b\right) + ab^3\mathrm{sgn}\left(a\tan(xe + d) + b\right)\right)(xe + d)e^{(-1)} - \frac{1}{2}\left(a^4\mathrm{sgn}\left(a\tan(xe + d) + b\right) - b^4\mathrm{sgn}\left(a\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] -2*(a^3*b*sgn(a*tan(x*e + d) + b) + a*b^3*sgn(a*tan(x*e + d) + b))*(x*e + d
)*e^(-1) - 1/2*(a^4*sgn(a*tan(x*e + d) + b) - b^4*sgn(a*tan(x*e + d) + b))*
e^(-1)*log(tan(x*e + d)^2 + 1) + 1/6*(2*a^3*b*e^2*sgn(a*tan(x*e + d) + b)*t
an(x*e + d)^3 + 3*a^4*e^2*sgn(a*tan(x*e + d) + b)*tan(x*e + d)^2 + 9*a^2*b^
2*e^2*sgn(a*tan(x*e + d) + b)*tan(x*e + d)^2 + 12*a^3*b*e^2*sgn(a*tan(x*e +
 d) + b)*tan(x*e + d) + 18*a*b^3*e^2*sgn(a*tan(x*e + d) + b)*tan(x*e + d))*
e^(-3)
```

3.515 $\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex}$

Optimal. Leaf size=122

$$\frac{a^2b\tan(d+ex)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}{e\left(a^2\tan(d+ex)+ab\right)}-\frac{\left(a^2+b^2\right)\log(\cos(d+ex))\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+e}}{e(a\tan(d+ex)+b)}$$

```
[Out] -(((a^2 + b^2)*Log[Cos[d + e*x]]*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d
+ e*x]^2])/(e*(b + a*Tan[d + e*x]))) + (a^2*b*Tan[d + e*x]*Sqrt[b^2 + 2*a*b
*Tan[d + e*x] + a^2*Tan[d + e*x]^2])/(e*(a*b + a^2*Tan[d + e*x]))
```

Rubi [A] time = 0.100776, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3710, 3525, 3475}

$$\frac{a^2b\tan(d+ex)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}{e\left(a^2\tan(d+ex)+ab\right)}-\frac{\left(a^2+b^2\right)\log(\cos(d+ex))\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+e}}{e(a\tan(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2
],x]
```

```
[Out] -(((a^2 + b^2)*Log[Cos[d + e*x]]*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d
+ e*x]^2])/(e*(b + a*Tan[d + e*x]))) + (a^2*b*Tan[d + e*x]*Sqrt[b^2 + 2*a*b
*Tan[d + e*x] + a^2*Tan[d + e*x]^2])/(e*(a*b + a^2*Tan[d + e*x]))
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\int \left(2ab+2a\right.}{2ab+2a^2\tan(d+}$$

$$= \frac{a^2b\tan(d+ex)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+}}{e\left(ab+a^2\tan(d+ex)\right)}$$

$$= -\frac{\left(a^2+b^2\right)\log(\cos(d+ex))\sqrt{b^2+2ab\tan(d+ex)+}}{e(b+a\tan(d+ex))}$$

Mathematica [A] time = 0.288498, size = 58, normalized size = 0.48

$$\frac{\sqrt{(a\tan(d+ex)+b)^2}\left(ab\tan(d+ex)-\left(a^2+b^2\right)\log(\cos(d+ex))\right)}{e(a\tan(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
e*x]^2],x]
```

```
[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-((a^2 + b^2)*Log[Cos[d + e*x]]) + a*b*Tan[d
 + e*x]))/(e*(b + a*Tan[d + e*x]))
```

Maple [C] time = 0.095, size = 75, normalized size = 0.6

$$\frac{\mathrm{csgn}\left(b+a\tan\left(ex+d\right)\right)\left(\ln\left(a^2\left(\tan\left(ex+d\right)\right)^2+a^2\right)a^2+\ln\left(a^2\left(\tan\left(ex+d\right)\right)^2+a^2\right)b^2+2\,ab\tan\left(ex+d\right)+2\,b^2\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x)
```

```
[Out] 1/2/e*csgn(b+a*tan(e*x+d))*(ln(a^2*tan(e*x+d)^2+a^2)*a^2+ln(a^2*tan(e*x+d)^
2+a^2)*b^2+2*a*b*tan(e*x+d)+2*b^2)
```

Maxima [A] time = 1.55801, size = 88, normalized size = 0.72

$$\frac{\left(2\,(ex+d)b+a\log\left(\tan\left(ex+d\right)^{2}+1\right)\right)a-\left(2\,(ex+d)a-b\log\left(\tan\left(ex+d\right)^{2}+1\right)-2\,a\tan\left(ex+d\right)\right)b}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="maxima")
```

```
[Out] 1/2*((2*(e*x + d)*b + a*log(tan(e*x + d)^2 + 1))*a - (2*(e*x + d)*a - b*log
(tan(e*x + d)^2 + 1) - 2*a*tan(e*x + d))*b)/e
```

Fricas [A] time = 1.73161, size = 95, normalized size = 0.78

$$\frac{2\,ab\tan\left(ex+d\right)-\left(a^{2}+b^{2}\right)\log\left(\frac{1}{\tan(ex+d)^{2}+1}\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*tan(e*x + d) - (a^2 + b^2)*log(1/(tan(e*x + d)^2 + 1)))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan\left(d+ex\right))\sqrt{\left(a\tan\left(d+ex\right)+b\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2)*(a+b*tan(e*x+d)
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))*sqrt((a*tan(d + e*x) + b)**2), x)
```

Giac [A] time = 1.25722, size = 100, normalized size = 0.82

$$abe^{(-1)}\mathrm{sgn}\left(a\tan\left(xe+d\right)+b\right)\tan\left(xe+d\right)+\frac{1}{2}\left(a^2\mathrm{sgn}\left(a\tan\left(xe+d\right)+b\right)+b^2\mathrm{sgn}\left(a\tan\left(xe+d\right)+b\right)\right)e^{(-1)}\log\left(\tan\left(xe\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="giac")
```

```
[Out] a*b*e^(-1)*sgn(a*tan(x*e + d) + b)*tan(x*e + d) + 1/2*(a^2*sgn(a*tan(x*e +
d) + b) + b^2*sgn(a*tan(x*e + d) + b))*e^(-1)*log(tan(x*e + d)^2 + 1)
```

3.516 $\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$

Optimal. Leaf size=138

$$\frac{2bx\left(a^2\tan(d+ex)+ab\right)}{\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}+\frac{\left(a^2-b^2\right)\left(a\tan(d+ex)+b\right)\log(a\sin(d+ex)+b\cos(d+ex))}{e\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

```
[Out] ((a^2 - b^2)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x]))/((a
^2 + b^2)*e*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2]) + (2*b*x*(
a*b + a^2*Tan[d + e*x]))/((a^2 + b^2)*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2])
```

Rubi [A] time = 0.188173, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3710, 3531, 3530}

$$\frac{2bx\left(a^2\tan(d+ex)+ab\right)}{\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}+\frac{\left(a^2-b^2\right)\left(a\tan(d+ex)+b\right)\log(a\sin(d+ex)+b\cos(d+ex))}{e\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2
],x]
```

```
[Out] ((a^2 - b^2)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x]))/((a
^2 + b^2)*e*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2]) + (2*b*x*(
a*b + a^2*Tan[d + e*x]))/((a^2 + b^2)*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2])
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx = \frac{\left(2ab+2a^2\tan(d+ex)\right)\int \frac{a+b\tan(d+ex)}{2ab+2a^2\tan(d+ex)}\,dx}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}$$

$$= \frac{2bx\left(ab+a^2\tan(d+ex)\right)}{\left(a^2+b^2\right)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}} + \frac{\left(\left(a^2-b^2\right)\left(2ab+\right.\right.}{2a\left(a^2+b^2\right)\sqrt{b}}$$

$$= \frac{\left(a^2-b^2\right)\log(b\cos(d+ex)+a\sin(d+ex))(b+a\tan(d+ex))}{\left(a^2+b^2\right)e\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}} + \frac{}{\left(a^2+\right.}$$

Mathematica [A] time = 0.703782, size = 88, normalized size = 0.64

$$\frac{(a\tan(d+ex)+b)\left(4ab\tan^{-1}(\tan(d+ex))-\left(a^2-b^2\right)\left(\log\left(\sec^2(d+ex)\right)-2\log(a\tan(d+ex)+b)\right)\right)}{2e\left(a^2+b^2\right)\sqrt{(a\tan(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
e*x]^2],x]
```

```
[Out] ((4*a*b*ArcTan[Tan[d + e*x]] - (a^2 - b^2)*(Log[Sec[d + e*x]^2] - 2*Log[b +
 a*Tan[d + e*x]]))*(b + a*Tan[d + e*x]))/(2*(a^2 + b^2)*e*Sqrt[(b + a*Tan[d
 + e*x])^2])
```

Maple [A] time = 0.067, size = 114, normalized size = 0.8

$$\frac{(b+a\tan(ex+d))\left(2\,\ln(b+a\tan(ex+d))\,a^2-2\,\ln(b+a\tan(ex+d))\,b^2-\ln\left(1+(\tan(ex+d))^2\right)a^2+\ln\left(1+(\tan\right.\right.}{2\,e\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] 1/2/e*(b+a*tan(e*x+d))*(2*ln(b+a*tan(e*x+d))*a^2-2*ln(b+a*tan(e*x+d))*b^2-l
n(1+tan(e*x+d)^2)*a^2+ln(1+tan(e*x+d)^2)*b^2+4*a*b*arctan(tan(e*x+d)))/((b+
a*tan(e*x+d))^2)^(1/2)/(a^2+b^2)
```

Maxima [A] time = 1.51937, size = 185, normalized size = 1.34

$$\frac{a\left(\frac{2\,(ex+d)b}{a^2+b^2}+\frac{2\,a\log(a\tan(ex+d)+b)}{a^2+b^2}-\frac{a\log\left(\tan(ex+d)^2+1\right)}{a^2+b^2}\right)+\left(\frac{2\,(ex+d)a}{a^2+b^2}-\frac{2\,b\log(a\tan(ex+d)+b)}{a^2+b^2}+\frac{b\log\left(\tan(ex+d)^2+1\right)}{a^2+b^2}\right)b}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] 1/2*(a*(2*(e*x + d)*b/(a^2 + b^2) + 2*a*log(a*tan(e*x + d) + b)/(a^2 + b^2)
 - a*log(tan(e*x + d)^2 + 1)/(a^2 + b^2)) + (2*(e*x + d)*a/(a^2 + b^2) - 2*
b*log(a*tan(e*x + d) + b)/(a^2 + b^2) + b*log(tan(e*x + d)^2 + 1)/(a^2 + b^
2))*b)/e
```

Fricas [A] time = 1.65239, size = 163, normalized size = 1.18

$$\frac{4\,abex+\left(a^2-b^2\right)\log\left(\frac{a^2\tan(ex+d)^2+2\,ab\tan(ex+d)+b^2}{\tan(ex+d)^2+1}\right)}{2\left(a^2+b^2\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*e*x + (a^2 - b^2)*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
 b^2)/(tan(e*x + d)^2 + 1)))/((a^2 + b^2)*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \tan (d + ex)}{\sqrt{(a \tan (d + ex) + b)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))/sqrt((a*tan(d + e*x) + b)**2), x)
```

Giac [B] time = 2.01922, size = 748, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/2*(2*(pi*sgn(tan(1/2*x*e + 1/2*d)) + 2*arctan(1/2*(tan(1/2*x*e + 1/2*d)^
2 - 1)/tan(1/2*x*e + 1/2*d)))*a*b/(a^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*
tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2
*d) - b) + b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (a^2 - b^2)*
log((1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 + 4)/(a^2*sgn(-b*tan(
1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^
2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a
*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/
2*d) - b)) + 2*(a^2*b - b^3)*log(abs(-b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x
*e + 1/2*d)) - 2*a))/(a^2*b*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e
 + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) +
b^3*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/
2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)))*e^(-1)
```

3.517 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(a^2-b^2\right)(a\tan(d+ex)+b)}{2e\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}-\frac{4bx\left(a^2-b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}{a^2\left(a^2+b^2\right)^3\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}-$$

```
[Out] -((a^2 - b^2)*(b + a*Tan[d + e*x]))/(2*(a^2 + b^2)*e*(b^2 + 2*a*b*Tan[d + e
*x] + a^2*Tan[d + e*x]^2)^(3/2)) - ((a^4 - 6*a^2*b^2 + b^4)*Log[b*Cos[d + e
*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x])^3)/((a^2 + b^2)^3*e*(b^2 + 2*a*b
*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)) - (4*b*(a^2 - b^2)*x*(a*b + a^2*
Tan[d + e*x])^3)/(a^2*(a^2 + b^2)^3*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
 e*x]^2)^(3/2)) - (b*(3*a^2 - b^2)*(a*b + a^2*Tan[d + e*x])^3)/((a^2 + b^2)
^2*e*(a^3*b + a^4*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2)^(3/2))
```

Rubi [A] time = 0.401571, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {3710, 3529, 3531, 3530}

$$-\frac{\left(a^2-b^2\right)(a\tan(d+ex)+b)}{2e\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}-\frac{4bx\left(a^2-b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}{a^2\left(a^2+b^2\right)^3\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3
/2),x]
```

```
[Out] -((a^2 - b^2)*(b + a*Tan[d + e*x]))/(2*(a^2 + b^2)*e*(b^2 + 2*a*b*Tan[d + e
*x] + a^2*Tan[d + e*x]^2)^(3/2)) - ((a^4 - 6*a^2*b^2 + b^4)*Log[b*Cos[d + e
*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x])^3)/((a^2 + b^2)^3*e*(b^2 + 2*a*b
*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)) - (4*b*(a^2 - b^2)*x*(a*b + a^2*
Tan[d + e*x])^3)/(a^2*(a^2 + b^2)^3*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
 e*x]^2)^(3/2)) - (b*(3*a^2 - b^2)*(a*b + a^2*Tan[d + e*x])^3)/((a^2 + b^2)
^2*e*(a^3*b + a^4*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2)^(3/2))
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\tan(d+ex)\right)^3 \int \frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^3}\,dx}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} + \frac{(2a}{4a^2\,(a}$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} - \frac{}{(a^2+}$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} - \frac{}{(a^2+}$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} - \frac{}{(a^2+}$$

Mathematica [C] time = 3.18623, size = 268, normalized size = 0.85

$$\frac{(a\tan(d+ex)+b)^3\left(b\left(\frac{2a\left(2b\log(a\tan(d+ex)+b)-\frac{a^2+b^2}{a\tan(d+ex)+b}\right)}{\left(a^2+b^2\right)^2}+\frac{i\log(-\tan(d+ex)+i)}{(a-ib)^2}-\frac{i\log(\tan(d+ex)+i)}{(a+ib)^2}\right)+(a-b)(a+b)\left(\frac{a\left(-\frac{(a^2+b^2)}{}\right.}{}\right.\right.}{2ae\left((a\tan(d+ex)+b)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^(3/2),x]
```

```
[Out] ((b + a*Tan[d + e*x])^3*(b*((I*Log[I - Tan[d + e*x]])/(a - I*b)^2 - (I*Log[
I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*b*Log[b + a*Tan[d + e*x]] - (a^2 +
 b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^2) + (a - b)*(a + b)*(Log[I - Tan[
d + e*x]]/(a - I*b)^3 + Log[I + Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3
*b^2)*Log[b + a*Tan[d + e*x]] - ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e
*x]))/(b + a*Tan[d + e*x])^2))/(a^2 + b^2)^3)))/(2*a*e*((b + a*Tan[d + e*x]
)^2)^(3/2))
```

Maple [B] time = 0.094, size = 622, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/2/e*(a^6+2*ln(b+a*tan(e*x+d))*b^6-ln(1+tan(e*x+d)^2)*b^6+3*a^2*b^4+7*a^4
*b^2-ln(1+tan(e*x+d)^2)*tan(e*x+d)^2*a^6-3*b^6-12*ln(b+a*tan(e*x+d))*a^2*b^
4-ln(1+tan(e*x+d)^2)*a^4*b^2+6*ln(1+tan(e*x+d)^2)*a^2*b^4+8*arctan(tan(e*x+
d))*a^3*b^3-8*arctan(tan(e*x+d))*a*b^5+2*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^
6-2*tan(e*x+d)*a*b^5+6*tan(e*x+d)*a^5*b+4*tan(e*x+d)*a^3*b^3+2*ln(b+a*tan(e
*x+d))*a^4*b^2+4*ln(b+a*tan(e*x+d))*tan(e*x+d)*a^5*b-24*ln(b+a*tan(e*x+d))*
tan(e*x+d)*a^3*b^3+4*ln(b+a*tan(e*x+d))*tan(e*x+d)*a*b^5-2*ln(1+tan(e*x+d)^
2)*tan(e*x+d)*a^5*b+12*ln(1+tan(e*x+d)^2)*tan(e*x+d)*a^3*b^3-2*ln(1+tan(e*x
+d)^2)*tan(e*x+d)*a*b^5+16*arctan(tan(e*x+d))*tan(e*x+d)*a^4*b^2-16*arctan(
tan(e*x+d))*tan(e*x+d)*a^2*b^4-12*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^4*b^2+2
*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^2*b^4+6*ln(1+tan(e*x+d)^2)*tan(e*x+d)^2*
a^4*b^2-ln(1+tan(e*x+d)^2)*tan(e*x+d)^2*a^2*b^4+8*arctan(tan(e*x+d))*tan(e*
x+d)^2*a^5*b-8*arctan(tan(e*x+d))*tan(e*x+d)^2*a^3*b^3)*(b+a*tan(e*x+d))/(a
^2+b^2)^3/((b+a*tan(e*x+d))^2)^(3/2)
```

Maxima [A] time = 1.54713, size = 672, normalized size = 2.13

$$-\frac{\left(\frac{2\left(3a^2b-b^3\right)(ex+d)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\left(a^3-3ab^2\right)\log(a\tan(ex+d)+b)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(a^3-3ab^2\right)\log\left(\tan(ex+d)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4a^2b\tan(ex+d)+a^3+5ab^2}{a^4b^2+2a^2b^4+b^6+\left(a^6+2a^4b^2+a^2b^4\right)\tan(ex+d)^2+2\left(a^5b+2a\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] -1/2*((2*(3*a^2*b - b^3)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*
(a^3 - 3*a*b^2)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (4*a^2*b*tan(e*x + d) + a^3 + 5*a*b^2)/(a^4*b^2 + 2*a^2*b^4 + b^6 + (
a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*t
an(e*x + d)))*a + (2*(a^3 - 3*a*b^2)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) - 2*(3*a^2*b - b^3)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6) + (3*a^2*b - b^3)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3
```

```
*a^2*b^4 + b^6) + (a^2*b - 3*b^3 + 2*(a^3 - a*b^2)*tan(e*x + d))/(a^4*b^2 +
 2*a^2*b^4 + b^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b +
2*a^3*b^3 + a*b^5)*tan(e*x + d)))*b)/e
```

Fricas [A] time = 1.73784, size = 764, normalized size = 2.42

$$-\frac{a^6 + 8\,a^4b^2 - 5\,a^2b^4 + 8\left(a^3b^3 - ab^5\right)ex + \left(a^6 - 8\,a^4b^2 + 3\,a^2b^4 + 8\left(a^5b - a^3b^3\right)ex\right)\tan\left(ex + d\right)^2 + \left(a^4b^2 - 6\,a^2b^4 + b^6\right.}{2\left(\left(a^8 + 3\,a^6b^2 + 3\,a^4b^4 + a^2b^6\right)e\tan\left(e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(a^6 + 8*a^4*b^2 - 5*a^2*b^4 + 8*(a^3*b^3 - a*b^5)*e*x + (a^6 - 8*a^4*
b^2 + 3*a^2*b^4 + 8*(a^5*b - a^3*b^3)*e*x)*tan(e*x + d)^2 + (a^4*b^2 - 6*a^
2*b^4 + b^6 + (a^6 - 6*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b - 6*a^3
*b^3 + a*b^5)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
b^2)/(tan(e*x + d)^2 + 1)) + 4*(2*a^5*b - 3*a^3*b^3 + a*b^5 + 4*(a^4*b^2 -
a^2*b^4)*e*x)*tan(e*x + d))/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*e*tan(
e*x + d)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*e*tan(e*x + d) + (a^
6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\tan\left(d + ex\right)}{\left(\left(a\tan\left(d + ex\right) + b\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))/((a*tan(d + e*x) + b)**2)**(3/2), x)
```

Giac [B] time = 3.00569, size = 2121, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/2*(4*(a^3*b - a*b^3)*(pi*sgn(tan(1/2*x*e + 1/2*d)) + 2*arctan(1/2*(tan(1/
2*x*e + 1/2*d)^2 - 1)/tan(1/2*x*e + 1/2*d)))/(a^6*sgn(-b*tan(1/2*x*e + 1/2*
d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/
2*x*e + 1/2*d) - b) + 3*a^4*b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2
*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b
) + 3*a^2*b^4*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^6*sgn(-b*tan
(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (a^4 - 6*a^2*b^2 + b^4)*log((1/tan(1/
2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 + 4)/(a^6*sgn(-b*tan(1/2*x*e + 1/2
*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1
/2*x*e + 1/2*d) - b) + 3*a^4*b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/
2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) -
b) + 3*a^2*b^4*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^6*sgn(-b*ta
n(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d
)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) + 2*(a^4*b - 6*a^2*b^3 + b^5)*log(abs(
-b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) - 2*a))/(a^6*b*sgn(-b*ta
n(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d
)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + 3*a^4*b^3*sgn(-b*tan(1/2*x*e + 1/2*d)
^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*
x*e + 1/2*d) - b) + 3*a^2*b^5*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x
*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)
+ b^7*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(
1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (3*a^4*b^4*(1/tan(1/2
*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 - 18*a^2*b^6*(1/tan(1/2*x*e + 1/2*d
) - tan(1/2*x*e + 1/2*d))^2 + 3*b^8*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e +
 1/2*d))^2 + 4*a^7*b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) + 28*a
^5*b^3*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) - 68*a^3*b^5*(1/tan(
1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) + 4*a*b^7*(1/tan(1/2*x*e + 1/2*d)
- tan(1/2*x*e + 1/2*d)) + 4*a^8 + 40*a^6*b^2 - 60*a^4*b^4)/((a^6*b^2*sgn(-b
*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/
2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + 3*a^4*b^4*sgn(-b*tan(1/2*x*e + 1/2
*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1
/2*x*e + 1/2*d) - b) + 3*a^2*b^6*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/
```

```
2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) -
b) + b^8*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*t
an(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b))*(b*(1/tan(1/2*x*e +
1/2*d) - tan(1/2*x*e + 1/2*d)) + 2*a)^2))*e^(-1)
```

3.518 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2$

Optimal. Leaf size=184

$$\frac{a\left(50a^2b^2+4a^4+19b^4\right)\tan(d+ex)}{6e}+\frac{b\left(56a^2b^2+19a^4+8b^4\right)\tanh^{-1}(\sin(d+ex))}{8e}+\frac{a^2b\left(41a^2+26b^2\right)\tan(d+ex)\sec}{24e}$$

```
[Out] a*b^4*x + (b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]])/(8*e) + (
a*(4*a^4 + 50*a^2*b^2 + 19*b^4)*Tan[d + e*x])/(6*e) + (a^2*b*(41*a^2 + 26*b
^2)*Sec[d + e*x]*Tan[d + e*x])/(24*e) + ((4*a^2 + 7*b^2)*(a*b + a^2*Sec[d +
 e*x])^2*Tan[d + e*x])/(12*a*e) + (b*(a*b + a^2*Sec[d + e*x])^3*Tan[d + e*x
])/(4*a^2*e)
```

Rubi [A] time = 0.429861, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4172, 3918, 4056, 4048, 3770, 3767, 8}

$$\frac{a\left(50a^2b^2+4a^4+19b^4\right)\tan(d+ex)}{6e}+\frac{b\left(56a^2b^2+19a^4+8b^4\right)\tanh^{-1}(\sin(d+ex))}{8e}+\frac{a^2b\left(41a^2+26b^2\right)\tan(d+ex)\sec}{24e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,
x]
```

```
[Out] a*b^4*x + (b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]])/(8*e) + (
a*(4*a^4 + 50*a^2*b^2 + 19*b^4)*Tan[d + e*x])/(6*e) + (a^2*b*(41*a^2 + 26*b
^2)*Sec[d + e*x]*Tan[d + e*x])/(24*e) + ((4*a^2 + 7*b^2)*(a*b + a^2*Sec[d +
 e*x])^2*Tan[d + e*x])/(12*a*e) + (b*(a*b + a^2*Sec[d + e*x])^3*Tan[d + e*x
])/(4*a^2*e)
```

Rule 4172

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx = \frac{\int \left(2ab+2a^2\sec(d+ex)\right)^4(a+b\sec(d+ex))\,dx}{16a^4}$$
$$= \frac{b\left(ab+a^2\sec(d+ex)\right)^3\tan(d+ex)}{4a^2e} + \frac{\int \left(2ab+2a^2\right.}{}$$
$$= \frac{\left(4a^2+7b^2\right)\left(ab+a^2\sec(d+ex)\right)^2\tan(d+ex)}{12ae} + \frac{b\left(\right.}{}$$
$$= \frac{a^2b\left(41a^2+26b^2\right)\sec(d+ex)\tan(d+ex)}{24e} + \frac{\left(4a^2+\right.}{}$$
$$= ab^4x + \frac{a^2b\left(41a^2+26b^2\right)\sec(d+ex)\tan(d+ex)}{24e} +$$
$$= ab^4x + \frac{b\left(19a^4+56a^2b^2+8b^4\right)\tanh^{-1}(\sin(d+ex))}{8e}$$
$$= ab^4x + \frac{b\left(19a^4+56a^2b^2+8b^4\right)\tanh^{-1}(\sin(d+ex))}{8e}$$

Mathematica [A] time = 0.80197, size = 130, normalized size = 0.71

$$\frac{8a^3\left(a^2+4b^2\right)\tan^3(d+ex)+3b\left(56a^2b^2+19a^4+8b^4\right)\tanh^{-1}(\sin(d+ex))+3a\tan(d+ex)\left(ab\left(19a^2+24b^2\right)\sec(d+e\right.}{24e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^2,x]
```

```
[Out] (24*a*b^4*e*x + 3*b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]] + 3
*a*(8*(a^4 + 10*a^2*b^2 + 4*b^4) + a*b*(19*a^2 + 24*b^2)*Sec[d + e*x] + 2*a
^3*b*Sec[d + e*x]^3)*Tan[d + e*x] + 8*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3)/(24
*e)
```

Maple [A] time = 0.073, size = 246, normalized size = 1.3

$$ab^4x+\frac{ab^4d}{e}+7\,\frac{a^2b^3\ln(\sec(ex+d)+\tan(ex+d))}{e}+\frac{26\,a^3b^2\tan(ex+d)}{3\,e}+\frac{19\,a^4b\sec(ex+d)\tan(ex+d)}{8\,e}+\frac{19\,a^4b\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x)
```

```
[Out] a*b^4*x+1/e*a*b^4*d+7/e*a^2*b^3*ln(sec(e*x+d)+tan(e*x+d))+26/3/e*a^3*b^2*ta
n(e*x+d)+19/8/e*a^4*b*sec(e*x+d)*tan(e*x+d)+19/8/e*a^4*b*ln(sec(e*x+d)+tan(
e*x+d))+2/3/e*a^5*tan(e*x+d)+1/3/e*a^5*tan(e*x+d)*sec(e*x+d)^2+1/e*b^5*ln(s
ec(e*x+d)+tan(e*x+d))+4/e*a*b^4*tan(e*x+d)+3/e*a^2*b^3*sec(e*x+d)*tan(e*x+d
)+4/3/e*a^3*b^2*tan(e*x+d)*sec(e*x+d)^2+1/4/e*a^4*b*tan(e*x+d)*sec(e*x+d)^3
```

Maxima [A] time = 1.03341, size = 404, normalized size = 2.2

$$16\left(\tan\left(ex+d\right)^{3}+3\,\tan\left(ex+d\right)\right)a^{5}+64\left(\tan\left(ex+d\right)^{3}+3\,\tan\left(ex+d\right)\right)a^{3}b^{2}+48\left(ex+d\right)ab^{4}-3\,a^{4}b\left(\frac{2\left(3\,\sin(ex+d)^{3}-5\right.}{\sin(ex+d)^{4}-2\,\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/48*(16*(tan(e*x + d)^3 + 3*tan(e*x + d))*a^5 + 64*(tan(e*x + d)^3 + 3*tan
(e*x + d))*a^3*b^2 + 48*(e*x + d)*a*b^4 - 3*a^4*b*(2*(3*sin(e*x + d)^3 - 5*
sin(e*x + d))/(sin(e*x + d)^4 - 2*sin(e*x + d)^2 + 1) - 3*log(sin(e*x + d)
+ 1) + 3*log(sin(e*x + d) - 1)) - 48*a^4*b*(2*sin(e*x + d)/(sin(e*x + d)^2
- 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1)) - 72*a^2*b^3*(2*sin(e
*x + d)/(sin(e*x + d)^2 - 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1
)) + 192*a^2*b^3*log(sec(e*x + d) + tan(e*x + d)) + 48*b^5*log(sec(e*x + d)
 + tan(e*x + d)) + 288*a^3*b^2*tan(e*x + d) + 192*a*b^4*tan(e*x + d))/e
```

Fricas [A] time = 1.97682, size = 481, normalized size = 2.61

$$48\,ab^{4}ex\cos\left(ex+d\right)^{4}+3\left(19\,a^{4}b+56\,a^{2}b^{3}+8\,b^{5}\right)\cos\left(ex+d\right)^{4}\log\left(\sin\left(ex+d\right)+1\right)-3\left(19\,a^{4}b+56\,a^{2}b^{3}+8\,b^{5}\right)\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] 1/48*(48*a*b^4*e*x*cos(e*x + d)^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(e
*x + d)^4*log(sin(e*x + d) + 1) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(e*x
```

```
 + d)^4*log(-sin(e*x + d) + 1) + 2*(6*a^4*b + 16*(a^5 + 13*a^3*b^2 + 6*a*b^
4)*cos(e*x + d)^3 + 3*(19*a^4*b + 24*a^2*b^3)*cos(e*x + d)^2 + 8*(a^5 + 4*a
^3*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(d + ex))(a\sec(d + ex) + b)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
```

```
[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**4, x)
```

Giac [B] time = 1.28002, size = 635, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/24*(24*(x*e + d)*a*b^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1/
2*x*e + 1/2*d) + 1)) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*x*
e + 1/2*d) - 1)) - 2*(24*a^5*tan(1/2*x*e + 1/2*d)^7 - 63*a^4*b*tan(1/2*x*e
+ 1/2*d)^7 + 240*a^3*b^2*tan(1/2*x*e + 1/2*d)^7 - 72*a^2*b^3*tan(1/2*x*e +
1/2*d)^7 + 96*a*b^4*tan(1/2*x*e + 1/2*d)^7 - 40*a^5*tan(1/2*x*e + 1/2*d)^5
+ 39*a^4*b*tan(1/2*x*e + 1/2*d)^5 - 592*a^3*b^2*tan(1/2*x*e + 1/2*d)^5 + 72
*a^2*b^3*tan(1/2*x*e + 1/2*d)^5 - 288*a*b^4*tan(1/2*x*e + 1/2*d)^5 + 40*a^5
*tan(1/2*x*e + 1/2*d)^3 + 39*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 592*a^3*b^2*tan
(1/2*x*e + 1/2*d)^3 + 72*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + 288*a*b^4*tan(1/2
*x*e + 1/2*d)^3 - 24*a^5*tan(1/2*x*e + 1/2*d) - 63*a^4*b*tan(1/2*x*e + 1/2*
d) - 240*a^3*b^2*tan(1/2*x*e + 1/2*d) - 72*a^2*b^3*tan(1/2*x*e + 1/2*d) - 9
6*a*b^4*tan(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)^4)*e^(-1)
```

3.519 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)$

Optimal. Leaf size=76

$$\frac{a\left(a^2+2b^2\right)\tan(d+ex)}{e}+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a^2b\tan(d+ex)\sec(d+ex)}{2e}+ab^2x$$

```
[Out] a*b^2*x + (b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]])/(2*e) + (a*(a^2 + 2*b^2
)*Tan[d + e*x])/e + (a^2*b*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rubi [A] time = 0.0783303, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4048, 3770, 3767, 8}

$$\frac{a\left(a^2+2b^2\right)\tan(d+ex)}{e}+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a^2b\tan(d+ex)\sec(d+ex)}{2e}+ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x]
```

```
[Out] a*b^2*x + (b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]])/(2*e) + (a*(a^2 + 2*b^2
)*Tan[d + e*x])/e + (a^2*b*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)dx = \frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}+\frac{1}{2}\int\left(2ab^2+b\left(5a^2+2\right.\right.$$
$$=ab^2x+\frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}+\left(a\left(a^2+2b^2\right)\right)\int$$
$$=ab^2x+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a^2b\sec(\cdots}{\cdots}$$
$$=ab^2x+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a\left(a^2+2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.265345, size = 64, normalized size = 0.84

$$\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))+a\tan(d+ex)\left(2a^2+ab\sec(d+ex)+4b^2\right)+2ab^2ex}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2),x]
```

```
[Out] (2*a*b^2*e*x + b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]] + a*(2*a^2 + 4*b^2 +
 a*b*Sec[d + e*x])*Tan[d + e*x])/(2*e)
```

Maple [A] time = 0.039, size = 110, normalized size = 1.5

$$ab^2x+\frac{ab^2d}{e}+\frac{5\,a^2b\ln\left(\sec\left(ex+d\right)+\tan\left(ex+d\right)\right)}{2\,e}+\frac{a^3\tan\left(ex+d\right)}{e}+\frac{b^3\ln\left(\sec\left(ex+d\right)+\tan\left(ex+d\right)\right)}{e}+2\,\frac{ab^2\tan\left(\cdots\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x)
```

```
[Out] a*b^2*x+1/e*a*b^2*d+5/2/e*a^2*b*ln(sec(e*x+d)+tan(e*x+d))+1/e*a^3*tan(e*x+d
)+1/e*b^3*ln(sec(e*x+d)+tan(e*x+d))+2*a*b^2*tan(e*x+d)/e+1/2*a^2*b*sec(e*x+
d)*tan(e*x+d)/e
```

Maxima [A] time = 1.0208, size = 170, normalized size = 2.24

$$\frac{4\,(ex+d)ab^2 - a^2b\left(\frac{2\,\sin(ex+d)}{\sin(ex+d)^2-1} - \log\left(\sin\left(ex+d\right)+1\right) + \log\left(\sin\left(ex+d\right)-1\right)\right) + 8\,a^2b\log\left(\sec\left(ex+d\right)+\tan\left(ex+d\right)\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*(e*x + d)*a*b^2 - a^2*b*(2*sin(e*x + d)/(sin(e*x + d)^2 - 1) - log(s
in(e*x + d) + 1) + log(sin(e*x + d) - 1)) + 8*a^2*b*log(sec(e*x + d) + tan(
e*x + d)) + 4*b^3*log(sec(e*x + d) + tan(e*x + d)) + 4*a^3*tan(e*x + d) + 8
*a*b^2*tan(e*x + d))/e
```

Fricas [A] time = 1.74779, size = 305, normalized size = 4.01

$$\frac{4\,ab^2ex\cos\left(ex+d\right)^2 + \left(5\,a^2b+2\,b^3\right)\cos\left(ex+d\right)^2\log\left(\sin\left(ex+d\right)+1\right) - \left(5\,a^2b+2\,b^3\right)\cos\left(ex+d\right)^2\log\left(-\sin\left(ex+d\right)\right.}{4\,e\cos\left(ex+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(4*a*b^2*e*x*cos(e*x + d)^2 + (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(sin(
e*x + d) + 1) - (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(-sin(e*x + d) + 1) + 2
*(a^2*b + 2*(a^3 + 2*a*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec\left(d+ex\right)\right)\left(a\sec\left(d+ex\right)+b\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**2, x)
```

Giac [B] time = 1.22314, size = 258, normalized size = 3.39

$$\frac{1}{2}\left(2\,(xe+d)ab^2+\left(5\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+1\right|\right)-\left(5\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-1\right|\right)-\frac{2\left(2\,a^3\tan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] 1/2*(2*(x*e + d)*a*b^2 + (5*a^2*b + 2*b^3)*log(abs(tan(1/2*x*e + 1/2*d) + 1
)) - (5*a^2*b + 2*b^3)*log(abs(tan(1/2*x*e + 1/2*d) - 1)) - 2*(2*a^3*tan(1/
2*x*e + 1/2*d)^3 - a^2*b*tan(1/2*x*e + 1/2*d)^3 + 4*a*b^2*tan(1/2*x*e + 1/2
*d)^3 - 2*a^3*tan(1/2*x*e + 1/2*d) - a^2*b*tan(1/2*x*e + 1/2*d) - 4*a*b^2*t
an(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)^2)*e^(-1)
```

3.520 $\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

Optimal. Leaf size=92

$$-\frac{a^2\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e}+\frac{ax}{b^2}$$

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2]
)/Sqrt[a + b]])/(b^2*e) - (a^2*Tan[d + e*x])/(b*e*(a*b + a^2*Sec[d + e*x]))
```

Rubi [A] time = 0.303862, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4172, 3923, 3919, 3831, 2659, 205}

$$-\frac{a^2\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e}+\frac{ax}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2]
)/Sqrt[a + b]])/(b^2*e) - (a^2*Tan[d + e*x])/(b*e*(a*b + a^2*Sec[d + e*x]))
```

Rule 4172

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \left(4a^2\right)\int \frac{a+b\sec(d+ex)}{\left(2ab+2a^2\sec(d+ex)\right)^2}\,dx$$

$$= -\frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} + \frac{\int \frac{4a^3\left(a^2-b^2\right)+4a^2b\left(a^2-b^2\right)\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{2ab\left(a^2-b^2\right)}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{\left(2a\left(a^2-b^2\right)\right)\int \frac{\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{b^2}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{\left(a^2-b^2\right)\int \frac{1}{1+\frac{b\cos(d+ex)}{a}}\,dx}{ab^2}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{b}{a}+\left(1-\frac{b}{a}\right)x^2}\,dx, x$$

$$= \frac{ax}{b^2} - \frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)}$$

Mathematica [A] time = 0.38539, size = 97, normalized size = 1.05

$$\frac{2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)+\frac{a(ad+aex-b\sin(d+ex)+b(d+ex)\cos(d+ex))}{a+b\cos(d+ex)}}{b^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2),x]
```

```
[Out] (2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]] +
 (a*(a*d + a*e*x + b*(d + e*x)*Cos[d + e*x] - b*Sin[d + e*x]))/(a + b*Cos[d
 + e*x]))/(b^2*e)
```

Maple [A] time = 0.093, size = 163, normalized size = 1.8

$$2\,\frac{a\arctan\left(\tan\left(d/2+1/2\,ex\right)\right)}{b^2e} - 2\,\frac{a\tan\left(d/2+1/2\,ex\right)}{be\left(a\left(\tan\left(d/2+1/2\,ex\right)\right)^2-b\left(\tan\left(d/2+1/2\,ex\right)\right)^2+a+b\right)} - 2\,\frac{a^2}{b^2e\sqrt{(a-b)(a+b)}}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x)
```

```
[Out] 2/e*a/b^2*arctan(tan(1/2*d+1/2*e*x))-2/e/b*tan(1/2*d+1/2*e*x)*a/(a*tan(1/2*
d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)-2/e/b^2/((a-b)*(a+b))^(1/2)*arctan
(tan(1/2*d+1/2*e*x)*(a-b)/((a-b)*(a+b))^(1/2))*a^2+2/e/((a-b)*(a+b))^(1/2)*
arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01331, size = 670, normalized size = 7.28

$$\left[\frac{2\,abex\cos\left(ex+d\right)+2\,a^2ex-2\,ab\sin\left(ex+d\right)+\sqrt{-a^2+b^2}(b\cos\left(ex+d\right)+a)\log\left(\frac{2\,ab\cos(ex+d)+\left(2\,a^2-b^2\right)\cos(ex+d)^2+2\,\sqrt{-a^2+}}{b^2\cos(ex+d)^2+2\,ab\cos}\right.}{2\left(b^3e\cos\left(ex+d\right)+ab^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(2*a*b*e*x*cos(e*x + d) + 2*a^2*e*x - 2*a*b*sin(e*x + d) + sqrt(-a^2 +
 b^2)*(b*cos(e*x + d) + a)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x
+ d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2
)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b^3*e*cos(e*x + d) + a
*b^2*e), (a*b*e*x*cos(e*x + d) + a^2*e*x - a*b*sin(e*x + d) - sqrt(a^2 - b^
2)*(b*cos(e*x + d) + a)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e
*x + d))))/(b^3*e*cos(e*x + d) + a*b^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sec(d + ex)}{(a\sec(d + ex) + b)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**2, x)
```

Giac [A] time = 1.23847, size = 196, normalized size = 2.13

$$\left(\frac{(xe + d)a}{b^2} - \frac{2\,a\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)}{\left(a\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^2 - b\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^2 + a + b\right)b} - \frac{2\left(\pi\left\lfloor\frac{xe+d}{2\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a - 2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,xe+\right.}{}\right.\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] ((x*e + d)*a/b^2 - 2*a*tan(1/2*x*e + 1/2*d)/((a*tan(1/2*x*e + 1/2*d)^2 - b*
tan(1/2*x*e + 1/2*d)^2 + a + b)*b) - 2*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sg
n(2*a - 2*b) + arctan((a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d))/sqr
t(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)*e^(-1)
```

3.521 $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{\left(a^2-2b^2\right)\left(-a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^4e(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(-11a^2b^2+6a^4+11b^4\right)\tan(d+ex)}{6b^3e\left(a^2-b^2\right)^2(a\sec(d+ex)+b)}-\frac{a\left(3a^2-5b^2\right)\tan}{6b^2e\left(a^2-b^2\right)(a\sec(}$$

```
[Out] (a*x)/b^4 - ((a^2 - 2*b^2)*(2*a^4 - a^2*b^2 + b^4)*ArcTan[(Sqrt[a - b]*Tan[
(d + e*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*e) - (a*(3*a^2
 - 5*b^2)*Tan[d + e*x])/(6*b^2*(a^2 - b^2)*e*(b + a*Sec[d + e*x])^2) - (a*(
6*a^4 - 11*a^2*b^2 + 11*b^4)*Tan[d + e*x])/(6*b^3*(a^2 - b^2)^2*e*(b + a*Se
c[d + e*x])) - (a^4*Tan[d + e*x])/(3*b*e*(a*b + a^2*Sec[d + e*x])^3)
```

Rubi [A] time = 0.832768, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4172, 3923, 4060, 3919, 3831, 2659, 205}

$$-\frac{\left(a^2-2b^2\right)\left(-a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^4e(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(-11a^2b^2+6a^4+11b^4\right)\tan(d+ex)}{6b^3e\left(a^2-b^2\right)^2(a\sec(d+ex)+b)}-\frac{a\left(3a^2-5b^2\right)\tan}{6b^2e\left(a^2-b^2\right)(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,
x]
```

```
[Out] (a*x)/b^4 - ((a^2 - 2*b^2)*(2*a^4 - a^2*b^2 + b^4)*ArcTan[(Sqrt[a - b]*Tan[
(d + e*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*e) - (a*(3*a^2
 - 5*b^2)*Tan[d + e*x])/(6*b^2*(a^2 - b^2)*e*(b + a*Sec[d + e*x])^2) - (a*(
6*a^4 - 11*a^2*b^2 + 11*b^4)*Tan[d + e*x])/(6*b^3*(a^2 - b^2)^2*e*(b + a*Se
c[d + e*x])) - (a^4*Tan[d + e*x])/(3*b*e*(a*b + a^2*Sec[d + e*x])^3)
```

Rule 4172

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
```

```
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx = \left(16a^4\right)\int \frac{a+b\sec(d+ex)}{\left(2ab+2a^2\sec(d+ex)\right)^4}\,dx$$

$$= -\frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3} + \frac{(2a)\int \frac{12a^3\left(a^2-b^2\right)+12a^2b\left(a^2-b^2\right)\sec(d+ex)-8a^3(}{\left(2ab+2a^2\sec(d+ex)\right)^3}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3} + \frac{\int}{}$$

$$= -\frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3} - \frac{a^2(}{6b^3}$$

$$= \frac{ax}{b^4} - \frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3} -$$

$$= \frac{ax}{b^4} - \frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3} -$$

$$= \frac{ax}{b^4} - \frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3} -$$

$$= \frac{ax}{b^4} - \frac{\left(a^2-2b^2\right)\left(2a^4-a^2b^2+b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}e} - \frac{a(3a}{6b^2(a^2-}$$

Mathematica [A] time = 1.57569, size = 276, normalized size = 1.2

$$\frac{\sec^3(d+ex)(a+b\cos(d+ex))(a+b\sec(d+ex))\left(\frac{a^2b\left(7a^2-9b^2\right)\sin(d+ex)(a+b\cos(d+ex))}{(a-b)(a+b)} - \frac{ab\left(-23a^2b^2+11a^4+18b^4\right)\sin(d+ex)(a+b\cos(d+e}{(a-b)^2(a+b)^2}\right.}{6b^4e(a\cos(d+ex)+b)(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^2,x]
```

```
[Out] ((a + b*Cos[d + e*x])*Sec[d + e*x]^3*(a + b*Sec[d + e*x])*(6*a*(d + e*x)*(a
 + b*Cos[d + e*x])^3 + (6*(-2*a^6 + 5*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*ArcTanh[
((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^3)/(-a^2
 + b^2)^(5/2) - 2*a^3*b*Sin[d + e*x] + (a^2*b*(7*a^2 - 9*b^2)*(a + b*Cos[d
+ e*x])*Sin[d + e*x])/((a - b)*(a + b)) - (a*b*(11*a^4 - 23*a^2*b^2 + 18*b^
4)*(a + b*Cos[d + e*x])^2*Sin[d + e*x])/((a - b)^2*(a + b)^2)))/(6*b^4*e*(b
 + a*Cos[d + e*x])*(b + a*Sec[d + e*x])^4)
```

Maple [B] time = 0.113, size = 1118, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x)
```

```
[Out] 2/e*a/b^4*arctan(tan(1/2*d+1/2*e*x))-2/e/b^3/(a*tan(1/2*d+1/2*e*x)^2-b*tan(
1/2*d+1/2*e*x)^2+a+b)^3*a^5/(a^2+2*a*b+b^2)*tan(1/2*d+1/2*e*x)^5+1/e/b^2/(a
*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^4/(a^2+2*a*b+b^2)*tan
(1/2*d+1/2*e*x)^5+4/e/b/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)
^3*a^3/(a^2+2*a*b+b^2)*tan(1/2*d+1/2*e*x)^5-3/e/(a*tan(1/2*d+1/2*e*x)^2-b*t
an(1/2*d+1/2*e*x)^2+a+b)^3*a^2/(a^2+2*a*b+b^2)*tan(1/2*d+1/2*e*x)^5-6/e*b/(
a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a/(a^2+2*a*b+b^2)*tan(
1/2*d+1/2*e*x)^5-4/e/b^3/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b
)^3*a^5/(a-b)/(a+b)*tan(1/2*d+1/2*e*x)^3+32/3/e/b/(a*tan(1/2*d+1/2*e*x)^2-b
*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^3/(a-b)/(a+b)*tan(1/2*d+1/2*e*x)^3-12/e*b/(a
*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a/(a-b)/(a+b)*tan(1/2*d
+1/2*e*x)^3-2/e/b^3/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a
^5/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)-1/e/b^2/(a*tan(1/2*d+1/2*e*x)^2-b*tan
(1/2*d+1/2*e*x)^2+a+b)^3*a^4/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)+4/e/b/(a*ta
n(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^3/(a^2-2*a*b+b^2)*tan(1/
2*d+1/2*e*x)+3/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^2/
(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)-6/e*b/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*
d+1/2*e*x)^2+a+b)^3*a/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)-2/e/b^4/(a^4-2*a^2
*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a-b)*(a+b))
^(1/2))*a^6+5/e/b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*
d+1/2*e*x)*(a-b)/((a-b)*(a+b))^(1/2))*a^4-3/e/(a^4-2*a^2*b^2+b^4)/((a-b)*(a
```

```
+b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a-b)*(a+b))^(1/2))*a^2+2/e*b^2
/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((
a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.05475, size = 2871, normalized size = 12.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] [1/12*(12*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x + d)^3 + 36
*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2 + 36*(a^9*b
 - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 12*(a^10 - 3*a^8*b^2
 + 3*a^6*b^4 - a^4*b^6)*e*x + 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6
+ (2*a^6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^3 + 3*(2*a^7*b^2
 - 5*a^5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(e*x + d)^2 + 3*(2*a^8*b - 5*a^6*b^3
 + 3*a^4*b^5 - 2*a^2*b^7)*cos(e*x + d))*sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x
 + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) +
 b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) +
a^2)) - 2*(6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 11*a^3*b^7 + (11*a^7*b^3 - 3
4*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(e*x + d)^2 + 3*(5*a^8*b^2 - 15*a^6*b
^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(e*x + d))*sin(e*x + d))/((a^6*b^7 - 3*a^4*
b^9 + 3*a^2*b^11 - b^13)*e*cos(e*x + d)^3 + 3*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*
b^10 - a*b^12)*e*cos(e*x + d)^2 + 3*(a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*
b^11)*e*cos(e*x + d) + (a^9*b^4 - 3*a^7*b^6 + 3*a^5*b^8 - a^3*b^10)*e), 1/6
*(6*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x + d)^3 + 18*(a^8*
```

```
b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2 + 18*(a^9*b - 3*a
^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 6*(a^10 - 3*a^8*b^2 + 3*a^
6*b^4 - a^4*b^6)*e*x - 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6 + (2*a^
6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^3 + 3*(2*a^7*b^2 - 5*a^
5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(e*x + d)^2 + 3*(2*a^8*b - 5*a^6*b^3 + 3*a^
4*b^5 - 2*a^2*b^7)*cos(e*x + d))*sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) +
b)/(sqrt(a^2 - b^2)*sin(e*x + d))) - (6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 1
1*a^3*b^7 + (11*a^7*b^3 - 34*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(e*x + d)^
2 + 3*(5*a^8*b^2 - 15*a^6*b^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(e*x + d))*sin(e
*x + d))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*e*cos(e*x + d)^3 + 3*(a
^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*e*cos(e*x + d)^2 + 3*(a^8*b^5 - 3
*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*e*cos(e*x + d) + (a^9*b^4 - 3*a^7*b^6 + 3*
a^5*b^8 - a^3*b^10)*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sec(d + ex)}{(a\sec(d + ex) + b)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
```

```
[Out] Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**4, x)
```

Giac [B] time = 1.33189, size = 662, normalized size = 2.88

$$\frac{1}{3}\left(\frac{3\left(2a^6 - 5a^4b^2 + 3a^2b^4 - 2b^6\right)\left(\pi\left\lfloor\frac{xe+d}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - b\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4 - 2a^2b^6 + b^8\right)\sqrt{a^2-b^2}} + \frac{3(xe+d)}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/3*(3*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(x*e + d)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1
```

```
/2*d))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*
(x*e + d)*a/b^4 - (6*a^7*tan(1/2*x*e + 1/2*d)^5 - 15*a^6*b*tan(1/2*x*e + 1/
2*d)^5 + 30*a^4*b^3*tan(1/2*x*e + 1/2*d)^5 - 12*a^3*b^4*tan(1/2*x*e + 1/2*d
)^5 - 27*a^2*b^5*tan(1/2*x*e + 1/2*d)^5 + 18*a*b^6*tan(1/2*x*e + 1/2*d)^5 +
 12*a^7*tan(1/2*x*e + 1/2*d)^3 - 44*a^5*b^2*tan(1/2*x*e + 1/2*d)^3 + 68*a^3
*b^4*tan(1/2*x*e + 1/2*d)^3 - 36*a*b^6*tan(1/2*x*e + 1/2*d)^3 + 6*a^7*tan(1
/2*x*e + 1/2*d) + 15*a^6*b*tan(1/2*x*e + 1/2*d) - 30*a^4*b^3*tan(1/2*x*e +
1/2*d) - 12*a^3*b^4*tan(1/2*x*e + 1/2*d) + 27*a^2*b^5*tan(1/2*x*e + 1/2*d)
+ 18*a*b^6*tan(1/2*x*e + 1/2*d))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*x*
e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + a + b)^3))*e^(-1)
```

3.522 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)$

Optimal. Leaf size=359

$$\frac{a^4b^3x\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{\left(a^2\sec(d+ex)+ab\right)^3}+\frac{a^5\left(3a^2+5b^2\right)\tan(d+ex)\sec(d+ex)\left(a^2\sec^2(d+ex)+2ab\sec(d\right.}{6e\left(a^2\sec(d+ex)+ab\right)^3}$$

```
[Out] ((a^4 + 9*a^2*b^2 + 2*b^4)*ArcTanh[Sin[d + e*x]]*(b^2 + 2*a*b*Sec[d + e*x]
+ a^2*Sec[d + e*x]^2)^(3/2))/(2*e*(b + a*Sec[d + e*x])^3) + (a^4*b^3*x*(b^2
 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))/(a*b + a^2*Sec[d + e*x])
^3 + (a^4*b*(11*a^2 + 8*b^2)*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
)^(3/2)*Tan[d + e*x])/(3*e*(a*b + a^2*Sec[d + e*x])^3) + (a^5*(3*a^2 + 5*b^
2)*Sec[d + e*x]*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d
 + e*x])/(6*e*(a*b + a^2*Sec[d + e*x])^3) + (b*(a^2*b + a^3*Sec[d + e*x])^2
*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d + e*x])/(3*e*(
a*b + a^2*Sec[d + e*x])^3)
```

Rubi [A] time = 0.287014, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4174, 3918, 4048, 3770, 3767, 8}

$$\frac{a^4b^3x\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{\left(a^2\sec(d+ex)+ab\right)^3}+\frac{a^5\left(3a^2+5b^2\right)\tan(d+ex)\sec(d+ex)\left(a^2\sec^2(d+ex)+2ab\sec(d\right.}{6e\left(a^2\sec(d+ex)+ab\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3
/2),x]
```


```
[Out] ((a^4 + 9*a^2*b^2 + 2*b^4)*ArcTanh[Sin[d + e*x]]*(b^2 + 2*a*b*Sec[d + e*x]
+ a^2*Sec[d + e*x]^2)^(3/2))/(2*e*(b + a*Sec[d + e*x])^3) + (a^4*b^3*x*(b^2
 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))/(a*b + a^2*Sec[d + e*x])
^3 + (a^4*b*(11*a^2 + 8*b^2)*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
)^(3/2)*Tan[d + e*x])/(3*e*(a*b + a^2*Sec[d + e*x])^3) + (a^5*(3*a^2 + 5*b^
2)*Sec[d + e*x]*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d
 + e*x])/(6*e*(a*b + a^2*Sec[d + e*x])^3) + (b*(a^2*b + a^3*Sec[d + e*x])^2
*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d + e*x])/(3*e*(
a*b + a^2*Sec[d + e*x])^3)
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\,dx=\frac{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\int\left(2ab\right.}{\left(2ab+2a^2\sec(\right.}$$

$$=\frac{b\left(a^2b+a^3\sec(d+ex)\right)^2\left(b^2+2ab\sec(d+ex)+\right.}{3e\left(ab+a^2\sec(d+ex\right.}$$

$$=\frac{a^5\left(3a^2+5b^2\right)\sec(d+ex)\left(b^2+2ab\sec(d+ex)+\right.}{6e\left(ab+a^2\sec(d+e\right.}$$

$$=\frac{a^4b^3x\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}{\left(ab+a^2\sec(d+ex)\right)^3}$$

$$=\frac{\left(a^4+9a^2b^2+2b^4\right)\tanh^{-1}(\sin(d+ex))\left(b^2+2ab\right.}{2e(b+a\sec(d+ex}$$

$$=\frac{\left(a^4+9a^2b^2+2b^4\right)\tanh^{-1}(\sin(d+ex))\left(b^2+2ab\right.}{2e(b+a\sec(d+ex}$$

Mathematica [A] time = 0.837774, size = 128, normalized size = 0.36

$$\frac{\cos(d+ex)\sqrt{(a\sec(d+ex)+b)^2}\left(3\left(9a^2b^2+a^4+2b^4\right)\tanh^{-1}(\sin(d+ex))+3a\tan(d+ex)\left(a\left(a^2+3b^2\right)\sec(d+ex)+\right.\right.}{6e(a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*(6*a*b^3*e*x + 3*(a^4 + 9*a^2*b^
2 + 2*b^4)*ArcTanh[Sin[d + e*x]] + 3*a*(8*a^2*b + 6*b^3 + a*(a^2 + 3*b^2)*S
ec[d + e*x])*Tan[d + e*x] + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(a + b*Cos[d + e*
x]))
```

Maple [A] time = 0.264, size = 387, normalized size = 1.1

$$\frac{1}{6\,e\,(b\cos(ex+d)+a)^3}\left(3\,\ln\left(\frac{\sin(ex+d)+1-\cos(ex+d)}{\sin(ex+d)}\right)(\cos(ex+d))^3a^4+27\,\ln\left(\frac{\sin(ex+d)+1-\cos(ex+d)}{\sin(ex+d)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x)
```

```
[Out] 1/6/e*(3*ln((sin(e*x+d)+1-cos(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^4+27*ln((s
in(e*x+d)+1-cos(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^2*b^2+6*ln((sin(e*x+d)+1
-cos(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*b^4-3*ln(-(cos(e*x+d)-1+sin(e*x+d))/s
in(e*x+d))*cos(e*x+d)^3*a^4-27*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*co
s(e*x+d)^3*a^2*b^2-6*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3
*b^4+6*cos(e*x+d)^3*(e*x+d)*a*b^3+22*sin(e*x+d)*cos(e*x+d)^2*a^3*b+18*sin(e
*x+d)*cos(e*x+d)^2*a*b^3+3*sin(e*x+d)*cos(e*x+d)*a^4+9*sin(e*x+d)*cos(e*x+d
)*a^2*b^2+2*a^3*b*sin(e*x+d))*((b*cos(e*x+d)+a)^2/cos(e*x+d)^2)^(3/2)/(b*co
s(e*x+d)+a)^3
```

Maxima [A] time = 1.66739, size = 594, normalized size = 1.65

$$3\left(4\,b^{3}\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)+\left(a^{3}+6\,ab^{2}\right)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-\left(a^{3}+6\,ab^{2}\right)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)-\frac{2\left(\frac{\left(a^{3}+6\,a^{2}b\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(a^{3}-\cdots\right.}{\cdots}\right.}{\frac{2\,\sin(ex+d)^{2}}{(\cos(ex+d)+1)^{2}}-\frac{\sin(\cdots}{(\cos(e\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/6*(3*(4*b^3*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + (a^3 + 6*a*b^2)*log
(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - (a^3 + 6*a*b^2)*log(sin(e*x + d)/(c
os(e*x + d) + 1) - 1) - 2*((a^3 + 6*a^2*b)*sin(e*x + d)/(cos(e*x + d) + 1)
+ (a^3 - 6*a^2*b)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(2*sin(e*x + d)^2/(c
os(e*x + d) + 1)^2 - sin(e*x + d)^4/(cos(e*x + d) + 1)^4 - 1))*a + (3*(3*a^
2*b + 2*b^3)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - 3*(3*a^2*b + 2*b^3)
*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1) - 2*(3*(2*a^3 + 3*a^2*b + 6*a*b^2
)*sin(e*x + d)/(cos(e*x + d) + 1) - 4*(a^3 + 9*a*b^2)*sin(e*x + d)^3/(cos(e
*x + d) + 1)^3 + 3*(2*a^3 - 3*a^2*b + 6*a*b^2)*sin(e*x + d)^5/(cos(e*x + d)
 + 1)^5)/(3*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 3*sin(e*x + d)^4/(cos(e*x
 + d) + 1)^4 + sin(e*x + d)^6/(cos(e*x + d) + 1)^6 - 1))*b)/e
```

Fricas [A] time = 2.20469, size = 394, normalized size = 1.1

$$\frac{12\,ab^3ex\cos\left(ex+d\right)^3+3\left(a^4+9\,a^2b^2+2\,b^4\right)\cos\left(ex+d\right)^3\log\left(\sin\left(ex+d\right)+1\right)-3\left(a^4+9\,a^2b^2+2\,b^4\right)\cos\left(ex+d\right)^3\log}{12\,e\cos\left(ex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(12*a*b^3*e*x*cos(e*x + d)^3 + 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x + d
)^3*log(sin(e*x + d) + 1) - 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x + d)^3*log(
-sin(e*x + d) + 1) + 2*(2*a^3*b + 2*(11*a^3*b + 9*a*b^3)*cos(e*x + d)^2 + 3
*(a^4 + 3*a^2*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(d+ex))\left((a\sec(d+ex)+b)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*((a*sec(d + e*x) + b)**2)**(3/2), x)
```

Giac [A] time = 1.66667, size = 880, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/6*(6*(x*e + d)*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 3*(a^4*sgn(
b*cos(x*e + d)^2 + a*cos(x*e + d)) + 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos
(x*e + d)) + 2*b^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(abs(tan(1/2*
```

```
x*e + 1/2*d) + 1)) - 3*(a^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 9*a^2*
b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 2*b^4*sgn(b*cos(x*e + d)^2 + a
*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) - 1)) + 2*(3*a^4*sgn(b*cos(x*e
 + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 - 24*a^3*b*sgn(b*cos(x*e +
 d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 + 9*a^2*b^2*sgn(b*cos(x*e +
d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 - 18*a*b^3*sgn(b*cos(x*e + d)
^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 + 40*a^3*b*sgn(b*cos(x*e + d)^2
 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^3 + 36*a*b^3*sgn(b*cos(x*e + d)^2 +
 a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^3 - 3*a^4*sgn(b*cos(x*e + d)^2 + a*co
s(x*e + d))*tan(1/2*x*e + 1/2*d) - 24*a^3*b*sgn(b*cos(x*e + d)^2 + a*cos(x*
e + d))*tan(1/2*x*e + 1/2*d) - 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e +
 d))*tan(1/2*x*e + 1/2*d) - 18*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))
*tan(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)^3)*e^(-1)
```

3.523 $\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}$

Optimal. Leaf size=173

$$\frac{a^2bx\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{a^2\sec(d+ex)+ab}+\frac{a^2b\tan(d+ex)\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{e\left(a^2\sec(d+ex)+ab\right)}+\frac{\left(a^2+b^2\right)\tan$$

```
[Out] ((a^2 + b^2)*ArcTanh[Sin[d + e*x]]*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[
d + e*x]^2])/(e*(b + a*Sec[d + e*x])) + (a^2*b*x*Sqrt[b^2 + 2*a*b*Sec[d + e
*x] + a^2*Sec[d + e*x]^2])/(a*b + a^2*Sec[d + e*x]) + (a^2*b*Sqrt[b^2 + 2*a
*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2]*Tan[d + e*x])/(e*(a*b + a^2*Sec[d + e
*x]))
```

Rubi [A] time = 0.118901, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4174, 3914, 3767, 8, 3770}

$$\frac{a^2bx\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{a^2\sec(d+ex)+ab}+\frac{a^2b\tan(d+ex)\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{e\left(a^2\sec(d+ex)+ab\right)}+\frac{\left(a^2+b^2\right)\tan$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
],x]
```


```
[Out] ((a^2 + b^2)*ArcTanh[Sin[d + e*x]]*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[
d + e*x]^2])/(e*(b + a*Sec[d + e*x])) + (a^2*b*x*Sqrt[b^2 + 2*a*b*Sec[d + e
*x] + a^2*Sec[d + e*x]^2])/(a*b + a^2*Sec[d + e*x]) + (a^2*b*Sqrt[b^2 + 2*a
*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2]*Tan[d + e*x])/(e*(a*b + a^2*Sec[d + e
*x]))
```


Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\int \left(2ab+2a^2 \right.}{2ab+2a^2\sec(d+ex}$$

$$= \frac{a^2bx\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}{ab+a^2\sec(d+ex)} + \frac{\left(2a^2b \right.}{}$$

$$= \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(d+ex))\sqrt{b^2+2ab\sec(d+ex)+}}{e(b+a\sec(d+ex))}$$

$$= \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(d+ex))\sqrt{b^2+2ab\sec(d+ex)+}}{e(b+a\sec(d+ex))}$$

Mathematica [A] time = 0.263301, size = 67, normalized size = 0.39

$$\frac{\cos(d+ex)\sqrt{(a\sec(d+ex)+b)^2}\left(\left(a^2+b^2\right)\tanh^{-1}(\sin(d+ex))+ab(\tan(d+ex)+ex)\right)}{e(a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2],x]
```

```
[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*((a^2 + b^2)*ArcTanh[Sin[d + e*x
]] + a*b*(e*x + Tan[d + e*x])))/(e*(a + b*Cos[d + e*x]))
```

Maple [A] time = 0.238, size = 208, normalized size = 1.2

$$\frac{1}{e\left(b\cos\left(ex+d\right)+a\right)}\left(\cos\left(ex+d\right)\ln\left(\frac{\sin\left(ex+d\right)+1-\cos\left(ex+d\right)}{\sin\left(ex+d\right)}\right)a^{2}+\cos\left(ex+d\right)\ln\left(\frac{\sin\left(ex+d\right)+1-\cos\left(ex+\right.}{\sin\left(ex+d\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x)
```

```
[Out] 1/e*(cos(e*x+d)*ln((sin(e*x+d)+1-cos(e*x+d))/sin(e*x+d))*a^2+cos(e*x+d)*ln(
(sin(e*x+d)+1-cos(e*x+d))/sin(e*x+d))*b^2-cos(e*x+d)*ln(-(cos(e*x+d)-1+sin(
e*x+d))/sin(e*x+d))*a^2-cos(e*x+d)*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d)
)*b^2+cos(e*x+d)*(e*x+d)*a*b+a*b*sin(e*x+d))*((b*cos(e*x+d)+a)^2/cos(e*x+d)
^2)^(1/2)/(b*cos(e*x+d)+a)
```

Maxima [A] time = 1.5922, size = 221, normalized size = 1.28

$$\frac{\left(2\,b\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)+a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)\right)a+\left(b\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-b\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+\right.\right.\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] ((2*b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + a*log(sin(e*x + d)/(cos(e*x
 + d) + 1) + 1) - a*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1))*a + (b*log(si
n(e*x + d)/(cos(e*x + d) + 1) + 1) - b*log(sin(e*x + d)/(cos(e*x + d) + 1)
- 1) - 2*a*sin(e*x + d)/((sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 1)*(cos(e*x
 + d) + 1)))*b)/e
```

Fricas [A] time = 2.0364, size = 225, normalized size = 1.3

$$\frac{2\,abex\cos(ex+d)+\left(a^2+b^2\right)\cos(ex+d)\log(\sin(ex+d)+1)-\left(a^2+b^2\right)\cos(ex+d)\log(-\sin(ex+d)+1)+2\,ab\sin}{2\,e\cos(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*e*x*cos(e*x + d) + (a^2 + b^2)*cos(e*x + d)*log(sin(e*x + d) + 1
) - (a^2 + b^2)*cos(e*x + d)*log(-sin(e*x + d) + 1) + 2*a*b*sin(e*x + d))/(
e*cos(e*x + d))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(d+ex))\sqrt{(a\sec(d+ex)+b)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*sqrt((a*sec(d + e*x) + b)**2), x)
```

Giac [A] time = 1.36005, size = 302, normalized size = 1.75

$$\left((xe+d)ab\mathrm{sgn}\left(b\cos(xe+d)^2+a\cos(xe+d)\right)-\frac{2\,ab\mathrm{sgn}\left(b\cos(xe+d)^2+a\cos(xe+d)\right)\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)}{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2-1}+\left(a^2\mathrm{sgn}\left(b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] ((x*e + d)*a*b*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) - 2*a*b*sgn(b*cos(x*e
 + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)/(tan(1/2*x*e + 1/2*d)^2 - 1)
 + (a^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + b^2*sgn(b*cos(x*e + d)^2 +
 a*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) + 1)) - (a^2*sgn(b*cos(x*e +
 d)^2 + a*cos(x*e + d)) + b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(a
bs(tan(1/2*x*e + 1/2*d) - 1)))*e^(-1)
```

3.524 $\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx$

Optimal. Leaf size=142

$$\frac{x\left(a^2\sec(d+ex)+ab\right)}{b\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a\sec(d+ex)+b)}{be\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

```
[Out] (-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a +
b]]*(b + a*Sec[d + e*x]))/(b*e*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2]) + (x*(a*b + a^2*Sec[d + e*x]))/(b*Sqrt[b^2 + 2*a*b*Sec[d + e*x] +
a^2*Sec[d + e*x]^2])
```

Rubi [A] time = 0.213063, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4174, 3919, 3831, 2659, 205}

$$\frac{x\left(a^2\sec(d+ex)+ab\right)}{b\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a\sec(d+ex)+b)}{be\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a +
b]]*(b + a*Sec[d + e*x]))/(b*e*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2]) + (x*(a*b + a^2*Sec[d + e*x]))/(b*Sqrt[b^2 + 2*a*b*Sec[d + e*x] +
a^2*Sec[d + e*x]^2])
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx = \frac{\left(2ab+2a^2\sec(d+ex)\right)\int \frac{a+b\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{\left(\left(2a^3-2ab^2\right)\left(2ab+2a^2\sec\right.\right.}{2ab\sqrt{b^2+2ab\sec(d}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{\left(\left(2a^3-2ab^2\right)\left(2ab+2a^2\sec\right.\right.}{4a^3b\sqrt{b^2+2ab\sec(d-}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{\left(\left(2a^3-2ab^2\right)\left(2ab+2a^2\sec\right.\right.}{2a^3be\sqrt{b}$$

$$= -\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+ex))}{be\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} + \frac{}{b\sqrt{b^2+}}$$

Mathematica [A] time = 0.392007, size = 92, normalized size = 0.65

$$\frac{\sec(d+ex)(a+b\cos(d+ex))\left(2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)+a(d+ex)\right)}{be\sqrt{(a\sec(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2],x]
```

```
[Out] ((a*(d + e*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt
[-a^2 + b^2]])*(a + b*Cos[d + e*x])*Sec[d + e*x])/(b*e*Sqrt[(b + a*Sec[d +
e*x])^2])
```

Maple [A] time = 0.238, size = 157, normalized size = 1.1

$$\frac{b\cos(ex+d)+a}{be\cos(ex+d)}\left(a(ex+d)\sqrt{(a-b)(a+b)}+2\arctan\left(\frac{(\cos(ex+d)-1)(a-b)}{\sin(ex+d)\sqrt{(a-b)(a+b)}}\right)a^2-2\arctan\left(\frac{(\cos(ex+d)-}{\sin(ex+d)\sqrt{(a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x)
```

```
[Out] 1/e/b/((a-b)*(a+b))^(1/2)*(b*cos(e*x+d)+a)*(a*(e*x+d)*((a-b)*(a+b))^(1/2)+2
*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*a^2-2*arctan((
cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*b^2)/cos(e*x+d)/((b*cos
(e*x+d)+a)^2/cos(e*x+d)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17424, size = 421, normalized size = 2.96

$$\left[\frac{2\,aex+\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(ex+d)+\left(2\,a^2-b^2\right)\cos(ex+d)^2+2\,\sqrt{-a^2+b^2}(a\cos(ex+d)+b)\sin(ex+d)-a^2+2\,b^2}{b^2\cos(ex+d)^2+2\,ab\cos(ex+d)+a^2}\right)}{2\,be},\ \frac{aex-\sqrt{a^2-b^2}\arctan\left(-\cdots\right.}{be}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/2*(2*a*e*x + sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*co
s(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 +
 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b*e), (a*e*x - s
qrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d)))
)/(b*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sec(d + ex)}{\sqrt{(a\sec(d + ex) + b)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/sqrt((a*sec(d + e*x) + b)**2), x)
```

Giac [A] time = 1.55967, size = 265, normalized size = 1.87

$$-\left(\frac{\left(xe - 2\,\pi\left\lfloor\frac{xe+d}{2\,\pi} + \frac{1}{2}\right\rfloor + d\right)a}{b\mathrm{sgn}\left(a\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^4 - b\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^4 + 2\,b\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^2 - a - b\right)} - \frac{2\,\sqrt{a^2 - b^2}\arctan\left(\cdots\right.}{b\mathrm{sgn}\left(a\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^4 - b\tan\left(\frac{1}{2}\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] -((x*e - 2*pi*floor(1/2*(x*e + d)/pi + 1/2) + d)*a/(b*sgn(a*tan(1/2*x*e + 1
/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) -
 2*sqrt(a^2 - b^2)*arctan((a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d))
/sqrt(a^2 - b^2))/(b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^
4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)))*e^(-1)
```

3.525 $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=330

$$\frac{x\left(a^2\sec(d+ex)+ab\right)^3}{a^2b^3\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-\frac{\left(-3a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a\sec(d+ex)+b)^3}{b^3e(a-b)^{3/2}(a+b)^{3/2}\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-$$

```
[Out] -(((2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a
 + b]]*(b + a*Sec[d + e*x])^3)/((a - b)^(3/2)*b^3*(a + b)^(3/2)*e*(b^2 + 2*
a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))) + (x*(a*b + a^2*Sec[d + e*x]
)^3)/(a^2*b^3*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)) - ((a*
b + a^2*Sec[d + e*x])*Tan[d + e*x])/(2*b*e*(b^2 + 2*a*b*Sec[d + e*x] + a^2*
Sec[d + e*x]^2)^(3/2)) - ((2*a^2 - 3*b^2)*(a*b + a^2*Sec[d + e*x])^3*Tan[d
+ e*x])/(2*b^2*(a^2 - b^2)*e*(a^2*b + a^3*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d
+ e*x] + a^2*Sec[d + e*x]^2)^(3/2))
```

Rubi [A] time = 0.566333, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4174, 3923, 4060, 3919, 3831, 2659, 205}

$$\frac{x\left(a^2\sec(d+ex)+ab\right)^3}{a^2b^3\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-\frac{\left(-3a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a\sec(d+ex)+b)^3}{b^3e(a-b)^{3/2}(a+b)^{3/2}\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3
/2),x]
```


```
[Out] -(((2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a
 + b]]*(b + a*Sec[d + e*x])^3)/((a - b)^(3/2)*b^3*(a + b)^(3/2)*e*(b^2 + 2*
a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))) + (x*(a*b + a^2*Sec[d + e*x]
)^3)/(a^2*b^3*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)) - ((a*
b + a^2*Sec[d + e*x])*Tan[d + e*x])/(2*b*e*(b^2 + 2*a*b*Sec[d + e*x] + a^2*
Sec[d + e*x]^2)^(3/2)) - ((2*a^2 - 3*b^2)*(a*b + a^2*Sec[d + e*x])^3*Tan[d
+ e*x])/(2*b^2*(a^2 - b^2)*e*(a^2*b + a^3*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d
+ e*x] + a^2*Sec[d + e*x]^2)^(3/2))
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\sec(d+ex)\right)^3\int \frac{a+b\sec(d+ex)}{\left(2ab+2a^2\sec(d+ex)\right)^3}\,dx}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\left(ab+a^2\sec(d+ex)\right)\tan(d+ex)}{2be\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} + \frac{\left(2ab+2a^2\sec(d\right.}{16a^3b\,(\ldots}$$

$$= -\frac{\left(ab+a^2\sec(d+ex)\right)\tan(d+ex)}{2be\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(2a^2-3b^2\right.}{2ab^2\left(a^2-b^2\right)e\,(\ldots}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(ab+a^2\mathrm{se}\right.}{2be\left(b^2+2ab\sec\right.}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(ab+a^2\mathrm{se}\right.}{2be\left(b^2+2ab\sec\right.}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(ab+a^2\mathrm{se}\right.}{2be\left(b^2+2ab\sec\right.}$$

$$= -\frac{\left(2a^4-3a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+ex))^3}{(a-b)^{3/2}b^3(a+b)^{3/2}e\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \ldots$$

Mathematica [A] time = 1.01106, size = 216, normalized size = 0.65

$$\frac{\sec^2(d+ex)(a+b\cos(d+ex))(a+b\sec(d+ex))\left(\frac{ab\left(3a^2-4b^2\right)\sin(d+ex)(a+b\cos(d+ex))}{(b-a)(a+b)}+\frac{2\left(-3a^2b^2+2a^4+2b^4\right)(a+b\cos(d+ex))^2\tanh^{-1}\left(\frac{(b-\dots}{\dots}\right)}{\left(b^2-a^2\right)^{3/2}}\dots\right)}{2b^3e(a\cos(d+ex)+b)\left((a\sec(d+ex)+b)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^(3/2),x]
```

```
[Out] ((a + b*Cos[d + e*x])*Sec[d + e*x]^2*(a + b*Sec[d + e*x])*(2*a*(d + e*x)*(a
 + b*Cos[d + e*x])^2 + (2*(2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan
[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^2)/(-a^2 + b^2)^(3/2)
 + a^2*b*Sin[d + e*x] + (a*b*(3*a^2 - 4*b^2)*(a + b*Cos[d + e*x])*Sin[d + e
*x])/((-a + b)*(a + b))))/(2*b^3*e*(b + a*Cos[d + e*x])*((b + a*Sec[d + e*x
])^2)^(3/2))
```

Maple [B] time = 0.21, size = 756, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/2/e/((a-b)*(a+b))^(1/2)/(a^2-b^2)/b^3*(b*cos(e*x+d)+a)*(-4*cos(e*x+d)^2*
arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*a^4*b^2+6*cos(e
*x+d)^2*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*a^2*b^4
-4*cos(e*x+d)^2*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))
*b^6-2*cos(e*x+d)^2*((a-b)*(a+b))^(1/2)*(e*x+d)*a^3*b^2+2*cos(e*x+d)^2*((a-
b)*(a+b))^(1/2)*(e*x+d)*a*b^4+3*sin(e*x+d)*cos(e*x+d)*((a-b)*(a+b))^(1/2)*a
^3*b^2-4*sin(e*x+d)*cos(e*x+d)*((a-b)*(a+b))^(1/2)*a*b^4-8*cos(e*x+d)*arcta
n((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*a^5*b+12*cos(e*x+d)*
arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*a^3*b^3-8*cos(e
*x+d)*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*a*b^5-4*c
os(e*x+d)*((a-b)*(a+b))^(1/2)*(e*x+d)*a^4*b+4*cos(e*x+d)*((a-b)*(a+b))^(1/2
)*(e*x+d)*a^2*b^3+2*((a-b)*(a+b))^(1/2)*a^4*b*sin(e*x+d)-3*((a-b)*(a+b))^(1
/2)*a^2*b^3*sin(e*x+d)-4*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b
```

```
))^(1/2))*a^6+6*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))
*a^4*b^2-4*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a-b)*(a+b))^(1/2))*a^2*
b^4-2*(e*x+d)*((a-b)*(a+b))^(1/2)*a^5+2*((a-b)*(a+b))^(1/2)*(e*x+d)*a^3*b^2
)/cos(e*x+d)^3/((b*cos(e*x+d)+a)^2/cos(e*x+d)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.61807, size = 1739, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*x*cos(e*x + d)^2 + 8*(a^6*b - 2*a^4
*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 4*(a^7 - 2*a^5*b^2 + a^3*b^4)*e*x + (2*a
^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*cos(e*x + d)^2
 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(e*x + d))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos
(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(
e*x + d) + a^2)) - 2*(2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*
b^4 + 4*a*b^6)*cos(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*e*c
os(e*x + d)^2 + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (a^6*b^3 -
 2*a^4*b^5 + a^2*b^7)*e), 1/2*(2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*x*cos(e*x
+ d)^2 + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 2*(a^7 - 2*a^5*
b^2 + a^3*b^4)*e*x - (2*a^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^
4 + 2*b^6)*cos(e*x + d)^2 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(e*x + d))
*sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d)
)) - (2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*c
```

```
os(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*e*cos(e*x + d)^2 +
2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (a^6*b^3 - 2*a^4*b^5 + a^2
*b^7)*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sec(d + ex)}{\left((a\sec(d + ex) + b)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/((a*sec(d + e*x) + b)**2)**(3/2), x)
```

Giac [A] time = 1.88619, size = 768, normalized size = 2.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] ((2*a^4 - 3*a^2*b^2 + 2*b^4)*arctan((a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e
 + 1/2*d))/sqrt(a^2 - b^2))/((a^2*b^3*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(
1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b^5*sgn(a*tan(1/
2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 -
a - b))*sqrt(a^2 - b^2)) + (2*a^4*tan(1/2*x*e + 1/2*d)^3 - 3*a^3*b*tan(1/2*
x*e + 1/2*d)^3 - 3*a^2*b^2*tan(1/2*x*e + 1/2*d)^3 + 4*a*b^3*tan(1/2*x*e + 1
/2*d)^3 + 2*a^4*tan(1/2*x*e + 1/2*d) + 3*a^3*b*tan(1/2*x*e + 1/2*d) - 3*a^2
*b^2*tan(1/2*x*e + 1/2*d) - 4*a*b^3*tan(1/2*x*e + 1/2*d))/((a^2*b^2*sgn(a*t
an(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - a - b) - b^4*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 +
 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2
*x*e + 1/2*d)^2 + a + b)^2) - (x*e - 2*pi*floor(1/2*(x*e + d)/pi + 1/2) + d
)*a/(b^3*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(
1/2*x*e + 1/2*d)^2 - a - b)))*e^(-1)
```

3.526 $\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

```
[Out] (I/2)*(Cos[x] - I*Sin[x])^2
```

Rubi [A] time = 0.0400325, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4385}

$$\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

```
[Out] (I/2)*(Cos[x] - I*Sin[x])^2
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx=\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

Mathematica [A] time = 0.0054046, size = 19, normalized size = 1.12

$$\frac{1}{2}\sin(2x)+\frac{1}{2}i\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

```
[Out] (I/2)*Cos[2*x] + Sin[2*x]/2
```

Maple [A] time = 0.065, size = 8, normalized size = 0.5

$$(\tan(x) - i)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)
```

```
[Out] 1/(tan(x)-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.78768, size = 24, normalized size = 1.41

$$\frac{1}{2} i e^{(-2i x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*x)
```

Sympy [A] time = 0.128324, size = 8, normalized size = 0.47

$$\frac{ie^{-2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)
```

```
[Out] I*exp(-2*I*x)/2
```

Giac [A] time = 1.13732, size = 19, normalized size = 1.12

$$-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)-i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(tan(1/2*x) - I)^2
```

3.527 $\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx$

Optimal. Leaf size=17

$$-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

```
[Out] (-I/2)/(Cos[x] - I*Sin[x])^2
```

Rubi [A] time = 0.0365433, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4385}

$$-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
```

```
[Out] (-I/2)/(Cos[x] - I*Sin[x])^2
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx=-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

Mathematica [A] time = 0.0048979, size = 19, normalized size = 1.12

$$\frac{1}{2}\sin(2x)-\frac{1}{2}i\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
```

```
[Out] (-I/2)*Cos[2*x] + Sin[2*x]/2
```

Maple [A] time = 0.057, size = 8, normalized size = 0.5

$$(\tan(x) + i)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)
```

```
[Out] 1/(tan(x)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.88227, size = 24, normalized size = 1.41

$$-\frac{1}{2} i e^{(2 i x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*I*e^(2*I*x)
```

Sympy [A] time = 0.10255, size = 10, normalized size = 0.59

$$-\frac{ie^{2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)
```

```
[Out] -I*exp(2*I*x)/2
```

Giac [A] time = 1.14414, size = 19, normalized size = 1.12

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)+i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(tan(1/2*x) + I)^2
```

3.528 $\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=6

$$\log(\sin(x)+\cos(x))$$

```
[Out] Log[Cos[x] + Sin[x]]
```

Rubi [A] time = 0.0225762, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3133}

$$\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] Log[Cos[x] + Sin[x]]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx=\log(\cos(x)+\sin(x))$$

Mathematica [A] time = 0.0262814, size = 6, normalized size = 1.

$$\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] Log[Cos[x] + Sin[x]]
```

Maple [A] time = 0.022, size = 7, normalized size = 1.2

$$\ln(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] ln(cos(x)+sin(x))
```

Maxima [A] time = 0.989582, size = 8, normalized size = 1.33

$$\log(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] log(cos(x) + sin(x))
```

Fricas [A] time = 1.88883, size = 41, normalized size = 6.83

$$\frac{1}{2}\log(2\cos(x)\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] time = 0.138288, size = 7, normalized size = 1.17

$$\log(\sin(x) + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] log(sin(x) + cos(x))
```

Giac [B] time = 1.14849, size = 22, normalized size = 3.67

$$-\frac{1}{2}\log\left(\tan(x)^2 + 1\right) + \log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(tan(x)^2 + 1) + log(abs(tan(x) + 1))
```

3.529 $\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=47

$$\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 +
 c^2)
```

Rubi [A] time = 0.0405832, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 +
 c^2)
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A] time = 0.119037, size = 39, normalized size = 0.83

$$\frac{x(bB + cC) + (Bc - bC)\log(b\cos(x) + c\sin(x))}{b^2 + c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [B] time = 0.059, size = 111, normalized size = 2.4

$$\frac{\ln(c\tan(x) + b)\,Bc}{b^2 + c^2} - \frac{\ln(c\tan(x) + b)\,bC}{b^2 + c^2} - \frac{\ln\left(1 + (\tan(x))^2\right)Bc}{2\,b^2 + 2\,c^2} + \frac{\ln\left(1 + (\tan(x))^2\right)bC}{2\,b^2 + 2\,c^2} + \frac{B\arctan(\tan(x))\,b}{b^2 + c^2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)*ln(c*tan(x)+b)*B*c-1/(b^2+c^2)*ln(c*tan(x)+b)*b*C-1/2/(b^2+c^2)
*ln(1+tan(x)^2)*B*c+1/2/(b^2+c^2)*ln(1+tan(x)^2)*b*C+1/(b^2+c^2)*B*arctan(t
an(x))*b+1/(b^2+c^2)*C*arctan(tan(x))*c
```

Maxima [B] time = 1.51117, size = 244, normalized size = 5.19

$$B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2 + c^2} + \frac{c\log\left(-b - \frac{2\,c\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2 + c^2} - \frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b^2 + c^2}\right) + C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2 + c^2} - \frac{b\log\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) -
 b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2
```

```
) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2))
```

Fricas [A] time = 1.98651, size = 139, normalized size = 2.96

$$\frac{2\,(Bb+Cc)x-(Cb-Bc)\log\left(2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2\right)}{2\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*co
s(x)^2 + c^2))/(b^2 + c^2)
```

Sympy [A] time = 2.45975, size = 360, normalized size = 7.66

$$\begin{cases} \tilde{\infty}\,(B\log(\sin(x))+Cx) \\ \frac{Bx-C\log(\cos(x))}{b} \\ -\frac{iBx\sin(x)}{-2c\sin(x)+2ic\cos(x)}-\frac{Bx\cos(x)}{-2c\sin(x)+2ic\cos(x)}-\frac{B\sin(x)}{-2c\sin(x)+2ic\cos(x)}-\frac{Cx\sin(x)}{-2c\sin(x)+2ic\cos(x)}+\frac{iCx\cos(x)}{-2c\sin(x)+2ic\cos(x)}-\frac{iC\sin(x)}{-2c\sin(x)+2ic\cos(x)} \\ -\frac{iBx\sin(x)}{2c\sin(x)+2ic\cos(x)}+\frac{Bx\cos(x)}{2c\sin(x)+2ic\cos(x)}+\frac{B\sin(x)}{2c\sin(x)+2ic\cos(x)}+\frac{Cx\sin(x)}{2c\sin(x)+2ic\cos(x)}+\frac{iCx\cos(x)}{2c\sin(x)+2ic\cos(x)}-\frac{iC\sin(x)}{2c\sin(x)+2ic\cos(x)} \\ \frac{Bbx}{b^2+c^2}+\frac{Bc\log\left(\frac{b\cos(x)}{c}+\sin(x)\right)}{b^2+c^2}-\frac{Cb\log\left(\frac{b\cos(x)}{c}+\sin(x)\right)}{b^2+c^2}+\frac{Ccx}{b^2+c^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(B*log(sin(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*x - C*log(c
os(x)))/b, Eq(c, 0)), (-I*B*x*sin(x)/(-2*c*sin(x) + 2*I*c*cos(x)) - B*x*cos
(x)/(-2*c*sin(x) + 2*I*c*cos(x)) - B*sin(x)/(-2*c*sin(x) + 2*I*c*cos(x)) -
C*x*sin(x)/(-2*c*sin(x) + 2*I*c*cos(x)) + I*C*x*cos(x)/(-2*c*sin(x) + 2*I*c
*cos(x)) - I*C*sin(x)/(-2*c*sin(x) + 2*I*c*cos(x)), Eq(b, -I*c)), (-I*B*x*s
in(x)/(2*c*sin(x) + 2*I*c*cos(x)) + B*x*cos(x)/(2*c*sin(x) + 2*I*c*cos(x))
+ B*sin(x)/(2*c*sin(x) + 2*I*c*cos(x)) + C*x*sin(x)/(2*c*sin(x) + 2*I*c*cos
(x)) + I*C*x*cos(x)/(2*c*sin(x) + 2*I*c*cos(x)) - I*C*sin(x)/(2*c*sin(x) +
2*I*c*cos(x)), Eq(b, I*c)), (B*b*x/(b**2 + c**2) + B*c*log(b*cos(x)/c + sin
(x))/(b**2 + c**2) - C*b*log(b*cos(x)/c + sin(x))/(b**2 + c**2) + C*c*x/(b*
```

```
*2 + c**2), True))
```

Giac [A] time = 1.16139, size = 104, normalized size = 2.21

$$\frac{(Bb+Cc)x}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan(x)^2+1\right)}{2\left(b^2+c^2\right)}-\frac{\left(Cbc-Bc^2\right)\log\left(|c\tan(x)+b|\right)}{b^2c+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) + 1/2*(C*b - B*c)*log(tan(x)^2 + 1)/(b^2 + c^2) -
 (C*b*c - B*c^2)*log(abs(c*tan(x) + b))/(b^2*c + c^3)
```

3.530 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.0678324, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3153, 3074, 206}

$$-\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= -\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\,\mathrm{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)}{b^2+c^2}$$

$$= -\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.214847, size = 75, normalized size = 1.01

$$\frac{bC-Bc}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2(bB+cC)\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2
) + (-(B*c) + b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Maple [A] time = 0.092, size = 113, normalized size = 1.5

$$-2\,\frac{1}{b\left(\tan(x/2)\right)^2-2\,c\tan(x/2)-b}\left(-\frac{c\,(Bc-bC)\tan(x/2)}{b\left(b^2+c^2\right)}-\frac{Bc-bC}{b^2+c^2}\right)+2\,\frac{bB+Cc}{\left(b^2+c^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{2\,b\tan(x/2)-2\,c}{\sqrt{b^2+c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] -2*(-c*(B*c-C*b)/b/(b^2+c^2)*tan(1/2*x)-(B*c-C*b)/(b^2+c^2))/(b*tan(1/2*x)^
2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)
-2*c)/(b^2+c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03862, size = 466, normalized size = 6.3

$$\frac{2\,Cb^{3} - 2\,Bb^{2}c + 2\,Cbc^{2} - 2\,Bc^{3} + \sqrt{b^{2}+c^{2}}\left(\left(Bb^{2} + Cbc\right)\cos\left(x\right) + \left(Bbc + Cc^{2}\right)\sin\left(x\right)\right)\log\left(-\frac{2\,bc\cos(x)\sin(x) + \left(b^{2}-c^{2}\right)\cos(x)^{2} -}{2\,bc\cos(x)\sin(x) +}\right.}{2\left(\left(b^{5} + 2\,b^{3}c^{2} + bc^{4}\right)\cos\left(x\right) + \left(b^{4}c + 2\,b^{2}c^{3} + c^{5}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 -
c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b
*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)))/((b^5 + 2*b^3*c^2 + b*c^4)
*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19957, size = 178, normalized size = 2.41

$$-\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Cbc\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+Cb^2-Bbc\right)}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(
1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(C*b*c*tan(1/2*x)
- B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*ta
n(1/2*x) - b))
```

3.531 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=66

$$\frac{\sin(x)(bB+cC)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

```
[Out] -(B*c - b*C)/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) + ((b*B + c*C)*Sin[x])
/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.056569, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3156, 12, 3075}

$$\frac{\sin(x)(bB+cC)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(B*c - b*C)/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) + ((b*B + c*C)*Sin[x])
/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx &= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2(bB+cC)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}\\
&= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{(bB+cC)\int \frac{1}{(b\cos(x)+c\sin(x))^2}\,dx}{b^2+c^2}\\
&= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{(bB+cC)\sin(x)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.174264, size = 64, normalized size = 0.97

$$\frac{C\left(b^2+c^2\right)+b\sin(2x)(bB+cC)-c\cos(2x)(bB+cC)}{2b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((b^2 + c^2)*C - c*(b*B + c*C)*Cos[2*x] + b*(b*B + c*C)*Sin[2*x])/(2*b*(b^2
 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

Maple [A] time = 0.098, size = 37, normalized size = 0.6

$$-\frac{C}{c^2\left(c\tan(x)+b\right)}-\frac{Bc-bC}{2\,c^2\left(c\tan(x)+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -C/c^2/(c*tan(x)+b)-1/2*(B*c-C*b)/c^2/(c*tan(x)+b)^2
```

Maxima [B] time = 1.07834, size = 269, normalized size = 4.08

$$\frac{2\,B\left(\frac{b\sin(x)}{\cos(x)+1}+\frac{c\sin(x)^2}{(\cos(x)+1)^2}-\frac{b\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^4+\frac{4\,b^3c\sin(x)}{\cos(x)+1}-\frac{4\,b^3c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^4\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\left(b^4-2\,b^2c^2\right)\sin(x)^2}{(\cos(x)+1)^2}}+\frac{2\,C\sin(x)^2}{\left(b^3+\frac{4\,b^2c\sin(x)}{\cos(x)+1}-\frac{4\,b^2c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\left(b^3-2\,bc^2\right)\sin(x)^2}{(\cos(x)+1)^2}\right)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x)
 + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/(cos(x) + 1)
^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2/(cos(x) + 1
)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2*c*sin(x)^3/
(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*sin(x)^2/(
cos(x) + 1)^2)*(cos(x) + 1)^2)
```

Fricas [B] time = 1.89731, size = 333, normalized size = 5.05

$$\frac{Cb^3+Bb^2c+3\,Cbc^2-Bc^3-4\left(Bb^2c+Cbc^2\right)\cos(x)^2+2\left(Bb^3+Cb^2c-Bbc^2-Cc^3\right)\cos(x)\sin(x)}{2\left(b^4c^2+2\,b^2c^4+c^6+\left(b^6+b^4c^2-b^2c^4-c^6\right)\cos(x)^2+2\left(b^5c+2\,b^3c^3+bc^5\right)\cos(x)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*b^3 + B*b^2*c + 3*C*b*c^2 - B*c^3 - 4*(B*b^2*c + C*b*c^2)*cos(x)^2 +
 2*(B*b^3 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x)*sin(x))/(b^4*c^2 + 2*b^2*c^4
+ c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b
*c^5)*cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2102, size = 35, normalized size = 0.53

$$-\frac{2\,Cc\tan(x)+Cb+Bc}{2\,(c\tan(x)+b)^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*c*tan(x) + C*b + B*c)/((c*tan(x) + b)^2*c^2)
```

3.532 $\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=84

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c
^2]])/Sqrt[b^2 + c^2] + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rubi [A] time = 0.0584858, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3136, 3074, 206}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c
^2]])/Sqrt[b^2 + c^2] + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx &= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx\\ &= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\,\mathrm{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos\right.\\ &= \frac{(bB+cC)x}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}\end{aligned}$$

Mathematica [A] time = 0.219391, size = 78, normalized size = 0.93

$$\frac{2A\sqrt{b^2+c^2}\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)+x(bB+cC)+(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + 2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c
^2]] + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [B] time = 0.06, size = 222, normalized size = 2.6

$$\frac{Bc}{b^2+c^2}\ln\left(b\left(\tan\left(\frac{x}{2}\right)\right)^2-2\,c\tan(x/2)-b\right)-\frac{bC}{b^2+c^2}\ln\left(b\left(\tan\left(\frac{x}{2}\right)\right)^2-2\,c\tan(x/2)-b\right)+2\,\frac{Ab^2}{\left(b^2+c^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)*B*c*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)-1/(b^2+c^2)*b*C*ln(b*ta
n(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-
2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2
*c)/(b^2+c^2)^(1/2))*A*c^2-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+C/(b^2+c^2)*b*l
n(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))+2*C/(b^2+c^2)*c*arctan
(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13355, size = 379, normalized size = 4.51

$$\frac{\sqrt{b^2+c^2}A\log\left(-\frac{2bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2b^2-c^2+2\sqrt{b^2+c^2}(c\cos(x)-b\sin(x))}{2bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2}\right)+2(Bb+Cc)x-(Cb-Bc)\log\left(2bc\cos(x)\sin\right.}{2\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2
*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*co
s(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2))/(b^2 + c^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.29173, size = 200, normalized size = 2.38

$$-\frac{A\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}+\frac{(Bb+Cc)x}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{b^2+c^2}-\frac{(Cb-Bc)\log\left(\left|b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\,\mathrm{ta}\right.\right.}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] -A*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2
*c + 2*sqrt(b^2 + c^2)))/sqrt(b^2 + c^2) + (B*b + C*c)*x/(b^2 + c^2) + (C*b
 - B*c)*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - (C*b - B*c)*log(abs(b*tan(1/2*x
)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2)
```

3.533 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*S
in[x]))
```

Rubi [A] time = 0.0569903, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3153, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```



```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*S
in[x]))
```


Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(bB+cC)\int\frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\,\mathrm{Subst}\left(\int\frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)}{b^2+c^2}$$

$$= -\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.240411, size = 92, normalized size = 1.08

$$\frac{A\left(b^2+c^2\right)\sin(x)+b(bC-Bc)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2(bB+cC)\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2
) + (b*(-(B*c) + b*C) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*
Sin[x]))
```

Maple [A] time = 0.09, size = 124, normalized size = 1.5

$$2\,\frac{1}{b\left(\tan(x/2)\right)^2-2\,c\tan(x/2)-b}\left(-\frac{\left(Ab^2+Ac^2-Bc^2+Cbc\right)\tan(x/2)}{b\left(b^2+c^2\right)}+\frac{Bc-bC}{b^2+c^2}\right)+2\,\frac{bB+Cc}{\left(b^2+c^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{2\,b\,t

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*b^2+A*c^2-B*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)+(B*c-C*b)/(b^2+c^2))/(
b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2
*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29003, size = 544, normalized size = 6.4

$$\frac{2\,Cb^3 - 2\,Bb^2c + 2\,Cbc^2 - 2\,Bc^3 + \sqrt{b^2+c^2}\left(\left(Bb^2 + Cbc\right)\cos\left(x\right) + \left(Bbc + Cc^2\right)\sin\left(x\right)\right)\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-}{2\,bc\cos(x)\sin(x)+}\right.}{2\left(\left(b^5 + 2\,b^3c^2 + bc^4\right)\cos\left(x\right) + \left(b^4c + 2\,b^2c^3 + c^5\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 -
c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b
*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x
) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c
+ 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19079, size = 203, normalized size = 2.39

$$-\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Cbc\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+Cb^2-B\right.}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(
1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x)
+ C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/(
(b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))
```

3.534 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c\cos(x)(bB+cC)-b\sin(x)(bB+cC)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) -
 (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])
^2) - (c*(b*B + c*C)*Cos[x] - b*(b*B + c*C)*Sin[x])/((b^2 + c^2)^2*(b*Cos[x
] + c*Sin[x]))
```

Rubi [A] time = 0.124475, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3156, 3153, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c\cos(x)(bB+cC)-b\sin(x)(bB+cC)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) -
 (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])
^2) - (c*(b*B + c*C)*Cos[x] - b*(b*B + c*C)*Sin[x])/((b^2 + c^2)^2*(b*Cos[x
] + c*Sin[x]))
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int\frac{2(bB+cC)+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$

$$= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-b(bB+cC)\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}+\frac{A\int\frac{}{b\,\mathrm{c}}}{2(}$$

$$= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-b(bB+cC)\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}-\frac{A\,\mathrm{Subs}}{}$$

$$= -\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-}{\left(b^2+c^2\right)^2(b\cos}$$

Mathematica [A] time = 0.584843, size = 122, normalized size = 0.95

$$\frac{Ab^2\sin(x)-Abc\cos(x)+b^2B\sin(2x)+b^2C-c\cos(2x)(bB+cC)+bcC\sin(2x)+c^2C}{2b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{A\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^2*C +
 c^2*C - A*b*c*Cos[x] - c*(b*B + c*C)*Cos[2*x] + A*b^2*Sin[x] + b^2*B*Sin[2
*x] + b*c*C*Sin[2*x])/(2*b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

Maple [A] time = 0.105, size = 218, normalized size = 1.7

$$-2\,\frac{1}{\left(b\left(\tan\left(x/2\right)\right)^2-2\,c\tan\left(x/2\right)-b\right)^2}\left(-1/2\,\frac{\left(Ab^2+2\,Ac^2-2\,Bb^2-2\,Bc^2\right)\left(\tan\left(x/2\right)\right)^3}{\left(b^2+c^2\right)b}-1/2\,\frac{\left(Ab^2c-2\,Ac^3+2\,Bb^2c+2\right.}{\left(b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^
2*c-2*A*c^3+2*B*b^2*c+2*B*c^3+2*C*b^3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2
-1/2*(A*b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^
2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*
tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26349, size = 728, normalized size = 5.64

$$\frac{2\,Cb^3+2\,Bb^2c+6\,Cbc^2-2\,Bc^3-8\left(Bb^2c+Cbc^2\right)\cos\left(x\right)^2+\left(2\,Abc\cos\left(x\right)\sin\left(x\right)+Ac^2+\left(Ab^2-Ac^2\right)\cos\left(x\right)^2\right)\sqrt{b^2-}}{4\left(b^4c^2+2\,b^2c^4+c^6+\left(b^6+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*C*b^3 + 2*B*b^2*c + 6*C*b*c^2 - 2*B*c^3 - 8*(B*b^2*c + C*b*c^2)*cos(
x)^2 + (2*A*b*c*cos(x)*sin(x) + A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2
+ c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*s
qrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*co
s(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2 + 2*(B*b^3
 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x))*sin(x))/(b^4*c^2 + 2*b^2*c^4 + c^6 +
(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b*c^5)*co
s(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29618, size = 365, normalized size = 2.83

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3-2\,Bb^3\tan\left(\frac{1}{2}\,x\right)^3+2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3-2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)^3+2\,Cb^3\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B
*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + 2*C*b
^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 + 2*C*b*c^2
*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2
*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x) + 2*B*b*c^2*tan(1/2*x) - A*
b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2)
```

3.535 $\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=115

$$-\frac{2\left(abB-A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

```
[Out] (b*B*x)/(b^2 + c^2) - (2*(a*b*B - A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + (B*c*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rubi [A] time = 0.129708, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3138, 3124, 618, 204}

$$-\frac{2\left(abB-A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (b*B*x)/(b^2 + c^2) - (2*(a*b*B - A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + (B*c*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{abB}{b^2+c^2}\right)\int\frac{1}{a+b\cos(x)+c\sin(x)}\,dx$$

$$= \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{abB}{b^2+c^2}\right)\right)\text{Subst}\left(\int\frac{1}{a+b+2cx+(a\cdots}\right.$$

$$= \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{abB}{b^2+c^2}\right)\right)\text{Subst}\left(\int\frac{1}{-4\left(a^2-b^2-c^2\cdots}\right.$$

$$= \frac{bBx}{b^2+c^2}+\frac{2\left(A-\frac{abB}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A] time = 0.269046, size = 95, normalized size = 0.83

$$\frac{B(c\log(a+b\cos(x)+c\sin(x))+bx)-\frac{2\left(A\left(b^2+c^2\right)-abB\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((-2*(-(a*b*B) + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 +
b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + B*(b*x + c*Log[a + b*Cos[x] + c*Sin[x
```

```
]]))/(b^2 + c^2)
```

Maple [B] time = 0.053, size = 544, normalized size = 4.7

$$\frac{aBc}{\left(b^2+c^2\right)(a-b)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2-b\left(\tan\left(\frac{x}{2}\right)\right)^2+2c\tan(x/2)+a+b\right)-\frac{bBc}{\left(b^2+c^2\right)(a-b)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^2-b\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*
c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*
B*c+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^
2-b^2-c^2)^(1/2))*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)
*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+c^2)
/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2
))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c
)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan
(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B-B/(b^2+c^2
)*c*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.79789, size = 1377, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*b - A*b^2 - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4
- c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*
c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b
^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b
*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))
- 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*c
os(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a
^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 - A*c^2)*sq
rt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2
- b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x)))
 - 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*
cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(
a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16656, size = 240, normalized size = 2.09

$$\frac{Bbx}{b^2+c^2}+\frac{Bc\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2\,c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}-\frac{Bc\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}+\frac{2\left(Bab-Ab^2-Ac^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] B*b*x/(b^2 + c^2) + B*c*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + 2*(B*a*b
```

```
- A*b^2 - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*
(b^2 + c^2))
```

3.536 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=113

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

```
[Out] (2*(a*A - b*B)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.106341, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3155, 3124, 618, 204}

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(aA-bB)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$
$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(2(aA-bB))\,\text{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx,x,\right.}{a^2-b^2-c^2}$$
$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(4(aA-bB))\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx,x,\right.}{a^2-b^2-c^2}$$
$$= \frac{2(aA-bB)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.287751, size = 118, normalized size = 1.04

$$\frac{\sin(x)\left(A\left(b^2+c^2\right)-abB\right)+c(aA-bB)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^
2 + b^2 + c^2)^(3/2) + ((a*A - b*B)*c + (-(a*b*B) + A*(b^2 + c^2))*Sin[x])/
(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Maple [B] time = 0.094, size = 254, normalized size = 2.3

$$2\frac{1}{a(\tan(x/2))^2 - b(\tan(x/2))^2 + 2c\tan(x/2) + a + b}\left(-\frac{\left(aAb - Ab^2 - Ac^2 - a^2B + abB + Bc^2\right)\tan(x/2)}{a^3 - a^2b - ab^2 - ac^2 + b^3 + bc^2} + \frac{(a}{a^3 - a^2b - a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)
*tan(1/2*x)+(A*a-B*b)*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^2-
b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b
)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*A-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2
*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88085, size = 2691, normalized size = 23.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*c^5 - 4*(B*a^2 - B*b^2)*c^3 + (A*a^2*b^2 - B*a*b^3 + (A*a^2 - B*
a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)*c^
3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b
^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2
 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^
3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) -
 (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*
a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)
)) + 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(A*c^5 - (A*a^2 + B*a*b - 2*A*b^
2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(x) - 2*(B*a^3*b^2 -
 A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2)
*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^
5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*
b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 -
3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)
*sin(x)), -(B*c^5 - 2*(B*a^2 - B*b^2)*c^3 - (A*a^2*b^2 - B*a*b^3 + (A*a^2 -
 B*a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)
*c^3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(
x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*c
os(x) + (a^2*b - b^3 - b*c^2)*sin(x))) + (B*a^4 - 2*B*a^2*b^2 + B*b^4)*c +
(A*c^5 - (A*a^2 + B*a*b - 2*A*b^2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A
*b^4)*c)*cos(x) - (B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a
^2*b + B*a*b^2 - 2*A*b^3)*c^2)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6
 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a
^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*
c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17346, size = 282, normalized size = 2.5

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Aab\tan\left(\frac{1}{2}x\right)-Bab\,\mathrm{t}\right.}{\left(a^3-a^2b-ab^2+b^3-ac^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b)/(a^2 - b^2 - c^2)^(3/2) +
 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*tan(1/2*
x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + A*a*c - B*b*c)/((a^3 - a^2*b - a
*b^2 + b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*
x) + a + b))
```

3.537 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=200

$$\frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}+$$

```
[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*B*c + (3*a*A
- 2*b*B)*c*Cos[x] - (3*a*A*b - a^2*B - 2*b^2*B)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.254899, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3158, 3153, 3124, 618, 204}

$$\frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*B*c + (3*a*A
- 2*b*B)*c*Cos[x] - (3*a*A*b - a^2*B - 2*b^2*B)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cos(x)+Ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx}{2\left(a^2-b^2-c^2\right)}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)-\left(3aAb-a^2\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)-\left(3aAb-a^2\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)-\left(3aAb-a^2\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+}$$

$$= \frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc+Ac\cos(x)-(Ab-aB)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+}$$

Mathematica [A] time = 0.803585, size = 326, normalized size = 1.63

$$\frac{-2bc\cos(x)\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)+c\cos(2x)\left(a^2(-b)B+3aA\left(b^2+c^2\right)-2bB\left(b^2+c^2\right)\right)-8a^2Ab^2\sin(x)-12a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c +
 9*a^2*b*B*c - 3*a*A*c^3 - 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*Cos[x]
 + c*(-(a^2*b*B) + 3*a*A*(b^2 + c^2) - 2*b*B*(b^2 + c^2))*Cos[2*x] - 8*a^2*
A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] - 12*a^
2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^2*Sin[x] - 3*a*A*b^3*Sin[2*
x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x] + 2*b^2*B
*c^2*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] time = 0.115, size = 1109, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b^4
-4*B*b^2*c^2-2*B*c^4)/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan
(1/2*x)^3+1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2+7*A*a^2*c^2-6*A*a*b^3-6*A*
a*b*c^2+A*b^4-A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2-4*B*a^2*c^2-
9*B*a*b^3+2*B*b^4+4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c
^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^
2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c
^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^
2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5-4*B*b^3*c^2-2*B*b*c^4)/(a^4-2*a
^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)+1/2*c*(4*A*a
^4-3*A*a^2*b^2-A*a^2*c^2-A*b^4-A*b^2*c^2-5*B*a^3*b+5*B*a*b^3+2*B*a*b*c^2)/(
a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2
-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2+2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*
c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c
^2)^(1/2))*a^2*A+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2
)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*b^2+1/(a
^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2
*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-3/(a^4-2*a^2*b^2-2*a^2*c^
2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c
)/(a^2-b^2-c^2)^(1/2))*a*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.65881, size = 7119, normalized size = 35.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*c^7 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(3*B*a^4 - 3*A*a^3*b
- 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 4*((3*A*a*b - 2*B*b^2)*c^5 - (3*A*
a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B
*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 +
A*a^2*b^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*
b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6
 + A*b^4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a
^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A
*a*b^3)*c^2)*cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 +
(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2
 + 2*A*b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*sin(x))*sqrt
(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2
*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*
b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos
(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x
))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a
^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^2 + 3*A*a^3
*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(A*c^7 - (5*A*a^2 - B*a*b - 3
*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3
- (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)*
cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 -
 A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (4*A*a^4*b + B*a^3
*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4*b^3 - 3*A*a^3*b^4 +
 B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6 + (3*A*a^3 - B*a^2*b
 - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2)*cos(x))*
sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)
*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^
4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^
6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^
4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)
*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b
 - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3
+ 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(
a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^
3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3
)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5
- 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)),
1/2*(B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (3*B*a^4 - 3*A*a^3*b - 5*B*a
^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*((3*A*a*b - 2*B*b^2)*c^5 - (3*A*a^3*b -
 B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^
```

```
4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b
^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*b + 4*A
*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 + A*b^
4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3
- 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)
*c^2)*cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + (2*A*a^
3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2 + 2*A*
b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*sin(x))*sqrt(a^2 -
b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c
^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6
 - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + (A*c^7
- (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B
*a*b^3 + 3*A*b^4)*c^3 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4
- B*a*b^5 - A*b^6)*c)*cos(x) + (2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A
*a^2*b^5 - B*a*b^6 - A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4
- (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4
*b^3 - 3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6
+ (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*
B*b^5)*c^2)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^
10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3
*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 +
 (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^
4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2
*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9
- a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 +
 (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3
- 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a
^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^
9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*
a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*
c)*cos(x))*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31697, size = 1569, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*A*a^2 - 3*B*a*b + A*b^2 + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
+ (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(1/2*x)^3 +
 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b^3*tan(1/2
*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b^4*tan(1/2
*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^
3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*t
an(1/2*x)^3 - A*a*b^2*c^2*tan(1/2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b
^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2
*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 + 4
*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2
- 9*B*a^3*b*c*tan(1/2*x)^2 + 13*A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*t
an(1/2*x)^2 - 6*A*a*b^3*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c
*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2*x)^2 - 4*B*a^2
*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*tan(1/2*x)^2 + 4*B
*b^2*c^3*tan(1/2*x)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a
^5*tan(1/2*x) - 4*A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + 5*A*a^3*b^2*t
an(1/2*x) + B*a^3*b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x) + B*a^2*b^3*tan(1
/2*x) - 5*A*a*b^4*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + A*b^5*tan(1/2*x) + 2*
B*b^5*tan(1/2*x) + 11*A*a^3*c^2*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 3*A*a
^2*b*c^2*tan(1/2*x) - 8*B*a^2*b*c^2*tan(1/2*x) - 7*A*a*b^2*c^2*tan(1/2*x) +
 8*B*a*b^2*c^2*tan(1/2*x) - A*b^3*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) -
 2*A*a*c^4*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) - 2*A*b*c^4*tan(1/2*x) + 2*B*b
*c^4*tan(1/2*x) + 4*A*a^4*c - 5*B*a^3*b*c - 3*A*a^2*b^2*c + 5*B*a*b^3*c - A
*b^4*c - A*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4
*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2
+ 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)
^2 + 2*c*tan(1/2*x) + a + b)^2)
```

3.538 $\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=84

$$\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}+\frac{iB\cos(x)}{2a}$$

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + ((I/2)*B*Cos[x])/a + ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rubi [A] time = 0.0449218, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3132}

$$\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}+\frac{iB\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + ((I/2)*B*Cos[x])/a + ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-bB)x}{2a^2}+\frac{iB\cos(x)}{2a}+\frac{i\left(2aAb-a^2B-b^2B\right)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+\frac{B\sin(x)}{2a}$$

Mathematica [A] time = 0.209883, size = 147, normalized size = 1.75

$$\frac{2\left(a^2B-2aAb+b^2B\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)-ia^2B\log\left(a^2+2ab\cos(x)+b^2\right)-ib^2B\log}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
b)*Cot[x/2])/(a - b)] + (2*I)*a*b*B*Cos[x] + (2*I)*a*A*b*Log[a^2 + b^2 + 2*
a*b*Cos[x]] - I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] - I*b^2*B*Log[a^2 + b^2
 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

Maple [B] time = 0.08, size = 153, normalized size = 1.8

$$\frac{iA}{a}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)-\frac{\frac{i}{2}B}{b}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)-\frac{\frac{i}{2}bB}{a^2}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] I/a*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*A-1/2*I/b*ln(I*a+I*b+a*tan(1/2*x)
-b*tan(1/2*x))*B-1/2*I/a^2*b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B+1/2*I*
B/b*ln(tan(1/2*x)+I)-I/a*ln(tan(1/2*x)-I)*A+1/2*I/a^2*ln(tan(1/2*x)-I)*b*B+
B/a/(tan(1/2*x)-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.00701, size = 173, normalized size = 2.06

$$\frac{\left(i\,Bab+\left(2\,Aab-Bb^{2}\right)xe^{(i\,x)}+\left(-i\,Ba^{2}+2i\,Aab-i\,Bb^{2}\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*B*a*b + (2*A*a*b - B*b^2)*x*e^(I*x) + (-I*B*a^2 + 2*I*A*a*b - I*B*b^
2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

Sympy [A] time = 1.7146, size = 58, normalized size = 0.69

$$\left(\frac{iA}{a}-\frac{iB}{2b}-\frac{iBb}{2a^{2}}\right)\log\left(\frac{a}{b}+e^{ix}\right)+\frac{2Aax+iBae^{-ix}-Bbx}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] (I*A/a - I*B/(2*b) - I*B*b/(2*a**2))*log(a/b + exp(I*x)) + (2*A*a*x + I*B*a
*exp(-I*x) - B*b*x)/(2*a**2)
```

Giac [B] time = 1.13287, size = 209, normalized size = 2.49

$$-\frac{2\left(Ba^{3}-2\,Aa^{2}b-Ba^{2}b+2\,Aab^{2}+Bab^{2}-Bb^{3}\right)\log\left(-a\tan\left(\frac{1}{2}\,x\right)+b\tan\left(\frac{1}{2}\,x\right)-i\,a-i\,b\right)}{-4i\,a^{3}b+4i\,a^{2}b^{2}}+\frac{i\,B\log\left(\tan\left(\frac{1}{2}\,x\right)+i\right)}{2\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(B*a^3 - 2*A*a^2*b - B*a^2*b + 2*A*a*b^2 + B*a*b^2 - B*b^3)*log(-a*tan(1
/2*x) + b*tan(1/2*x) - I*a - I*b)/(-4*I*a^3*b + 4*I*a^2*b^2) + 1/2*I*B*log(
tan(1/2*x) + I)/b - 1/2*(2*I*A*a - I*B*b)*log(tan(1/2*x) - I)/a^2 - 1/2*(-2
```

```
*I*A*a*tan(1/2*x) + I*B*b*tan(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x
) - I))
```

3.539 $\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=84

$$-\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}-\frac{iB\cos(x)}{2a}$$

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - ((I/2)*B*Cos[x])/a - ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rubi [A] time = 0.0424567, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3132}

$$-\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}-\frac{iB\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - ((I/2)*B*Cos[x])/a - ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA-bB)x}{2a^2}-\frac{iB\cos(x)}{2a}-\frac{i\left(2aAb-a^2B-b^2B\right)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}+\frac{B\sin(x)}{2a}$$

Mathematica [A] time = 0.182585, size = 147, normalized size = 1.75

$$\frac{2\left(a^2B-2aAb+b^2B\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)+ia^2B\log\left(a^2+2ab\cos(x)+b^2\right)+ib^2B\log}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
b)*Cot[x/2])/(a - b)] - (2*I)*a*b*B*Cos[x] - (2*I)*a*A*b*Log[a^2 + b^2 + 2*
a*b*Cos[x]] + I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] + I*b^2*B*Log[a^2 + b^2
 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

Maple [B] time = 0.08, size = 284, normalized size = 3.4

$$\frac{iA}{a}\ln\left(\tan\left(\frac{x}{2}\right)+i\right)-\frac{\frac{i}{2}bB}{a^2}\ln\left(\tan\left(\frac{x}{2}\right)+i\right)+\frac{B}{a}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-1}+\frac{iA}{-a+b}\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)-\frac{iA}{a(-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] I/a*ln(tan(1/2*x)+I)*A-1/2*I/a^2*ln(tan(1/2*x)+I)*b*B+B/a/(tan(1/2*x)+I)+I/
(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A-I/a*b/(-a+b)*ln(I*a+I*b-a*ta
n(1/2*x)+b*tan(1/2*x))*A-1/2*I*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2
*x))*B+1/2*I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B-1/2*I/a*b/(-a+b
)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+1/2*I/a^2*b^2/(-a+b)*ln(I*a+I*b-a
*tan(1/2*x)+b*tan(1/2*x))*B-1/2*I*B/b*ln(tan(1/2*x)-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.89394, size = 132, normalized size = 1.57

$$\frac{Ba^2x - i\,Babe^{(i\,x)} + \left(i\,Ba^2 - 2i\,Aab + i\,Bb^2\right)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*x - I*B*a*b*e^(I*x) + (I*B*a^2 - 2*I*A*a*b + I*B*b^2)*log((a*e^(
I*x) + b)/a))/(a^2*b)
```

Sympy [A] time = 0.884295, size = 51, normalized size = 0.61

$$\left(-\frac{iA}{a} + \frac{iB}{2b} + \frac{iBb}{2a^2}\right)\log\left(e^{ix} + \frac{b}{a}\right) + \frac{Bax - iBbe^{ix}}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] (-I*A/a + I*B/(2*b) + I*B*b/(2*a**2))*log(exp(I*x) + b/a) + (B*a*x - I*B*b*
exp(I*x))/(2*a*b)
```

Giac [B] time = 1.15591, size = 209, normalized size = 2.49

$$-\frac{2\left(Ba^3 - 2\,Aa^2b - Ba^2b + 2\,Aab^2 + Bab^2 - Bb^3\right)\log\left(-a\tan\left(\frac{1}{2}\,x\right) + b\tan\left(\frac{1}{2}\,x\right) + i\,a + i\,b\right)}{4i\,a^3b - 4i\,a^2b^2} - \frac{i\,B\log\left(\tan\left(\frac{1}{2}\,x\right) - i\right)}{2\,b} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(B*a^3 - 2*A*a^2*b - B*a^2*b + 2*A*a*b^2 + B*a*b^2 - B*b^3)*log(-a*tan(1
/2*x) + b*tan(1/2*x) + I*a + I*b)/(4*I*a^3*b - 4*I*a^2*b^2) - 1/2*I*B*log(t
an(1/2*x) - I)/b - 1/2*(-2*I*A*a + I*B*b)*log(tan(1/2*x) + I)/a^2 - 1/2*(2*
I*A*a*tan(1/2*x) - I*B*b*tan(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x)
 + I))
```

3.540 $\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=116

$$\frac{2\left(A\left(b^2+c^2\right)-acC\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

```
[Out] (c*C*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*c*C)*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*C*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rubi [A] time = 0.107879, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3137, 3124, 618, 204}

$$\frac{2\left(A\left(b^2+c^2\right)-acC\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*C*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*c*C)*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*C*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{acC}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx$$

$$= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{acC}{b^2+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b+2cx+\ldots}\right.$$

$$= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{acC}{b^2+c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-\ldots\right.}\right.$$

$$= \frac{cCx}{b^2+c^2}+\frac{2\left(A-\frac{acC}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A] time = 0.277802, size = 96, normalized size = 0.83

$$\frac{C(cx-b\log(a+b\cos(x)+c\sin(x)))-\frac{2\left(A\left(b^2+c^2\right)-acC\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((-2*(A*(b^2 + c^2) - a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2
 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + C*(c*x - b*Log[a + b*Cos[x] + c*Sin[x]])
```

```
)/(b^2 + c^2)
```

Maple [B] time = 0.06, size = 542, normalized size = 4.7

$$-\frac{abC}{\left(b^{2}+c^{2}\right)(a-b)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^{2}-b\left(\tan\left(\frac{x}{2}\right)\right)^{2}+2c\tan\left(x/2\right)+a+b\right)+\frac{b^{2}C}{\left(b^{2}+c^{2}\right)(a-b)}\ln\left(a\left(\tan\left(\frac{x}{2}\right)\right)^{2}-b\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] -1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*b
*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b
^2*C+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a
^2-b^2-c^2)^(1/2))*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b
)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+c^2
)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/
2))*C*b*c+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*
c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arcta
n(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2*C+C/(b^2+c^
2)*b*ln(1+tan(1/2*x)^2)+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.85079, size = 1374, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b^2 - C*a*c + A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 -
 c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2
*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c
)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^
3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*
cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) -
 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*co
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^
2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), 1/2*(2*(A*b^2 - C*a*c + A*c^2)*sqrt
(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 -
b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*co
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^
2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15184, size = 239, normalized size = 2.06

$$\frac{Ccx}{b^2+c^2} - \frac{Cb\log\left(-a\tan\left(\frac{1}{2}x\right)^2 + b\tan\left(\frac{1}{2}x\right)^2 - 2c\tan\left(\frac{1}{2}x\right) - a - b\right)}{b^2+c^2} + \frac{Cb\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} - \frac{2\left(Ab^2 - Cac + Ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] C*c*x/(b^2 + c^2) - C*b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - 2*(A*b^2
```

```
- C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*
(b^2 + c^2))
```

3.541 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=114

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

```
[Out] (2*(a*A - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.100546, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3154, 3124, 618, 204}

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```



```
[Out] (2*(a*A - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```


Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(aA-cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(2(aA-cC))\,\text{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx,x,\right.}{a^2-b^2-c^2}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(4(aA-cC))\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx,x,\right.}{a^2-b^2-c^2}$$

$$= \frac{2(aA-cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.342689, size = 123, normalized size = 1.08

$$\frac{a^2(-C)+\sin(x)\left(A\left(b^2+c^2\right)-acC\right)+aAc+b^2C}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{2(aA-cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^
2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b^2*C + (A*(b^2 + c^2) - a*c*C)*Sin
[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Maple [B] time = 0.091, size = 255, normalized size = 2.2

$$2\frac{1}{a(\tan(x/2))^2 - b(\tan(x/2))^2 + 2c\tan(x/2) + a + b}\left(-\frac{(aAb - Ab^2 - Ac^2 + acC - Cbc)\tan(x/2)}{a^3 - a^2b - ab^2 - ac^2 + b^3 + bc^2} + \frac{aAc - a^2C}{a^3 - a^2b - ab^2 -}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1
/2*x)+(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^
2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a
-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*A-2/(a^2-b^2-c^2)^(3/2)*arctan(1
/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.94021, size = 2739, normalized size = 24.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 4*(C*a^2*b - C*b^3)*c
^2 - (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*b^3 - C*b^3*c + A*
a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*c^3 - C*c^4)*sin(x
))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 -
 (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x)
- 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2
)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x
)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(C*a*c^4 - A*c^5 + (A*a
^2 - 2*A*b^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c)*cos(x) -
 2*(A*a^2*b^3 - A*b^5 + C*a*b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3)*c^2 - (C*
a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 -
 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^
7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x)
 + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 -
2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - 2*(
C*a^2*b - C*b^3)*c^2 + (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*
b^3 - C*b^3*c + A*a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*
c^3 - C*c^4)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x)
 + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b
- b^3 - b*c^2)*sin(x))) + (C*a*c^4 - A*c^5 + (A*a^2 - 2*A*b^2)*c^3 - (C*a^3
 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c)*cos(x) - (A*a^2*b^3 - A*b^5 + C*a*
b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3)*c^2 - (C*a^3*b - C*a*b^3)*c)*sin(x))/
(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3
*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3
)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^
5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16627, size = 278, normalized size = 2.44

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Cc)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}-\frac{2\left(Aab\tan\left(\frac{1}{2}\,x\right)-Ab^2\tan\left(\frac{1}{2}\,x\right)+Ca\right.}{\left(a^3-a^2b-ab^2+b^3-ac^2+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - C*c)/(a^2 - b^2 - c^2)^(3/2) -
 2*(A*a*b*tan(1/2*x) - A*b^2*tan(1/2*x) + C*a*c*tan(1/2*x) - C*b*c*tan(1/2*
x) - A*c^2*tan(1/2*x) + C*a^2 - C*b^2 - A*a*c)/((a^3 - a^2*b - a*b^2 + b^3
- a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)
)
```

3.542 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=200

$$\frac{\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+abC}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}-$$

```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) - (a*b*C - (3*a*A*
c - a^2*C - 2*c^2*C)*Cos[x] + b*(3*a*A - 2*c*C)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.249437, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3157, 3153, 3124, 618, 204}

$$\frac{\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+abC}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) - (a*b*C - (3*a*A*
c - a^2*C - 2*c^2*C)*Cos[x] + b*(3*a*A - 2*c*C)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-cC)+Ab\cos(x)+(Ac-aC)\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx}{2\left(a^2-b^2-c^2\right)}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{abC-\left(3aAc-a^2C-2c^2C\right)\cos(x)+b(3 \cdots}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+ \cdots}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{abC-\left(3aAc-a^2C-2c^2C\right)\cos(x)+b(3 \cdots}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+ \cdots}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{abC-\left(3aAc-a^2C-2c^2C\right)\cos(x)+b(3 \cdots}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+ \cdots}$$

$$= \frac{\left(2a^2A+A\left(b^2+c^2\right)-3acC\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} - \frac{bC-(Ac-aC)\cos(x)+Ab\,\mathrm{s}\cdots}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c \cdots}$$

Mathematica [A] time = 0.857815, size = 361, normalized size = 1.8

$$\frac{-2bc\cos(x)\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)-c\cos(2x)\left(a^2cC-3aA\left(b^2+c^2\right)+2cC\left(b^2+c^2\right)\right)-8a^2Ab^2\sin(x)-12a^2Ac^2 \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -((((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c -
 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C +
2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*Cos[x] - c*(-3*a*A*(b^2
 + c^2) + a^2*c*C + 2*c*(b^2 + c^2)*C)*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*
b^4*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 4*a^3*c*C*Sin[x] +
2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A*b^3*Sin[2*x] - 3*a*A*b*c^2*Si
n[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x])/(4*b
*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] time = 0.125, size = 1088, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^2
+b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c+13*A*a^2
*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*c^5-2*C*a^
5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b^4-13
*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-2*a^2*
c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*
b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^
4-A*b^5+A*b^3*c^2+2*A*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c+4*C*a^2*c^3
+5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-
2*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-A*b^4*c-A*b^2*
c^3-2*C*a^5+4*C*a^3*b^2-C*a^3*c^2-2*C*a*b^4+C*a*b^2*c^2)/(a^4-2*a^2*b^2-2*a
^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2
*c*tan(1/2*x)+a+b)^2+2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2
-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a^2*A+
1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*b^2+1/(a^4-2*a^2*b^2-2*a^
2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c
^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(
1/2))*a*c*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.91415, size = 7318, normalized size = 36.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 + 6*A*a*b*c^5 - 6*C*b
*c^6 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 - 6*(A*a^3*b - 2*A*a*b^3)*c^3 - 2*(2*C*a
^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 - 4*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b -
 4*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^
5)*c^2 - 3*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 - (2*A*a^4*b^2 + A*a^2*b^4 - 3
*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a
*b^2)*c^3 + (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*
C*a*b^4*c + A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2
 + 2*(2*A*a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2
*(A*a^3*b + A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c
^5 - 2*(A*a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 +
 3*C*a*b*c^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)
*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2
+ 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*
b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*si
n(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 +
(a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^
2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 6*(A*a^3*b^
3 - A*a*b^5)*c + 2*(C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b^2)*c^5 + (C*a^3 + 2*
C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^
2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)*c)*cos(x) - 2*(4*A*a
^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*A*a^2*b - 3*A*b^3)*
c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*b^3 + 3*A*b^5)*c^2
- (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3*A*a*b^6 - 3*A*a*b^
4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*(A*a^3 - A*a*b^2)*c
^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*s
in(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*
c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4
 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6
 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4
)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*
cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b
- 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 +
 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a
^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3
 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)
```

```
*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 -
 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1
/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 + 3*A*a*b*c^5 - 3*C*b*c^6 +
 (4*C*a^2*b - 7*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 - (2*C*a^4*b - 7*C
*a^2*b^3 + 5*C*b^5)*c^2 - 2*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b - 4*C*b^3)*
c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - 3
*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 + (2*A*a^4*b^2 + A*a^2*b^4 - 3*C*a^3*b^2
*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3
+ (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*C*a*b^4*c
+ A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*
a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2*(A*a^3*b
+ A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - 2*(A*
a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c
^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)*cos(x))*s
in(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*
sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)
*sin(x))) - 3*(A*a^3*b^3 - A*a*b^5)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b
^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3
- (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)
*c)*cos(x) - (4*A*a^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*
A*a^2*b - 3*A*b^3)*c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*
b^3 + 3*A*b^5)*c^2 - (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3
*A*a*b^6 - 3*A*a*b^4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*
(A*a^3 - A*a*b^2)*c^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*
C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10
 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (
a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4
- 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a
^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 -
a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (
a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 -
4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3
*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9
- (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^
4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)
*cos(x))*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35178, size = 1423, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*A*a^2 + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
- (4*A*a^4*b*tan(1/2*x)^3 - 11*A*a^3*b^2*tan(1/2*x)^3 + 9*A*a^2*b^3*tan(1/2
*x)^3 - A*a*b^4*tan(1/2*x)^3 - A*b^5*tan(1/2*x)^3 + 3*C*a^4*c*tan(1/2*x)^3
- 9*C*a^3*b*c*tan(1/2*x)^3 + 9*C*a^2*b^2*c*tan(1/2*x)^3 - 3*C*a*b^3*c*tan(1
/2*x)^3 - 5*A*a^3*c^2*tan(1/2*x)^3 + 7*A*a^2*b*c^2*tan(1/2*x)^3 + A*a*b^2*c
^2*tan(1/2*x)^3 - 3*A*b^3*c^2*tan(1/2*x)^3 + 2*A*a*c^4*tan(1/2*x)^3 - 2*A*b
*c^4*tan(1/2*x)^3 + 2*C*a^5*tan(1/2*x)^2 - 2*C*a^4*b*tan(1/2*x)^2 - 4*C*a^3
*b^2*tan(1/2*x)^2 + 4*C*a^2*b^3*tan(1/2*x)^2 + 2*C*a*b^4*tan(1/2*x)^2 - 2*C
*b^5*tan(1/2*x)^2 - 4*A*a^4*c*tan(1/2*x)^2 + 12*A*a^3*b*c*tan(1/2*x)^2 - 13
*A*a^2*b^2*c*tan(1/2*x)^2 + 6*A*a*b^3*c*tan(1/2*x)^2 - A*b^4*c*tan(1/2*x)^2
 + 5*C*a^3*c^2*tan(1/2*x)^2 - 14*C*a^2*b*c^2*tan(1/2*x)^2 + 13*C*a*b^2*c^2*
tan(1/2*x)^2 - 4*C*b^3*c^2*tan(1/2*x)^2 - 7*A*a^2*c^3*tan(1/2*x)^2 + 6*A*a*
b*c^3*tan(1/2*x)^2 + A*b^2*c^3*tan(1/2*x)^2 + 2*C*a*c^4*tan(1/2*x)^2 - 2*C*
b*c^4*tan(1/2*x)^2 + 2*A*c^5*tan(1/2*x)^2 + 4*A*a^4*b*tan(1/2*x) - 5*A*a^3*
b^2*tan(1/2*x) - 3*A*a^2*b^3*tan(1/2*x) + 5*A*a*b^4*tan(1/2*x) - A*b^5*tan(
1/2*x) + 5*C*a^4*c*tan(1/2*x) - 5*C*a^3*b*c*tan(1/2*x) - 5*C*a^2*b^2*c*tan(
1/2*x) + 5*C*a*b^3*c*tan(1/2*x) - 11*A*a^3*c^2*tan(1/2*x) + 3*A*a^2*b*c^2*t
an(1/2*x) + 7*A*a*b^2*c^2*tan(1/2*x) + A*b^3*c^2*tan(1/2*x) + 4*C*a^2*c^3*t
an(1/2*x) - 4*C*a*b*c^3*tan(1/2*x) + 2*A*a*c^4*tan(1/2*x) + 2*A*b*c^4*tan(1
/2*x) + 2*C*a^5 - 4*C*a^3*b^2 + 2*C*a*b^4 - 4*A*a^4*c + 3*A*a^2*b^2*c + A*b
^4*c + C*a^3*c^2 - C*a*b^2*c^2 + A*a^2*c^3 + A*b^2*c^3)/((a^6 - 2*a^5*b - a
^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*
a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*
tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)^2)
```

3.543 $\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=85

$$\frac{(a^2(-C)+2iaAb+b^2C)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-ibC)}{2a^2}+\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

```
[Out] ((2*a*A - I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) + (((2*I)*a*A*b - a^2*C + b^
2*C)*Log[a + b*Cos[x] + I*b*Sin[x]])/(2*a^2*b) + ((I/2)*C*Sin[x])/a
```

Rubi [A] time = 0.0458097, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3131}

$$\frac{(a^2(-C)+2iaAb+b^2C)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-ibC)}{2a^2}+\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) + (((2*I)*a*A*b - a^2*C + b^
2*C)*Log[a + b*Cos[x] + I*b*Sin[x]])/(2*a^2*b) + ((I/2)*C*Sin[x])/a
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-ibC)x}{2a^2}-\frac{C\cos(x)}{2a}+\frac{(2iaAb-a^2C+b^2C)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+\frac{iC\sin(x)}{2a}$$

Mathematica [A] time = 0.260401, size = 152, normalized size = 1.79

$$\frac{\left(-2ia^2C-4aAb+2ib^2C\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)-a^2C\log\left(a^2+2ab\cos(x)+b^2\right)+b^2C}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x - I*a^2*C*x - I*b^2*C*x + (-4*a*A*b - (2*I)*a^2*C + (2*I)*b^2*C)
*ArcTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] + (2*I)*a*A*b*Log[a^2
+ b^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2
 + b^2 + 2*a*b*Cos[x]] + (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

Maple [B] time = 0.082, size = 151, normalized size = 1.8

$$-\frac{C}{2\,b}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)+\frac{bC}{2\,a^2}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)+\frac{iA}{a}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] -1/2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+1/2/a^2*b*ln(I*a+I*b+a*tan(1
/2*x)-b*tan(1/2*x))*C+I/a*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*A+1/2*C/b*l
n(tan(1/2*x)+I)+I*C/a/(tan(1/2*x)-I)-I/a*ln(tan(1/2*x)-I)*A-1/2/a^2*ln(tan(
1/2*x)-I)*b*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.92754, size = 167, normalized size = 1.96

$$-\frac{\left(Cab-\left(2\,Aab-i\,Cb^2\right)xe^{(i\,x)}+\left(Ca^2-2i\,Aab-Cb^2\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(C*a*b - (2*A*a*b - I*C*b^2)*x*e^(I*x) + (C*a^2 - 2*I*A*a*b - C*b^2)*e
^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

Sympy [A] time = 1.75927, size = 54, normalized size = 0.64

$$\left(\frac{iA}{a}-\frac{C}{2b}+\frac{Cb}{2a^2}\right)\log\left(\frac{a}{b}+e^{ix}\right)+\frac{2Aax-Cae^{-ix}-iCbx}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] (I*A/a - C/(2*b) + C*b/(2*a**2))*log(a/b + exp(I*x)) + (2*A*a*x - C*a*exp(-
I*x) - I*C*b*x)/(2*a**2)
```

Giac [B] time = 1.17244, size = 212, normalized size = 2.49

$$\frac{2\left(i\,Ca^3+2\,Aa^2b-i\,Ca^2b-2\,Aab^2-i\,Cab^2+i\,Cb^3\right)\log\left(-a\tan\left(\frac{1}{2}\,x\right)+b\tan\left(\frac{1}{2}\,x\right)-i\,a-i\,b\right)}{-4i\,a^3b+4i\,a^2b^2}+\frac{C\log\left(\tan\left(\frac{1}{2}\,x\right)+i\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(I*C*a^3 + 2*A*a^2*b - I*C*a^2*b - 2*A*a*b^2 - I*C*a*b^2 + I*C*b^3)*log(-
a*tan(1/2*x) + b*tan(1/2*x) - I*a - I*b)/(-4*I*a^3*b + 4*I*a^2*b^2) + 1/2*C
*log(tan(1/2*x) + I)/b - 1/2*(2*I*A*a + C*b)*log(tan(1/2*x) - I)/a^2 - 1/2*
```

```
(-2*I*A*a*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*I*C*a + I*C*b)/(a^2*(tan(
1/2*x) - I))
```

3.544 $\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{(a^2C+2iaAb-b^2C)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA+ibC)}{2a^2}-\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

```
[Out] ((2*a*A + I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) - (((2*I)*a*A*b + a^2*C - b^
2*C)*Log[a + b*Cos[x] - I*b*Sin[x]])/(2*a^2*b) - ((I/2)*C*Sin[x])/a
```

Rubi [A] time = 0.0462409, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3131}

$$-\frac{(a^2C+2iaAb-b^2C)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA+ibC)}{2a^2}-\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A + I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) - (((2*I)*a*A*b + a^2*C - b^
2*C)*Log[a + b*Cos[x] - I*b*Sin[x]])/(2*a^2*b) - ((I/2)*C*Sin[x])/a
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA+ibC)x}{2a^2}-\frac{C\cos(x)}{2a}-\frac{(2iaAb+a^2C-b^2C)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-\frac{iC\sin(x)}{2a}$$

Mathematica [A] time = 0.238415, size = 152, normalized size = 1.79

$$\frac{2i\left(a^2C+2iaAb-b^2C\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)-a^2C\log\left(a^2+2ab\cos(x)+b^2\right)+b^2C\log}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + I*a^2*C*x + I*b^2*C*x + (2*I)*((2*I)*a*A*b + a^2*C - b^2*C)*Ar
cTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] - (2*I)*a*A*b*Log[a^2 + b
^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2 +
b^2 + 2*a*b*Cos[x]] - (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

Maple [B] time = 0.077, size = 280, normalized size = 3.3

$$\frac{-iC}{a}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-1}+\frac{iA}{a}\ln\left(\tan\left(\frac{x}{2}\right)+i\right)-\frac{bC}{2\,a^2}\ln\left(\tan\left(\frac{x}{2}\right)+i\right)+\frac{Ca}{2\,b\,(-a+b)}\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] -I*C/a/(tan(1/2*x)+I)+I/a*ln(tan(1/2*x)+I)*A-1/2/a^2*ln(tan(1/2*x)+I)*b*C+1
/2*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/(-a+b)*ln(I*a+I*b
-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(
1/2*x))*C+1/2/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C+I/(-a+
b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A-I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/
2*x)+b*tan(1/2*x))*A+1/2*C/b*ln(tan(1/2*x)-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.08965, size = 127, normalized size = 1.49

$$\frac{i\,Ca^2x - Cabe^{(i\,x)} - \left(Ca^2 + 2i\,Aab - Cb^2\right)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*C*a^2*x - C*a*b*e^(I*x) - (C*a^2 + 2*I*A*a*b - C*b^2)*log((a*e^(I*x)
 + b)/a))/(a^2*b)
```

Sympy [A] time = 0.976919, size = 48, normalized size = 0.56

$$\left(-\frac{iA}{a} - \frac{C}{2b} + \frac{Cb}{2a^2}\right)\log\left(e^{ix} + \frac{b}{a}\right) + \frac{iCax - Cbe^{ix}}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] (-I*A/a - C/(2*b) + C*b/(2*a**2))*log(exp(I*x) + b/a) + (I*C*a*x - C*b*exp(
I*x))/(2*a*b)
```

Giac [B] time = 1.12915, size = 212, normalized size = 2.49

$$\frac{2\left(-i\,Ca^3 + 2\,Aa^2b + i\,Ca^2b - 2\,Aab^2 + i\,Cab^2 - i\,Cb^3\right)\log\left(-a\tan\left(\frac{1}{2}\,x\right) + b\tan\left(\frac{1}{2}\,x\right) + i\,a + i\,b\right)}{4i\,a^3b - 4i\,a^2b^2} + \frac{C\log\left(\tan\left(\frac{1}{2}\,x\right) -\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(-I*C*a^3 + 2*A*a^2*b + I*C*a^2*b - 2*A*a*b^2 + I*C*a*b^2 - I*C*b^3)*log(
-a*tan(1/2*x) + b*tan(1/2*x) + I*a + I*b)/(4*I*a^3*b - 4*I*a^2*b^2) + 1/2*C
*log(tan(1/2*x) - I)/b - 1/2*(-2*I*A*a + C*b)*log(tan(1/2*x) + I)/a^2 - 1/2
*(2*I*A*a*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a + 2*I*C*a - I*C*b)/(a^2*(tan(
1/2*x) + I))
```

3.545 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=119

$$-\frac{2a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (2*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2]
)/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + ((B*c - b*C
)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rubi [A] time = 0.113433, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3136, 3124, 618, 204}

$$-\frac{2a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (2*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2]
)/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + ((B*c - b*C
)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(a(bB+cC))\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(2a(bB+cC))\,\text{Subst}\left(\int \frac{1}{a+b+2cx+(a \ldots}{b^2+c^2}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{(4a(bB+cC))\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c \ldots}{b^2+ \ldots}$$

$$= \frac{(bB+cC)x}{b^2+c^2}-\frac{2a(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A] time = 0.352565, size = 98, normalized size = 0.82

$$\frac{\frac{2a(bB+cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+(Bc-bC)\log(a+b\cos(x)+c\sin(x))+x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (2*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2
+ b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Log[a + b*Cos[x] + c*Si
n[x]])/(b^2 + c^2)
```

Maple [B] time = 0.057, size = 824, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*
c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*
B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*
a*b*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b
)*b^2*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)
/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(
a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+
c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^
(1/2))*C*b*c-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*ar
ctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(b^2
+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)
^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*t
an(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2*C-B/(b^2+c^2)*c*ln(1+tan(1/
2*x)^2)+C/(b^2+c^2)*b*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))
+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66707, size = 1507, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*b + C*a*c)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 -
(a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3
 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x
))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c
^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x)
+ (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a
^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 -
C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^
2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 -
 2*b^2)*c^2), -1/2*(2*(B*a*b + C*a*c)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*co
s(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)
*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C
*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 -
 B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos
(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16191, size = 252, normalized size = 2.12

$$\frac{(Bb+Cc)x}{b^2+c^2} - \frac{(Cb-Bc)\log\left(-a\tan\left(\frac{1}{2}x\right)^2 + b\tan\left(\frac{1}{2}x\right)^2 - 2\,c\tan\left(\frac{1}{2}x\right) - a - b\right)}{b^2+c^2} + \frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} + \frac{2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^
2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)
/(b^2 + c^2) + 2*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a
^2 - b^2 - c^2)*(b^2 + c^2))
```

3.546 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=110

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}$$

```
[Out] (-2*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2
- b^2 - c^2)^(3/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/((a^2 - b^2 - c^
2)*(a + b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.0963721, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 3124, 618, 204}

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (-2*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2
- b^2 - c^2)^(3/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/((a^2 - b^2 - c^
2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(2(bB+cC))\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx,x,\right.}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(4(bB+cC))\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx,x,\right.}{a^2-b^2-c^2}$$

$$= -\frac{2(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.35717, size = 116, normalized size = 1.05

$$-\frac{a^2C+a\sin(x)(bB+cC)-b^2C+bBc}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (-2*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a
^2 + b^2 + c^2)^(3/2) - (b*B*c + a^2*C - b^2*C + a*(b*B + c*C)*Sin[x])/(b*(
-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Maple [B] time = 0.1, size = 255, normalized size = 2.3

$$-2\,\frac{1}{a\,(\tan(x/2))^2-b\,(\tan(x/2))^2+2\,c\tan(x/2)+a+b}\left(-\frac{\left(a^2B-abB-Bc^2-acC+Cbc\right)\tan(x/2)}{a^3-a^2b-ab^2-ac^2+b^3+bc^2}+\frac{bBc+a^2C}{a^3-a^2b-ab^2-a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] -2*(-(B*a^2-B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(
1/2*x)+(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)
^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(
a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*b*B-2/(a^2-b^2-c^2)^(3/2)*arctan(
1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.79401, size = 2799, normalized size = 25.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 + (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C
*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*
c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4
 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)
*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (
b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*
b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))
 - 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(B*a*b*c^3 + C*a*c^4 - (C*a^3 - C*
a*b^2)*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + 2*(B*a^3*b^2 - B*a*b^4 - B*a*b
^2*c^2 - C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 +
a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (
a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*
b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 +
3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3
 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*a^2*b - C*b^3)*c^
2 - (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c
^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt
(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 -
b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
 (B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + (B*a*b*c^3 + C*a*c^4 - (C*a^3 - C*a*b^2)
*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + (B*a^3*b^2 - B*a*b^4 - B*a*b^2*c^2 -
 C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 +
a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3
- 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*
b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c
^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17682, size = 277, normalized size = 2.52

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Bb+Cc)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Bab\tan\left(\frac{1}{2}x\right)-Cac\tan\right.}{\left(a^3-a^2b-ab^2+b^3-ac^2+bc^2\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(B*b + C*c)/(a^2 - b^2 - c^2)^(3/2) +
2*(B*a^2*tan(1/2*x) - B*a*b*tan(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x
) - B*c^2*tan(1/2*x) - C*a^2 + C*b^2 - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 -
 a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))
```

3.547 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=197

$$-\frac{3a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\cos(x)\left(C\left(a^2+2c^2\right)+2bBc\right)+\sin(x)\left(a^2B+2b(bB+cC)\right)+a(Bc-bC)}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}+\frac{}{2(}$$

```
[Out] (-3*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^
2 - b^2 - c^2)^(5/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/(2*(a^2 - b^2
- c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*(B*c - b*C) - (2*b*B*c + (a^2 + 2*
c^2)*C)*Cos[x] + (a^2*B + 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(
a + b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.232421, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3156, 3153, 3124, 618, 204}

$$-\frac{3a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\cos(x)\left(C\left(a^2+2c^2\right)+2bBc\right)+\sin(x)\left(a^2B+2b(bB+cC)\right)+a(Bc-bC)}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}+\frac{}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```


```
[Out] (-3*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^
2 - b^2 - c^2)^(5/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/(2*(a^2 - b^2
- c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*(B*c - b*C) - (2*b*B*c + (a^2 + 2*
c^2)*C)*Cos[x] + (a^2*B + 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(
a + b*Cos[x] + c*Sin[x]))
```


Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
```

```
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{2(bB+cC)-aB\cos(x)-aC\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx}{2\left(a^2-b^2-c^2\right)}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)-\left(2bBc+\left(a^2+2c^2\right)C\right)\cos(x}{2\left(a^2-b^2-c^2\right)^2(a+b\,\mathrm{co}}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)-\left(2bBc+\left(a^2+2c^2\right)C\right)\cos(x}{2\left(a^2-b^2-c^2\right)^2(a+b\,\mathrm{co}}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)-\left(2bBc+\left(a^2+2c^2\right)C\right)\cos(x}{2\left(a^2-b^2-c^2\right)^2(a+b\,\mathrm{co}}$$

$$= -\frac{3a(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(}{}$$

Mathematica [A] time = 0.821103, size = 311, normalized size = 1.58

$$\frac{-c\cos(2x)\left(a^2+2\left(b^2+c^2\right)\right)(bB+cC)+a^2b^2B\sin(2x)-4a^2b^2C+9a^2bBc+4a^3bB\sin(x)+a^2bcC\sin(2x)+5a^2c^2C+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (3*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-
a^2 + b^2 + c^2)^(5/2) + (9*a^2*b*B*c + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5
*a^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C + 6*a*b*c*(b*B + c*C)*Cos[x] - c*(a^2 +
2*(b^2 + c^2))*(b*B + c*C)*Cos[2*x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] +
 8*a*b*B*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin
[x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*
c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x])/(4*b*(-a^2 + b^2 +
c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] time = 0.127, size = 881, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2-4*B*a^2*c^2-3*B*a*b^3+2*B*b^4+4*B*b
^2*c^2+2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^2+
b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3-1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b
^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c+4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*
b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b^4-13*C*a*b^2*c
^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*
b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(2*B*a^5-3*B*a^4*b+B*a^3*b^2-
4*B*a^3*c^2+B*a^2*b^3-8*B*a^2*b*c^2-3*B*a*b^4+8*B*a*b^2*c^2+2*B*a*c^4+2*B*b
^5+4*B*b^3*c^2+2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c-4*C*a^2*c^3-5*
C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a
*b+b^2)*tan(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c-2*B*b*c^3+2*C*a^4-4*C*a^2*b
^2+C*a^2*c^2+2*C*b^4-C*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)
/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2-3/(a
^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2
*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B-3/(a^4-2*a^2*b^2-2*a^2*c^
2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c
)/(a^2-b^2-c^2)^(1/2))*a*c*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.5243, size = 6728, normalized size = 34.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7
 - 2*(3*B*a^2 - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B*a^4 - 5*B
*a^2*b^2 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*B*
b^2*c^5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 -
 (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)
*cos(x)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a
*b^2)*c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*
a*c^5)*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)
*cos(x) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b
^4*c + C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(-a^2 + b
^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4
 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*
c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*
b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))
*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 +
 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^2 + 2*B*a^2*b^4 + B*b
^6)*c + 2*(B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2)*c^4 + (B*a^3*b + 2*B*a
*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - B*a^3*b^3 -
B*a*b^5)*c)*cos(x) + 2*(2*B*a^5*b^2 - B*a^3*b^4 - B*a*b^6 - B*a*b^2*c^4 - C
*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2*B*a*b^4)*c^2 + (2*C*a
^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^5 - 2*B*b^7 + 2*B*b*c^
6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*b^3)*c^4 - (C*a^4 + 2*
C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)
*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2
 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 +
4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 -
 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b
^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 +
b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 +
 (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6
*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*
c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*
a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*
b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9
*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*
sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 - 3*C*b*c^6 + B*c
^7 - (3*B*a^2 - B*b^2)*c^5 + (4*C*a^2*b - 7*C*b^3)*c^4 + (3*B*a^4 - 5*B*a^2
*b^2 - B*b^4)*c^3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*B*b^2*c^
5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 - (C*a^
4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)*cos(x
)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a*b^2)*
```

```
c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*a*c^5)
*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)*cos(x
) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b^4*c +
 C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^
2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^
3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a
^4*b^2 + 2*B*a^2*b^4 + B*b^6)*c + (B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2
)*c^4 + (B*a^3*b + 2*B*a*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (
2*B*a^5*b - B*a^3*b^3 - B*a*b^5)*c)*cos(x) + (2*B*a^5*b^2 - B*a^3*b^4 - B*a
*b^6 - B*a*b^2*c^4 - C*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2
*B*a*b^4)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^
5 - 2*B*b^7 + 2*B*b*c^6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*
b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C
*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a
^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*
a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 -
 b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*
c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(
a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b
^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a
*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^
9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 -
(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*
b^7 - b^9)*c)*cos(x))*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32147, size = 1396, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a^4 - 2*a^2*b^2 + b^
4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2)) + (2*B*a^5*tan(1/2*
x)^3 - 5*B*a^4*b*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(
1/2*x)^3 + 5*B*a*b^4*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*tan(1/
2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*a*b^3*
c*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 + 4*
B*a*b^2*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^
3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b*tan(1/2*x)^2
+ 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a*b^4*tan(1/2*x
)^2 + 2*C*b^5*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x
)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + 2*B*b^4*c*ta
n(1/2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2*x)^2 - 13*C*
a*b^2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 - 4*B*a^2*c^3*tan(1/2*x)^
2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*b*c^4*tan(1/2*x
)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + B*
a^3*b^2*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + 2*B*b^5*
tan(1/2*x) - 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x) + 5*C*a^2*b^2*c*
tan(1/2*x) - 5*C*a*b^3*c*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 8*B*a^2*b*c^
2*tan(1/2*x) + 8*B*a*b^2*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) - 4*C*a^2*
c^3*tan(1/2*x) + 4*C*a*b*c^3*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) + 2*B*b*c^4*
tan(1/2*x) - 2*C*a^5 + 4*C*a^3*b^2 - 2*C*a*b^4 - 5*B*a^3*b*c + 5*B*a*b^3*c
- C*a^3*c^2 + C*a*b^2*c^2 + 2*B*a*b*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*
b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2 + 2*b
^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 +
2*c*tan(1/2*x) + a + b)^2)
```

3.548 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{\left(a^2(C+iB)+ib^2(B+iC)\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}-\frac{bx(B+iC)}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

```
[Out] -(b*(B + I*C)*x)/(2*a^2) - ((I*b^2*(B + I*C) + a^2*(I*B + C))*Log[a + b*Cos
[x] + I*b*Sin[x]])/(2*a^2*b) + ((I*B - C)*(Cos[x] - I*Sin[x]))/(2*a)
```

Rubi [A] time = 0.0775119, antiderivative size = 87, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3130}

$$-\frac{\left(\frac{ib^2(B+iC)}{a^2}+iB+C\right)\log(a+ib\sin(x)+b\cos(x))}{2b}-\frac{bx(B+iC)}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] -(b*(B + I*C)*x)/(2*a^2) - ((I*B + (I*b^2*(B + I*C))/a^2 + C)*Log[a + b*Cos
[x] + I*b*Sin[x]])/(2*b) + ((I*B - C)*(Cos[x] - I*Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=-\frac{b(B+iC)x}{2a^2}-\frac{\left(iB+\frac{ib^2(B+iC)}{a^2}+C\right)\log(a+b\cos(x)+ib\sin(x))}{2b}+\frac{(iB-C)(\cos(x)-i\sin(x))}{2a}$$

Mathematica [B] time = 0.296928, size = 195, normalized size = 2.12

$$\frac{x\left(a^2B-ia^2C-b^2B-ib^2C\right)}{4a^2b}-\frac{i\left(a^2B-ia^2C+b^2B+ib^2C\right)\log\left(a^2+2ab\cos(x)+b^2\right)}{4a^2b}-\frac{\left(a^2B-ia^2C+b^2B+ib^2C\right)\tan}{2a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((a^2*B - b^2*B - I*a^2*C - I*b^2*C)*x)/(4*a^2*b) - ((a^2*B + b^2*B - I*a^2
*C + I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(-(a*Sin[x/2]) + b*Sin[x/2])])/(2*a
^2*b) + ((I/2)*(B + I*C)*Cos[x])/a - ((I/4)*(a^2*B + b^2*B - I*a^2*C + I*b^
2*C)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B + I*C)*Sin[x])/(2*a)
```

Maple [B] time = 0.079, size = 212, normalized size = 2.3

$$-\frac{C}{2b}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)+\frac{bC}{2a^2}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)-\frac{\frac{i}{2}B}{b}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\,t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] -1/2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+1/2/a^2*b*ln(I*a+I*b+a*tan(1
/2*x)-b*tan(1/2*x))*C-1/2*I/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B-1/2*I
/a^2*b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B+1/2*C/b*ln(tan(1/2*x)+I)+1/2
*I*B/b*ln(tan(1/2*x)+I)+I*C/a/(tan(1/2*x)-I)+B/a/(tan(1/2*x)-I)+1/2*I/a^2*l
n(tan(1/2*x)-I)*b*B-1/2/a^2*ln(tan(1/2*x)-I)*b*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima"
)
```

[Out] Exception raised: RuntimeError

Fricas [A] time = 2.08525, size = 178, normalized size = 1.93

$$-\frac{\left((B+i\,C)b^2xe^{(i\,x)}-(i\,B-C)ab-\left((-i\,B-C)a^2+(-i\,B+C)b^2\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas"
)
```

```
[Out] -1/2*((B + I*C)*b^2*x*e^(I*x) - (I*B - C)*a*b - ((-I*B - C)*a^2 + (-I*B + C
)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

Sympy [A] time = 1.93881, size = 75, normalized size = 0.82

$$\frac{iBae^{-ix}-Bbx-Cae^{-ix}-iCbx}{2a^2}+\frac{\left(-iBa^2-iBb^2-Ca^2+Cb^2\right)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] (I*B*a*exp(-I*x) - B*b*x - C*a*exp(-I*x) - I*C*b*x)/(2*a**2) + (-I*B*a**2 -
 I*B*b**2 - C*a**2 + C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

Giac [B] time = 1.16486, size = 238, normalized size = 2.59

$$-\frac{2\left(Ba^3-i\,Ca^3-Ba^2b+i\,Ca^2b+Bab^2+i\,Cab^2-Bb^3-i\,Cb^3\right)\log\left(-a\tan\left(\frac{1}{2}\,x\right)+b\tan\left(\frac{1}{2}\,x\right)-i\,a-i\,b\right)}{-4i\,a^3b+4i\,a^2b^2}-\frac{(-i\,B-C)\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(B*a^3 - I*C*a^3 - B*a^2*b + I*C*a^2*b + B*a*b^2 + I*C*a*b^2 - B*b^3 - I
*C*b^3)*log(-a*tan(1/2*x) + b*tan(1/2*x) - I*a - I*b)/(-4*I*a^3*b + 4*I*a^2
*b^2) - 1/2*(-I*B - C)*log(tan(1/2*x) + I)/b - 1/2*(-I*B*b + C*b)*log(tan(1
/2*x) - I)/a^2 - 1/2*(I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*B*a - 2*I*C*a +
 B*b + I*C*b)/(a^2*(tan(1/2*x) - I))
```

3.549 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=90

$$\frac{\left(ia^2(B+iC)+b^2(C+iB)\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}-\frac{bx(B-iC)}{2a^2}-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

```
[Out] -(b*(B - I*C)*x)/(2*a^2) + ((I*a^2*(B + I*C) + b^2*(I*B + C))*Log[a + b*Cos
[x] - I*b*Sin[x]])/(2*a^2*b) - ((I*B + C)*(Cos[x] + I*Sin[x]))/(2*a)
```

Rubi [A] time = 0.07837, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3130}

$$-\frac{bx(B-iC)}{2a^2}+\frac{1}{2}\left(\frac{b(C+iB)}{a^2}+\frac{i(B+iC)}{b}\right)\log(a-ib\sin(x)+b\cos(x))-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] -(b*(B - I*C)*x)/(2*a^2) + (((I*(B + I*C))/b + (b*(I*B + C))/a^2)*Log[a + b
*Cos[x] - I*b*Sin[x]])/2 - ((I*B + C)*(Cos[x] + I*Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=-\frac{b(B-iC)x}{2a^2}+\frac{1}{2}\left(\frac{i(B+iC)}{b}+\frac{b(iB+C)}{a^2}\right)\log(a+b\cos(x)-ib\sin(x))-\frac{(iB+C)(\cos(x)}{2a}$$

Mathematica [B] time = 0.276705, size = 195, normalized size = 2.17

$$\frac{x\left(a^2B+ia^2C-b^2B+ib^2C\right)}{4a^2b}+\frac{i\left(a^2B+ia^2C+b^2B-ib^2C\right)\log\left(a^2+2ab\cos(x)+b^2\right)}{4a^2b}+\frac{\left(a^2B+ia^2C+b^2B-ib^2C\right)\text{ta}}{2a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((a^2*B - b^2*B + I*a^2*C + I*b^2*C)*x)/(4*a^2*b) + ((a^2*B + b^2*B + I*a^2
*C - I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(a*Sin[x/2] - b*Sin[x/2])])/(2*a^2*
b) - ((I/2)*(B - I*C)*Cos[x])/a + ((I/4)*(a^2*B + b^2*B + I*a^2*C - I*b^2*C
)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B - I*C)*Sin[x])/(2*a)
```

Maple [B] time = 0.085, size = 388, normalized size = 4.3

$$\frac{-iC}{a}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-1}+\frac{B}{a}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-1}-\frac{\frac{i}{2}bB}{a^2}\ln\left(\tan\left(\frac{x}{2}\right)+i\right)-\frac{bC}{2a^2}\ln\left(\tan\left(\frac{x}{2}\right)+i\right)+\frac{Ca}{2b(-a+b)}\ln\left(ia+ib-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] -I*C/a/(tan(1/2*x)+I)+B/a/(tan(1/2*x)+I)-1/2*I/a^2*ln(tan(1/2*x)+I)*b*B-1/2
/a^2*ln(tan(1/2*x)+I)*b*C+1/2*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*
x))*C-1/2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/a*b/(-a+b)*ln(
I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C+1/2/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/
2*x)+b*tan(1/2*x))*C-1/2*I*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))
*B+1/2*I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B-1/2*I/a*b/(-a+b)*ln
(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+1/2*I/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan
(1/2*x)+b*tan(1/2*x))*B+1/2*C/b*ln(tan(1/2*x)-I)-1/2*I*B/b*ln(tan(1/2*x)-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.05515, size = 153, normalized size = 1.7

$$\frac{(B+iC)a^2x+(-iB-C)abe^{(ix)}+\left((iB-C)a^2+(iB+C)b^2\right)\log\left(\frac{ae^{(ix)}+b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas"
)
```

```
[Out] 1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 + (I*B + C)*
b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

Sympy [A] time = 1.06271, size = 75, normalized size = 0.83

$$\frac{Bax-iBbe^{ix}+iCax-Cbe^{ix}}{2ab}+\frac{\left(iBa^2+iBb^2-Ca^2+Cb^2\right)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] (B*a*x - I*B*b*exp(I*x) + I*C*a*x - C*b*exp(I*x))/(2*a*b) + (I*B*a**2 + I*B
*b**2 - C*a**2 + C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)
```

Giac [B] time = 1.16851, size = 238, normalized size = 2.64

$$-\frac{2\left(Ba^3+iCa^3-Ba^2b-iCa^2b+Bab^2-iCab^2-Bb^3+iCb^3\right)\log\left(-a\tan\left(\frac{1}{2}x\right)+b\tan\left(\frac{1}{2}x\right)+ia+ib\right)}{4ia^3b-4ia^2b^2}-\frac{(iB-C)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(B*a^3 + I*C*a^3 - B*a^2*b - I*C*a^2*b + B*a*b^2 - I*C*a*b^2 - B*b^3 + I
*C*b^3)*log(-a*tan(1/2*x) + b*tan(1/2*x) + I*a + I*b)/(4*I*a^3*b - 4*I*a^2*
b^2) - 1/2*(I*B - C)*log(tan(1/2*x) - I)/b - 1/2*(I*B*b + C*b)*log(tan(1/2*
x) + I)/a^2 - 1/2*(-I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*B*a + 2*I*C*a + B
*b - I*C*b)/(a^2*(tan(1/2*x) + I))
```

3.550 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=131

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(A\left(b^2+c^2\right)-a(bB+cC)\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*(b*B + c*C))*ArcTan[(c
+ (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c
^2)) + ((B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rubi [A] time = 0.126317, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3136, 3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(A\left(b^2+c^2\right)-a(bB+cC)\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*(b*B + c*C))*ArcTan[(c
+ (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c
^2)) + ((B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\int \frac{}{a+b\cos}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\right)\text{Subst}\Big($$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\right)\text{Subst}\Big($$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{2\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c}{b^2+c^2}$$

Mathematica [A] time = 0.333339, size = 110, normalized size = 0.84

$$\frac{\frac{2\left(a(bB+cC)-A\left(b^2+c^2\right)\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+(Bc-bC)\log(a+b\cos(x)+c\sin(x))+x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (2*(-(A*(b^2 + c^2)) + a*(b*B + c*C))*ArcTanh[(c + (a - b)
*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Lo
g[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [B] time = 0.056, size = 954, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*
c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*
B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*
a*b*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b
)*b^2*C+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)
/(a^2-b^2-c^2)^(1/2))*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(
a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+
c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^
(1/2))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2
*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*b*c-2/(b^2+c^2)/(a^2-b^2-c
^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b
)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(
a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(
a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
*c/(a-b)*b^2*C-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+C/(b^2+c^2)*b*ln(1+tan(1/2*
x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.98718, size = 1550, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas"
)
```

```
[Out] [-1/2*((B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2
- 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos
(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b
 - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos
(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)
)/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*s
in(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*
a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^
2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^
4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(
a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b
^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*
b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*c
os(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 +
(a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1498, size = 269, normalized size = 2.05

$$\frac{(Bb+Cc)x}{b^2+c^2} - \frac{(Cb-Bc)\log\left(-a\tan\left(\frac{1}{2}x\right)^2 + b\tan\left(\frac{1}{2}x\right)^2 - 2c\tan\left(\frac{1}{2}x\right) - a - b\right)}{b^2+c^2} + \frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} + \frac{2(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^
2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)
/(b^2 + c^2) + 2*(B*a*b - A*b^2 + C*a*c - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*
sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2
- c^2)))/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2))
```

3.551 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=127

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)(aA-bB-cC)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

```
[Out] (2*(a*A - b*B - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/
(a^2 - b^2 - c^2)^(3/2) + (B*c - b*C + (A*c - a*C)*Cos[x] - (A*b - a*B)*Sin
[x])/((a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rubi [A] time = 0.123414, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3153, 3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)(aA-bB-cC)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/
(a^2 - b^2 - c^2)^(3/2) + (B*c - b*C + (A*c - a*C)*Cos[x] - (A*b - a*B)*Sin
[x])/((a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))} + \frac{(aA-bB-cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))} + \frac{(2(aA-bB-cC))\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx \cdots}\right.}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))} - \frac{(4(aA-bB-cC))\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2 \cdots\right.}\right.}{a^2-b^2 \cdots}$$

$$= \frac{2(aA-bB-cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}} + \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(\cdots}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.436297, size = 137, normalized size = 1.08

$$\frac{a^2(-C)+\sin(x)\left(A\left(b^2+c^2\right)-a(bB+cC)\right)+aAc+b(bC-Bc)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))} + \frac{2(aA-bB-cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]]
)/(-a^2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b*(-(B*c) + b*C) + (A*(b^2 +
c^2) - a*(b*B + c*C))*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x
]))
```

Maple [B] time = 0.102, size = 329, normalized size = 2.6

$$2\,\frac{1}{a\left(\tan\left(x/2\right)\right)^{2}-b\left(\tan\left(x/2\right)\right)^{2}+2\,c\tan\left(x/2\right)+a+b}\left(-\frac{\left(aAb-Ab^{2}-Ac^{2}-a^{2}B+abB+Bc^{2}+acC-Cbc\right)\tan\left(x/2\right)}{a^{3}-a^{2}b-ab^{2}-ac^{2}+b^{3}+bc^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^
2+b^3+b*c^2)*tan(1/2*x)+(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^
3+b*c^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2/(a^2-b^2-c^2
)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*A-2/(a^2
-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*b*
B-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(
1/2))*C*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.05675, size = 3236, normalized size = 25.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="frica
s")
```

```
[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 - (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C
*a*c^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A*a*
b - B*b^2)*c^2)*cos(x) - (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^2 -
B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2
 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a
*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*s
in(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 +
 (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b
^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^4 -
 2*B*a^2*b^2 + B*b^4)*c + 2*(C*a*c^4 - A*c^5 + (A*a^2 + B*a*b - 2*A*b^2)*c^
3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(
x) + 2*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 + A*b*c^4 - (A*
a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 -
2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*
b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^
4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 -
4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b -
 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*a^2*b
 - C*b^3)*c^2 + (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C*a*c^3 + (A*a^2 - B*a*b
)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A*a*b - B*b^2)*c^2)*cos(x)
- (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(
a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b
^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + (C*a*c^4 - A*c^5 + (A*a^2 + B*a*b - 2*A*b
^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c
)*cos(x) + (B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 + A*b*c^4 -
 (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^
2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 +
3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 +
 (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^
4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18353, size = 325, normalized size = 2.56

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb-Cc)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Aab\tan\left(\frac{1}{2}x\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac"
)
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b - C*c)/(a^2 - b^2 - c^2)^(
3/2) + 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*ta
n(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*t
an(1/2*x) - C*a^2 + C*b^2 + A*a*c - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*
c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))
```

3.552 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=237

$$\frac{\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+}$$

```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTan[(c + (a - b)*Tan[x/2])/
Sqrt[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c - b*C + (A*c - a*C)*
Cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])
^2) + (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B + c*C))*Cos[x] - (3*a*A*
b - a^2*B - 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*Cos[x] +
 c*Sin[x]))
```

Rubi [A] time = 0.276504, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3156, 3153, 3124, 618, 204}

$$\frac{\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```


```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTan[(c + (a - b)*Tan[x/2])/
Sqrt[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c - b*C + (A*c - a*C)*
Cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])
^2) + (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B + c*C))*Cos[x] - (3*a*A*
b - a^2*B - 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*Cos[x] +
 c*Sin[x]))
```


Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
```

```
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-bB-cC)+(Ab-aB)\cos(x)+(Ac-aC)\,\mathrm{si}}{(a+b\cos(x)+c\sin(x))^2}}{2\left(a^2-b^2-c^2\right)}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+\left(3aAc-a^2C-2c(bB\right.}{2\left(a^2-b^2-c\right.}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+\left(3aAc-a^2C-2c(bB\right.}{2\left(a^2-b^2-c\right.}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+\left(3aAc-a^2C-2c(bB\right.}{2\left(a^2-b^2-c\right.}$$

$$= \frac{\left(2a^2A+A\left(b^2+c^2\right)-3a(bB+cC)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc-bC+(Ac-aC)\cos(}{2\left(a^2-b^2-c^2\right)(a+b}$$

Mathematica [A] time = 1.19407, size = 452, normalized size = 1.91

$$\frac{-2bc\cos(x)\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)-c\cos(2x)\left(a^2(bB+cC)-3aA\left(b^2+c^2\right)+2\left(b^2+c^2\right)(bB+cC)\right)-8a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTanh[(c + (a - b)*Tan[x/2
])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A
*b^2*c + 9*a^2*b*B*c - 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*
c^2*C + 4*b^2*c^2*C + 2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B +
 c*C))*Cos[x] - c*(-3*a*A*(b^2 + c^2) + a^2*(b*B + c*C) + 2*(b^2 + c^2)*(b*
B + c*C))*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x]
 + 2*a*b^3*B*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^
2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A
*b^3*Sin[2*x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x
] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^
3*C*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] time = 0.13, size = 1422, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b^4
-4*B*b^2*c^2-2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a-b)/(a^4-2*a^2*b^
2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c+13*
A*a^2*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*c^5+2
*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c+4*B*b
^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a
^2*b*c^2-2*C*a*b^4-13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/
(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/
2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7
*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^
2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B
*b^5-4*B*b^3*c^2-2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c+4*C*a^2*c^3+
5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-A*b^4*c-A*b^2*c
^3-5*B*a^3*b*c+5*B*a*b^3*c+2*B*a*b*c^3-2*C*a^5+4*C*a^3*b^2-C*a^3*c^2-2*C*a*
b^4+C*a*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2
))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2+2/(a^4-2*a^2*b^2-2*
a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*
x)+2*c)/(a^2-b^2-c^2)^(1/2))*a^2*A+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2
+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)
^(1/2))*A*b^2+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-3/(a^4-
2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a
-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B-3/(a^4-2*a^2*b^2-2*a^2*c^2+b
^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(
a^2-b^2-c^2)^(1/2))*a*c*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.721, size = 8820, normalized size = 37.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="frica
s")
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7
 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B
*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*
C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*C*b*c^6 - (3*A*a*b - 2*B*b^2)*c^5 - (C*a^2*
b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 - (C*a
^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*
A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 - 3
*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 - 3*(C
*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^
4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a
*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3 -
 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + (2*A*a
^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*
c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 - (2*A*a^3*b^2 - 3*B*
a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - (2*A*a^2*b
- 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*s
in(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c
^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos
(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(
2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a
*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*c
os(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^
2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(C*a*c^6 + A*c^7 -
 (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a
^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a
*b^4)*c^2 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 -
A*b^6)*c)*cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 -
 B*a*b^6 - A*b^7 - C*a*b*c^5 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^
```

4 - (C*a^3*b + 2*C*a*b^3)*c^3 - (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B
*a*b^4 + 3*A*b^5)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 -
3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 + 2*C*c^7 - (3*A*a - 2*B*b)*c
^6 - (C*a^2 - 2*C*b^2)*c^5 + (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4
- (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2 + (C*a^4*b^2
+ C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6
- a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 -
3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^
6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^
2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 -
3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^
3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 +
2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a
*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6
*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*
b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5
- (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a
^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5
- C*b^7 - 3*C*b*c^6 + B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (4*C*a^2*b
- 7*C*b^3)*c^4 + (3*B*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^
3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*C*b*c^6 - (3*A*a*b - 2*B
*b^2)*c^5 - (C*a^2*b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 +
4*B*b^4)*c^3 - (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b
^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3*
b^3 + A*a^2*b^4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b +
2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2
- 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c
+ A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2
+ 2*(2*A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 +
A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2
*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 -
(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*
b*c^5 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 +
A*b^5)*c)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*s
in(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a
^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2
*b^4 - 3*A*a*b^5 + B*b^6)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - B*a*b - 3*A*b^2
)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a
*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - 4*
A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)*cos(x) + (2*B*a^5
*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 - A*b^7 - C*a*b*c^5
- A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (C*a^3*b + 2*C*a*b^3)*c^3
- (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (2*C*
a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 - 3*A*a^3*b^4 + B*a^2*b^5 + 3*A
*a*b^6 - 2*B*b^7 + 2*C*c^7 - (3*A*a - 2*B*b)*c^6 - (C*a^2 - 2*C*b^2)*c^5 +

```
(3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B*
a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos
(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2
*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^
6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a
^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 +
 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)
*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*
a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5
*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7
+ 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^
6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b -
4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2
*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(
x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34866, size = 2033, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac"
)
```

```
[Out] -(2*A*a^2 - 3*B*a*b + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 -
c^2)))/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^
2 - c^2)) + (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(
```

1/2*x)^3 + 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b ^3*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b ^4*tan(1/2*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*tan (1/2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*a*b ^3*c*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7 *A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 - A*a*b^2*c^2*tan(1/ 2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^ 2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^ 4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b* tan(1/2*x)^2 + 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a* b^4*tan(1/2*x)^2 + 2*C*b^5*tan(1/2*x)^2 + 4*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4* c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x)^2 + 13* A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 6*A*a*b^3*c*tan(1/ 2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/ 2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2*x)^2 - 13*C*a*b^ 2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2*x)^2 - 4*B*a^2*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*tan(1/2*x)^ 2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*b*c^4*tan(1/2*x)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 4* A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + 5*A*a^3*b^2*tan(1/2*x) + B*a^3* b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 5*A*a*b^4* tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + A*b^5*tan(1/2*x) + 2*B*b^5*tan(1/2*x) - 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x) + 5*C*a^2*b^2*c*tan(1/2*x) - 5*C*a*b^3*c*tan(1/2*x) + 11*A*a^3*c^2*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 3*A*a^2*b*c^2*tan(1/2*x) - 8*B*a^2*b*c^2*tan(1/2*x) - 7*A*a*b^2*c^2*tan(1 /2*x) + 8*B*a*b^2*c^2*tan(1/2*x) - A*b^3*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1 /2*x) - 4*C*a^2*c^3*tan(1/2*x) + 4*C*a*b*c^3*tan(1/2*x) - 2*A*a*c^4*tan(1/2 *x) + 2*B*a*c^4*tan(1/2*x) - 2*A*b*c^4*tan(1/2*x) + 2*B*b*c^4*tan(1/2*x) - 2*C*a^5 + 4*C*a^3*b^2 - 2*C*a*b^4 + 4*A*a^4*c - 5*B*a^3*b*c - 3*A*a^2*b^2*c + 5*B*a*b^3*c - A*b^4*c - C*a^3*c^2 + C*a*b^2*c^2 - A*a^2*c^3 + 2*B*a*b*c^ 3 - A*b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c ^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)^2)

3.553 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=105

$$\frac{i\left(a^2(-(B-iC))+2aAb-b^2(B+iC)\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-b(B+iC))}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

```
[Out] ((2*a*A - b*(B + I*C))*x)/(2*a^2) + ((I/2)*(2*a*A*b - a^2*(B - I*C) - b^2*(
B + I*C))*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + ((I*B - C)*(Cos[x] - I*
Sin[x]))/(2*a)
```

Rubi [A] time = 0.0736377, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3130}

$$\frac{i\left(a^2(-(B-iC))+2aAb-b^2(B+iC)\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-b(B+iC))}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*(B + I*C))*x)/(2*a^2) + ((I/2)*(2*a*A*b - a^2*(B - I*C) - b^2*(
B + I*C))*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + ((I*B - C)*(Cos[x] - I*
Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-b(B+iC))x}{2a^2}+\frac{i\left(2aAb-a^2(B-iC)-b^2(B+iC)\right)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}\cdot$$

Mathematica [A] time = 0.412856, size = 165, normalized size = 1.57

$$\frac{x\left(a^2(B-iC)+2aAb-b^2(B+iC)\right)+\left(a^2(-C-iB)+2iaAb+b^2(C-iB)\right)\log\left(a^2+2ab\cos(x)+b^2\right)+2\left(a^2(B-iC)-\right.}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A*b + a^2*(B - I*C) - b^2*(B + I*C))*x + 2*(-2*a*A*b + a^2*(B - I*C)
+ b^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)] + (2*I)*a*b*(B + I*C)*C
os[x] + ((2*I)*a*A*b + a^2*((-I)*B - C) + b^2*((-I)*B + C))*Log[a^2 + b^2 +
 2*a*b*Cos[x]] + 2*a*b*(B + I*C)*Sin[x])/(4*a^2*b)
```

Maple [B] time = 0.08, size = 257, normalized size = 2.5

$$-\frac{C}{2\,b}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)+\frac{bC}{2\,a^2}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)+\frac{iA}{a}\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\,t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] -1/2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+1/2/a^2*b*ln(I*a+I*b+a*tan(1
/2*x)-b*tan(1/2*x))*C+I/a*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*A-1/2*I/b*l
n(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B-1/2*I/a^2*b*ln(I*a+I*b+a*tan(1/2*x)-
b*tan(1/2*x))*B+1/2*C/b*ln(tan(1/2*x)+I)+1/2*I*B/b*ln(tan(1/2*x)+I)+I*C/a/(
tan(1/2*x)-I)+B/a/(tan(1/2*x)-I)-I/a*ln(tan(1/2*x)-I)*A+1/2*I/a^2*ln(tan(1/
2*x)-I)*b*B-1/2/a^2*ln(tan(1/2*x)-I)*b*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.14681, size = 209, normalized size = 1.99

$$\frac{\left((iB - C)ab + \left(2\,Aab - (B + i\,C)b^2\right)xe^{(i\,x)} + \left((-i\,B - C)a^2 + 2i\,Aab + (-i\,B + C)b^2\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*B - C)*a*b + (2*A*a*b - (B + I*C)*b^2)*x*e^(I*x) + ((-I*B - C)*a^2
+ 2*I*A*a*b + (-I*B + C)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2
*b)
```

Sympy [A] time = 2.33136, size = 87, normalized size = 0.83

$$\left(\frac{iA}{a} - \frac{iB}{2b} - \frac{iBb}{2a^2} - \frac{C}{2b} + \frac{Cb}{2a^2}\right)\log\left(\frac{a}{b} + e^{ix}\right) + \frac{2Aax + iBae^{-ix} - Bbx - Cae^{-ix} - iCbx}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] (I*A/a - I*B/(2*b) - I*B*b/(2*a**2) - C/(2*b) + C*b/(2*a**2))*log(a/b + exp
(I*x)) + (2*A*a*x + I*B*a*exp(-I*x) - B*b*x - C*a*exp(-I*x) - I*C*b*x)/(2*a
**2)
```

Giac [B] time = 1.12666, size = 278, normalized size = 2.65

$$-\frac{2\left(Ba^3 - i\,Ca^3 - 2\,Aa^2b - Ba^2b + i\,Ca^2b + 2\,Aab^2 + Bab^2 + i\,Cab^2 - Bb^3 - i\,Cb^3\right)\log\left(-a\tan\left(\frac{1}{2}\,x\right) + b\tan\left(\frac{1}{2}\,x\right) - i\,a -\right.}{-4i\,a^3b + 4i\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac"
)
```

```
[Out] -2*(B*a^3 - I*C*a^3 - 2*A*a^2*b - B*a^2*b + I*C*a^2*b + 2*A*a*b^2 + B*a*b^2
 + I*C*a*b^2 - B*b^3 - I*C*b^3)*log(-a*tan(1/2*x) + b*tan(1/2*x) - I*a - I*
b)/(-4*I*a^3*b + 4*I*a^2*b^2) - 1/2*(-I*B - C)*log(tan(1/2*x) + I)/b - 1/2*
(2*I*A*a - I*B*b + C*b)*log(tan(1/2*x) - I)/a^2 - 1/2*(-2*I*A*a*tan(1/2*x)
+ I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*B*a - 2*I*C*a + B*b + I*C*b
)/(a^2*(tan(1/2*x) - I))
```

3.554 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=103

$$-\frac{i\left(a^2(-(B+iC))+2aAb-b^2(B-iC)\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB+ibC)}{2a^2}-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

```
[Out] ((2*a*A - b*B + I*b*C)*x)/(2*a^2) - ((I/2)*(2*a*A*b - b^2*(B - I*C) - a^2*(
B + I*C))*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) - ((I*B + C)*(Cos[x] + I*
Sin[x]))/(2*a)
```

Rubi [A] time = 0.0732889, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3130}

$$-\frac{i\left(a^2(-(B+iC))+2aAb-b^2(B-iC)\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB+ibC)}{2a^2}-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B + I*b*C)*x)/(2*a^2) - ((I/2)*(2*a*A*b - b^2*(B - I*C) - a^2*(
B + I*C))*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) - ((I*B + C)*(Cos[x] + I*
Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA-bB+ibC)x}{2a^2}-\frac{i\left(2aAb-b^2(B-iC)-a^2(B+iC)\right)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-$$

Mathematica [A] time = 0.421403, size = 167, normalized size = 1.62

$$\frac{\frac{\left(ia^2(B+iC)-2iaAb+b^2(C+iB)\right)\log\left(a^2+2ab\cos(x)+b^2\right)}{b}+\frac{2\left(a^2(B+iC)-2aAb+b^2(B-iC)\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)}{b}+x\left(\frac{a^2(B+iC)}{b}+2aA-b(B-iC)\right)+2}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*(B - I*C) + (a^2*(B + I*C))/b)*x + (2*(-2*a*A*b + b^2*(B - I*C)
 + a^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)])/b - (2*I)*a*(B - I*C)
*Cos[x] + (((-2*I)*a*A*b + I*a^2*(B + I*C) + b^2*(I*B + C))*Log[a^2 + b^2 +
 2*a*b*Cos[x]])/b + 2*a*(B - I*C)*Sin[x])/(4*a^2)
```

Maple [B] time = 0.082, size = 475, normalized size = 4.6

$$\frac{\frac{i}{2}B}{-a+b}\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)+\frac{B}{a}\left(\tan\left(\frac{x}{2}\right)+i\right)^{-1}+\frac{\frac{i}{2}b^2B}{a^2(-a+b)}\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] 1/2*I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+B/a/(tan(1/2*x)+I)+1/2
*I/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+I/(-a+b)*ln(I*a+I
*b-a*tan(1/2*x)+b*tan(1/2*x))*A-1/2/a^2*ln(tan(1/2*x)+I)*b*C+1/2*a/b/(-a+b)
*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)
+b*tan(1/2*x))*C-1/2/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C+1/2
/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2*I*a/b/(-a+b)*ln
(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B-I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+
b*tan(1/2*x))*A+I/a*ln(tan(1/2*x)+I)*A-1/2*I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/
2*x)+b*tan(1/2*x))*B-I*C/a/(tan(1/2*x)+I)-1/2*I*B/b*ln(tan(1/2*x)-I)+1/2*C/
b*ln(tan(1/2*x)-I)-1/2*I/a^2*ln(tan(1/2*x)+I)*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.06735, size = 169, normalized size = 1.64

$$\frac{(B+iC)a^2x+(-iB-C)abe^{(ix)}+\left((iB-C)a^2-2iAab+(iB+C)b^2\right)\log\left(\frac{ae^{(ix)}+b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="frica
s")
```

```
[Out] 1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 - 2*I*A*a*b
+ (I*B + C)*b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

Sympy [A] time = 1.38012, size = 80, normalized size = 0.78

$$\left(-\frac{iA}{a}+\frac{iB}{2b}+\frac{iBb}{2a^2}-\frac{C}{2b}+\frac{Cb}{2a^2}\right)\log\left(e^{ix}+\frac{b}{a}\right)+\frac{Bax-iBbe^{ix}+iCax-Cbe^{ix}}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] (-I*A/a + I*B/(2*b) + I*B*b/(2*a**2) - C/(2*b) + C*b/(2*a**2))*log(exp(I*x)
 + b/a) + (B*a*x - I*B*b*exp(I*x) + I*C*a*x - C*b*exp(I*x))/(2*a*b)
```

Giac [B] time = 1.14179, size = 278, normalized size = 2.7

$$-\frac{2\left(Ba^3+iCa^3-2Aa^2b-Ba^2b-iCa^2b+2Aab^2+Bab^2-iCab^2-Bb^3+iCb^3\right)\log\left(-a\tan\left(\frac{1}{2}x\right)+b\tan\left(\frac{1}{2}x\right)+ia-\right.}{4ia^3b-4ia^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac"
)
```

```
[Out] -2*(B*a^3 + I*C*a^3 - 2*A*a^2*b - B*a^2*b - I*C*a^2*b + 2*A*a*b^2 + B*a*b^2
 - I*C*a*b^2 - B*b^3 + I*C*b^3)*log(-a*tan(1/2*x) + b*tan(1/2*x) + I*a + I*
b)/(4*I*a^3*b - 4*I*a^2*b^2) - 1/2*(I*B - C)*log(tan(1/2*x) - I)/b - 1/2*(-
2*I*A*a + I*B*b + C*b)*log(tan(1/2*x) + I)/a^2 - 1/2*(2*I*A*a*tan(1/2*x) -
I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*B*a + 2*I*C*a + B*b - I*C*b)/
(a^2*(tan(1/2*x) + I))
```

3.555 $\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=24

$$-\frac{c\cos(x)-b\sin(x)}{a+b\cos(x)+c\sin(x)}$$

```
[Out] -((c*Cos[x] - b*Sin[x])/(a + b*Cos[x] + c*Sin[x]))
```

Rubi [B] time = 0.0675144, antiderivative size = 68, normalized size of antiderivative = 2.83, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3150}

$$-\frac{c\cos(x)\left(a^2-b^2-c^2\right)-b\sin(x)\left(a^2-b^2-c^2\right)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((c*(a^2 - b^2 - c^2)*Cos[x] - b*(a^2 - b^2 - c^2)*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x])))
```

Rule 3150

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

Rubi steps

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx=-\frac{c\left(a^2-b^2-c^2\right)\cos(x)-b\left(a^2-b^2-c^2\right)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.0943512, size = 32, normalized size = 1.33

$$\frac{ac+b^2\sin(x)+c^2\sin(x)}{b(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2
,x]
```

```
[Out] (a*c + b^2*Sin[x] + c^2*Sin[x])/(b*(a + b*Cos[x] + c*Sin[x]))
```

Maple [B] time = 0.102, size = 70, normalized size = 2.9

$$-2\,\frac{1}{a\left(\tan\left(x/2\right)\right)^{2}-b\left(\tan\left(x/2\right)\right)^{2}+2\,c\tan\left(x/2\right)+a+b}\left(-\frac{\left(ab-b^{2}-c^{2}\right)\tan\left(x/2\right)}{a-b}+\frac{ac}{a-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] -2*(-(a*b-b^2-c^2)/(a-b)*tan(1/2*x)+a*c/(a-b))/(a*tan(1/2*x)^2-b*tan(1/2*x)
^2+2*c*tan(1/2*x)+a+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94278, size = 68, normalized size = 2.83

$$-\frac{c\cos(x) - b\sin(x)}{b\cos(x) + c\sin(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="fricas")
```

```
[Out] -(c*cos(x) - b*sin(x))/(b*cos(x) + c*sin(x) + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2+c**2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18427, size = 92, normalized size = 3.83

$$\frac{2\left(ab\tan\left(\frac{1}{2}x\right) - b^2\tan\left(\frac{1}{2}x\right) - c^2\tan\left(\frac{1}{2}x\right) - ac\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2 - b\tan\left(\frac{1}{2}x\right)^2 + 2c\tan\left(\frac{1}{2}x\right) + a + b\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="giac")
```

```
[Out] 2*(a*b*tan(1/2*x) - b^2*tan(1/2*x) - c^2*tan(1/2*x) - a*c)/((a*tan(1/2*x)^2
 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)*(a - b))
```

3.556 $\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=390

$$-\frac{2\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{105\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(161a^2 \cdots\right.}{\cdots}$$

```
[Out] (2*(161*a^2*d + 63*(b^2 + c^2)*d + 15*a^3*e + 145*a*(b^2 + c^2)*e)*Elliptic
E[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a +
 b*Cos[x] + c*Sin[x]])/(105*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 +
c^2])]) - (2*(a^2 - b^2 - c^2)*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Ellip
ticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[
(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(105*Sqrt[a + b*Cos[x] +
c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(5/2)*(c*e*Cos[x] - b*e*Sin[x]))/
7 - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*(7*d + 5*a*e)*Cos[x] - b*(7*d + 5
*a*e)*Sin[x]))/35 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(56*a*d + 15*a^2*e
+ 25*(b^2 + c^2)*e)*Cos[x] - b*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Sin[x
]))/105
```

Rubi [A] time = 0.887684, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{105\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(161a^2d+15a^3e \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] (2*(161*a^2*d + 63*(b^2 + c^2)*d + 15*a^3*e + 145*a*(b^2 + c^2)*e)*Elliptic
E[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a +
 b*Cos[x] + c*Sin[x]])/(105*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 +
c^2])]) - (2*(a^2 - b^2 - c^2)*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Ellip
ticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[
(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(105*Sqrt[a + b*Cos[x] +
c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(5/2)*(c*e*Cos[x] - b*e*Sin[x]))/
```

```
7 - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*(7*d + 5*a*e)*Cos[x] - b*(7*d + 5
*a*e)*Sin[x]))/35 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(56*a*d + 15*a^2*e
+ 25*(b^2 + c^2)*e)*Cos[x] - b*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Sin[x
]))/105
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
```

```
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x)) + \frac{2\int(}{}$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x)) - \frac{2}{35}(a$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x)) - \frac{2}{35}(a$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x)) - \frac{2}{35}(a$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x)) - \frac{2}{35}(a$$

$$= \frac{2\left(161a^2d+63\left(b^2+c^2\right)d+15a^3e+145a\left(b^2+c^2\right)e\right)E\left(\frac{1}{2}\right.}{105\sqrt{\frac{a+b\cos(x}{a+\sqrt{b}}}}$$

Mathematica [C] time = 6.90226, size = 7823, normalized size = 20.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 16.03, size = 3502, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)*((b^6*e+3*b^4*c^2*e+3*b^
2*c^4*e+c^6*e)*(-2/7/(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))^2*(cos(x-arctan(-b
,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+12/35/(b^2+c^2)*a*sin
(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)+a))^(1/2)-2/3*(5/7+24/35/(b^2+c^2)*a^2)/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,
c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2*(-4/35/(b^2+c^2)*a^2
+5/21)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2
+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^
(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a
))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^
(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(-48*a^3-4
4*a*b^2-44*a*c^2)/(105*(b^2+c^2)^(1/2)*b^2+105*(b^2+c^2)^(1/2)*c^2)*(1/(b^2
+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1
/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^
(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/
(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/
(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+
EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(
1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+(3*(b^2+c^2)^(1/2)*
a*b^4*e+6*(b^2+c^2)^(1/2)*a*b^2*c^2*e+3*(b^2+c^2)^(1/2)*a*c^4*e+(b^2+c^2)^(
1/2)*b^4*d+2*(b^2+c^2)^(1/2)*b^2*c^2*d+(b^2+c^2)^(1/2)*c^4*d)*(-2/5/(b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-
arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(x-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/
2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/
2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1
+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-ar
ctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(
b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+
```

c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+
c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(
1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(
cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(
b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+E
llipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+(3*a^2*b^4*e+6*a^2*b
^2*c^2*e+3*a^2*c^4*e+3*a*b^4*d+6*a*b^2*c^2*d+3*a*c^4*d)*(-2/3/(b^2+c^2)^(1/
2)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/
3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^
2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)
^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)
))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1
/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)
))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/
2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+
(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c
^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1
/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))
^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(
-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/
2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^
2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+2*((b^2
+c^2)^(1/2)*a^3*b^2*e+(b^2+c^2)^(1/2)*a^3*c^2*e+(b^2+c^2)^(3/2)*a^2*d+2*a^2
*b^2*d*(b^2+c^2)^(1/2)+2*a^2*c^2*d*(b^2+c^2)^(1/2))*(1/(b^2+c^2)^(1/2)*a-1)
*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-s
in(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-
arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b
,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*
a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^
2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^
2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*a^3*b^2*d*(1/(b^2+c^2)^(1/2)*a-1)*
((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-si
n(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-a
rctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arc
tan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2
/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)
/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2
))+2*a^3*c^2*d*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c
))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)
/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b
^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a

```
*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((
-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^
2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-
arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (be\cos(x) + ce\sin(x) + d)(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(\left(\left(b^3 - 3\,bc^2\right)e\cos(x)^3 + 2\,ac^2e + \left(\left(b^2 - c^2\right)d + 2\left(ab^2 - ac^2\right)e\right)\cos(x)^2 + \left(a^2 + c^2\right)d + \left(2\,abd + \left(a^2b + 3\,bc^2\right)e\right)\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] integral(((b^3 - 3*b*c^2)*e*cos(x)^3 + 2*a*c^2*e + ((b^2 - c^2)*d + 2*(a*b^
2 - a*c^2)*e)*cos(x)^2 + (a^2 + c^2)*d + (2*a*b*d + (a^2*b + 3*b*c^2)*e)*co
s(x) + ((3*b^2*c - c^3)*e*cos(x)^2 + 2*a*c*d + (a^2*c + c^3)*e + 2*(2*a*b*c
*e + b*c*d)*cos(x))*sin(x))*sqrt(b*cos(x) + c*sin(x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (be\cos(x) + ce\sin(x) + d)(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

3.557 $\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=294

$$-\frac{2\left(a^2-b^2-c^2\right)(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\cdots}{\cdots}$$

```
[Out] (2*(20*a*d + 3*a^2*e + 9*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*
Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(15*
Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^
2)*(5*d + 3*a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + S
qrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(15
*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*e*C
os[x] - b*e*Sin[x]))/5 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(5*d + 3*a*e)*
Cos[x] - b*(5*d + 3*a*e)*Sin[x]))/15
```

Rubi [A] time = 0.555243, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\left(a^2-b^2-c^2\right)(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\sqrt{a+b\cos\cdots}}{15\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```



```
[Out] (2*(20*a*d + 3*a^2*e + 9*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*
Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(15*
Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^
2)*(5*d + 3*a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + S
qrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(15
*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*e*C
os[x] - b*e*Sin[x]))/5 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(5*d + 3*a*e)*
Cos[x] - b*(5*d + 3*a*e)*Sin[x]))/15
```


Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)) + \frac{2\int \sqrt{}}{}$$

$$= -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)) - \frac{2}{15}\sqrt{a}$$

$$= -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)) - \frac{2}{15}\sqrt{a}$$

$$= -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)) - \frac{2}{15}\sqrt{a}$$

$$= \frac{2\left(20ad+3a^2e+9\left(b^2+c^2\right)e\right)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c}}{a+\sqrt{b^2+}}\right.}{15\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.55543, size = 5218, normalized size = 17.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 10.746, size = 2238, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*((b^4*e+2*b^2*c^2*
e+c^4*e)*(-2/5/(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(x-arcta
n(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c^2)^(1
/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
)^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(
1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/5+8/15/(
b^2+c^2)*a^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/
(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^
2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-
b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(
x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2
+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))
)+(2*(b^2+c^2)^(1/2)*a*b^2*e+2*(b^2+c^2)^(1/2)*a*c^2*e+(b^2+c^2)^(1/2)*b^2*
d+(b^2+c^2)^(1/2)*c^2*d)*(-2/3/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*((b^2
+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-a
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan
(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2
*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2
)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((
-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(
x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arc
tan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c)
)^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1
)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))
^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^
(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+2*(a^2*b^2*e+a^2*c^2*e+2*a*b^2*d+2*a*c
^2*d)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
)^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan
(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1
```

```
/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c
^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*a^2*d
*(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b
,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/
2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+
(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))
-a*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(
((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(
b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(
x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (be\cos(x) + ce\sin(x) + d)(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(b^2 - c^2\right)e\cos(x)^2 + c^2e + ad + (abe + bd)\cos(x) + (2\,bce\cos(x) + ace + cd)\sin(x)\right)\sqrt{b\cos(x) + c\sin(x) + a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] integral(((b^2 - c^2)*e*cos(x)^2 + c^2*e + a*d + (a*b*e + b*d)*cos(x) + (2*
b*c*e*cos(x) + a*c*e + c*d)*sin(x))*sqrt(b*cos(x) + c*sin(x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (be\cos(x) + ce\sin(x) + d)(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

3.558 $\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=229

$$-\frac{2e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(\right.\right.}{3\sqrt{\frac{a+b\cos(x)+c\sin}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] (2*(3*d + a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqr
t[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*Sqrt[(a + b*Cos[x] + c*Sin
[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)*e*EllipticF[(x - ArcTan
[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] +
c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*Sq
rt[a + b*Cos[x] + c*Sin[x]]*(c*e*Cos[x] - b*e*Sin[x]))/3
```

Rubi [A] time = 0.332188, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-}\right.\right.}{3\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```


```
[Out] (2*(3*d + a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqr
t[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*Sqrt[(a + b*Cos[x] + c*Sin
[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)*e*EllipticF[(x - ArcTan
[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] +
c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*Sq
rt[a + b*Cos[x] + c*Sin[x]]*(c*e*Cos[x] - b*e*Sin[x]))/3
```


Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
```

```
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x)) + \frac{2\int \frac{\frac{1}{2}a(3ad}{}}{}$$

$$= -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x)) - \frac{1}{3}\left(\left(a^2-b\right.\right.$$

$$= -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x)) + \frac{\left((3d+ae\right.}{}$$

$$= \frac{2(3d+ae)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\,\mathrm{s}}}{3\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.34291, size = 3006, normalized size = 13.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*b*(3*d + a*e))/(3*c) - (2*c*e*Cos[x])/3 +
 (2*b*e*Sin[x])/3) + (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^
2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]
*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^
2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b
^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[
(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*S
qrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]
)/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(Sqrt[1 + b^2/c^2]*c) + (2*b^2*e*AppellF
1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqr
t[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*
c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c
))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)
/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sq
rt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/
```

(3*Sqrt[1 + b^2/c^2]*c) + (2*c*e*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1
 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2
/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b
^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sq
rt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*
Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c
]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[
b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]))/(3*Sqrt[1 + b^2/c^2]) + (b^2*d*(-((
c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan
[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt
[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1
+ c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^
2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(
b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sq
rt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])
/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x -
 ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]
))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/c + c*d*(-((c*Appel
lF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])
/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^
2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b
^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2
)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c
^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a +
b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan
[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt
[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]) + (a*b^2*e*(-((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2
]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2
] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(
b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(3*c) + (a*c*e*(-((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2
]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2
] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(
b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqr

```
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/3
```

Maple [B] time = 8.033, size = 1460, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(((b^2+c^2)^(1/2)*
b^2*e+(b^2+c^2)^(1/2)*c^2*e)*(-2/3/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*(
(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin
(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-ar
ctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c
))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^
(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2
))/(a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1
)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-
sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x
-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-
b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)
*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c
^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+2*(a*b^2*e+a*c^2*e+b^2*d+c^2*d)*(1
/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^
2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/
2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*
((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-
a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1
/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*a*d*(b^2+c^2)
^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2
+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^
(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^
```

```
2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(
1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-
b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\cos(x) + c\sin(x)}\,(be\cos(x) + ce\sin(x) + d)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] Integral(sqrt(a + b*cos(x) + c*sin(x))*(b*e*cos(x) + c*e*sin(x) + d), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

3.559 $\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$

Optimal. Leaf size=180

$$\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] (2*e*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^
2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqr
t[b^2 + c^2])] + (2*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 +
 c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2
+ c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]]
```

Rubi [A] time = 0.186792, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3149, 3119, 2653, 3127, 2661}

$$\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

```
[Out] (2*e*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^
2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqr
t[b^2 + c^2])] + (2*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 +
 c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2
+ c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx = e\int \sqrt{a+b\cos(x)+c\sin(x)}\,dx + (d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$$

$$= \frac{\left(e\sqrt{a+b\cos(x)+c\sin(x)}\right)\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}\,dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{\left((d-ae)\sqrt{\frac{a+b\,\ldots}{}}\right.}{}$$

$$= \frac{2eE\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{2(d-ae)F\left(\frac{1}{2}\left(x-\tan\ldots\right.\right.}{\sqrt{a+\ldots}}$$

Mathematica [C] time = 6.26099, size = 1319, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

```
[Out] (2*b*e*Sqrt[a + b*Cos[x] + c*Sin[x]])/c + (2*d*AppellF1[1/2, 1/2, 1/2, 3/2,
 -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a
/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]
])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/
c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[
b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[
x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*S
in[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(Sqrt[1 + b^2/c^2]*c)
 + (b^2*e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*
Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))),
 -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1
- a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sq
rt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])
/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - Ar
cTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x -
 ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^
2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqr
t[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/c + c
*e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x -
 ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a +
```

```
 b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*
Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b
*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/
b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan
[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*
Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c
^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]])
```

Maple [B] time = 6.956, size = 777, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2*e+c^2*e)*(
1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c
^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1
/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^
(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)
*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))
-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(
1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*d*(b^2+c^2)^
(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2
)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1
/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b
,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{be\cos(x) + ce\sin(x) + d}{\sqrt{b\cos(x) + c\sin(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{be\cos(x) + ce\sin(x) + d}{\sqrt{b\cos(x) + c\sin(x) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{be\cos(x) + ce\sin(x) + d}{\sqrt{a + b\cos(x) + c\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(1/2),x)
```

```
[Out] Integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(a + b*cos(x) + c*sin(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{be\cos(x) + ce\sin(x) + d}{\sqrt{b\cos(x) + c\sin(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

3.560 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=250

$$\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2e\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\right.}{\sqrt{a+b\cos(x)+c\sin(x)}}$$

```
[Out] (2*(d - a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[
b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/((a^2 - b^2 - c^2)*Sqrt[(a + b*
Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) + (2*e*EllipticF[(x - ArcTan[b,
c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Si
n[x])/(a + Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]] + (2*(c*(d - a*
e)*Cos[x] - b*(d - a*e)*Sin[x]))/((a^2 - b^2 - c^2)*Sqrt[a + b*Cos[x] + c*S
in[x]])
```

Rubi [A] time = 0.323051, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2e\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right.}{\sqrt{a+b\cos(x)+c\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

```
[Out] (2*(d - a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[
b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/((a^2 - b^2 - c^2)*Sqrt[(a + b*
Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) + (2*e*EllipticF[(x - ArcTan[b,
c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Si
n[x])/(a + Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]] + (2*(c*(d - a*
e)*Cos[x] - b*(d - a*e)*Sin[x]))/((a^2 - b^2 - c^2)*Sqrt[a + b*Cos[x] + c*S
in[x]])
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx = \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}} - \frac{2\int \frac{\frac{1}{2}\left(-ad+\left(b^2+c^2\right)e\right)-\frac{1}{2}b(d-ae)\cos(x)-\frac{1}{2}c(d-ae)\sin(}{\sqrt{a+b\cos(x)+c\sin(x)}}}{a^2-b^2-c^2}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}} + e\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx + \frac{(d-ae}{}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}} + \frac{\left((d-ae)\sqrt{a+b\cos(x)+c\sin(x)}\right)\int\sqrt{\frac{}{a+}}}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\,c}{a}}}$$

$$= \frac{2(d-ae)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{2eF\left(\frac{1}{2}\left(x-\tan\right.\right.}{}$$

Mathematica [C] time = 6.48662, size = 3176, normalized size = 12.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

```
[Out] Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*(b^2 + c^2)*(-d + a*e))/(b*c*(-a^2 + b^2
+ c^2)) - (2*(-(a*c*d) + a^2*c*e - b^2*d*Sin[x] - c^2*d*Sin[x] + a*b^2*e*Si
n[x] + a*c^2*e*Sin[x]))/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))) -
 (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + Arc
Tan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqr
t[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 +
 b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*S
qrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*S
qrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c
^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2
```

+ c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*b^2*e*AppellF1
[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt
[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c
Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2](-1 - a/(Sqrt[1 + b^2/c^2]*c))*c)
)]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/
c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(
b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqr
t[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(
Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*c*e*AppellF1[1/2, 1/2, 1/2, 3/
2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/
c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[
b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTa
n[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Si
n[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]
*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*
(-a^2 + b^2 + c^2)) - (b^2*d*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b
*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqr
t[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqr
t[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Co
s[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c
^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2
+ c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b
*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - Ar
cTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)) - (c*d*(-((c*AppellF1[-1/2, -1/2, -1/2
, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2
]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTa
n[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2
+ c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a +
b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2]
+ b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b
^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2)
- (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/
b^2]*Cos[x - ArcTan[c/b]]]))/(-a^2 + b^2 + c^2) + (a*b^2*e*(-((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2
]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2
] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(
b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]

```
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)) + (a*c*e
*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b
*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sq
rt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqr
t[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*S
qrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]
]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c
/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]))) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Co
s[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2
/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(-a^2 + b^2 +
c^2)
```

Maple [B] time = 9.936, size = 2596, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2+c^2)^(1/2)
*e*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b
^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2
)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2
)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(
1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(
1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2)
)/(a+(b^2+c^2)^(1/2)))^(1/2))-(b^2+c^2)*cos(x-arctan(-b,c))^2*(a*e-d)/(a^2-
b^2-c^2)/(-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2)^
(1/2)-a*(b^2+c^2)^(1/2)*(a*e-d)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-a
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan
(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*si
n(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2)^(1/2)*EllipticF(((-(b^2+c^2)^(1
/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))
/(a+(b^2+c^2)^(1/2)))^(1/2))-(a*e-d)*(b^2+c^2)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(
1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(
1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*(
(1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^
```

```
2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2)^(1/2)*((-1/(b^2+
c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^
2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+Ellip
ticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),
((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+(1/2*a*e-1/2*d)*(1/(b^2+c
^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1
/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-
(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2)^(1/2)*(b^2+
c^2)^(1/2)/a*EllipticPi(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+
c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2
+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(x-a
rctan(-b,c))^2/(a^2-b^2-c^2)*(a*e-d)/(-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)-a)*cos(x-arctan(-b,c))^2*(b^2+c^2))^(1/2)-a*(b^2+c^2)*(a*e-d)/(a^2-b^2-c^
2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b
^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2
)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2
)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2*
(b^2+c^2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b
^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-2*(-
1/2*(b^2+c^2)^(3/2)*(a*e-d)/(a^2-b^2-c^2)+1/2*(b^2+c^2)^(1/2)*(2*b^2+2*c^2)
/(a^2-b^2-c^2)*(a*e-d))*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-ar
ctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c
^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/
2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*c
os(x-arctan(-b,c))^2*(b^2+c^2))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE((
(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b
^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin
(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^
2+c^2)^(1/2)))^(1/2)))-1/2*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)*(1/(b^2+c^2)^(1/2)
*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)
*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+s
in(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2*(b^2+c^2))^(1/2)/a*El
lipticPi(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a
+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b,c))+c^
2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}}{2\,ab\cos(x) + \left(b^2 - c^2\right)\cos(x)^2 + a^2 + c^2 + 2\,(bc\cos(x) + ac)\sin(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a)/(2*a*b
*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

3.561 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

Optimal. Leaf size=378

$$-\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+}}{3\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a}{}}}$$

```
[Out] (2*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqr
t[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*(a^2
 - b^2 - c^2)^2*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2
*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^
2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*(a^2 -
 b^2 - c^2)*Sqrt[a + b*Cos[x] + c*Sin[x]]) + (2*(c*(d - a*e)*Cos[x] - b*(d
- a*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^(3/2)) + (2*
(c*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*Cos[x] - b*(4*a*d - a^2*e - 3*(b^2 + c
^2)*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)^2*Sqrt[a + b*Cos[x] + c*Sin[x]])
```

Rubi [A] time = 0.561126, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3156, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}}{3\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(x)+c}{a+\sqrt{b^2+}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
```


```
[Out] (2*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqr
t[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*(a^2
 - b^2 - c^2)^2*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2
*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^
2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*(a^2 -
 b^2 - c^2)*Sqrt[a + b*Cos[x] + c*Sin[x]]) + (2*(c*(d - a*e)*Cos[x] - b*(d
- a*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^(3/2)) + (2*
(c*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*Cos[x] - b*(4*a*d - a^2*e - 3*(b^2 + c
```

```
^2)*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)^2*Sqrt[a + b*Cos[x] + c*Sin[x]])
```

<u>Rule 3156</u>

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

<u>Rule 3149</u>

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

<u>Rule 3119</u>

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

<u>Rule 2653</u>

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

<u>Rule 3127</u>

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
```

```
0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx = \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(ad-\left(b^2+c^2\right)e\right)+\frac{1}{2}b(d-ae)\cos(x)+\frac{1}{2}c(d-ae}{(a+b\cos(x)+c\sin(x))^{3/2}}}{3\left(a^2-b^2-c^2\right)}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2\left(c\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)\cos(x)\right.}{3\left(a^2-b^2-c^2\right)^2\sqrt{a}}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2\left(c\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)\cos(x)\right.}{3\left(a^2-b^2-c^2\right)^2\sqrt{a}}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2\left(c\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)\cos(x)\right.}{3\left(a^2-b^2-c^2\right)^2\sqrt{a}}$$

$$= \frac{2\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(}}{3\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.84663, size = 5554, normalized size = 14.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 43.325, size = 3164, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(1/4/a/(a^2-b^2-c^
2)*(a*e-d)*(b^2+c^2)^(3/2)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-ar
ctan(-b,c))+a))^(1/2)/(b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))-a*(b
^2+c^2)^(1/2))-1/3/(a^2-b^2-c^2)*(a*e-d)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(x-arctan(-b,c))+a))^(1/2)/(sin(x-arctan(-b,c))+1/(b^2+c^2)^(1/2)*
a)^2-1/3*(b^2+c^2)*cos(x-arctan(-b,c))^2/(a^2-b^2-c^2)^2*(a^2*e+3*b^2*e+3*c
^2*e-4*a*d)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))
^(1/2)+2*(1/24*(a*e-d)*(b^2+c^2)^(1/2)/(a^2-b^2-c^2)-1/6*a*(b^2+c^2)^(1/2)*
(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(
b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-
arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arcta
n(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^
2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/
2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/
(a+(b^2+c^2)^(1/2)))^(1/2))+2*(1/8*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-b^2
-c^2)-1/6*(b^2+c^2)*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+
c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(
1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(
cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(
b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+E
llipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+1/8*(a^3*b^2*e+a^3*c^
2*e+3*a*b^4*e+6*a*b^2*c^2*e+3*a*c^4*e-5*a^2*b^2*d-5*a^2*c^2*d+b^4*d+2*b^2*c
^2*d+c^4*d)/a^2/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-a
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan
(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2
*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticPi(((-(b^2+c^2)^(1/
2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1
/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))
-1/4*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-b^2-c^2)*(cos(x-arctan(-b,c))^2*(
b^2+c^2)*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)/(b^2*sin(x-arctan(-
```

```
b,c))+c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))-1/3/(a^2-b^2-c^2)*(a*e-d)/
(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(x-arc
tan(-b,c))+a))^(1/2)/(sin(x-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2+1/3*(b^2+c
^2)^(1/2)*(-b^2-c^2)*cos(x-arctan(-b,c))^2/(a^2-b^2-c^2)^2*(a^2*e+3*b^2*e+3
*c^2*e-4*a*d)/(cos(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(x-arcta
n(-b,c))+a))^(1/2)+2*(7/24*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/(a^2-b^2-c^2)-1/6*
a*(b^2+c^2)*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/
2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/
2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1
+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-ar
ctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*Elli
pticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)
,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(-1/8*(a*b^2*e+a*c^2*e-
b^2*d-c^2*d)*(b^2+c^2)^(1/2)/a/(a^2-b^2-c^2)+1/6*(b^2+c^2)^(3/2)*(a^2*e+3*b
^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2-1/6*(b^2+c^2)^(1/2)*(2*b^2+2*c^2)/(a^2-
b^2-c^2)^2*(a^2*e+3*b^2*e+3*c^2*e-4*a*d))*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arcta
n(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,
c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*(b^
2+c^2)*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*
a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^
2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^
2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))-1/8*(a^3*b^2*e+a^3*c^2*e+3*a*b^4*e+6
*a*b^2*c^2*e+3*a*c^4*e-5*a^2*b^2*d-5*a^2*c^2*d+b^4*d+2*b^2*c^2*d+c^4*d)/a^2
/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c
))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)
/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b
^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin
(x-arctan(-b,c))+a))^(1/2)*EllipticPi(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1
/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))
/((b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+
c^2)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
```

```
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}}{(b^3 - 3\,bc^2)\cos(x)^3 + a^3 + 3\,ac^2 + 3\,(ab^2 - ac^2)\cos(x)^2 + 3\,(a^2b + bc^2)\cos(x) + (6\,abc\cos(x) + 3\,a^2c + c^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a)/((b^3
 - 3*b*c^2)*cos(x)^3 + a^3 + 3*a*c^2 + 3*(a*b^2 - a*c^2)*cos(x)^2 + 3*(a^2*b
 + b*c^2)*cos(x) + (6*a*b*c*cos(x) + 3*a^2*c + c^3 + (3*b^2*c - c^3)*cos(x)
^2)*sin(x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

3.562 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$

Optimal. Leaf size=84

$$\frac{2(Ac-aC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{ce\sqrt{a^2-c^2}}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

```
[Out] (C*x)/c + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/
(c*Sqrt[a^2 - c^2]*e) + (B*Log[a + c*Sin[d + e*x]])/(c*e)
```

Rubi [A] time = 0.151861, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4376, 2735, 2660, 618, 204, 2668, 31}

$$\frac{2(Ac-aC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{ce\sqrt{a^2-c^2}}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

```
[Out] (C*x)/c + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/
(c*Sqrt[a^2 - c^2]*e) + (B*Log[a + c*Sin[d + e*x]])/(c*e)
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx = B\int \frac{\cos(d+ex)}{a+c\sin(d+ex)}\,dx + \int \frac{A+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$$

$$= \frac{Cx}{c} - \frac{(-Ac+aC)\int \frac{1}{a+c\sin(d+ex)}\,dx}{c} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, c\sin(d+ex)\right)}{ce}$$

$$= \frac{Cx}{c} + \frac{B\log(a+c\sin(d+ex))}{ce} + \frac{(2(Ac-aC))\,\mathrm{Subst}\left(\int \frac{1}{a+2cx+ax^2}\,dx, x, \tan\left(\right.\right.}{ce}$$

$$= \frac{Cx}{c} + \frac{B\log(a+c\sin(d+ex))}{ce} - \frac{(4(Ac-aC))\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-c^2)-x^2}\,dx, x, 2c\right.}{ce}$$

$$= \frac{Cx}{c} + \frac{2(Ac-aC)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{c\sqrt{a^2-c^2}e} + \frac{B\log(a+c\sin(d+ex))}{ce}$$

Mathematica [A] time = 0.262016, size = 80, normalized size = 0.95

$$\frac{\frac{2(Ac-aC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\sqrt{a^2-c^2}} + B\log(a+c\sin(d+ex)) + C(d+ex)}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

```
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^
2]])/Sqrt[a^2 - c^2] + B*Log[a + c*Sin[d + e*x]])/(c*e)
```

Maple [B] time = 0.059, size = 178, normalized size = 2.1

$$\frac{B}{ce}\ln\left(a\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2 + 2c\tan(d/2+1/2\,ex) + a\right) + 2\frac{A}{e\sqrt{a^2-c^2}}\arctan\left(1/2\,\frac{2a\tan(d/2+1/2\,ex)+2c}{\sqrt{a^2-c^2}}\right) - 2\frac{Ca}{ce\sqrt{a^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)
```

```
[Out] 1/e/c*B*ln(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)+2/e/(a^2-c^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*A-2/e/c/(a^2-
c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*C*a-1/e
/c*B*ln(1+tan(1/2*d+1/2*e*x)^2)+2/e/c*C*arctan(tan(1/2*d+1/2*e*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63804, size = 763, normalized size = 9.08

$$\left[\frac{2\left(Ca^2 - Cc^2\right)ex + (Ca - Ac)\sqrt{-a^2 + c^2}\log\left(\frac{\left(2\,a^2-c^2\right)\cos(ex+d)^2-2\,ac\sin(ex+d)-a^2-c^2+2\,(a\cos(ex+d)\sin(ex+d)+c\cos(ex+d))\sqrt{-a^2+c^2}}{c^2\cos(ex+d)^2-2\,ac\sin(ex+d)-a^2-c^2}\right) + \cdots}{2\left(a^2c - c^3\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(C*a^2 - C*c^2)*e*x + (C*a - A*c)*sqrt(-a^2 + c^2)*log(((2*a^2 - c^
2)*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(
e*x + d) + c*cos(e*x + d))*sqrt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*si
n(e*x + d) - a^2 - c^2)) + (B*a^2 - B*c^2)*log(-c^2*cos(e*x + d)^2 + 2*a*c*
sin(e*x + d) + a^2 + c^2))/((a^2*c - c^3)*e), 1/2*(2*(C*a^2 - C*c^2)*e*x +
2*(C*a - A*c)*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)
*cos(e*x + d))) + (B*a^2 - B*c^2)*log(-c^2*cos(e*x + d)^2 + 2*a*c*sin(e*x +
 d) + a^2 + c^2))/((a^2*c - c^3)*e)]
```

Sympy [A] time = 52.2945, size = 1151, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cos(d) + C*sin(d))/sin(d), Eq(a, 0) & Eq(c, 0) & Eq
(e, 0)), (2*A*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) + 2*B*log(tan(d
/2 + e*x/2) - 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) - 2*B*log(ta
n(d/2 + e*x/2) - 1)/(c*e*tan(d/2 + e*x/2) - c*e) - B*log(tan(d/2 + e*x/2)**
2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) + B*log(tan(d/2 + e*x/
2)**2 + 1)/(c*e*tan(d/2 + e*x/2) - c*e) + C*e*x*tan(d/2 + e*x/2)/(c*e*tan(d
/2 + e*x/2) - c*e) - C*e*x/(c*e*tan(d/2 + e*x/2) - c*e) + 2*C*tan(d/2 + e*x
/2)/(c*e*tan(d/2 + e*x/2) - c*e), Eq(a, -c)), (2*A*tan(d/2 + e*x/2)/(c*e*ta
n(d/2 + e*x/2) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(c*e
*tan(d/2 + e*x/2) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)/(c*e*tan(d/2 + e*x
/2) + c*e) - B*log(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 +
 e*x/2) + c*e) - B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) + c*e
) + C*e*x*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x/(c*e*tan(d/
2 + e*x/2) + c*e) - 2*C*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e), Eq(a
, c)), ((A*x + B*sin(d + e*x)/e - C*cos(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B
*cos(d) + C*sin(d))/(a + c*sin(d)), Eq(e, 0)), ((A*log(tan(d/2 + e*x/2))/e
- B*log(tan(d/2 + e*x/2)**2 + 1)/e + B*log(tan(d/2 + e*x/2))/e + C*x)/c, Eq
(a, 0)), (-A*c*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt(-a**2 +
 c**2)/a)/(a**2*c*e - c**3*e) + A*c*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2)
 + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - B*a**2*log(tan(d/2 + e
*x/2)**2 + 1)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2) + c/a - sqr
t(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2) + c/a
+ sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*c**2*log(tan(d/2 + e*x/2)**
2 + 1)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a - sqrt(-a**2
 + c**2)/a)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a + sqrt(
-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + C*a**2*e*x/(a**2*c*e - c**3*e) + C*a
*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt(-a**2 + c**2)/a)/(a**
2*c*e - c**3*e) - C*a*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a + sqrt(
-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - C*c**2*e*x/(a**2*c*e - c**3*e), True
))
```

Giac [A] time = 1.13396, size = 190, normalized size = 2.26

$$\left(\frac{(xe+d)C}{c}+\frac{B\log\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+2\,c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right)}{c}-\frac{B\log\left(\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}+1\right)}{c}-\frac{2\left(\pi\left\lfloor\frac{xe+d}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{s}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="giac
")
```

```
[Out] ((x*e + d)*C/c + B*log(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d)
+ a)/c - B*log(tan(1/2*x*e + 1/2*d)^2 + 1)/c - 2*(pi*floor(1/2*(x*e + d)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2)))*(C*a
 - A*c)/(sqrt(a^2 - c^2)*c))*e^(-1)
```

3.563 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=118

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{3/2}}+\frac{(Ac-aC)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

```
[Out] (2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/((a^2 - c^
2)^(3/2)*e) - B/(c*e*(a + c*Sin[d + e*x])) + ((A*c - a*C)*Cos[d + e*x])/((a
^2 - c^2)*e*(a + c*Sin[d + e*x]))
```

Rubi [A] time = 0.156626, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{3/2}}+\frac{(Ac-aC)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```



```
[Out] (2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/((a^2 - c^
2)^(3/2)*e) - B/(c*e*(a + c*Sin[d + e*x])) + ((A*c - a*C)*Cos[d + e*x])/((a
^2 - c^2)*e*(a + c*Sin[d + e*x]))
```


Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^2}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} + \frac{\int \frac{-aA+cC}{a+c\sin(d+ex)}\,dx}{-a^2+c^2} + \frac{B\,\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx,x,c\,\text{s}\right.}{ce}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} + \frac{(aA-cC)\int \frac{1}{a+c\sin(d\text{-}}}{a^2-c^2}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} + \frac{(2(aA-cC))\,\text{Subst}\,(}{}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} - \frac{(4(aA-cC))\,\text{Subst}\,(}{}$$

$$= \frac{2(aA-cC)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{3/2}e} - \frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\,\text{co}}{\left(a^2-c^2\right)e(a+c}$$

Mathematica [A] time = 0.460344, size = 114, normalized size = 0.97

$$\frac{\frac{B\left(a^2-c^2\right)-c(Ac-aC)\cos(d+ex)}{c(c-a)(a+c)(a+c\sin(d+ex))} + \frac{2(aA-cC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{3/2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```

```
[Out] ((2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/(a^2 - c^
2)^(3/2) + (B*(a^2 - c^2) - c*(A*c - a*C)*Cos[d + e*x])/(c*(-a + c)*(a + c)
*(a + c*Sin[d + e*x])))/e
```

Maple [B] time = 0.16, size = 426, normalized size = 3.6

$$2\,\frac{\tan(d/2+1/2\,ex)\,Ac^2}{e\left(a\left(\tan(d/2+1/2\,ex)\right)^2+2\,c\tan(d/2+1/2\,ex)+a\right)a\left(a^2-c^2\right)} + 2\,\frac{a\tan(d/2+1/2\,ex)\,B}{e\left(a\left(\tan(d/2+1/2\,ex)\right)^2+2\,c\tan(d/2+1/2\,ex)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x)
```

```
[Out] 2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)/a/(a^2-c^2)*tan(1/2*d
+1/2*e*x)*A*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)*a/(a^
2-c^2)*tan(1/2*d+1/2*e*x)*B-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e
*x)+a)/a/(a^2-c^2)*tan(1/2*d+1/2*e*x)*B*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c
*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*tan(1/2*d+1/2*e*x)*c*C+2/e/(a*tan(1/2*d+1/
2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*A*c-2/e/(a*tan(1/2*d+1/2*e*x)^
2+2*c*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*C*a+2/e/(a^2-c^2)^(3/2)*arctan(1/2*(2
*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*a*A-2/e/(a^2-c^2)^(3/2)*arctan(
1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*C*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.76902, size = 995, normalized size = 8.43

$$\left[-\frac{2\,Ba^4 - 4\,Ba^2c^2 + 2\,Bc^4 + \left(Aa^2c - Cac^2 + \left(Aac^2 - Cc^3\right)\sin\left(ex+d\right)\right)\sqrt{-a^2+c^2}\log\left(\frac{\left(2\,a^2-c^2\right)\cos(ex+d)^2 - 2\,ac\sin(ex+d) - a^2 - \cdots}{c^2\cos(ex+d)^2 \cdots}\right.}{2\left(\left(a^4c^2 - 2\,a^2c^4 + c^6\right)e\sin\left(ex+d\right) + \left(a^5c - \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*B*a^4 - 4*B*a^2*c^2 + 2*B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*
c^3)*sin(e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a
```

```
*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x +
d))*sqrt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)
) + 2*(C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(e*x + d))/((a^4*c^2 - 2*a
^2*c^4 + c^6)*e*sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e), -(B*a^4 - 2*
B*a^2*c^2 + B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*c^3)*sin(e*x + d))*sq
rt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)*cos(e*x + d)))
+ (C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(e*x + d))/((a^4*c^2 - 2*a^2*c
^4 + c^6)*e*sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14629, size = 252, normalized size = 2.14

$$2\left(\frac{\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c}{\sqrt{a^2-c^2}}\right)\right)(Aa-Cc)}{\left(a^2-c^2\right)^{\frac{3}{2}}}+\frac{Ba^2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-Cac\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+Ac^2\tan}{\left(a^3-ac^2\right)\left(a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="gi
ac")
```

```
[Out] 2*((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x*e + 1/2*d
) + c)/sqrt(a^2 - c^2)))*(A*a - C*c)/(a^2 - c^2)^(3/2) + (B*a^2*tan(1/2*x*e
 + 1/2*d) - C*a*c*tan(1/2*x*e + 1/2*d) + A*c^2*tan(1/2*x*e + 1/2*d) - B*c^2
*tan(1/2*x*e + 1/2*d) - C*a^2 + A*a*c)/((a^3 - a*c^2)*(a*tan(1/2*x*e + 1/2*
d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a)))*e^(-1)
```

3.564 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=185

$$\frac{\left(2a^2A-3acC+Ac^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{5/2}}+\frac{\left(a^2(-C)+3aAc-2c^2C\right)\cos(d+ex)}{2e\left(a^2-c^2\right)^2(a+c\sin(d+ex))}+\frac{(Ac-aC)\cos(d+ex)}{2e\left(a^2-c^2\right)(a+c\sin(d+ex))^2}$$

```
[Out] ((2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2
]])/((a^2 - c^2)^(5/2)*e) - B/(2*c*e*(a + c*Sin[d + e*x])^2) + ((A*c - a*C)
*Cos[d + e*x])/(2*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^2) + ((3*a*A*c - a^2*C
 - 2*c^2*C)*Cos[d + e*x])/(2*(a^2 - c^2)^2*e*(a + c*Sin[d + e*x]))
```

Rubi [A] time = 0.246184, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{\left(2a^2A-3acC+Ac^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{5/2}}+\frac{\left(a^2(-C)+3aAc-2c^2C\right)\cos(d+ex)}{2e\left(a^2-c^2\right)^2(a+c\sin(d+ex))}+\frac{(Ac-aC)\cos(d+ex)}{2e\left(a^2-c^2\right)(a+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

```
[Out] ((2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2
]])/((a^2 - c^2)^(5/2)*e) - B/(2*c*e*(a + c*Sin[d + e*x])^2) + ((A*c - a*C)
*Cos[d + e*x])/(2*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^2) + ((3*a*A*c - a^2*C
 - 2*c^2*C)*Cos[d + e*x])/(2*(a^2 - c^2)^2*e*(a + c*Sin[d + e*x]))
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^3}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} - \frac{\int \frac{-2(aA-cC)+(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx}{2\left(a^2-c^2\right)} + \frac{B\,\mathrm{Subst}}{}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc-a^2C\right.}{2\left(a^2-c^2\right)^2 e}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc-a^2C\right.}{2\left(a^2-c^2\right)^2 e}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc-a^2C\right.}{2\left(a^2-c^2\right)^2 e}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc-a^2C\right.}{2\left(a^2-c^2\right)^2 e}$$

$$= \frac{\left(2a^2A+Ac^2-3acC\right)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{5/2}e} - \frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{}{2}$$

Mathematica [A] time = 0.956972, size = 174, normalized size = 0.94

$$\frac{\frac{B\left(c^2-a^2\right)+c(Ac-aC)\cos(d+ex)}{c(a-c)(a+c)(a+c\sin(d+ex))^2} + \frac{2\left(2a^2A-3acC+Ac^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{5/2}} - \frac{\left(a^2C-3aAc+2c^2C\right)\cos(d+ex)}{(a-c)^2(a+c)^2(a+c\sin(d+ex))}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

```
[Out] ((2*(2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 -
c^2]])/(a^2 - c^2)^(5/2) + (B*(-a^2 + c^2) + c*(A*c - a*C)*Cos[d + e*x])/((
a - c)*c*(a + c)*(a + c*Sin[d + e*x])^2) - ((-3*a*A*c + a^2*C + 2*c^2*C)*Co
s[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])))/(2*e)
```

Maple [B] time = 0.147, size = 1891, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x)
```

```
[Out] -2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4
)*C*a^3-1/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*
c^2+c^4)*A*c^3+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2*a^3/
(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*B+4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*ta
n(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*A*a^2*c-1/e/(a*tan(1/2*d+1/2*e*x)
^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*C*a*c^2+2/e/(a^4-2*a^2*c
^2+c^4)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(
1/2))*a^2*A+1/e/(a^4-2*a^2*c^2+c^4)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*A*c^2+7/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan
(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^2*A*c^3-4/e/(a*
tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*tan(1/
2*d+1/2*e*x)^2*B*c^3-4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^
2/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*C*c^3+2/e/(a*tan(1/2*d+1/2*e*x)^2+
2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a^3*tan(1/2*d+1/2*e*x)^3*B-
2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)
*a^3*tan(1/2*d+1/2*e*x)^2*C-5/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e
*x)+a)^2*a^2/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*C*c+5/e/(a*tan(1/2*d+1/
2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a*tan(1/2*d+1/2*e*
x)^3*A*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a
^2*c^2+c^4)/a*tan(1/2*d+1/2*e*x)^3*A*c^4-3/e/(a^4-2*a^2*c^2+c^4)/(a^2-c^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*a*c*C-4/e/(a
*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a*tan
(1/2*d+1/2*e*x)^3*B*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+
a)^2/(a^4-2*a^2*c^2+c^4)/a*tan(1/2*d+1/2*e*x)^3*B*c^4-3/e/(a*tan(1/2*d+1/2*
e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a^2*tan(1/2*d+1/2*e*
x)^3*C*c+4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2
*c^2+c^4)*a^2*tan(1/2*d+1/2*e*x)^2*A*c-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(
1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a^2*tan(1/2*d+1/2*e*x)^2*A*c^5+2/e/
(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a^2
*tan(1/2*d+1/2*e*x)^2*B*c+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x
)+a)^2/(a^4-2*a^2*c^2+c^4)/a^2*tan(1/2*d+1/2*e*x)^2*B*c^5-5/e/(a*tan(1/2*d+
1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a*tan(1/2*d+1/2*
e*x)^2*C*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2
*a^2*c^2+c^4)/a*tan(1/2*d+1/2*e*x)^2*C*c^4+11/e/(a*tan(1/2*d+1/2*e*x)^2+2*c
*tan(1/2*d+1/2*e*x)+a)^2*a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*A*c^2-2/e
/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/a/(a^4-2*a^2*c^2+c^4)*
```

```
tan(1/2*d+1/2*e*x)*A*c^4-4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)
+a)^2*a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*B*c^2+2/e/(a*tan(1/2*d+1/2*e
*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*
B*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.33691, size = 1891, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*B*a^6 - 6*B*a^4*c^2 + 6*B*a^2*c^4 - 2*B*c^6 + 2*(C*a^4*c^2 - 3*A*a^
3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x + d)*sin(e*x + d) + (2*A*a
^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2*c^3 - 3*C*a
*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A*a*c^4)*sin(
e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*sin(e*
x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x + d))*sqrt(
-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)) + 2*(2*
C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^4 - C*a*c^5 - A*c^6)*cos(e*x
+ d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)*e*cos(e*x + d)^2 - 2*(a^7*c^
2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(e*x + d) - (a^8*c - 2*a^6*c^3 + 2*
a^2*c^7 - c^9)*e), 1/2*(B*a^6 - 3*B*a^4*c^2 + 3*B*a^2*c^4 - B*c^6 + (C*a^4*
c^2 - 3*A*a^3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x + d)*sin(e*x +
 d) + (2*A*a^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2
*c^3 - 3*C*a*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A
*a*c^4)*sin(e*x + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^
```

```
2 - c^2)*cos(e*x + d))) + (2*C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^
4 - C*a*c^5 - A*c^6)*cos(e*x + d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)
*e*cos(e*x + d)^2 - 2*(a^7*c^2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(e*x +
 d) - (a^8*c - 2*a^6*c^3 + 2*a^2*c^7 - c^9)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22202, size = 805, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="gi
ac")
```

```
[Out] ((2*A*a^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2))))/((a^4 - 2*a^2*c^2 + c^4)
*sqrt(a^2 - c^2)) + (2*B*a^5*tan(1/2*x*e + 1/2*d)^3 - 3*C*a^4*c*tan(1/2*x*e
 + 1/2*d)^3 + 5*A*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 4*B*a^3*c^2*tan(1/2*x*e
+ 1/2*d)^3 - 2*A*a*c^4*tan(1/2*x*e + 1/2*d)^3 + 2*B*a*c^4*tan(1/2*x*e + 1/2
*d)^3 - 2*C*a^5*tan(1/2*x*e + 1/2*d)^2 + 4*A*a^4*c*tan(1/2*x*e + 1/2*d)^2 +
 2*B*a^4*c*tan(1/2*x*e + 1/2*d)^2 - 5*C*a^3*c^2*tan(1/2*x*e + 1/2*d)^2 + 7*
A*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 4*B*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 2*C
*a*c^4*tan(1/2*x*e + 1/2*d)^2 - 2*A*c^5*tan(1/2*x*e + 1/2*d)^2 + 2*B*c^5*ta
n(1/2*x*e + 1/2*d)^2 + 2*B*a^5*tan(1/2*x*e + 1/2*d) - 5*C*a^4*c*tan(1/2*x*e
 + 1/2*d) + 11*A*a^3*c^2*tan(1/2*x*e + 1/2*d) - 4*B*a^3*c^2*tan(1/2*x*e + 1
/2*d) - 4*C*a^2*c^3*tan(1/2*x*e + 1/2*d) - 2*A*a*c^4*tan(1/2*x*e + 1/2*d) +
 2*B*a*c^4*tan(1/2*x*e + 1/2*d) - 2*C*a^5 + 4*A*a^4*c - C*a^3*c^2 - A*a^2*c
^3)/((a^6 - 2*a^4*c^2 + a^2*c^4)*(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*
e + 1/2*d) + a)^2))*e^(-1)
```

3.565 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=258

$$\frac{\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{7/2}}+\frac{\left(11a^2Ac-2a^3C-13ac^2C+4Ac^3\right)\cos(d+ex)}{6e\left(a^2-c^2\right)^3(a+c\sin(d+ex))}+\frac{\left(-2a^2C+5\right.}{6e\left(a^2-c\right.}$$

```
[Out] ((2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/
Sqrt[a^2 - c^2]])/((a^2 - c^2)^(7/2)*e) - B/(3*c*e*(a + c*Sin[d + e*x])^3)
+ ((A*c - a*C)*Cos[d + e*x])/(3*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^3) + ((5
*a*A*c - 2*a^2*C - 3*c^2*C)*Cos[d + e*x])/(6*(a^2 - c^2)^2*e*(a + c*Sin[d +
 e*x])^2) + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/(6
*(a^2 - c^2)^3*e*(a + c*Sin[d + e*x]))
```

Rubi [A] time = 0.404632, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{7/2}}+\frac{\left(11a^2Ac-2a^3C-13ac^2C+4Ac^3\right)\cos(d+ex)}{6e\left(a^2-c^2\right)^3(a+c\sin(d+ex))}+\frac{\left(-2a^2C+5\right.}{6e\left(a^2-c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```



```
[Out] ((2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/
Sqrt[a^2 - c^2]])/((a^2 - c^2)^(7/2)*e) - B/(3*c*e*(a + c*Sin[d + e*x])^3)
+ ((A*c - a*C)*Cos[d + e*x])/(3*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^3) + ((5
*a*A*c - 2*a^2*C - 3*c^2*C)*Cos[d + e*x])/(6*(a^2 - c^2)^2*e*(a + c*Sin[d +
 e*x])^2) + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/(6
*(a^2 - c^2)^3*e*(a + c*Sin[d + e*x]))
```


Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
```

```
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^4}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} - \frac{\int \frac{-3(aA-cC)+2(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx}{3\left(a^2-c^2\right)} + \frac{B\,\mathrm{Subs}}{}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{\left(5aAc-2a^2C\right.}{6\left(a^2-c^2\right)^2 e}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{\left(5aAc-2a^2C\right.}{6\left(a^2-c^2\right)^2 e}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{\left(5aAc-2a^2C\right.}{6\left(a^2-c^2\right)^2 e}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{\left(5aAc-2a^2C\right.}{6\left(a^2-c^2\right)^2 e}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{\left(5aAc-2a^2C\right.}{6\left(a^2-c^2\right)^2 e}$$

$$= \frac{\left(2a^3A+3aAc^2-4a^2cC-c^3C\right)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{7/2}e} - \frac{B}{3ce(a+c\sin(d+}$$

Mathematica [A] time = 2.7688, size = 244, normalized size = 0.95

$$\frac{\frac{2B\left(c^2-a^2\right)+2c(Ac-aC)\cos(d+ex)}{c(a-c)(a+c)(a+c\sin(d+ex))^3} + \frac{6\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{7/2}} + \frac{\left(11a^2Ac-2a^3C-13ac^2C+4Ac^3\right)\cos(d+ex)}{(a-c)^3(a+c)^3(a+c\sin(d+ex))} + \frac{\left(-2a^2C+5aAc-\right.}{(a-c)^2(a+c)^2(}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2
])/Sqrt[a^2 - c^2]])/(a^2 - c^2)^(7/2) + (2*B*(-a^2 + c^2) + 2*c*(A*c - a*C
)*Cos[d + e*x])/((a - c)*c*(a + c)*(a + c*Sin[d + e*x])^3) + ((5*a*A*c - 2*
a^2*C - 3*c^2*C)*Cos[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])^2)
 + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/((a - c)^3*
(a + c)^3*(a + c*Sin[d + e*x])))/(6*e)
```

Maple [B] time = 0.166, size = 5051, normalized size = 19.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.83786, size = 3085, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(4*B*a^8 - 16*B*a^6*c^2 + 24*B*a^4*c^4 - 16*B*a^2*c^6 + 4*B*c^8 - 2*(
2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 + 4*A*
c^8)*cos(e*x + d)^3 + 6*(2*C*a^6*c^2 - 9*A*a^5*c^3 + 7*C*a^4*c^4 + 8*A*a^3*
c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(e*x + d)*sin(e*x + d) + 3*(2*A*a^
6*c - 4*C*a^5*c^2 + 9*A*a^4*c^3 - 13*C*a^3*c^4 + 9*A*a^2*c^5 - 3*C*a*c^6 -
3*(2*A*a^4*c^3 - 4*C*a^3*c^4 + 3*A*a^2*c^5 - C*a*c^6)*cos(e*x + d)^2 + (6*A
*a^5*c^2 - 12*C*a^4*c^3 + 11*A*a^3*c^4 - 7*C*a^2*c^5 + 3*A*a*c^6 - C*c^7 -
(2*A*a^3*c^4 - 4*C*a^2*c^5 + 3*A*a*c^6 - C*c^7)*cos(e*x + d)^2)*sin(e*x + d
))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*sin(e*x + d)
- a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x + d))*sqrt(-a^2 +
c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)) + 12*(C*a^7*c
- 3*A*a^6*c^2 + C*a^5*c^3 + 2*A*a^4*c^4 - 2*C*a*c^7 + A*c^8)*cos(e*x + d))/
(3*(a^9*c^3 - 4*a^7*c^5 + 6*a^5*c^7 - 4*a^3*c^9 + a*c^11)*e*cos(e*x + d)^2
- (a^11*c - a^9*c^3 - 6*a^7*c^5 + 14*a^5*c^7 - 11*a^3*c^9 + 3*a*c^11)*e + (
(a^8*c^4 - 4*a^6*c^6 + 6*a^4*c^8 - 4*a^2*c^10 + c^12)*e*cos(e*x + d)^2 - (3
*a^10*c^2 - 11*a^8*c^4 + 14*a^6*c^6 - 6*a^4*c^8 - a^2*c^10 + c^12)*e)*sin(e
*x + d)), 1/6*(2*B*a^8 - 8*B*a^6*c^2 + 12*B*a^4*c^4 - 8*B*a^2*c^6 + 2*B*c^8
 - (2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 +
4*A*c^8)*cos(e*x + d)^3 + 3*(2*C*a^6*c^2 - 9*A*a^5*c^3 + 7*C*a^4*c^4 + 8*A*
a^3*c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(e*x + d)*sin(e*x + d) + 3*(2*
A*a^6*c - 4*C*a^5*c^2 + 9*A*a^4*c^3 - 13*C*a^3*c^4 + 9*A*a^2*c^5 - 3*C*a*c^
6 - 3*(2*A*a^4*c^3 - 4*C*a^3*c^4 + 3*A*a^2*c^5 - C*a*c^6)*cos(e*x + d)^2 +
(6*A*a^5*c^2 - 12*C*a^4*c^3 + 11*A*a^3*c^4 - 7*C*a^2*c^5 + 3*A*a*c^6 - C*c^
7 - (2*A*a^3*c^4 - 4*C*a^2*c^5 + 3*A*a*c^6 - C*c^7)*cos(e*x + d)^2)*sin(e*x
 + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)*cos(e*
x + d))) + 6*(C*a^7*c - 3*A*a^6*c^2 + C*a^5*c^3 + 2*A*a^4*c^4 - 2*C*a*c^7 +
 A*c^8)*cos(e*x + d))/(3*(a^9*c^3 - 4*a^7*c^5 + 6*a^5*c^7 - 4*a^3*c^9 + a*c
^11)*e*cos(e*x + d)^2 - (a^11*c - a^9*c^3 - 6*a^7*c^5 + 14*a^5*c^7 - 11*a^3
*c^9 + 3*a*c^11)*e + ((a^8*c^4 - 4*a^6*c^6 + 6*a^4*c^8 - 4*a^2*c^10 + c^12)
*e*cos(e*x + d)^2 - (3*a^10*c^2 - 11*a^8*c^4 + 14*a^6*c^6 - 6*a^4*c^8 - a^2
*c^10 + c^12)*e)*sin(e*x + d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27571, size = 1809, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(2*A*a^3 - 4*C*a^2*c + 3*A*a*c^2 - C*c^3)*(pi*floor(1/2*(x*e + d)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2)))/((a^
6 - 3*a^4*c^2 + 3*a^2*c^4 - c^6)*sqrt(a^2 - c^2)) + (6*B*a^8*tan(1/2*x*e +
1/2*d)^5 - 12*C*a^7*c*tan(1/2*x*e + 1/2*d)^5 + 27*A*a^6*c^2*tan(1/2*x*e + 1
/2*d)^5 - 18*B*a^6*c^2*tan(1/2*x*e + 1/2*d)^5 - 3*C*a^5*c^3*tan(1/2*x*e + 1
/2*d)^5 - 18*A*a^4*c^4*tan(1/2*x*e + 1/2*d)^5 + 18*B*a^4*c^4*tan(1/2*x*e +
1/2*d)^5 + 6*A*a^2*c^6*tan(1/2*x*e + 1/2*d)^5 - 6*B*a^2*c^6*tan(1/2*x*e + 1
/2*d)^5 - 6*C*a^8*tan(1/2*x*e + 1/2*d)^4 + 18*A*a^7*c*tan(1/2*x*e + 1/2*d)^
4 + 12*B*a^7*c*tan(1/2*x*e + 1/2*d)^4 - 42*C*a^6*c^2*tan(1/2*x*e + 1/2*d)^4
 + 81*A*a^5*c^3*tan(1/2*x*e + 1/2*d)^4 - 36*B*a^5*c^3*tan(1/2*x*e + 1/2*d)^
4 - 33*C*a^4*c^4*tan(1/2*x*e + 1/2*d)^4 - 36*A*a^3*c^5*tan(1/2*x*e + 1/2*d)
^4 + 36*B*a^3*c^5*tan(1/2*x*e + 1/2*d)^4 + 6*C*a^2*c^6*tan(1/2*x*e + 1/2*d)
^4 + 12*A*a*c^7*tan(1/2*x*e + 1/2*d)^4 - 12*B*a*c^7*tan(1/2*x*e + 1/2*d)^4
+ 12*B*a^8*tan(1/2*x*e + 1/2*d)^3 - 36*C*a^7*c*tan(1/2*x*e + 1/2*d)^3 + 108
*A*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 - 28*B*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 - 8
4*C*a^5*c^3*tan(1/2*x*e + 1/2*d)^3 + 42*A*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 +
12*B*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 - 34*C*a^3*c^5*tan(1/2*x*e + 1/2*d)^3 -
 8*A*a^2*c^6*tan(1/2*x*e + 1/2*d)^3 + 12*B*a^2*c^6*tan(1/2*x*e + 1/2*d)^3 +
 4*C*a*c^7*tan(1/2*x*e + 1/2*d)^3 + 8*A*c^8*tan(1/2*x*e + 1/2*d)^3 - 8*B*c^
8*tan(1/2*x*e + 1/2*d)^3 - 12*C*a^8*tan(1/2*x*e + 1/2*d)^2 + 36*A*a^7*c*tan
(1/2*x*e + 1/2*d)^2 + 12*B*a^7*c*tan(1/2*x*e + 1/2*d)^2 - 60*C*a^6*c^2*tan(
1/2*x*e + 1/2*d)^2 + 120*A*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 - 36*B*a^5*c^3*ta
n(1/2*x*e + 1/2*d)^2 - 84*C*a^4*c^4*tan(1/2*x*e + 1/2*d)^2 - 18*A*a^3*c^5*t
an(1/2*x*e + 1/2*d)^2 + 36*B*a^3*c^5*tan(1/2*x*e + 1/2*d)^2 + 6*C*a^2*c^6*t
an(1/2*x*e + 1/2*d)^2 + 12*A*a*c^7*tan(1/2*x*e + 1/2*d)^2 - 12*B*a*c^7*tan(
1/2*x*e + 1/2*d)^2 + 6*B*a^8*tan(1/2*x*e + 1/2*d) - 24*C*a^7*c*tan(1/2*x*e
+ 1/2*d) + 81*A*a^6*c^2*tan(1/2*x*e + 1/2*d) - 18*B*a^6*c^2*tan(1/2*x*e + 1
/2*d) - 57*C*a^5*c^3*tan(1/2*x*e + 1/2*d) - 12*A*a^4*c^4*tan(1/2*x*e + 1/2*
d) + 18*B*a^4*c^4*tan(1/2*x*e + 1/2*d) + 6*C*a^3*c^5*tan(1/2*x*e + 1/2*d) +
 6*A*a^2*c^6*tan(1/2*x*e + 1/2*d) - 6*B*a^2*c^6*tan(1/2*x*e + 1/2*d) - 6*C*
a^8 + 18*A*a^7*c - 10*C*a^6*c^2 - 5*A*a^5*c^3 + C*a^4*c^4 + 2*A*a^3*c^5)/((
```

```
a^9 - 3*a^7*c^2 + 3*a^5*c^4 - a^3*c^6)*(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(
1/2*x*e + 1/2*d) + a)^3))*e^(-1)
```

3.566 $\int (a + b\cos(c + dx)\sin(c + dx))^m \, dx$

Optimal. Leaf size=131

$$-\frac{\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx)}{2a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)}{\sqrt{2}d\sqrt{\sin(2c+2dx)+1}}$$

```
[Out] -((AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[2*c + 2*d*x])/2, (b*(1 - Sin[2*c +
2*d*x]))/(2*a + b)]*Cos[2*c + 2*d*x]*(a + (b*Sin[2*c + 2*d*x])/2)^m)/(Sqrt[
2]*d*Sqrt[1 + Sin[2*c + 2*d*x]]*((2*a + b*Sin[2*c + 2*d*x])/(2*a + b))^m))
```

Rubi [A] time = 0.112209, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2665, 139, 138}

$$-\frac{\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx)}{2a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)}{\sqrt{2}d\sqrt{\sin(2c+2dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

```
[Out] -((AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[2*c + 2*d*x])/2, (b*(1 - Sin[2*c +
2*d*x]))/(2*a + b)]*Cos[2*c + 2*d*x]*(a + (b*Sin[2*c + 2*d*x])/2)^m)/(Sqrt[
2]*d*Sqrt[1 + Sin[2*c + 2*d*x]]*((2*a + b*Sin[2*c + 2*d*x])/(2*a + b))^m))
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\,dx$$

$$= \frac{\cos(2c+2dx)\,\text{Subst}\left(\int \frac{\left(a+\frac{bx}{2}\right)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(2c+2dx)\right)}{2d\sqrt{1-\sin(2c+2dx)}\sqrt{1+\sin(2c+2dx)}}$$

$$= \frac{\left(\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(-\frac{a+\frac{1}{2}b\sin(2c+2dx)}{-a-\frac{b}{2}}\right)^{-m}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-\frac{b}{2}}\right.}{\sqrt{1-}}\right.}{2d\sqrt{1-\sin(2c+2dx)}\sqrt{1+\sin(2c+2dx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)\cos(2c+2dx)\left(a+\frac{1}{2}b\sin\right.}{\sqrt{2}d\sqrt{1+\sin(2c+2dx)}}$$

Mathematica [A] time = 0.610215, size = 145, normalized size = 1.11

$$\frac{\sec(2(c+dx))\sqrt{-\frac{b(\sin(2(c+dx))-1)}{2a+b}}\sqrt{\frac{b(\sin(2(c+dx))+1)}{b-2a}}\left(a+\frac{1}{2}b\sin(2(c+dx))\right)^{m+1}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2a+b\sin(2(c+dx))}{2a-b},\frac{2a+b\,\mathrm{s}}{}\right.}{bd(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sin[2*(c + d*x)])/(2*a - b), (2*
a + b*Sin[2*(c + d*x)])/(2*a + b)]*Sec[2*(c + d*x)]*Sqrt[-((b*(-1 + Sin[2*(
c + d*x)]))/(2*a + b))]*Sqrt[(b*(1 + Sin[2*(c + d*x)]))/(-2*a + b)]*(a + (b
*Sin[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

Maple [F] time = 0.521, size = 0, normalized size = 0.

$$\int \left(a + b\cos\left(dx + c\right)\sin\left(dx + c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx + c\right)\sin\left(dx + c\right) + a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx + c\right)\sin\left(dx + c\right) + a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

3.567 $\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-\frac{b\left(16a^2+b^2\right)\cos(2c+2dx)}{24d}+\frac{1}{8}ax\left(8a^2+3b^2\right)-\frac{5ab^2\sin(2c+2dx)\cos(2c+2dx)}{48d}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2$$

```
[Out] (a*(8*a^2 + 3*b^2)*x)/8 - (b*(16*a^2 + b^2)*Cos[2*c + 2*d*x])/(24*d) - (5*a
*b^2*Cos[2*c + 2*d*x]*Sin[2*c + 2*d*x])/(48*d) - (b*Cos[2*c + 2*d*x]*(2*a +
 b*Sin[2*c + 2*d*x])^2)/(48*d)
```

Rubi [A] time = 0.0823962, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2666, 2656, 2734}

$$-\frac{b\left(16a^2+b^2\right)\cos(2c+2dx)}{24d}+\frac{1}{8}ax\left(8a^2+3b^2\right)-\frac{5ab^2\sin(2c+2dx)\cos(2c+2dx)}{48d}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]
```

```
[Out] (a*(8*a^2 + 3*b^2)*x)/8 - (b*(16*a^2 + b^2)*Cos[2*c + 2*d*x])/(24*d) - (5*a
*b^2*Cos[2*c + 2*d*x]*Sin[2*c + 2*d*x])/(48*d) - (b*Cos[2*c + 2*d*x]*(2*a +
 b*Sin[2*c + 2*d*x])^2)/(48*d)
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3 dx$$

$$= -\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d} + \frac{1}{3}\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)\left(\frac{1}{2}\left(6a^2\right.\right.$$

$$= \frac{1}{8}a\left(8a^2+3b^2\right)x - \frac{b\left(16a^2+b^2\right)\cos(2c+2dx)}{24d} - \frac{5ab^2\cos(2c+2dx)\sin(2c+2}{48d}$$

Mathematica [A] time = 0.282551, size = 75, normalized size = 0.7

$$\frac{6a\left(4\left(8a^2+3b^2\right)(c+dx)-3b^2\sin(4(c+dx))\right)-9\left(16a^2b+b^3\right)\cos(2(c+dx))+b^3\cos(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]
```

```
[Out] (-9*(16*a^2*b + b^3)*Cos[2*(c + d*x)] + b^3*Cos[6*(c + d*x)] + 6*a*(4*(8*a^
2 + 3*b^2)*(c + d*x) - 3*b^2*Sin[4*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.061, size = 106, normalized size = 1.

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6}-\frac{(\cos(dx+c))^4}{12}\right)+3\,ab^2\left(-1/4\,\sin(dx+c)(\cos(dx+c))^3+1/8\,\sin(dx+c)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/4*s
in(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c)-3/2*cos(d*x
```

```
+c)^2*a^2*b+a^3*(d*x+c))
```

Maxima [A] time = 1.00551, size = 108, normalized size = 1.01

$$a^3x - \frac{3\,a^2b\cos\left(dx+c\right)^2}{2\,d} + \frac{3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)ab^2}{32\,d} - \frac{\left(2\,\sin\left(dx+c\right)^6 - 3\,\sin\left(dx+c\right)^4\right)b^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3/2*a^2*b*cos(d*x + c)^2/d + 3/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*
a*b^2/d - 1/12*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*b^3/d
```

Fricas [A] time = 2.4859, size = 228, normalized size = 2.13

$$\frac{4\,b^3\cos\left(dx+c\right)^6 - 6\,b^3\cos\left(dx+c\right)^4 - 36\,a^2b\cos\left(dx+c\right)^2 + 3\left(8\,a^3+3\,ab^2\right)dx - 9\left(2\,ab^2\cos\left(dx+c\right)^3 - ab^2\cos\left(dx+c\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*cos(d*x + c)^6 - 6*b^3*cos(d*x + c)^4 - 36*a^2*b*cos(d*x + c)^2
 + 3*(8*a^3 + 3*a*b^2)*d*x - 9*(2*a*b^2*cos(d*x + c)^3 - a*b^2*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [A] time = 4.36334, size = 190, normalized size = 1.78

$$\begin{cases} a^3x + \frac{3a^2b\sin^2\left(c+dx\right)}{2d} + \frac{3ab^2x\sin^4\left(c+dx\right)}{8} + \frac{3ab^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4} + \frac{3ab^2x\cos^4\left(c+dx\right)}{8} + \frac{3ab^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d} - \frac{3ab^2\sin\left(c+dx\right)\cos}{8d} \\ x\left(a+b\sin\left(c\right)\cos\left(c\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sin(c + d*x)**2/(2*d) + 3*a*b**2*x*sin(c + d*x
)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*cos(c +
d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*sin(c +
d*x)*cos(c + d*x)**3/(8*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**4/(4*d) - b
**3*cos(c + d*x)**6/(12*d), Ne(d, 0)), (x*(a + b*sin(c)*cos(c))**3, True))
```

Giac [A] time = 1.13105, size = 101, normalized size = 0.94

$$\frac{b^3 \cos(6\,dx + 6\,c)}{192\,d} - \frac{3\,ab^2 \sin(4\,dx + 4\,c)}{32\,d} + \frac{1}{8}\left(8\,a^3 + 3\,ab^2\right)x - \frac{3\left(16\,a^2b + b^3\right)\cos(2\,dx + 2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*b^3*cos(6*d*x + 6*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(8*a^3 +
 3*a*b^2)*x - 3/64*(16*a^2*b + b^3)*cos(2*d*x + 2*c)/d
```

3.568 $\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}x\left(8a^2+b^2\right)-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\sin(2c+2dx)\cos(2c+2dx)}{16d}$$

```
[Out] ((8*a^2 + b^2)*x)/8 - (a*b*Cos[2*c + 2*d*x])/(2*d) - (b^2*Cos[2*c + 2*d*x]*
Sin[2*c + 2*d*x])/(16*d)
```

Rubi [A] time = 0.0342727, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2666, 2644}

$$\frac{1}{8}x\left(8a^2+b^2\right)-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\sin(2c+2dx)\cos(2c+2dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

```
[Out] ((8*a^2 + b^2)*x)/8 - (a*b*Cos[2*c + 2*d*x])/(2*d) - (b^2*Cos[2*c + 2*d*x]*
Sin[2*c + 2*d*x])/(16*d)
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2\,dx$$
$$= \frac{1}{8}\left(8a^2+b^2\right)x - \frac{ab\cos(2c+2dx)}{2d} - \frac{b^2\cos(2c+2dx)\sin(2c+2dx)}{16d}$$

Mathematica [A] time = 0.157421, size = 48, normalized size = 0.79

$$-\frac{-4\left(8a^2+b^2\right)(c+dx)+16ab\cos(2(c+dx))+b^2\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

```
[Out] -(-4*(8*a^2 + b^2)*(c + d*x) + 16*a*b*Cos[2*(c + d*x)] + b^2*Sin[4*(c + d*x
)])/(32*d)
```

Maple [A] time = 0.049, size = 69, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-(\cos(dx+c))^2ab+a^2(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)-cos(d*x+c)^2*a*b+a^2*(d*x+c))
```

Maxima [A] time = 0.998978, size = 65, normalized size = 1.07

$$a^2x - \frac{ab\cos(dx+c)^2}{d} + \frac{(4\,dx+4\,c-\sin(4\,dx+4\,c))b^2}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - a*b*cos(d*x + c)^2/d + 1/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*b^2/d
```

Fricas [A] time = 2.41165, size = 146, normalized size = 2.39

$$-\frac{8\,ab\cos\left(dx+c\right)^{2}-\left(8\,a^{2}+b^{2}\right)dx+\left(2\,b^{2}\cos\left(dx+c\right)^{3}-b^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*b*cos(d*x + c)^2 - (8*a^2 + b^2)*d*x + (2*b^2*cos(d*x + c)^3 - b^
2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.15868, size = 129, normalized size = 2.11

$$\begin{cases} a^2x+\frac{ab\sin^2(c+dx)}{d}+\frac{b^2x\sin^4(c+dx)}{8}+\frac{b^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{b^2x\cos^4(c+dx)}{8}+\frac{b^2\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{8d} & \text{fo} \\ x\left(a+b\sin(c)\cos(c)\right)^2 & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*sin(c + d*x)**2/d + b**2*x*sin(c + d*x)**4/8 + b**2
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**4/8 + b**2*sin(
c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), N
e(d, 0)), (x*(a + b*sin(c)*cos(c))**2, True))
```

Giac [A] time = 1.13387, size = 62, normalized size = 1.02

$$\frac{1}{8}\left(8\,a^{2}+b^{2}\right)x-\frac{ab\cos\left(2\,dx+2\,c\right)}{2\,d}-\frac{b^{2}\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d - 1/32*b^2*sin(4*d*x + 4*c
)/d
```

3.569 $\int (a+b\cos(c+dx)\sin(c+dx))\,dx$

Optimal. Leaf size=20

$$ax+\frac{b\sin^2(c+dx)}{2d}$$

```
[Out] a*x + (b*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0155313, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2564, 30}

$$ax+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] a*x + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))\,dx = ax+b\int \cos(c+dx)\sin(c+dx)\,dx$$
$$= ax+\frac{b\,\text{Subst}(\int x\,dx, x, \sin(c+dx))}{d}$$
$$= ax+\frac{b\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.0080549, size = 38, normalized size = 1.9

$$ax+\frac{b\sin(2c)\sin(2dx)}{4d}-\frac{b\cos(2c)\cos(2dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] a*x - (b*Cos[2*c]*Cos[2*d*x])/(4*d) + (b*Sin[2*c]*Sin[2*d*x])/(4*d)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.

$$ax+\frac{b\,(\sin(dx+c))^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] a*x+1/2*b*sin(d*x+c)^2/d
```

Maxima [A] time = 1.00264, size = 24, normalized size = 1.2

$$ax-\frac{b\cos(dx+c)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x - 1/2*b*cos(d*x + c)^2/d
```

Fricas [A] time = 2.30785, size = 49, normalized size = 2.45

$$\frac{2\,adx - b\cos\left(dx + c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x - b*cos(d*x + c)^2)/d
```

Sympy [A] time = 0.199445, size = 24, normalized size = 1.2

$$ax + b\left(\begin{cases} \frac{\sin^2(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x\sin(c)\cos(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*sin(c)*cos(c), True
))
```

Giac [A] time = 1.12474, size = 24, normalized size = 1.2

$$ax + \frac{b\sin\left(dx + c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*sin(d*x + c)^2/d
```

3.570 $\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx$

Optimal. Leaf size=48

$$\frac{2\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

```
[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

Rubi [A] time = 0.0661284, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx &= \int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{a+bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(2c+2dx)\right)\right)}{d}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{-4a^2+b^2-x^2}\,dx, x, b+2a\tan\left(\frac{1}{2}(2c+2dx)\right)\right)}{d}\\ &= \frac{2\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}\,d}\end{aligned}$$

Mathematica [A] time = 0.0763346, size = 48, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

Maple [A] time = 0.067, size = 45, normalized size = 0.9

$$2\,\frac{1}{d\sqrt{4\,a^2-b^2}}\arctan\left(\frac{b+2\,a\tan(dx+c)}{\sqrt{4\,a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)
```

```
[Out] 2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.41256, size = 653, normalized size = 13.6

$$\Bigg[-\frac{\sqrt{-4\,a^2+b^2}\,\log\Bigg(-\frac{2\,(8\,a^2-b^2)\cos(dx+c)^4-4\,ab\cos(dx+c)\sin(dx+c)-2\,(8\,a^2-b^2)\cos(dx+c)^2+2\,a^2-b^2+(2\,b\cos(dx+c)^2+4\,(2\,a\cos(dx+c)^3-a\cos(dx+}{b^2\cos(dx+c)^4-b^2\cos(dx+c)^2-2\,ab\cos(dx+c)\sin(dx+c)-a^2}}{2\,(4\,a^2-b^2)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d
*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*
cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*
sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*
x + c)*sin(d*x + c) - a^2))/((4*a^2 - b^2)*d), -arctan(-(4*a*cos(d*x + c)*s
in(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2
+ b^2))/(sqrt(4*a^2 - b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13529, size = 82, normalized size = 1.71

$$\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)}{\sqrt{4\,a^2-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqrt
(4*a^2 - b^2)))/(sqrt(4*a^2 - b^2)*d)
```

3.571 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx$

Optimal. Leaf size=95

$$\frac{8a\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{3/2}}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))}$$

```
[Out] (8*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2 - b^2)^(3/2)
*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]))
```

Rubi [A] time = 0.108601, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2666, 2664, 12, 2660, 618, 204}

$$\frac{8a\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{3/2}}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]
```

```
[Out] (8*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2 - b^2)^(3/2)
*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]))
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}\,dx$$
$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} + \frac{4\int \frac{a}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2}$$
$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} + \frac{(4a)\int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2}$$
$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} + \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{a+bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(2c+\right.\right.}{\left(4a^2-b^2\right)d}$$
$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} - \frac{(8a)\,\mathrm{Subst}\left(\int \frac{1}{-4a^2+b^2-x^2}\,dx, x, b+2a\tan\right.}{\left(4a^2-b^2\right)d}$$
$$= \frac{8a\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\left(4a^2-b^2\right)^{3/2}d} + \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))}$$

Mathematica [A] time = 0.413638, size = 94, normalized size = 0.99

$$\frac{2\left(\frac{4a\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{\left(4a^2-b^2\right)^{3/2}} + \frac{b\cos(2(c+dx))}{(2a-b)(2a+b)(2a+b\sin(2(c+dx)))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]
```

```
[Out] (2*((4*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a^2 - b^2)^(3
/2) + (b*Cos[2*(c + d*x)])/((2*a - b)*(2*a + b)*(2*a + b*Sin[2*(c + d*x)]))
))/d
```

Maple [A] time = 0.113, size = 139, normalized size = 1.5

$$\frac{b^2\tan(dx+c)}{d\left((\tan(dx+c))^2a+b\tan(dx+c)+a\right)a\left(4a^2-b^2\right)} + 2\frac{b}{d\left((\tan(dx+c))^2a+b\tan(dx+c)+a\right)\left(4a^2-b^2\right)} + 8\frac{}{\left(4a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x)
```

```
[Out] 1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)*b^2/a/(4*a^2-b^2)*tan(d*x+c)+2/d/(tan(d
*x+c)^2*a+b*tan(d*x+c)+a)*b/(4*a^2-b^2)+8*a*arctan((b+2*a*tan(d*x+c))/(4*a^
2-b^2)^(1/2))/(4*a^2-b^2)^(3/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72315, size = 1116, normalized size = 11.75

$$\left[-\frac{4\,a^2b-b^3-2\left(4\,a^2b-b^3\right)\cos\left(dx+c\right)^2-2\left(ab\cos\left(dx+c\right)\sin\left(dx+c\right)+a^2\right)\sqrt{-4\,a^2+b^2}\log\left(\frac{2\left(8\,a^2-b^2\right)\cos(dx+c)^4-4\,ab\cos\right.}{\left.\right.}\right.}{\left(16\,a^4b-8\,a^2b^3+b^5\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-(4*a^2*b - b^3 - 2*(4*a^2*b - b^3)*cos(d*x + c)^2 - 2*(a*b*cos(d*x + c)*s
in(d*x + c) + a^2)*sqrt(-4*a^2 + b^2)*log((2*(8*a^2 - b^2)*cos(d*x + c)^4 -
 4*a*b*cos(d*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 -
 b^2 - (2*b*cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*
x + c) - b)*sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 -
2*a*b*cos(d*x + c)*sin(d*x + c) - a^2)))/((16*a^4*b - 8*a^2*b^3 + b^5)*d*co
s(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a*b^4)*d), -(4*a^2*b - b^3
- 2*(4*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b*cos(d*x + c)*sin(d*x + c) + a^2
)*sqrt(4*a^2 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*a^2
- b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)))/((16*a^4*b - 8*a^2*
```

```
b^3 + b^5)*d*cos(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1771, size = 157, normalized size = 1.65

$$\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)a}{\left(4\,a^2-b^2\right)^{\frac{3}{2}}}+\frac{b^2\tan(dx+c)+2\,ab}{\left(4\,a^3-ab^2\right)\left(a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqr
t(4*a^2 - b^2)))*a/(4*a^2 - b^2)^(3/2) + (b^2*tan(d*x + c) + 2*a*b)/((4*a^3
 - a*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

3.572 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3} dx$

Optimal. Leaf size=149

$$\frac{4\left(8a^2+b^2\right)\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{5/2}}+\frac{12ab\cos(2c+2dx)}{d\left(4a^2-b^2\right)^2(2a+b\sin(2c+2dx))}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

```
[Out] (4*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2
- b^2)^(5/2)*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c
+ 2*d*x])^2) + (12*a*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)^2*d*(2*a + b*Sin[2*
c + 2*d*x]))
```

Rubi [A] time = 0.177361, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2666, 2664, 2754, 12, 2660, 618, 204}

$$\frac{4\left(8a^2+b^2\right)\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{5/2}}+\frac{12ab\cos(2c+2dx)}{d\left(4a^2-b^2\right)^2(2a+b\sin(2c+2dx))}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

```
[Out] (4*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2
- b^2)^(5/2)*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c
+ 2*d*x])^2) + (12*a*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)^2*d*(2*a + b*Sin[2*
c + 2*d*x]))
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
```

```
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3}\,dx$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} - \frac{2\int \frac{-2a+\frac{1}{2}b\sin(2c+2dx)}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}\,dx}{4a^2-b^2}$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+2dx))} + \frac{8\int}{}$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+2dx))} + \frac{(2(}{}$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+2dx))} + \frac{(2(}{}$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+2dx))} - \frac{(4(}{}$$

$$= \frac{4(8a^2+b^2)\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{5/2}\,d} + \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{}{(4a^2}$$

Mathematica [A] time = 0.950039, size = 120, normalized size = 0.81

$$\frac{2\left(\frac{2(8a^2+b^2)\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{5/2}} + \frac{b\cos(2(c+dx))(16a^2+6ab\sin(2(c+dx))-b^2)}{(b^2-4a^2)^2(2a+b\sin(2(c+dx)))^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

```
[Out] (2*((2*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a
^2 - b^2)^(5/2) + (b*Cos[2*(c + d*x)]*(16*a^2 - b^2 + 6*a*b*Sin[2*(c + d*x)
]))/((-4*a^2 + b^2)^2*(2*a + b*Sin[2*(c + d*x)])^2)))/d
```

Maple [B] time = 0.148, size = 640, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x)
```

```
[Out] 10/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^2/(16*a^4-8*a^2*b^2+b^4)*a*tan(d*x
+c)^3-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^4/(16*a^4-8*a^2*b^2+b^4)/a*ta
n(d*x+c)^3+16/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b/(16*a^4-8*a^2*b^2+b^4)*
a^2*tan(d*x+c)^2+7/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^3/(16*a^4-8*a^2*b^
2+b^4)*tan(d*x+c)^2-1/2/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^5/(16*a^4-8*a
^2*b^2+b^4)/a^2*tan(d*x+c)^2+22/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^2*a/(
16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^4/
a/(16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)+16/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*
b/(16*a^4-8*a^2*b^2+b^4)*a^2-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^3/(16*
a^4-8*a^2*b^2+b^4)+32/d/(16*a^4-8*a^2*b^2+b^4)/(4*a^2-b^2)^(1/2)*arctan((b+
2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))*a^2+4/d/(16*a^4-8*a^2*b^2+b^4)/(4*a^2-b^
2)^(1/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.17366, size = 2152, normalized size = 14.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(64*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*cos(d*x
 + c)^2 - 2*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^
2 + b^4)*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*sq
rt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)*
sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*cos(d*x
+ c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*sqrt(-4*
a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*s
in(d*x + c) - a^2)) - 12*(2*(4*a^3*b^2 - a*b^4)*cos(d*x + c)^3 - (4*a^3*b^2
 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^
6 - b^8)*d*cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*
cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b^5 - a*b^7)*d*cos(d*x +
 c)*sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4 - a^2*b^6)*d), 1/2*(64
*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*cos(d*x + c)^2
- 4*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^2 + b^4)
*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(4*a^2
 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4
*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)) - 12*(2*(4*a^3*b^2 - a*b^4)*cos(
d*x + c)^3 - (4*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((64*a^6*b^2 -
 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4
 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b
^5 - a*b^7)*d*cos(d*x + c)*sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4
 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14886, size = 340, normalized size = 2.28

$$\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)\left(8\,a^2+b^2\right)}{\left(16\,a^4-8\,a^2b^2+b^4\right)\sqrt{4\,a^2-b^2}}+\frac{20\,a^3b^2\tan(dx+c)^3-2\,ab^4\tan(dx+c)^3+32\,a^4b\tan(dx+c)^2+14\,a^2b^3\tan(dx+c)^2-b^5\tan(dx+c)^2+44}{\left(16\,a^6-8\,a^4b^2+a^2b^4\right)\left(a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)
/sqrt(4*a^2 - b^2)))*(8*a^2 + b^2)/((16*a^4 - 8*a^2*b^2 + b^4)*sqrt(4*a^2 -
 b^2)) + (20*a^3*b^2*tan(d*x + c)^3 - 2*a*b^4*tan(d*x + c)^3 + 32*a^4*b*tan
(d*x + c)^2 + 14*a^2*b^3*tan(d*x + c)^2 - b^5*tan(d*x + c)^2 + 44*a^3*b^2*t
an(d*x + c) - 2*a*b^4*tan(d*x + c) + 32*a^4*b - 2*a^2*b^3)/((16*a^6 - 8*a^4
*b^2 + a^2*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)^2))/d
```

3.573 $\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=265

$$-\frac{2\sqrt{2}a(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{15d\sqrt{2a+b\sin(2c+2dx)}}+\frac{(92a^2+9b^2)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{60\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] (-2*Sqrt[2]*a*b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(15*d) - (
b*Cos[2*c + 2*d*x]*(2*a + b*Sin[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) + ((92*
a^2 + 9*b^2)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*
c + 2*d*x]])/(60*Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - (2
*Sqrt[2]*a*(4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2
*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(15*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rubi [A] time = 0.365848, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2666, 2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sqrt{2}a(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{15d\sqrt{2a+b\sin(2c+2dx)}}+\frac{(92a^2+9b^2)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{60\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{b\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*Sqrt[2]*a*b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(15*d) - (
b*Cos[2*c + 2*d*x]*(2*a + b*Sin[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) + ((92*
a^2 + 9*b^2)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*
c + 2*d*x]])/(60*Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - (2
*Sqrt[2]*a*(4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2
*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(15*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```


Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}\,dx$$

$$= -\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d} + \frac{2}{5}\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\left(\frac{1}{8}\left(2\right.\right.$$

$$= -\frac{2\sqrt{2}ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx)(2a+b\sin(2c+}{20\sqrt{2}d}$$

$$= -\frac{2\sqrt{2}ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx)(2a+b\sin(2c+}{20\sqrt{2}d}$$

$$= -\frac{2\sqrt{2}ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx)(2a+b\sin(2c+}{20\sqrt{2}d}$$

$$= -\frac{2\sqrt{2}ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx)(2a+b\sin(2c+}{20\sqrt{2}d}$$

Mathematica [A] time = 1.91393, size = 202, normalized size = 0.76

$$\frac{-32a\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)+2\left(92a^2b+184a^3+18ab^2+9b^3\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}E\left(c+d\right.}{120d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(184*a^3 + 92*a^2*b + 18*a*b^2 + 9*b^3)*EllipticE[c - Pi/4 + d*x, (2*b)/
(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] - 32*a*(4*a^2 - b^2)*
EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/
```

```
(2*a + b)] - b*(88*a^2*Cos[2*(c + d*x)] + b*(28*a + 3*b*Sin[2*(c + d*x)])*S
in[4*(c + d*x)]))/(120*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

Maple [B] time = 3.159, size = 1138, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/60*(240*a^4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b
/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin
(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+64*((2*a+b*sin(2*d*x+2
*c))/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-
b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c
))*b/(2*a-b))^(1/2)*a^3*b-24*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(
sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*El
lipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^2
-16*a*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b)
)^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2
*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-9*((2*a+b*sin(2*d*x+2*c))/
(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2
*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/
(2*a-b))^(1/2)*b^4-368*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2
*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2
*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^4+56*((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))
^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+
sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^2*b^2+9*((2*a+b*sin(2*d*x+2*c))/(2*a-b))
^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(
1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))
^(1/2)*b^4+3*b^4*sin(2*d*x+2*c)^4+28*a*b^3*sin(2*d*x+2*c)^3+44*a^2*b^2*sin(
2*d*x+2*c)^2-3*b^4*sin(2*d*x+2*c)^2-28*sin(2*d*x+2*c)*a*b^3-44*a^2*b^2)/b/c
os(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos(dx+c)^4 - b^2\cos(dx+c)^2 - 2\,ab\cos(dx+c)\sin(dx+c) - a^2\right)\sqrt{b\cos(dx+c)\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin
(d*x + c) - a^2)*sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.574 $\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=212

$$-\frac{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{6\sqrt{2}d\sqrt{2a+b\sin(2c+2dx)}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}+\frac{2\sqrt{2}a\sqrt{2a+b\sin(\cdots}}{3d\sqrt{\cdots}}$$

```
[Out] -(b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(6*Sqrt[2]*d) + (2*Sqr
t[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*
d*x]])/(3*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - ((4*a^2 - b^2)*El
lipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2
*a + b)])/(6*Sqrt[2]*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rubi [A] time = 0.219818, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2666, 2656, 2752, 2663, 2661, 2655, 2653}

$$-\frac{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{6\sqrt{2}d\sqrt{2a+b\sin(2c+2dx)}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}+\frac{2\sqrt{2}a\sqrt{2a+b\sin(2c+2dx\cdots}}{3d\sqrt{\frac{2a+b\sin(\cdots}{2a+\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(6*Sqrt[2]*d) + (2*Sqr
t[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*
d*x]])/(3*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - ((4*a^2 - b^2)*El
lipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2
*a + b)])/(6*Sqrt[2]*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
```

```
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}\,dx$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d} + \frac{2}{3}\int \frac{\frac{1}{8}\left(12a^2+b^2\right)+ab\sin(2c+2a}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d} + \frac{1}{3}(4a)\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\,dx$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d} + \frac{\left(4a\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\right)\int\sqrt{\frac{a}{a+\frac{b}{2}}}}{3\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}}$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d} + \frac{2\sqrt{2}aE\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)\sqrt{2a+b\,s}}{3d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Mathematica [A] time = 1.50225, size = 167, normalized size = 0.79

$$\frac{-\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)-b\cos(2(c+dx))(2a+b\sin(2(c+dx)))+8a(2a+b)\sqrt{\frac{2a+b\,s}{}}}{6d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-(b*Cos[2*(c + d*x)]*(2*a + b*Sin[2*(c + d*x)])) + 8*a*(2*a + b)*EllipticE
[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)
] - (4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*
Sin[2*(c + d*x)])/(2*a + b)])/(6*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

Maple [B] time = 2.825, size = 844, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(24*a^3*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(
2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2
*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+4*((2*a+b*sin(2*d*x+2*c)
)/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/
(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*
b/(2*a-b))^(1/2)*a^2*b-6*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*
x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(
((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^2*a-((2*a
+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(
1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b
))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-32*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(
1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/
2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(
1/2)*a^3+8*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d
*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*
a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a*b^2+b^3*sin(2*d*x+2*c)^
3+2*a*b^2*sin(2*d*x+2*c)^2-sin(2*d*x+2*c)*b^3-2*a*b^2)/b/cos(2*d*x+2*c)/(4*
a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.575 $\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] (EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
/(Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])
```

Rubi [A] time = 0.0608275, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2666, 2655, 2653}

$$\frac{\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
/(Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx=\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\,dx$$
$$=\frac{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\int \sqrt{\frac{a}{a+\frac{b}{2}}+\frac{b\sin(2c+2dx)}{2\left(a+\frac{b}{2}\right)}}\,dx}{\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}}$$
$$=\frac{E\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Mathematica [A] time = 0.114354, size = 75, normalized size = 0.99

$$\frac{(2a+b)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] ((2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(
c + d*x)])/(2*a + b)])/(d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

Maple [B] time = 2.24, size = 312, normalized size = 4.1

$$-\frac{2\,a-b}{b\cos(2\,dx+2\,c)\,d}\sqrt{\frac{2\,a+b\sin(2\,dx+2\,c)}{2\,a-b}}\sqrt{-\frac{(\sin(2\,dx+2\,c)-1)\,b}{2\,a+b}}\sqrt{-\frac{(1+\sin(2\,dx+2\,c))\,b}{2\,a-b}}\left(2\,\mathrm{EllipticE}\left(\sqrt{\frac{2\,a}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x)
```

```
[Out] -((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/
2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)/b*(2*a-b)*(2*EllipticE(((2*a+b*sin
(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a+EllipticE(((2*a+b*si
n(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b-2*a*EllipticF(((2*a
+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))-EllipticF(((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b)/cos(2*d*x+2*c)
/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin(c+dx)\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.576 $\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}$$

```
[Out] (Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c +
 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rubi [A] time = 0.0665995, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2666, 2663, 2661}

$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c +
 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}\,dx$$
$$=\frac{\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}\int \frac{1}{\sqrt{\frac{a}{a+\frac{b}{2}}+\frac{b\sin(2c+2dx)}{2\left(a+\frac{b}{2}\right)}}}\,dx}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}$$
$$=\frac{\sqrt{2}F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{d\sqrt{2a+b\sin(2c+2dx)}}$$

Mathematica [A] time = 0.140003, size = 70, normalized size = 0.92

$$\frac{\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{d\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])
/(2*a + b)])/(d*Sqrt[a + (b*Sin[2*(c + d*x)])/2])
```

Maple [A] time = 2.135, size = 165, normalized size = 2.2

$$2\,\frac{2\,a-b}{b\cos(2\,dx+2\,c)\sqrt{4\,a+2\,b\sin(2\,dx+2\,c)}d}\sqrt{\frac{2\,a+b\sin(2\,dx+2\,c)}{2\,a-b}}\sqrt{-\frac{(\sin(2\,dx+2\,c)-1)\,b}{2\,a+b}}\sqrt{-\frac{(1+\sin(2\,dx+2}{2\,a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(2*a-b)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*
a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d
*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))/b/cos(2*d*x+2*c)/(4*a+2*b*
sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sin\left(c+dx\right)\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.577 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}+\frac{2\sqrt{2}\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] (2*Sqrt[2]*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*Sqrt[2*a + b*Sin[2*c + 2*d*
x]]) + (2*Sqrt[2]*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*S
in[2*c + 2*d*x]])/((4*a^2 - b^2)*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b
)])
```

Rubi [A] time = 0.0942034, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2666, 2664, 21, 2655, 2653}

$$\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}+\frac{2\sqrt{2}\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[2]*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*Sqrt[2*a + b*Sin[2*c + 2*d*
x]]) + (2*Sqrt[2]*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*S
in[2*c + 2*d*x]])/((4*a^2 - b^2)*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b
)])
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
```

```
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}\,dx$$

$$= \frac{2\sqrt{2}b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} - \frac{8\int \frac{-\frac{a}{2}-\frac{1}{4}b\sin(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}dx}{4a^2-b^2}$$

$$= \frac{2\sqrt{2}b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} + \frac{4\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2}$$

$$= \frac{2\sqrt{2}b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} + \frac{\left(4\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\right)\int \sqrt{\frac{a}{a+\frac{b}{2}} + \frac{b\sin}{2}}}{\left(4a^2-b^2\right)\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}}$$

$$= \frac{2\sqrt{2}b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} + \frac{2\sqrt{2}E\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+}}{\left(4a^2-b^2\right)d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Mathematica [A] time = 0.434222, size = 101, normalized size = 0.71

$$\frac{2\left((2a+b)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)+b\cos(2(c+dx))\right)}{d\left(4a^2-b^2\right)\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (2*(b*Cos[2*(c + d*x)] + (2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b
)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)]))/((4*a^2 - b^2)*d*Sqrt[a + (
b*Sin[2*(c + d*x)])/2])
```

Maple [B] time = 3.159, size = 570, normalized size = 4.

$$4\frac{1}{\left(4a^2-b^2\right)b\cos(2dx+2c)\sqrt{4a+2b\sin(2dx+2c)}d}\left(4a^2\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x)
```

```
[Out] 4/b*(4*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2
*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*
d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))-((2*a+b*sin(2*d*x+2*c))/(
2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*
a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(
2*a-b))^(1/2)*b^2-4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)
-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^2+((2*a+b*sin(
2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)
,((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*
d*x+2*c))*b/(2*a-b))^(1/2)*b^2-sin(2*d*x+2*c)^2*b^2+b^2)/(4*a^2-b^2)/cos(2*
d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^4-b^2\cos\left(dx+c\right)^2-2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*cos(d*x + c)*sin(d*x + c) + a)/(b^2*cos(d*x + c)^4 - b^2*c
os(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

3.578 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=295

$$-\frac{4\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{3d\left(4a^2-b^2\right)\sqrt{2a+b\sin(2c+2dx)}}+\frac{32\sqrt{2}ab\cos(2c+2dx)}{3d\left(4a^2-b^2\right)^2\sqrt{2a+b\sin(2c+2dx)}}+\frac{4\sqrt{2}b\cos(2c+2}{3d\left(4a^2-b^2\right)(2a+b\sin(}$$

```
[Out] (4*Sqrt[2]*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]
)^(3/2)) + (32*Sqrt[2]*a*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)^2*d*Sqrt[2*a
+ b*Sin[2*c + 2*d*x]]) + (32*Sqrt[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a
 + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(3*(4*a^2 - b^2)^2*d*Sqrt[(2*a + b*S
in[2*c + 2*d*x])/(2*a + b)]) - (4*Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(
2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(3*(4*a^2 - b^2)*d*Sq
rt[2*a + b*Sin[2*c + 2*d*x]])
```

Rubi [A] time = 0.300671, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2666, 2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{32\sqrt{2}ab\cos(2c+2dx)}{3d\left(4a^2-b^2\right)^2\sqrt{2a+b\sin(2c+2dx)}}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}-\frac{4\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+}\right.}{3d\left(4a^2-b^2\right)\sqrt{2a+b\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2]*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]
)^(3/2)) + (32*Sqrt[2]*a*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)^2*d*Sqrt[2*a
+ b*Sin[2*c + 2*d*x]]) + (32*Sqrt[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a
 + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(3*(4*a^2 - b^2)^2*d*Sqrt[(2*a + b*S
in[2*c + 2*d*x])/(2*a + b)]) - (4*Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(
2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(3*(4*a^2 - b^2)*d*Sq
rt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
```

x]

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}}\,dx$$

$$= \frac{4\sqrt{2}b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} - \frac{8\int \frac{-\frac{3a}{2}+\frac{1}{4}b\sin(2c+2dx)}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}\,dx}{3\left(4a^2-b^2\right)}$$

$$= \frac{4\sqrt{2}b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}ab\cos(2c+2dx)}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c+2dx}}$$

$$= \frac{4\sqrt{2}b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}ab\cos(2c+2dx)}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c+2dx}}$$

$$= \frac{4\sqrt{2}b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}ab\cos(2c+2dx)}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c+2dx}}$$

$$= \frac{4\sqrt{2}b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}ab\cos(2c+2dx)}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c+2dx}}$$

Mathematica [A] time = 1.5317, size = 201, normalized size = 0.68

$$-\frac{4\sqrt{2}\left((2a-b)(2a+b)^2\left(\frac{2a+b\sin(2(c+dx))}{2a+b}\right)^{3/2}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)+b\cos(2(c+dx))\left(-20a^2-8ab\sin(2(c+dx))\right)\right.}{3d\left(b^2-4a^2\right)^2(2a+b\sin(2(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-4*Sqrt[2]*((-8*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*(2*a + b*Sin[
2*(c + d*x)])^2)/Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] + (2*a - b)*(2*
a + b)^2*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*((2*a + b*Sin[2*(c + d*
x)])/(2*a + b))^(3/2) + b*Cos[2*(c + d*x)]*(-20*a^2 + b^2 - 8*a*b*Sin[2*(c
+ d*x)])))/(3*(-4*a^2 + b^2)^2*d*(2*a + b*Sin[2*(c + d*x)])^(3/2))
```

Maple [B] time = 3.382, size = 1554, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x)
```

```
[Out] 8/3*(8*sin(2*d*x+2*c)*cos(2*d*x+2*c)^2*a*b^3-(-b/(2*a-b)*sin(2*d*x+2*c)-b/(
2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(b/(2*a-b)*sin(2*
d*x+2*c)+2*a/(2*a-b))^(1/2)*b*(32*EllipticE((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(
2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^3-8*EllipticE((b/(2*a-b)*sin(2*d*x
+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a*b^2-24*EllipticF((b/(2*
a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^3-4*Ellip
ticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*
a^2*b+6*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*
a+b))^(1/2))*a*b^2+EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),(
(2*a-b)/(2*a+b))^(1/2))*b^3)*sin(2*d*x+2*c)+(20*a^2*b^2-b^4)*cos(2*d*x+2*c)
^2+48*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin
(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d
*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a^4+8*
(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin(2*d*x
+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c
)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a^3*b-12*(b/
(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin(2*d*x+2*
c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c)+b
/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a^2*b^2-2*(b/(2
*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)
+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(
2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a*b^3-64*(b/(2*a-
b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^
(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*EllipticE((b/(2*a-b)*sin(
```

```
2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^4+16*(b/(2*a-b)*si
n(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)
*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*EllipticE((b/(2*a-b)*sin(2*d*x
+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^2*b^2)/(2*a+b*sin(2*d*x
+2*c))/(4*a^2-b^2)^2/b/cos(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\cos(dx+c)\sin(dx+c)+a}}{3\,ab^2\cos(dx+c)^4-3\,ab^2\cos(dx+c)^2-a^3+\left(b^3\cos(dx+c)^5-b^3\cos(dx+c)^3-3\,a^2b\cos(dx+c)\right)\sin(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*cos(d*x + c)*sin(d*x + c) + a)/(3*a*b^2*cos(d*x + c)^4 - 3
*a*b^2*cos(d*x + c)^2 - a^3 + (b^3*cos(d*x + c)^5 - b^3*cos(d*x + c)^3 - 3*
a^2*b*cos(d*x + c))*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

3.579 $\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=461

$$-\frac{3x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{3ix\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3ix\text{PolyLog}\left(3,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+}\right.}{2\sqrt{4a^2-b^2}}$$

```
[Out] ((-I)*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
- b^2] + (I*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2]
)])/(2*Sqrt[4*a^2 - b^2]) + (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt
[4*a^2 - b^2])])/(2*Sqrt[4*a^2 - b^2]) - (((3*I)/2)*x*PolyLog[3, (I*b*E^((2
*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (((3*I)/2)*x*PolyLo
g[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (3*P
olyLog[4, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*a^2 - b^2
]) - (3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*
a^2 - b^2])
```

Rubi [A] time = 0.629673, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4584, 3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{3ix\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3ix\text{PolyLog}\left(3,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+}\right.}{2\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cos[x]*Sin[x]),x]
```


```
[Out] ((-I)*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
- b^2] + (I*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2]
)])/(2*Sqrt[4*a^2 - b^2]) + (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt
[4*a^2 - b^2])])/(2*Sqrt[4*a^2 - b^2]) - (((3*I)/2)*x*PolyLog[3, (I*b*E^((2
*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (((3*I)/2)*x*PolyLo
g[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (3*P
olyLog[4, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*a^2 - b^2
]) - (3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*
a^2 - b^2])
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sin[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^3}{a+\frac{1}{2}b\sin(2x)}\,dx$$

$$= 2\int \frac{e^{2ix}x^3}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{2ix}x^3}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} + \frac{(2ib)\int \frac{e^{2ix}x^3}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{(3i)\int x^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)dx}{\sqrt{4a^2-b^2}} - \cdots$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \cdots$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \cdots$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \cdots$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \cdots$$

Mathematica [A] time = 0.863064, size = 340, normalized size = 0.74

$$\frac{-6x^2\text{PolyLog}\left(2,-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)+6x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)-6ix\text{PolyLog}\left(3,-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)+6ix\text{PolyLog}\left(3,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2+}}\right.}{4\sqrt{4a^2-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-4*I)*x^3*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (4*I)*x
^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - 6*x^2*PolyLog[2,
((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + 6*x^2*PolyLog[2, (I*b*E^
((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (6*I)*x*PolyLog[3, ((-I)*b*E^((2*I)
*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (6*I)*x*PolyLog[3, (I*b*E^((2*I)*x))/(2*
a + Sqrt[4*a^2 - b^2])] + 3*PolyLog[4, ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*
a^2 - b^2])] - 3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/(
4*Sqrt[4*a^2 - b^2])
```

Maple [B] time = 0.148, size = 2282, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cos(x)*sin(x)),x)
```

```
[Out] -2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*
a-(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x^3-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a
-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2)*a*x^4+12/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+
b)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))
)*a^2*x-3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*polylog(3,b*exp(2
*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x+3/(8*a^2-2*b^2)/(-2*I*a-(-(2
*a+b)*(2*a-b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/
2)))*(-(2*a+b)*(2*a-b))^(1/2)*a+6/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^
(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*(-(2*a+b)*
(2*a-b))^(1/2)*a*x^2-6/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*poly
log(2,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*(-(2*a+b)*(2*a-b))^(1
/2)*a*x^2+I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*b^2*x^4+I/(8*a^
2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*b^2*x^4-4*I/(8*a^2-2*b^2)/(-2*I*
a-(-(2*a+b)*(2*a-b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*
a-b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*a^2-
```

```
3/2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*polylog(4,b*exp(2*I*x
)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*b^2-4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b
)*(2*a-b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2
))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a^2-3/2*I/(8*a
^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+
(-(2*a+b)*(2*a-b))^(1/2)))*b^2-3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(
1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*(-(2*a+b)*(
2*a-b))^(1/2)*a+8/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*ex
p(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a^2*x^3-2/(8*a^2-2*b^2)/(-2*I*a
+(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/
2)))*b^2*x^3+2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*(-(2*a+b)*(2
*a-b))^(1/2)*a*x^4+12/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polyl
og(3,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a^2*x-3/(8*a^2-2*b^2)/
(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*
(2*a-b))^(1/2)))*b^2*x+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*ln
(1-b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*a^2*x^3-4*I/(8*a^2-2*b^2
)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*
a-b))^(1/2)))*(-(2*a+b)*(2*a-b))^(1/2)*a*x^3-6*I/(8*a^2-2*b^2)/(-2*I*a-(-(2
*a+b)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/
2)))*(-(2*a+b)*(2*a-b))^(1/2)*a*x+6*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-
b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*(-(2*a
+b)*(2*a-b))^(1/2)*a*x+4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*
ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*(-(2*a+b)*(2*a-b))^(1/
2)*a*x^3-12*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*e
xp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a^2*x^2+3*I/(8*a^2-2*b^2)/(-2*
I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a
-b))^(1/2)))*b^2*x^2-12*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*p
olylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*a^2*x^2+3*I/(8*a^2
-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-
(2*a+b)*(2*a-b))^(1/2)))*b^2*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

Fricas [C] time = 5.08906, size = 7900, normalized size = 17.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x)
- 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2
- b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/
2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b
^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x^3*
sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x)
+ I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) +
 2*I*a)/b) + 2*b)/b) - 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*co
s(x) + 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqr
t(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^3*sqrt(-(4*a^2
 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x)
)*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) +
2*b)/b) + 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*cos(x) + 4*a*si
n(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4
*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*l
og(1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2
 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b
*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*c
os(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/
b^2) - 2*I*a)/b) + 2*b)/b) + 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*
((4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)
/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 6*I*b*x
^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*
cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/
b^2) + 2*I*a)/b) + 2*b)/b + 1) + 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-
1/2*((4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 -
b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 6*
I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) + 4*a*sin(x) +
2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
 b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*d
ilog(-1/2*((4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*
a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1)
 - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) + 4*a*sin(
x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a
^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^
```

2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(
-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b
 + 1) - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a
sin(x) - 2(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt
(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) + 12*b*x*sqrt(-(4*a^2 - b^2)
/b^2)*polylog(3, 1/2*(4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))
*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b)
 + 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x
) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^
2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1
/2*(4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^
2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*b*x*sqrt(-(4
*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) +
 I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) +
2*I*a)/b)/b) + 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(4*I*a*cos(x)
 - 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b
*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) + 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*
polylog(3, 1/2*(-4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt
(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 12*
b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(4*I*a*cos(x) + 4*a*sin(x) + 2*
(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b
^2)/b^2) - 2*I*a)/b)/b) - 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-
4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b
^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 12*I*b*sqrt(-(4*a^2
 - b^2)/b^2)*polylog(4, 1/2*(4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*
sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)
/b)/b) - 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(-4*I*a*cos(x) - 4*
a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt
(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polyl
og(4, 1/2*(4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a
^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*I*b*s
qrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(-4*I*a*cos(x) + 4*a*sin(x) + 2*(b*c
os(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/
b^2) + 2*I*a)/b)/b) + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(4*I*a
*cos(x) - 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*
sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) + 12*I*b*sqrt(-(4*a^2 - b^2
)/b^2)*polylog(4, 1/2*(-4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) + I*b*sin(x
))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b
) + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(4*I*a*cos(x) + 4*a*sin(
x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*
a^2 - b^2)/b^2) - 2*I*a)/b)/b) + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4,
 1/2*(-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 -
 b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b))/(4*a^2 - b^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b\sin(x)\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(x**3/(a + b*sin(x)*cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

3.580 $\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=340

$$-\frac{x\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{x\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}-\frac{i\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{i\text{PolyLog}\left(3,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix^2}{}$$

```
[Out] ((-I)*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
- b^2] + (I*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/
Sqrt[4*a^2 - b^2] + (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2
])])/Sqrt[4*a^2 - b^2] - ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*
a^2 - b^2])])/Sqrt[4*a^2 - b^2] + ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a
+ Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2]
```

Rubi [A] time = 0.536579, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4584, 3323, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{x\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}-\frac{i\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{i\text{PolyLog}\left(3,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[x]*Sin[x]),x]
```


```
[Out] ((-I)*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
- b^2] + (I*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/
Sqrt[4*a^2 - b^2] + (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2
])])/Sqrt[4*a^2 - b^2] - ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*
a^2 - b^2])])/Sqrt[4*a^2 - b^2] + ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a
+ Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2]
```


Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sin[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^2}{a+\frac{1}{2}b\sin(2x)}\,dx$$

$$= 2\int \frac{e^{2ix}x^2}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{2ix}x^2}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} + \frac{(2ib)\int \frac{e^{2ix}x^2}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{(2i)\int x\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)dx}{\sqrt{4a^2-b^2}} - \cdots$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}} \cdots\right)}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}} \cdots\right)}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}} \cdots\right)}{\sqrt{4a^2-b^2}}$$

Mathematica [A] time = 0.745741, size = 256, normalized size = 0.75

$$-\frac{i\left(-2ix\text{PolyLog}\left(2,-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)+2ix\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)+\text{PolyLog}\left(3,-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)-\text{PolyLog}\left(3,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a} \cdots\right)\right)}{2\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I/2)*(2*x^2*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] - 2*x^
2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (2*I)*x*PolyLog[2,
 ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (2*I)*x*PolyLog[2, (I*b
*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] + PolyLog[3, ((-I)*b*E^((2*I)*x))/
(-2*a + Sqrt[4*a^2 - b^2])] - PolyLog[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^
2 - b^2])]))/Sqrt[4*a^2 - b^2]
```

Maple [B] time = 0.112, size = 1782, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(x)*sin(x)),x)
```

```
[Out] 8/3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2
)*a*x^3-16/3*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*a^2*x^3+2*I/
(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*
I*a+(-(2*a+b)*(2*a-b))^(1/2)))*(-(2*a+b)*(2*a-b))^(1/2)*a-4*I/(8*a^2-2*b^2)
/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a
-b))^(1/2)))*(-(2*a+b)*(2*a-b))^(1/2)*a*x^2+8/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+
b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a^2
*x^2-2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-
2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x^2+4/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*
(2*a-b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*(
-(2*a+b)*(2*a-b))^(1/2)*a*x-2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1
/2))*(-(2*a+b)*(2*a-b))^(1/2)*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a
-b))^(1/2)))*a+4/3*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*b^2*x^
3-8*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*exp(2*I*x
)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a^2*x+4/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b
)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))
*a^2-1/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*
x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*b^2-8/3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+
b)*(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2)*a*x^3+4/3*I/(8*a^2-2*b^2)/(-2*I
*a-(-(2*a+b)*(2*a-b))^(1/2))*b^2*x^3-8*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2
*a-b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*a^2
*x+2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*exp(2*I*
x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+
b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*a^2
*x^2-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-
2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x^2-4/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*
(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2)*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(
2*a+b)*(2*a-b))^(1/2)))*a*x+2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1
/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x-16/3*I
/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*a^2*x^3+4*I/(8*a^2-2*b^2)/
(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-
b))^(1/2)))*(-(2*a+b)*(2*a-b))^(1/2)*a*x^2+4/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b
)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))
*a^2-1/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*polylog(3,b*exp(2*I*
x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

Fricas [C] time = 4.78926, size = 5913, normalized size = 17.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x)
- 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2
- b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/
2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b
^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x^2*
sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x)
+ I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) +
 2*I*a)/b) + 2*b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*co
s(x) + 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqr
t(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^2*sqrt(-(4*a^2
 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x)
)*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) +
2*b)/b) + 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*cos(x) + 4*a*si
n(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4
*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*l
og(1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2
 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b
*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*c
os(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/
b^2) - 2*I*a)/b) + 2*b)/b) + 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((
4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b
```

```
^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 4*I*b*x*s
qrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(
x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2)
 + 2*I*a)/b) + 2*b)/b + 1) + 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((
4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b
^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 4*I*b*x*
sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos
(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^
2) + 2*I*a)/b) + 2*b)/b + 1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*
((4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)
/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 4*I*b*x
*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) + 4*a*sin(x) - 2*(b*co
s(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^
2) - 2*I*a)/b) + 2*b)/b + 1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*
((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)
/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 4*I*b*
x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*c
os(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/
b^2) - 2*I*a)/b) + 2*b)/b + 1) + 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/
2*(4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2
)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 4*b*sqrt(-(4*a^2
- b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*
sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)
/b)/b) - 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(4*I*a*cos(x) - 4*a*si
n(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(
4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3,
1/2*(-4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 -
b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 4*b*sqrt(-(4*
a^2 - b^2)/b^2)*polylog(3, 1/2*(4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) + I
*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I
*a)/b)/b) + 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) + 4*
a*sin(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt
(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(
3, 1/2*(4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2
- b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 4*b*sqrt(-(
4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x)
- I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) -
 2*I*a)/b)/b))/(4*a^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b \sin(x) \cos(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(x**2/(a + b*sin(x)*cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

3.581 $\int \frac{x}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=225

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

```
[Out] ((-I)*x*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 -
b^2] + (I*x*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^
2 - b^2] - PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])]/(2*Sqrt[
4*a^2 - b^2]) + PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])]/(2*
Sqrt[4*a^2 - b^2])
```

Rubi [A] time = 0.318976, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4584, 3323, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I)*x*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 -
b^2] + (I*x*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^
2 - b^2] - PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])]/(2*Sqrt[
4*a^2 - b^2]) + PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])]/(2*
Sqrt[4*a^2 - b^2])
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sin[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
```

```
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x}{a+\frac{1}{2}b\sin(2x)}\,dx$$
$$= 2\int \frac{e^{2ix}x}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx$$
$$= -\frac{(2ib)\int \frac{e^{2ix}x}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} + \frac{(2ib)\int \frac{e^{2ix}x}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}$$
$$= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{i\int\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)dx}{\sqrt{4a^2-b^2}} - \frac{i\int\log\left(1-\right.}{\sqrt{4}}$$
$$= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{ibx}{2a-\sqrt{4a^2-b^2}}\right)}{x}\,dx,x,e^{2ix}\right)}{2\sqrt{4a^2-b^2}} - $$
$$= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}$$

Mathematica [B] time = 1.43413, size = 788, normalized size = 3.5

$$\frac{1}{2}\left(\frac{\pi\tan^{-1}\left(\frac{2a\tan(x)+b}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{i\left(\text{PolyLog}\left(2,\frac{\left(2a-i\sqrt{b^2-4a^2}\right)\left(-\sqrt{b^2-4a^2}\cot\left(x+\frac{\pi}{4}\right)+2a+b\right)}{b\left(\sqrt{b^2-4a^2}\cot\left(x+\frac{\pi}{4}\right)+2a+b\right)}\right)-\text{PolyLog}\left(2,\frac{\left(2a+i\sqrt{b^2-4a^2}\right)\left(-\sqrt{b^2-4a^2}\cot\left(x+\right.\right.}{b\left(\sqrt{b^2-4a^2}\cot\left(x+\frac{\pi}{4}\right)+2a+\right.}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((Pi*ArcTan[(b + 2*a*Tan[x])/Sqrt[4*a^2 - b^2]])/Sqrt[4*a^2 - b^2] + (2*Arc
Cos[(-2*a)/b]*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] + (Pi -
 4*x)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] - (ArcCos[(-2*a
)/b] + (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2
*a + b)*(-2*a + b - I*Sqrt[-4*a^2 + b^2])*(1 + I*Cot[Pi/4 + x]))/(b*(2*a +
b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))] - (ArcCos[(-2*a)/b] - (2*I)*ArcTanh
[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2*a + b)*((2*I)*a - I
*b + Sqrt[-4*a^2 + b^2])*(I + Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b
^2]*Cot[Pi/4 + x]))] + (ArcCos[(-2*a)/b] + (2*I)*(ArcTanh[((2*a - b)*Cot[Pi
/4 + x])/Sqrt[-4*a^2 + b^2]] + ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^
```

```
2 + b^2]]))*Log[((-1)^(1/4)*Sqrt[-4*a^2 + b^2])/(2*Sqrt[b]*E^(I*x)*Sqrt[a +
 b*Cos[x]*Sin[x]])] + (ArcCos[(-2*a)/b] - (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4
 + x])/Sqrt[-4*a^2 + b^2]] - (2*I)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-
4*a^2 + b^2]])*Log[-((-1)^(3/4)*Sqrt[-4*a^2 + b^2]*E^(I*x))/(2*Sqrt[b]*Sqrt
[a + b*Cos[x]*Sin[x]])] + I*(PolyLog[2, ((2*a - I*Sqrt[-4*a^2 + b^2])*(2*a
+ b - Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]*C
ot[Pi/4 + x]))] - PolyLog[2, ((2*a + I*Sqrt[-4*a^2 + b^2])*(2*a + b - Sqrt[
-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x
]))]))/Sqrt[-4*a^2 + b^2])/2
```

Maple [B] time = 0.111, size = 1284, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] 4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2
)*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a*x-4/(8*a^2-2*b^2)/
(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2)*a*x^2+8/(8*a^2-2
*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a+b)
*(2*a-b))^(1/2)))*a^2*x+2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*(
-(2*a+b)*(2*a-b))^(1/2)*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(
1/2)))*a-2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x
)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x-4*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a
+b)*(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2)*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2
*a+b)*(2*a-b))^(1/2)))*a*x-8*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/
2))*a^2*x^2+2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*b^2*x^2-8*I
/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*a^2*x^2-4*I/(8*a^2-2*b^2)/
(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*
(2*a-b))^(1/2)))*a^2-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*ln(1
-b*exp(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*b^2*x+4/(8*a^2-2*b^2)/(-2*
I*a+(-(2*a+b)*(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2)*a*x^2-2/(8*a^2-2*b^2
)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*(-(2*a+b)*(2*a-b))^(1/2)*polylog(2,b*ex
p(2*I*x)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2)))*a+8/(8*a^2-2*b^2)/(-2*I*a+(-(2*
a+b)*(2*a-b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a
^2*x+2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*b^2*x^2+I/(8*a^2-2
*b^2)/(-2*I*a-(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2
*a+b)*(2*a-b))^(1/2)))*b^2-4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/
2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2)))*a^2+I/(8*a^2-
2*b^2)/(-2*I*a+(-(2*a+b)*(2*a-b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(
```

```
2*a+b)*(2*a-b))^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cos(x)*sin(x) + a), x)
```

Fricas [B] time = 4.88477, size = 3969, normalized size = 17.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x) -
2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 -
b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((
-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/
b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x*sqrt(-
(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) + I*b*
sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a
)/b) + 2*b)/b) - 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*cos(x) + 4
*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sq
rt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x*sqrt(-(4*a^2 - b^2)/b^
2)*log(1/2*((4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4
*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) +
2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I*a*cos(x) + 4*a*sin(x) - 2*(b*
cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/
b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a
*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*
sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x*sqrt(-(4*a^
2 - b^2)/b^2)*log(1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(
x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)
```

```
 + 2*b)/b) + 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a
*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(
-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 2*I*b*sqrt(-(4*a^2 - b^2)/b
^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqr
t(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/
b + 1) + 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) - 4*a*sin
(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4
*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)
*dilog(-1/2*((-4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-
(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b
+ 1) - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) - 4*a*sin(x
) + 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^
2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*di
log(-1/2*((-4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) + I*b*sin(x))*sqrt(-(4*
a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1)
 - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) +
2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
 b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog
(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2
 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1))/
(4*a^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a+b\sin(x)\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(x/(a + b*sin(x)*cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cos(x)\sin(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(x)*sin(x) + a), x)
```

3.582 $\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(a + (b*Sin[2*x])/2)), x]
```

Rubi [A] time = 0.0853994, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Cos[x]*Sin[x])),x]
```

```
[Out] Defer[Int][1/(x*(a + (b*Sin[2*x])/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx=\int \frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)}\,dx$$

Mathematica [A] time = 1.62831, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Cos[x]*Sin[x])),x]
```

```
[Out] Integrate[1/(x*(a + b*Cos[x]*Sin[x])), x]
```

Maple [A] time = 0.107, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cos\left(x\right)\sin\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] int(1/x/(a+b*cos(x)*sin(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(x\right)\sin\left(x\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{bx\cos\left(x\right)\sin\left(x\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*cos(x)*sin(x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(x\right)\cos\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(1/(x*(a + b*sin(x)*cos(x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(x\right)\sin\left(x\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

3.583 $\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$

Optimal. Leaf size=78

$$\frac{b^2(1-n)\text{Unintegrable}\left((bx)^{-n}\sin^{n-2}(ax),x\right)}{a^2c^2}+\frac{b(bx)^{1-n}\sin^{n-1}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}$$

```
[Out] (b*(b*x)^(1 - n)*Sin[a*x]^(-1 + n))/(a^2*(a*c^2*x*Cos[a*x] - c^2*Sin[a*x]))
 + (b^2*(1 - n)*Unintegrable[Sin[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi [A] time = 0.155194, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

```
[Out] (b*(b*x)^(1 - n)*Sin[a*x]^(-1 + n))/(a^2*(a*c^2*x*Cos[a*x] - c^2*Sin[a*x]))
 + (b^2*(1 - n)*Defer[Int][Sin[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi steps

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx=\frac{b(bx)^{1-n}\sin^{-1+n}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}+\frac{\left(b^2(1-n)\right)\int (bx)^{-n}\sin^{-2+n}(ax)\,dx}{a^2c^2}$$

Mathematica [A] time = 5.51458, size = 0, normalized size = 0.

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

```
[Out] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2, x]
```

Maple [A] time = 0.951, size = 0, normalized size = 0.

$$\int \frac{(bx)^{2-n} (\sin (ax))^n}{(acx \cos (ax) - c \sin (ax))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

```
[Out] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^{-n+2} \sin (ax)^n}{(acx \cos (ax) - c \sin (ax))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="maxima")
```

```
[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx)^{-n+2} \sin (ax)^n}{2\,ac^2x \cos (ax) \sin (ax) - \left(a^2c^2x^2 - c^2\right) \cos (ax)^2 - c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(b*x)^(-n + 2)*sin(a*x)^n/(2*a*c^2*x*cos(a*x)*sin(a*x) - (a^2*c^2
*x^2 - c^2)*cos(a*x)^2 - c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(2-n)*sin(a*x)**n/(a*c*x*cos(a*x)-c*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^{-n+2}\sin(ax)^n}{(acx\cos(ax) - c\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="giac")
```

```
[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

3.584 $\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$

Optimal. Leaf size=78

$$\frac{b^2(1-n)\text{Unintegrable}\left((bx)^{-n}\cos^{n-2}(ax),x\right)}{a^2c^2}-\frac{b(bx)^{1-n}\cos^{n-1}(ax)}{a^2\left(ac^2x\sin(ax)+c^2\cos(ax)\right)}$$

```
[Out] -((b*(b*x)^(1 - n)*Cos[a*x]^(-1 + n))/(a^2*(c^2*Cos[a*x] + a*c^2*x*Sin[a*x]
))) + (b^2*(1 - n)*Unintegrable[Cos[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi [A] time = 0.146015, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

```
[Out] -((b*(b*x)^(1 - n)*Cos[a*x]^(-1 + n))/(a^2*(c^2*Cos[a*x] + a*c^2*x*Sin[a*x]
))) + (b^2*(1 - n)*Defer[Int][Cos[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi steps

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx=-\frac{b(bx)^{1-n}\cos^{-1+n}(ax)}{a^2\left(c^2\cos(ax)+ac^2x\sin(ax)\right)}+\frac{\left(b^2(1-n)\right)\int (bx)^{-n}\cos^{-2+n}(ax)\,dx}{a^2c^2}$$

Mathematica [A] time = 4.91937, size = 0, normalized size = 0.

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

```
[Out] Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2, x]
```

Maple [A] time = 0.837, size = 0, normalized size = 0.

$$\int \frac{(bx)^{2-n} (\cos(ax))^n}{(c\cos(ax) + acx\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

```
[Out] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^{-n+2} \cos(ax)^n}{(acx\sin(ax) + c\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="maxima")
```

```
[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx)^{-n+2}\cos(ax)^n}{a^2c^2x^2 + 2\,ac^2x\cos(ax)\sin(ax) - \left(a^2c^2x^2 - c^2\right)\cos(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="fricas")
```

```
[Out] integral((b*x)^(-n + 2)*cos(a*x)^n/(a^2*c^2*x^2 + 2*a*c^2*x*cos(a*x)*sin(a*
x) - (a^2*c^2*x^2 - c^2)*cos(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(2-n)*cos(a*x)**n/(c*cos(a*x)+a*c*x*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^{-n+2}\cos(ax)^n}{(acx\sin(ax)+c\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="giac")
```

```
[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

3.585 $\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=175

$$-\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\sin^4(ax)}{a^2x^5}+\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}+\frac{a^2}{x}+\frac{32a^2\sin^4(ax)}{3x}-\frac{10a^2\sin^2(ax)}{x}-\frac{4\sin^4(ax)}{3x^3}$$

```
[Out] a^2/x + (a*Cos[a*x]*Sin[a*x])/x^2 + Sin[a*x]^2/x^3 - (10*a^2*Sin[a*x]^2)/x
+ (Cos[a*x]*Sin[a*x]^3)/(a*x^4) - (8*a*Cos[a*x]*Sin[a*x]^3)/(3*x^2) + Sin[a
*x]^4/(a^2*x^5) - (4*Sin[a*x]^4)/(3*x^3) + (32*a^2*Sin[a*x]^4)/(3*x) + Sin[
a*x]^5/(a^2*x^5*(a*x*Cos[a*x] - Sin[a*x])) - (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```

Rubi [A] time = 0.296565, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4598, 3314, 30, 3313, 12, 3299}

$$-\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\sin^4(ax)}{a^2x^5}+\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}+\frac{a^2}{x}+\frac{32a^2\sin^4(ax)}{3x}-\frac{10a^2\sin^2(ax)}{x}-\frac{4\sin^4(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```


```
[Out] a^2/x + (a*Cos[a*x]*Sin[a*x])/x^2 + Sin[a*x]^2/x^3 - (10*a^2*Sin[a*x]^2)/x
+ (Cos[a*x]*Sin[a*x]^3)/(a*x^4) - (8*a*Cos[a*x]*Sin[a*x]^3)/(3*x^2) + Sin[a
*x]^4/(a^2*x^5) - (4*Sin[a*x]^4)/(3*x^3) + (32*a^2*Sin[a*x]^4)/(3*x) + Sin[
a*x]^5/(a^2*x^5*(a*x*Cos[a*x] - Sin[a*x])) - (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```


Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
```

```
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))} - \frac{5\int \frac{\sin^4(ax)}{x^6}\,dx}{a^2}$$
$$= \frac{\cos(ax)\sin^3(ax)}{ax^4} + \frac{\sin^4(ax)}{a^2x^5} + \frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))} - 3\int \frac{\sin^2(ax)}{x^4}\,dx + 4\int$$
$$= \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \frac{8a\cos(ax)\sin^3(ax)}{3x^2} + \frac{\sin^4(ax)}{a^2x^5} -$$
$$= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \frac{8a\cos(ax)\sin}{3x^2}$$
$$= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \frac{8a\cos(ax)\sin}{3x^2}$$
$$= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \frac{8a\cos(ax)\sin}{3x^2}$$

Mathematica [A] time = 1.47984, size = 198, normalized size = 1.13

$$\frac{-32a^3x^3\text{Si}(2ax)(ax\cos(ax)-\sin(ax)) + 256a^3x^3\text{Si}(4ax)(ax\cos(ax)-\sin(ax)) - 12a^2x^2\sin(ax) + 44a^2x^2\sin(3ax) - 24a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (8*a*x*Cos[a*x] - 8*a^3*x^3*Cos[a*x] - 12*a*x*Cos[3*a*x] + 24*a^3*x^3*Cos[3
*a*x] + 4*a*x*Cos[5*a*x] + 32*a^3*x^3*Cos[5*a*x] + 10*Sin[a*x] - 12*a^2*x^2
*Sin[a*x] - 5*Sin[3*a*x] + 44*a^2*x^2*Sin[3*a*x] + Sin[5*a*x] - 24*a^2*x^2*
Sin[5*a*x] - 32*a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[2*a*x] + 256*
a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(a*x*Cos[a*x]
 - Sin[a*x]))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{(\sin(ax))^6}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.73354, size = 478, normalized size = 2.73

$$\frac{4\left(8a^3x^3+ax\right)\cos(ax)^5-2\left(17a^3x^3+4ax\right)\cos(ax)^3+\left(16a^4x^4\operatorname{Si}(4ax)-2a^4x^4\operatorname{Si}(2ax)+5a^3x^3+4ax\right)\cos(ax)-\cdots}{3\left(ax^4\cos(ax)-x^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*(8*a^3*x^3 + a*x)*cos(a*x)^5 - 2*(17*a^3*x^3 + 4*a*x)*cos(a*x)^3 + (
16*a^4*x^4*sin_integral(4*a*x) - 2*a^4*x^4*sin_integral(2*a*x) + 5*a^3*x^3
+ 4*a*x)*cos(a*x) - (16*a^3*x^3*sin_integral(4*a*x) - 2*a^3*x^3*sin_integra
l(2*a*x) + (24*a^2*x^2 - 1)*cos(a*x)^4 + 5*a^2*x^2 - (29*a^2*x^2 - 2)*cos(a
*x)^2 - 1)*sin(a*x))/(a*x^4*cos(a*x) - x^3*sin(a*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**6/x**4/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.58733, size = 9918, normalized size = 56.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x
)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2
*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan
(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2*
a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*ta
n(2*a*x)^2*tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)
^2*tan(a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(
a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2
 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^8*x^8*sin_i
ntegral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(
4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*
x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))
*tan(2*a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*t
an(2*a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*ta
n(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 +
32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^8
*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*i
mag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_
part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integ
ral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(a*
x)^2*tan(1/2*a*x)^2 + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^
2*tan(a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*
a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_integral(-2*a*x))*
tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4
*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*
x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*ta
n(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^
8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 4*a^8*x^8*imag_part(cos
_integral(-2*a*x))*tan(2*a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x)
)*tan(2*a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^8*x^8*si
n_integral(2*a*x)*tan(2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*
tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^8*x^
```

8*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^8*x^8*sin_integral(2*a*x)*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 64*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - 20*a^7*x^7*tan(2*a*x)^2*tan(1/2*a*x)^2 + 20*a^7*x^7*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x)) + 4*a^8*x^8*imag_part(cos_integral(2*a*x)) - 4*a^8*x^8*imag_part(cos_integral(-2*a*x)) + 32*a^8*x^8*imag_part(cos_integral(-4*a*x)) - 64*a^8*x^8*sin_integral(4*a*x) + 8*a^8*x^8*sin_integral(2*a*x) - 64*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 - 24*a^6*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 64*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^6*x^6*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 + 64*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^

2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^6*x^6*ta
n(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 20*a^7*x^7*tan(2*a*x)^2 - 20*a^7*x^7*t
an(a*x)^2 + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(1/2*a*x) - 8*a^7*
x^7*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_i
ntegral(-2*a*x))*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4*a*x))*
tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*x)*tan(1/2*a*x) - 16*a^7*x^7*si
n_integral(2*a*x)*tan(1/2*a*x) + 128*a^5*x^5*imag_part(cos_integral(4*a*x))
*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^5*x^5*imag_part(cos_integral(2
*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_inte
gral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 128*a^5*x^5*imag_part(
cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 256*a^5*x^5*si
n_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^5*x^5*sin_int
egral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 12*a^7*x^7*tan(1/2*a*x)
^2 - 24*a^5*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_pa
rt(cos_integral(4*a*x))*tan(2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(2*a
*x))*tan(2*a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2
+ 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 - 128*a^6*x^6*sin
_integral(4*a*x)*tan(2*a*x)^2 + 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2
 + 4*a^6*x^6*tan(2*a*x)^2*tan(a*x) - 64*a^6*x^6*imag_part(cos_integral(4*a*
x))*tan(a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 8*a^
6*x^6*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 64*a^6*x^6*imag_part(cos
_integral(-4*a*x))*tan(a*x)^2 - 128*a^6*x^6*sin_integral(4*a*x)*tan(a*x)^2
+ 16*a^6*x^6*sin_integral(2*a*x)*tan(a*x)^2 - 8*a^6*x^6*tan(2*a*x)*tan(a*x)
^2 - 40*a^6*x^6*tan(2*a*x)^2*tan(1/2*a*x) + 40*a^6*x^6*tan(a*x)^2*tan(1/2*a
*x) + 64*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 8*a^6*x^6*
imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 8*a^6*x^6*imag_part(cos_int
egral(-2*a*x))*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*
tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 16*a^6*x^
6*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 8*a^6*x^6*tan(2*a*x)*tan(1/2*a*x)^2
- 4*a^6*x^6*tan(a*x)*tan(1/2*a*x)^2 + 32*a^4*x^4*imag_part(cos_integral(4*a
*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integ
ral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(co
s_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*ima
g_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^
4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^4*x^
4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 12*a^7*x^7 +
 24*a^5*x^5*tan(2*a*x)^2*tan(a*x)^2 + 128*a^5*x^5*imag_part(cos_integral(4*
a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 16*a^5*x^5*imag_part(cos_integral(2*a*x))
*tan(2*a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan
(2*a*x)^2*tan(1/2*a*x) - 128*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(2*
a*x)^2*tan(1/2*a*x) + 256*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*
a*x) - 32*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^5*x^5
*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 128*a^5*x^5*imag_part(cos_integral(4*
a*x))*tan(a*x)^2*tan(1/2*a*x) - 16*a^5*x^5*imag_part(cos_integral(2*a*x))*t
an(a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(a*x

)^2*tan(1/2*a*x) - 128*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*t
an(1/2*a*x) + 256*a^5*x^5*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) - 32*
a^5*x^5*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*tan(2*a*x)
*tan(a*x)^2*tan(1/2*a*x) - 36*a^5*x^5*tan(2*a*x)^2*tan(1/2*a*x)^2 + 36*a^5*
x^5*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(4*a*x)) +
 8*a^6*x^6*imag_part(cos_integral(2*a*x)) - 8*a^6*x^6*imag_part(cos_integra
l(-2*a*x)) + 64*a^6*x^6*imag_part(cos_integral(-4*a*x)) - 128*a^6*x^6*sin_i
ntegral(4*a*x) + 16*a^6*x^6*sin_integral(2*a*x) - 8*a^6*x^6*tan(2*a*x) + 4*
a^6*x^6*tan(a*x) - 32*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*t
an(a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^
2 - 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 32*
a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 64*a^4*x^
4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^4*x^4*sin_integral(2*a*
x)*tan(2*a*x)^2*tan(a*x)^2 + 24*a^6*x^6*tan(1/2*a*x) - 48*a^4*x^4*tan(2*a*x
)^2*tan(a*x)^2*tan(1/2*a*x) + 32*a^4*x^4*imag_part(cos_integral(4*a*x))*tan
(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(2*a
*x)^2*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)
^2*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2
*tan(1/2*a*x)^2 + 64*a^4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^
2 - 8*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 2*a^4*x^4*t
an(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 + 32*a^4*x^4*imag_part(cos_integral(4*a
*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integral(2*a*x))*t
an(a*x)^2*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*
x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(a*x)^2
*tan(1/2*a*x)^2 + 64*a^4*x^4*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2
- 8*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 13*a^4*x^4*tan(
2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 36*a^5*x^5*tan(2*a*x)^2 - 36*a^5*x^5*tan
(a*x)^2 + 128*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(1/2*a*x) - 16*a^5*
x^5*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_
integral(-2*a*x))*tan(1/2*a*x) - 128*a^5*x^5*imag_part(cos_integral(-4*a*x)
)*tan(1/2*a*x) + 256*a^5*x^5*sin_integral(4*a*x)*tan(1/2*a*x) - 32*a^5*x^5*
sin_integral(2*a*x)*tan(1/2*a*x) + 16*a^5*x^5*tan(2*a*x)*tan(1/2*a*x) - 8*a
^5*x^5*tan(a*x)*tan(1/2*a*x) + 64*a^3*x^3*imag_part(cos_integral(4*a*x))*ta
n(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 8*a^3*x^3*imag_part(cos_integral(2*a*x
))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^3*x^3*imag_part(cos_integral(
-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag_part(cos_in
tegral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_inte
gral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(
2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 24*a^5*x^5*tan(1/2*a*x)^2 - 3
*a^3*x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_
integral(4*a*x))*tan(2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(2*a*x))*ta
n(2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2 + 32*a^
4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 - 64*a^4*x^4*sin_integra
l(4*a*x)*tan(2*a*x)^2 + 8*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2 + 2*a^4*
x^4*tan(2*a*x)^2*tan(a*x) - 32*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(a

*x)^2 + 4*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^4*x^4*ima
g_part(cos_integral(-2*a*x))*tan(a*x)^2 + 32*a^4*x^4*imag_part(cos_integral
(-4*a*x))*tan(a*x)^2 - 64*a^4*x^4*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^4*x^
4*sin_integral(2*a*x)*tan(a*x)^2 - 13*a^4*x^4*tan(2*a*x)*tan(a*x)^2 - 72*a^
4*x^4*tan(2*a*x)^2*tan(1/2*a*x) + 72*a^4*x^4*tan(a*x)^2*tan(1/2*a*x) + 32*a
^4*x^4*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(
cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(-2*a
*x))*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(1/2*a*
x)^2 + 64*a^4*x^4*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 8*a^4*x^4*sin_integr
al(2*a*x)*tan(1/2*a*x)^2 + 13*a^4*x^4*tan(2*a*x)*tan(1/2*a*x)^2 - 2*a^4*x^4
*tan(a*x)*tan(1/2*a*x)^2 - 24*a^5*x^5 + 3*a^3*x^3*tan(2*a*x)^2*tan(a*x)^2 +
 64*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^
3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^3*x^3*
imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag
_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_int
egral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan
(2*a*x)^2*tan(1/2*a*x) - 4*a^3*x^3*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 64*
a^3*x^3*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^3*x^3*
imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^3*x^3*imag_par
t(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag_part(cos_
integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_integral(4*a*x)
*tan(a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x) + 26*a^3*x^3*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - 15*a^3*x^3*tan(2*a
*x)^2*tan(1/2*a*x)^2 + 24*a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*im
ag_part(cos_integral(4*a*x)) + 4*a^4*x^4*imag_part(cos_integral(2*a*x)) - 4
*a^4*x^4*imag_part(cos_integral(-2*a*x)) + 32*a^4*x^4*imag_part(cos_integra
l(-4*a*x)) - 64*a^4*x^4*sin_integral(4*a*x) + 8*a^4*x^4*sin_integral(2*a*x)
 - 13*a^4*x^4*tan(2*a*x) + 2*a^4*x^4*tan(a*x) + 48*a^4*x^4*tan(1/2*a*x) - 3
0*a^2*x^2*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 10*a^2*x^2*tan(2*a*x)^2*ta
n(a*x)*tan(1/2*a*x)^2 + 5*a^2*x^2*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 15
*a^3*x^3*tan(2*a*x)^2 - 24*a^3*x^3*tan(a*x)^2 + 64*a^3*x^3*imag_part(cos_in
tegral(4*a*x))*tan(1/2*a*x) - 8*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(
1/2*a*x) + 8*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^3*
x^3*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^3*x^3*sin_integral
(4*a*x)*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x) + 26*a^3
*x^3*tan(2*a*x)*tan(1/2*a*x) - 4*a^3*x^3*tan(a*x)*tan(1/2*a*x) + 12*a^3*x^3
*tan(1/2*a*x)^2 - 3*a*x*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 10*a^2*x^2
*tan(2*a*x)^2*tan(a*x) - 5*a^2*x^2*tan(2*a*x)*tan(a*x)^2 - 54*a^2*x^2*tan(2
*a*x)^2*tan(1/2*a*x) + 24*a^2*x^2*tan(a*x)^2*tan(1/2*a*x) + 5*a^2*x^2*tan(2
*a*x)*tan(1/2*a*x)^2 - 10*a^2*x^2*tan(a*x)*tan(1/2*a*x)^2 - 12*a^3*x^3 + 3*
a*x*tan(2*a*x)^2*tan(a*x)^2 - 20*a*x*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 1
0*a*x*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + a*x*tan(2*a*x)^2*tan(1/2*a*x)^2
- 4*a*x*tan(a*x)^2*tan(1/2*a*x)^2 - 5*a^2*x^2*tan(2*a*x) + 10*a^2*x^2*tan(a
*x) - 6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a*x*tan(2*a*x)^2 + 4*a*x*tan
(a*x)^2 + 10*a*x*tan(2*a*x)*tan(1/2*a*x) - 20*a*x*tan(a*x)*tan(1/2*a*x) + 2

```
*tan(2*a*x)^2*tan(1/2*a*x) - 8*tan(a*x)^2*tan(1/2*a*x))/(a^5*x^8*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x)^2 - a^5*x^8*tan(2*a*x)^2*tan(a*x)^2 + a^5*x^8*ta
n(2*a*x)^2*tan(1/2*a*x)^2 + a^5*x^8*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^7*t
an(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a^5*x^8*tan(2*a*x)^2 - a^5*x^8*tan(a*
x)^2 + a^5*x^8*tan(1/2*a*x)^2 + 2*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a
*x)^2 + 2*a^4*x^7*tan(2*a*x)^2*tan(1/2*a*x) + 2*a^4*x^7*tan(a*x)^2*tan(1/2*
a*x) - a^5*x^8 - 2*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2 + 2*a^3*x^6*tan(2*a*x)^2
*tan(1/2*a*x)^2 + 2*a^3*x^6*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^7*tan(1/2*a
*x) + 4*a^2*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^6*tan(2*a*x)
^2 - 2*a^3*x^6*tan(a*x)^2 + 2*a^3*x^6*tan(1/2*a*x)^2 + a*x^4*tan(2*a*x)^2*t
an(a*x)^2*tan(1/2*a*x)^2 + 4*a^2*x^5*tan(2*a*x)^2*tan(1/2*a*x) + 4*a^2*x^5*
tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^6 - a*x^4*tan(2*a*x)^2*tan(a*x)^2 + a*x^4
*tan(2*a*x)^2*tan(1/2*a*x)^2 + a*x^4*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^2*x^5*
tan(1/2*a*x) + 2*x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a*x^4*tan(2*a*x
)^2 - a*x^4*tan(a*x)^2 + a*x^4*tan(1/2*a*x)^2 + 2*x^3*tan(2*a*x)^2*tan(1/2*
a*x) + 2*x^3*tan(a*x)^2*tan(1/2*a*x) - a*x^4 + 2*x^3*tan(1/2*a*x))
```

3.586 $\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=131

$$-\frac{1}{8}a^2\mathrm{Si}(ax)+\frac{27}{8}a^2\mathrm{Si}(3ax)+\frac{\sin^3(ax)}{a^2x^4}+\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{3\sin^3(ax)}{2x^2}+\frac{\sin(ax)}{x^2}+\frac{\sin^2(ax)\cos(ax)}{ax^3}+\frac{a\cos(}{x}$$

```
[Out] (a*Cos[a*x])/x + Sin[a*x]/x^2 + (Cos[a*x]*Sin[a*x]^2)/(a*x^3) - (9*a*Cos[a*
x]*Sin[a*x]^2)/(2*x) + Sin[a*x]^3/(a^2*x^4) - (3*Sin[a*x]^3)/(2*x^2) + Sin[
a*x]^4/(a^2*x^4*(a*x*Cos[a*x] - Sin[a*x])) - (a^2*SinIntegral[a*x])/8 + (27
*a^2*SinIntegral[3*a*x])/8
```

Rubi [A] time = 0.227389, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4598, 3314, 3297, 3299, 3312}

$$-\frac{1}{8}a^2\mathrm{Si}(ax)+\frac{27}{8}a^2\mathrm{Si}(3ax)+\frac{\sin^3(ax)}{a^2x^4}+\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{3\sin^3(ax)}{2x^2}+\frac{\sin(ax)}{x^2}+\frac{\sin^2(ax)\cos(ax)}{ax^3}+\frac{a\cos(}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```


```
[Out] (a*Cos[a*x])/x + Sin[a*x]/x^2 + (Cos[a*x]*Sin[a*x]^2)/(a*x^3) - (9*a*Cos[a*
x]*Sin[a*x]^2)/(2*x) + Sin[a*x]^3/(a^2*x^4) - (3*Sin[a*x]^3)/(2*x^2) + Sin[
a*x]^4/(a^2*x^4*(a*x*Cos[a*x] - Sin[a*x])) - (a^2*SinIntegral[a*x])/8 + (27
*a^2*SinIntegral[3*a*x])/8
```


Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
```

```
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - \frac{4\int \frac{\sin^3(ax)}{x^5}\,dx}{a^2}$$

$$= \frac{\cos(ax)\sin^2(ax)}{ax^3} + \frac{\sin^3(ax)}{a^2x^4} + \frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - 2\int \frac{\sin(ax)}{x^3}\,dx + 3\int$$

$$= \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} - \frac{3\sin^3(ax)}{2x^2} +$$

$$= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} -$$

$$= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} -$$

$$= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} -$$

Mathematica [A] time = 0.996785, size = 142, normalized size = 1.08

$$\frac{-2a^2x^2\mathrm{Si}(ax)(ax\cos(ax)-\sin(ax))+54a^2x^2\mathrm{Si}(3ax)(ax\cos(ax)-\sin(ax))-a^2x^2+8a^2x^2\cos(2ax)+9a^2x^2\cos(4ax)+1}{16x^2(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (3 - a^2*x^2 - 4*Cos[2*a*x] + 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a^2*x^2
*Cos[4*a*x] + 12*a*x*Sin[2*a*x] - 6*a*x*Sin[4*a*x] - 2*a^2*x^2*(a*x*Cos[a*x
] - Sin[a*x])*SinIntegral[a*x] + 54*a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])*SinIn
tegral[3*a*x])/(16*x^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{(\sin(ax))^5}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.49318, size = 385, normalized size = 2.94

$$\frac{4\left(9\,a^{2}x^{2}+1\right)\cos\left(ax\right)^{4}-4\left(7\,a^{2}x^{2}+2\right)\cos\left(ax\right)^{2}+\left(27\,a^{3}x^{3}\,\mathrm{Si}\left(3\,ax\right)-a^{3}x^{3}\,\mathrm{Si}\left(ax\right)\right)\cos\left(ax\right)-\left(24\,ax\cos\left(ax\right)^{3}+27\,a^{2}\cdots\right.}{8\left(ax^{3}\cos\left(ax\right)-x^{2}\sin\left(ax\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(9*a^2*x^2 + 1)*cos(a*x)^4 - 4*(7*a^2*x^2 + 2)*cos(a*x)^2 + (27*a^3*
x^3*sin_integral(3*a*x) - a^3*x^3*sin_integral(a*x))*cos(a*x) - (24*a*x*cos
(a*x)^3 + 27*a^2*x^2*sin_integral(3*a*x) - a^2*x^2*sin_integral(a*x) - 24*a
*x*cos(a*x))*sin(a*x) + 4)/(a*x^3*cos(a*x) - x^2*sin(a*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**5/x**3/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.85612, size = 5636, normalized size = 43.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(27*a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + a
^7*x^7*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^7
*x^7*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^7
*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^7*x^7*sin_inte
gral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^7*x^7*imag_part(cos_integral
```

(3*a*x))*tan(1/2*a*x)^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(1/2*a*x)
^4 + a^7*x^7*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^7*x^7*imag
_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^7*x^7*sin_integral(3*a*x)
*tan(1/2*a*x)^4 - 2*a^7*x^7*sin_integral(a*x)*tan(1/2*a*x)^4 + 54*a^6*x^6*i
mag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^6*x^6*ima
g_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^6*x^6*imag_pa
rt(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 54*a^6*x^6*imag_part
(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^6*x^6*sin_inte
gral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^6*x^6*sin_integral(a*x)*tan
(3/2*a*x)^2*tan(1/2*a*x)^3 - 16*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*
a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^7*x^7*imag_part(c
os_integral(a*x))*tan(3/2*a*x)^2 - a^7*x^7*imag_part(cos_integral(-a*x))*ta
n(3/2*a*x)^2 + 27*a^7*x^7*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 -
54*a^7*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 2*a^7*x^7*sin_integral(a*x)
*tan(3/2*a*x)^2 + 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*
tan(1/2*a*x)^4 - 2*a^5*x^5*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 + 2*a^5*x^5*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2
*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2
*a*x)^4 + 108*a^5*x^5*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 4
*a^5*x^5*sin_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^6*x^6*imag_
part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 2*a^6*x^6*imag_part
(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^6*x^6*imag_part(cos_i
ntegral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 54*a^6*x^6*imag_part(cos_integ
ral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 108*a^6*x^6*sin_integral(3*a*x)*
tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a^6*x^6*sin_integral(a*x)*tan(3/2*a*x)^2*ta
n(1/2*a*x) - 4*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^2 + 54*a^6*x^6*imag_part
(cos_integral(3*a*x))*tan(1/2*a*x)^3 - 2*a^6*x^6*imag_part(cos_integral(a*x
))*tan(1/2*a*x)^3 + 2*a^6*x^6*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3
- 54*a^6*x^6*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 + 108*a^6*x^6*s
in_integral(3*a*x)*tan(1/2*a*x)^3 - 4*a^6*x^6*sin_integral(a*x)*tan(1/2*a*x
)^3 + 20*a^6*x^6*tan(1/2*a*x)^4 - 27*a^7*x^7*imag_part(cos_integral(3*a*x))
 + a^7*x^7*imag_part(cos_integral(a*x)) - a^7*x^7*imag_part(cos_integral(-a
*x)) + 27*a^7*x^7*imag_part(cos_integral(-3*a*x)) - 54*a^7*x^7*sin_integral
(3*a*x) + 2*a^7*x^7*sin_integral(a*x) - 36*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a
*x)^3 + 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - 2*a^5*x^
5*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 + 2*a^5*x^5*imag_part(cos_int
egral(-a*x))*tan(1/2*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*ta
n(1/2*a*x)^4 + 108*a^5*x^5*sin_integral(3*a*x)*tan(1/2*a*x)^4 - 4*a^5*x^5*s
in_integral(a*x)*tan(1/2*a*x)^4 + 12*a^5*x^5*tan(3/2*a*x)*tan(1/2*a*x)^4 +
20*a^6*x^6*tan(3/2*a*x)^2 + 54*a^6*x^6*imag_part(cos_integral(3*a*x))*tan(1
/2*a*x) - 2*a^6*x^6*imag_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^6*x^6*i
mag_part(cos_integral(-a*x))*tan(1/2*a*x) - 54*a^6*x^6*imag_part(cos_integr
al(-3*a*x))*tan(1/2*a*x) + 108*a^6*x^6*sin_integral(3*a*x)*tan(1/2*a*x) - 4
*a^6*x^6*sin_integral(a*x)*tan(1/2*a*x) - 4*a^6*x^6*tan(1/2*a*x)^2 + 108*a^
4*x^4*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^4*

```
x^4*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^4*x^4*
imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 108*a^4*x^4*i
mag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 216*a^4*x^4*
sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 8*a^4*x^4*sin_integral(
a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 32*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + 2*a^5*x^5*i
mag_part(cos_integral(a*x))*tan(3/2*a*x)^2 - 2*a^5*x^5*imag_part(cos_integr
al(-a*x))*tan(3/2*a*x)^2 + 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*tan(3
/2*a*x)^2 - 108*a^5*x^5*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 4*a^5*x^5*sin_
integral(a*x)*tan(3/2*a*x)^2 - 36*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 36*
a^5*x^5*tan(1/2*a*x)^3 + 27*a^3*x^3*imag_part(cos_integral(3*a*x))*tan(3/2*
a*x)^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2
*tan(1/2*a*x)^4 + a^3*x^3*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 - 27*a^3*x^3*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 + 54*a^3*x^3*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
 2*a^3*x^3*sin_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 16*a^6*x^6 + 1
08*a^4*x^4*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a
^4*x^4*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^4*x^4
*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 108*a^4*x^4*im
ag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 216*a^4*x^4*sin
_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*sin_integral(a*x)*
tan(3/2*a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^2 + 108
*a^4*x^4*imag_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 - 4*a^4*x^4*imag_par
t(cos_integral(a*x))*tan(1/2*a*x)^3 + 4*a^4*x^4*imag_part(cos_integral(-a*x
))*tan(1/2*a*x)^3 - 108*a^4*x^4*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x
)^3 + 216*a^4*x^4*sin_integral(3*a*x)*tan(1/2*a*x)^3 - 8*a^4*x^4*sin_integr
al(a*x)*tan(1/2*a*x)^3 + 24*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x)^3 + 32*a^4*x^
4*tan(1/2*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(3*a*x)) + 2*a^5*x^5*im
ag_part(cos_integral(a*x)) - 2*a^5*x^5*imag_part(cos_integral(-a*x)) + 54*a
^5*x^5*imag_part(cos_integral(-3*a*x)) - 108*a^5*x^5*sin_integral(3*a*x) +
4*a^5*x^5*sin_integral(a*x) - 12*a^5*x^5*tan(3/2*a*x) + 36*a^5*x^5*tan(1/2*
a*x) - 48*a^3*x^3*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 27*a^3*x^3*imag_part(cos_
integral(3*a*x))*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(a*x))*tan(
1/2*a*x)^4 + a^3*x^3*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^3*
x^3*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^3*x^3*sin_integra
l(3*a*x)*tan(1/2*a*x)^4 - 2*a^3*x^3*sin_integral(a*x)*tan(1/2*a*x)^4 + 16*a
^3*x^3*tan(3/2*a*x)*tan(1/2*a*x)^4 + 32*a^4*x^4*tan(3/2*a*x)^2 + 108*a^4*x^
4*imag_part(cos_integral(3*a*x))*tan(1/2*a*x) - 4*a^4*x^4*imag_part(cos_int
egral(a*x))*tan(1/2*a*x) + 4*a^4*x^4*imag_part(cos_integral(-a*x))*tan(1/2*
a*x) - 108*a^4*x^4*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x) + 216*a^4*x
^4*sin_integral(3*a*x)*tan(1/2*a*x) - 8*a^4*x^4*sin_integral(a*x)*tan(1/2*a
*x) + 24*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x) - 8*a^4*x^4*tan(1/2*a*x)^2 + 54*
a^2*x^2*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^
2*x^2*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^2*x^
2*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 54*a^2*x^2*
```

```
imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^2*x^2
*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^2*x^2*sin_integral
(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 32*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x
)^4 - 27*a^3*x^3*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^3*x^3*im
ag_part(cos_integral(a*x))*tan(3/2*a*x)^2 - a^3*x^3*imag_part(cos_integral(
-a*x))*tan(3/2*a*x)^2 + 27*a^3*x^3*imag_part(cos_integral(-3*a*x))*tan(3/2*
a*x)^2 - 54*a^3*x^3*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 2*a^3*x^3*sin_inte
gral(a*x)*tan(3/2*a*x)^2 - 80*a^3*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 80*a^3*
x^3*tan(1/2*a*x)^3 - 32*a^4*x^4 + 54*a^2*x^2*imag_part(cos_integral(3*a*x))
*tan(3/2*a*x)^2*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(3
/2*a*x)^2*tan(1/2*a*x) + 2*a^2*x^2*imag_part(cos_integral(-a*x))*tan(3/2*a*
x)^2*tan(1/2*a*x) - 54*a^2*x^2*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)
^2*tan(1/2*a*x) + 108*a^2*x^2*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*
x) - 4*a^2*x^2*sin_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x) - 60*a^2*x^2*t
an(3/2*a*x)^2*tan(1/2*a*x)^2 + 54*a^2*x^2*imag_part(cos_integral(3*a*x))*ta
n(1/2*a*x)^3 - 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^
2*x^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 - 54*a^2*x^2*imag_part(c
os_integral(-3*a*x))*tan(1/2*a*x)^3 + 108*a^2*x^2*sin_integral(3*a*x)*tan(1
/2*a*x)^3 - 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x)^3 + 32*a^2*x^2*tan(3/2
*a*x)*tan(1/2*a*x)^3 - 4*a^2*x^2*tan(1/2*a*x)^4 - 27*a^3*x^3*imag_part(cos_
integral(3*a*x)) + a^3*x^3*imag_part(cos_integral(a*x)) - a^3*x^3*imag_part
(cos_integral(-a*x)) + 27*a^3*x^3*imag_part(cos_integral(-3*a*x)) - 54*a^3*
x^3*sin_integral(3*a*x) + 2*a^3*x^3*sin_integral(a*x) - 16*a^3*x^3*tan(3/2*
a*x) + 48*a^3*x^3*tan(1/2*a*x) - 12*a*x*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a
*x*tan(3/2*a*x)*tan(1/2*a*x)^4 - 4*a^2*x^2*tan(3/2*a*x)^2 + 54*a^2*x^2*imag
_part(cos_integral(3*a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(
a*x))*tan(1/2*a*x) + 2*a^2*x^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x) -
 54*a^2*x^2*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x) + 108*a^2*x^2*sin_
integral(3*a*x)*tan(1/2*a*x) - 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x) + 3
2*a^2*x^2*tan(3/2*a*x)*tan(1/2*a*x) - 60*a^2*x^2*tan(1/2*a*x)^2 - 44*a*x*ta
n(3/2*a*x)^2*tan(1/2*a*x) + 44*a*x*tan(1/2*a*x)^3 - 32*a^2*x^2 - 24*tan(3/2
*a*x)^2*tan(1/2*a*x)^2 + 8*tan(3/2*a*x)*tan(1/2*a*x)^3 - 4*a*x*tan(3/2*a*x)
 + 12*a*x*tan(1/2*a*x) + 8*tan(3/2*a*x)*tan(1/2*a*x) - 24*tan(1/2*a*x)^2)/(
a^5*x^7*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + a^5*x^7*tan(1/2*a*x)^4 + 2*a^4*x^6*
tan(3/2*a*x)^2*tan(1/2*a*x)^3 - a^5*x^7*tan(3/2*a*x)^2 + 2*a^3*x^5*tan(3/2*
a*x)^2*tan(1/2*a*x)^4 + 2*a^4*x^6*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^4*x^6*t
an(1/2*a*x)^3 - a^5*x^7 + 2*a^3*x^5*tan(1/2*a*x)^4 + 2*a^4*x^6*tan(1/2*a*x)
 + 4*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^3*x^5*tan(3/2*a*x)^2 + a*x
^3*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 4*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x) +
4*a^2*x^4*tan(1/2*a*x)^3 - 2*a^3*x^5 + a*x^3*tan(1/2*a*x)^4 + 4*a^2*x^4*tan
(1/2*a*x) + 2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - a*x^3*tan(3/2*a*x)^2 + 2*
x^2*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*x^2*tan(1/2*a*x)^3 - a*x^3 + 2*x^2*tan(
1/2*a*x))
```

3.587 $\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=80

$$\frac{\sin^2(ax)}{a^2x^3}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\mathrm{Si}(2ax)+\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\sin^2(ax)}{x}+\frac{1}{x}$$

```
[Out] x^(-1) + (Cos[a*x]*Sin[a*x])/(a*x^2) + Sin[a*x]^2/(a^2*x^3) - (2*Sin[a*x]^2
)/x + Sin[a*x]^3/(a^2*x^3*(a*x*Cos[a*x] - Sin[a*x])) + 2*a*SinIntegral[2*a*
x]
```

Rubi [A] time = 0.130805, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4598, 3314, 30, 3313, 12, 3299}

$$\frac{\sin^2(ax)}{a^2x^3}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\mathrm{Si}(2ax)+\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\sin^2(ax)}{x}+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] x^(-1) + (Cos[a*x]*Sin[a*x])/(a*x^2) + Sin[a*x]^2/(a^2*x^3) - (2*Sin[a*x]^2
)/x + Sin[a*x]^3/(a^2*x^3*(a*x*Cos[a*x] - Sin[a*x])) + 2*a*SinIntegral[2*a*
x]
```

Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
```

```
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\int \frac{\sin^2(ax)}{x^4}\,dx}{a^2}\\
&= \frac{\cos(ax)\sin(ax)}{ax^2}+\frac{\sin^2(ax)}{a^2x^3}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2\int \frac{\sin^2(ax)}{x^2}\,dx-\int \frac{1}{x^2}\,d\\
&= \frac{1}{x}+\frac{\cos(ax)\sin(ax)}{ax^2}+\frac{\sin^2(ax)}{a^2x^3}-\frac{2\sin^2(ax)}{x}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+(4a)\int \frac{\mathrm{s}}{}\\
&= \frac{1}{x}+\frac{\cos(ax)\sin(ax)}{ax^2}+\frac{\sin^2(ax)}{a^2x^3}-\frac{2\sin^2(ax)}{x}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+(2a)\int \frac{\mathrm{s}}{}\\
&= \frac{1}{x}+\frac{\cos(ax)\sin(ax)}{ax^2}+\frac{\sin^2(ax)}{a^2x^3}-\frac{2\sin^2(ax)}{x}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\mathrm{Si}(2a
\end{aligned}$$

Mathematica [A] time = 0.85417, size = 77, normalized size = 0.96

$$\frac{8ax\text{Si}(2ax)(ax\cos(ax)-\sin(ax))+3\sin(ax)-\sin(3ax)+2ax\cos(ax)+2ax\cos(3ax)}{4x(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (2*a*x*Cos[a*x] + 2*a*x*Cos[3*a*x] + 3*Sin[a*x] - Sin[3*a*x] + 8*a*x*(a*x*C
os[a*x] - Sin[a*x])*SinIntegral[2*a*x])/(4*x*(a*x*Cos[a*x] - Sin[a*x]))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{(\sin(ax))^4}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.22975, size = 209, normalized size = 2.61

$$\frac{2\,ax\cos(ax)^3+\left(2\,a^2x^2\,\text{Si}(2\,ax)-ax\right)\cos(ax)-\left(2\,ax\,\text{Si}(2\,ax)+\cos(ax)^2-1\right)\sin(ax)}{ax^2\cos(ax)-x\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*cos(a*x)^3 + (2*a^2*x^2*sin_integral(2*a*x) - a*x)*cos(a*x) - (2*a*x
*sin_integral(2*a*x) + cos(a*x)^2 - 1)*sin(a*x))/(a*x^2*cos(a*x) - x*sin(a*
x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^4(ax)}{x^2 (ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**4/x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(sin(a*x)**4/(x**2*(a*x*cos(a*x) - sin(a*x))**2), x)
```

Giac [C] time = 1.50151, size = 1395, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] (a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4
*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^4*sin_
integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(
2*a*x))*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2
*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(2*
a*x))*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)
^2 + 2*a^4*x^4*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^3*x^3*imag_part(cos
_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^3*imag_part(cos_integra
l(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)*tan(a*x)
^2*tan(1/2*a*x) - a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos
_integral(2*a*x)) + a^4*x^4*imag_part(cos_integral(-2*a*x)) - 2*a^4*x^4*sin
_integral(2*a*x) + a^2*x^2*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/
```

```
2*a*x)^2 - a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^
2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*tan(a
*x)^2 + 2*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^3*x^3*i
mag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)
*tan(1/2*a*x) + a^3*x^3*tan(1/2*a*x)^2 - a^2*x^2*imag_part(cos_integral(2*a
*x))*tan(a*x)^2 + a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2*a^
2*x^2*sin_integral(2*a*x)*tan(a*x)^2 - 2*a^2*x^2*tan(a*x)^2*tan(1/2*a*x) +
a^2*x^2*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 - a^2*x^2*imag_part(c
os_integral(-2*a*x))*tan(1/2*a*x)^2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(1/2
*a*x)^2 + a^2*x^2*tan(a*x)*tan(1/2*a*x)^2 - a^3*x^3 + 2*a*x*imag_part(cos_i
ntegral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a*x*imag_part(cos_integral(-2*a
*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2
*a*x) - a^2*x^2*imag_part(cos_integral(2*a*x)) + a^2*x^2*imag_part(cos_inte
gral(-2*a*x)) - 2*a^2*x^2*sin_integral(2*a*x) - a^2*x^2*tan(a*x) + 2*a^2*x^
2*tan(1/2*a*x) + 2*a*x*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a*x*
imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*ta
n(1/2*a*x) + 2*a*x*tan(a*x)*tan(1/2*a*x) + a*x*tan(1/2*a*x)^2 - 2*tan(a*x)^
2*tan(1/2*a*x) - a*x)/(a^3*x^4*tan(a*x)^2*tan(1/2*a*x)^2 - a^3*x^4*tan(a*x)
^2 + a^3*x^4*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^4 +
 a*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(1/2*a*x) - a*x^2*tan(a*x)^
2 + a*x^2*tan(1/2*a*x)^2 + 2*x*tan(a*x)^2*tan(1/2*a*x) - a*x^2 + 2*x*tan(1/
2*a*x))
```

3.588 $\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=56

$$\frac{\sin(ax)}{a^2x^2}+\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}+\mathrm{Si}(ax)+\frac{\cos(ax)}{ax}$$

```
[Out] Cos[a*x]/(a*x) + Sin[a*x]/(a^2*x^2) + Sin[a*x]^2/(a^2*x^2*(a*x*Cos[a*x] - S
in[a*x])) + SinIntegral[a*x]
```

Rubi [A] time = 0.101601, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {4598, 3297, 3299}

$$\frac{\sin(ax)}{a^2x^2}+\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}+\mathrm{Si}(ax)+\frac{\cos(ax)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/(a*x) + Sin[a*x]/(a^2*x^2) + Sin[a*x]^2/(a^2*x^2*(a*x*Cos[a*x] - S
in[a*x])) + SinIntegral[a*x]
```

Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} - \frac{2\int \frac{\sin(ax)}{x^3}\,dx}{a^2} \\
&= \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} - \frac{\int \frac{\cos(ax)}{x^2}\,dx}{a} \\
&= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} + \int \frac{\sin(ax)}{x}\,dx \\
&= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} + \mathrm{Si}(ax)
\end{aligned}$$

Mathematica [C] time = 7.54569, size = 242, normalized size = 4.32

$$-ie\,\mathrm{CosIntegral}(-ax+i)(ax\cos(ax)-\sin(ax)) + ie\,\mathrm{CosIntegral}(ax+i)(ax\cos(ax)-\sin(ax)) - ie\,\mathrm{ExpIntegralEi}(-1$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (1 + Cos[2*a*x] + I*a*E*x*Cos[a*x]*ExpIntegralEi[-1 - I*a*x] - I*a*E*x*Cos[
a*x]*ExpIntegralEi[-1 + I*a*x] - I*E*CosIntegral[I - a*x]*(a*x*Cos[a*x] - S
in[a*x]) + I*E*CosIntegral[I + a*x]*(a*x*Cos[a*x] - Sin[a*x]) - I*E*ExpInte
gralEi[-1 - I*a*x]*Sin[a*x] + I*E*ExpIntegralEi[-1 + I*a*x]*Sin[a*x] + 2*a*
x*Cos[a*x]*SinIntegral[a*x] - 2*Sin[a*x]*SinIntegral[a*x] + a*E*x*Cos[a*x]*
SinIntegral[I - a*x] - E*Sin[a*x]*SinIntegral[I - a*x] - a*E*x*Cos[a*x]*Sin
Integral[I + a*x] + E*Sin[a*x]*SinIntegral[I + a*x])/(2*a*x*Cos[a*x] - 2*Si
n[a*x])
```

Maple [C] time = 0.666, size = 108, normalized size = 1.9

$$\frac{\frac{i}{2}e^{iax}}{-1+iax} + \frac{i}{2}\mathrm{Ei}(1,-iax) + \frac{\frac{i}{2}e^{-iax}}{1+iax} - \frac{i}{2}\mathrm{Ei}(1,iax) + 2\,\frac{e^{iax}}{(ax+i)(ax-i)\left(axe^{2\,iax}+ie^{2\,iax}+ax-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] 1/2*I*exp(I*a*x)/(-1+I*a*x)+1/2*I*Ei(1,-I*a*x)+1/2*I*exp(-I*a*x)/(1+I*a*x)-
1/2*I*Ei(1,I*a*x)+2*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)+I*exp(2*I*
a*x)+a*x-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.08375, size = 138, normalized size = 2.46

$$\frac{ax\cos(ax)\operatorname{Si}(ax) + \cos(ax)^2 - \sin(ax)\operatorname{Si}(ax)}{ax\cos(ax) - \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a*x*cos(a*x)*sin_integral(a*x) + cos(a*x)^2 - sin(a*x)*sin_integral(a*x))/
(a*x*cos(a*x) - sin(a*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**3/x/(a*x*cos(a*x)-sin(a*x))**2,x)
```

[Out] Timed out

Giac [C] time = 1.28642, size = 670, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*x^3*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a^3*x^3*imag_par
t(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a^3*x^3*sin_integral(a*x)*tan(1/2*
a*x)^4 + 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 - 2*a^2*x^2*
imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*a^2*x^2*sin_integral(a*x)*
tan(1/2*a*x)^3 - 2*a^2*x^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(
a*x)) + a^3*x^3*imag_part(cos_integral(-a*x)) - 2*a^3*x^3*sin_integral(a*x)
 + a*x*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a*x*imag_part(cos_inte
gral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*sin_integral(a*x)*tan(1/2*a*x)^4 + 2*a^2
*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_in
tegral(-a*x))*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x) + 4*a
^2*x^2*tan(1/2*a*x)^2 - 2*a^2*x^2 + 2*imag_part(cos_integral(a*x))*tan(1/2*
a*x)^3 - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*sin_integral(a*
x)*tan(1/2*a*x)^3 - 4*tan(1/2*a*x)^4 - a*x*imag_part(cos_integral(a*x)) + a
*x*imag_part(cos_integral(-a*x)) - 2*a*x*sin_integral(a*x) + 2*imag_part(co
s_integral(a*x))*tan(1/2*a*x) - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x
) + 4*sin_integral(a*x)*tan(1/2*a*x) - 4)/(a^3*x^3*tan(1/2*a*x)^4 + 2*a^2*x
^2*tan(1/2*a*x)^3 - a^3*x^3 + a*x*tan(1/2*a*x)^4 + 2*a^2*x^2*tan(1/2*a*x) +
 2*tan(1/2*a*x)^3 - a*x + 2*tan(1/2*a*x))
```

3.589 $\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

```
[Out] 1/(a^2*x) + Sin[a*x]/(a^2*x*(a*x*Cos[a*x] - Sin[a*x]))
```

Rubi [A] time = 0.0241799, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {4596}

$$\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] 1/(a^2*x) + Sin[a*x]/(a^2*x*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 4596

```
Int[Sin[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] + Simp[Sin[a*x]/(a*d*x*(d*x*Cos[a*x] + c
*Sin[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rubi steps

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

Mathematica [A] time = 0.297996, size = 24, normalized size = 0.69

$$\frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] Cos[a*x]/(a^2*x*Cos[a*x] - a*Sin[a*x])
```

Maple [B] time = 0.454, size = 77, normalized size = 2.2

$$\left(\frac{1}{a}\left(\tan\left(\frac{ax}{2}\right)\right)^4+\frac{1}{a}\left(\tan\left(\frac{ax}{2}\right)\right)^6-a^{-1}-\frac{1}{a}\left(\tan\left(\frac{ax}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{ax}{2}\right)\right)^2\right)^{-2}\left(ax\left(\tan\left(\frac{ax}{2}\right)\right)^2-ax+2\tan(1/2\,ax)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] (1/a*tan(1/2*a*x)^4+1/a*tan(1/2*a*x)^6-1/a-1/a*tan(1/2*a*x)^2)/(1+tan(1/2*a
*x)^2)^2/(a*x*tan(1/2*a*x)^2-a*x+2*tan(1/2*a*x))
```

Maxima [B] time = 1.06221, size = 154, normalized size = 4.4

$$\frac{ax\cos(2\,ax)^2+ax\sin(2\,ax)^2+2\,ax\cos(2\,ax)+ax-2\,\sin(2\,ax)}{\left(a^2x^2+\left(a^2x^2+1\right)\cos(2\,ax)^2-4\,ax\sin(2\,ax)+\left(a^2x^2+1\right)\sin(2\,ax)^2+2\left(a^2x^2-1\right)\cos(2\,ax)+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 + 2*a*x*cos(2*a*x) + a*x - 2*sin(2*a*x
))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1
)*sin(2*a*x)^2 + 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

Fricas [A] time = 2.08256, size = 54, normalized size = 1.54

$$\frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] cos(a*x)/(a^2*x*cos(a*x) - a*sin(a*x))
```

Sympy [A] time = 3.59789, size = 20, normalized size = 0.57

$$\frac{\cos(ax)}{a^2x\cos(ax) - a\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] cos(a*x)/(a**2*x*cos(a*x) - a*sin(a*x))
```

Giac [A] time = 1.14914, size = 53, normalized size = 1.51

$$\frac{\tan\left(\frac{1}{2}\,ax\right)^2 - 1}{a^2x\tan\left(\frac{1}{2}\,ax\right)^2 - a^2x + 2\,a\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] (tan(1/2*a*x)^2 - 1)/(a^2*x*tan(1/2*a*x)^2 - a^2*x + 2*a*tan(1/2*a*x))
```

3.590 $\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=20

$$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

```
[Out] 1/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rubi [A] time = 0.037818, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6686}

$$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] 1/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

Mathematica [A] time = 0.0309856, size = 20, normalized size = 1.

$$-\frac{1}{a^2(\sin(ax)-ax\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] -(1/(a^2*(-(a*x*Cos[a*x]) + Sin[a*x])))
```

Maple [A] time = 0.035, size = 21, normalized size = 1.1

$$\frac{1}{a^2 (ax \cos (ax) - \sin (ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] 1/a^2/(a*x*cos(a*x)-sin(a*x))
```

Maxima [A] time = 1.01165, size = 27, normalized size = 1.35

$$\frac{1}{(ax \cos (ax) - \sin (ax))a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] 1/((a*x*cos(a*x) - sin(a*x))*a^2)
```

Fricas [A] time = 1.96554, size = 47, normalized size = 2.35

$$\frac{1}{a^3 x \cos (ax) - a^2 \sin (ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/(a^3*x*cos(a*x) - a^2*sin(a*x))
```

Sympy [A] time = 3.4626, size = 19, normalized size = 0.95

$$\frac{1}{a^3x\cos(ax) - a^2\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] 1/(a**3*x*cos(a*x) - a**2*sin(a*x))
```

Giac [B] time = 1.16951, size = 57, normalized size = 2.85

$$-\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^2 + 1\right)}{a^3x\tan\left(\frac{1}{2}\,ax\right)^2 - a^3x + 2\,a^2\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*a*x)^2 + 1)/(a^3*x*tan(1/2*a*x)^2 - a^3*x + 2*a^2*tan(1/2*a*x))
```

3.591 $\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=35

$$\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\cot(ax)}{a^3}$$

```
[Out] -(Cot[a*x]/a^3) + (x*Csc[a*x])/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rubi [A] time = 0.0388193, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4594, 3767, 8}

$$\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\cot(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] -(Cot[a*x]/a^3) + (x*Csc[a*x])/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 4594

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[1/d^2, In
t[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))} + \frac{\int \csc^2(ax)\,dx}{a^2}$$
$$= \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))} - \frac{\text{Subst}(\int 1\,dx, x, \cot(ax))}{a^3}$$
$$= -\frac{\cot(ax)}{a^3} + \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

Mathematica [A] time = 0.464316, size = 32, normalized size = 0.91

$$\frac{ax\sin(ax)+\cos(ax)}{a^3(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (Cos[a*x] + a*x*Sin[a*x])/(a^3*(a*x*Cos[a*x] - Sin[a*x]))
```

Maple [A] time = 0.316, size = 54, normalized size = 1.5

$$\left(\frac{1}{a^3}\left(\tan\left(\frac{ax}{2}\right)\right)^2 - a^{-3} - 2\,\frac{x\tan(1/2\,ax)}{a^2}\right)\left(ax\left(\tan\left(\frac{ax}{2}\right)\right)^2 - ax + 2\,\tan(1/2\,ax)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] (1/a^3*tan(1/2*a*x)^2-1/a^3-2*x/a^2*tan(1/2*a*x))/(a*x*tan(1/2*a*x)^2-a*x+2
*tan(1/2*a*x))
```

Maxima [B] time = 1.02527, size = 135, normalized size = 3.86

$$\frac{2\left(2\,ax\cos(2\,ax)+\left(a^2x^2-1\right)\sin(2\,ax)\right)}{\left(a^2x^2+\left(a^2x^2+1\right)\cos(2\,ax)^2-4\,ax\sin(2\,ax)+\left(a^2x^2+1\right)\sin(2\,ax)^2+2\left(a^2x^2-1\right)\cos(2\,ax)+1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] 2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*c
os(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 + 2*(a^2*x^2 -
1)*cos(2*a*x) + 1)*a^3)
```

Fricas [A] time = 2.13257, size = 80, normalized size = 2.29

$$\frac{ax\sin(ax)+\cos(ax)}{a^4x\cos(ax)-a^3\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a*x*sin(a*x) + cos(a*x))/(a^4*x*cos(a*x) - a^3*sin(a*x))
```

Sympy [B] time = 5.4182, size = 112, normalized size = 3.2

$$-\frac{2ax\tan\left(\frac{ax}{2}\right)}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)}+\frac{\tan^2\left(\frac{ax}{2}\right)}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)}-\frac{1}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] -2*a*x*tan(a*x/2)/(a**4*x*tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2)) + tan
(a*x/2)**2/(a**4*x*tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2)) - 1/(a**4*x*
tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2))
```

Giac [A] time = 1.12798, size = 72, normalized size = 2.06

$$-\frac{2\,ax\tan\left(\frac{1}{2}\,ax\right)-\tan\left(\frac{1}{2}\,ax\right)^2+1}{a^4x\tan\left(\frac{1}{2}\,ax\right)^2-a^4x+2\,a^3\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -(2*a*x*tan(1/2*a*x) - tan(1/2*a*x)^2 + 1)/(a^4*x*tan(1/2*a*x)^2 - a^4*x +
2*a^3*tan(1/2*a*x))
```

3.592 $\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=104

$$\frac{i\text{PolyLog}\left(2,-e^{iax}\right)}{a^4}-\frac{i\text{PolyLog}\left(2,e^{iax}\right)}{a^4}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\csc(ax)}{a^4}-\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{x\cot(ax)\csc(ax)}{a^3}$$

```
[Out] (-2*x*ArcTanh[E^(I*a*x)])/a^3 - Csc[a*x]/a^4 - (x*Cot[a*x]*Csc[a*x])/a^3 +
(I*PolyLog[2, -E^(I*a*x)])/a^4 - (I*PolyLog[2, E^(I*a*x)])/a^4 + (x^2*Csc[a
*x]^2)/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rubi [A] time = 0.0912974, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4600, 4185, 4183, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-e^{iax}\right)}{a^4}-\frac{i\text{PolyLog}\left(2,e^{iax}\right)}{a^4}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\csc(ax)}{a^4}-\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{x\cot(ax)\csc(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```


```
[Out] (-2*x*ArcTanh[E^(I*a*x)])/a^3 - Csc[a*x]/a^4 - (x*Cot[a*x]*Csc[a*x])/a^3 +
(I*PolyLog[2, -E^(I*a*x)])/a^4 - (I*PolyLog[2, E^(I*a*x)])/a^4 + (x^2*Csc[a
*x]^2)/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```


Rule 4600

```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Csc[a*x]^(n
+ 1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[(
b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && Eq
Q[a*c + d, 0] && EqQ[m, n + 2]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{2\int x\csc^3(ax)\,dx}{a^2}\\
&= -\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{\int x\csc(ax)\,dx}{a^2}\\
&= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\int \log\left(1\right.}{}\\
&= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{i\,\mathrm{Subst}}{}\\
&= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{i\mathrm{Li}_2\left(-e^{iax}\right)}{a^4}-\frac{i\mathrm{Li}_2\left(e^{iax}\right)}{a^4}+\frac{x}{a^2(ax\,c}
\end{aligned}$$

Mathematica [A] time = 1.01867, size = 157, normalized size = 1.51

$$\frac{i(ax\cot(ax)-1)\mathrm{PolyLog}\left(2,-e^{iax}\right)-i(ax\cot(ax)-1)\mathrm{PolyLog}\left(2,e^{iax}\right)+a^2x^2\csc(ax)+a^2x^2\log\left(1-e^{iax}\right)\cot(ax)-a}{a^4(ax\cot(ax)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (Csc[a*x] + a^2*x^2*Csc[a*x] - a*x*Log[1 - E^(I*a*x)] + a^2*x^2*Cot[a*x]*Lo
g[1 - E^(I*a*x)] + a*x*Log[1 + E^(I*a*x)] - a^2*x^2*Cot[a*x]*Log[1 + E^(I*a
*x)] + I*(-1 + a*x*Cot[a*x])*PolyLog[2, -E^(I*a*x)] - I*(-1 + a*x*Cot[a*x])
*PolyLog[2, E^(I*a*x)])/(a^4*(-1 + a*x*Cot[a*x]))
```

Maple [F] time = 1.249, size = 0, normalized size = 0.

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.33218, size = 771, normalized size = 7.41

$$\frac{2\,a^2x^2 - (i\,ax\cos(ax) - i\,\sin(ax))\mathrm{Li}_2(\cos(ax) + i\,\sin(ax)) - (-i\,ax\cos(ax) + i\,\sin(ax))\mathrm{Li}_2(\cos(ax) - i\,\sin(ax)) - (i\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2 - (I*a*x*cos(a*x) - I*sin(a*x))*dilog(cos(a*x) + I*sin(a*x))
 - (-I*a*x*cos(a*x) + I*sin(a*x))*dilog(cos(a*x) - I*sin(a*x)) - (I*a*x*cos
(a*x) - I*sin(a*x))*dilog(-cos(a*x) + I*sin(a*x)) - (-I*a*x*cos(a*x) + I*si
n(a*x))*dilog(-cos(a*x) - I*sin(a*x)) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*l
og(cos(a*x) + I*sin(a*x) + 1) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(cos(a
*x) - I*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) + I
*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) - I*sin(a*
x) + 1) + 2)/(a^5*x*cos(a*x) - a^4*sin(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(x**3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))^2, x)
```

3.593 $\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=127

$$-\frac{2i\text{PolyLog}\left(2,e^{2iax}\right)}{a^5}-\frac{2ix^2}{a^3}-\frac{2x^2\cot(ax)}{a^3}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}-\frac{\cot(}{a}$$

```
[Out] ((-2*I)*x^2)/a^3 - Cot[a*x]/a^5 - (2*x^2*Cot[a*x])/a^3 - (x*Csc[a*x]^2)/a^4
 - (x^2*Cot[a*x]*Csc[a*x]^2)/a^3 + (4*x*Log[1 - E^((2*I)*a*x)])/a^4 - ((2*I
)*PolyLog[2, E^((2*I)*a*x)])/a^5 + (x^3*Csc[a*x]^3)/(a^2*(a*x*Cos[a*x] - Si
n[a*x]))
```

Rubi [A] time = 0.181463, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4600, 4186, 3767, 8, 4184, 3717, 2190, 2279, 2391}

$$-\frac{2i\text{PolyLog}\left(2,e^{2iax}\right)}{a^5}-\frac{2ix^2}{a^3}-\frac{2x^2\cot(ax)}{a^3}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}-\frac{\cot(}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```



```
[Out] ((-2*I)*x^2)/a^3 - Cot[a*x]/a^5 - (2*x^2*Cot[a*x])/a^3 - (x*Csc[a*x]^2)/a^4
 - (x^2*Cot[a*x]*Csc[a*x]^2)/a^3 + (4*x*Log[1 - E^((2*I)*a*x)])/a^4 - ((2*I
)*PolyLog[2, E^((2*I)*a*x)])/a^5 + (x^3*Csc[a*x]^3)/(a^2*(a*x*Cos[a*x] - Si
n[a*x]))
```


Rule 4600

```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Csc[a*x]^(n
+ 1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[(
b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && Eq
Q[a*c + d, 0] && EqQ[m, n + 2]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{3\int x^2\csc^4(ax)\,dx}{a^2}$$

$$= -\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{\int\csc^2(ax)\,dx}{a^4}+\frac{2\int x^2$$

$$= -\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\text{Subst}(\int$$

$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin$$

$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}$$

$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}$$

$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}$$

Mathematica [A] time = 1.06375, size = 102, normalized size = 0.8

$$\frac{-2ia\left(a^2x^2+\text{PolyLog}\left(2,e^{2iax}\right)\right)+a^3\left(-x^2\right)\cot(ax)+\frac{\left(a^2x^2+1\right)^2\sin(ax)}{x(ax\cos(ax)-\sin(ax))}+a^2x+4a^2x\log\left(1-e^{2iax}\right)+\frac{1}{x}}{a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (x^(-1) + a^2*x - a^3*x^2*Cot[a*x] + 4*a^2*x*Log[1 - E^((2*I)*a*x)] - (2*I)
*a*(a^2*x^2 + PolyLog[2, E^((2*I)*a*x)]) + ((1 + a^2*x^2)^2*Sin[a*x])/(x*(a
*x*Cos[a*x] - Sin[a*x])))/a^6
```

Maple [A] time = 0.357, size = 172, normalized size = 1.4

$$\frac{-2i\left(2ia^2x^2e^{2iax}+2x^3a^3-2ix^2a^2-axe^{2iax}+ie^{2iax}+ax-i\right)}{\left(e^{2iax}-1\right)\left(axe^{2iax}+ie^{2iax}+ax-i\right)a^5}-\frac{4ix^2}{a^3}+4\frac{x\ln\left(e^{iax}+1\right)}{a^4}-\frac{4i\text{polylog}\left(2,-e^{iax}\right)}{a^5}+4\frac{x\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] -2*I*(2*I*a^2*x^2*exp(2*I*a*x)+2*x^3*a^3-2*I*a^2*x^2-a*x*exp(2*I*a*x)+I*exp
(2*I*a*x)+a*x-I)/(exp(2*I*a*x)-1)/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)/a
^5-4*I/a^3*x^2+4/a^4*x*ln(exp(I*a*x)+1)-4*I/a^5*polylog(2,-exp(I*a*x))+4/a^
4*x*ln(1-exp(I*a*x))-4*I/a^5*polylog(2,exp(I*a*x))
```

Maxima [B] time = 1.2702, size = 821, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (4*a^2*x^2 + 8*I*a*x*cos(2*a*x) - 8*a*x*sin(2*a*x) - 4*I*a*x - (4
*a^2*x^2 + 4*I*a*x)*cos(4*a*x) + 4*(-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(s
in(a*x), cos(a*x) + 1) - (4*a^2*x^2 + 8*I*a*x*cos(2*a*x) - 8*a*x*sin(2*a*x)
 - 4*I*a*x - (4*a^2*x^2 + 4*I*a*x)*cos(4*a*x) - 4*(I*a^2*x^2 - a*x)*sin(4*a
*x))*arctan2(sin(a*x), -cos(a*x) + 1) + 4*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x)
- (4*I*a^2*x^2 + 2*a*x - 2*I)*cos(2*a*x) - (4*a*x - (4*a*x + 4*I)*cos(4*a*x
) - 4*(I*a*x - 1)*sin(4*a*x) + 8*I*cos(2*a*x) - 8*sin(2*a*x) - 4*I)*dilog(-
e^(I*a*x)) - (4*a*x - (4*a*x + 4*I)*cos(4*a*x) - 4*(I*a*x - 1)*sin(4*a*x) +
 8*I*cos(2*a*x) - 8*sin(2*a*x) - 4*I)*dilog(e^(I*a*x)) - (2*I*a^2*x^2 - 4*a
*x*cos(2*a*x) - 4*I*a*x*sin(2*a*x) + 2*a*x - 2*(I*a^2*x^2 - a*x)*cos(4*a*x)
 + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(a*x)^2 + sin(a*x)^2 + 2*cos(a*
x) + 1) - (2*I*a^2*x^2 - 4*a*x*cos(2*a*x) - 4*I*a*x*sin(2*a*x) + 2*a*x - 2*
(I*a^2*x^2 - a*x)*cos(4*a*x) + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(a*
x)^2 + sin(a*x)^2 - 2*cos(a*x) + 1) - (-4*I*a^3*x^3 + 4*a^2*x^2)*sin(4*a*x)
 + (4*a^2*x^2 - 2*I*a*x - 2)*sin(2*a*x) - 2*I)/((I*a*x + (-I*a*x + 1)*cos(4
*a*x) + (a*x + I)*sin(4*a*x) - 2*cos(2*a*x) - 2*I*sin(2*a*x) + 1)*a^5)
```

Fricas [B] time = 2.43328, size = 1095, normalized size = 8.62

$$\frac{a^3x^3 - \left(2\,a^3x^3 + ax\right)\cos\left(ax\right)^2 + \left(2\,a^2x^2 + 1\right)\cos\left(ax\right)\sin\left(ax\right) + ax + \left(-2i\,ax\cos\left(ax\right)\sin\left(ax\right) - 2i\,\cos\left(ax\right)^2 + 2i\right)\mathrm{Li}_2\left(\mathrm{c}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x)
 + a*x + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2 + 2*I)*dilog(cos(a*x)
 + I*sin(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2 - 2*I)*dilog(c
os(a*x) - I*sin(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2 - 2*I)*
dilog(-cos(a*x) + I*sin(a*x)) + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^
2 + 2*I)*dilog(-cos(a*x) - I*sin(a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x
*cos(a*x)^2 - a*x)*log(cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin
(a*x) + a*x*cos(a*x)^2 - a*x)*log(cos(a*x) - I*sin(a*x) + 1) + 2*(a^2*x^2*c
os(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) + I*sin(a*x) + 1) +
2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) - I*sin(
a*x) + 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2 - a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*csc(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \csc(ax)^2}{(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x) - sin(a*x))^2, x)
```

3.594 $\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=176

$$\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}+\frac{a^2}{x}+\frac{32a^2\cos^4(ax)}{3x}-\frac{10a^2\cos^2(ax)}{x}-\frac{4\cos^4(a}{3x^3}$$

```
[Out] a^2/x + Cos[a*x]^2/x^3 - (10*a^2*Cos[a*x]^2)/x + Cos[a*x]^4/(a^2*x^5) - (4*
Cos[a*x]^4)/(3*x^3) + (32*a^2*Cos[a*x]^4)/(3*x) - (a*Cos[a*x]*Sin[a*x])/x^2
 - (Cos[a*x]^3*Sin[a*x])/(a*x^4) + (8*a*Cos[a*x]^3*Sin[a*x])/(3*x^2) - Cos[
a*x]^5/(a^2*x^5*(Cos[a*x] + a*x*Sin[a*x])) + (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```

Rubi [A] time = 0.299207, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4599, 3314, 30, 3313, 12, 3299}

$$\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}+\frac{a^2}{x}+\frac{32a^2\cos^4(ax)}{3x}-\frac{10a^2\cos^2(ax)}{x}-\frac{4\cos^4(a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```


```
[Out] a^2/x + Cos[a*x]^2/x^3 - (10*a^2*Cos[a*x]^2)/x + Cos[a*x]^4/(a^2*x^5) - (4*
Cos[a*x]^4)/(3*x^3) + (32*a^2*Cos[a*x]^4)/(3*x) - (a*Cos[a*x]*Sin[a*x])/x^2
 - (Cos[a*x]^3*Sin[a*x])/(a*x^4) + (8*a*Cos[a*x]^3*Sin[a*x])/(3*x^2) - Cos[
a*x]^5/(a^2*x^5*(Cos[a*x] + a*x*Sin[a*x])) + (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```


Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
```

```
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax) + ax\sin(ax))^2}\,dx = -\frac{\cos^5(ax)}{a^2x^5(\cos(ax) + ax\sin(ax))} - \frac{5\int \frac{\cos^4(ax)}{x^6}\,dx}{a^2}$$
$$= \frac{\cos^4(ax)}{a^2x^5} - \frac{\cos^3(ax)\sin(ax)}{ax^4} - \frac{\cos^5(ax)}{a^2x^5(\cos(ax) + ax\sin(ax))} - 3\int \frac{\cos^2(ax)}{x^4}\,dx + 4\int$$
$$= \frac{\cos^2(ax)}{x^3} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} - \frac{a\cos(ax)\sin(ax)}{x^2} - \frac{\cos^3(ax)\sin(ax)}{ax^4} + \frac{8a\cos^3}{}$$
$$= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2\cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} + \frac{32a^2\cos^4(ax)}{3x} - \frac{a\cos(ax}{}$$
$$= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2\cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} + \frac{32a^2\cos^4(ax)}{3x} - \frac{a\cos(ax}{}$$
$$= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2\cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} + \frac{32a^2\cos^4(ax)}{3x} - \frac{a\cos(ax}{}$$

Mathematica [A] time = 1.2424, size = 194, normalized size = 1.1

$$\frac{32a^3x^3\text{Si}(2ax)(ax\sin(ax) + \cos(ax)) + 256a^3x^3\text{Si}(4ax)(ax\sin(ax) + \cos(ax)) - 8a^3x^3\sin(ax) - 24a^3x^3\sin(3ax) + 32a^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] (-10*Cos[a*x] + 12*a^2*x^2*Cos[a*x] - 5*Cos[3*a*x] + 44*a^2*x^2*Cos[3*a*x]
- Cos[5*a*x] + 24*a^2*x^2*Cos[5*a*x] + 8*a*x*Sin[a*x] - 8*a^3*x^3*Sin[a*x]
+ 12*a*x*Sin[3*a*x] - 24*a^3*x^3*Sin[3*a*x] + 4*a*x*Sin[5*a*x] + 32*a^3*x^3
*Sin[5*a*x] + 32*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x] + 256
*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(Cos[a*x] +
a*x*Sin[a*x]))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{(\cos(ax))^6}{x^4(\cos(ax) + ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.9505, size = 421, normalized size = 2.39

$$-\frac{19\,a^2x^2\cos(ax)^3-\left(24\,a^2x^2-1\right)\cos(ax)^5-2\left(8\,a^3x^3\,\mathrm{Si}(4\,ax)+a^3x^3\,\mathrm{Si}(2\,ax)\right)\cos(ax)-\left(16\,a^4x^4\,\mathrm{Si}(4\,ax)+2\,a^4x^4\,\mathrm{Si}\right.}{3\left(ax^4\sin(ax)+x^3\cos(ax)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(19*a^2*x^2*cos(a*x)^3 - (24*a^2*x^2 - 1)*cos(a*x)^5 - 2*(8*a^3*x^3*si
n_integral(4*a*x) + a^3*x^3*sin_integral(2*a*x))*cos(a*x) - (16*a^4*x^4*sin
_integral(4*a*x) + 2*a^4*x^4*sin_integral(2*a*x) - 30*a^3*x^3*cos(a*x)^2 +
3*a^3*x^3 + 4*(8*a^3*x^3 + a*x)*cos(a*x)^4)*sin(a*x))/(a*x^4*sin(a*x) + x^3
*cos(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^6(ax)}{x^4\left(ax\sin(ax)+\cos(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**6/x**4/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**6/(x**4*(a*x*sin(a*x) + cos(a*x))**2), x)
```

Giac [C] time = 2.494, size = 9827, normalized size = 55.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(64*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^
2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan
(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x
)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*
tan(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a
*x)^2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2
*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2
*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x
))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integr
al(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integra
l(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2*
a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_inte
gral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2
*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x)
)*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*ta
n(2*a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(
1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 64*a^
8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*im
ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_in
tegral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*t
an(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*
a*x) + 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 +
4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 4*a^7*x^
7*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 32*a^7*x^7*imag
_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*sin_integr
al(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^7*x^7*sin_integral(2*a*x)*tan(2*a*x
)^2*tan(a*x)^2 - 40*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^7*x
^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*i
mag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_
part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_pa
```

```
rt(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integ
ral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2*a*x)*tan(
2*a*x)^2*tan(1/2*a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x
)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(a*x)^2*ta
n(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2
*a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x
)^2 - 64*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*
sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_in
tegral(4*a*x))*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(
1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^8*
x^8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^8*x^8*sin_integral
(4*a*x)*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x) + 128*a^
6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) +
 16*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*
a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*t
an(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(
a*x)^2*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)
^2*tan(1/2*a*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*ta
n(1/2*a*x) + 20*a^6*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7
*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^7*x^7*imag_part(cos_inte
gral(2*a*x))*tan(2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(2
*a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 + 64*a^7*
x^7*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^7*x^7*sin_integral(2*a*x)*tan(2*
a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x)^2 + 4*a^7*x^7*i
mag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^7*x^7*imag_part(cos_integral
(-2*a*x))*tan(a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^
2 + 64*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^7*x^7*sin_integral(2*a*
x)*tan(a*x)^2 - 24*a^7*x^7*tan(2*a*x)^2*tan(1/2*a*x) + 24*a^7*x^7*tan(a*x)^
2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 -
 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_p
art(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integra
l(-4*a*x))*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integral(4*a*x)*tan(1/2*a*x)^2 -
 8*a^7*x^7*sin_integral(2*a*x)*tan(1/2*a*x)^2 - 64*a^5*x^5*imag_part(cos_in
tegral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_part
(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^5*x^5*im
ag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a
^5*x^5*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)
^2 - 128*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2
 - 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 -
20*a^6*x^6*tan(2*a*x)^2*tan(a*x)^2 + 128*a^6*x^6*imag_part(cos_integral(4*a
*x))*tan(2*a*x)^2*tan(1/2*a*x) + 16*a^6*x^6*imag_part(cos_integral(2*a*x))*
tan(2*a*x)^2*tan(1/2*a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(
2*a*x)^2*tan(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a
*x)^2*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a
*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^6*x^6*
```

tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 128*a^6*x^6*imag_part(cos_integral(4*a
*x))*tan(a*x)^2*tan(1/2*a*x) + 16*a^6*x^6*imag_part(cos_integral(2*a*x))*ta
n(a*x)^2*tan(1/2*a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(a*x)
^2*tan(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*ta
n(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) + 32*a
^6*x^6*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 16*a^6*x^6*tan(2*a*x)*
tan(a*x)^2*tan(1/2*a*x) + 12*a^6*x^6*tan(2*a*x)^2*tan(1/2*a*x)^2 - 12*a^6*x
^6*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(4*a*x)) +
4*a^7*x^7*imag_part(cos_integral(2*a*x)) - 4*a^7*x^7*imag_part(cos_integral
(-2*a*x)) - 32*a^7*x^7*imag_part(cos_integral(-4*a*x)) + 64*a^7*x^7*sin_int
egral(4*a*x) + 8*a^7*x^7*sin_integral(2*a*x) + 64*a^5*x^5*imag_part(cos_int
egral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(2*
a*x))*tan(2*a*x)^2*tan(a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*t
an(2*a*x)^2*tan(a*x)^2 - 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(2*a
*x)^2*tan(a*x)^2 + 128*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2
+ 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 40*a^7*x^7*tan(1
/2*a*x) - 72*a^5*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^5*x^5*imag
_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_par
t(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 8*a^5*x^5*imag_part(co
s_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_
integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 128*a^5*x^5*sin_integral(4*
a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x
)^2*tan(1/2*a*x)^2 - 4*a^5*x^5*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 - 64*a^
5*x^5*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*
imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^5*x^5*imag_p
art(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(
cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 128*a^5*x^5*sin_integral(
4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 16*a^5*x^5*sin_integral(2*a*x)*tan(a*x)^
2*tan(1/2*a*x)^2 - 8*a^5*x^5*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 12*a^6*
x^6*tan(2*a*x)^2 + 12*a^6*x^6*tan(a*x)^2 + 128*a^6*x^6*imag_part(cos_integr
al(4*a*x))*tan(1/2*a*x) + 16*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(1/2
*a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 128*a^6*x
^6*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(
4*a*x)*tan(1/2*a*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(1/2*a*x) + 16*a^6*
x^6*tan(2*a*x)*tan(1/2*a*x) + 8*a^6*x^6*tan(a*x)*tan(1/2*a*x) + 64*a^4*x^4*
imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^4
*x^4*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) -
8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a
*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*ta
n(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/
2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x
) - 20*a^6*x^6*tan(1/2*a*x)^2 + 36*a^4*x^4*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*
a*x)^2 + 64*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 8*a^5*x^5
*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 8*a^5*x^5*imag_part(cos_inte
gral(-2*a*x))*tan(2*a*x)^2 - 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan

(2*a*x)^2 + 128*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2 + 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2 + 4*a^5*x^5*tan(2*a*x)^2*tan(a*x) + 64*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(a*x)^2 + 128*a^5*x^5*sin_integral(4*a*x)*tan(a*x)^2 + 16*a^5*x^5*sin_integral(2*a*x)*tan(a*x)^2 + 8*a^5*x^5*tan(2*a*x)*tan(a*x)^2 - 48*a^5*x^5*tan(2*a*x)^2*tan(1/2*a*x) + 48*a^5*x^5*tan(a*x)^2*tan(1/2*a*x) - 64*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x)^2 - 128*a^5*x^5*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 16*a^5*x^5*sin_integral(2*a*x)*tan(1/2*a*x)^2 - 8*a^5*x^5*tan(2*a*x)*tan(1/2*a*x)^2 - 4*a^5*x^5*tan(a*x)*tan(1/2*a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 20*a^6*x^6 - 36*a^4*x^4*tan(2*a*x)^2*tan(a*x)^2 + 64*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 4*a^4*x^4*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 64*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 26*a^4*x^4*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 24*a^4*x^4*tan(2*a*x)^2*tan(1/2*a*x)^2 - 24*a^4*x^4*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_integral(4*a*x)) + 8*a^5*x^5*imag_part(cos_integral(2*a*x)) - 8*a^5*x^5*imag_part(cos_integral(-2*a*x)) - 64*a^5*x^5*imag_part(cos_integral(-4*a*x)) + 128*a^5*x^5*sin_integral(4*a*x) + 16*a^5*x^5*sin_integral(2*a*x) + 8*a^5*x^5*tan(2*a*x) + 4*a^5*x^5*tan(a*x) + 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 64*a^3*x^3*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 72*a^5*x^5*tan(1/2*a*x) - 30*a^3*x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(

-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 2*a^3*x^3*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 13*a^3*x^3*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 24*a^4*x^4*tan(2*a*x)^2 + 24*a^4*x^4*tan(a*x)^2 + 64*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(1/2*a*x) + 8*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(1/2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(1/2*a*x) + 26*a^4*x^4*tan(2*a*x)*tan(1/2*a*x) + 4*a^4*x^4*tan(a*x)*tan(1/2*a*x) - 36*a^4*x^4*tan(1/2*a*x)^2 + 27*a^2*x^2*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 + 64*a^3*x^3*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2 + 2*a^3*x^3*tan(2*a*x)^2*tan(a*x) + 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(a*x)^2 + 64*a^3*x^3*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2 + 13*a^3*x^3*tan(2*a*x)*tan(a*x)^2 - 6*a^3*x^3*tan(2*a*x)^2*tan(1/2*a*x) + 24*a^3*x^3*tan(a*x)^2*tan(1/2*a*x) - 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x)^2 - 13*a^3*x^3*tan(2*a*x)*tan(1/2*a*x)^2 - 2*a^3*x^3*tan(a*x)*tan(1/2*a*x)^2 + 36*a^4*x^4 - 27*a^2*x^2*tan(2*a*x)^2*tan(a*x)^2 + 20*a^2*x^2*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 10*a^2*x^2*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 15*a^2*x^2*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(4*a*x)) + 4*a^3*x^3*imag_part(cos_integral(2*a*x)) - 4*a^3*x^3*imag_part(cos_integral(-2*a*x)) - 32*a^3*x^3*imag_part(cos_integral(-4*a*x)) + 64*a^3*x^3*sin_integral(4*a*x) + 8*a^3*x^3*sin_integral(2*a*x) + 13*a^3*x^3*tan(2*a*x) + 2*a^3*x^3*tan(a*x) + 48*a^3*x^3*tan(1/2*a*x) + 2*a*x*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 10*a*x*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 - 5*a*x*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 15*a^2*x^2*tan(2*a*x)^2 + 10*a^2*x^2*tan(2*a*x)*tan(1/2*a*x) + 20*a^2*x^2*tan(a*x)*tan(1/2*a*x) - 12*a^2*x^2*tan(1/2*a*x)^2 - tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 10*a*x*tan(2*a*x)^2*tan(a*x) + 5*a*x*tan(2*a*x)*tan(a*x)^2 - 6*a*x*tan(2*a*x)^2*tan(1/2*a*x) - 5*a*x*tan(2*a*x)*tan(1/2*a*x)^2 - 10*a*x*tan(a*x)*t

```
an(1/2*a*x)^2 + 12*a^2*x^2 + tan(2*a*x)^2*tan(a*x)^2 + 3*tan(2*a*x)^2*tan(1
/2*a*x)^2 + 5*a*x*tan(2*a*x) + 10*a*x*tan(a*x) - 8*a*x*tan(1/2*a*x) - 3*tan
(2*a*x)^2 + 4*tan(1/2*a*x)^2 - 4)/(2*a^5*x^8*tan(2*a*x)^2*tan(a*x)^2*tan(1/
2*a*x) - a^4*x^7*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^5*x^8*tan(2*a
*x)^2*tan(1/2*a*x) + 2*a^5*x^8*tan(a*x)^2*tan(1/2*a*x) + a^4*x^7*tan(2*a*x)
^2*tan(a*x)^2 - a^4*x^7*tan(2*a*x)^2*tan(1/2*a*x)^2 - a^4*x^7*tan(a*x)^2*ta
n(1/2*a*x)^2 + 2*a^5*x^8*tan(1/2*a*x) + 4*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2*t
an(1/2*a*x) + a^4*x^7*tan(2*a*x)^2 + a^4*x^7*tan(a*x)^2 - a^4*x^7*tan(1/2*a
*x)^2 - 2*a^2*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^6*tan(2*
a*x)^2*tan(1/2*a*x) + 4*a^3*x^6*tan(a*x)^2*tan(1/2*a*x) + a^4*x^7 + 2*a^2*x
^5*tan(2*a*x)^2*tan(a*x)^2 - 2*a^2*x^5*tan(2*a*x)^2*tan(1/2*a*x)^2 - 2*a^2*
x^5*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^6*tan(1/2*a*x) + 2*a*x^4*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x) + 2*a^2*x^5*tan(2*a*x)^2 + 2*a^2*x^5*tan(a*x)^2
- 2*a^2*x^5*tan(1/2*a*x)^2 - x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2
*a*x^4*tan(2*a*x)^2*tan(1/2*a*x) + 2*a*x^4*tan(a*x)^2*tan(1/2*a*x) + 2*a^2*
x^5 + x^3*tan(2*a*x)^2*tan(a*x)^2 - x^3*tan(2*a*x)^2*tan(1/2*a*x)^2 - x^3*t
an(a*x)^2*tan(1/2*a*x)^2 + 2*a*x^4*tan(1/2*a*x) + x^3*tan(2*a*x)^2 + x^3*ta
n(a*x)^2 - x^3*tan(1/2*a*x)^2 + x^3)
```

3.595 $\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{8}a^2\text{CosIntegral}(ax)-\frac{27}{8}a^2\text{CosIntegral}(3ax)+\frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}-\frac{3\cos^3(ax)}{2x^2}+\frac{\cos(ax)}{x^2}-\text{s}$$

```
[Out] Cos[a*x]/x^2 + Cos[a*x]^3/(a^2*x^4) - (3*Cos[a*x]^3)/(2*x^2) - (a^2*CosInte
gral[a*x])/8 - (27*a^2*CosIntegral[3*a*x])/8 - (a*Sin[a*x])/x - (Cos[a*x]^2
*Sin[a*x])/(a*x^3) + (9*a*Cos[a*x]^2*Sin[a*x])/(2*x) - Cos[a*x]^4/(a^2*x^4*
(Cos[a*x] + a*x*Sin[a*x]))
```

Rubi [A] time = 0.226077, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4599, 3314, 3297, 3302, 3312}

$$-\frac{1}{8}a^2\text{CosIntegral}(ax)-\frac{27}{8}a^2\text{CosIntegral}(3ax)+\frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}-\frac{3\cos^3(ax)}{2x^2}+\frac{\cos(ax)}{x^2}-\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```


```
[Out] Cos[a*x]/x^2 + Cos[a*x]^3/(a^2*x^4) - (3*Cos[a*x]^3)/(2*x^2) - (a^2*CosInte
gral[a*x])/8 - (27*a^2*CosIntegral[3*a*x])/8 - (a*Sin[a*x])/x - (Cos[a*x]^2
*Sin[a*x])/(a*x^3) + (9*a*Cos[a*x]^2*Sin[a*x])/(2*x) - Cos[a*x]^4/(a^2*x^4*
(Cos[a*x] + a*x*Sin[a*x]))
```


Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
```

```
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^4(ax)}{a^2x^4(\cos(ax)+ax\sin(ax))} - \frac{4\int \frac{\cos^3(ax)}{x^5}\,dx}{a^2} \\
&= \frac{\cos^3(ax)}{a^2x^4} - \frac{\cos^2(ax)\sin(ax)}{ax^3} - \frac{\cos^4(ax)}{a^2x^4(\cos(ax)+ax\sin(ax))} - 2\int \frac{\cos(ax)}{x^3}\,dx + 3\int \frac{\mathrm{c}}{-} \\
&= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2x^4} - \frac{3\cos^3(ax)}{2x^2} - \frac{\cos^2(ax)\sin(ax)}{ax^3} + \frac{9a\cos^2(ax)\sin(ax)}{2x} - \frac{}{a^2x^4(\mathrm{co}} \\
&= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2x^4} - \frac{3\cos^3(ax)}{2x^2} + 9a^2\mathrm{Ci}(ax) - \frac{a\sin(ax)}{x} - \frac{\cos^2(ax)\sin(ax)}{ax^3} + \frac{9a\,\mathrm{co}}{} \\
&= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2x^4} - \frac{3\cos^3(ax)}{2x^2} + 10a^2\mathrm{Ci}(ax) - \frac{a\sin(ax)}{x} - \frac{\cos^2(ax)\sin(ax)}{ax^3} + \frac{9a\,\mathrm{c}}{} \\
&= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2x^4} - \frac{3\cos^3(ax)}{2x^2} - \frac{1}{8}a^2\mathrm{Ci}(ax) - \frac{27}{8}a^2\mathrm{Ci}(3ax) - \frac{a\sin(ax)}{x} - \frac{\cos^2(ax}{ax}
\end{aligned}
$$

Mathematica [A] time = 0.808307, size = 136, normalized size = 1.03

$$-\frac{2a^2x^2\text{CosIntegral}(ax)(ax\sin(ax)+\cos(ax))+54a^2x^2\text{CosIntegral}(3ax)(ax\sin(ax)+\cos(ax))-a^2x^2-8a^2x^2\cos(2}{16x^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] -(3 - a^2*x^2 + 4*Cos[2*a*x] - 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a^2*x^
2*Cos[4*a*x] + 2*a^2*x^2*CosIntegral[a*x]*(Cos[a*x] + a*x*Sin[a*x]) + 54*a^
2*x^2*CosIntegral[3*a*x]*(Cos[a*x] + a*x*Sin[a*x]) - 12*a*x*Sin[2*a*x] - 6*
a*x*Sin[4*a*x])/(16*x^2*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{(\cos(ax))^5}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.59391, size = 535, normalized size = 4.05

$$\frac{88\,a^2x^2\cos(ax)^2 - 8\left(9\,a^2x^2+1\right)\cos(ax)^4 - 16\,a^2x^2 - \left(27\,a^2x^2\,\mathrm{Ci}(3\,ax) + a^2x^2\,\mathrm{Ci}(ax) + a^2x^2\,\mathrm{Ci}(-ax) + 27\,a^2x^2\,\mathrm{Ci}(-3}{16\left(ax^3\sin(ax) + x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(88*a^2*x^2*cos(a*x)^2 - 8*(9*a^2*x^2 + 1)*cos(a*x)^4 - 16*a^2*x^2 - (
27*a^2*x^2*cos_integral(3*a*x) + a^2*x^2*cos_integral(a*x) + a^2*x^2*cos_in
tegral(-a*x) + 27*a^2*x^2*cos_integral(-3*a*x))*cos(a*x) - (27*a^3*x^3*cos_
integral(3*a*x) + a^3*x^3*cos_integral(a*x) + a^3*x^3*cos_integral(-a*x) +
27*a^3*x^3*cos_integral(-3*a*x) - 48*a*x*cos(a*x)^3)*sin(a*x))/(a*x^3*sin(a
*x) + x^2*cos(a*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**5/x**3/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.73133, size = 4226, normalized size = 32.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -1/16*(54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x
)^3 + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3
+ 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 5
4*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2
```

7*a^6*x^6*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^6*x^6*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 54*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 - 27*a^6*x^6*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^6*x^6*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(1/2*a*x) - 8*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^2 - 72*a^6*x^6*tan(3/2*a*x)*tan(1/2*a*x)^3 + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 27*a^6*x^6*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^6*x^6*real_part(cos_integral(a*x))*tan(3/2*a*x)^2 + a^6*x^6*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2 + 27*a^6*x^6*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 - 12*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 36*a^5*x^5*tan(3/2*a*x)*tan(1/2*a*x)^4 - 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 54*a^4*x^4*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 72*a^6*x^6*tan(3/2*a*x)*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 8*a^6*x^6*tan(1/2*a*x)^2 + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 + 8*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^6*x^6*real_part(cos_integral(3*a*x)) + a^6*x^6*real_part(cos_integral(a*x)) + a^6*x^6*real_part(cos_integral(-a*x)) + 27*a^6*x^6*real_part(cos_integral(-3*a*x)) - 12*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 12*a^5*x^5*tan(1/2*a*x)^3 - 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 54*a^4*x^4*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 108*a^5*x^5*real_part(cos_integral(3*a*x)) *tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(1/2*a*x) + 4*a^5 *x^5*real_part(cos_integral(-a*x))*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos _integral(-3*a*x))*tan(1/2*a*x) - 4*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^2 - 128*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integra l(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^4*x^4*tan(1/2*a*x)^4 - 36*a^5*x^5*ta n(3/2*a*x) + 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + 2*a ^4*x^4*real_part(cos_integral(a*x))*tan(3/2*a*x)^2 + 2*a^4*x^4*real_part(co s_integral(-a*x))*tan(3/2*a*x)^2 + 54*a^4*x^4*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 + 12*a^5*x^5*tan(1/2*a*x) + 64*a^3*x^3*tan(3/2*a*x)*tan(1 /2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/ 2*a*x)^4 - a^2*x^2*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) ^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 4*a^4*x^4*tan(3/2*a*x)^2 - 128*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x) + 54*a^3*x ^3*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^3*x^3*r eal_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^3*x^3*real_pa rt(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^3*x^3*real_part(c os_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a^4*x^4*tan(1/2*a*x)^2 + 54*a^3*x^3*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*rea l_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral (-a*x))*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(1/2 *a*x)^3 + 32*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^4*x^4*real_part(c os_integral(3*a*x)) + 2*a^4*x^4*real_part(cos_integral(a*x)) + 2*a^4*x^4*re al_part(cos_integral(-a*x)) + 54*a^4*x^4*real_part(cos_integral(-3*a*x)) - 32*a^3*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 32*a^3*x^3*tan(1/2*a*x)^3 - 27*a^2 *x^2*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_ integral(a*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(1 /2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 8*a ^4*x^4 + 54*a^3*x^3*real_part(cos_integral(3*a*x))*tan(1/2*a*x) + 2*a^3*x^3 *real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^3*x^3*real_part(cos_integr al(-a*x))*tan(1/2*a*x) + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(1/2 *a*x) + 24*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^2 - 56*a^2*x^2*tan(3/2*a*x)* tan(1/2*a*x)^3 + 16*a^2*x^2*tan(1/2*a*x)^4 - 64*a^3*x^3*tan(3/2*a*x) + 27*a ^2*x^2*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^2*x^2*real_part(co s_integral(a*x))*tan(3/2*a*x)^2 + a^2*x^2*real_part(cos_integral(-a*x))*tan (3/2*a*x)^2 + 27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 + 1 2*a*x*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 28*a*x*tan(3/2*a*x)*tan(1/2*a*x)^4 + 16*a^2*x^2*tan(3/2*a*x)^2 - 56*a^2*x^2*tan(3/2*a*x)*tan(1/2*a*x) + 24*a^2*x ^2*tan(1/2*a*x)^2 + 8*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^2*x^2*real_part(cos_integral(3*a*x)) + a^2*x^2*real_part(cos_integral(a*x)) + a^2*x^2*real_ part(cos_integral(-a*x)) + 27*a^2*x^2*real_part(cos_integral(-3*a*x)) - 20* a*x*tan(3/2*a*x)^2*tan(1/2*a*x) + 20*a*x*tan(1/2*a*x)^3 + 32*a^2*x^2 - 12*t

```
an(3/2*a*x)^2*tan(1/2*a*x)^2 + 4*tan(1/2*a*x)^4 - 28*a*x*tan(3/2*a*x) - 12*
a*x*tan(1/2*a*x) + 4*tan(3/2*a*x)^2 - 12*tan(1/2*a*x)^2 + 8)/(2*a^5*x^7*tan
(3/2*a*x)^2*tan(1/2*a*x)^3 - a^4*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 2*a^5*
x^7*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^5*x^7*tan(1/2*a*x)^3 - a^4*x^6*tan(1/
2*a*x)^4 + 2*a^5*x^7*tan(1/2*a*x) + 4*a^3*x^5*tan(3/2*a*x)^2*tan(1/2*a*x)^3
 + a^4*x^6*tan(3/2*a*x)^2 - 2*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 4*a^3
*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^3*x^5*tan(1/2*a*x)^3 + a^4*x^6 - 2*a
^2*x^4*tan(1/2*a*x)^4 + 4*a^3*x^5*tan(1/2*a*x) + 2*a*x^3*tan(3/2*a*x)^2*tan
(1/2*a*x)^3 + 2*a^2*x^4*tan(3/2*a*x)^2 - x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^4
+ 2*a*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a*x^3*tan(1/2*a*x)^3 + 2*a^2*x^4
- x^2*tan(1/2*a*x)^4 + 2*a*x^3*tan(1/2*a*x) + x^2*tan(3/2*a*x)^2 + x^2)
```

3.596 $\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=80

$$\frac{\cos^2(ax)}{a^2x^3} - \frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))} - 2a\text{Si}(2ax) - \frac{\sin(ax)\cos(ax)}{ax^2} - \frac{2\cos^2(ax)}{x} + \frac{1}{x}$$

```
[Out] x^(-1) + Cos[a*x]^2/(a^2*x^3) - (2*Cos[a*x]^2)/x - (Cos[a*x]*Sin[a*x])/(a*x
^2) - Cos[a*x]^3/(a^2*x^3*(Cos[a*x] + a*x*Sin[a*x])) - 2*a*SinIntegral[2*a*
x]
```

Rubi [A] time = 0.128426, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4599, 3314, 30, 3313, 12, 3299}

$$\frac{\cos^2(ax)}{a^2x^3} - \frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))} - 2a\text{Si}(2ax) - \frac{\sin(ax)\cos(ax)}{ax^2} - \frac{2\cos^2(ax)}{x} + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] x^(-1) + Cos[a*x]^2/(a^2*x^3) - (2*Cos[a*x]^2)/x - (Cos[a*x]*Sin[a*x])/(a*x
^2) - Cos[a*x]^3/(a^2*x^3*(Cos[a*x] + a*x*Sin[a*x])) - 2*a*SinIntegral[2*a*
x]
```

Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
```

```
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}-\frac{3\int\frac{\cos^2(ax)}{x^4}\,dx}{a^2}\\
&= \frac{\cos^2(ax)}{a^2x^3}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}+2\int\frac{\cos^2(ax)}{x^2}\,dx-\int\frac{1}{x^2}\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}+(4a)\int\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}-(2a)\int\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}-2a\mathrm{Si}(2
\end{aligned}
$$

Mathematica [A] time = 0.684764, size = 71, normalized size = 0.89

$$-\frac{8ax\text{Si}(2ax)(ax\sin(ax)+\cos(ax))-2ax\sin(ax)+2ax\sin(3ax)+3\cos(ax)+\cos(3ax)}{4x(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] -(3*Cos[a*x] + Cos[3*a*x] - 2*a*x*Sin[a*x] + 2*a*x*Sin[3*a*x] + 8*a*x*(Cos[
a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x])/(4*x*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{(\cos(ax))^4}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.29183, size = 203, normalized size = 2.54

$$-\frac{2\,ax\cos(ax)\,\text{Si}(2\,ax)+\cos(ax)^3+\left(2\,a^2x^2\,\text{Si}(2\,ax)+2\,ax\cos(ax)^2-ax\right)\sin(ax)}{ax^2\sin(ax)+x\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*x*cos(a*x)*sin_integral(2*a*x) + cos(a*x)^3 + (2*a^2*x^2*sin_integral
(2*a*x) + 2*a*x*cos(a*x)^2 - a*x)*sin(a*x))/(a*x^2*sin(a*x) + x*cos(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^4(ax)}{x^2(ax\sin(ax)+\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**4/x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**4/(x**2*(a*x*sin(a*x) + cos(a*x))**2), x)
```

Giac [C] time = 1.45223, size = 1346, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -(2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^4*
x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^4*x^4*sin
_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_integral(2
*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*
tan(a*x)^2*tan(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x)^2 + 2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^4*x^
4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^4*x^4*sin_integral(2*a
*x)*tan(1/2*a*x) + a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a^3*
x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a^3*x^3*sin_integral(2*a
*x)*tan(a*x)^2 - 2*a^3*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_
integral(2*a*x))*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*t
an(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^2*x^2*im
ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^2*x^2*imag_part(
```

```
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(2*a*
x)*tan(a*x)^2*tan(1/2*a*x) + a^2*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*im
ag_part(cos_integral(2*a*x)) - a^3*x^3*imag_part(cos_integral(-2*a*x)) + 2*
a^3*x^3*sin_integral(2*a*x) + 2*a^3*x^3*tan(1/2*a*x) - a*x*imag_part(cos_in
tegral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*
x))*tan(a*x)^2*tan(1/2*a*x)^2 - 2*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x)^2 - a^2*x^2*tan(a*x)^2 + 2*a^2*x^2*imag_part(cos_integral(2*a*x))*ta
n(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^2
*x^2*sin_integral(2*a*x)*tan(1/2*a*x) + 2*a^2*x^2*tan(a*x)*tan(1/2*a*x) - a
^2*x^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a*x
*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a*x*sin_integral(2*a*x)*tan
(a*x)^2 - 2*a*x*tan(a*x)^2*tan(1/2*a*x) - a*x*imag_part(cos_integral(2*a*x)
)*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 2*a
*x*sin_integral(2*a*x)*tan(1/2*a*x)^2 - a*x*tan(a*x)*tan(1/2*a*x)^2 + a^2*x
^2 + a*x*imag_part(cos_integral(2*a*x)) - a*x*imag_part(cos_integral(-2*a*x
)) + 2*a*x*sin_integral(2*a*x) + a*x*tan(a*x) - tan(1/2*a*x)^2 + 1)/(2*a^3*
x^4*tan(a*x)^2*tan(1/2*a*x) - a^2*x^3*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^3*x^4
*tan(1/2*a*x) + a^2*x^3*tan(a*x)^2 - a^2*x^3*tan(1/2*a*x)^2 + 2*a*x^2*tan(a
*x)^2*tan(1/2*a*x) + a^2*x^3 - x*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a*x^2*tan(1/
2*a*x) + x*tan(a*x)^2 - x*tan(1/2*a*x)^2 + x)
```

3.597 $\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=56

$$\frac{\cos(ax)}{a^2x^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}+\text{CosIntegral}(ax)-\frac{\sin(ax)}{ax}$$

```
[Out] Cos[a*x]/(a^2*x^2) + CosIntegral[a*x] - Sin[a*x]/(a*x) - Cos[a*x]^2/(a^2*x^
2*(Cos[a*x] + a*x*Sin[a*x]))
```

Rubi [A] time = 0.09317, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4599, 3297, 3302}

$$\frac{\cos(ax)}{a^2x^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}+\text{CosIntegral}(ax)-\frac{\sin(ax)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/(a^2*x^2) + CosIntegral[a*x] - Sin[a*x]/(a*x) - Cos[a*x]^2/(a^2*x^
2*(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}-\frac{2\int \frac{\cos(ax)}{x^3}\,dx}{a^2}\\ &= \frac{\cos(ax)}{a^2x^2}-\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}+\frac{\int \frac{\sin(ax)}{x^2}\,dx}{a}\\ &= \frac{\cos(ax)}{a^2x^2}-\frac{\sin(ax)}{ax}-\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}+\int \frac{\cos(ax)}{x}\,dx\\ &= \frac{\cos(ax)}{a^2x^2}+\mathrm{Ci}(ax)-\frac{\sin(ax)}{ax}-\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}\end{aligned}$$

Mathematica [C] time = 7.47208, size = 237, normalized size = 4.23

$-eax\mathrm{CosIntegral}(ax+i)\sin(ax)-e\mathrm{CosIntegral}(ax+i)\cos(ax)+2\mathrm{CosIntegral}(ax)(ax\sin(ax)+\cos(ax))-e\mathrm{CosInt}$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] (-1 + Cos[2*a*x] - E*Cos[a*x]*CosIntegral[I + a*x] + E*Cos[a*x]*ExpIntegral
Ei[-1 - I*a*x] + E*Cos[a*x]*ExpIntegralEi[-1 + I*a*x] - a*E*x*CosIntegral[I
 + a*x]*Sin[a*x] + a*E*x*ExpIntegralEi[-1 - I*a*x]*Sin[a*x] + a*E*x*ExpInte
gralEi[-1 + I*a*x]*Sin[a*x] + 2*CosIntegral[a*x]*(Cos[a*x] + a*x*Sin[a*x])
- E*CosIntegral[I - a*x]*(Cos[a*x] + a*x*Sin[a*x]) - I*E*Cos[a*x]*SinIntegr
al[I - a*x] - I*a*E*x*Sin[a*x]*SinIntegral[I - a*x] - I*E*Cos[a*x]*SinInteg
ral[I + a*x] - I*a*E*x*Sin[a*x]*SinIntegral[I + a*x])/(2*(Cos[a*x] + a*x*Si
n[a*x]))
```

Maple [C] time = 1.171, size = 106, normalized size = 1.9

$$-\frac{e^{iax}}{-2+2\,iax}-\frac{\mathrm{Ei}\left(1,-iax\right)}{2}+\frac{e^{-iax}}{2+2\,iax}-\frac{\mathrm{Ei}\left(1,iax\right)}{2}-\frac{2\,ie^{iax}}{\left(ax+i\right)\left(ax-i\right)\left(axe^{2\,iax}-ax+ie^{2\,iax}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] -1/2*exp(I*a*x)/(-1+I*a*x)-1/2*Ei(1,-I*a*x)+1/2*exp(-I*a*x)/(1+I*a*x)-1/2*E
i(1,I*a*x)-2*I*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a
*x)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.21616, size = 219, normalized size = 3.91

$$\frac{(\text{Ci}(ax) + \text{Ci}(-ax))\cos(ax) + 2\cos(ax)^2 + (ax\,\text{Ci}(ax) + ax\,\text{Ci}(-ax))\sin(ax) - 2}{2\,(ax\sin(ax) + \cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(a*x) + cos_integral(-a*x))*cos(a*x) + 2*cos(a*x)^2 + (a*
x*cos_integral(a*x) + a*x*cos_integral(-a*x))*sin(a*x) - 2)/(a*x*sin(a*x) +
 cos(a*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**3/x/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.28893, size = 494, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*real
_part(cos_integral(-a*x))*tan(1/2*a*x)^3 - a^2*x^2*real_part(cos_integral(a
*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4
+ 2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^3*x^3*real_part
(cos_integral(-a*x))*tan(1/2*a*x) - 8*a^2*x^2*tan(1/2*a*x)^2 + 2*a*x*real_p
art(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a*x*real_part(cos_integral(-a*x))
*tan(1/2*a*x)^3 + a^2*x^2*real_part(cos_integral(a*x)) + a^2*x^2*real_part(
cos_integral(-a*x)) - real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - real_pa
rt(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*real_part(cos_integral(a*x))*
tan(1/2*a*x) + 2*a*x*real_part(cos_integral(-a*x))*tan(1/2*a*x) - 2*tan(1/2
*a*x)^4 - 12*tan(1/2*a*x)^2 + real_part(cos_integral(a*x)) + real_part(cos_
integral(-a*x)) - 2)/(2*a^3*x^3*tan(1/2*a*x)^3 - a^2*x^2*tan(1/2*a*x)^4 + 2
*a^3*x^3*tan(1/2*a*x) + 2*a*x*tan(1/2*a*x)^3 + a^2*x^2 - tan(1/2*a*x)^4 + 2
*a*x*tan(1/2*a*x) + 1)
```

3.598 $\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(ax\sin(ax)+\cos(ax))}$$

```
[Out] 1/(a^2*x) - Cos[a*x]/(a^2*x*(Cos[a*x] + a*x*Sin[a*x]))
```

Rubi [A] time = 0.0224119, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4597}

$$\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] 1/(a^2*x) - Cos[a*x]/(a^2*x*(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 4597

```
Int[Cos[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] - Simp[Cos[a*x]/(a*d*x*(d*x*Sin[a*x] + c
*Cos[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

Rubi steps

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(\cos(ax)+ax\sin(ax))}$$

Mathematica [A] time = 0.220609, size = 22, normalized size = 0.65

$$\frac{\sin(ax)}{a(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] Sin[a*x]/(a*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [B] time = 0.663, size = 70, normalized size = 2.1

$$\left(2\,\frac{\tan(1/2\,ax)}{a}+4\,\frac{(\tan(1/2\,ax))^3}{a}+2\,\frac{(\tan(1/2\,ax))^5}{a}\right)\left(1+\left(\tan\left(\frac{ax}{2}\right)\right)^2\right)^{-2}\left(2\,\tan(1/2\,ax)\,xa-\left(\tan\left(\frac{ax}{2}\right)\right)^2+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] (2/a*tan(1/2*a*x)+4/a*tan(1/2*a*x)^3+2/a*tan(1/2*a*x)^5)/(1+tan(1/2*a*x)^2)
^2/(2*tan(1/2*a*x)*x*a-tan(1/2*a*x)^2+1)
```

Maxima [B] time = 1.06031, size = 154, normalized size = 4.53

$$\frac{ax\cos(2\,ax)^2+ax\sin(2\,ax)^2-2\,ax\cos(2\,ax)+ax+2\,\sin(2\,ax)}{\left(a^2x^2+\left(a^2x^2+1\right)\cos(2\,ax)^2+4\,ax\sin(2\,ax)+\left(a^2x^2+1\right)\sin(2\,ax)^2-2\left(a^2x^2-1\right)\cos(2\,ax)+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 - 2*a*x*cos(2*a*x) + a*x + 2*sin(2*a*x
))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1
)*sin(2*a*x)^2 - 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

Fricas [A] time = 2.02329, size = 54, normalized size = 1.59

$$\frac{\sin(ax)}{a^2x\sin(ax)+a\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] sin(a*x)/(a^2*x*sin(a*x) + a*cos(a*x))
```

Sympy [A] time = 3.09642, size = 20, normalized size = 0.59

$$\frac{\sin(ax)}{a^2x\sin(ax) + a\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] sin(a*x)/(a**2*x*sin(a*x) + a*cos(a*x))
```

Giac [A] time = 1.14777, size = 43, normalized size = 1.26

$$\frac{2\tan\left(\frac{1}{2}ax\right)}{2a^2x\tan\left(\frac{1}{2}ax\right) - a\tan\left(\frac{1}{2}ax\right)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 2*tan(1/2*a*x)/(2*a^2*x*tan(1/2*a*x) - a*tan(1/2*a*x)^2 + a)
```

3.599 $\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

```
[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

Rubi [A] time = 0.0559611, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {6686}

$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$$

Mathematica [A] time = 0.0200806, size = 19, normalized size = 1.

$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

Maple [A] time = 0.043, size = 20, normalized size = 1.1

$$-\frac{1}{a^2(\cos(ax) + ax\sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] -1/a^2/(cos(a*x)+a*x*sin(a*x))
```

Maxima [A] time = 0.989304, size = 26, normalized size = 1.37

$$-\frac{1}{(ax\sin(ax) + \cos(ax))a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -1/((a*x*sin(a*x) + cos(a*x))*a^2)
```

Fricas [A] time = 1.99532, size = 49, normalized size = 2.58

$$-\frac{1}{a^3x\sin(ax) + a^2\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/(a^3*x*sin(a*x) + a^2*cos(a*x))
```

Sympy [A] time = 3.69455, size = 20, normalized size = 1.05

$$-\frac{1}{a^3x\sin(ax) + a^2\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] -1/(a**3*x*sin(a*x) + a**2*cos(a*x))
```

Giac [B] time = 1.16989, size = 54, normalized size = 2.84

$$-\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^2+1\right)}{2\,a^3x\tan\left(\frac{1}{2}\,ax\right)-a^2\tan\left(\frac{1}{2}\,ax\right)^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*a*x)^2 + 1)/(2*a^3*x*tan(1/2*a*x) - a^2*tan(1/2*a*x)^2 + a^2)
```

3.600 $\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=33

$$\frac{\tan(ax)}{a^3} - \frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

```
[Out] -((x*Sec[a*x])/(a^2*(Cos[a*x] + a*x*Sin[a*x]))) + Tan[a*x]/a^3
```

Rubi [A] time = 0.0379801, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4595, 3767, 8}

$$\frac{\tan(ax)}{a^3} - \frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -((x*Sec[a*x])/(a^2*(Cos[a*x] + a*x*Sin[a*x]))) + Tan[a*x]/a^3
```

Rule 4595

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> -Simp[x/(a*d*Cos[a*x]*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[1/d^2, I
nt[1/Cos[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{\int \sec^2(ax)\,dx}{a^2}$$
$$= -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))} - \frac{\mathrm{Subst}(\int 1\,dx, x, -\tan(ax))}{a^3}$$
$$= -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{\tan(ax)}{a^3}$$

Mathematica [A] time = 0.417018, size = 31, normalized size = 0.94

$$\frac{\sin(ax)-ax\cos(ax)}{a^3(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] (-(a*x*Cos[a*x]) + Sin[a*x])/(a^3*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [A] time = 0.372, size = 53, normalized size = 1.6

$$\left(\frac{x}{a^2}\left(\tan\left(\frac{ax}{2}\right)\right)^2 - \frac{x}{a^2} + 2\,\frac{\tan(1/2\,ax)}{a^3}\right)\left(2\,\tan(1/2\,ax)\,xa - \left(\tan\left(\frac{ax}{2}\right)\right)^2 + 1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] (x/a^2*tan(1/2*a*x)^2-x/a^2+2/a^3*tan(1/2*a*x))/(2*tan(1/2*a*x)*x*a-tan(1/2
*a*x)^2+1)
```

Maxima [B] time = 1.02277, size = 135, normalized size = 4.09

$$-\frac{2\left(2\,ax\cos(2\,ax) + \left(a^2x^2-1\right)\sin(2\,ax)\right)}{\left(a^2x^2 + \left(a^2x^2+1\right)\cos(2\,ax)^2 + 4\,ax\sin(2\,ax) + \left(a^2x^2+1\right)\sin(2\,ax)^2 - 2\left(a^2x^2-1\right)\cos(2\,ax) + 1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*
cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 - 2*(a^2*x^2 -
 1)*cos(2*a*x) + 1)*a^3)
```

Fricas [A] time = 1.97615, size = 81, normalized size = 2.45

$$-\frac{ax\cos(ax) - \sin(ax)}{a^4x\sin(ax) + a^3\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -(a*x*cos(a*x) - sin(a*x))/(a^4*x*sin(a*x) + a^3*cos(a*x))
```

Sympy [B] time = 5.1095, size = 109, normalized size = 3.3

$$\frac{ax\tan^2\left(\frac{ax}{2}\right)}{2a^4x\tan\left(\frac{ax}{2}\right) - a^3\tan^2\left(\frac{ax}{2}\right) + a^3} - \frac{ax}{2a^4x\tan\left(\frac{ax}{2}\right) - a^3\tan^2\left(\frac{ax}{2}\right) + a^3} + \frac{2\tan\left(\frac{ax}{2}\right)}{2a^4x\tan\left(\frac{ax}{2}\right) - a^3\tan^2\left(\frac{ax}{2}\right) + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] a*x*tan(a*x/2)**2/(2*a**4*x*tan(a*x/2) - a**3*tan(a*x/2)**2 + a**3) - a*x/(
2*a**4*x*tan(a*x/2) - a**3*tan(a*x/2)**2 + a**3) + 2*tan(a*x/2)/(2*a**4*x*t
an(a*x/2) - a**3*tan(a*x/2)**2 + a**3)
```

Giac [A] time = 1.15183, size = 70, normalized size = 2.12

$$\frac{ax\tan\left(\frac{1}{2}ax\right)^2 - ax + 2\tan\left(\frac{1}{2}ax\right)}{2a^4x\tan\left(\frac{1}{2}ax\right) - a^3\tan\left(\frac{1}{2}ax\right)^2 + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] (a*x*tan(1/2*a*x)^2 - a*x + 2*tan(1/2*a*x))/(2*a^4*x*tan(1/2*a*x) - a^3*tan
(1/2*a*x)^2 + a^3)
```

3.601 $\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=110

$$\frac{i\text{PolyLog}\left(2,-ie^{iax}\right)}{a^4}-\frac{i\text{PolyLog}\left(2,ie^{iax}\right)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}-\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}+\frac{x\tan(ax)\sec(ax)}{a^3}$$

```
[Out] ((-2*I)*x*ArcTan[E^(I*a*x)])/a^3 + (I*PolyLog[2, (-I)*E^(I*a*x)])/a^4 - (I*
PolyLog[2, I*E^(I*a*x)])/a^4 - Sec[a*x]/a^4 - (x^2*Sec[a*x]^2)/(a^2*(Cos[a*
x] + a*x*Sin[a*x])) + (x*Sec[a*x]*Tan[a*x])/a^3
```

Rubi [A] time = 0.0934006, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4601, 4185, 4181, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{iax}\right)}{a^4}-\frac{i\text{PolyLog}\left(2,ie^{iax}\right)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}-\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}+\frac{x\tan(ax)\sec(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x*ArcTan[E^(I*a*x)])/a^3 + (I*PolyLog[2, (-I)*E^(I*a*x)])/a^4 - (I*
PolyLog[2, I*E^(I*a*x)])/a^4 - Sec[a*x]/a^4 - (x^2*Sec[a*x]^2)/(a^2*(Cos[a*
x] + a*x*Sin[a*x])) + (x*Sec[a*x]*Tan[a*x])/a^3
```

Rule 4601

```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Sec[a*x]^(n
 + 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[
(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, n + 2]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{2\int x\sec^3(ax)\,dx}{a^2}$$
$$= -\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(ax)}{a^3}+\frac{\int x\sec(ax)\,dx}{a^2}$$
$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(ax)}{a^3}-\frac{\int\log\left(1-\right.}{a}$$
$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(ax)}{a^3}+\frac{i\,\mathrm{Subst}\left(\right.}{}$$
$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}+\frac{i\mathrm{Li}_2\left(-ie^{iax}\right)}{a^4}-\frac{i\mathrm{Li}_2\left(ie^{iax}\right)}{a^4}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+$$

Mathematica [A] time = 1.14046, size = 176, normalized size = 1.6

$$-\frac{-i(ax\tan(ax)+1)\mathrm{PolyLog}\left(2,-ie^{iax}\right)+i(ax\tan(ax)+1)\mathrm{PolyLog}\left(2,ie^{iax}\right)+a^2x^2\sec(ax)-a^2x^2\log\left(1-ie^{iax}\right)\tan(ax}{a^4(ax\tan(ax)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -((-(a*x*Log[1 - I*E^(I*a*x)]) + a*x*Log[1 + I*E^(I*a*x)] + Sec[a*x] + a^2*
x^2*Sec[a*x] - a^2*x^2*Log[1 - I*E^(I*a*x)]*Tan[a*x] + a^2*x^2*Log[1 + I*E^
(I*a*x)]*Tan[a*x] - I*PolyLog[2, (-I)*E^(I*a*x)]*(1 + a*x*Tan[a*x]) + I*Pol
yLog[2, I*E^(I*a*x)]*(1 + a*x*Tan[a*x]))/(a^4*(1 + a*x*Tan[a*x])))
```

Maple [F] time = 1.057, size = 0, normalized size = 0.

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.32017, size = 772, normalized size = 7.02

$$-\frac{2\,a^2x^2 - (-i\,ax\sin(ax) - i\,\cos(ax))\mathrm{Li}_2(i\,\cos(ax) + \sin(ax)) - (-i\,ax\sin(ax) - i\,\cos(ax))\mathrm{Li}_2(i\,\cos(ax) - \sin(ax)) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2 - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) + sin(a*x
)) - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) - sin(a*x)) - (I*a*x*s
in(a*x) + I*cos(a*x))*dilog(-I*cos(a*x) + sin(a*x)) - (I*a*x*sin(a*x) + I*c
os(a*x))*dilog(-I*cos(a*x) - sin(a*x)) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*
log(I*cos(a*x) + sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(I*co
s(a*x) - sin(a*x) + 1) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x)
+ sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x) - sin(a
*x) + 1) + 2)/(a^5*x*sin(a*x) + a^4*cos(a*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sec(ax)}{(ax\sin(ax) + \cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(x**3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sec(ax)}{(ax\sin(ax) + \cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))^2, x)
```

3.602 $\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{2i\text{PolyLog}\left(2,-e^{2iax}\right)}{a^5}-\frac{2ix^2}{a^3}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\tan(ax)\sec^2(ax)}{a^3}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}+$$

```
[Out] ((-2*I)*x^2)/a^3 + (4*x*Log[1 + E^((2*I)*a*x)])/a^4 - ((2*I)*PolyLog[2, -E^
((2*I)*a*x)])/a^5 - (x*Sec[a*x]^2)/a^4 - (x^3*Sec[a*x]^3)/(a^2*(Cos[a*x] +
a*x*Sin[a*x])) + Tan[a*x]/a^5 + (2*x^2*Tan[a*x])/a^3 + (x^2*Sec[a*x]^2*Tan[
a*x])/a^3
```

Rubi [A] time = 0.183291, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4601, 4186, 3767, 8, 4184, 3719, 2190, 2279, 2391}

$$-\frac{2i\text{PolyLog}\left(2,-e^{2iax}\right)}{a^5}-\frac{2ix^2}{a^3}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\tan(ax)\sec^2(ax)}{a^3}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x^2)/a^3 + (4*x*Log[1 + E^((2*I)*a*x)])/a^4 - ((2*I)*PolyLog[2, -E^
((2*I)*a*x)])/a^5 - (x*Sec[a*x]^2)/a^4 - (x^3*Sec[a*x]^3)/(a^2*(Cos[a*x] +
a*x*Sin[a*x])) + Tan[a*x]/a^5 + (2*x^2*Tan[a*x])/a^3 + (x^2*Sec[a*x]^2*Tan[
a*x])/a^3
```

Rule 4601

```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Sec[a*x]^(n
 + 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[
(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, n + 2]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{3\int x^2\sec^4(ax)\,dx}{a^2}$$
$$= -\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x^2\sec^2(ax)\tan(ax)}{a^3}+\frac{\int \sec^2(ax)\,dx}{a^4}+\frac{2\int x}{}$$
$$= -\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\sec^2(ax)\tan(ax)}{a^3}-\frac{\text{Subst}}{}$$
$$= -\frac{2ix^2}{a^3}-\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\tan(ax)}{a^5}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\sec^2(ax}{a^3}$$
$$= -\frac{2ix^2}{a^3}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}-\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\tan(ax)}{a^5}+\frac{2x^2\text{t}}{}$$
$$= -\frac{2ix^2}{a^3}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}-\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\tan(ax)}{a^5}+\frac{2x^2\text{t}}{}$$
$$= -\frac{2ix^2}{a^3}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}-\frac{2i\text{Li}_2\left(-e^{2iax}\right)}{a^5}-\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+$$

Mathematica [A] time = 1.08472, size = 130, normalized size = 1.05

$$\frac{-2i(ax\tan(ax)+1)\text{PolyLog}\left(2,-e^{2iax}\right)-ax\left(a^2x^2+2iax-4\log\left(1+e^{2iax}\right)+1\right)+a^3x^3\tan^2(ax)+\left(-2ia^3x^3+2a^2x^2+\right.}{a^5(ax\tan(ax)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] (-(a*x*(1 + (2*I)*a*x + a^2*x^2 - 4*Log[1 + E^((2*I)*a*x)])) + (1 + 2*a^2*x
^2 - (2*I)*a^3*x^3 + 4*a^2*x^2*Log[1 + E^((2*I)*a*x)])*Tan[a*x] + a^3*x^3*T
an[a*x]^2 - (2*I)*PolyLog[2, -E^((2*I)*a*x)]*(1 + a*x*Tan[a*x]))/(a^5*(1 +
a*x*Tan[a*x]))
```

Maple [A] time = 0.353, size = 141, normalized size = 1.1

$$\frac{-2\,i\left(-2\,ia^2x^2\mathrm{e}^{2\,iax}+2\,x^3a^3-2\,ia^2x^2+ax\mathrm{e}^{2\,iax}-i\mathrm{e}^{2\,iax}+ax-i\right)}{\left(1+\mathrm{e}^{2\,iax}\right)\left(ax\mathrm{e}^{2\,iax}-ax+i\mathrm{e}^{2\,iax}+i\right)a^5}-\frac{4\,ix^2}{a^3}+4\,\frac{x\ln\left(1+\mathrm{e}^{2\,iax}\right)}{a^4}-\frac{2\,i\mathrm{polylog}\left(2,-\mathrm{e}^{2\,iax}\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] -2*I*(-2*I*a^2*x^2*exp(2*I*a*x)+2*x^3*a^3-2*I*a^2*x^2+a*x*exp(2*I*a*x)-I*ex
p(2*I*a*x)+a*x-I)/(1+exp(2*I*a*x))/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I)/
a^5-4*I/a^3*x^2+4*x*ln(1+exp(2*I*a*x))/a^4-2*I*polylog(2,-exp(2*I*a*x))/a^5
```

Maxima [B] time = 1.65258, size = 514, normalized size = 4.15

$$-\frac{2\,ax+\left(4\,a^2x^2-8i\,ax\cos\left(2\,ax\right)+8\,ax\sin\left(2\,ax\right)-4i\,ax-\left(4\,a^2x^2+4i\,ax\right)\cos\left(4\,ax\right)+4\left(-i\,a^2x^2+ax\right)\sin\left(4\,ax\right)\right)\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (4*a^2*x^2 - 8*I*a*x*cos(2*a*x) + 8*a*x*sin(2*a*x) - 4*I*a*x - (4
*a^2*x^2 + 4*I*a*x)*cos(4*a*x) + 4*(-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(s
in(2*a*x), cos(2*a*x) + 1) + 4*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x) - (-4*I*a^2
*x^2 - 2*a*x + 2*I)*cos(2*a*x) - (2*a*x - (2*a*x + 2*I)*cos(4*a*x) - 2*(I*a
*x - 1)*sin(4*a*x) - 4*I*cos(2*a*x) + 4*sin(2*a*x) - 2*I)*dilog(-e^(2*I*a*x
)) - (2*I*a^2*x^2 + 4*a*x*cos(2*a*x) + 4*I*a*x*sin(2*a*x) + 2*a*x - 2*(I*a^
2*x^2 - a*x)*cos(4*a*x) + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(2*a*x)^
2 + sin(2*a*x)^2 + 2*cos(2*a*x) + 1) - (-4*I*a^3*x^3 + 4*a^2*x^2)*sin(4*a*x
) - (4*a^2*x^2 - 2*I*a*x - 2)*sin(2*a*x) - 2*I)/((I*a*x + (-I*a*x + 1)*cos(
4*a*x) + (a*x + I)*sin(4*a*x) + 2*cos(2*a*x) + 2*I*sin(2*a*x) + 1)*a^5)
```

Fricas [B] time = 2.5673, size = 1014, normalized size = 8.18

$$\frac{a^3x^3-\left(2\,a^3x^3+ax\right)\cos\left(ax\right)^2+\left(2\,a^2x^2+1\right)\cos\left(ax\right)\sin\left(ax\right)+\left(2i\,ax\cos\left(ax\right)\sin\left(ax\right)+2i\,\cos\left(ax\right)^2\right)\mathrm{Li}_2\left(i\,\cos\left(ax\right)+\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x)
 + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2)*dilog(I*cos(a*x) + sin(a*x)
) + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2)*dilog(I*cos(a*x) - sin(a*
x)) + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2)*dilog(-I*cos(a*x) + sin
(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2)*dilog(-I*cos(a*x) - s
in(a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*x) +
sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*
x) - sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(-I*
cos(a*x) + sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*l
og(-I*cos(a*x) - sin(a*x) + 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sec(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sec{(ax)}^2}{(ax\sin{(ax)} + \cos{(ax)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*sec(a*x)^2/(a*x*sin(a*x) + cos(a*x))^2, x)
```

3.603 $\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=157

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35bc}-\frac{4\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{35b}-\frac{2\cdots}{5b\sqrt{\cdots}}$$

```
[Out] (-2*c*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sec[2*a +
2*b*x]^3*Tan[2*a + 2*b*x])/(7*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (4*Sqrt[-c
 + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(35*b) - (6*(-c + c*Sec[2*a + 2*b*
x])^(3/2)*Tan[2*a + 2*b*x])/(35*b*c)
```

Rubi [A] time = 0.445167, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3803, 3800, 4001, 3792}

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35bc}-\frac{4\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{35b}-\frac{2\cdots}{5b\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (-2*c*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sec[2*a +
2*b*x]^3*Tan[2*a + 2*b*x])/(7*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (4*Sqrt[-c
 + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(35*b) - (6*(-c + c*Sec[2*a + 2*b*
x])^(3/2)*Tan[2*a + 2*b*x])/(35*b*c)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^4(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$

$$= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{6}{7}\int \sec^3(2a+2bx)\sqrt{-c+c\,\mathrm{se}$$

$$= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{6(-c+c\sec(2a+2bx))^{3/2}\tan}{35bc}$$

$$= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{4\sqrt{-c+c\sec(2a+2bx)}\tan(2}{35b}$$

$$= -\frac{2c\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - $$

Mathematica [A] time = 0.234057, size = 64, normalized size = 0.41

$$-\frac{(7\cos(3(a+bx))+2\cos(7(a+bx)))\csc(a+bx)\sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -((7*Cos[3*(a + b*x)] + 2*Cos[7*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b*x)]^3
*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(35*b)
```

Maple [A] time = 0.642, size = 98, normalized size = 0.6

$$-\frac{\sqrt{2}\sqrt{4}\cos(bx+a)\left(128\left(\cos(bx+a)\right)^{6}-224\left(\cos(bx+a)\right)^{4}+140\left(\cos(bx+a)\right)^{2}-35\right)}{70\,b\sin(bx+a)\left(2\left(\cos(bx+a)\right)^{2}-1\right)^{3}}\sqrt{\frac{c\left(\sin(bx+a)\right)^{2}}{2\left(\cos(bx+a)\right)^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/70*2^(1/2)/b*4^(1/2)*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2
)*(128*cos(b*x+a)^6-224*cos(b*x+a)^4+140*cos(b*x+a)^2-35)/sin(b*x+a)/(2*cos
(b*x+a)^2-1)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.05686, size = 273, normalized size = 1.74

$$-\frac{\sqrt{2}\left(35\tan(bx+a)^{6}-35\tan(bx+a)^{4}+49\tan(bx+a)^{2}-9\right)\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}}{35\left(b\tan(bx+a)^{7}-3\,b\tan(bx+a)^{5}+3\,b\tan(bx+a)^{3}-b\tan(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/35*sqrt(2)*(35*tan(b*x + a)^6 - 35*tan(b*x + a)^4 + 49*tan(b*x + a)^2 -
9)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^7 - 3*b*tan
(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.604 $\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=110

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}+\frac{2\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15b}+\frac{7c\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] (7*c*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (2*Sqrt[-c +
c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b*c)
```

Rubi [A] time = 0.2762, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3800, 4001, 3792}

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}+\frac{2\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15b}+\frac{7c\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (7*c*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (2*Sqrt[-c +
c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b*c)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^3(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= \frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5bc} + \frac{2\int \sec(2a+2bx)\sqrt{-c}$$
$$= \frac{2\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{15b} + \frac{(-c+c\sec(2a+2bx))^3}{5bc}$$
$$= \frac{7c\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{2\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2}{15b}$$

Mathematica [A] time = 0.18082, size = 62, normalized size = 0.56

$$\frac{(5\cos(a+bx)+2\cos(5(a+bx)))\csc(a+bx)\sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((5*Cos[a + b*x] + 2*Cos[5*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b*x)]^2*Sqrt
[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(15*b)
```

Maple [A] time = 0.466, size = 88, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{4}\cos(bx+a)\left(32\,(\cos(bx+a))^4-40\,(\cos(bx+a))^2+15\right)}{30\,b\sin(bx+a)\left(2\,(\cos(bx+a))^2-1\right)^2}\sqrt{\frac{c\,(\sin(bx+a))^2}{2\,(\cos(bx+a))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/30*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)
*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+15)/sin(b*x+a)/(2*cos(b*x+a)^2-1)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.09083, size = 216, normalized size = 1.96

$$\frac{\sqrt{2}\left(15\,\tan\left(bx+a\right)^{4}-10\,\tan\left(bx+a\right)^{2}+7\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{15\left(b\tan\left(bx+a\right)^{5}-2\,b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/15*sqrt(2)*(15*tan(b*x + a)^4 - 10*tan(b*x + a)^2 + 7)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*tan(b
*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.605 $\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=72

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{c\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] -(c*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (Sqrt[-c + c*Se
c[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

Rubi [A] time = 0.198944, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4397, 3798, 3792}

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{c\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(c*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (Sqrt[-c + c*Se
c[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^2(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$

$$= \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3b} - \frac{1}{3}\int \sec(2a+2bx)\sqrt{-c\ \cdots}$$

$$= -\frac{c\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2b\cdots}{3b}$$

Mathematica [A] time = 0.181313, size = 44, normalized size = 0.61

$$\frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}(\tan(2(a+bx))-\cot(a+bx))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]*(-Cot[a + b*x] + Tan[2*(a + b*x)]))/
(3*b)
```

Maple [A] time = 0.445, size = 78, normalized size = 1.1

$$-\frac{\sqrt{2}\sqrt{4}\cos(bx+a)\left(4(\cos(bx+a))^2-3\right)}{6b\sin(bx+a)\left(2(\cos(bx+a))^2-1\right)}\sqrt{\frac{c(\sin(bx+a))^2}{2(\cos(bx+a))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/6*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)
*(4*cos(b*x+a)^2-3)/sin(b*x+a)/(2*cos(b*x+a)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/3*(6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(3/4)*b*sqrt(c)*integrate(-(((cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8
*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(
4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)
*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12
*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2
(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b*x + 4*a
), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x
 + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*
x + 8*a)*sin(4*b*x + 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1)) - (cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x + 8*a)*cos(4
*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*x + 4*a) + 2*
sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(3/2*arctan2(sin(4*b*x + 4*a), c
os(4*b*x + 4*a))))/(((2*(2*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(12*b*x
+ 12*a) + cos(12*b*x + 12*a)^2 + 4*cos(8*b*x + 8*a)^2 + 4*cos(8*b*x + 8*a)*
cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(2*sin(8*b*x + 8*a) + sin(4*b*x +
 4*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 + 4*sin(8*b*x + 8*a)^2 + 4
*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*arctan2(si
n(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (2*(2*cos(8*b*x + 8*a) + cos(4*
b*x + 4*a))*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 + 4*cos(8*b*x + 8*a)^
2 + 4*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(2*sin(8*b
*x + 8*a) + sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 + 4
*sin(8*b*x + 8*a)^2 + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a
)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*(cos(4*b*
x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)), x) + sqrt
(c)*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))/((cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(3/4)*b)
```

Fricas [A] time = 2.03655, size = 159, normalized size = 2.21

$$-\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(3\tan(bx+a)^2-1\right)}{3\left(b\tan(bx+a)^3-b\tan(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/3*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(3*tan(b*x + a)^2
 - 1)/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.606 $\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=33

$$\frac{c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] (c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.0649942, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4397, 3792}

$$\frac{c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx &= \int \sec(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx\\ &= \frac{c\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.0853419, size = 30, normalized size = 0.91

$$\frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cot[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

Maple [A] time = 0.357, size = 52, normalized size = 1.6

$$\frac{\sqrt{2}\sqrt{4}\cos(bx+a)}{2b\sin(bx+a)}\sqrt{\frac{c(\sin(bx+a))^2}{2(\cos(bx+a))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/2*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)/
sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] (2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)
*b*sqrt(c)*integrate(-(((cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*
a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*arct
an2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(8*b*x
 + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2(sin(4*b*x + 4*
a), -cos(4*b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x +
4*a))) + ((cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x +
```

```
 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(8*b
*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(8*b*x + 8*a)*sin(4*b*
x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/(((cos
(8*b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
+ sin(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*
a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(8*
b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + s
in(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^
2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*(cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)), x) - sqrt(c
)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))/((cos(4*b*x +
4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*b)
```

Fricas [A] time = 2.14543, size = 96, normalized size = 2.91

$$\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.607 $\int \sqrt{c \tan(a+bx) \tan(2(a+bx))} \, dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b}$$

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/b)
```

Rubi [A] time = 0.0404964, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4397, 3774, 207}

$$-\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/b)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$
$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$

Mathematica [A] time = 0.129006, size = 73, normalized size = 1.62

$$-\frac{\sqrt{\cos(2(a+bx))}\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -((ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a + b
*x)]]*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(Sqrt[2]*b))
```

Maple [B] time = 0.317, size = 136, normalized size = 3.

$$-\frac{\sqrt{4}\sin(bx+a)}{2b(-1+\cos(bx+a))}\sqrt{\frac{c\left(1-(\cos(bx+a))^2\right)}{2(\cos(bx+a))^2-1}}\sqrt{\frac{2(\cos(bx+a))^2-1}{(\cos(bx+a)+1)^2}}\text{Artanh}\left(\frac{\sqrt{2}\cos(bx+a)\sqrt{4}(-1+\cos(bx+a}{2(\sin(bx+a))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/2/b*4^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(
1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2))/(-1+cos(b*x+a))
```

Maxima [B] time = 1.72376, size = 581, normalized size = 12.91

$$\sqrt{c}\left(\log\left(4\sqrt{\cos(4\,bx+4\,a)^2+\sin(4\,bx+4\,a)^2+2\,\cos(4\,bx+4\,a)+1}\cos\left(\frac{1}{2}\arctan(\sin(4\,bx+4\,a),\cos(4\,bx+4\,a)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c)*(log(4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b
*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 +
 4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*s
in(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + 8*(cos(4*b*x +
4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*cos(1/2*arctan2
(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + 4) - log(cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2
 + sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2) + 2*(cos(4*b
*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(2*b*x
 + 2*a)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + sin(2*b*
x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)))))/b
```

Fricas [A] time = 2.38249, size = 525, normalized size = 11.67

$$\left[\frac{\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3-4\,\sqrt{2}\left(\tan(bx+a)^4-4\,\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}+17\,c\tan(bx+a)}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}\right)}{4\,b},\frac{\sqrt{-c}\arctan\left(\frac{2\,\sqrt{2}\sqrt{-\frac{c\tan(bx+a)}{\tan(bx+a)^2-}}}{c\tan(bx+a)^3}\right.}{2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(
b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 -
 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(
```

```
b*x + a)))/b, 1/2*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x
 + a)))/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.608 $\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.13946, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4397, 3805, 3774, 207}

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{2}\int \sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b}$$
$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} - \frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

Mathematica [A] time = 0.244687, size = 92, normalized size = 1.1

$$-\frac{\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(\cos(a+bx)+\cos(3(a+bx))-\sqrt{2}\sqrt{\cos(2(a+bx))}\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -((Cos[a + b*x] - Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*
x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Tan[a
+ b*x]*Tan[2*(a + b*x)]])/(4*b)
```

Maple [B] time = 0.424, size = 387, normalized size = 4.6

$$\frac{\sqrt{4}\sin(bx+a)}{2b(-1+\cos(bx+a))}\sqrt{\frac{c\left(1-(\cos(bx+a))^2\right)}{2(\cos(bx+a))^2-1}}\sqrt{\frac{2(\cos(bx+a))^2-1}{(\cos(bx+a)+1)^2}}\text{Artanh}\left(\frac{\sqrt{2}\cos(bx+a)\sqrt{4}(-1+\cos(bx+a))}{2(\sin(bx+a))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/2/b*4^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1
/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
))/(-1+cos(b*x+a))-1/8*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1)
)^(1/2)*sin(b*x+a)*(2^(1/2)*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2
/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a
))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))-4*cos(b*x+a)^3
+2*cos(b*x+a))/(-1+cos(b*x+a)^2)
```

Maxima [B] time = 2.10735, size = 1416, normalized size = 16.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/16*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2*b*x
+ 2*a) + (cos(2*b*x + 2*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*
x + 4*a) - 1)))*sqrt(c) - sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4
*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))
^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(
1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(s
qrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1
/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x +
 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1)) + 1) + log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
```

```
 + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*s
in(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4
*b*x + 4*a) - 1))) + 1) - log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
+ 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2
*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1))) + 1)))/b
```

Fricas [B] time = 2.40093, size = 921, normalized size = 10.96

$$\left[\frac{\left(\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3+4\,\sqrt{2}\left(\tan(bx+a)^4-4\,\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}+17\,c\tan(bx+a}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}\right.}{8\left(b\tan(bx+a)^3+b\tan(bx+a)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/8*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c
*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c
*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x
 + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) + 4*sqrt(2)*sqrt(-c*tan(b*x + a
)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*tan(b
*x + a)), -1/4*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*s
qrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(
c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*tan(b*x + a
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \tan(2\,bx + 2\,a)\tan(bx + a)}\cos(2\,bx + 2\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a), x)
```

3.609 $\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=129

$$\frac{3c\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

```
[Out] (-3*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(8*b) + (3*c*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c
*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.214331, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3805, 3774, 207}

$$\frac{3c\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (-3*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(8*b) + (3*c*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c
*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos^2(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{3}{4}\int \cos(2a+2bx)\sqrt{-c+c\sec(}$$
$$= \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} + \frac{3}{8}\int$$
$$= \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{(3c)\,\mathrm{S}}{}$$
$$= -\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b} + \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\,\mathrm{co}}{4}$$

Mathematica [A] time = 0.256139, size = 105, normalized size = 0.81

$$\frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(2(-\sin(2(a+bx))+\sin(4(a+bx))+\cot(a+bx))-3\sqrt{2}\sqrt{\cos(2(a+bx))}\csc(a+bx)\tan\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((-3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Co
s[2*(a + b*x)]]*Csc[a + b*x] + 2*(Cot[a + b*x] - Sin[2*(a + b*x)] + Sin[4*(
a + b*x)]))*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

Maple [B] time = 0.451, size = 649, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/2/b*4^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(
1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2))/(-1+cos(b*x+a))+1/4*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(1/2)*sin(b*x+a)*(2^(1/2)*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^
2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+
a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))-4*cos(b*x+a)^
3+2*cos(b*x+a))/(-1+cos(b*x+a)^2)-1/32*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2
*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*(-16*cos(b*x+a)^5+3*2^(1/2)*cos(b*x+a)*(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4
^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2))+3*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(
1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2))-4*cos(b*x+a)^3+6*cos(b*x+a))/(-1+cos(b*x+a)^2)
```

Maxima [B] time = 2.26503, size = 1918, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/64*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
^(1/4)*((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*
x + 4*a) - (cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) - co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*x + 4*a) -
(cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) - ((cos(4*b*x + 4*a) - 2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*
x + 4*a) - 1)) + sin(4*b*x + 4*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
```

```
b*x + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))*sqr
t(c) - 3*sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))
^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x
 + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arct
an2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(sqrt(cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) + 1) + log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*
arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x
 + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos
(4*b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))
^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
+ 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4
)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a),
-cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))
)) + 1) - log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2
 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arc
tan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x +
4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)
*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) - 2
*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(
cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))
+ 1)))/b
```

Fricas [A] time = 2.48251, size = 1092, normalized size = 8.47

$$\left[\frac{3\left(\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3-4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan}{\tan(b}}}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right.}{32\left(b\tan(bx+a)^5+2b\tan(bx+a)^3+b\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/32*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c
*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b
*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*
tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(
2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*
x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a)), 1
/16*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2
*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*
sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(5*tan(b*x + a)
^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b
*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.610 $\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=176

$$-\frac{5c\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}+\frac{5c\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a}{\sqrt{c\sec(2a+}}\right.}{16b}$$

```
[Out] (5*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(16*b) - (5*c*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
5*c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
 - (c*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2*a + 2*b*x
]])
```

Rubi [A] time = 0.288151, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3805, 3774, 207}

$$-\frac{5c\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}+\frac{5c\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a}{\sqrt{c\sec(2a+}}\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (5*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(16*b) - (5*c*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
5*c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
 - (c*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2*a + 2*b*x
]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos^3(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$

$$= -\frac{c\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} - \frac{5}{6}\int \cos^2(2a+2bx)\sqrt{-c+c\,\mathrm{se}$$

$$= \frac{5c\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}}$$

$$= -\frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} - c$$

$$= -\frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} - c$$

$$= \frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b} - \frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\,\mathrm{c}}{2}$$

Mathematica [A] time = 0.306241, size = 116, normalized size = 0.66

$$\frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}\Big(30\sin(2(a+bx)) - 2\sin(4(a+bx)) + 4\sin(6(a+bx)) - 26\cot(a+bx) + 15\sqrt{2}\sqrt{\cos(2(a}}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((-26*Cot[a + b*x] + 15*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(
a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] + 30*Sin[2*(a + b*x)] - 2*S
in[4*(a + b*x)] + 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]
)/(96*b)
```

Maple [B] time = 0.413, size = 921, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/2/b*4^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1
/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
))/(-1+cos(b*x+a))-3/8*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1)
)^(1/2)*sin(b*x+a)*(2^(1/2)*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2
/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a
))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))-4*cos(b*x+a)^3
+2*cos(b*x+a))/(-1+cos(b*x+a)^2)+3/32*2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*
cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*(-16*cos(b*x+a)^5+3*2^(1/2)*cos(b*x+a)*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^
(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2))+3*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1
/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(co
s(b*x+a)+1)^2)^(1/2))-4*cos(b*x+a)^3+6*cos(b*x+a))/(-1+cos(b*x+a)^2)-1/192*
2^(1/2)/b*4^(1/2)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*(-12
8*cos(b*x+a)^7-16*cos(b*x+a)^5+15*2^(1/2)*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a)
)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+15*2^(1/2)*((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1
/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
))-20*cos(b*x+a)^3+30*cos(b*x+a))/(-1+cos(b*x+a)^2)
```

Maxima [B] time = 3.38579, size = 3150, normalized size = 17.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/384*(8*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(s
in(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos
(6*b*x + 6*a))) - 1))*sin(6*b*x + 6*a) + (cos(6*b*x + 6*a) + 1)*sin(3/2*arc
tan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2
(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqrt(c) + 12*(cos(2/3*arctan2(
sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a),
cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)
)) + 1)^(1/4)*((sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 5*si
n(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))))*cos(1/2*arctan2(sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x +
 6*a), cos(6*b*x + 6*a))) - 1)) + (cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*
b*x + 6*a))) - 3*cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 4)*
sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(
2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqrt(c) + 15*sqrt(c
)*(log(sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/
3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*
b*x + 6*a), cos(6*b*x + 6*a))) + 1)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
 + 6*a))) - 1))^2 + sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)
))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*a
rctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*
a), cos(6*b*x + 6*a))) - 1))^2 + 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6
*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 +
2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arcta
n2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)) + 1) - log(sqrt(cos(2/3*arcta
n2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a
), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6
*a))) + 1)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*
a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2 + sqrt(
cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a),
cos(6*b*x + 6*a))) + 1)*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), c
os(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) -
1))^2 - 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b
```

```
*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(
6*b*x + 6*a))) - 1)) + 1) + log(((cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b
*x + 6*a)))^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*cos
(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2 + (cos(1/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), co
s(6*b*x + 6*a)))^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6
*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^
2)*sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*ar
ctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x
+ 6*a), cos(6*b*x + 6*a))) + 1) + 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(
6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 +
 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arc
tan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))*sin(1/3*arctan2(sin(6*b*x +
 6*a), cos(6*b*x + 6*a))) + cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6
*a)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))),
 -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))) + 1) - log(((
cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(1/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6
*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b
*x + 6*a))) - 1))^2 + (cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))
^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*sin(1/2*arctan
2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2)*sqrt(cos(2/3*arctan2(sin(6*b*x
+ 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
+ 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1) -
2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2
(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a
), cos(6*b*x + 6*a))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x
 + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x +
 6*a))) - 1))*sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + cos(1/
3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))*sin(1/2*arctan2(sin(2/3*arct
an2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a)
, cos(6*b*x + 6*a))) - 1))) + 1)))/b
```

Fricas [A] time = 2.50927, size = 1260, normalized size = 7.16

$$\left[\frac{15 \left(\tan\left(bx+a\right)^{7} + 3 \tan\left(bx+a\right)^{5} + 3 \tan\left(bx+a\right)^{3} + \tan\left(bx+a\right)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^{5} - 14\,c\tan(bx+a)^{3} + 4\sqrt{2}\left(\tan(bx+a)^{4} - 4\,\mathrm{t}\cdots\right.}{\tan(bx+a)^{5} + 2\tan(b\cdots}\cdots\right.}{192\left(b\tan\left(bx+a\right)^{7} + 3\,b\tan\left(bx+a\cdots\right.\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(15*(tan(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x
+ a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan
(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan
(b*x + a))) + 4*sqrt(2)*(33*tan(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b*x +
a)^2 - 13)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7
+ 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a)), -1/96*(15*(tan
(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*
arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)
^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(33*tan
(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b*x + a)^2 - 13)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(
b*x + a)^3 + b*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.611 $\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17c^2\tan(2a+2bx)\sec^3(2a+2bx)}{63b\sqrt{c\sec(2a+2bx)-c}}+\frac{34c^2\tan(2a+2bx)}{45b\sqrt{c\sec(2a+2bx)-c}}+\frac{34\tan(2a+2bx)(c\,s}{10}$$

```
[Out] (34*c^2*Tan[2*a + 2*b*x])/(45*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (17*c^2*Se
c[2*a + 2*b*x]^3*Tan[2*a + 2*b*x])/(63*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Sec[2*a + 2*b*x]^4*Tan[2*a + 2*b*x])/(9*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) + (68*c*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(315*b) + (34*(-c
 + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(105*b)
```

Rubi [A] time = 0.527679, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4397, 3814, 21, 3803, 3800, 4001, 3792}

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17c^2\tan(2a+2bx)\sec^3(2a+2bx)}{63b\sqrt{c\sec(2a+2bx)-c}}+\frac{34c^2\tan(2a+2bx)}{45b\sqrt{c\sec(2a+2bx)-c}}+\frac{34\tan(2a+2bx)(c\,s}{10}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```


```
[Out] (34*c^2*Tan[2*a + 2*b*x])/(45*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (17*c^2*Se
c[2*a + 2*b*x]^3*Tan[2*a + 2*b*x])/(63*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Sec[2*a + 2*b*x]^4*Tan[2*a + 2*b*x])/(9*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) + (68*c*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(315*b) + (34*(-c
 + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(105*b)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^4(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{9}(2c)\int \frac{\sec^4(2a+2bx)\left(\frac{17c}{2}\right.}{\sqrt{-c+c\,\mathrm{se}}}$$
$$= \frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{9}(17c)\int \sec^4(2a+2bx)\sqrt{-c}$$
$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+2bx)\tan(2a}{9b\sqrt{-c+c\sec(2a+2}}$$
$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+2bx)\tan(2a}{9b\sqrt{-c+c\sec(2a+2}}$$
$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+2bx)\tan(2a}{9b\sqrt{-c+c\sec(2a+2}}$$
$$= \frac{34c^2\tan(2a+2bx)}{45b\sqrt{-c+c\sec(2a+2bx)}} - \frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}}$$

Mathematica [A] time = 0.367165, size = 85, normalized size = 0.41

$$\frac{\cot(a+bx)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\left(188\cot(a+bx)\cot(2(a+bx))+35\sec^3(2(a+bx))-50\sec^2(2(a+bx))+5\right.}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Cot[a + b*x]*(-84 + 188*Cot[a + b*x]*Cot[2*(a + b*x)] + 52*Sec[2*(a + b*x)
] - 50*Sec[2*(a + b*x)]^2 + 35*Sec[2*(a + b*x)]^3)*(c*Tan[a + b*x]*Tan[2*(a
 + b*x)])^(3/2))/(315*b)
```

Maple [A] time = 0.519, size = 105, normalized size = 0.5

$$\frac{2\sqrt{2}\left(2176\,(\cos(bx+a))^8-4896\,(\cos(bx+a))^6+4284\,(\cos(bx+a))^4-1785\,(\cos(bx+a))^2+315\right)\cos(bx+a)}{315\,b\left(2\,(\cos(bx+a))^2-1\right)^3(\sin(bx+a))^3}\left(\frac{c}{2\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2/315*2^(1/2)/b*(2176*cos(b*x+a)^8-4896*cos(b*x+a)^6+4284*cos(b*x+a)^4-1785
*cos(b*x+a)^2+315)*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/(2*
cos(b*x+a)^2-1)^3/sin(b*x+a)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.06705, size = 348, normalized size = 1.67

$$\frac{2\sqrt{2}\left(315\,c\tan\left(bx+a\right)^{8}-525\,c\tan\left(bx+a\right)^{6}+819\,c\tan\left(bx+a\right)^{4}-423\,c\tan\left(bx+a\right)^{2}+94\,c\right)\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}}{315\left(b\tan\left(bx+a\right)^{9}-4\,b\tan\left(bx+a\right)^{7}+6\,b\tan\left(bx+a\right)^{5}-4\,b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/315*sqrt(2)*(315*c*tan(b*x + a)^8 - 525*c*tan(b*x + a)^6 + 819*c*tan(b*x
+ a)^4 - 423*c*tan(b*x + a)^2 + 94*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))/(b*tan(b*x + a)^9 - 4*b*tan(b*x + a)^7 + 6*b*tan(b*x + a)^5 - 4*b*t
an(b*x + a)^3 + b*tan(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.612 $\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=148

$$-\frac{76c^2\tan(2a+2bx)}{105b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}+\frac{2\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35b}+\frac{19c\,\mathrm{ta}}{}$$

```
[Out] (-76*c^2*Tan[2*a + 2*b*x])/(105*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (19*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(105*b) + (2*(-c + c*Sec[2*a
+ 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(35*b) + ((-c + c*Sec[2*a + 2*b*x])^(5/2)
*Tan[2*a + 2*b*x])/(7*b*c)
```

Rubi [A] time = 0.347543, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3800, 4001, 3793, 3792}

$$-\frac{76c^2\tan(2a+2bx)}{105b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}+\frac{2\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35b}+\frac{19c\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-76*c^2*Tan[2*a + 2*b*x])/(105*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (19*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(105*b) + (2*(-c + c*Sec[2*a
+ 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(35*b) + ((-c + c*Sec[2*a + 2*b*x])^(5/2)
*Tan[2*a + 2*b*x])/(7*b*c)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^3(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{(-c+c\sec(2a+2bx))^{5/2}\tan(2a+2bx)}{7bc} + \frac{2\int \sec(2a+2bx)(-c$$
$$= \frac{2(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{35b} + \frac{(-c+c\sec(2a+2bx}{7b}$$
$$= \frac{19c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{105b} + \frac{2(-c+c\sec(2a+2b}{35}$$
$$= -\frac{76c^2\tan(2a+2bx)}{105b\sqrt{-c+c\sec(2a+2bx)}} + \frac{19c\sqrt{-c+c\sec(2a+2bx)}\tan(2}{105b}$$

Mathematica [A] time = 0.220233, size = 73, normalized size = 0.49

$$-\frac{\cot(a+bx)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\left(76\cot(a+bx)\cot(2(a+bx))-15\sec^2(2(a+bx))+24\sec(2(a+bx))-2\right.}{105b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -(Cot[a + b*x]*(-28 + 76*Cot[a + b*x]*Cot[2*(a + b*x)] + 24*Sec[2*(a + b*x)
] - 15*Sec[2*(a + b*x)]^2)*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/(105*b)
```

Maple [A] time = 0.398, size = 95, normalized size = 0.6

$$-\frac{2\sqrt{2}\left(416\left(\cos\left(bx+a\right)\right)^{6}-728\left(\cos\left(bx+a\right)\right)^{4}+455\left(\cos\left(bx+a\right)\right)^{2}-105\right)\cos\left(bx+a\right)}{105\,b\left(2\left(\cos\left(bx+a\right)\right)^{2}-1\right)^{2}\left(\sin\left(bx+a\right)\right)^{3}}\left(\frac{c\left(\sin\left(bx+a\right)\right)^{2}}{2\left(\cos\left(bx+a\right)\right)^{2}-1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2/105*2^(1/2)/b*(416*cos(b*x+a)^6-728*cos(b*x+a)^4+455*cos(b*x+a)^2-105)*c
os(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/(2*cos(b*x+a)^2-1)^2/si
n(b*x+a)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.05706, size = 290, normalized size = 1.96

$$-\frac{2\sqrt{2}\left(105\,c\tan\left(bx+a\right)^{6}-140\,c\tan\left(bx+a\right)^{4}+133\,c\tan\left(bx+a\right)^{2}-38\,c\right)\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}}{105\left(b\tan\left(bx+a\right)^{7}-3\,b\tan\left(bx+a\right)^{5}+3\,b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/105*sqrt(2)*(105*c*tan(b*x + a)^6 - 140*c*tan(b*x + a)^4 + 133*c*tan(b*x
 + a)^2 - 38*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a
)^7 - 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.613 $\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{4c^2\tan(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{5b}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}$$

```
[Out] (4*c^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c*Sqrt[-c +
 c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(5*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b)
```

Rubi [A] time = 0.268276, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3798, 3793, 3792}

$$\frac{4c^2\tan(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{5b}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (4*c^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c*Sqrt[-c +
 c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(5*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
```

```
/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^2(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5b} - \frac{3}{5}\int \sec(2a+2bx)(-c$$
$$= -\frac{c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{5b} + \frac{(-c+c\sec(2a+2bx)}{5b}$$
$$= \frac{4c^2\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2b}{5b}$$

Mathematica [A] time = 0.237816, size = 59, normalized size = 0.54

$$\frac{\cot(a+bx)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}(4\cot(a+bx)\cot(2(a+bx))+\sec(2(a+bx))-2)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Cot[a + b*x]*(-2 + 4*Cot[a + b*x]*Cot[2*(a + b*x)] + Sec[2*(a + b*x)])*(c*
Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/(5*b)
```

Maple [A] time = 0.34, size = 85, normalized size = 0.8

$$\frac{2\sqrt{2}\left(12\,(\cos(bx+a))^4-15\,(\cos(bx+a))^2+5\right)\cos(bx+a)}{5\,b\left(2\,(\cos(bx+a))^2-1\right)(\sin(bx+a))^3}\left(\frac{c\,(\sin(bx+a))^2}{2\,(\cos(bx+a))^2-1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2/5*2^(1/2)/b*(12*cos(b*x+a)^4-15*cos(b*x+a)^2+5)*cos(b*x+a)*(c*sin(b*x+a)^
2/(2*cos(b*x+a)^2-1))^(3/2)/(2*cos(b*x+a)^2-1)/sin(b*x+a)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.0322, size = 220, normalized size = 2.

$$\frac{2\sqrt{2}\left(5\,c\tan\left(bx+a\right)^{4}-5\,c\tan\left(bx+a\right)^{2}+2\,c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{5\left(b\tan\left(bx+a\right)^{5}-2\,b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/5*sqrt(2)*(5*c*tan(b*x + a)^4 - 5*c*tan(b*x + a)^2 + 2*c)*sqrt(-c*tan(b*x
 + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*ta
n(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.614 $\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] (-4*c^2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sqrt[-c
+ c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

Rubi [A] time = 0.109576, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4397, 3793, 3792}

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```


```
[Out] (-4*c^2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sqrt[-c
+ c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3b} - \frac{1}{3}(4c)\int \sec(2a+2bx)\surd$$
$$= -\frac{4c^2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2}{3b}$$

Mathematica [A] time = 0.16986, size = 51, normalized size = 0.68

$$-\frac{\cot(a+bx)(4\cot(a+bx)\cot(2(a+bx))-1)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -(Cot[a + b*x]*(-1 + 4*Cot[a + b*x]*Cot[2*(a + b*x)])*(c*Tan[a + b*x]*Tan[2
*(a + b*x)])^(3/2))/(3*b)
```

Maple [A] time = 0.429, size = 61, normalized size = 0.8

$$-\frac{2\sqrt{2}\left(5(\cos(bx+a))^2-3\right)\cos(bx+a)}{3b(\sin(bx+a))^3}\left(\frac{c(\sin(bx+a))^2}{2(\cos(bx+a))^2-1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2/3*2^(1/2)/b*(5*cos(b*x+a)^2-3)*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^
2-1))^(3/2)/sin(b*x+a)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.99335, size = 165, normalized size = 2.2

$$-\frac{2\sqrt{2}\left(3\,c\tan\left(bx+a\right)^{2}-2\,c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{3\left(b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] -2/3*sqrt(2)*(3*c*tan(b*x + a)^2 - 2*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a
)^2 - 1))/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.615 $\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=80

$$\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

```
[Out] (c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/b + (c^2*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.0597835, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4397, 3775, 21, 3774, 207}

$$\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/b + (c^2*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx &= \int (-c+c\sec(2a+2bx))^{3/2}\,dx\\
&= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (2c)\int \frac{-\frac{c}{2}+\frac{1}{2}c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx\\
&= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - c\int \sqrt{-c+c\sec(2a+2bx)}\,dx\\
&= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}\\
&= \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} + \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.157503, size = 86, normalized size = 1.08

$$\frac{c\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(2\cot(a+bx)+\sqrt{2}\sqrt{\cos(2(a+bx))}\csc(a+bx)\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(2*Cot[a + b*x] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a +
 b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x])*Sqrt[c*Tan[a + b*x]*Tan[2*(a
```

```
+ b*x)]])/(2*b)
```

Maple [B] time = 0.514, size = 253, normalized size = 3.2

$$\frac{\sqrt{2}\left(2\,(\cos(bx+a))^{2}-1\right)}{b\left(\sqrt{2}-2\right)\left(2+\sqrt{2}\right)(\sin(bx+a))^{3}}\left(\sqrt{2}\cos(bx+a)\sqrt{\frac{2\,(\cos(bx+a))^{2}-1}{(\cos(bx+a)+1)^{2}}}\mathrm{Artanh}\left(\frac{\sqrt{2}\cos(bx+a)\sqrt{4}\,(-1+\cos(bx}{2\,(\sin(bx+a))^{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2^(1/2)/b/(2^(1/2)-2)/(2+2^(1/2))*(2*cos(b*x+a)^2-1)*(2^(1/2)*cos(b*x+a)*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^
(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2
)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2))-2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)
/sin(b*x+a)^3
```

Maxima [B] time = 2.1117, size = 1778, normalized size = 22.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1
/4)*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(c
os(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*ar
ctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x + 4*a)^2 +
 sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b
*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*log(sqrt(cos(4*b*x + 4*a)^2 + s
in(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a
), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2
 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
```

```
*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1
) + c*log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + s
in(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arctan2
(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
+ 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)*sqr
t(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) + 2*(co
s(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(
1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*b*x
+ 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))) + 1)
 - c*log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + si
n(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arctan2(
sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x + 4*a),
 -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x +
 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)*sqrt
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) - 2*(cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(1
/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*b*x +
 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b
*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))) + 1))
*sqrt(c) + 8*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/
2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + c*cos(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), co
s(4*b*x + 4*a))) + c*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1)))*sqrt(c))/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)^(1/4)*b)
```

Fricas [A] time = 2.42018, size = 770, normalized size = 9.62

$$\left[\frac{c^{\frac{3}{2}} \log\left(-\frac{c \tan(bx+a)^5 - 14\, c \tan(bx+a)^3 + 4\, \sqrt{2}\left(\tan(bx+a)^4 - 4\, \tan(bx+a)^2 + 3\right)\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}}\sqrt{c} + 17\, c \tan(bx+a)}{\tan(bx+a)^5 + 2\, \tan(bx+a)^3 + \tan(bx+a)}\right) \tan(bx+a) + 4\, \sqrt{2}\sqrt{-\frac{c \tan(b}{\tan(bx+}}}{4\, b \tan(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c^(3/2)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan
(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan
(b*x + a)))*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*c)/(b*tan(b*x + a)), -1/2*(sqrt(-c)*c*arctan(2*sqrt(2)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x +
a)^3 - 3*c*tan(b*x + a)))*tan(b*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*c)/(b*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.616 $\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=86

$$\frac{c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}$$

```
[Out] (-3*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(2*b) + (c^2*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.222264, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4397, 3814, 21, 3805, 3774, 207}

$$\frac{c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(2*b) + (c^2*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= -\frac{c^2\sin(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (2c)\int \frac{\cos(2a+2bx)\left(-\frac{3c}{2}+\frac{3}{2}c\sec\ldots\right.}{\sqrt{-c+c\sec(2a+2\ldots}}$$

$$= -\frac{c^2\sin(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (3c)\int \cos(2a+2bx)\sqrt{-c+c\sec(2\ldots}$$

$$= \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{2}(3c)\int \sqrt{-c+c\sec(2a+2bx)}\,dx$$

$$= \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\left(3c^2\right)\mathrm{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan\ldots}{\sqrt{-c+c\,\mathrm{s}\ldots}}\right.}{2b}$$

$$= -\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} + \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

Mathematica [A] time = 0.249089, size = 93, normalized size = 1.08

$$\frac{c\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(\cos(a+bx)+\cos(3(a+bx))-3\sqrt{2}\sqrt{\cos(2(a+bx))}\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(Cos[a + b*x] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a +
 b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Tan
[a + b*x]*Tan[2*(a + b*x)]])/(4*b)
```

Maple [B] time = 0.474, size = 518, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2^(1/2)/b/(2^(1/2)-2)/(2+2^(1/2))*(2*cos(b*x+a)^2-1)*(2^(1/2)*cos(b*x+a)*(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4
^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/
2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2))-2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2
)/sin(b*x+a)^3-2*2^(1/2)/b/(2^(1/2)-2)^3/(2+2^(1/2))^3*(2*cos(b*x+a)^2-1)*(
2^(1/2)*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*
2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*si
n(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3
```

Maxima [B] time = 2.07666, size = 1428, normalized size = 16.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] -1/16*(4*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2
*b*x + 2*a) + (c*cos(2*b*x + 2*a) + c)*sin(1/2*arctan2(sin(4*b*x + 4*a), -c
os(4*b*x + 4*a) - 1)))*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b
*x + 4*a) + 1)^(1/4)*sqrt(c) - 3*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos
(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*lo
g(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*
x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1)) + 1) + c*log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*
x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1))*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1))) + 1) - c*log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
+ 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1))) + 1))*sqrt(c))/b
```

Fricas [B] time = 2.54091, size = 941, normalized size = 10.94

$$\left[\frac{3\left(c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3-4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}+17c\tan(bx}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right.}{8\left(b\tan(bx+a)^3+b\tan(bx+a)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/8*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5
- 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*s
qrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(t
an(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(2)*(c*tan(b*x +
a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^3 +
 b*tan(b*x + a)), 1/4*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(-c)*arcta
n(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(c*tan(b*x +
 a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^3
+ b*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.617 $\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=133

$$-\frac{7c^2\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

```
[Out] (7*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(8*b) - (7*c^2*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.256441, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3813, 21, 3805, 3774, 207}

$$-\frac{7c^2\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (7*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(8*b) - (7*c^2*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos^2(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{2}c\int \frac{\cos(2a+2bx)\left(\frac{7c}{2}-\frac{7}{2}c\right.}{\sqrt{-c+c\sec(2a}}$$

$$= \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{4}(7c)\int \cos(2a+2bx)\sqrt{-c}$$

$$= -\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} -$$

$$= -\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} +$$

$$= \frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b} - \frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} +$$

Mathematica [A] time = 0.287793, size = 105, normalized size = 0.79

$$\frac{c\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(-5\cos(a+bx)-6\cos(3(a+bx))+\cos(5(a+bx))+7\sqrt{2}\sqrt{\cos(2(a+bx))}\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\right)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(-5*Cos[a + b*x] + 7*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(
a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]] - 6*Cos[3*(a + b*x)] + Cos[5*(a + b*x)])
*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

Maple [B] time = 0.455, size = 792, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2^(1/2)/b/(2^(1/2)-2)/(2+2^(1/2))*(2*cos(b*x+a)^2-1)*(2^(1/2)*cos(b*x+a)*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^
(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2
)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2))-2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)
/sin(b*x+a)^3+4*2^(1/2)/b/(2^(1/2)-2)^3/(2+2^(1/2))^3*(2*cos(b*x+a)^2-1)*(2
^(1/2)*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2
^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*c
os(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin
(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3-2*2^(1/2)/b/(2+2^(1/2))^5/
(2^(1/2)-2)^5*(2*cos(b*x+a)^2-1)*(16*cos(b*x+a)^5+9*2^(1/2)*cos(b*x+a)*((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1
/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
))+9*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2
)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2))-12*cos(b*x+a)^3+18*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b
*x+a)^2-1))^(3/2)/sin(b*x+a)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.37623, size = 1126, normalized size = 8.47

$$\left[\frac{7\left(c\tan(bx+a)^5+2\,c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3+4\,\sqrt{2}\left(\tan(bx+a)^4-4\,\tan(bx+a)^2+3\right)\sqrt{-}}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}\right.}{32\left(b\tan(bx+a)^5+2\,b\tan(bx+a)^3+b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/32*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*l
og(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4
*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) +
 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) + 4
*sqrt(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(
b*x + a)), -1/16*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a)
)*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(t
an(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt
(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x +
 a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.618 $\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=182

$$\frac{11c^2\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^2\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan}{\sqrt{c}\sec(}\right.}{16b}$$

```
[Out] (-11*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x
]]])/(16*b) + (11*c^2*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) - (11*c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a +
2*b*x]]) + (c^2*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2
*a + 2*b*x]])
```

Rubi [A] time = 0.31209, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3813, 21, 3805, 3774, 207}

$$\frac{11c^2\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^2\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan}{\sqrt{c}\sec(}\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-11*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x
]]])/(16*b) + (11*c^2*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) - (11*c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a +
2*b*x]]) + (c^2*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2
*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
```

```
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos^3(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{c^2\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{3}c\int \frac{\cos^2(2a+2bx)\left(\frac{11c}{2}-\right.}{\sqrt{-c+c\sec}}$$
$$= \frac{c^2\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{6}(11c)\int \cos^2(2a+2bx)\sqrt{-}$$
$$= -\frac{11c^2\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos^2(2a+2bx)\sin(2a}{6b\sqrt{-c+c\sec(2a+2}}$$
$$= \frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} - \frac{11c^2\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}}$$
$$= \frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} - \frac{11c^2\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}}$$
$$= -\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b} + \frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}}$$

Mathematica [A] time = 0.249013, size = 117, normalized size = 0.64

$$\frac{c\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(-42\sin(2(a+bx))+14\sin(4(a+bx))-4\sin(6(a+bx))+38\cot(a+bx)-33\sqrt{2}\sqrt{co}\right.}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(38*Cot[a + b*x] - 33*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*
(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] - 42*Sin[2*(a + b*x)] + 14
*Sin[4*(a + b*x)] - 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)
]])/(96*b)
```

Maple [B] time = 0.457, size = 1078, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2^(1/2)/b/(2^(1/2)-2)/(2+2^(1/2))*(2*cos(b*x+a)^2-1)*(2^(1/2)*cos(b*x+a)*(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4
^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/
2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2))-2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2
)/sin(b*x+a)^3-6*2^(1/2)/b/(2^(1/2)-2)^3/(2+2^(1/2))^3*(2*cos(b*x+a)^2-1)*(
2^(1/2)*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*
2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2))+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*si
n(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3+6*2^(1/2)/b/(2+2^(1/2))^5
/(2^(1/2)-2)^5*(2*cos(b*x+a)^2-1)*(16*cos(b*x+a)^5+9*2^(1/2)*cos(b*x+a)*((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(
1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2))+9*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/
2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2))-12*cos(b*x+a)^3+18*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(
b*x+a)^2-1))^(3/2)/sin(b*x+a)^3-4/3*2^(1/2)/b/(2^(1/2)-2)^7/(2+2^(1/2))^7*(
2*cos(b*x+a)^2-1)*(128*cos(b*x+a)^7-80*cos(b*x+a)^5+75*2^(1/2)*cos(b*x+a)*(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4
^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2))+75*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*2^
(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2))-100*cos(b*x+a)^3+150*cos(b*x+a))*(c*sin(b*x+a)^2/(2
*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.56592, size = 1310, normalized size = 7.2

$$\left[\frac{33\left(c\tan\left(bx+a\right)^{7}+3\,c\tan\left(bx+a\right)^{5}+3\,c\tan\left(bx+a\right)^{3}+c\tan\left(bx+a\right)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^{5}-14\,c\tan(bx+a)^{3}-4\,\sqrt{2}\left(\tan(bx\cdots\right.}{\tan(bx+a)^{5}+\cdots}\right.}{192\left(b\tan\left(bx+a\right)^{7}+3\,b\tan\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 + c*
tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt
(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)
^3 + tan(b*x + a))) - 4*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4
- 31*c*tan(b*x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))
/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x +
a)), 1/96*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 +
 c*tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
+ a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x
+ a))) - 2*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4 - 31*c*tan(b*
x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x +
 a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.619 $\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=175

$$\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15bc}+\frac{14\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (14*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a +
 2*b*x]]) + (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b*c)
```

Rubi [A] time = 0.599777, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3822, 4010, 4001, 3795, 207}

$$\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15bc}+\frac{14\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```


```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (14*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a +
 2*b*x]]) + (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b*c)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
```

```
] && IntegerQ[2*n]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec^4(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\int \frac{\sec^2(2a+2bx)(4c+c\sec(2a+2bx))}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{5c}$$

$$= \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{15bc} + \frac{2\int -$$

$$= \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+}{1}$$

$$= \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+}{1}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan}{5b\sqrt{-c+c\sec(}$$

Mathematica [A] time = 0.726858, size = 112, normalized size = 0.64

$$\frac{\sin(a+bx)\cos(a+bx)\sec^3(2(a+bx))\left(4\cos(2(a+bx)) + 26\cos(4(a+bx)) + 30\cos^2(2(a+bx))\tan^{-1}\left(\sqrt{\tan^2(a+b}\right.\right.}{30b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cos[a + b*x]*Sec[2*(a + b*x)]^3*Sin[a + b*x]*(38 + 4*Cos[2*(a + b*x)] + 26
*Cos[4*(a + b*x)] + 30*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Cos[2*(a + b*x)]^2
*Sqrt[-1 + Tan[a + b*x]^2]))/(30*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [B] time = 0.577, size = 980, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/120*2^(1/2)/b*4^(1/2)/(-3+2*2^(1/2))^3/(3+2*2^(1/2))^3*(-1+cos(b*x+a))*(1
20*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)^6+120*cos(b*x+a)^6*ln(-2*(
cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos
(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+208*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^6+208*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^5-180*arctanh(1/2*4^(1/2)*(2*cos(b*x
+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2))*cos(b*x+a)^4-180*cos(b*x+a)^4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-200*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)*cos(b*x+a)^4-200*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b
*x+a)^3+90*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2
/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)^2+90*cos(b*x+a)^2*
ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a
)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)
+60*cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+60*cos(b*x+a)*
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-15*arctanh(1/2*4^(1/2)*(2*cos(b
*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2))-15*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2
*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin
(b*x+a)^2))/(2*cos(b*x+a)^2-1)^3/sin(b*x+a)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2
-1))^(1/2)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2\,bx + 2\,a)^4}{\sqrt{c\tan(2\,bx + 2\,a)\tan(bx + a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^4/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A] time = 2.43407, size = 994, normalized size = 5.68

$$\left[\frac{4\sqrt{2}\left(15\tan\left(bx+a\right)^4-20\tan\left(bx+a\right)^2+17\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}+\frac{15\sqrt{2}\left(c\tan(bx+a)^5-2c\tan(bx+a)^3+c\tan(bx+a)\right)\log\left(\frac{\tan(bx+a)^3-\frac{2}{}}{}\right)}{\sqrt{c}}}{60\left(bc\tan\left(bx+a\right)^5-2\,bc\tan\left(bx+a\right)^3+bc\tan\left(bx+a\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/60*(4*sqrt(2)*(15*tan(b*x + a)^4 - 20*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1)) + 15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*
x + a)^3 + c*tan(b*x + a))*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x
 + a)^3)/sqrt(c))/(b*c*tan(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x
+ a)), -1/30*(15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*x + a)^3 + c*tan(b*x
 + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(
b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a)) - 2*sqrt(2)*(15*tan(b*x + a)^4 - 2
0*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*t
an(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.620 $\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=129

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}+\frac{2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b*c)
```

Rubi [A] time = 0.35947, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3800, 4001, 3795, 207}

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}+\frac{2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b*c)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec^3(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3bc} + \frac{2\int \frac{\sec(2a+2bx)\left(\frac{c}{2}+c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{3c}$$

$$= \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3bc} + \int \frac{\sec(2}{\sqrt{-c+c\,\text{s}}}$$

$$= \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3bc} - \frac{\text{Subst}\left(\int \frac{}{-2}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2b}}{3bc}$$

Mathematica [A] time = 0.392146, size = 89, normalized size = 0.69

$$\frac{\cos^2(a+bx)\csc(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(3\sqrt{\tan^2(a+bx)-1}\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)+2\sec(2(a+\right.}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cos[a + b*x]^2*Csc[2*(a + b*x)]*(2 + 2*Sec[2*(a + b*x)] + 3*ArcTan[Sqrt[-1
 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a
 + b*x)]])/(3*b*c)
```

Maple [B] time = 0.506, size = 673, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/24*2^(1/2)/b*4^(1/2)/(3+2*2^(1/2))^2/(-3+2*2^(1/2))^2*(-1+cos(b*x+a))*(1
2*cos(b*x+a)^4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1
)/sin(b*x+a)^2)+12*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(
b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)^4+8*((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^4+8*((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3-12*cos(b*x+a)^2*ln(-2*(cos(b*x+a)^2*((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-12*arctanh(1/2*4^(1/2)*
(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2))*cos(b*x+a)^2+3*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+3*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(
b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/(2*cos
(b*x+a)^2-1)^2/sin(b*x+a)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*si
n(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(2\,bx + 2\,a\right)^{3}}{\sqrt{c\tan\left(2\,bx + 2\,a\right)\tan\left(bx + a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^3/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A] time = 2.37962, size = 749, normalized size = 5.81

$$\left[\frac{\frac{3\sqrt{2}\left(c\tan(bx+a)^3 - c\tan(bx+a)\right)\log\left(\frac{\tan(bx+a)^3 - \frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}} - 2\tan(bx+a)}{\tan(bx+a)^3}\right)}{\sqrt{c}} - 8\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{12\left(bc\tan(bx+a)^3 - bc\tan(bx+a)\right)}, -\frac{3\sqrt{2}\left(c\tan(bx+a)^3 - \cdots\right.}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/12*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*log((tan(b*x + a)^3 -
2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c)
 - 2*tan(b*x + a))/tan(b*x + a)^3)/sqrt(c) - 8*sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^3 - b*c*tan(b*x + a)), -1/6*(3*
sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a
)) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x +
 a)^3 - b*c*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.621 $\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=88

$$\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + Tan[2*a + 2*b*x]/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)
```

Rubi [A] time = 0.237719, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3798, 3795, 207}

$$\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + Tan[2*a + 2*b*x]/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx &= \int \frac{\sec^2(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx \\
&= \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} + \int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx \\
&= \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.257214, size = 67, normalized size = 0.76

$$
\frac{\left(\sqrt{\tan^2(a+bx)-1}\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)+2\right)\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a
 + b*x)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [B] time = 0.467, size = 478, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/b*(cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(
b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*c
os(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2))+ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-
2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/si
n(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+arctanh(1/2*4^(1/2)
*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*cos(b*x+a))*s
in(b*x+a)/(2*cos(b*x+a)^2-1)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(2\,bx+2\,a\right)^{2}}{\sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A] time = 2.40652, size = 626, normalized size = 7.11

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{\tan(bx+a)^{3}-\frac{2\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^{3}}\right)\tan\left(bx+a\right)+4\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}}{4\,bc\tan\left(bx+a\right)},-\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\,t\ldots}{ta\ldots}}}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(c)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b
*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)
^3)*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/
(b*c*tan(b*x + a)), -1/2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))*tan(b
*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(
b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.622 $\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c]))
```

Rubi [A] time = 0.0761244, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4397, 3795, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}}$$

Mathematica [A] time = 0.147745, size = 64, normalized size = 1.16

$$\frac{\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2]*Tan[2*(a + b*x
)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [B] time = 0.388, size = 236, normalized size = 4.3

$$\frac{\sqrt{2}\sqrt{4}(\cos(bx+a)+1)}{8b\sin(bx+a)c}\sqrt{\frac{2(\cos(bx+a))^2-1}{(\cos(bx+a)+1)^2}}\sqrt{\frac{c\left(1-(\cos(bx+a))^2\right)}{2(\cos(bx+a))^2-1}}\left(\ln\left(-2\frac{1}{(\sin(bx+a))^2}\left((\cos(bx+a))^2\sqrt{\frac{2(c}{(co}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/8*2^(1/2)/b*4^(1/2)*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(ln(-2*(cos(b*x+a)^2*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+arctanh(1/2*4^(1/2)*(
```

```
2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)))/sin(b*x+a)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2bx+2a)}{\sqrt{c\tan(2bx+2a)\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A] time = 2.27136, size = 386, normalized size = 7.02

$$\left[\frac{\sqrt{2}\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)}{4b\sqrt{c}}, -\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/(b*sq
rt(c)), -1/2*sqrt(2)*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)
^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError
```

3.623 $\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*Sqrt[c
]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])]/(Sqrt[2]*b*Sqrt[c])
```

Rubi [A] time = 0.0899419, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4397, 3776, 3774, 207, 3795}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```


```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*Sqrt[c
]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])]/(Sqrt[2]*b*Sqrt[c])
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx &= \int \frac{1}{\sqrt{-c+c\sec(2a+2bx)}}\,dx \\
&= -\frac{\int \sqrt{-c+c\sec(2a+2bx)}\,dx}{c} + \int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx \\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} + \frac{\operatorname{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2}{\sqrt{-c+c\sec(2a}}\right.}{b} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.332494, size = 94, normalized size = 0.94

$$\frac{\tan(a+bx)\left(2\tanh^{-1}\left(\frac{1}{2}\sqrt{2-2\tan^2(a+bx)}\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)\right)}{b\sqrt{2-2\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((2*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a
+ b*x]^2]])*Tan[a + b*x])/(b*Sqrt[2 - 2*Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]
*Tan[2*(a + b*x)]])
```

Maple [B] time = 0.363, size = 301, normalized size = 3.

$$-\frac{\sqrt{2}\sqrt{4}\left(\cos\left(bx+a\right)+1\right)}{8\,b\sin\left(bx+a\right)c}\sqrt{\frac{2\left(\cos\left(bx+a\right)\right)^{2}-1}{\left(\cos\left(bx+a\right)+1\right)^{2}}}\sqrt{\frac{c\left(1-\left(\cos\left(bx+a\right)\right)^{2}\right)}{2\left(\cos\left(bx+a\right)\right)^{2}-1}}\left(2\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\cos\left(bx+a\right)\sqrt{4}\left(-1+c\right.}{\left(\sin\left(bx+a\right)\right)^{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/8*2^(1/2)/b*4^(1/2)*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(2*2^(1/2)*arctanh(1/2
*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2))-ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)+1)/sin(b*x+a)^2)-arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a
)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/sin(b*x+a)/
c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14032, size = 807, normalized size = 8.07

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}-2\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^{3}}\right)+2\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-3\right.}{\tan(bx+a)^{3}+\tan(bx+a)}\right.}{4\,bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan
(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x
+ a)^3) + 2*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(t
an(b*x + a)^3 + tan(b*x + a))))/(b*c), -1/2*(sqrt(2)*sqrt(-c)*arctan(sqrt(-
c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan
(b*x + a))) - 2*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))))/(b*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.624 $\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=138

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(2*b*Sqrt
[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*
b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(2*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]])
```

Rubi [A] time = 0.278831, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4397, 3823, 3904, 3887, 481, 206}

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```


```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(2*b*Sqrt
[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*
b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(2*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]])
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$
$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{-c-c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c}$$
$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{2}c\int \frac{\tan^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$
$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\,\text{Subst}\left(\int \frac{x^2}{(1-cx^2)(2-cx^2)}\,dx, x, -\frac{\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b}$$
$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, -\frac{\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2}}$$

Mathematica [A] time = 2.47319, size = 166, normalized size = 1.2

$$\frac{\tan(a+bx)\left(\sqrt{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-2\tan^2(a+bx)}\right) - \tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right) + \sqrt{2}\cos^2(a+bx)\sqrt{\frac{1}{\sec(2(a+bx))+1}}\left(\tan^{-1}\right.\right.}{2b\sqrt{1-\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Tan[a + b*x]*(Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - ArcTanh[Sqrt
[1 - Tan[a + b*x]^2]] + Sqrt[2]*Cos[a + b*x]^2*Sqrt[(1 + Sec[2*(a + b*x)])^
(-1)]*(2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sqrt[-1 + Tan
[a + b*x]^2])))/(2*b*Sqrt[1 - Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a
+ b*x)]])
```

Maple [B] time = 0.466, size = 1030, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/16*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(cos(b*x+a)^3*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)+4*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(3/2)*cos(b*x+a)^2+5*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/
2)*cos(b*x+a)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*arcta
nh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*2^(1/2)+6*cos(b*x+a)*((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)+4*arctanh(1/2*4^(1/2)*(2*co
s(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2))*cos(b*x+a)-6*2^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+c
os(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+4*((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*x
+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*
x+a)^2-1))^(1/2)/sin(b*x+a)^3+1/8*2^(1/2)/b*4^(1/2)*(cos(b*x+a)+1)*((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))
^(1/2)*(2*2^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/si
n(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))-ln(-2*(cos(b*x+a)^2
*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-arctanh(1/2*4^(1/2
)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)))/sin(b*x+a)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2\,bx + 2\,a)}{\sqrt{c\tan(2\,bx + 2\,a)\tan(bx + a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A] time = 2.2922, size = 1274, normalized size = 9.23

$$\left[\frac{\sqrt{2}\left(\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}-2\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^{3}}\right)+\left(\tan\left(bx+a\right)^{3}+\tan\left(b\ldots\right.\right.}{4\left(bc\tan\left(bx+a\right)^{3}+b\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3
 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt
(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + (tan(b*x + a)^3 + tan(b*x + a))*s
qrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3
 + tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*
(tan(b*x + a)^2 - 1))/(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1/2*(sqrt(2
)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - (tan(
b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) +
sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/
(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError
```

3.625 $\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=182

$$\frac{\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] (7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*b*
Sqrt[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(8*b*Sqrt[-c + c*Sec[2*a
 + 2*b*x]]) + (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])
```

Rubi [A] time = 0.369753, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4397, 3823, 4022, 3920, 3774, 207, 3795}

$$\frac{\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*b*
Sqrt[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(8*b*Sqrt[-c + c*Sec[2*a
 + 2*b*x]]) + (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
```

```
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos^2(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{\cos(2a+2bx)(-c-3c\sec(2a+2bx))}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{-\frac{7c^2}{2}-\frac{1}{2}c^2\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,a}{4c^2}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{7\int \sqrt{-c+c\sec(2a+}}{8c}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\,\mathrm{Subst}\left(\int \frac{1}{-c+x^2}\,dx,\right.}{8l}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+}}$$

Mathematica [A] time = 3.07411, size = 186, normalized size = 1.02

$$\frac{\tan(a+bx)\left(7\sqrt{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-2\tan^2(a+bx)}\right) - 7\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right) + \sqrt{2}\cos^2(a+bx)\sec(2(a+bx))\sqrt{\frac{}{\mathrm{se}}}\right.}{8b\sqrt{1-\tan^2(a+bx)}\sqrt{c\tan(a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Tan[a + b*x]*(7*Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - 7*ArcTanh[
Sqrt[1 - Tan[a + b*x]^2]] + Sqrt[2]*Cos[a + b*x]^2*Sec[2*(a + b*x)]*Sqrt[(1
 + Sec[2*(a + b*x)])^(-1)]*(2*(1 + Cos[2*(a + b*x)] + Cos[4*(a + b*x)]) + A
rcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])))/(8*b*Sqrt[1 -
 Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [B] time = 0.462, size = 1835, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] -1/8*2^(1/2)/b*4^(1/2)*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(2*2^(1/2)*arctanh(1/2
*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2))-ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)+1)/sin(b*x+a)^2)-arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a
)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/sin(b*x+a)/
c-1/8*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(cos(b*x+a)^3*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)+4*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+
1)^2)^(3/2)*cos(b*x+a)^2+5*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3
/2)*cos(b*x+a)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*arct
anh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*2^(1/2)+6*cos(b*x+a)*((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(co
s(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)+4*arctanh(1/2*4^(1/2)*(2*c
os(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2))*cos(b*x+a)-6*2^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+
cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))+4*((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*
x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b
*x+a)^2-1))^(1/2)/sin(b*x+a)^3-1/64*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(-4
*cos(b*x+a)^5*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)-16*4^(1/2
)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^4-33*cos(b*x+a)^3*
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)-52*4^(1/2)*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^2-49*4^(1/2)*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)-18*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(3/2)+46*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/s
in(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*2^(1/2)
+46*2^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+
a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))-54*cos(b*x+a)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-32*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)-32*arctanh(1/2*4^(1/2)*(2*c
os(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2))*cos(b*x+a)-36*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-32*ln(
-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2
+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-32
*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b
```

```
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/sin(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2bx+2a)^2}{\sqrt{c\tan(2bx+2a)\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A] time = 2.32684, size = 1503, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)
*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(ta
n(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 7*(tan(b*x
+ a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3 + 2
*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*
sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(t
an(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1)))/(b*c*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1
/8*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*a
rctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqr
t(-c)/(c*tan(b*x + a))) - 7*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x +
a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)
)*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(tan(b*x + a)^4
 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c
*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.626 $\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=180

$$\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{12bc^2}-\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}+\frac{13\tan(2a+}{6bc\sqrt{c\sec(2a+}}$$

```
[Out] (-11*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(4*b*(
-c + c*Sec[2*a + 2*b*x])^(3/2)) + (13*Tan[2*a + 2*b*x])/(6*b*c*Sqrt[-c + c*
Sec[2*a + 2*b*x]]) + (7*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(12
*b*c^2)
```

Rubi [A] time = 0.513003, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3816, 4010, 4001, 3795, 207}

$$\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{12bc^2}-\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}+\frac{13\tan(2a+}{6bc\sqrt{c\sec(2a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```


```
[Out] (-11*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(4*b*(
-c + c*Sec[2*a + 2*b*x])^(3/2)) + (13*Tan[2*a + 2*b*x])/(6*b*c*Sqrt[-c + c*
Sec[2*a + 2*b*x]]) + (7*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(12
*b*c^2)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
```

```
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^4(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec^2(2a+2bx)\left(2c+\frac{7}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{7\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{12bc^2} + \frac{\int}{}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\sqrt{-c+c\sec(2}{}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\sqrt{-c+c\sec(2}{}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan}{6bc\sqrt{-c+}}$$

Mathematica [A] time = 1.36145, size = 100, normalized size = 0.56

$$-\frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(\csc^2(a+bx)((19\cos(4(a+bx))+11)\sec(2(a+bx))-24)-66\tan^{-1}\left(\sqrt{\tan}\right.\right.}{48bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -(Cot[a + b*x]*(Csc[a + b*x]^2*(-24 + (11 + 19*Cos[4*(a + b*x)])*Sec[2*(a +
 b*x)]) - 66*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*S
qrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(48*b*c^2)
```

Maple [B] time = 0.454, size = 1211, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 1/96*2^(1/2)/b*4^(1/2)/(-3+2*2^(1/2))^3/(3+2*2^(1/2))^3*(-1+cos(b*x+a))^2*(
132*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b
*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a
)^2)*cos(b*x+a)^5+152*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a
)^5+132*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)^5-132*cos(b*x+a)^4*ln
(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^
2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-1
32*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)^4-132*ln(-2*(cos(b*x+a)^2*
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*c
os(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)^3-200*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3-132*arctanh(1/2*4^(1
/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2))*cos(b*x+a)^3+132*cos(b*x+a)^2*ln(-2*(cos(b*x+a)^2*((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+132*arctanh(1/2*4^(1/2)*(
2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2))*cos(b*x+a)^2+33*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)+54*cos(b*x+a)*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)+33*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a
)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)-3
3*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x
+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^
2)-33*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/(2*cos(b*x+a)^2-1)^2/((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/s
in(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2\,bx+2\,a)^4}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^4/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.90706, size = 886, normalized size = 4.92

$$\left[\frac{33\sqrt{2}\left(\tan(bx+a)^5-\tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right)+2\sqrt{2}\left(27\tan(bx+\cdots\right.}{48\left(bc^2\tan(bx+a)^5-bc^2\tan(bx+a)^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/48*(33*sqrt(2)*(tan(b*x + a)^5 - tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x
+ a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1
)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 2*sqrt(2)*(27*tan(b*x + a)^
4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b
*c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3), -1/24*(33*sqrt(2)*(tan(b*x + a
)^5 - tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(27*tan(b
*x + a)^4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 -
 1)))/(b*c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.627 $\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}+\frac{\tan(2a+2bx)}{bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] (-7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2)) + Tan[2*a + 2*b*x]/(b*c*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.306361, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3799, 4001, 3795, 207}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}+\frac{\tan(2a+2bx)}{bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2)) + Tan[2*a + 2*b*x]/(b*c*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^3(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec(2a+2bx)\left(\frac{3c}{2}+2c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2b}}}{4c}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{-2c+x^2}\right.}{}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a}{bc\sqrt{-c+c\,\text{se}}}$$

Mathematica [A] time = 0.625027, size = 94, normalized size = 0.73

$$\frac{\tan(2(a+bx))\left(4\sec(2(a+bx))+7\sin^2(a+bx)\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\sec(2(a+bx))-5\right)}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ((-5 + 4*Sec[2*(a + b*x)] + 7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a +
b*x)]*Sin[a + b*x]^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*T
an[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

Maple [B] time = 0.453, size = 930, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -1/16*2^(1/2)/b*(7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^
3+7*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*cos(
b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2))*cos(b*x+a)^3+7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+
1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x
+a)^2+7*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*
cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2))*cos(b*x+a)^2-7*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)-7*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/
2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2))+20*cos(b*x+a)^3-7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)-7*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+
a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)-18*cos(b*x+a))*sin(b*x+a)/(2*cos(b*x+a)^2-1)^2/(c*sin(b*
x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2bx+2a)^3}{(c\tan(2bx+2a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^3/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.85703, size = 711, normalized size = 5.55

$$\left[\frac{7\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(9\tan(bx+a)^2\right.}{16bc^2\tan(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(7*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(7*sqrt(2)*sqr
t(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a
)^2/(tan(b*x + a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.628 $\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2))
```

Rubi [A] time = 0.236104, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3797, 3795, 207}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx &= \int \frac{\sec^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{3\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c} \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{4bc} \\
&= -\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.609946, size = 84, normalized size = 0.9

$$\frac{\tan(2(a+bx))\left(3\sin^2(a+bx)\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\sec(2(a+bx))-1\right)}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ((-1 + 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*
Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*Tan[a + b*x]*Tan[2*(a
+ b*x)])^(3/2))
```

Maple [B] time = 0.423, size = 433, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -1/32*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(2*cos(b*x+a)*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)+3*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)+3*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^
2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))
*cos(b*x+a)-3*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)
/sin(b*x+a)^2)-3*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*
x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/(c*sin(b*x+a)^2/(2*cos
(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2\,bx+2\,a)^2}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.79174, size = 706, normalized size = 7.59

$$\left[\frac{3\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-\right.}{16\,bc^2\tan(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(3*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(3*sqrt(2)*sqrt(
-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1
)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.629 $\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]
/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3
/2))
```

Rubi [A] time = 0.12124, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4397, 3796, 3795, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]
/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3
/2))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx &= \int \frac{\sec(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx\\ &= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}\\ &= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{4bc}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.651006, size = 83, normalized size = 0.89

$$-\frac{\tan(2(a+bx))\left(\sin^2(a+bx)\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\sec(2(a+bx))+1\right)}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -((1 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*Sq
rt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*Tan[a + b*x]*Tan[2*(a +
b*x)])^(3/2))
```

Maple [B] time = 0.406, size = 599, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 1/32*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^3*(2*4^(1/2)*((2*cos(b*x+a)^2-1)/(co
s(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^2+4*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(3/2)*cos(b*x+a)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/
2)-6*cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)^2*
ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a
)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)
-arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)^2+2*cos(b*x+a)*((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)+ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x
+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^
2)+arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^
(3/2)/sin(b*x+a)^5/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2\,bx+2\,a)}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.98484, size = 699, normalized size = 7.52

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right.}{16\,bc^2\tan(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/16*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(ta
n(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x
 + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), 1/8*(sqrt(2)*sqrt(-c)*a
rctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqr
t(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sqrt(2)*sqrt(-c*tan(b*x + a)^2/(ta
n(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError
```

3.630 $\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{bc^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*c^(3
/2))) + (5*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a
+ 2*b*x])^(3/2))
```

Rubi [A] time = 0.142754, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4397, 3777, 3920, 3774, 207, 3795}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{bc^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*c^(3
/2))) + (5*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a
+ 2*b*x])^(3/2))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{2c+\frac{1}{2}c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \sqrt{-c+c\sec(2a+2bx)}\,dx}{c^2} - \frac{5\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2}}}{4c}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{bc} + \frac{5\,\text{Su}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{bc^{3/2}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+}{4b(-c+c\sec(2a}$$

Mathematica [A] time = 3.85748, size = 196, normalized size = 1.42

$$-\frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{-\left(\tan^2(a+bx)-1\right)^2}+\cot^2(a+bx)\left(\cos(2(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

```
[Out] -(Cot[a + b*x]*(4*Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2]*Cos[2*(a +
b*x)]*Sec[a + b*x]^2 - 4*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]]*Cos[2*(a + b*x)]
*Sec[a + b*x]^2 + Cot[a + b*x]^2*(Cos[2*(a + b*x)]*Sec[a + b*x]^2)^(3/2) +
ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-(-1 + Tan[a + b*x]^2)^2])*Sqrt[c*Ta
n[a + b*x]*Tan[2*(a + b*x)]])/(8*b*c^2*Sqrt[1 - Tan[a + b*x]^2])
```

Maple [B] time = 0.388, size = 561, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -1/32*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(8*arctanh(1/2*2^(1/2)*cos(b*x+a)
*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2))*cos(b*x+a)*2^(1/2)-8*2^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)
*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))-
5*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x
+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^
2)*cos(b*x+a)-5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x
+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)+2*cos(b*x+a)*
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+5*ln(-2*(cos(b*x+a)^2*((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)
^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*cos
(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*tan(2*b*x + 2*a)*tan(b*x + a))^(-3/2), x)
```

Fricas [A] time = 2.97344, size = 1148, normalized size = 8.32

$$\left[\frac{5\,\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan\left(bx+a\right)^3+8\,\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\,\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\cdots\right.}{\tan(bx+a)^3+\cdots}\right.\cdots}{16\,bc^2\tan\left(bx+a\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(5*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 8*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqr
t(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*
c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a)))*tan(b*x + a)^3 + 2*sqrt(2)
*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*
tan(b*x + a)^3), 1/8*(5*sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan
(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x +
 a)^3 - 8*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sq
rt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b
*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.631 $\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2bc^{3/2}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{3\sin(2a+2bx)}{4bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(2*b
*c^(3/2)) + (9*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[
2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Sin[2*a + 2*b*x]/(4*b*(-c + c*Sec[
2*a + 2*b*x])^(3/2)) - (3*Sin[2*a + 2*b*x])/(4*b*c*Sqrt[-c + c*Sec[2*a + 2*
b*x]])
```

Rubi [A] time = 0.319622, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {4397, 3817, 4022, 3920, 3774, 207, 3795}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2bc^{3/2}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{3\sin(2a+2bx)}{4bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(2*b
*c^(3/2)) + (9*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[
2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Sin[2*a + 2*b*x]/(4*b*(-c + c*Sec[
2*a + 2*b*x])^(3/2)) - (3*Sin[2*a + 2*b*x])/(4*b*c*Sqrt[-c + c*Sec[2*a + 2*
b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
```

```
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\cos(2a+2bx)\left(3c+\frac{3}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{3c^2+\frac{3}{2}c^2\sec(2a+2bx}{\sqrt{-c+c\sec(2a+2bx)}}}{2c^3}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{3\int\sqrt{-c+c\sec(2}{2c^2}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{3\,\text{Subst}\left(\int\frac{1}{-c+x^2}\right.}{}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2bc^{3/2}} + \frac{9\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\sin(2a}{4b(-c+c\sec(}$$

Mathematica [A] time = 6.19842, size = 342, normalized size = 1.92

$$\frac{\tan^2(a+bx)\tan^2(2(a+bx))\left(\frac{1}{2}\sin(2(a+bx))-\frac{1}{4}\cot(a+bx)-\frac{1}{8}\cot(a+bx)\csc^2(a+bx)\right)}{b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}} - \frac{3\tan^{\frac{3}{2}}(a+bx)\tan^{\frac{3}{2}}(2(a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*Tan[a + b*x]^(3/2)*((ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*S
ec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a +
b*x)]])/(1 + Tan[a + b*x]^2)^2 + (Sqrt[2]*(2*ArcTanh[Sqrt[2 - 2*Tan[a + b*x
]^2]/2] - Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]])*Cos[2*(a + b*x)]*Csc[a
 + b*x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[Tan[2*(a + b*x)]])/(Sqrt[1
 - Tan[a + b*x]^2]*(1 + Tan[a + b*x]^2)))*Tan[2*(a + b*x)]^(3/2))/(8*b*(c*T
an[a + b*x]*Tan[2*(a + b*x)])^(3/2)) + ((-Cot[a + b*x]/4 - (Cot[a + b*x]*Cs
c[a + b*x]^2)/8 + Sin[2*(a + b*x)]/2)*Tan[a + b*x]^2*Tan[2*(a + b*x)]^2)/(b
*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

Maple [B] time = 0.43, size = 1157, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 1/32*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(8*arctanh(1/2*2^(1/2)*cos(b*x+a)*
4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2))*cos(b*x+a)*2^(1/2)-8*2^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*
(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))-5
*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+
a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2
)*cos(b*x+a)-5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+
a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)+2*cos(b*x+a)*(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+5*ln(-2*(cos(b*x+a)^2*((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^
2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*cos(
b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-1/16*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(
-4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3+10*arctanh(1/2*
2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*2^(1/2)-10*2^(1/2)*arctanh(1/2*2^(1/2)
*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2))-7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)+6*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2)-7*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*
x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)+7*ln(-2*(cos
(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*
x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+7*arctanh
(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)/(c
*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2\,bx + 2\,a)}{(c\tan(2\,bx + 2\,a)\tan(bx + a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.91226, size = 1372, normalized size = 7.71

$$\left[\frac{9\sqrt{2}\left(\tan\left(bx+a\right)^5+\tan\left(bx+a\right)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)+12\left(\tan\left(bx+a\right)^5+\mathrm{t}\right.}{16\left(bc^2\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/16*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x +
 a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)
*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 12*(tan(b*x + a)^5 + tan(b*x
 + a)^3)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(
b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^
2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^5
+ b*c^2*tan(b*x + a)^3), 1/8*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*s
qrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2
 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 12*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqr
t(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(
b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) + sqrt(2)*(5*tan(b*x + a)^4 - 4*
tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*ta
n(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError
```

3.632 $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8bc^{3/2}}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{7\sin(2a+2bx)}{8bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{2bc\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] (-19*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*
b*c^(3/2)) + (13*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Se
c[2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x])/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3/2)) - (7*Sin[2*a + 2*b*x])/(8*b*c*Sqr
t[-c + c*Sec[2*a + 2*b*x]]) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(2*b*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]])
```

Rubi [A] time = 0.496741, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4397, 3817, 4022, 3920, 3774, 207, 3795}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8bc^{3/2}}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{7\sin(2a+2bx)}{8bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{2bc\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*
b*c^(3/2)) + (13*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Se
c[2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x])/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3/2)) - (7*Sin[2*a + 2*b*x])/(8*b*c*Sqr
t[-c + c*Sec[2*a + 2*b*x]]) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(2*b*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
```

```
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\cos^2(2a+2bx)\left(4c+\frac{5}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\cos(2a+2bx)\sin(2a+2bx)}{2bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{\cos(2a+2bx)(7c}{\sqrt{-c+c\,s}}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\cos(2a+2bx)\sin}{2bc\sqrt{-c+c\sec(}}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\cos(2a+2bx)\sin}{2bc\sqrt{-c+c\sec(}}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\cos(2a+2bx)\sin}{2bc\sqrt{-c+c\sec(}}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8bc^{3/2}} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\cos(2a+2b}{4b(-c+c\,\mathrm{se}}$$

Mathematica [A] time = 6.24743, size = 356, normalized size = 1.52

$$\frac{\tan^2(a+bx)\tan^2(2(a+bx))\left(\frac{7}{8}\sin(2(a+bx))+\frac{1}{8}\sin(4(a+bx))-\frac{5}{8}\cot(a+bx)-\frac{1}{8}\cot(a+bx)\csc^2(a+bx)\right)}{b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}} + \frac{\tan^{\frac{3}{2}}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Tan[a + b*x]^(3/2)*((-7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*S
ec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a +
b*x)]])/(1 + Tan[a + b*x]^2)^2 - (19*(2*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/
2] - Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]])*Cos[2*(a + b*x)]*Csc[a + b*
x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[Tan[2*(a + b*x)]])/(Sqrt[2]*Sqr
t[1 - Tan[a + b*x]^2]*(1 + Tan[a + b*x]^2)))*Tan[2*(a + b*x)]^(3/2))/(16*b*
(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2)) + (((-5*Cot[a + b*x])/8 - (Cot[a +
 b*x]*Csc[a + b*x]^2)/8 + (7*Sin[2*(a + b*x)])/8 + Sin[4*(a + b*x)]/8)*Tan[
```

```
a + b*x]^2*Tan[2*(a + b*x)]^2)/(b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

Maple [B] time = 0.395, size = 1787, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 1/32*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(8*((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)*cos(b*x+a)^5+14*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*co
s(b*x+a)^3-51*arctanh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*
x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*2^(1/2)+36*l
n(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)
^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*
cos(b*x+a)-30*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+36*arc
tanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)+51*2^(1/2)*arctanh(1/2*2^(1/2)*c
os(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2))-36*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)+1)/sin(b*x+a)^2)-36*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)
/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/((2*cos(b*x+a)^
2-1)/(cos(b*x+a)+1)^2)^(3/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(
b*x+a)^3-1/32*2^(1/2)/b*4^(1/2)*(-1+cos(b*x+a))^2*(8*arctanh(1/2*2^(1/2)*co
s(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2))*cos(b*x+a)*2^(1/2)-8*2^(1/2)*arctanh(1/2*2^(1/2)*cos(b*x+a)
*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2))-5*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2
*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin
(b*x+a)^2)*cos(b*x+a)-5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)
/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)+2*cos
(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+5*ln(-2*(cos(b*x+a)^2*(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2
)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)+1/8*2^(1/2)/b*4^(1/2)*(-1+cos(b*x
+a))^2*(-4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3+10*arct
anh(1/2*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*2^(1/2)-10*2^(1/2)*arctanh(1/2
```

```
*2^(1/2)*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2))-7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)+6*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)-7*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1
)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)+7*ln
(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^
2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+7
*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/sin(b*x+a)^2/((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(3/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(2\,bx+2\,a\right)^{2}}{\left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.91411, size = 1596, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(13*sqrt(2)*(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt
(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*
(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 19*(tan(
b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x + a)
^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2
 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt
```

```
(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4 - 8*tan(b*x + a)^2 - 1)*sqrt(-c*ta
n(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^7 + 2*b*c^2*tan(b*x
 + a)^5 + b*c^2*tan(b*x + a)^3), 1/8*(13*sqrt(2)*(tan(b*x + a)^7 + 2*tan(b*
x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x
+ a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 19*(tan(b*x
+ a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(1/2*sqrt(2)*sqr
t(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*
tan(b*x + a))) + sqrt(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4 - 8*tan(b*x +
 a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a
)^7 + 2*b*c^2*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.633 $\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

```
[Out] (-2*Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]])
```

Rubi [A] time = 0.0868706, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4390, 30}

$$-\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
```

```
[Out] (-2*Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]])
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = \frac{\sin(x)\int \frac{\csc^2(x)}{\sqrt{\tan(x)}}\,dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$
$$= \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \tan(x)\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$
$$= -\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Mathematica [A] time = 0.0331401, size = 16, normalized size = 1.

$$-\frac{1}{3}\sqrt{\sin(2x)}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
```

```
[Out] -(Cot[x]*Csc[x]*Sqrt[Sin[2*x]])/3
```

Maple [C] time = 0.09, size = 119, normalized size = 7.4

$$\frac{1}{6}\sqrt{-\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(4\sqrt{1+\tan(x/2)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}\text{EllipticF}\left(\sqrt{1+\text{t}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/sin(2*x)^(1/2),x)
```

```
[Out] 1/6*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(4*(1+
tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+
tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)+tan(1/2*x)^4-1)/(tan(1/2*x)*(tan(
1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

Fricas [B] time = 2.51186, size = 97, normalized size = 6.06

$$\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x) + \cos(x)^2 - 1}{3\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + cos(x)^2 - 1)/(cos(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)**(1/2),x)
```

```
[Out] Integral(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

3.634 $\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx$

Optimal. Leaf size=69

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}}-\frac{5\sin(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\sqrt{\sin(2x)}\sqrt{\tan(x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

```
[Out] Cos[x]/(2*Sqrt[Sin[2*x]]) + (Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]]) - (5*ArcTanh
[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x])/(2*Sqrt[2]*Sqrt[Sin[2*x]]*Sqrt[Tan[x]])
```

Rubi [A] time = 0.363889, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4390, 898, 1262, 207}

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}}-\frac{5\sin(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\sqrt{\sin(2x)}\sqrt{\tan(x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```


```
[Out] Cos[x]/(2*Sqrt[Sin[2*x]]) + (Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]]) - (5*ArcTanh
[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x])/(2*Sqrt[2]*Sqrt[Sin[2*x]]*Sqrt[Tan[x]])
```


Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
```

```
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx &= \frac{\sin(x)\int \frac{\csc^3(x)\sec(x)\sqrt{\tan(x)}}{-2+\tan(x)}\,dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{\sin(x)\,\text{Subst}\left(\int \frac{1+x^2}{(-2+x)x^{5/2}}\,dx, x, \tan(x)\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{(2\sin(x))\,\text{Subst}\left(\int \frac{1+x^4}{x^4(-2+x^2)}\,dx, x, \sqrt{\tan(x)}\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{(2\sin(x))\,\text{Subst}\left(\int \left(-\frac{1}{2x^4}-\frac{1}{4x^2}+\frac{5}{4(-2+x^2)}\right)dx, x, \sqrt{\tan(x)}\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}+\frac{(5\sin(x))\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin(x)}{2\sqrt{2}\sqrt{\sin(2x)}\sqrt{\tan(x)}}
\end{aligned}$$

Mathematica [C] time = 5.90324, size = 119, normalized size = 1.72

$$\frac{1}{4}\sqrt{\sin(2x)}\left(5\sqrt{\frac{\cos(x)}{2\cos(x)-2}}\sqrt{\tan\left(\frac{x}{2}\right)}\sec(x)\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right),-1\right)+\Pi\left(-\frac{2}{-1+\sqrt{5}};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```

```
[Out] (Sqrt[Sin[2*x]]*((1 + (2*Cot[x])/3)*Csc[x] + 5*Sqrt[Cos[x]/(-2 + 2*Cos[x])]
*(EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1] + EllipticPi[-2/(-1 + Sqrt[5]), -
ArcSin[1/Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1 + Sqrt[5])/2, -ArcSin[1/Sqrt
[Tan[x/2]]], -1])*Sec[x]*Sqrt[Tan[x/2]]))/4
```

Maple [C] time = 0.141, size = 396, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x)
```

```
[Out] -1/480*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(-140*(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x))^(1
/2),1/2*2^(1/2))*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)+240*(t
an(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+tan(
1/2*x))^(1/2),1/2*2^(1/2))*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2
*x)+2^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(
1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3)*(1+tan
(1/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/
2*x)^2-1))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3
/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)+40*(tan(1/
2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^4+120*tan(1/2*x)^3*(tan(1/2*x)^3-ta
n(1/2*x))^(1/2)-120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)-40*(tan(1/2*
x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)^2 \sec(x)}{(\tan(x) - 2)\sqrt{\sin(2x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="maxima")
```

```
[Out] integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

Fricas [B] time = 2.64763, size = 431, normalized size = 6.25

$$-\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}(2\cos(x)+3\sin(x))-4\cos(x)^2-15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x)+3\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/48*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 - 1
5*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x))
 + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos(
x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) +
4)/(cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*sec(x)/sin(2*x)**(1/2)/(-2+tan(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)^2\sec(x)}{(\tan(x)-2)\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="giac")
```

```
[Out] integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

3.635 $\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=79

$$\frac{\sin^2(x)\cos^3(x)}{2\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{3\sin^{\frac{5}{2}}(2x)}-\frac{5\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

```
[Out] (Cos[x]^4*Sin[x])/(3*Sin[2*x]^(5/2)) + (Cos[x]^3*Sin[x]^2)/(2*Sin[2*x]^(5/2
)) - (5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(2*Sqrt[2]*Sin[2*x]^(5/2)*T
an[x]^(5/2))
```

Rubi [A] time = 0.567016, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4390, 898, 1262, 207}

$$\frac{\sin^2(x)\cos^3(x)}{2\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{3\sin^{\frac{5}{2}}(2x)}-\frac{5\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```


```
[Out] (Cos[x]^4*Sin[x])/(3*Sin[2*x]^(5/2)) + (Cos[x]^3*Sin[x]^2)/(2*Sin[2*x]^(5/2
)) - (5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(2*Sqrt[2]*Sin[2*x]^(5/2)*T
an[x]^(5/2))
```


Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
```

```
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)\sin(x)}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx &= \frac{\sin^5(x)\int \frac{\csc^2(x)\sqrt{\tan(x)}}{\sin^2(x)-\sin(2x)}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\sin^5(x)\,\mathrm{Subst}\left(\int \frac{-1-x^2}{(2-x)x^{5/2}}\,dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\mathrm{Subst}\left(\int \frac{-1-x^4}{x^4\left(2-x^2\right)}\,dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\mathrm{Subst}\left(\int \left(-\frac{1}{2x^4}-\frac{1}{4x^2}+\frac{5}{4\left(-2+x^2\right)}\right)dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)}+\frac{\left(5\sin^5(x)\right)\mathrm{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{2\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}
\end{aligned}$$

Mathematica [C] time = 4.80679, size = 139, normalized size = 1.76

$$-\frac{\sqrt{\sin(2x)}\csc(x)(2\cos(x)-\sin(x))\left(-5\sqrt{\frac{\cos(x)}{2\cos(x)-2}}\sqrt{\tan\left(\frac{x}{2}\right)}\sec(x)\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right),-1\right)+\Pi\left(-\frac{2}{-1+\sqrt{5}};-\sin\right.\right.\right.}{16(2\cot(x)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] -(Csc[x]*(2*Cos[x] - Sin[x])*Sqrt[Sin[2*x]]*(-((3 + 2*Cot[x])*Csc[x])/3 - 5
*Sqrt[Cos[x]/(-2 + 2*Cos[x])]*(EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1] + El
lipticPi[-2/(-1 + Sqrt[5]), -ArcSin[1/Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1
 + Sqrt[5])/2, -ArcSin[1/Sqrt[Tan[x/2]]], -1])*Sec[x]*Sqrt[Tan[x/2]]))/(16*
(-1 + 2*Cot[x]))
```

Maple [C] time = 0.116, size = 396, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)
```

```
[Out] -1/1920*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(-140*(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x))^(
1/2),1/2*2^(1/2))*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)+240*(
tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+tan
(1/2*x))^(1/2),1/2*2^(1/2))*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/
2*x)+2^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^
(1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3)*(1+ta
n(1/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1
/2*x)^2-1))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+
3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)+40*(tan(1
/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^4+120*tan(1/2*x)^3*(tan(1/2*x)^3-t
an(1/2*x))^(1/2)-120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)-40*(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.65727, size = 432, normalized size = 5.47

$$-\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}(2\cos(x)+3\sin(x))-4\cos(x)^2-15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x)+3\,\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/192*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 -
15*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x)
) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos
(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) +
 4)/(cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^2 \sin(x)}{\left(\sin(x)^2 - \sin(2x)\right)\sin(2x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(cos(x)^2*sin(x)/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

3.636 $\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=95

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) + (Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (3*Cos
[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (3*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x
]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

Rubi [A] time = 0.575766, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4390, 898, 1262, 207}

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) + (Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (3*Cos
[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (3*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x
]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
```

```
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(x)\cos(2x)}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx &= \frac{\sin^5(x)\int \frac{\cos(2x)\csc^2(x)}{\left(\sin^2(x)-\sin(2x)\right)\sqrt{\tan(x)}}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\sin^5(x)\,\text{Subst}\left(\int \frac{-1+x^2}{(2-x)x^{7/2}}\,dx,x,\tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\text{Subst}\left(\int \frac{-1+x^4}{x^6\left(2-x^2\right)}\,dx,x,\sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\text{Subst}\left(\int \left(-\frac{1}{2x^6}-\frac{1}{4x^4}+\frac{3}{8x^2}-\frac{3}{8\left(-2+x^2\right)}\right)dx,x,\sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}-\frac{\left(3\sin^5(x)\right)\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx,x,\right.}{4\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}
\end{aligned}
$$

Mathematica [C] time = 15.3218, size = 188, normalized size = 1.98

$$\frac{1}{960}\sqrt{\sin(2x)}\sec(x)\left(-45\sqrt{2}\sqrt{\frac{\cos(x)}{\cos(x)-1}}\sqrt{\tan\left(\frac{x}{2}\right)}\text{EllipticF}\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right),-1\right)+20\cot^2(x)-114\cot(x)+24\,c$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Sec[x]*Sqrt[Sin[2*x]]*(-114*Cot[x] + 20*Cot[x]^2 + 24*Cot[x]*Csc[x]^2 - 45
*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1]
*Sqrt[Tan[x/2]] - 45*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticPi[-2/(-1 +
 Sqrt[5]), -ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]] - 45*Sqrt[2]*Sqrt[
Cos[x]/(-1 + Cos[x])]*EllipticPi[(-1 + Sqrt[5])/2, -ArcSin[1/Sqrt[Tan[x/2]]
], -1]*Sqrt[Tan[x/2]]))/960
```

Maple [C] time = 0.187, size = 761, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)
```

```
[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(1772*(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x))^(1
/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(1+tan(1/
2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)^2-4464*(tan(1/2*x)*(tan(1/2*x)^2
-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/
2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(1+tan(1/2*x))^(1/2)*(
-tan(1/2*x))^(1/2)*tan(1/2*x)^2+24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan
(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^6+3*2^(1/2)*(tan(1/
2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(t
an(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*sum((6*_alpha^3+7*_alpha^2+6*_alpha+1)*(
_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x)
)^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^
(1/2))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_
Z+1))*tan(1/2*x)^2-40*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(
1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^5-1920*tan(1/2*x)^4*(tan(1/2*x)^
3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)-24*tan(1/2*x)^4*(ta
```

```
n(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))
^(1/2)-1272*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1
/2*x)-1)*(1+tan(1/2*x)))^(1/2)-24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(
1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2+1272*(tan(1/2*x)^3
-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2
*x)^2+40*tan(1/2*x)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(tan(1
/2*x)*(tan(1/2*x)^2-1))^(1/2)+24*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))
^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
/(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.63272, size = 501, normalized size = 5.27

$$-\frac{45\left(\cos\left(x\right)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos\left(x\right)\sin\left(x\right)}(4\cos\left(x\right)+3\sin\left(x\right))+\frac{1}{2}\cos\left(x\right)^2+\frac{7}{2}\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\right)\sin\left(x\right)-45\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/1920*(45*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) +
 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 45*(cos(x)^2
- 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)
*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(57*cos(x)^2 + 10*cos(x)*sin(x) - 45)*sqr
t(cos(x)*sin(x)) + 268*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*cos(2*x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2x)\cos(x)^3}{\left(\sin(x)^2 - \sin(2x)\right)\sin(2x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate(cos(2*x)*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

3.637 $\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=30

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^{n+1}}{d(n+1)}$$

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^(1 + n)/(d*(1 + n))
```

Rubi [A] time = 0.0592225, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^(1 + n)/(d*(1 + n))
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx))^{1+n}}{d(1+n)}$$

Mathematica [A] time = 1.12582, size = 51, normalized size = 1.7

$$\frac{\sec(c+dx)(a\sin(2(c+dx))+2b)(a\sin(c+dx)+b\sec(c+dx))^n}{2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(2*b + a*Sin[2*(c + d*x)]
))/(2*d*(1 + n))
```

Maple [A] time = 0.078, size = 31, normalized size = 1.

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^{1+n}}{d(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] (b*sec(d*x+c)+a*sin(d*x+c))^(1+n)/d/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.09156, size = 150, normalized size = 5.

$$\frac{(a\cos(dx+c)\sin(dx+c)+b)\left(\frac{a\cos(dx+c)\sin(dx+c)+b}{\cos(dx+c)}\right)^{n}}{(dn+d)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] (a*cos(d*x + c)*sin(d*x + c) + b)*((a*cos(d*x + c)*sin(d*x + c) + b)/cos(d*
x + c))^n/((d*n + d)*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)\tan(dx+c) + a\cos(dx+c))(b\sec(dx+c) + a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c)*tan(d*x + c) + a*cos(d*x + c))*(b*sec(d*x + c) +
a*sin(d*x + c))^n, x)
```

3.638 $\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^4}{4d}$$

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^4/(4*d)
```

Rubi [A] time = 0.0441515, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^4/(4*d)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx))^4}{4d}$$

Mathematica [B] time = 6.55879, size = 938, normalized size = 36.08

$$\frac{a^4\cos(4c)\cos(4dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\cos^5(c+dx)}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}-\frac{4a^3\cos(2c$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

```
[Out] (8*b^4*Cos[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b
*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*
b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (a^4*Cos[4*c]*Cos[4*d*x]*Co
s[c + d*x]^5*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c
+ d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c
 + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (16*a*b^2*Cos[c + d*x]^3*Sec[c]*(3
*a*Cos[c] + 2*b*Sin[c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x]
 + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x]
+ 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (4*a^3*Cos[2*d*x]*Cos[c
 + d*x]^5*(a*Cos[2*c] + 4*b*Sin[2*c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(
a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos
[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (32*a*b^3
*Cos[c + d*x]^2*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[
c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c +
 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (32*a^3*b*Cos[c
 + d*x]^4*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*
x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x
] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (4*a^3*Cos[c + d*x]^5
*(-4*b*Cos[2*c] + a*Sin[2*c])*Sin[2*d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^
3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*
Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (a^4*C
os[c + d*x]^5*Sin[4*c]*Sin[4*d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Co
s[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c
 + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3)
```

Maple [B] time = 0.2, size = 137, normalized size = 5.3

$$\frac{a^4 (\sin (dx+c))^4}{4 d}+\frac{a^3 b (\sin (dx+c))^5}{d \cos (dx+c)}+\frac{a^3 b \cos (dx+c) (\sin (dx+c))^3}{d}+\frac{3 a^2 b^2 (\tan (dx+c))^2}{2 d}+\frac{a b^3 (\sin (dx+c))^3}{d (\cos (dx+c))^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] 1/4/d*a^4*sin(d*x+c)^4+1/d*a^3*b*sin(d*x+c)^5/cos(d*x+c)+1/d*a^3*b*cos(d*x+
c)*sin(d*x+c)^3+3/2/d*a^2*b^2*tan(d*x+c)^2+1/d*a*b^3*sin(d*x+c)^3/cos(d*x+c
```

```
)^3+1/d*a*b^3*tan(d*x+c)+1/4/d*b^4/cos(d*x+c)^4
```

Maxima [A] time = 0.963945, size = 32, normalized size = 1.23

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] 1/4*(b*sec(d*x + c) + a*sin(d*x + c))^4/d
```

Fricas [B] time = 2.62149, size = 292, normalized size = 11.23

$$\frac{8\,a^4\cos(dx+c)^8-16\,a^4\cos(dx+c)^6+5\,a^4\cos(dx+c)^4+48\,a^2b^2\cos(dx+c)^2+8\,b^4-32\left(a^3b\cos(dx+c)^5-a^3b\,\mathrm{co}\right.}{32\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a^4*cos(d*x + c)^8 - 16*a^4*cos(d*x + c)^6 + 5*a^4*cos(d*x + c)^4 +
 48*a^2*b^2*cos(d*x + c)^2 + 8*b^4 - 32*(a^3*b*cos(d*x + c)^5 - a^3*b*cos(d
*x + c)^3 - a*b^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [A] time = 65.903, size = 129, normalized size = 4.96

$$\begin{cases}\frac{a^4\sin^4(c+dx)}{4d}+\frac{a^3b\sin^3(c+dx)\sec(c+dx)}{d}+\frac{3a^2b^2\sin^2(c+dx)\sec^2(c+dx)}{2d}+\frac{ab^3\sin(c+dx)\sec^3(c+dx)}{d}+\frac{b^4\sec^4(c+dx)}{4d} & \text{for } d\neq 0\\ x\,(a\sin(c)+b\sec(c))^3\,(a\cos(c)+b\tan(c)\sec(c)) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**4/(4*d) + a**3*b*sin(c + d*x)**3*sec(c + d*x)
/d + 3*a**2*b**2*sin(c + d*x)**2*sec(c + d*x)**2/(2*d) + a*b**3*sin(c + d*x
)*sec(c + d*x)**3/d + b**4*sec(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) +
 b*sec(c))**3*(a*cos(c) + b*tan(c)*sec(c)), True))
```

Giac [B] time = 1.29462, size = 192, normalized size = 7.38

$$\frac{b^4 \tan(dx+c)^4 + 4\,ab^3 \tan(dx+c)^3 + 6\,a^2b^2 \tan(dx+c)^2 + 2\,b^4 \tan(dx+c)^2 + 4\,a^3b \tan(dx+c) + 4\,ab^3 \tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] 1/4*(b^4*tan(d*x + c)^4 + 4*a*b^3*tan(d*x + c)^3 + 6*a^2*b^2*tan(d*x + c)^2
 + 2*b^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) - (4*
a^3*b*tan(d*x + c)^3 + 2*a^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + a^4)/(
tan(d*x + c)^2 + 1)^2)/d
```

3.639 $\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^3}{3d}$$

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^3/(3*d)
```

Rubi [A] time = 0.0429013, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^3/(3*d)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx))^3}{3d}$$

Mathematica [A] time = 1.27789, size = 31, normalized size = 1.19

$$\frac{\sec^3(c+dx)(a\sin(2(c+dx))+2b)^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^3*(2*b + a*Sin[2*(c + d*x)])^3)/(24*d)
```

Maple [B] time = 0.172, size = 118, normalized size = 4.5

$$\frac{a^3 (\sin(dx+c))^3}{3d} + \frac{a^2 b (\sin(dx+c))^4}{d\cos(dx+c)} + \frac{a^2 b (\sin(dx+c))^2 \cos(dx+c)}{d} + \frac{a b^2 (\sin(dx+c))^3}{d (\cos(dx+c))^2} + \frac{a b^2 \sin(dx+c)}{d} + \frac{\quad}{3d (\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] 1/3/d*a^3*sin(d*x+c)^3+1/d*a^2*b*sin(d*x+c)^4/cos(d*x+c)+1/d*a^2*b*sin(d*x+
c)^2*cos(d*x+c)+1/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/d*a*b^2*sin(d*x+c)+1/
3/d*b^3/cos(d*x+c)^3
```

Maxima [A] time = 0.96345, size = 32, normalized size = 1.23

$$\frac{(b\sec(dx+c) + a\sin(dx+c))^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] 1/3*(b*sec(d*x + c) + a*sin(d*x + c))^3/d
```

Fricas [B] time = 2.60107, size = 217, normalized size = 8.35

$$-\frac{3 a^2 b \cos(dx+c)^4 - 3 a^2 b \cos(dx+c)^2 - b^3 + \left(a^3 \cos(dx+c)^5 - a^3 \cos(dx+c)^3 - 3 a b^2 \cos(dx+c)\right)\sin(dx+c)}{3 d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*b*cos(d*x + c)^4 - 3*a^2*b*cos(d*x + c)^2 - b^3 + (a^3*cos(d*x
+ c)^5 - a^3*cos(d*x + c)^3 - 3*a*b^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^3)
```

Sympy [A] time = 18.2675, size = 100, normalized size = 3.85

$$\begin{cases} \frac{a^3 \sin^3(c+dx)}{3d} + \frac{a^2 b \sin^2(c+dx) \sec(c+dx)}{d} + \frac{a b^2 \sin(c+dx) \sec^2(c+dx)}{d} + \frac{b^3 \sec^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x\left(a \sin(c) + b \sec(c)\right)^2 \left(a \cos(c) + b \tan(c) \sec(c)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**3/(3*d) + a**2*b*sin(c + d*x)**2*sec(c + d*x)
/d + a*b**2*sin(c + d*x)*sec(c + d*x)**2/d + b**3*sec(c + d*x)**3/(3*d), Ne
(d, 0)), (x*(a*sin(c) + b*sec(c))**2*(a*cos(c) + b*tan(c)*sec(c)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.640 $\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^2}{2d}$$

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^2/(2*d)
```

Rubi [A] time = 0.0280635, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[
c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^2/(2*d)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx))^2}{2d}$$

Mathematica [B] time = 0.0368757, size = 67, normalized size = 2.58

$$-\frac{a^2\cos^2(c+dx)}{2d}-\frac{ab\tan^{-1}(\tan(c+dx))}{d}+\frac{ab\tan(c+dx)}{d}+abx+\frac{b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x
]*Tan[c + d*x]),x]
```

```
[Out] a*b*x - (a*b*ArcTan[Tan[c + d*x]])/d - (a^2*Cos[c + d*x]^2)/(2*d) + (b^2*Se
c[c + d*x]^2)/(2*d) + (a*b*Tan[c + d*x])/d
```

Maple [B] time = 0.119, size = 57, normalized size = 2.2

$$\frac{1}{d}\left(-\frac{(\cos(dx+c))^2 a^2}{2}+ab(\tan(dx+c)-dx-c)+ab(dx+c)+\frac{b^2}{2(\cos(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/2*cos(d*x+c)^2*a^2+a*b*(tan(d*x+c)-d*x-c)+a*b*(d*x+c)+1/2*b^2/cos(d
*x+c)^2)
```

Maxima [A] time = 0.96683, size = 32, normalized size = 1.23

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="maxima")
```

```
[Out] 1/2*(b*sec(d*x + c) + a*sin(d*x + c))^2/d
```

Fricas [B] time = 2.39611, size = 150, normalized size = 5.77

$$-\frac{2a^2\cos(dx+c)^4-a^2\cos(dx+c)^2-4ab\cos(dx+c)\sin(dx+c)-2b^2}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*
x + c) - 2*b^2)/(d*cos(d*x + c)^2)
```

Sympy [A] time = 5.01278, size = 73, normalized size = 2.81

$$\begin{cases} \frac{a^2\sin^2(c+dx)}{2d} + \frac{ab\sin(c+dx)\sec(c+dx)}{d} + \frac{b^2\sec^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a\sin(c) + b\sec(c))(a\cos(c) + b\tan(c)\sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**2/(2*d) + a*b*sin(c + d*x)*sec(c + d*x)/d + b
**2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + b*sec(c))*(a*cos(c) +
b*tan(c)*sec(c)), True))
```

Giac [A] time = 1.18359, size = 61, normalized size = 2.35

$$\frac{b^2\tan(dx+c)^2 + 2\,ab\tan(dx+c) - \frac{a^2}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) - a^2/(tan(d*x + c)^2 + 1))/d
```

3.641 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(a\sin(c+dx)+b\sec(c+dx))}{d}$$

```
[Out] Log[b*Sec[c + d*x] + a*Sin[c + d*x]]/d
```

Rubi [A] time = 0.0484212, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4383}

$$\frac{\log(a\sin(c+dx)+b\sec(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x]),x]
```

```
[Out] Log[b*Sec[c + d*x] + a*Sin[c + d*x]]/d
```

Rule 4383

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], Ac
tivateTrig[u], x]}, Simp[q*Log[RemoveContent[ActivateTrig[y], x]], x] /;  !
FalseQ[q]] /;  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx=\frac{\log(b\sec(c+dx)+a\sin(c+dx))}{d}$$

Mathematica [A] time = 0.462226, size = 29, normalized size = 1.32

$$\frac{\log(a\sin(2(c+dx))+2b)-\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x]),x]
```

```
[Out] (-Log[Cos[c + d*x]] + Log[2*b + a*Sin[2*(c + d*x)]])/d
```

Maple [A] time = 0.09, size = 23, normalized size = 1.1

$$\frac{\ln\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x)
```

```
[Out] ln(b*sec(d*x+c)+a*sin(d*x+c))/d
```

Maxima [A] time = 0.965781, size = 30, normalized size = 1.36

$$\frac{\log\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="maxima")
```

```
[Out] log(b*sec(d*x + c) + a*sin(d*x + c))/d
```

Fricas [A] time = 2.76279, size = 85, normalized size = 3.86

$$\frac{\log\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\right)-\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="fricas")
```

```
[Out] (log(a*cos(d*x + c)*sin(d*x + c) + b) - log(-cos(d*x + c)))/d
```

Sympy [A] time = 7.52807, size = 63, normalized size = 2.86

$$\begin{cases} x\tan(c) & \text{for } a = 0 \wedge d = 0 \\ \frac{\log(\tan^2(c+dx)+1)}{2d} & \text{for } a = 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{a\sin(c)+b\sec(c)} & \text{for } d = 0 \\ \frac{\log\left(\sin(c+dx)+\frac{b\sec(c+dx)}{a}\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x)
```

```
[Out] Piecewise((x*tan(c), Eq(a, 0) & Eq(d, 0)), (log(tan(c + d*x)**2 + 1)/(2*d),
 Eq(a, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0)
), (log(sin(c + d*x) + b*sec(c + d*x)/a)/d, True))
```

Giac [A] time = 1.30741, size = 57, normalized size = 2.59

$$\frac{2\log\left(b\tan(dx+c)^2 + a\tan(dx+c) + b\right) - \log\left(\tan(dx+c)^2 + 1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(b*tan(d*x + c)^2 + a*tan(d*x + c) + b) - log(tan(d*x + c)^2 + 1)
)/d
```

3.642 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{d(a\sin(c+dx)+b\sec(c+dx))}$$

```
[Out] -(1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])))
```

Rubi [A] time = 0.0441496, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$-\frac{1}{d(a\sin(c+dx)+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x])^2,x]
```

```
[Out] -(1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])))
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx=-\frac{1}{d(b\sec(c+dx)+a\sin(c+dx))}$$

Mathematica [A] time = 0.310056, size = 27, normalized size = 1.12

$$-\frac{2\cos(c+dx)}{d(a\sin(2(c+dx))+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Cos[c + d*x])/(d*(2*b + a*Sin[2*(c + d*x)]))
```

Maple [A] time = 0.139, size = 25, normalized size = 1.

$$-\frac{1}{d\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(b*sec(d*x+c)+a*sin(d*x+c))
```

Maxima [A] time = 0.984597, size = 32, normalized size = 1.33

$$-\frac{1}{\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sec(d*x + c) + a*sin(d*x + c))*d)
```

Fricas [A] time = 2.35497, size = 72, normalized size = 3.

$$-\frac{\cos\left(dx+c\right)}{ad\cos\left(dx+c\right)\sin\left(dx+c\right)+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="fricas")
```

```
[Out] -cos(d*x + c)/(a*d*cos(d*x + c)*sin(d*x + c) + b*d)
```

Sympy [A] time = 22.5336, size = 49, normalized size = 2.04

$$\begin{cases} -\frac{1}{ad\sin(c+dx)+bd\sec(c+dx)} & \text{for } d\neq 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{(a\sin(c)+b\sec(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)**2,x)
```

```
[Out] Piecewise((-1/(a*d*sin(c + d*x) + b*d*sec(c + d*x)), Ne(d, 0)), (x*(a*cos(c
) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c))**2, True))
```

Giac [B] time = 1.23862, size = 146, normalized size = 6.08

$$\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\right)}{\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2\,b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="giac")
```

```
[Out] 2*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/
2*c) - b)/((b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan
(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)*b*d)
```

3.643 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2d(a\sin(c+dx)+b\sec(c+dx))^2}$$

```
[Out] -1/(2*d*(b*Sec[c + d*x] + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.045177, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$-\frac{1}{2d(a\sin(c+dx)+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x])^3,x]
```

```
[Out] -1/(2*d*(b*Sec[c + d*x] + a*Sin[c + d*x])^2)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx = -\frac{1}{2d(b\sec(c+dx)+a\sin(c+dx))^2}$$

Mathematica [A] time = 0.727144, size = 29, normalized size = 1.12

$$-\frac{2\cos^2(c+dx)}{d(a\sin(2(c+dx))+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*Cos[c + d*x]^2)/(d*(2*b + a*Sin[2*(c + d*x)])^2)
```

Maple [A] time = 0.171, size = 25, normalized size = 1.

$$-\frac{1}{2\,d\,(b\sec(dx+c)+a\sin(dx+c))^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2
```

Maxima [A] time = 0.982554, size = 32, normalized size = 1.23

$$-\frac{1}{2\,(b\sec(dx+c)+a\sin(dx+c))^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*sec(d*x + c) + a*sin(d*x + c))^2*d)
```

Fricas [B] time = 2.98695, size = 149, normalized size = 5.73

$$\frac{\cos(dx+c)^{2}}{2\left(a^{2}d\cos(dx+c)^{4}-a^{2}d\cos(dx+c)^{2}-2\,abd\cos(dx+c)\sin(dx+c)-b^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="fricas")
```

```
[Out] 1/2*cos(d*x + c)^2/(a^2*d*cos(d*x + c)^4 - a^2*d*cos(d*x + c)^2 - 2*a*b*d*c
os(d*x + c)*sin(d*x + c) - b^2*d)
```

Sympy [A] time = 62.2404, size = 80, normalized size = 3.08

$$\begin{cases} -\frac{1}{2a^2d\sin^2(c+dx)+4abd\sin(c+dx)\sec(c+dx)+2b^2d\sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{(a\sin(c)+b\sec(c))^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)**3,x)
```

```
[Out] Piecewise((-1/(2*a**2*d*sin(c + d*x)**2 + 4*a*b*d*sin(c + d*x)*sec(c + d*x)
 + 2*b**2*d*sec(c + d*x)**2), Ne(d, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a
*sin(c) + b*sec(c))**3, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.644 $\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}(\sin(a+bx)F(c,d,\cos(a+bx),r,s),x)$$

```
[Out] CannotIntegrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]
```

Rubi [A] time = 0.0128501, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
```

```
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cos[a + b*x]]/b)
```

Rubi steps

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = -\frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\cos(a+bx)\right)}{b}$$

Mathematica [A] time = 0.0426402, size = 0, normalized size = 0.

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
```

```
[Out] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]
```

Maple [A] time = 0.025, size = 0, normalized size = 0.

$$\int F(c,d,\cos(bx+a),r,s)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

```
[Out] int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\cos(bx+a),r,s)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F(c,d,\cos(bx+a),r,s)\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

```
[Out] Integral(F(c, d, cos(a + b*x), r, s)*sin(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\cos(bx+a),r,s)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

3.645 $\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$

Optimal. Leaf size=20

$$\text{CannotIntegrate}(\cos(a+bx)F(c,d,\sin(a+bx),r,s),x)$$

```
[Out] CannotIntegrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]
```

Rubi [A] time = 0.0127099, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]

[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Sin[a + b*x]]/b
```

Rubi steps

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\sin(a+bx)\right)}{b}$$

Mathematica [A] time = 0.0351948, size = 0, normalized size = 0.

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]

[Out] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]
```

Maple [A] time = 0.013, size = 0, normalized size = 0.

$$\int \cos \left(bx+a\right) F\left(c,d,\sin \left(bx+a\right),r,s\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

```
[Out] int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(c,d,\sin \left(bx+a\right),r,s\right) \cos \left(bx+a\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F\left(c,d,\sin \left(bx+a\right),r,s\right) \cos \left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\sin \left(a+bx\right),r,s) \cos \left(a+bx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, sin(a + b*x), r, s)*cos(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\sin(bx+a),r,s)\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="giac")
```

```
[Out] integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

3.646 $\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) \, dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\sec^2(a + bx)F(c, d, \tan(a + bx), r, s), x\right)$$

```
[Out] CannotIntegrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]
```

Rubi [A] time = 0.0161296, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Tan[a + b*x]]/b
```

Rubi steps

$$\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) \, dx = \frac{\text{Subst}\left(\int F(c, d, x, r, s) \, dx, x, \tan(a + bx)\right)}{b}$$

Mathematica [A] time = 0.0776369, size = 0, normalized size = 0.

$$\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]
```

Maple [A] time = 0.026, size = 0, normalized size = 0.

$$\int F\left(c,d,\tan\left(bx+a\right),r,s\right)\left(\sec\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

```
[Out] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(c,d,\tan\left(bx+a\right),r,s\right)\sec\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F\left(c,d,\tan\left(bx+a\right),r,s\right)\sec\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\tan\left(a+bx\right),r,s)\sec^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)**2,x)
```

```
[Out] Integral(F(c, d, tan(a + b*x), r, s)*sec(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F(c, d, \tan(bx + a), r, s) \sec(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

3.647 $\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\csc^2(a+bx)F(c,d,\cot(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]
```

Rubi [A] time = 0.0166877, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
```

```
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cot[a + b*x]]/b)
```

Rubi steps

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = -\frac{\text{Subst}(\int F(c,d,x,r,s)\,dx,x,\cot(a+bx))}{b}$$

Mathematica [A] time = 0.0760555, size = 0, normalized size = 0.

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
```

```
[Out] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]
```

Maple [A] time = 0.028, size = 0, normalized size = 0.

$$\int \left(\csc \left(bx+a\right)\right)^{2} F\left(c,d,\cot \left(bx+a\right),r,s\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

```
[Out] int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(c,d,\cot \left(bx+a\right),r,s\right) \csc \left(bx+a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F\left(c,d,\cot \left(bx+a\right),r,s\right) \csc \left(bx+a\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\cot (a+bx),r,s) \csc^{2}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*F(c,d,cot(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, cot(a + b*x), r, s)*csc(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F(c,d,\cot(bx+a),r,s)\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="giac")
```

```
[Out] integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

3.648 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\cos(x))}{b}$$

[Out] -(Log[a + b*Cos[x]]/b)

Rubi [A] time = 0.0223243, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a + b*Cos[x]),x]

[Out] -(Log[a + b*Cos[x]]/b)

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+b\cos(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}\\ &= -\frac{\log(a+b\cos(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0182928, size = 12, normalized size = 1.

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x]),x]
```

```
[Out] -(Log[a + b*Cos[x]]/b)
```

Maple [A] time = 0.007, size = 13, normalized size = 1.1

$$-\frac{\ln\left(a+b\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)),x)
```

```
[Out] -ln(a+b*cos(x))/b
```

Maxima [A] time = 0.968939, size = 16, normalized size = 1.33

$$-\frac{\log\left(b\cos\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] -log(b*cos(x) + a)/b
```

Fricas [A] time = 2.06818, size = 31, normalized size = 2.58

$$-\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] -log(-b*cos(x) - a)/b
```

Sympy [A] time = 0.386041, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b}+\cos(x)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x)
```

```
[Out] Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

Giac [A] time = 1.07803, size = 18, normalized size = 1.5

$$-\frac{\log\left(|b\cos(x)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -log(abs(b*cos(x) + a))/b
```

3.649 $\int (a+b\cos(x))^n \sin(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$$

```
[Out] -((a + b*Cos[x])^(1 + n)/(b*(1 + n)))
```

Rubi [A] time = 0.0245843, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 32}

$$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x])^n*Sin[x],x]
```

```
[Out] -((a + b*Cos[x])^(1 + n)/(b*(1 + n)))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(x))^n \sin(x)\,dx &= -\frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\cos(x)\right)}{b}\\ &= -\frac{(a+b\cos(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0322533, size = 19, normalized size = 0.95

$$-\frac{(a+b\cos(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x])^n*Sin[x],x]
```

```
[Out] -((a + b*Cos[x])^(1 + n)/(b + b*n))
```

Maple [A] time = 0.005, size = 21, normalized size = 1.1

$$-\frac{(a+b\cos(x))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x))^n*sin(x),x)
```

```
[Out] -(a+b*cos(x))^(1+n)/b/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07964, size = 59, normalized size = 2.95

$$-\frac{(b\cos(x)+a)(b\cos(x)+a)^{n}}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="fricas")
```

```
[Out] -(b*cos(x) + a)*(b*cos(x) + a)^n/(b*n + b)
```

Sympy [A] time = 2.5132, size = 63, normalized size = 3.15

$$\begin{cases} -\frac{\cos(x)}{a} & \text{for } b = 0 \wedge n = -1 \\ -a^n \cos(x) & \text{for } b = 0 \\ -\frac{\log\left(\frac{a}{b} + \cos(x)\right)}{b} & \text{for } n = -1 \\ -\frac{a(a+b\cos(x))^n}{bn+b} - \frac{b(a+b\cos(x))^n \cos(x)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))**n*sin(x),x)
```

```
[Out] Piecewise((-cos(x)/a, Eq(b, 0) & Eq(n, -1)), (-a**n*cos(x), Eq(b, 0)), (-lo
g(a/b + cos(x))/b, Eq(n, -1)), (-a*(a + b*cos(x))**n/(b*n + b) - b*(a + b*c
os(x))**n*cos(x)/(b*n + b), True))
```

Giac [A] time = 1.10384, size = 27, normalized size = 1.35

$$-\frac{(b\cos(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="giac")
```

```
[Out] -(b*cos(x) + a)^(n + 1)/(b*(n + 1))
```

3.650 $\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=5

$$-\sinh^{-1}(\cos(x))$$

```
[Out] -ArcSinh[Cos[x]]
```

Rubi [A] time = 0.022591, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3190, 215}

$$-\sinh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 + Cos[x]^2],x]

[Out] -ArcSinh[Cos[x]]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \cos(x)\right)\\ &= -\sinh^{-1}(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.0208214, size = 5, normalized size = 1.

$$-\sinh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] -ArcSinh[Cos[x]]
```

Maple [A] time = 0.013, size = 6, normalized size = 1.2

$$-\mathrm{Arcsinh}\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)^2)^(1/2),x)
```

```
[Out] -arcsinh(cos(x))
```

Maxima [A] time = 1.42716, size = 7, normalized size = 1.4

$$-\operatorname{arsinh}\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(cos(x))
```

Fricas [B] time = 2.1824, size = 112, normalized size = 22.4

$$\frac{1}{4}\,\log\left(8\,\cos\left(x\right)^{4} + 8\,\cos\left(x\right)^{2} - 4\left(2\,\cos\left(x\right)^{3} + \cos\left(x\right)\right)\sqrt{\cos\left(x\right)^{2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 + 8*cos(x)^2 - 4*(2*cos(x)^3 + cos(x))*sqrt(cos(x)^2 + 1
) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10103, size = 19, normalized size = 3.8

$$\log\left(\sqrt{\cos(x)^2+1}-\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(sqrt(cos(x)^2 + 1) - cos(x))
```

3.651 $\int \cos(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=5

$$-\sin(\cos(x))$$

```
[Out] -Sin[Cos[x]]
```

Rubi [A] time = 0.0090848, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4335, 2637}

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(\cos(x))\sin(x)\,dx &= -\operatorname{Subst}\left(\int \cos(x)\,dx, x, \cos(x)\right)\\ &= -\sin(\cos(x))\end{aligned}$$

Mathematica [A] time = 2.71167, size = 5, normalized size = 1.

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Maple [A] time = 0.008, size = 6, normalized size = 1.2

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

Maxima [A] time = 0.963075, size = 7, normalized size = 1.4

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
```

```
[Out] -sin(cos(x))
```

Fricas [B] time = 2.01724, size = 59, normalized size = 11.8

$$\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
```

```
[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

Sympy [A] time = 0.533696, size = 5, normalized size = 1.

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

Giac [A] time = 1.09581, size = 7, normalized size = 1.4

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="giac")
```

```
[Out] -sin(cos(x))
```

3.652 $\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx$

Optimal. Leaf size=28

$$\frac{\cos(x)}{4}-\frac{1}{2}\cos(x)\sin^2(\cos(x))-\frac{1}{4}\cos(\cos(x))\sin(\cos(x))$$

```
[Out] Cos[x]/4 - (Cos[Cos[x]]*Sin[Cos[x]])/4 - (Cos[x]*Sin[Cos[x]]^2)/2
```

Rubi [A] time = 0.0255663, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4335, 3443, 2635, 8}

$$\frac{\cos(x)}{4}-\frac{1}{2}\cos(x)\sin^2(\cos(x))-\frac{1}{4}\cos(\cos(x))\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
```

```
[Out] Cos[x]/4 - (Cos[Cos[x]]*Sin[Cos[x]])/4 - (Cos[x]*Sin[Cos[x]]^2)/2
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx &= -\operatorname{Subst}(\int x\cos(x)\sin(x)\,dx, x, \cos(x))\\ &= -\frac{1}{2}\cos(x)\sin^2(\cos(x)) + \frac{1}{2}\operatorname{Subst}\left(\int \sin^2(x)\,dx, x, \cos(x)\right)\\ &= -\frac{1}{4}\cos(\cos(x))\sin(\cos(x)) - \frac{1}{2}\cos(x)\sin^2(\cos(x)) + \frac{1}{4}\operatorname{Subst}(\int 1\,dx, x, \cos(x))\\ &= \frac{\cos(x)}{4} - \frac{1}{4}\cos(\cos(x))\sin(\cos(x)) - \frac{1}{2}\cos(x)\sin^2(\cos(x))\end{aligned}$$

Mathematica [A] time = 1.51206, size = 21, normalized size = 0.75

$$\frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
```

```
[Out] (Cos[x]*Cos[2*Cos[x]])/4 - Sin[2*Cos[x]]/8
```

Maple [A] time = 0.009, size = 23, normalized size = 0.8

$$\frac{(\cos(\cos(x)))^2\cos(x)}{2} - \frac{\cos(\cos(x))\sin(\cos(x))}{4} - \frac{\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)
```

```
[Out] 1/2*cos(cos(x))^2*cos(x)-1/4*cos(cos(x))*sin(cos(x))-1/4*cos(x)
```

Maxima [A] time = 0.96901, size = 23, normalized size = 0.82

$$\frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="maxima")
```

```
[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

Fricas [B] time = 2.05015, size = 219, normalized size = 7.82

$$\frac{1}{2}\cos(x)\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)^2 + \frac{1}{4}\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) - \frac{1}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^2 + 1/4*cos((tan(1/2*
x)^2 - 1)/(tan(1/2*x)^2 + 1))*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1)) -
1/4*cos(x)
```

Sympy [A] time = 7.89546, size = 34, normalized size = 1.21

$$-\frac{\sin^2(\cos(x))\cos(x)}{4} - \frac{\sin(\cos(x))\cos(\cos(x))}{4} + \frac{\cos(x)\cos^2(\cos(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)
```

```
[Out] -sin(cos(x))**2*cos(x)/4 - sin(cos(x))*cos(cos(x))/4 + cos(x)*cos(cos(x))**
2/4
```

Giac [A] time = 1.08866, size = 23, normalized size = 0.82

$$\frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="giac")
```

```
[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

3.653 $\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2}\sin(\cos(x))+\frac{1}{44}\sin(11\cos(x))+\frac{1}{52}\sin(13\cos(x))$$

```
[Out] -Sin[Cos[x]]/2 + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

Rubi [A] time = 0.0464201, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4335, 4354, 2637}

$$-\frac{1}{2}\sin(\cos(x))+\frac{1}{44}\sin(11\cos(x))+\frac{1}{52}\sin(13\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
```

```
[Out] -Sin[Cos[x]]/2 + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = -\text{Subst}\left(\int \cos(x)\sin^2(6x)\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{\cos(x)}{2} - \frac{1}{4}\cos(11x) - \frac{1}{4}\cos(13x)\right)dx, x, \cos(x)\right)$$
$$= \frac{1}{4}\text{Subst}(\int \cos(11x)\,dx, x, \cos(x)) + \frac{1}{4}\text{Subst}(\int \cos(13x)\,dx, x, \cos(x)) - \frac{1}{2}\text{S}$$
$$= -\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

Mathematica [A] time = 4.68363, size = 26, normalized size = 1.

$$-\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
```

```
[Out] -Sin[Cos[x]]/2 + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

Maple [A] time = 0.067, size = 21, normalized size = 0.8

$$-\frac{\sin(\cos(x))}{2} + \frac{\sin(11\cos(x))}{44} + \frac{\sin(13\cos(x))}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x)
```

```
[Out] -1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))
```

Maxima [A] time = 0.961604, size = 27, normalized size = 1.04

$$\frac{1}{52}\sin(13\cos(x)) + \frac{1}{44}\sin(11\cos(x)) - \frac{1}{2}\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

Fricas [B] time = 2.37467, size = 494, normalized size = 19.

$$-\frac{4}{143}\left(2816\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{12}-6912\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{10}+6048\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{8}-2240\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2}{\tan\left(\frac{1}{2}x\right)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="fricas")
```

```
[Out] -4/143*(2816*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^12 - 6912*cos((tan(
1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^10 + 6048*cos((tan(1/2*x)^2 - 1)/(tan(1/2
*x)^2 + 1))^8 - 2240*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^6 + 315*cos
((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^4 - 9*cos((tan(1/2*x)^2 - 1)/(tan(1
/2*x)^2 + 1))^2 - 18)*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

Sympy [B] time = 70.251, size = 54, normalized size = 2.08

$$-\frac{71\sin(\cos(x))\sin^2(6\cos(x))}{143}-\frac{72\sin(\cos(x))\cos^2(6\cos(x))}{143}+\frac{12\sin(6\cos(x))\cos(\cos(x))\cos(6\cos(x))}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))**2,x)
```

```
[Out] -71*sin(cos(x))*sin(6*cos(x))**2/143 - 72*sin(cos(x))*cos(6*cos(x))**2/143
+ 12*sin(6*cos(x))*cos(cos(x))*cos(6*cos(x))/143
```

Giac [A] time = 1.09441, size = 27, normalized size = 1.04

$$\frac{1}{52}\sin(13\cos(x))+\frac{1}{44}\sin(11\cos(x))-\frac{1}{2}\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

3.654 $\int \cos^3(x) \left(a+b\cos^2(x)\right)^3 \sin(x)\,dx$

Optimal. Leaf size=36

$$\frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}$$

```
[Out] (a*(a + b*Cos[x]^2)^4)/(8*b^2) - (a + b*Cos[x]^2)^5/(10*b^2)
```

Rubi [A] time = 0.0872477, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4335, 266, 43}

$$\frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

```
[Out] (a*(a + b*Cos[x]^2)^4)/(8*b^2) - (a + b*Cos[x]^2)^5/(10*b^2)
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx &= -\operatorname{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\cos(x)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^3\,dx,x,\cos^2(x)\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\cos^2(x)\right)\right)\\ &= \frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}\end{aligned}$$

Mathematica [B] time = 0.301158, size = 137, normalized size = 3.81

$$\frac{1}{32}\left(-12a^2b\cos^4(x)-4a^2b\cos(3x)\cos^3(x)-4a^3\cos(2x)-a^3\cos(4x)-8ab^2\cos^6(x)-\frac{1}{32}ab^2(48\cos(2x)+36\cos(4x)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

```
[Out] (-12*a^2*b*Cos[x]^4 - 8*a*b^2*Cos[x]^6 - 2*b^3*Cos[x]^8 - 4*a^3*Cos[2*x] -
4*a^2*b*Cos[x]^3*Cos[3*x] - a^3*Cos[4*x] - (a*b^2*(48*Cos[2*x] + 36*Cos[4*x
] + 16*Cos[6*x] + 3*Cos[8*x]))/32 - (b^3*(140*Cos[2*x] + 100*Cos[4*x] + 50*
Cos[6*x] + 15*Cos[8*x] + 2*Cos[10*x]))/320)/32
```

Maple [A] time = 0.009, size = 40, normalized size = 1.1

$$-\frac{b^3(\cos(x))^{10}}{10}-\frac{3ab^2(\cos(x))^8}{8}-\frac{a^2b(\cos(x))^6}{2}-\frac{(\cos(x))^4a^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x)
```

```
[Out] -1/10*b^3*cos(x)^10-3/8*a*b^2*cos(x)^8-1/2*a^2*b*cos(x)^6-1/4*cos(x)^4*a^3
```

Maxima [B] time = 0.980933, size = 139, normalized size = 3.86

$$\frac{1}{10} b^3 \sin(x)^{10} - \frac{1}{8} \left(3 ab^2 + 4 b^3\right) \sin(x)^8 + \frac{1}{2} \left(a^2 b + 3 ab^2 + 2 b^3\right) \sin(x)^6 - \frac{1}{4} \left(a^3 + 6 a^2 b + 9 ab^2 + 4 b^3\right) \sin(x)^4 + \frac{1}{2} \left(a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sin(x)^10 - 1/8*(3*a*b^2 + 4*b^3)*sin(x)^8 + 1/2*(a^2*b + 3*a*b^2
+ 2*b^3)*sin(x)^6 - 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*sin(x)^4 + 1/2*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(x)^2
```

Fricas [A] time = 2.21531, size = 111, normalized size = 3.08

$$-\frac{1}{10} b^3 \cos(x)^{10} - \frac{3}{8} ab^2 \cos(x)^8 - \frac{1}{2} a^2 b \cos(x)^6 - \frac{1}{4} a^3 \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="fricas")
```

```
[Out] -1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*cos
(x)^4
```

Sympy [B] time = 16.4708, size = 97, normalized size = 2.69

$$\frac{a^3 \sin^4(x)}{4} + \frac{a^3 \sin^2(x) \cos^2(x)}{2} + \frac{a^2 b \sin^6(x)}{2} + \frac{3a^2 b \sin^4(x) \cos^2(x)}{2} + \frac{3a^2 b \sin^2(x) \cos^4(x)}{2} - \frac{3ab^2 \cos^8(x)}{8} - \frac{b^3 \cos^1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*(a+b*cos(x)**2)**3*sin(x),x)
```

```
[Out] a**3*sin(x)**4/4 + a**3*sin(x)**2*cos(x)**2/2 + a**2*b*sin(x)**6/2 + 3*a**2
*b*sin(x)**4*cos(x)**2/2 + 3*a**2*b*sin(x)**2*cos(x)**4/2 - 3*a*b**2*cos(x)
**8/8 - b**3*cos(x)**10/10
```

Giac [A] time = 1.08834, size = 53, normalized size = 1.47

$$-\frac{1}{10}\,b^3\cos\left(x\right)^{10}-\frac{3}{8}\,ab^2\cos\left(x\right)^8-\frac{1}{2}\,a^2b\cos\left(x\right)^6-\frac{1}{4}\,a^3\cos\left(x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="giac")
```

```
[Out] -1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*cos
(x)^4
```

3.655 $\int \sin(3x)\sin(\cos(3x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{3}\cos(\cos(3x))$$

[Out] Cos[Cos[3*x]]/3

Rubi [A] time = 0.0107249, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4335, 2638}

$$\frac{1}{3}\cos(\cos(3x))$$

Antiderivative was successfully verified.

[In] Int[Sin[3*x]*Sin[Cos[3*x]],x]

[Out] Cos[Cos[3*x]]/3

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(3x)\sin(\cos(3x))\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \sin(x)\,dx, x, \cos(3x)\right)\right)\\ &= \frac{1}{3}\cos(\cos(3x))\end{aligned}$$

Mathematica [A] time = 2.75389, size = 9, normalized size = 1.

$$\frac{1}{3}\cos(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]*Sin[Cos[3*x]],x]
```

```
[Out] Cos[Cos[3*x]]/3
```

Maple [A] time = 0.006, size = 8, normalized size = 0.9

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(cos(3*x)),x)
```

```
[Out] 1/3*cos(cos(3*x))
```

Maxima [A] time = 0.960353, size = 9, normalized size = 1.

$$\frac{1}{3}\cos(\cos(3x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="maxima")
```

```
[Out] 1/3*cos(cos(3*x))
```

Fricas [B] time = 1.99499, size = 65, normalized size = 7.22

$$\frac{1}{3}\cos\left(\frac{\tan\left(\frac{3}{2}x\right)^2-1}{\tan\left(\frac{3}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="fricas")
```

```
[Out] 1/3*cos((tan(3/2*x)^2 - 1)/(tan(3/2*x)^2 + 1))
```

Sympy [A] time = 0.532767, size = 7, normalized size = 0.78

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x)
```

```
[Out] cos(cos(3*x))/3
```

Giac [A] time = 1.10921, size = 9, normalized size = 1.

$$\frac{1}{3}\cos(\cos(3x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="giac")
```

```
[Out] 1/3*cos(cos(3*x))
```

3.656 $\int e^{\cos(1+3x)} \cos(1+3x) \sin(1+3x) \, dx$

Optimal. Leaf size=31

$$\frac{1}{3}e^{\cos(3x+1)} - \frac{1}{3}e^{\cos(3x+1)}\cos(3x+1)$$

```
[Out] E^Cos[1 + 3*x]/3 - (E^Cos[1 + 3*x]*Cos[1 + 3*x])/3
```

Rubi [A] time = 0.0226498, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4335, 2176, 2194}

$$\frac{1}{3}e^{\cos(3x+1)} - \frac{1}{3}e^{\cos(3x+1)}\cos(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
```

```
[Out] E^Cos[1 + 3*x]/3 - (E^Cos[1 + 3*x]*Cos[1 + 3*x])/3
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx = -\left(\frac{1}{3}\,\mathrm{Subst}\left(\int e^x x\,dx, x, \cos(1+3x)\right)\right)$$
$$= -\frac{1}{3}e^{\cos(1+3x)}\cos(1+3x) + \frac{1}{3}\,\mathrm{Subst}\left(\int e^x\,dx, x, \cos(1+3x)\right)$$
$$= \frac{1}{3}e^{\cos(1+3x)} - \frac{1}{3}e^{\cos(1+3x)}\cos(1+3x)$$

Mathematica [A] time = 0.121238, size = 24, normalized size = 0.77

$$\frac{2}{3}\sin^2\left(\frac{1}{2}(3x+1)\right)e^{\cos(3x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
```

```
[Out] (2*E^Cos[1 + 3*x]*Sin[(1 + 3*x)/2]^2)/3
```

Maple [A] time = 0.005, size = 26, normalized size = 0.8

$$\frac{e^{\cos(1+3x)}}{3} - \frac{e^{\cos(1+3x)}\cos(1+3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)
```

```
[Out] 1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)
```

Maxima [A] time = 0.976038, size = 23, normalized size = 0.74

$$-\frac{1}{3}(\cos(3x+1)-1)e^{(\cos(3x+1))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="maxima")
```

```
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

Fricas [A] time = 1.90291, size = 57, normalized size = 1.84

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

Sympy [A] time = 0.819937, size = 26, normalized size = 0.84

$$-\frac{e^{\cos(3x+1)}\cos(3x+1)}{3}+\frac{e^{\cos(3x+1)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)
```

```
[Out] -exp(cos(3*x + 1))*cos(3*x + 1)/3 + exp(cos(3*x + 1))/3
```

Giac [A] time = 1.10971, size = 23, normalized size = 0.74

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="giac")
```

```
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

3.657 $\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx$

Optimal. Leaf size=9

$$-\frac{1}{3}\sin^{-1}\left(\cos^3(x)\right)$$

```
[Out] -ArcSin[Cos[x]^3]/3
```

Rubi [A] time = 0.0718241, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4335, 275, 216}

$$-\frac{1}{3}\sin^{-1}\left(\cos^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

```
[Out] -ArcSin[Cos[x]^3]/3
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-x^6}}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cos^3(x)\right)\right)\\
&= -\frac{1}{3}\sin^{-1}\left(\cos^3(x)\right)
\end{aligned}
$$

Mathematica [C] time = 2.25209, size = 162, normalized size = 18.

$$
-\frac{i\sin(x)\cos^2(x)\sqrt{1-\frac{2i\tan^2(x)}{\sqrt{3}-3i}}\sqrt{1+\frac{2i\tan^2(x)}{\sqrt{3}+3i}}\,\Pi\left(\frac{3}{2}+\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{3}}}\tan(x)\right)|\frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{-\frac{i}{\sqrt{3}-3i}}\sqrt{1-\cos^6(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

```
[Out] ((-I)*Cos[x]^2*EllipticPi[3/2 + (I/2)*Sqrt[3], I*ArcSinh[Sqrt[(-2*I)/(-3*I
+ Sqrt[3])]*Tan[x]], (3*I - Sqrt[3])/(3*I + Sqrt[3])]*Sin[x]*Sqrt[1 - ((2*I
)*Tan[x]^2)/(-3*I + Sqrt[3])]*Sqrt[1 + ((2*I)*Tan[x]^2)/(3*I + Sqrt[3])])/(
Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt[3])]*Sqrt[1 - Cos[x]^6])
```

Maple [F] time = 0.518, size = 0, normalized size = 0.

$$
\int (\cos(x))^2\sin(x)\frac{1}{\sqrt{1-(\cos(x))^6}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
```

```
[Out] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
```

Maxima [B] time = 1.44395, size = 24, normalized size = 2.67

$$\frac{1}{3}\arctan\left(\frac{\sqrt{-\cos(x)^6+1}}{\cos(x)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(sqrt(-cos(x)^6 + 1)/cos(x)^3)
```

Fricas [B] time = 2.85505, size = 82, normalized size = 9.11

$$\frac{1}{6}\arctan\left(\frac{2\sqrt{-\cos(x)^6+1}\cos(x)^3}{2\cos(x)^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(2*sqrt(-cos(x)^6 + 1)*cos(x)^3/(2*cos(x)^6 - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(1-cos(x)**6)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09013, size = 9, normalized size = 1.

$$-\frac{1}{3}\arcsin\left(\cos(x)^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*arcsin(cos(x)^3)
```

3.658 $\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx$

Optimal. Leaf size=71

$$\frac{2(1-5\cos(x))^{9/2}}{28125}-\frac{8(1-5\cos(x))^{7/2}}{21875}-\frac{88(1-5\cos(x))^{5/2}}{15625}+\frac{64(1-5\cos(x))^{3/2}}{3125}+\frac{1152\sqrt{1-5\cos(x)}}{3125}$$

```
[Out] (1152*Sqrt[1 - 5*Cos[x]])/3125 + (64*(1 - 5*Cos[x])^(3/2))/3125 - (88*(1 -
5*Cos[x])^(5/2))/15625 - (8*(1 - 5*Cos[x])^(7/2))/21875 + (2*(1 - 5*Cos[x])
^(9/2))/28125
```

Rubi [A] time = 0.0658991, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2668, 697}

$$\frac{2(1-5\cos(x))^{9/2}}{28125}-\frac{8(1-5\cos(x))^{7/2}}{21875}-\frac{88(1-5\cos(x))^{5/2}}{15625}+\frac{64(1-5\cos(x))^{3/2}}{3125}+\frac{1152\sqrt{1-5\cos(x)}}{3125}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

```
[Out] (1152*Sqrt[1 - 5*Cos[x]])/3125 + (64*(1 - 5*Cos[x])^(3/2))/3125 - (88*(1 -
5*Cos[x])^(5/2))/15625 - (8*(1 - 5*Cos[x])^(7/2))/21875 + (2*(1 - 5*Cos[x])
^(9/2))/28125
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx = \frac{\text{Subst}\left(\int \frac{(25-x^2)^2}{\sqrt{1+x}}\,dx, x, -5\cos(x)\right)}{3125}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{576}{\sqrt{1+x}} + 96\sqrt{1+x} - 44(1+x)^{3/2} - 4(1+x)^{5/2} + (1+x)^{7/2}\right) dx, x, -5\cos(x)\right)}{3125}$$
$$= \frac{1152\sqrt{1-5\cos(x)}}{3125} + \frac{64(1-5\cos(x))^{3/2}}{3125} - \frac{88(1-5\cos(x))^{5/2}}{15625} - \frac{8(1-5\cos(x))^{7/2}}{21875} + \frac{2(1-5\cos}{2812}$$

Mathematica [A] time = 0.15787, size = 59, normalized size = 0.83

$$\frac{180607\left(\sqrt{1-5\cos(x)}-1\right)}{562500} + \sqrt{1-5\cos(x)}\left(-\frac{6772\cos(x)}{196875} - \frac{2227\cos(2x)}{39375} + \frac{4\cos(3x)}{1575} + \frac{1}{180}\cos(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

```
[Out] (180607*(-1 + Sqrt[1 - 5*Cos[x]]))/562500 + Sqrt[1 - 5*Cos[x]]*((-6772*Cos[
x])/196875 - (2227*Cos[2*x])/39375 + (4*Cos[3*x])/1575 + Cos[4*x]/180)
```

Maple [A] time = 1.081, size = 49, normalized size = 0.7

$$\frac{32}{984375}\sqrt{10\,(\sin(x/2))^2-4}\left(21875\,(\sin(x/2))^8 - 46250\,(\sin(x/2))^6 + 17175\,(\sin(x/2))^4 + 9160\,(\sin(x/2))^2 + 7328\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(1-5*cos(x))^(1/2),x)
```

```
[Out] 32/984375*(10*sin(1/2*x)^2-4)^(1/2)*(21875*sin(1/2*x)^8-46250*sin(1/2*x)^6+
17175*sin(1/2*x)^4+9160*sin(1/2*x)^2+7328)
```

Maxima [A] time = 0.985314, size = 69, normalized size = 0.97

$$\frac{2}{28125}(-5\cos(x)+1)^{\frac{9}{2}} - \frac{8}{21875}(-5\cos(x)+1)^{\frac{7}{2}} - \frac{88}{15625}(-5\cos(x)+1)^{\frac{5}{2}} + \frac{64}{3125}(-5\cos(x)+1)^{\frac{3}{2}} + \frac{1152}{3125}\sqrt{-5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/28125*(-5*cos(x) + 1)^(9/2) - 8/21875*(-5*cos(x) + 1)^(7/2) - 88/15625*(-
5*cos(x) + 1)^(5/2) + 64/3125*(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos
(x) + 1)
```

Fricas [A] time = 2.1865, size = 140, normalized size = 1.97

$$\frac{2}{984375}\left(21875\cos\left(x\right)^{4}+5000\cos\left(x\right)^{3}-77550\cos\left(x\right)^{2}-20680\cos\left(x\right)+188603\right)\sqrt{-5\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/984375*(21875*cos(x)^4 + 5000*cos(x)^3 - 77550*cos(x)^2 - 20680*cos(x) +
188603)*sqrt(-5*cos(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(1-5*cos(x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11028, size = 101, normalized size = 1.42

$$\frac{2}{28125}\left(5\cos\left(x\right)-1\right)^{4}\sqrt{-5\cos\left(x\right)+1}+\frac{8}{21875}\left(5\cos\left(x\right)-1\right)^{3}\sqrt{-5\cos\left(x\right)+1}-\frac{88}{15625}\left(5\cos\left(x\right)-1\right)^{2}\sqrt{-5\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/28125*(5*cos(x) - 1)^4*sqrt(-5*cos(x) + 1) + 8/21875*(5*cos(x) - 1)^3*sqr
t(-5*cos(x) + 1) - 88/15625*(5*cos(x) - 1)^2*sqrt(-5*cos(x) + 1) + 64/3125*
(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos(x) + 1)
```

3.659 $\int e^{n\cos(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=18

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

```
[Out] -(E^(n*Cos[a + b*x])/(b*n))
```

Rubi [A] time = 0.0144725, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4335, 2194}

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
```

```
[Out] -(E^(n*Cos[a + b*x])/(b*n))
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(a+bx)}\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{e^{n\cos(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.0510995, size = 18, normalized size = 1.

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
```

```
[Out] -(E^(n*Cos[a + b*x])/(b*n))
```

Maple [A] time = 0.005, size = 18, normalized size = 1.

$$-\frac{e^{n\cos(bx+a)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(b*x+a),x)
```

```
[Out] -exp(n*cos(b*x+a))/b/n
```

Maxima [A] time = 0.984035, size = 23, normalized size = 1.28

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -e^(n*cos(b*x + a))/(b*n)
```

Fricas [A] time = 2.00534, size = 36, normalized size = 2.

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -e^(n*cos(b*x + a))/(b*n)
```

Sympy [A] time = 0.512365, size = 39, normalized size = 2.17

$$\begin{cases} x \sin(a) & \text{for } b = 0 \wedge n = 0 \\ x e^{n \cos(a)} \sin(a) & \text{for } b = 0 \\ -\frac{\cos(a+bx)}{b} & \text{for } n = 0 \\ -\frac{e^{n \cos(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cos(a))*sin(a), Eq(b, 0
)), (-cos(a + b*x)/b, Eq(n, 0)), (-exp(n*cos(a + b*x))/(b*n), True))
```

Giac [A] time = 1.09581, size = 23, normalized size = 1.28

$$-\frac{e^{(n \cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -e^(n*cos(b*x + a))/(b*n)
```

3.660 $\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

```
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

Rubi [A] time = 0.0149893, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4335, 2194}

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]
```

```
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx &= -\frac{\operatorname{Subst}\left(\int e^{nx}\,dx, x, \cos(c(a+bx))\right)}{bc}\\ &= -\frac{e^{n\cos(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.236888, size = 23, normalized size = 1.

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]
```

```
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

Maple [A] time = 0.014, size = 24, normalized size = 1.

$$-\frac{\mathrm{e}^{n\cos(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)
```

```
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 0.98337, size = 31, normalized size = 1.35

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 1.88868, size = 45, normalized size = 1.96

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Sympy [A] time = 9.43634, size = 54, normalized size = 2.35

$$\begin{cases} xe^{n\cos(ac)}\sin(ac) & \text{for}\ b=0 \\ 0 & \text{for}\ c=0 \\ \begin{cases} x\sin(ac) & \text{for}\ b=0 \\ 0 & \text{for}\ c=0 \\ -\frac{\cos(ac+bcx)}{bc} & \text{otherwise} \end{cases} & \text{for}\ n=0 \\ -\frac{e^{n\cos(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)
```

```
[Out] Piecewise((x*exp(n*cos(a*c))*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (Piecewise
((x*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (-cos(a*c + b*c*x)/(b*c), True)), E
q(n, 0)), (-exp(n*cos(a*c + b*c*x))/(b*c*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cos(bcx+ac))}\sin((bx+a)c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos(b*c*x + a*c))*sin((b*x + a)*c), x)
```

3.661 $\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx$

Optimal. Leaf size=24

$$-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

```
[Out] -(E^(n*Cos[a*c + b*c*x])/(b*c*n))
```

Rubi [A] time = 0.0144949, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4335, 2194}

$$-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
```

```
[Out] -(E^(n*Cos[a*c + b*c*x])/(b*c*n))
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cos(ac+bcx)\right)}{bc}\\ &= -\frac{e^{n\cos(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.0425428, size = 23, normalized size = 0.96

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
```

```
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

Maple [A] time = 0.01, size = 24, normalized size = 1.

$$-\frac{e^{n\cos(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)
```

```
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 0.960272, size = 31, normalized size = 1.29

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 2.0295, size = 45, normalized size = 1.88

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Sympy [A] time = 2.76905, size = 51, normalized size = 2.12

$$\begin{cases} 0 & \text{for}\: b = 0 \wedge c = 0 \wedge n = 0 \\ x e^{n \cos{\left(a c \right)}} \sin{\left(a c \right)} & \text{for}\: b = 0 \\ 0 & \text{for}\: c = 0 \\ - \frac{\cos{\left(a c + b c x \right)}}{b c} & \text{for}\: n = 0 \\ - \frac{e^{n \cos{\left(a c + b c x \right)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*cos(a*c))*sin(a*c),
 Eq(b, 0)), (0, Eq(c, 0)), (-cos(a*c + b*c*x)/(b*c), Eq(n, 0)), (-exp(n*cos
(a*c + b*c*x))/(b*c*n), True))
```

Giac [A] time = 1.14893, size = 31, normalized size = 1.29

$$-\frac{e^{\left(n \cos\left(bcx + ac\right)\right)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

3.662 $\int e^{n\cos(a+bx)}\tan(a+bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\text{ExpIntegralEi}(n\cos(a+bx))}{b}$$

```
[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

Rubi [A] time = 0.0215174, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\text{Ei}(n\cos(a+bx))}{b}$$

Mathematica [A] time = 0.0350774, size = 14, normalized size = 1.

$$-\frac{\text{ExpIntegralEi}(n\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

Maple [A] time = 0.01, size = 16, normalized size = 1.1

$$\frac{\text{Ei}\left(1, -n\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*tan(b*x+a),x)
```

```
[Out] 1/b*Ei(1,-n*cos(b*x+a))
```

Maxima [A] time = 1.03419, size = 19, normalized size = 1.36

$$-\frac{\text{Ei}\left(n\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*x + a))/b
```

Fricas [A] time = 2.08915, size = 31, normalized size = 2.21

$$-\frac{\mathrm{Ei}\left(n\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] -Ei(n*cos(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x)
```

```
[Out] Integral(exp(n*cos(a + b*x))*tan(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cos(bx+a))}\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos(b*x + a))*tan(b*x + a), x)
```

3.663 $\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx$

Optimal. Leaf size=19

$$-\frac{\text{ExpIntegralEi}(n\cos(c(a+bx)))}{bc}$$

```
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Rubi [A] time = 0.0219697, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx = -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cos(c(a+bx))\right)}{bc}$$
$$= -\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}$$

Mathematica [A] time = 0.0607347, size = 19, normalized size = 1.

$$-\frac{\text{ExpIntegralEi}(n\cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Maple [A] time = 0.02, size = 22, normalized size = 1.2

$$\frac{\text{Ei}\left(1, -n\cos\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
```

```
[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))
```

Maxima [A] time = 1.08073, size = 27, normalized size = 1.42

$$-\frac{\text{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Fricas [A] time = 2.3288, size = 42, normalized size = 2.21

$$-\frac{\mathrm{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\cos\left(bcx+ac\right)\right)}\tan\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos(b*c*x + a*c))*tan((b*x + a)*c), x)
```

3.664 $\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx$

Optimal. Leaf size=20

$$-\frac{\text{ExpIntegralEi}(n\cos(ac+bcx))}{bc}$$

```
[Out] -(ExpIntegralEi[n*Cos[a*c + b*c*x]]/(b*c))
```

Rubi [A] time = 0.0231347, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n\cos(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a*c + b*c*x]]/(b*c))
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx=-\frac{\text{Subst}\left(\int\frac{e^{nx}}{x}\,dx,x,\cos(ac+bcx)\right)}{bc}$$
$$=-\frac{\text{Ei}(n\cos(ac+bcx))}{bc}$$

Mathematica [A] time = 0.0578868, size = 19, normalized size = 0.95

$$-\frac{\text{ExpIntegralEi}(n\cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]

[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Maple [A] time = 0.013, size = 22, normalized size = 1.1

$$\frac{\text{Ei}\left(1,-n\cos\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)

[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))
```

Maxima [A] time = 1.07013, size = 27, normalized size = 1.35

$$-\frac{\text{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Fricas [A] time = 1.88893, size = 42, normalized size = 2.1

$$-\frac{\operatorname{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cos((bx+a)c))}\tan\left(bcx+ac\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos((b*x + a)*c))*tan(b*c*x + a*c), x)
```

3.665 $\int \frac{\cos(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sin(x))}{b}$$

```
[Out] Log[a + b*Sin[x]]/b
```

Rubi [A] time = 0.0218965, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\sin(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}\\ &= \frac{\log(a+b\sin(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0062776, size = 11, normalized size = 1.

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Maple [A] time = 0.014, size = 12, normalized size = 1.1

$$\frac{\ln(a+b\sin(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)),x)
```

```
[Out] ln(a+b*sin(x))/b
```

Maxima [A] time = 0.944746, size = 15, normalized size = 1.36

$$\frac{\log(b\sin(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] log(b*sin(x) + a)/b
```

Fricas [A] time = 2.07247, size = 28, normalized size = 2.55

$$\frac{\log(b\sin(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] log(b*sin(x) + a)/b
```

Sympy [A] time = 0.359504, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log\left(\frac{a}{b}+\sin\left(x\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{\sin\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x)
```

```
[Out] Piecewise((log(a/b + sin(x))/b, Ne(b, 0)), (sin(x)/a, True))
```

Giac [A] time = 1.09398, size = 16, normalized size = 1.45

$$\frac{\log\left(|b\sin\left(x\right)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(x) + a))/b
```

3.666 $\int \cos(x)(a+b\sin(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$$

```
[Out] (a + b*Sin[x])^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0216845, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 32}

$$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(a + b*Sin[x])^n,x]
```

```
[Out] (a + b*Sin[x])^(1 + n)/(b*(1 + n))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(x)(a+b\sin(x))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\sin(x)\right)}{b}\\ &= \frac{(a+b\sin(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0204927, size = 18, normalized size = 0.95

$$\frac{(a+b\sin(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(a + b*Sin[x])^n,x]
```

```
[Out] (a + b*Sin[x])^(1 + n)/(b + b*n)
```

Maple [A] time = 0.007, size = 20, normalized size = 1.1

$$\frac{(a+b\sin(x))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(a+b*sin(x))^n,x)
```

```
[Out] (a+b*sin(x))^(1+n)/b/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06725, size = 58, normalized size = 3.05

$$\frac{(b\sin(x)+a)(b\sin(x)+a)^{n}}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="fricas")
```

```
[Out] (b*sin(x) + a)*(b*sin(x) + a)^n/(b*n + b)
```

Sympy [A] time = 2.56974, size = 56, normalized size = 2.95

$$\begin{cases} \frac{\sin(x)}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n \sin(x) & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(x)\right)}{b} & \text{for } n = -1 \\ \frac{a(a+b\sin(x))^n}{bn+b} + \frac{b(a+b\sin(x))^n \sin(x)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))**n,x)
```

```
[Out] Piecewise((sin(x)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sin(x), Eq(b, 0)), (log(a
/b + sin(x))/b, Eq(n, -1)), (a*(a + b*sin(x))**n/(b*n + b) + b*(a + b*sin(x
))**n*sin(x)/(b*n + b), True))
```

Giac [A] time = 1.08492, size = 26, normalized size = 1.37

$$\frac{(b\sin(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="giac")
```

```
[Out] (b*sin(x) + a)^(n + 1)/(b*(n + 1))
```

3.667 $\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} \, dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\sin(x))$$

```
[Out] ArcSinh[Sin[x]]
```

Rubi [A] time = 0.0227818, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3190, 215}

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \sin(x)\right) \\ &= \sinh^{-1}(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.0078914, size = 3, normalized size = 1.

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]]
```

Maple [A] time = 0.016, size = 4, normalized size = 1.3

$$\text{Arcsinh}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1+sin(x)^2)^(1/2),x)
```

```
[Out] arcsinh(sin(x))
```

Maxima [A] time = 1.4372, size = 4, normalized size = 1.33

$$\text{arsinh}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(sin(x))
```

Fricas [B] time = 2.17918, size = 119, normalized size = 39.67

$$\frac{1}{4}\log\left(8\cos\left(x\right)^{4}-4\left(2\cos\left(x\right)^{2}-3\right)\sqrt{-\cos\left(x\right)^{2}+2}\sin\left(x\right)-24\cos\left(x\right)^{2}+17\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 - 4*(2*cos(x)^2 - 3)*sqrt(-cos(x)^2 + 2)*sin(x) - 24*cos
(x)^2 + 17)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07965, size = 22, normalized size = 7.33

$$-\log\left(\sqrt{\sin(x)^2+1}-\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(sin(x)^2 + 1) - sin(x))
```

3.668 $\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx$

Optimal. Leaf size=7

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

```
[Out] ArcSin[Sin[x]/2]
```

Rubi [A] time = 0.0249788, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3190, 216}

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[4 - Sin[x]^2],x]

[Out] ArcSin[Sin[x]/2]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(x)\right)$$
$$= \sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

Mathematica [A] time = 0.0089002, size = 7, normalized size = 1.

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[4 - Sin[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/2]
```

Maple [A] time = 0.026, size = 6, normalized size = 0.9

$$\arcsin\left(\frac{\sin(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4-sin(x)^2)^(1/2),x)
```

```
[Out] arcsin(1/2*sin(x))
```

Maxima [A] time = 1.44222, size = 7, normalized size = 1.

$$\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*sin(x))
```

Fricas [B] time = 2.21549, size = 176, normalized size = 25.14

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\cos(x)^2+3}\left(\cos(x)^2+1\right)\sin(x)-4\cos(x)\sin(x)}{\cos(x)^4+6\cos(x)^2-3}\right)+\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*sin(x) - 4*cos(x)*sin(x))/(co
s(x)^4 + 6*cos(x)^2 - 3)) + 1/2*arctan(sin(x)/cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10703, size = 7, normalized size = 1.

$$\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/2*sin(x))
```

3.669 $\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)$$

[Out] ArcSin[Sin[3*x]/2]/3

Rubi [A] time = 0.0263306, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3190, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]

[Out] ArcSin[Sin[3*x]/2]/3

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(3x)\right)$$
$$= \frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)$$

Mathematica [A] time = 0.029751, size = 13, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]
```

```
[Out] ArcSin[Sin[3*x]/2]/3
```

Maple [A] time = 0.031, size = 10, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{\sin(3x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(4-sin(3*x)^2)^(1/2),x)
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

Maxima [A] time = 1.43736, size = 12, normalized size = 0.92

$$\frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

Fricas [B] time = 2.08545, size = 200, normalized size = 15.38

$$\frac{1}{6}\arctan\left(\frac{\sqrt{\cos(3x)^2+3}\left(\cos(3x)^2+1\right)\sin(3x)-4\cos(3x)\sin(3x)}{\cos(3x)^4+6\cos(3x)^2-3}\right)+\frac{1}{6}\arctan\left(\frac{\sin(3x)}{\cos(3x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan((sqrt(cos(3*x)^2 + 3)*(cos(3*x)^2 + 1)*sin(3*x) - 4*cos(3*x)*sin
(3*x))/(cos(3*x)^4 + 6*cos(3*x)^2 - 3)) + 1/6*arctan(sin(3*x)/cos(3*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(3x)}{\sqrt{-(\sin(3x)-2)(\sin(3x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)**2)**(1/2),x)
```

```
[Out] Integral(cos(3*x)/sqrt(-(sin(3*x) - 2)*(sin(3*x) + 2)), x)
```

Giac [A] time = 1.16606, size = 12, normalized size = 0.92

$$\frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

3.670 $\int \cos(x)\sqrt{1+\csc(x)}\,dx$

Optimal. Leaf size=21

$$\sin(x)\sqrt{\csc(x)+1}+\tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

```
[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

Rubi [A] time = 0.0239342, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3873, 47, 63, 207}

$$\sin(x)\sqrt{\csc(x)+1}+\tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sqrt{1+\csc(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \csc(x)\right)\\ &= \sqrt{1+\csc(x)}\sin(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \csc(x)\right)\\ &= \sqrt{1+\csc(x)}\sin(x) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\csc(x)}\right)\\ &= \tanh^{-1}\left(\sqrt{1+\csc(x)}\right) + \sqrt{1+\csc(x)}\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0151509, size = 21, normalized size = 1.

$$\sin(x)\sqrt{\csc(x)+1} + \tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

Maple [B] time = 0.034, size = 48, normalized size = 2.3

$$\frac{1}{2}\left(1+\sqrt{1+\csc(x)}\right)^{-1} + \frac{1}{2}\ln\left(1+\sqrt{1+\csc(x)}\right) + \frac{1}{2}\left(\sqrt{1+\csc(x)}-1\right)^{-1} - \frac{1}{2}\ln\left(\sqrt{1+\csc(x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+csc(x))^(1/2),x)
```

```
[Out] 1/2/(1+(1+csc(x))^(1/2))+1/2*ln(1+(1+csc(x))^(1/2))+1/2/((1+csc(x))^(1/2)-1
)-1/2*ln((1+csc(x))^(1/2)-1)
```

Maxima [B] time = 0.94912, size = 51, normalized size = 2.43

$$\sqrt{\frac{1}{\sin(x)}+1}\sin(x)+\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(1/sin(x) + 1)*sin(x) + 1/2*log(sqrt(1/sin(x) + 1) + 1) - 1/2*log(sqrt(
1/sin(x) + 1) - 1)
```

Fricas [B] time = 2.02988, size = 271, normalized size = 12.9

$$\sqrt{\frac{\sin(x)+1}{\sin(x)}}\sin(x)+\frac{1}{2}\log\left(\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\sin(x)+\sin(x)+1\right)}{\cos(x)+\sin(x)+1}\right)-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\sin(x)-\sin(x)-1\right)}{\cos(x)+\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt((sin(x) + 1)/sin(x))*sin(x) + 1/2*log(2*(sqrt((sin(x) + 1)/sin(x))*sin
(x) + sin(x) + 1)/(cos(x) + sin(x) + 1)) - 1/2*log(-2*(sqrt((sin(x) + 1)/si
n(x))*sin(x) - sin(x) - 1)/(cos(x) + sin(x) + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\csc(x)+1}\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(x) + 1)*cos(x), x)
```

Giac [B] time = 1.14278, size = 51, normalized size = 2.43

$$\frac{1}{2}\left(2\sqrt{\sin(x)^2+\sin(x)}-\log\left(\left|2\sqrt{\sin(x)^2+\sin(x)}-2\sin(x)-1\right|\right)\right)\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(sin(x)^2 + sin(x)) - log(abs(2*sqrt(sin(x)^2 + sin(x)) - 2*sin(
x) - 1)))*sgn(sin(x))
```

3.671 $\int \cos(x)\sqrt{4-\sin^2(x)}\,dx$

Optimal. Leaf size=28

$$2\sin^{-1}\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

```
[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

Rubi [A] time = 0.026376, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 195, 216}

$$2\sin^{-1}\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[4 - Sin[x]^2],x]
```

```
[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = \text{Subst}\left(\int \sqrt{4-x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(x)\right)$$
$$= 2\sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Mathematica [A] time = 0.0177558, size = 28, normalized size = 1.

$$2\sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[4 - Sin[x]^2],x]

[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

Maple [A] time = 0.023, size = 23, normalized size = 0.8

$$2\arcsin\left(1/2\sin(x)\right) + \frac{\sin(x)}{2}\sqrt{4-(\sin(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(4-sin(x)^2)^(1/2),x)

[Out] 2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)
```

Maxima [A] time = 1.49782, size = 30, normalized size = 1.07

$$\frac{1}{2}\sqrt{-\sin(x)^2+4}\sin(x) + 2\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

Fricas [B] time = 2.22431, size = 208, normalized size = 7.43

$$\frac{1}{2}\sqrt{\cos(x)^2+3}\sin(x)+\arctan\left(\frac{\sqrt{\cos(x)^2+3}\left(\cos(x)^2+1\right)\sin(x)-4\cos(x)\sin(x)}{\cos(x)^4+6\cos(x)^2-3}\right)+\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(cos(x)^2 + 3)*sin(x) + arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*s
in(x) - 4*cos(x)*sin(x))/(cos(x)^4 + 6*cos(x)^2 - 3)) + arctan(sin(x)/cos(x
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-(\sin(x)-2)(\sin(x)+2)}\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(sin(x) - 2)*(sin(x) + 2))*cos(x), x)
```

Giac [A] time = 1.09273, size = 30, normalized size = 1.07

$$\frac{1}{2}\sqrt{-\sin(x)^2+4}\sin(x)+2\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

3.672 $\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

```
[Out] (1 + Sin[x]^2)^(3/2)/3
```

Rubi [A] time = 0.0341567, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3198, 261}

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] (1 + Sin[x]^2)^(3/2)/3
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \text{Subst}\left(\int x\sqrt{1+x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3}\left(1+\sin^2(x)\right)^{3/2}$$

Mathematica [A] time = 0.0067136, size = 14, normalized size = 1.

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] (1 + Sin[x]^2)^(3/2)/3
```

Maple [A] time = 0.006, size = 11, normalized size = 0.8

$$\frac{1}{3}\left(1+(\sin(x))^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x)
```

```
[Out] 1/3*(1+sin(x)^2)^(3/2)
```

Maxima [A] time = 0.964293, size = 14, normalized size = 1.

$$\frac{1}{3}\left(\sin(x)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x)^2 + 1)^(3/2)
```

Fricas [A] time = 2.14527, size = 36, normalized size = 2.57

$$\frac{1}{3}\left(-\cos(x)^{2}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-cos(x)^2 + 2)^(3/2)
```

Sympy [B] time = 0.877731, size = 27, normalized size = 1.93

$$\frac{\sqrt{\sin^{2}(x)+1}\sin^{2}(x)}{3}+\frac{\sqrt{\sin^{2}(x)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)**2)**(1/2),x)
```

```
[Out] sqrt(sin(x)**2 + 1)*sin(x)**2/3 + sqrt(sin(x)**2 + 1)/3
```

Giac [A] time = 1.11555, size = 14, normalized size = 1.

$$\frac{1}{3}\left(\sin(x)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sin(x)^2 + 1)^(3/2)
```

3.673 $\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx$

Optimal. Leaf size=19

$$2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)+2\sin(x)}}\right)$$

```
[Out] 2*ArcTanh[Sin[x]/Sqrt[2*Sin[x] + Sin[x]^2]]
```

Rubi [A] time = 0.0316316, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3258, 620, 206}

$$2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)+2\sin(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]
```

```
[Out] 2*ArcTanh[Sin[x]/Sqrt[2*Sin[x] + Sin[x]^2]]
```

Rule 3258

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{2x+x^2}}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)\\ &= 2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.0181688, size = 40, normalized size = 2.11

$$\frac{2\sqrt{\sin(x)}\sqrt{\sin(x)+2}\sinh^{-1}\left(\frac{\sqrt{\sin(x)}}{\sqrt{2}}\right)}{\sqrt{\sin(x)(\sin(x)+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]
```

```
[Out] (2*ArcSinh[Sqrt[Sin[x]]/Sqrt[2]]*Sqrt[Sin[x]]*Sqrt[2 + Sin[x]])/Sqrt[Sin[x]
*(2 + Sin[x])]
```

Maple [A] time = 0.036, size = 17, normalized size = 0.9

$$\ln\left(1+\sin(x)+\sqrt{2\sin(x)+(\sin(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x)
```

```
[Out] ln(1+sin(x)+(2*sin(x)+sin(x)^2)^(1/2))
```

Maxima [A] time = 0.952721, size = 27, normalized size = 1.42

$$\log\left(2\sqrt{\sin\left(x\right)^{2}+2\,\sin\left(x\right)}+2\,\sin\left(x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(sin(x)^2 + 2*sin(x)) + 2*sin(x) + 2)
```

Fricas [B] time = 3.78977, size = 115, normalized size = 6.05

$$\frac{1}{2}\,\log\left(-2\,\cos\left(x\right)^{2}+2\sqrt{-\cos\left(x\right)^{2}+2\,\sin\left(x\right)+1}(\sin\left(x\right)+1)+4\,\sin\left(x\right)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(-2*cos(x)^2 + 2*sqrt(-cos(x)^2 + 2*sin(x) + 1)*(sin(x) + 1) + 4*sin
(x) + 3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09747, size = 27, normalized size = 1.42

$$-\log\left(-\sqrt{\sin(x)^2 + 2\sin(x)} + \sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(sin(x)^2 + 2*sin(x)) + sin(x) + 1)
```

3.674 $\int \cos(x)\cos(\sin(x))\,dx$

Optimal. Leaf size=3

$$\sin(\sin(x))$$

```
[Out] Sin[Sin[x]]
```

Rubi [A] time = 0.0084907, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4334, 2637}

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Sin[x]],x]
```

```
[Out] Sin[Sin[x]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(\sin(x))\,dx &= \text{Subst}\left(\int \cos(x)\,dx, x, \sin(x)\right)\\ &= \sin(\sin(x))\end{aligned}$$

Mathematica [A] time = 1.48304, size = 3, normalized size = 1.

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]],x]
```

```
[Out] Sin[Sin[x]]
```

Maple [A] time = 0.007, size = 4, normalized size = 1.3

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(sin(x)),x)
```

```
[Out] sin(sin(x))
```

Maxima [A] time = 0.969698, size = 4, normalized size = 1.33

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="maxima")
```

```
[Out] sin(sin(x))
```

Fricas [B] time = 2.15008, size = 51, normalized size = 17.

$$\sin\left(\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="fricas")
```

```
[Out] sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

Sympy [A] time = 0.540292, size = 3, normalized size = 1.

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x)
```

```
[Out] sin(sin(x))
```

Giac [A] time = 1.07498, size = 4, normalized size = 1.33

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="giac")
```

```
[Out] sin(sin(x))
```

3.675 $\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$

Optimal. Leaf size=4

$$\sin(\sin(\sin(x)))$$

```
[Out] Sin[Sin[Sin[x]]]
```

Rubi [A] time = 0.0216749, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4334, 2637}

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]

[Out] Sin[Sin[Sin[x]]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx &= \text{Subst}\left(\int \cos(x)\cos(\sin(x))\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \cos(x)\,dx, x, \sin(\sin(x))\right)\\ &= \sin(\sin(\sin(x)))\end{aligned}$$

Mathematica [A] time = 7.6869, size = 4, normalized size = 1.

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
```

```
[Out] Sin[Sin[Sin[x]]]
```

Maple [A] time = 0.012, size = 5, normalized size = 1.3

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

```
[Out] sin(sin(sin(x)))
```

Maxima [A] time = 0.952398, size = 5, normalized size = 1.25

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="maxima")
```

```
[Out] sin(sin(sin(x)))
```

Fricas [B] time = 2.15143, size = 116, normalized size = 29.

$$\sin\left(\frac{2\,\tan\left(\frac{\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)}{\tan\left(\frac{\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="fricas")
```

```
[Out] sin(2*tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))/(tan(tan(1/2*x)/(tan(1/2*x)^2 + 1)
)^2 + 1))
```

Sympy [A] time = 12.9891, size = 5, normalized size = 1.25

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

```
[Out] sin(sin(sin(x)))
```

Giac [A] time = 1.09896, size = 5, normalized size = 1.25

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="giac")
```

```
[Out] sin(sin(sin(x)))
```

3.676 $\int \cos(x)\sec(\sin(x))\,dx$

Optimal. Leaf size=4

$$\tanh^{-1}(\sin(\sin(x)))$$

```
[Out] ArcTanh[Sin[Sin[x]]]
```

Rubi [A] time = 0.0069563, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4334, 3770}

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[Sin[x]],x]
```

```
[Out] ArcTanh[Sin[Sin[x]]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(\sin(x))\,dx &= \text{Subst}\left(\int \sec(x)\,dx, x, \sin(x)\right)\\ &= \tanh^{-1}(\sin(\sin(x)))\end{aligned}$$

Mathematica [A] time = 0.0045549, size = 4, normalized size = 1.

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[Sin[x]],x]
```

```
[Out] ArcTanh[Sin[Sin[x]]]
```

Maple [A] time = 0.009, size = 9, normalized size = 2.3

$$\ln(\sec(\sin(x)) + \tan(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(sin(x)),x)
```

```
[Out] ln(sec(sin(x))+tan(sin(x)))
```

Maxima [B] time = 0.950847, size = 11, normalized size = 2.75

$$\log(\sec(\sin(x)) + \tan(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="maxima")
```

```
[Out] log(sec(sin(x)) + tan(sin(x)))
```

Fricas [B] time = 2.08479, size = 140, normalized size = 35.

$$\frac{1}{2}\log\left(\sin\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)+1\right)-\frac{1}{2}\log\left(-\sin\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)) + 1) - 1/2*log(-sin(2*tan(1/2*
x)/(tan(1/2*x)^2 + 1)) + 1)
```

Sympy [A] time = 1.72252, size = 10, normalized size = 2.5

$$\log(\tan(\sin(x)) + \sec(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x)
```

```
[Out] log(tan(sin(x)) + sec(sin(x)))
```

Giac [B] time = 1.08891, size = 39, normalized size = 9.75

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(\sin(x))} + \sin(\sin(x)) + 2\right|\right) - \frac{1}{4}\log\left(\left|\frac{1}{\sin(\sin(x))} + \sin(\sin(x)) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(sin(x)) + sin(sin(x)) + 2)) - 1/4*log(abs(1/sin(sin(x)) +
 sin(sin(x)) - 2))
```

3.677 $\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}$$

```
[Out] -(a*(a + b*Sin[x]^2)^4)/(8*b^2) + (a + b*Sin[x]^2)^5/(10*b^2)
```

Rubi [A] time = 0.076912, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3198, 266, 43}

$$\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]
```

```
[Out] -(a*(a + b*Sin[x]^2)^4)/(8*b^2) + (a + b*Sin[x]^2)^5/(10*b^2)
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx &= \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,\sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\sin^2(x)\right)\\
&= -\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}
\end{aligned}
$$

Mathematica [B] time = 0.42499, size = 128, normalized size = 3.56

$$\frac{-20\left(64a^3+24ab^2+7b^3\right)\cos(2x)+20\left(16a^3+18ab^2+5b^3\right)\cos(4x)+b\left(3840a^2\sin^4(x)-1280a^2\sin(3x)\sin^3(x)+256\right.}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]
```

```
[Out] (-20*(64*a^3 + 24*a*b^2 + 7*b^3)*Cos[2*x] + 20*(16*a^3 + 18*a*b^2 + 5*b^3)*
Cos[4*x] + b*(-10*b*(16*a + 5*b)*Cos[6*x] + 15*b*(2*a + b)*Cos[8*x] - 2*b^2
*Cos[10*x] + 3840*a^2*Sin[x]^4 + 2560*a*b*Sin[x]^6 + 640*b^2*Sin[x]^8 - 128
0*a^2*Sin[x]^3*Sin[3*x]))/10240
```

Maple [A] time = 0.011, size = 40, normalized size = 1.1

$$\frac{b^3(\sin(x))^{10}}{10}+\frac{3ab^2(\sin(x))^8}{8}+\frac{a^2b(\sin(x))^6}{2}+\frac{(\sin(x))^4a^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x)
```

```
[Out] 1/10*b^3*sin(x)^10+3/8*a*b^2*sin(x)^8+1/2*a^2*b*sin(x)^6+1/4*sin(x)^4*a^3
```

Maxima [A] time = 0.982705, size = 53, normalized size = 1.47

$$\frac{1}{10}\,b^3\sin\left(x\right)^{10}+\frac{3}{8}\,ab^2\sin\left(x\right)^8+\frac{1}{2}\,a^2b\sin\left(x\right)^6+\frac{1}{4}\,a^3\sin\left(x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(
x)^4
```

Fricas [B] time = 2.22598, size = 258, normalized size = 7.17

$$-\frac{1}{10}\,b^3\cos\left(x\right)^{10}+\frac{1}{8}\left(3\,ab^2+4\,b^3\right)\cos\left(x\right)^8-\frac{1}{2}\left(a^2b+3\,ab^2+2\,b^3\right)\cos\left(x\right)^6+\frac{1}{4}\left(a^3+6\,a^2b+9\,ab^2+4\,b^3\right)\cos\left(x\right)^4-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*b^3*cos(x)^10 + 1/8*(3*a*b^2 + 4*b^3)*cos(x)^8 - 1/2*(a^2*b + 3*a*b^2
 + 2*b^3)*cos(x)^6 + 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(x)^4 - 1/2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2
```

Sympy [B] time = 16.575, size = 102, normalized size = 2.83

$$\frac{a^3\sin^4\left(x\right)}{4}+\frac{a^2b\sin^6\left(x\right)}{2}+\frac{3ab^2\sin^8\left(x\right)}{8}-\frac{b^3\sin^8\left(x\right)\cos^2\left(x\right)}{2}-b^3\sin^6\left(x\right)\cos^4\left(x\right)-b^3\sin^4\left(x\right)\cos^6\left(x\right)-\frac{b^3\sin^2\left(x\right)\cos}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3*(a+b*sin(x)**2)**3,x)
```

```
[Out] a**3*sin(x)**4/4 + a**2*b*sin(x)**6/2 + 3*a*b**2*sin(x)**8/8 - b**3*sin(x)*
*8*cos(x)**2/2 - b**3*sin(x)**6*cos(x)**4 - b**3*sin(x)**4*cos(x)**6 - b**3
*sin(x)**2*cos(x)**8/2 - b**3*cos(x)**10/10
```

<u>Giac</u> [A] time = 1.13995, size = 53, normalized size = 1.47

$$\frac{1}{10}\,b^3\sin\left(x\right)^{10}+\frac{3}{8}\,ab^2\sin\left(x\right)^8+\frac{1}{2}\,a^2b\sin\left(x\right)^6+\frac{1}{4}\,a^3\sin\left(x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(
x)^4
```

3.678 $\int e^{\sin(x)}\cos(x)\sin(x)\,dx$

Optimal. Leaf size=14

$$e^{\sin(x)}\sin(x) - e^{\sin(x)}$$

```
[Out] -E^Sin[x] + E^Sin[x]*Sin[x]
```

Rubi [A] time = 0.0163624, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4334, 2176, 2194}

$$e^{\sin(x)}\sin(x) - e^{\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sin[x]*Cos[x]*Sin[x],x]
```

```
[Out] -E^Sin[x] + E^Sin[x]*Sin[x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{\sin(x)}\cos(x)\sin(x)\,dx = \text{Subst}\left(\int e^x x\,dx, x, \sin(x)\right)$$
$$= e^{\sin(x)}\sin(x) - \text{Subst}\left(\int e^x\,dx, x, \sin(x)\right)$$
$$= -e^{\sin(x)} + e^{\sin(x)}\sin(x)$$

Mathematica [A] time = 0.0098843, size = 9, normalized size = 0.64

$$e^{\sin(x)}(\sin(x)-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Cos[x]*Sin[x],x]
```

```
[Out] E^Sin[x]*(-1 + Sin[x])
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$-e^{\sin(x)} + e^{\sin(x)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*cos(x)*sin(x),x)
```

```
[Out] -exp(sin(x))+exp(sin(x))*sin(x)
```

Maxima [A] time = 0.964944, size = 11, normalized size = 0.79

$$(\sin(x)-1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] (sin(x) - 1)*e^sin(x)
```

Fricas [A] time = 2.16587, size = 31, normalized size = 2.21

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] (sin(x) - 1)*e^sin(x)
```

Sympy [A] time = 0.837246, size = 12, normalized size = 0.86

$$e^{\sin(x)}\sin(x) - e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x)
```

```
[Out] exp(sin(x))*sin(x) - exp(sin(x))
```

Giac [A] time = 1.08536, size = 11, normalized size = 0.79

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] (sin(x) - 1)*e^sin(x)
```

3.679 $\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}} dx$

Optimal. Leaf size=25

$$-\frac{2\sin(x)}{\sqrt{\sin^3(x)}}-\frac{2}{3}\sqrt{\sin^3(x)}$$

```
[Out] (-2*Sin[x])/Sqrt[Sin[x]^3] - (2*Sqrt[Sin[x]^3])/3
```

Rubi [A] time = 0.0488918, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3207, 2564, 14}

$$-\frac{2\sin(x)}{\sqrt{\sin^3(x)}}-\frac{2}{3}\sqrt{\sin^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/Sqrt[Sin[x]^3],x]
```

```
[Out] (-2*Sin[x])/Sqrt[Sin[x]^3] - (2*Sqrt[Sin[x]^3])/3
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(x)}\,dx}{\sqrt{\sin^3(x)}}$$
$$= \frac{\sin^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1-x^2}{x^{3/2}}\,dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}}$$
$$= \frac{\sin^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}-\sqrt{x}\right)dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}}$$
$$= -\frac{2\sin(x)}{\sqrt{\sin^3(x)}}-\frac{2}{3}\sqrt{\sin^3(x)}$$

Mathematica [A] time = 0.0226984, size = 20, normalized size = 0.8

$$\frac{\sin(x)(\cos(2x)-7)}{3\sqrt{\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/Sqrt[Sin[x]^3],x]
```

```
[Out] ((-7 + Cos[2*x])*Sin[x])/(3*Sqrt[Sin[x]^3])
```

Maple [A] time = 0.577, size = 14, normalized size = 0.6

$$-\frac{2}{3}(\sin(x))^{\frac{3}{2}}-2\frac{1}{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(sin(x)^3)^(1/2),x)
```

```
[Out] -2/3*sin(x)^(3/2)-2/sin(x)^(1/2)
```

Maxima [A] time = 0.988047, size = 26, normalized size = 1.04

$$-\frac{2}{3}\sqrt{\sin(x)^3} - \frac{2\sin(x)}{\sqrt{\sin(x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(sin(x)^3) - 2*sin(x)/sqrt(sin(x)^3)
```

Fricas [A] time = 2.14133, size = 88, normalized size = 3.52

$$-\frac{2\left(\cos(x)^2-4\right)\sqrt{-\left(\cos(x)^2-1\right)\sin(x)}}{3\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)*sin(x))/(cos(x)^2 - 1)
```

Sympy [A] time = 3.24742, size = 36, normalized size = 1.44

$$-\frac{8\sin^3(x)}{3\sqrt{\sin^3(x)}} - \frac{2\sin(x)\cos^2(x)}{\sqrt{\sin^3(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(sin(x)**3)**(1/2),x)
```

```
[Out] -8*sin(x)**3/(3*sqrt(sin(x)**3)) - 2*sin(x)*cos(x)**2/sqrt(sin(x)**3)
```

Giac [A] time = 1.08893, size = 18, normalized size = 0.72

$$-\frac{2}{3}\sin(x)^{\frac{3}{2}}-\frac{2}{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sin(x)^(3/2) - 2/sqrt(sin(x))
```

3.680 $\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=10

$$2e^{\sqrt{\sin(x)}}$$

```
[Out] 2*E^Sqrt[Sin[x]]
```

Rubi [A] time = 0.026561, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4334, 2209}

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]

[Out] 2*E^Sqrt[Sin[x]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx &= \text{Subst}\left(\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx, x, \sin(x)\right)\\ &= 2e^{\sqrt{\sin(x)}}\end{aligned}$$

Mathematica [A] time = 0.0130626, size = 10, normalized size = 1.

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]
```

```
[Out] 2*E^Sqrt[Sin[x]]
```

Maple [A] time = 0.007, size = 8, normalized size = 0.8

$$2\,\mathrm{e}^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x)
```

```
[Out] 2*exp(sin(x)^(1/2))
```

Maxima [A] time = 0.964918, size = 9, normalized size = 0.9

$$2\,e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*e^sqrt(sin(x))
```

Fricas [A] time = 2.0343, size = 24, normalized size = 2.4

$$2\,e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*e^sqrt(sin(x))
```

Sympy [A] time = 0.595964, size = 8, normalized size = 0.8

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)**(1/2))*cos(x)/sin(x)**(1/2),x)
```

```
[Out] 2*exp(sqrt(sin(x)))
```

Giac [A] time = 1.10442, size = 9, normalized size = 0.9

$$2\,e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*e^sqrt(sin(x))
```

3.681 $\int e^{4+\sin(x)} \cos(x) \, dx$

Optimal. Leaf size=6

$$e^{\sin(x)+4}$$

```
[Out] E^(4 + Sin[x])
```

Rubi [A] time = 0.0095759, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4334, 2194}

$$e^{\sin(x)+4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4 + Sin[x])*Cos[x],x]
```

```
[Out] E^(4 + Sin[x])
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{4+\sin(x)} \cos(x) \, dx &= \text{Subst}\left(\int e^{4+x} \, dx, x, \sin(x)\right)\\ &= e^{4+\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.0105102, size = 6, normalized size = 1.

$$e^{\sin(x)+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4 + Sin[x])*Cos[x],x]
```

```
[Out] E^(4 + Sin[x])
```

Maple [A] time = 0.006, size = 6, normalized size = 1.

$$e^{4+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(4+sin(x))*cos(x),x)
```

```
[Out] exp(4+sin(x))
```

Maxima [A] time = 0.957979, size = 7, normalized size = 1.17

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="maxima")
```

```
[Out] e^(sin(x) + 4)
```

Fricas [A] time = 1.95384, size = 22, normalized size = 3.67

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="fricas")
```

```
[Out] e^(sin(x) + 4)
```

Sympy [A] time = 0.725248, size = 7, normalized size = 1.17

$$e^4 e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x)
```

```
[Out] exp(4)*exp(sin(x))
```

Giac [A] time = 1.11814, size = 7, normalized size = 1.17

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="giac")
```

```
[Out] e^(sin(x) + 4)
```

3.682 $\int e^{\cos(x)\sin(x)}\cos(2x)\,dx$

Optimal. Leaf size=10

$$e^{\frac{1}{2}\sin(2x)}$$

[Out] E^(Sin[2*x]/2)

Rubi [A] time = 0.0117849, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4356, 2194}

$$e^{\frac{1}{2}\sin(2x)}$$

Antiderivative was successfully verified.

[In] Int[E^(Cos[x]*Sin[x])*Cos[2*x],x]

[Out] E^(Sin[2*x]/2)

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\cos(x)\sin(x)}\cos(2x)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int e^{x/2}\,dx, x, \sin(2x)\right)\\ &= e^{\frac{1}{2}\sin(2x)}\end{aligned}$$

Mathematica [A] time = 0.0213795, size = 7, normalized size = 0.7

$$e^{\sin(x)\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x]*Sin[x])*Cos[2*x],x]
```

```
[Out] E^(Cos[x]*Sin[x])
```

Maple [A] time = 0.015, size = 7, normalized size = 0.7

$$e^{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x)*sin(x))*cos(2*x),x)
```

```
[Out] exp(cos(x)*sin(x))
```

Maxima [A] time = 3.1773, size = 9, normalized size = 0.9

$$e^{\left(\frac{1}{2}\sin(2x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="maxima")
```

```
[Out] e^(1/2*sin(2*x))
```

Fricas [A] time = 2.15711, size = 26, normalized size = 2.6

$$e^{(\cos(x)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="fricas")
```

```
[Out] e^(cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09902, size = 16, normalized size = 1.6

$$e^{\left(\frac{\tan(x)}{\tan(x)^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="giac")
```

```
[Out] e^(tan(x)/(tan(x)^2 + 1))
```

3.683 $\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx$

Optimal. Leaf size=10

$$2e^{\frac{\sin(x)}{2}}$$

```
[Out] 2*E^(Sin[x]/2)
```

Rubi [A] time = 0.0109039, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4356, 2194}

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
```

```
[Out] 2*E^(Sin[x]/2)
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx &= \text{Subst}\left(\int e^{x/2}\,dx, x, \sin(x)\right)\\ &= 2e^{\frac{\sin(x)}{2}}\end{aligned}$$

Mathematica [A] time = 0.0086436, size = 10, normalized size = 1.

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
```

```
[Out] 2*E^(Sin[x]/2)
```

Maple [A] time = 0.017, size = 13, normalized size = 1.3

$$2\,\mathrm{e}^{\cos(x/2)\sin(x/2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)
```

```
[Out] 2*exp(cos(1/2*x)*sin(1/2*x))
```

Maxima [A] time = 0.996401, size = 9, normalized size = 0.9

$$2\,e^{\left(\frac{1}{2}\,\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*sin(x))
```

Fricas [A] time = 2.05714, size = 39, normalized size = 3.9

$$2\,e^{\left(\cos\left(\frac{1}{2}\,x\right)\sin\left(\frac{1}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="fricas")
```

```
[Out] 2*e^(cos(1/2*x)*sin(1/2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\cos\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)
```

```
[Out] Integral(exp(sin(x/2)*cos(x/2))*cos(x), x)
```

Giac [B] time = 1.10507, size = 24, normalized size = 2.4

$$2\,e^{\left(\frac{\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="giac")
```

```
[Out] 2*e^(tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

3.684 $\int e^{n \sin(a+bx)} \cos(a+bx) \, dx$

Optimal. Leaf size=17

$$\frac{e^{n \sin(a+bx)}}{bn}$$

```
[Out] E^(n*Sin[a + b*x])/(b*n)
```

Rubi [A] time = 0.0126711, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4334, 2194}

$$\frac{e^{n \sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Cos[a + b*x],x]
```

```
[Out] E^(n*Sin[a + b*x])/(b*n)
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^{n \sin(a+bx)} \cos(a+bx) \, dx &= \frac{\operatorname{Subst}\left(\int e^{nx} \, dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{e^{n \sin(a+bx)}}{bn} \end{aligned}$$

Mathematica [A] time = 0.0167467, size = 17, normalized size = 1.

$$\frac{e^{n\sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Cos[a + b*x],x]
```

```
[Out] E^(n*Sin[a + b*x])/(b*n)
```

Maple [A] time = 0.006, size = 17, normalized size = 1.

$$\frac{\mathrm{e}^{n\sin(bx+a)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*cos(b*x+a),x)
```

```
[Out] exp(n*sin(b*x+a))/b/n
```

Maxima [A] time = 0.962259, size = 22, normalized size = 1.29

$$\frac{e^{(n\sin(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

Fricas [A] time = 2.02746, size = 35, normalized size = 2.06

$$\frac{e^{(n\sin(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

Sympy [A] time = 0.460416, size = 36, normalized size = 2.12

$$\begin{cases} x\cos{(a)} & \text{for } b = 0 \wedge n = 0 \\ xe^{n\sin{(a)}}\cos{(a)} & \text{for } b = 0 \\ \frac{\sin{(a+bx)}}{b} & \text{for } n = 0 \\ \frac{e^{n\sin{(a+bx)}}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x)
```

```
[Out] Piecewise((x*cos(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sin(a))*cos(a), Eq(b, 0
)), (sin(a + b*x)/b, Eq(n, 0)), (exp(n*sin(a + b*x))/(b*n), True))
```

Giac [A] time = 1.11749, size = 22, normalized size = 1.29

$$\frac{e^{(n\sin(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

3.685 $\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx$

Optimal. Leaf size=22

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

```
[Out] E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

Rubi [A] time = 0.0132438, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4334, 2194}

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]
```

```
[Out] E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx &= \frac{\operatorname{Subst}\left(\int e^{nx}\,dx, x, \sin(c(a+bx))\right)}{bc}\\ &= \frac{e^{n\sin(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.145826, size = 23, normalized size = 1.05

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]
```

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

Maple [A] time = 0.015, size = 23, normalized size = 1.1

$$\frac{e^{n\sin(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)
```

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 0.956044, size = 30, normalized size = 1.36

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 2.10694, size = 43, normalized size = 1.95

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Sympy [A] time = 8.67409, size = 51, normalized size = 2.32

$$\begin{cases} xe^{n\sin{(ac)}}\cos{(ac)} & \text{for}\: b = 0 \\ x & \text{for}\: c = 0 \\ \begin{cases} x\cos{(ac)} & \text{for}\: b = 0 \\ x & \text{for}\: c = 0 \\ \frac{\sin{(ac+bcx)}}{bc} & \text{otherwise} \end{cases} & \text{for}\: n = 0 \\ \frac{e^{n\sin{(ac+bcx)}}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)
```

```
[Out] Piecewise((x*exp(n*sin(a*c))*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (Piecewise
((x*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (sin(a*c + b*c*x)/(b*c), True)), Eq
(n, 0)), (exp(n*sin(a*c + b*c*x))/(b*c*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left((bx+a)c\right)e^{\left(n\sin\left(bcx+ac\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cos((b*x + a)*c)*e^(n*sin(b*c*x + a*c)), x)
```

3.686 $\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

Rubi [A] time = 0.0130337, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4334, 2194}

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sin(ac+bcx)\right)}{bc}\\ &= \frac{e^{n\sin(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.046366, size = 23, normalized size = 1.

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

Maple [A] time = 0.008, size = 23, normalized size = 1.

$$\frac{\mathrm{e}^{n\sin(bcx+ac)}}{cbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)
```

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

Maxima [A] time = 1.09945, size = 30, normalized size = 1.3

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Fricas [A] time = 2.05841, size = 43, normalized size = 1.87

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Sympy [A] time = 2.5739, size = 48, normalized size = 2.09

$$\begin{cases} x & \text{for}\: b = 0 \wedge c = 0 \wedge n = 0 \\ x e^{n \sin{(a c)}} \cos{(a c)} & \text{for}\: b = 0 \\ x & \text{for}\: c = 0 \\ \frac{\sin{(a c + b c x)}}{b c} & \text{for}\: n = 0 \\ \frac{e^{n \sin{(a c + b c x)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)
```

```
[Out] Piecewise((x, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*sin(a*c))*cos(a*c),
 Eq(b, 0)), (x, Eq(c, 0)), (sin(a*c + b*c*x)/(b*c), Eq(n, 0)), (exp(n*sin(a
*c + b*c*x))/(b*c*n), True))
```

Giac [A] time = 1.14956, size = 30, normalized size = 1.3

$$\frac{e^{\left(n \sin\left(b c x + a c\right)\right)}}{b c n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

3.687 $\int e^{n \sin(a+bx)} \cot(a+bx) \, dx$

Optimal. Leaf size=13

$$\frac{\text{ExpIntegralEi}(n \sin(a+bx))}{b}$$

```
[Out] ExpIntegralEi[n*Sin[a + b*x]]/b
```

Rubi [A] time = 0.0202975, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n \sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[a + b*x]]/b
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned} \int e^{n \sin(a+bx)} \cot(a+bx) \, dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x} \, dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{\text{Ei}(n \sin(a+bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.0424549, size = 13, normalized size = 1.

$$\frac{\text{ExpIntegralEi}(n\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[a + b*x]]/b
```

Maple [A] time = 0.011, size = 17, normalized size = 1.3

$$-\frac{\text{Ei}\left(1,-n\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*cot(b*x+a),x)
```

```
[Out] -1/b*Ei(1,-n*sin(b*x+a))
```

Maxima [A] time = 1.07049, size = 18, normalized size = 1.38

$$\frac{\text{Ei}\left(n\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*x + a))/b
```

Fricas [A] time = 2.16819, size = 30, normalized size = 2.31

$$\frac{\text{Ei}\left(n\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x)
```

```
[Out] Integral(exp(n*sin(a + b*x))*cot(a + b*x), x)
```

Giac [A] time = 1.08886, size = 18, normalized size = 1.38

$$\frac{\operatorname{Ei}(n\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="giac")
```

```
[Out] Ei(n*sin(b*x + a))/b
```

3.688 $\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx$

Optimal. Leaf size=18

$$\frac{\text{ExpIntegralEi}(n\sin(c(a+bx)))}{bc}$$

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Rubi [A] time = 0.0206306, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sin(c(a+bx))\right)}{bc}\\ &= \frac{\text{Ei}(n\sin(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.0673777, size = 18, normalized size = 1.

$$\frac{\text{ExpIntegralEi}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Maple [A] time = 0.025, size = 23, normalized size = 1.3

$$-\frac{\text{Ei}\left(1,-n\sin\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
```

```
[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))
```

Maxima [A] time = 1.06197, size = 26, normalized size = 1.44

$$\frac{\text{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Fricas [A] time = 2.05709, size = 41, normalized size = 2.28

$$\frac{\text{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sin(ac+bcx)}\cot(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
```

```
[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot\left((bx+a)c\right)e^{(n\sin(bcx+ac))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cot((b*x + a)*c)*e^(n*sin(b*c*x + a*c)), x)
```

3.689 $\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx$

Optimal. Leaf size=19

$$\frac{\text{ExpIntegralEi}(n\sin(ac+bcx))}{bc}$$

```
[Out] ExpIntegralEi[n*Sin[a*c + b*c*x]]/(b*c)
```

Rubi [A] time = 0.0203697, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n\sin(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[a*c + b*c*x]]/(b*c)
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sin(ac+bcx)\right)}{bc}\\ &= \frac{\text{Ei}(n\sin(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] time = 0.0600483, size = 18, normalized size = 0.95

$$\frac{\text{ExpIntegralEi}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Maple [A] time = 0.014, size = 23, normalized size = 1.2

$$-\frac{\text{Ei}\left(1,-n\sin\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
```

```
[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))
```

Maxima [A] time = 1.06385, size = 26, normalized size = 1.37

$$\frac{\text{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Fricas [A] time = 2.09454, size = 41, normalized size = 2.16

$$\frac{\text{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sin{(ac+bcx)}}\cot{(ac+bcx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
```

```
[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot\left(bcx+ac\right)e^{\left(n\sin\left((bx+a)c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(cot(b*c*x + a*c)*e^(n*sin((b*x + a)*c)), x)
```

3.690 $\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\tan(x))}{b}$$

```
[Out] Log[a + b*Tan[x]]/b
```

Rubi [A] time = 0.0341299, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$\frac{\log(a+b\tan(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Tan[x]),x]
```

```
[Out] Log[a + b*Tan[x]]/b
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tan(x)\right)}{b}\\ &= \frac{\log(a+b\tan(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0652282, size = 20, normalized size = 1.82

$$\frac{\log(a\cos(x)+b\sin(x))-\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Tan[x]),x]
```

```
[Out] (-Log[Cos[x]] + Log[a*Cos[x] + b*Sin[x]])/b
```

Maple [A] time = 0.027, size = 12, normalized size = 1.1

$$\frac{\ln(a+b\tan(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*tan(x)),x)
```

```
[Out] ln(a+b*tan(x))/b
```

Maxima [A] time = 0.953026, size = 15, normalized size = 1.36

$$\frac{\log(b\tan(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="maxima")
```

```
[Out] log(b*tan(x) + a)/b
```

Fricas [B] time = 2.25314, size = 107, normalized size = 9.73

$$\frac{\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)-\log\left(\cos(x)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(cos(x)^2))
/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{a + b\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(a + b*tan(x)), x)
```

Giac [A] time = 1.08348, size = 16, normalized size = 1.45

$$\frac{\log(|b\tan(x) + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(x) + a))/b
```

3.691 $\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rubi [A] time = 0.0307602, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3675, 206}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 - Tan[x]^2),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))$$

Mathematica [B] time = 0.005647, size = 23, normalized size = 2.09

$$\frac{1}{2}\log(\sin(x)+\cos(x)) - \frac{1}{2}\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 - Tan[x]^2),x]
```

```
[Out] -Log[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x]]/2
```

Maple [A] time = 0.029, size = 4, normalized size = 0.4

$$\text{Artanh}\,(\tan\,(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1-tan(x)^2),x)
```

```
[Out] arctanh(tan(x))
```

Maxima [A] time = 0.955928, size = 20, normalized size = 1.82

$$\frac{1}{2}\,\log\,(\tan\,(x)+1) - \frac{1}{2}\,\log\,(\tan\,(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

Fricas [B] time = 2.05936, size = 84, normalized size = 7.64

$$\frac{1}{4}\log\left(2\cos\left(x\right)\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sec^2\left(x\right)}{\tan^2\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1-tan(x)**2),x)
```

```
[Out] -Integral(sec(x)**2/(tan(x)**2 - 1), x)
```

Giac [A] time = 1.11568, size = 23, normalized size = 2.09

$$\frac{1}{2}\log\left(|\tan\left(x\right)+1|\right)-\frac{1}{2}\log\left(|\tan\left(x\right)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

3.692 $\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{3}-\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\cos^2(x)+1}\right)$$

```
[Out] x/3 - ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Cos[x]^2)]/3
```

Rubi [A] time = 0.0292568, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 203}

$$\frac{x}{3}-\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\cos^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(9 + Tan[x]^2),x]
```

```
[Out] x/3 - ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Cos[x]^2)]/3
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{x}{3} - \frac{1}{3}\tan^{-1}\left(\frac{2\cos(x)\sin(x)}{1+2\cos^2(x)}\right)$$

Mathematica [A] time = 0.0239948, size = 9, normalized size = 0.33

$$-\frac{1}{3}\tan^{-1}(3\cot(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(9 + Tan[x]^2),x]
```

```
[Out] -ArcTan[3*Cot[x]]/3
```

Maple [A] time = 0.031, size = 8, normalized size = 0.3

$$\frac{1}{3}\arctan\left(\frac{\tan(x)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(9+tan(x)^2),x)
```

```
[Out] 1/3*arctan(1/3*tan(x))
```

Maxima [A] time = 1.44673, size = 9, normalized size = 0.33

$$\frac{1}{3}\arctan\left(\frac{1}{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(1/3*tan(x))
```

Fricas [A] time = 2.13205, size = 70, normalized size = 2.59

$$-\frac{1}{6}\arctan\left(\frac{10\cos(x)^2-1}{6\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 1)/(cos(x)*sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{\tan^2(x)+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(9+tan(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(tan(x)**2 + 9), x)
```

Giac [A] time = 1.08678, size = 9, normalized size = 0.33

$$\frac{1}{3}\arctan\left(\frac{1}{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(1/3*tan(x))
```

3.693 $\int \sec^2(x)(a+b\tan(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$$

```
[Out] (a + b*Tan[x])^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0354784, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*(a + b*Tan[x])^n,x]
```

```
[Out] (a + b*Tan[x])^(1 + n)/(b*(1 + n))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)(a+b\tan(x))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\tan(x)\right)}{b}\\ &= \frac{(a+b\tan(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.17782, size = 18, normalized size = 0.95

$$\frac{(a+b\tan(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(a + b*Tan[x])^n,x]
```

```
[Out] (a + b*Tan[x])^(1 + n)/(b + b*n)
```

Maple [A] time = 0.021, size = 20, normalized size = 1.1

$$\frac{(a+b\tan(x))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^n,x)
```

```
[Out] (a+b*tan(x))^(1+n)/b/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10435, size = 101, normalized size = 5.32

$$\frac{(a\cos(x)+b\sin(x))\left(\frac{a\cos(x)+b\sin(x)}{\cos(x)}\right)^{n}}{(bn+b)\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="fricas")
```

```
[Out] (a*cos(x) + b*sin(x))*((a*cos(x) + b*sin(x))/cos(x))^n/((b*n + b)*cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(x))^n \sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**n,x)
```

```
[Out] Integral((a + b*tan(x))**n*sec(x)**2, x)
```

Giac [A] time = 1.14073, size = 26, normalized size = 1.37

$$\frac{(b\tan(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="giac")
```

```
[Out] (b*tan(x) + a)^(n + 1)/(b*(n + 1))
```

3.694 $\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx$

Optimal. Leaf size=4

$$x+\tan(x)$$

```
[Out] x + Tan[x]
```

Rubi [A] time = 0.0426197, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {203}

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]
```

```
[Out] x + Tan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx &= \text{Subst}\left(\int \left(1+\frac{1}{1+x^2}\right)\,dx,x,\tan(x)\right)\\
&= \tan(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= x+\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.0063568, size = 4, normalized size = 1.

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]
```

```
[Out] x + Tan[x]
```

Maple [A] time = 0.046, size = 5, normalized size = 1.3

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1+1/(1+tan(x)^2)),x)
```

```
[Out] x+tan(x)
```

Maxima [A] time = 1.45915, size = 5, normalized size = 1.25

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="maxima")
```

```
[Out] x + tan(x)
```

Fricas [B] time = 2.02879, size = 38, normalized size = 9.5

$$\frac{x\cos(x) + \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="fricas")
```

```
[Out] (x*cos(x) + sin(x))/cos(x)
```

Sympy [B] time = 0.930394, size = 27, normalized size = 6.75

$$\frac{x\sec^2(x)}{\tan^2(x)+1}+\frac{\tan(x)\sec^2(x)}{\tan^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1+1/(1+tan(x)**2)),x)
```

```
[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

Giac [A] time = 1.1106, size = 5, normalized size = 1.25

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="giac")
```

```
[Out] x + tan(x)
```

3.695 $\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^2(x)} dx$

Optimal. Leaf size=4

$$x + \tan(x)$$

```
[Out] x + Tan[x]
```

Rubi [A] time = 0.0629814, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3657, 3473, 8}

$$x + \tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
```

```
[Out] x + Tan[x]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^2(x)}\,dx = \int \left(2+\tan^2(x)\right)\,dx$$
$$= 2x + \int \tan^2(x)\,dx$$
$$= 2x + \tan(x) - \int 1\,dx$$
$$= x + \tan(x)$$

Mathematica [A] time = 0.0052156, size = 4, normalized size = 1.

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
```

```
[Out] x + Tan[x]
```

Maple [A] time = 0.053, size = 5, normalized size = 1.3

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x)
```

```
[Out] x+tan(x)
```

Maxima [A] time = 1.49087, size = 5, normalized size = 1.25

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="maxima")
```

```
[Out] x + tan(x)
```

Fricas [B] time = 2.12041, size = 38, normalized size = 9.5

$$\frac{x\cos(x) + \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="fricas")
```

```
[Out] (x*cos(x) + sin(x))/cos(x)
```

Sympy [B] time = 0.923632, size = 27, normalized size = 6.75

$$\frac{x\sec^2(x)}{\tan^2(x)+1} + \frac{\tan(x)\sec^2(x)}{\tan^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**2),x)
```

```
[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

Giac [A] time = 1.08675, size = 5, normalized size = 1.25

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="giac")
```

```
[Out] x + tan(x)
```

3.696 $\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx$

Optimal. Leaf size=33

$$x-\tan^{-1}\left(\frac{-2\cos^2(x)+\sin(x)\cos(x)+1}{\cos^2(x)+2\sin(x)\cos(x)+2}\right)$$

```
[Out] x - ArcTan[(1 - 2*Cos[x]^2 + Cos[x]*Sin[x])/(2 + Cos[x]^2 + 2*Cos[x]*Sin[x]
)]
```

Rubi [A] time = 0.0422412, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4342, 617, 204}

$$x-\tan^{-1}\left(\frac{-2\cos^2(x)+\sin(x)\cos(x)+1}{\cos^2(x)+2\sin(x)\cos(x)+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
```

```
[Out] x - ArcTan[(1 - 2*Cos[x]^2 + Cos[x]*Sin[x])/(2 + Cos[x]^2 + 2*Cos[x]*Sin[x]
)]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{2+2x+x^2}\,dx, x, \tan(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\tan(x)\right)\\ &= x-\tan^{-1}\left(\frac{1-2\cos^2(x)+\cos(x)\sin(x)}{2+\cos^2(x)+2\cos(x)\sin(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.0438872, size = 31, normalized size = 0.94

$$2\left(\frac{1}{4}\tan^{-1}(\sec(x)(\sin(x)+\cos(x)))-\frac{1}{4}\tan^{-1}\left(\frac{\cos(x)}{\sin(x)+\cos(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
```

```
[Out] 2*(-ArcTan[Cos[x]/(Cos[x] + Sin[x])]/4 + ArcTan[Sec[x]*(Cos[x] + Sin[x])]/4
)
```

Maple [A] time = 0.06, size = 6, normalized size = 0.2

$$\arctan(1+\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(2+2*tan(x)+tan(x)^2),x)
```

```
[Out] arctan(1+tan(x))
```

Maxima [A] time = 1.44625, size = 7, normalized size = 0.21

$$\arctan\left(\tan\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(tan(x) + 1)
```

Fricas [A] time = 2.04943, size = 117, normalized size = 3.55

$$-\frac{1}{2}\arctan\left(-\frac{3\cos\left(x\right)^{2} + 6\cos\left(x\right)\sin\left(x\right) + 1}{2\left(2\cos\left(x\right)^{2} - \cos\left(x\right)\sin\left(x\right) - 1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(-1/2*(3*cos(x)^2 + 6*cos(x)*sin(x) + 1)/(2*cos(x)^2 - cos(x)*si
n(x) - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(x\right)}{\tan^{2}\left(x\right) + 2\tan\left(x\right) + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(2+2*tan(x)+tan(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(tan(x)**2 + 2*tan(x) + 2), x)
```

Giac [A] time = 1.09298, size = 7, normalized size = 0.21

$$\arctan\left(\tan\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="giac")
```

```
[Out] arctan(tan(x) + 1)
```

3.697 $\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx$

Optimal. Leaf size=10

$$\log(\cot(x)+1)-\cot(x)$$

```
[Out] -Cot[x] + Log[1 + Cot[x]]
```

Rubi [A] time = 0.0479915, antiderivative size = 15, normalized size of antiderivative = 1.5, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4342, 44}

$$-\cot(x)-\log(\tan(x))+\log(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]

[Out] -Cot[x] - Log[Tan[x]] + Log[1 + Tan[x]]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, \tan(x)\right)$$
$$= -\cot(x)-\log(\tan(x))+\log(1+\tan(x))$$

Mathematica [A] time = 0.0374629, size = 16, normalized size = 1.6

$$-\cot(x)-\log(\sin(x))+\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]
```

```
[Out] -Cot[x] - Log[Sin[x]] + Log[Cos[x] + Sin[x]]
```

Maple [A] time = 0.055, size = 18, normalized size = 1.8

$$-(\tan(x))^{-1}-\ln(\tan(x))+\ln(1+\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(tan(x)^2+tan(x)^3),x)
```

```
[Out] -1/tan(x)-ln(tan(x))+ln(1+tan(x))
```

Maxima [A] time = 0.960251, size = 23, normalized size = 2.3

$$-\frac{1}{\tan(x)}+\log(\tan(x)+1)-\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="maxima")
```

```
[Out] -1/tan(x) + log(tan(x) + 1) - log(tan(x))
```

Fricas [B] time = 2.17885, size = 124, normalized size = 12.4

$$-\frac{\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)\sin(x)-\log(2\cos(x)\sin(x)+1)\sin(x)+2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(2*cos(x)*sin(x) + 1)*sin(x) + 2
*cos(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2(x)}{(\tan(x)+1)\tan^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(tan(x)**2+tan(x)**3),x)
```

```
[Out] Integral(sec(x)**2/((tan(x) + 1)*tan(x)**2), x)
```

Giac [A] time = 1.1344, size = 26, normalized size = 2.6

$$-\frac{1}{\tan(x)}+\log(|\tan(x)+1|)-\log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="giac")
```

```
[Out] -1/tan(x) + log(abs(tan(x) + 1)) - log(abs(tan(x)))
```

3.698 $\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx$

Optimal. Leaf size=10

$$\cot(x) + \log(1 - \cot(x))$$

```
[Out] Cot[x] + Log[1 - Cot[x]]
```

Rubi [A] time = 0.052689, antiderivative size = 15, normalized size of antiderivative = 1.5, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4342, 44}

$$\cot(x) + \log(1 - \tan(x)) - \log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
```

```
[Out] Cot[x] + Log[1 - Tan[x]] - Log[Tan[x]]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{(-1+x)x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx, x, \tan(x)\right)$$
$$= \cot(x)+\log(1-\tan(x))-\log(\tan(x))$$

Mathematica [A] time = 0.0372257, size = 16, normalized size = 1.6

$$\cot(x)-\log(\sin(x))+\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
```

```
[Out] Cot[x] + Log[Cos[x] - Sin[x]] - Log[Sin[x]]
```

Maple [A] time = 0.058, size = 16, normalized size = 1.6

$$(\tan(x))^{-1}-\ln(\tan(x))+\ln(\tan(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(-tan(x)^2+tan(x)^3),x)
```

```
[Out] 1/tan(x)-ln(tan(x))+ln(tan(x)-1)
```

Maxima [A] time = 0.963174, size = 20, normalized size = 2.

$$\frac{1}{\tan(x)}+\log(\tan(x)-1)-\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 1/tan(x) + log(tan(x) - 1) - log(tan(x))
```

Fricas [B] time = 2.17118, size = 126, normalized size = 12.6

$$-\frac{\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)\sin(x)-\log(-2\cos(x)\sin(x)+1)\sin(x)-2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(-2*cos(x)*sin(x) + 1)*sin(x) -
2*cos(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{(\tan(x)-1)\tan^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(-tan(x)**2+tan(x)**3),x)
```

```
[Out] Integral(sec(x)**2/((tan(x) - 1)*tan(x)**2), x)
```

Giac [A] time = 1.11066, size = 23, normalized size = 2.3

$$\frac{1}{\tan(x)}+\log(|\tan(x)-1|)-\log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="giac")
```

```
[Out] 1/tan(x) + log(abs(tan(x) - 1)) - log(abs(tan(x)))
```

3.699 $\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx$

Optimal. Leaf size=176

$$\frac{x}{3\ 2^{2/3}\sqrt[6]{3}}+\frac{\log\left(2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}-\frac{\tan^{-1}\left(\frac{-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\sin(x)\cos\cdots}{\left(6-4\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\sin(x)\cos(x)+4\cdots}\right)}{3\ 2^{2/3}\sqrt[6]{3}}$$

```
[Out] x/(3*2^(2/3)*3^(1/6)) - ArcTan[(6^(2/3) - 2*6^(2/3)*Cos[x]^2 + 2*(3 - 2*6^(
1/3))*Cos[x]*Sin[x])/(3*2^(2/3)*3^(1/6) + 4*6^(1/3) + (6 - 4*6^(1/3))*Cos[x
]^2 + 2*6^(2/3)*Cos[x]*Sin[x])]/(3*2^(2/3)*3^(1/6)) - Log[3^(1/3) - 2^(2/3)
*Tan[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tan[x] + 2*2^(1/3)*Tan
[x]^2]/(6*6^(2/3))
```

Rubi [A] time = 0.13967, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3675, 200, 31, 634, 617, 204, 628}

$$\frac{x}{3\ 2^{2/3}\sqrt[6]{3}}+\frac{\log\left(2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}-\frac{\tan^{-1}\left(\frac{-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\sin(x)\cos\cdots}{\left(6-4\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\sin(x)\cos(x)+4\cdots}\right)}{3\ 2^{2/3}\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(3 - 4*Tan[x]^3),x]
```

```
[Out] x/(3*2^(2/3)*3^(1/6)) - ArcTan[(6^(2/3) - 2*6^(2/3)*Cos[x]^2 + 2*(3 - 2*6^(
1/3))*Cos[x]*Sin[x])/(3*2^(2/3)*3^(1/6) + 4*6^(1/3) + (6 - 4*6^(1/3))*Cos[x
]^2 + 2*6^(2/3)*Cos[x]*Sin[x])]/(3*2^(2/3)*3^(1/6)) - Log[3^(1/3) - 2^(2/3)
*Tan[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tan[x] + 2*2^(1/3)*Tan
[x]^2]/(6*6^(2/3))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
```

```
|| EqQ[n^2, 16])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{3-4x^3}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{3}-2^{2/3}x}\,dx, x, \tan(x)\right)}{3\ 3^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{3}+2^{2/3}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tan(x)\right)}{3\ 3^{2/3}}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tan(x)\right)}{2\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{3}+4\sqrt[3]{2}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x}\right.}{6\ 6^{2/3}}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tan(x)+2\sqrt[3]{2}\tan^2(x)\right)}{6\ 6^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.}{6^{2/3}}$$
$$= \frac{x}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\tan^{-1}\left(\frac{6^{2/3}-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\cos(x)\sin(x)}{3\ 2^{2/3}\sqrt[6]{3}+4\sqrt[3]{6}+2\left(3-2\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\cos(x)\sin(x)}\right)}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3\right.}{}$$

Mathematica [A] time = 0.121195, size = 74, normalized size = 0.42

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{2\ 6^{2/3}\tan(x)+3}{3\sqrt{3}}\right)+\log\left(2\sqrt[3]{6}\tan^2(x)+6^{2/3}\tan(x)+3\right)-2\log\left(3-6^{2/3}\tan(x)\right)}{6\ 6^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(3 - 4*Tan[x]^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tan[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)*T
an[x]] + Log[3 + 6^(2/3)*Tan[x] + 2*6^(1/3)*Tan[x]^2])/(6*6^(2/3))
```

Maple [A] time = 0.054, size = 80, normalized size = 0.5

$$-\frac{\sqrt[3]{3}4^{\frac{2}{3}}}{36}\ln\left(\tan(x)-\frac{\sqrt[3]{3}4^{\frac{2}{3}}}{4}\right)+\frac{\sqrt[3]{3}4^{\frac{2}{3}}}{72}\ln\left((\tan(x))^2+\frac{\sqrt[3]{3}4^{\frac{2}{3}}\tan(x)}{4}+\frac{3^{\frac{2}{3}}\sqrt[3]{4}}{4}\right)+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}}{36}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,3^{2/3}\sqrt[3]{4}\tan(x)}{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(3-4*tan(x)^3),x)
```

```
[Out] -1/36*3^(1/3)*4^(2/3)*ln(tan(x)-1/4*3^(1/3)*4^(2/3))+1/72*3^(1/3)*4^(2/3)*l
n(tan(x)^2+1/4*3^(1/3)*4^(2/3)*tan(x)+1/4*3^(2/3)*4^(1/3))+1/36*3^(5/6)*4^(
2/3)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*4^(1/3)*tan(x)+1))
```

Maxima [A] time = 1.45544, size = 120, normalized size = 0.68

$$\frac{1}{36} \cdot 4^{\frac{2}{3}} 3^{\frac{5}{6}} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} 3^{\frac{1}{6}} \left(2 \cdot 4^{\frac{2}{3}} \tan(x) + 4^{\frac{1}{3}} 3^{\frac{1}{3}}\right)\right) + \frac{1}{72} \cdot 4^{\frac{2}{3}} 3^{\frac{1}{3}} \log\left(4^{\frac{2}{3}} \tan(x)^2 + 4^{\frac{1}{3}} 3^{\frac{1}{3}} \tan(x) + 3^{\frac{2}{3}}\right) - \frac{1}{36} \cdot 4^{\frac{2}{3}} 3^{\frac{1}{3}} \mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="maxima")
```

```
[Out] 1/36*4^(2/3)*3^(5/6)*arctan(1/12*4^(2/3)*3^(1/6)*(2*4^(2/3)*tan(x) + 4^(1/3
)*3^(1/3))) + 1/72*4^(2/3)*3^(1/3)*log(4^(2/3)*tan(x)^2 + 4^(1/3)*3^(1/3)*t
an(x) + 3^(2/3)) - 1/36*4^(2/3)*3^(1/3)*log(1/4*4^(2/3)*(4^(1/3)*tan(x) - 3
^(1/3)))
```

Fricas [B] time = 3.06919, size = 1635, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="fricas")
```

```
[Out] -1/36*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/108*36^(1/6)*(28*(36^(2/3)*sqrt
(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*cos(x)^6 - 4*(14*36^(2/3)*sqrt(3)*(-
1)^(2/3) + 36*36^(1/3)*sqrt(3) - 63*sqrt(3)*(-1)^(1/3))*cos(x)^4 + (37*36^(
2/3)*sqrt(3)*(-1)^(2/3) + 144*36^(1/3)*sqrt(3) + 144*sqrt(3)*(-1)^(1/3))*co
s(x)^2 - 6*(16*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*cos(x)^
5 - (24*36^(2/3)*sqrt(3)*(-1)^(2/3) - 7*36^(1/3)*sqrt(3) - 72*sqrt(3)*(-1)^
(1/3))*cos(x)^3 + 4*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 4*36^(1/3)*sqrt(3) + 9*s
qrt(3)*(-1)^(1/3))*cos(x))*sin(x) - 18*36^(1/3)*sqrt(3) - 144*sqrt(3)*(-1)^
(1/3))/(48*cos(x)^6 - 72*cos(x)^4 + 18*cos(x)^2 + 14*(cos(x)^5 - cos(x)^3)*
sin(x) + 3)) - 1/432*36^(2/3)*(-1)^(1/3)*log(-3*(2*36^(2/3)*(-1)^(1/3) - 8*
36^(1/3)*(-1)^(2/3) + 25)*cos(x)^4 + 3*(3*36^(2/3)*(-1)^(1/3) - 4*36^(1/3)*
(-1)^(2/3) + 32)*cos(x)^2 - 2*((4*36^(2/3)*(-1)^(1/3) + 9*36^(1/3)*(-1)^(2/
3))*cos(x)^3 - 4*(36^(2/3)*(-1)^(1/3) - 9)*cos(x))*sin(x) - 12*36^(1/3)*(-1
)^(2/3) - 48) + 1/216*36^(2/3)*(-1)^(1/3)*log(3*(2*36^(2/3)*(-1)^(1/3) + 8*
```

```
36^(1/3)*(-1)^(2/3) - 7)*cos(x)^2 + 2*(4*36^(2/3)*(-1)^(1/3) - 9*36^(1/3)*(
-1)^(2/3) + 36)*cos(x)*sin(x) - 3*36^(2/3)*(-1)^(1/3) - 12*36^(1/3)*(-1)^(2
/3) + 48)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sec^2(x)}{4\tan^3(x) - 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(3-4*tan(x)**3),x)
```

```
[Out] -Integral(sec(x)**2/(4*tan(x)**3 - 3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.700 $\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx$

Optimal. Leaf size=53

$$\frac{2x}{\sqrt{195}}-\frac{2\tan^{-1}\left(\frac{10\cos^2(x)+12\sin(x)\cos(x)-5}{12\cos^2(x)-10\sin(x)\cos(x)+\sqrt{195}+10}\right)}{\sqrt{195}}$$

```
[Out] (2*x)/Sqrt[195] - (2*ArcTan[(-5 + 10*Cos[x]^2 + 12*Cos[x]*Sin[x])/(10 + Sqr
t[195] + 12*Cos[x]^2 - 10*Cos[x]*Sin[x])])/Sqrt[195]
```

Rubi [A] time = 0.0656439, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4342, 618, 204}

$$\frac{2x}{\sqrt{195}}-\frac{2\tan^{-1}\left(\frac{10\cos^2(x)+12\sin(x)\cos(x)-5}{12\cos^2(x)-10\sin(x)\cos(x)+\sqrt{195}+10}\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
```

```
[Out] (2*x)/Sqrt[195] - (2*ArcTan[(-5 + 10*Cos[x]^2 + 12*Cos[x]*Sin[x])/(10 + Sqr
t[195] + 12*Cos[x]^2 - 10*Cos[x]*Sin[x])])/Sqrt[195]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{11-5x+5x^2}\,dx, x, \tan(x)\right)\\ &= -\left(2\,\text{Subst}\left(\int \frac{1}{-195-x^2}\,dx, x, -5+10\tan(x)\right)\right)\\ &= \frac{2x}{\sqrt{195}}+\frac{2\tan^{-1}\left(\frac{5-10\cos^2(x)-12\cos(x)\sin(x)}{10+\sqrt{195}+12\cos^2(x)-10\cos(x)\sin(x)}\right)}{\sqrt{195}}\end{aligned}$$

Mathematica [A] time = 0.0549315, size = 22, normalized size = 0.42

$$-\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}(1-2\tan(x))\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
```

```
[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tan[x])])/Sqrt[195]
```

Maple [A] time = 0.064, size = 18, normalized size = 0.3

$$\frac{2\sqrt{195}}{195}\arctan\left(\frac{(10\tan(x)-5)\sqrt{195}}{195}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x)
```

```
[Out] 2/195*195^(1/2)*arctan(1/195*(10*tan(x)-5)*195^(1/2))
```

Maxima [A] time = 1.46819, size = 23, normalized size = 0.43

$$\frac{2}{195}\sqrt{195}\arctan\left(\frac{1}{39}\sqrt{195}(2\tan(x)-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="maxima")
```

```
[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

Fricas [A] time = 2.10278, size = 188, normalized size = 3.55

$$\frac{1}{195}\sqrt{195}\arctan\left(-\frac{192\sqrt{195}\cos(x)^2-160\sqrt{195}\cos(x)\sin(x)-35\sqrt{195}}{195\left(10\cos(x)^2+12\cos(x)\sin(x)-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="fricas")
```

```
[Out] 1/195*sqrt(195)*arctan(-1/195*(192*sqrt(195)*cos(x)^2 - 160*sqrt(195)*cos(x
)*sin(x) - 35*sqrt(195))/(10*cos(x)^2 + 12*cos(x)*sin(x) - 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2(x)}{5\tan^2(x)-5\tan(x)+11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(11-5*tan(x)+5*tan(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(5*tan(x)**2 - 5*tan(x) + 11), x)
```

Giac [A] time = 1.12237, size = 23, normalized size = 0.43

$$\frac{2}{195}\sqrt{195}\arctan\left(\frac{1}{39}\sqrt{195}(2\tan(x)-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="giac")
```

```
[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

3.701 $\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx$

Optimal. Leaf size=28

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(c+d\tan(x))}{d^2}$$

```
[Out] -(((b*c - a*d)*Log[c + d*Tan[x]])/d^2) + (b*Tan[x])/d
```

Rubi [A] time = 0.0873822, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4342, 43}

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(c+d\tan(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*Tan[x]])/d^2) + (b*Tan[x])/d
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = \text{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \tan(x)\right)$$
$$= -\frac{(bc-ad)\log(c+d\tan(x))}{d^2}+\frac{b\tan(x)}{d}$$

Mathematica [A] time = 0.353691, size = 54, normalized size = 1.93

$$\frac{\cos(x)(a+b\tan(x))((bc-ad)(\log(\cos(x))-\log(c\cos(x)+d\sin(x)))+bd\tan(x))}{d^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
```

```
[Out] (Cos[x]*(a + b*Tan[x])*((b*c - a*d)*(Log[Cos[x]] - Log[c*Cos[x] + d*Sin[x]]
) + b*d*Tan[x]))/(d^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.042, size = 35, normalized size = 1.3

$$\frac{b\tan(x)}{d}+\frac{\ln(c+d\tan(x))\,a}{d}-\frac{\ln(c+d\tan(x))\,cb}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x)
```

```
[Out] b*tan(x)/d+1/d*ln(c+d*tan(x))*a-1/d^2*ln(c+d*tan(x))*c*b
```

Maxima [A] time = 0.958968, size = 38, normalized size = 1.36

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(d\tan(x)+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] b*tan(x)/d - (b*c - a*d)*log(d*tan(x) + c)/d^2
```

Fricas [B] time = 2.33689, size = 194, normalized size = 6.93

$$-\frac{(bc-ad)\cos(x)\log\left(2\,cd\cos(x)\sin(x)+\left(c^2-d^2\right)\cos(x)^2+d^2\right)-(bc-ad)\cos(x)\log\left(\cos(x)^2\right)-2\,bd\sin(x)}{2\,d^2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((b*c - a*d)*cos(x)*log(2*c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d
^2) - (b*c - a*d)*cos(x)*log(cos(x)^2) - 2*b*d*sin(x))/(d^2*cos(x))
```

Sympy [A] time = 3.57323, size = 29, normalized size = 1.04

$$\frac{b\tan(x)}{d}+\frac{(ad-bc)\left(\begin{cases}\frac{\tan(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise}\end{cases}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))/(c+d*tan(x)),x)
```

```
[Out] b*tan(x)/d + (a*d - b*c)*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))
/d, True))/d
```

Giac [A] time = 1.11106, size = 39, normalized size = 1.39

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(|d\tan(x)+c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="giac")

[Out] b*tan(x)/d - (b*c - a*d)*log(abs(d*tan(x) + c))/d^2
```

3.702 $\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{b\tan(x)(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}+\frac{(a+b\tan(x))^2}{2d}$$

```
[Out] ((b*c - a*d)^2*Log[c + d*Tan[x]])/d^3 - (b*(b*c - a*d)*Tan[x])/d^2 + (a + b
*Tan[x])^2/(2*d)
```

Rubi [A] time = 0.137769, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$-\frac{b\tan(x)(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}+\frac{(a+b\tan(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

```
[Out] ((b*c - a*d)^2*Log[c + d*Tan[x]])/d^3 - (b*(b*c - a*d)*Tan[x])/d^2 + (a + b
*Tan[x])^2/(2*d)
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \tan(x)\right)$$
$$= \frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}-\frac{b(bc-ad)\tan(x)}{d^2}+\frac{(a+b\tan(x))^2}{2d}$$

Mathematica [A] time = 0.555891, size = 62, normalized size = 1.17

$$\frac{b^2d^2\sec^2(x)-2\left(bd\tan(x)(bc-2ad)+(bc-ad)^2(\log(\cos(x))-\log(c\cos(x)+d\sin(x)))\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

```
[Out] (b^2*d^2*Sec[x]^2 - 2*((b*c - a*d)^2*(Log[Cos[x]] - Log[c*Cos[x] + d*Sin[x]
]) + b*d*(b*c - 2*a*d)*Tan[x]))/(2*d^3)
```

Maple [A] time = 0.059, size = 80, normalized size = 1.5

$$\frac{b^2(\tan(x))^2}{2d}+2\,\frac{ab\tan(x)}{d}-\frac{b^2\tan(x)\,c}{d^2}+\frac{\ln(c+d\tan(x))\,a^2}{d}-2\,\frac{\ln(c+d\tan(x))\,abc}{d^2}+\frac{\ln(c+d\tan(x))\,b^2c^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x)
```

```
[Out] 1/2*b^2/d*tan(x)^2+2*b/d*a*tan(x)-b^2/d^2*tan(x)*c+1/d*ln(c+d*tan(x))*a^2-2
/d^2*ln(c+d*tan(x))*a*b*c+1/d^3*ln(c+d*tan(x))*b^2*c^2
```

Maxima [A] time = 0.990692, size = 85, normalized size = 1.6

$$\frac{b^2d\tan(x)^2-2\left(b^2c-2\,abd\right)\tan(x)}{2\,d^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log(d\tan(x)+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*tan(x)^2 - 2*(b^2*c - 2*a*b*d)*tan(x))/d^2 + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*log(d*tan(x) + c)/d^3
```

Fricas [B] time = 2.40642, size = 302, normalized size = 5.7

$$\frac{b^2d^2 + \left(b^2c^2 - 2\,abcd + a^2d^2\right)\cos\left(x\right)^2\log\left(2\,cd\cos\left(x\right)\sin\left(x\right) + \left(c^2 - d^2\right)\cos\left(x\right)^2 + d^2\right) - \left(b^2c^2 - 2\,abcd + a^2d^2\right)\cos\left(x\right)}{2\,d^3\cos\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2*log(2*c*d*cos(x)*si
n(x) + (c^2 - d^2)*cos(x)^2 + d^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)
^2*log(cos(x)^2) - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x))/(d^3*cos(x)^2)
```

Sympy [A] time = 4.70279, size = 56, normalized size = 1.06

$$\frac{b^{2}\tan^{2}\left(x\right)}{2d} + \frac{\left(ad - bc\right)^{2}\left(\begin{cases}\frac{\tan\left(x\right)}{c} & \text{for}\: d = 0\\\frac{\log\left(c + d\tan\left(x\right)\right)}{d} & \text{otherwise}\end{cases}\right)}{d^{2}} + \frac{\left(2abd - b^{2}c\right)\tan\left(x\right)}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**2/(c+d*tan(x)),x)
```

```
[Out] b**2*tan(x)**2/(2*d) + (a*d - b*c)**2*Piecewise((tan(x)/c, Eq(d, 0)), (log(
c + d*tan(x))/d, True))/d**2 + (2*a*b*d - b**2*c)*tan(x)/d**2
```

Giac [A] time = 1.09683, size = 86, normalized size = 1.62

$$\frac{b^2d\tan\left(x\right)^2 - 2\,b^2c\tan\left(x\right) + 4\,abd\tan\left(x\right)}{2\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|d\tan\left(x\right) + c\right|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*tan(x)^2 - 2*b^2*c*tan(x) + 4*a*b*d*tan(x))/d^2 + (b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*log(abs(d*tan(x) + c))/d^3
```

3.703 $\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx$

Optimal. Leaf size=78

$$\frac{b\tan(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tan(x))^2}{2d^2}-\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{(a+b\tan(x))^3}{3d}$$

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tan[x]])/d^4) + (b*(b*c - a*d)^2*Tan[x])/d^3 - (
(b*c - a*d)*(a + b*Tan[x])^2)/(2*d^2) + (a + b*Tan[x])^3/(3*d)
```

Rubi [A] time = 0.149608, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$\frac{b\tan(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tan(x))^2}{2d^2}-\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{(a+b\tan(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tan[x]])/d^4) + (b*(b*c - a*d)^2*Tan[x])/d^3 - (
(b*c - a*d)*(a + b*Tan[x])^2)/(2*d^2) + (a + b*Tan[x])^3/(3*d)
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = \text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx, x, \tan(x)\right)$$
$$= -\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4} + \frac{b(bc-ad)^2\tan(x)}{d^3} - \frac{(bc-ad)(a+b\tan(x))^2}{2d^2} + \frac{(a+b\text{ta}}{3a}$$

Mathematica [A] time = 0.897265, size = 133, normalized size = 1.71

$$\frac{(a+b\tan(x))^3(c\cos(x)+d\sin(x))\left(bd^2(9a\sin(2x)(ad-bc)+b(9ad-3bc+2bd\tan(x)))+6\cos^2(x)(bc-ad)^3(\log(\cos(\right.}{6d^4(c+d\tan(x))(a\cos(x)+b\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

```
[Out] ((c*Cos[x] + d*Sin[x])*(a + b*Tan[x])^3*(6*(b*c - a*d)^3*Cos[x]^2*(Log[Cos[
x]] - Log[c*Cos[x] + d*Sin[x]]) - b^3*d*(-3*c^2 + d^2)*Sin[2*x] + b*d^2*(9*
a*(-(b*c) + a*d)*Sin[2*x] + b*(-3*b*c + 9*a*d + 2*b*d*Tan[x]))))/(6*d^4*(a*
Cos[x] + b*Sin[x])^3*(c + d*Tan[x]))
```

Maple [A] time = 0.075, size = 143, normalized size = 1.8

$$\frac{b^3(\tan(x))^3}{3d} + \frac{3b^2(\tan(x))^2a}{2d} - \frac{b^3(\tan(x))^2c}{2d^2} + 3\frac{a^2b\tan(x)}{d} - 3\frac{ab^2c\tan(x)}{d^2} + \frac{b^3c^2\tan(x)}{d^3} + \frac{\ln(c+d\tan(x))a^3}{d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x)
```

```
[Out] 1/3*b^3/d*tan(x)^3+3/2*b^2/d*tan(x)^2*a-1/2*b^3/d^2*tan(x)^2*c+3*b/d*a^2*ta
n(x)-3*b^2/d^2*a*c*tan(x)+b^3/d^3*c^2*tan(x)+1/d*ln(c+d*tan(x))*a^3-3/d^2*l
n(c+d*tan(x))*a^2*b*c+3/d^3*ln(c+d*tan(x))*a*b^2*c^2-1/d^4*ln(c+d*tan(x))*b
^3*c^3
```

Maxima [A] time = 0.976686, size = 159, normalized size = 2.04

$$\frac{2\,b^3d^2\tan{(x)}^3-3\left(b^3cd-3\,ab^2d^2\right)\tan{(x)}^2+6\left(b^3c^2-3\,ab^2cd+3\,a^2bd^2\right)\tan{(x)}}{6\,d^3}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*tan(x)^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*tan(x)^2 + 6*(b^3*c^2 -
 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x))/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*log(d*tan(x) + c)/d^4
```

Fricas [B] time = 2.98869, size = 459, normalized size = 5.88

$$-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\cos{(x)}^3\log\left(2\,cd\cos{(x)}\sin{(x)}+\left(c^2-d^2\right)\cos{(x)}^2+d^2\right)-3\left(b^3c^3-3\,ab^2c^2d+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(2*
c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d^2) - 3*(b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(cos(x)^2) + 3*(b^3*c*d^2 - 3*a*b^
2*d^3)*cos(x) - 2*(b^3*d^3 + (3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)
*d^3)*cos(x)^2)*sin(x))/(d^4*cos(x)^3)
```

Sympy [A] time = 7.21373, size = 95, normalized size = 1.22

$$\frac{b^3\tan^3{(x)}}{3d}+\frac{\left(3ab^2d-b^3c\right)\tan^2{(x)}}{2d^2}+\frac{(ad-bc)^3\left(\begin{cases}\frac{\tan{(x)}}{c} & \text{for } d=0\\ \frac{\log{(c+d\tan{(x)})}}{d} & \text{otherwise}\end{cases}\right)}{d^3}+\frac{\left(3a^2bd^2-3ab^2cd+b^3c^2\right)\tan{(x)}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**3/(c+d*tan(x)),x)
```

```
[Out] b**3*tan(x)**3/(3*d) + (3*a*b**2*d - b**3*c)*tan(x)**2/(2*d**2) + (a*d - b*
c)**3*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))/d, True))/d**3 + (
3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*tan(x)/d**3
```

Giac [A] time = 1.08839, size = 166, normalized size = 2.13

$$\frac{2\,b^3d^2\tan\left(x\right)^3-3\,b^3cd\tan\left(x\right)^2+9\,ab^2d^2\tan\left(x\right)^2+6\,b^3c^2\tan\left(x\right)-18\,ab^2cd\tan\left(x\right)+18\,a^2bd^2\tan\left(x\right)}{6\,d^3}-\frac{\left(b^3c^3-3\,ab^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*tan(x)^3 - 3*b^3*c*d*tan(x)^2 + 9*a*b^2*d^2*tan(x)^2 + 6*b^3
*c^2*tan(x) - 18*a*b^2*c*d*tan(x) + 18*a^2*b*d^2*tan(x))/d^3 - (b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(d*tan(x) + c))/d^4
```

3.704 $\int \frac{\sec^2(x)\tan^2(x)}{\left(2+\tan^3(x)\right)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

```
[Out] -1/(3*(2 + Tan[x]^3))
```

Rubi [A] time = 0.0760194, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4342, 261}

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

```
[Out] -1/(3*(2 + Tan[x]^3))
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sec^2(x)\tan^2(x)}{\left(2+\tan^3(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(2+x^3\right)^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{1}{3\left(2+\tan^3(x)\right)}$$

Mathematica [A] time = 0.0357123, size = 12, normalized size = 1.

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

```
[Out] -1/(3*(2 + Tan[x]^3))
```

Maple [A] time = 0.072, size = 11, normalized size = 0.9

$$-\frac{1}{6+3\,(\tan(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x)
```

```
[Out] -1/3/(2+tan(x)^3)
```

Maxima [A] time = 0.97347, size = 14, normalized size = 1.17

$$-\frac{1}{3\left(\tan(x)^3+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(tan(x)^3 + 2)
```

Fricas [B] time = 2.02154, size = 109, normalized size = 9.08

$$-\frac{\cos(x)^3 + 2\left(\cos(x)^2 - 1\right)\sin(x)}{15\left(2\cos(x)^3 - \left(\cos(x)^2 - 1\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(x)^3 + 2*(cos(x)^2 - 1)*sin(x))/(2*cos(x)^3 - (cos(x)^2 - 1)*sin
(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**2/(2+tan(x)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10688, size = 14, normalized size = 1.17

$$-\frac{1}{3\left(\tan(x)^3 + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="giac")
```

```
[Out] -1/3/(tan(x)^3 + 2)
```

3.705 $\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^9(x)}{3}+\frac{\tan^7(x)}{7}$$

```
[Out] Tan[x]^7/7 + Tan[x]^9/3 + (3*Tan[x]^11)/11 + Tan[x]^13/13
```

Rubi [A] time = 0.0920761, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 2607, 270}

$$\frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^9(x)}{3}+\frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

```
[Out] Tan[x]^7/7 + Tan[x]^9/3 + (3*Tan[x]^11)/11 + Tan[x]^13/13
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx=\int \sec^8(x)\tan^6(x)\,dx$$
$$=\text{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx,x,\tan(x)\right)$$
$$=\text{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx,x,\tan(x)\right)$$
$$=\frac{\tan^7(x)}{7}+\frac{\tan^9(x)}{3}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^{13}(x)}{13}$$

Mathematica [B] time = 0.0263524, size = 67, normalized size = 2.03

$$-\frac{16\tan(x)}{3003}+\frac{1}{13}\tan(x)\sec^{12}(x)-\frac{27}{143}\tan(x)\sec^{10}(x)+\frac{53}{429}\tan(x)\sec^8(x)-\frac{5\tan(x)\sec^6(x)}{3003}-\frac{2\tan(x)\sec^4(x)}{1001}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

```
[Out] (-16*Tan[x])/3003 - (8*Sec[x]^2*Tan[x])/3003 - (2*Sec[x]^4*Tan[x])/1001 - (
5*Sec[x]^6*Tan[x])/3003 + (53*Sec[x]^8*Tan[x])/429 - (27*Sec[x]^10*Tan[x])/
143 + (Sec[x]^12*Tan[x])/13
```

Maple [A] time = 0.022, size = 42, normalized size = 1.3

$$\frac{(\sin(x))^7}{7(\cos(x))^7}+\frac{(\sin(x))^9}{3(\cos(x))^9}+\frac{3(\sin(x))^{11}}{11(\cos(x))^{11}}+\frac{(\sin(x))^{13}}{13(\cos(x))^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x)
```

```
[Out] 1/7*sin(x)^7/cos(x)^7+1/3*sin(x)^9/cos(x)^9+3/11*sin(x)^11/cos(x)^11+1/13*s
in(x)^13/cos(x)^13
```

Maxima [A] time = 0.959681, size = 34, normalized size = 1.03

$$\frac{1}{13}\tan(x)^{13}+\frac{3}{11}\tan(x)^{11}+\frac{1}{3}\tan(x)^9+\frac{1}{7}\tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

Fricas [A] time = 2.2282, size = 158, normalized size = 4.79

$$-\frac{\left(16 \cos\left(x\right)^{12} + 8 \cos\left(x\right)^{10} + 6 \cos\left(x\right)^{8} + 5 \cos\left(x\right)^{6} - 371 \cos\left(x\right)^{4} + 567 \cos\left(x\right)^{2} - 231\right) \sin\left(x\right)}{3003 \cos\left(x\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3003*(16*cos(x)^12 + 8*cos(x)^10 + 6*cos(x)^8 + 5*cos(x)^6 - 371*cos(x)^
4 + 567*cos(x)^2 - 231)*sin(x)/cos(x)^13
```

Sympy [A] time = 92.6382, size = 27, normalized size = 0.82

$$\frac{\tan^{13}\left(x\right)}{13} + \frac{3\tan^{11}\left(x\right)}{11} + \frac{\tan^{9}\left(x\right)}{3} + \frac{\tan^{7}\left(x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**6*(1+tan(x)**2)**3,x)
```

```
[Out] tan(x)**13/13 + 3*tan(x)**11/11 + tan(x)**9/3 + tan(x)**7/7
```

Giac [A] time = 1.10194, size = 34, normalized size = 1.03

$$\frac{1}{13} \tan\left(x\right)^{13} + \frac{3}{11} \tan\left(x\right)^{11} + \frac{1}{3} \tan\left(x\right)^{9} + \frac{1}{7} \tan\left(x\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

3.706 $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^3(x)}\,dx$

Optimal. Leaf size=46

$$\frac{2x}{\sqrt{3}}+\log(\tan(x)+1)+\frac{2\tan^{-1}\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] (2*x)/Sqrt[3] + (2*ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] - 2*Cos[x]*Sin[x])]
)/Sqrt[3] + Log[1 + Tan[x]]
```

Rubi [A] time = 0.0887961, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4342, 1863, 31, 618, 204}

$$\frac{2x}{\sqrt{3}}+\log(\tan(x)+1)+\frac{2\tan^{-1}\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```



```
[Out] (2*x)/Sqrt[3] + (2*ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] - 2*Cos[x]*Sin[x])]
)/Sqrt[3] + Log[1 + Tan[x]]
```


Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^3(x)}\,dx &= \text{Subst}\left(\int \frac{2+x^2}{1+x^3}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan(x)\right)+\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tan(x)\right)\\ &= \log(1+\tan(x))-2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tan(x)\right)\\ &= \frac{2x}{\sqrt{3}}+\frac{2\tan^{-1}\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{\sqrt{3}}+\log(1+\tan(x))\end{aligned}$$

Mathematica [A] time = 0.227092, size = 32, normalized size = 0.7

$$-\frac{2\tan^{-1}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\log(\cos(x))+\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*Tan[x])/Sqrt[3]])/Sqrt[3] - Log[Cos[x]] + Log[Cos[x] + Si
n[x]]
```

Maple [A] time = 0.108, size = 24, normalized size = 0.5

$$\ln(1+\tan(x))+\frac{2\sqrt{3}}{3}\arctan\left(\frac{(-1+2\tan(x))\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x)
```

```
[Out] ln(1+tan(x))+2/3*3^(1/2)*arctan(1/3*(-1+2*tan(x))*3^(1/2))
```

Maxima [A] time = 1.46102, size = 31, normalized size = 0.67

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right)+\log(\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(tan(x) + 1)
```

Fricas [A] time = 2.40297, size = 174, normalized size = 3.78

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\left(2\cos(x)^{2}-1\right)}\right)-\frac{1}{2}\log\left(\cos(x)^{2}\right)+\frac{1}{2}\log(2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^2 - 1)
) - 1/2*log(cos(x)^2) + 1/2*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] time = 8.21175, size = 41, normalized size = 0.89

$$\frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\left(\tan\left(x\right)-\frac{1}{2}\right)}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}+\log\left(\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**3),x)
```

```
[Out] 2*sqrt(3)*(atan(2*sqrt(3)*(tan(x) - 1/2)/3) + pi*floor((x - pi/2)/pi))/3 +
log(tan(x) + 1)
```

Giac [A] time = 1.10334, size = 32, normalized size = 0.7

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan\left(x\right)-1)\right)+\log\left(\left|\tan\left(x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(abs(tan(x) + 1))
```

3.707 $\int \left(1+\cos^2(x)\right)\sec^2(x)\,dx$

Optimal. Leaf size=4

$$x+\tan(x)$$

[Out] x + Tan[x]

Rubi [A] time = 0.0182728, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x+\tan(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x]^2)*Sec[x]^2,x]

[Out] x + Tan[x]

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(1+\cos^2(x)\right)\sec^2(x)\,dx &= \tan(x)+\int 1\,dx\\ &= x+\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0023942, size = 4, normalized size = 1.

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)*Sec[x]^2,x]
```

```
[Out] x + Tan[x]
```

Maple [A] time = 0.032, size = 5, normalized size = 1.3

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)*sec(x)^2,x)
```

```
[Out] x+tan(x)
```

Maxima [A] time = 1.46473, size = 5, normalized size = 1.25

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="maxima")
```

```
[Out] x + tan(x)
```

Fricas [B] time = 1.95409, size = 38, normalized size = 9.5

$$\frac{x \cos(x) + \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (x*cos(x) + sin(x))/cos(x)
```

Sympy [A] time = 23.6743, size = 3, normalized size = 0.75

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)*sec(x)**2,x)
```

```
[Out] x + tan(x)
```

Giac [B] time = 1.12031, size = 20, normalized size = 5.

$$-\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="giac")
```

```
[Out] -pi*floor(x/pi + 1/2) + x + tan(x)
```

3.708 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

Optimal. Leaf size=21

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rubi [A] time = 0.115477, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {616, 31}

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{2-3x+x^2}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-2+x}\,dx,x,\tan(x)\right)-\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\tan(x)\right)\\
&= -\log(1-\tan(x))+\log(2-\tan(x))
\end{aligned}$$

Mathematica [A] time = 0.0308383, size = 29, normalized size = 1.38

$$2\left(\frac{1}{2}\log(2\cos(x)-\sin(x))-\frac{1}{2}\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] 2*(-Log[Cos[x] - Sin[x]]/2 + Log[2*Cos[x] - Sin[x]]/2)
```

Maple [A] time = 0.057, size = 14, normalized size = 0.7

$$\ln(-2+\tan(x))-\ln(\tan(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x)
```

```
[Out] ln(-2+tan(x))-ln(tan(x)-1)
```

Maxima [A] time = 0.961783, size = 18, normalized size = 0.86

$$-\log(\tan(x)-1)+\log(\tan(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

Fricas [A] time = 2.0796, size = 104, normalized size = 4.95

$$\frac{1}{2}\log\left(\frac{3}{4}\cos(x)^{2}-\cos(x)\sin(x)+\frac{1}{4}\right)-\frac{1}{2}\log(-2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{-3\tan(x) + \sec^2(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

Giac [A] time = 1.14781, size = 20, normalized size = 0.95

$$-\log(|\tan(x) - 1|) + \log(|\tan(x) - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

3.709 $\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx$

Optimal. Leaf size=9

$$\sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

```
[Out] ArcSin[Tan[x]/Sqrt[3]]
```

Rubi [A] time = 0.0467953, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4146, 216}

$$\sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]
```

```
[Out] ArcSin[Tan[x]/Sqrt[3]]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{3-x^2}}\,dx, x, \tan(x)\right)$$
$$= \sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

Mathematica [B] time = 0.0437544, size = 43, normalized size = 4.78

$$\frac{\sqrt{2\cos(2x)+1}\sec(x)\tan^{-1}\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)}{\sqrt{4-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]
```

```
[Out] (ArcTan[Sin[x]/Sqrt[3 - 4*Sin[x]^2]]*Sqrt[1 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 -
Sec[x]^2]
```

Maple [C] time = 0.164, size = 103, normalized size = 11.4

$$-\frac{\sqrt{3}\sqrt{2}\sqrt{6}\left(\sin(x)\right)^2}{9\cos(x)\left(-1+\cos(x)\right)}\sqrt{\frac{2\cos(x)-1}{1+\cos(x)}}\sqrt{\frac{1+2\cos(x)}{1+\cos(x)}}\left(\text{EllipticF}\left(\frac{\sqrt{3}\left(-1+\cos(x)\right)}{\sin(x)},\frac{1}{3}\right)-2\,\text{EllipticPi}\left(\frac{\sqrt{3}\left(-1+\cos(x)\right)}{\sin(x)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(4-sec(x)^2)^(1/2),x)
```

```
[Out] -1/9*3^(1/2)*2^(1/2)*((2*cos(x)-1)/(1+cos(x)))^(1/2)*6^(1/2)*((1+2*cos(x))/
(1+cos(x)))^(1/2)*(EllipticF(3^(1/2)*(-1+cos(x))/sin(x),1/3)-2*EllipticPi(3
^(1/2)*(-1+cos(x))/sin(x),1/3,1/3))*sin(x)^2/((4*cos(x)^2-1)/cos(x)^2)^(1/2
)/cos(x)/(-1+cos(x))
```

Maxima [A] time = 1.41891, size = 11, normalized size = 1.22

$$\arcsin\left(\frac{1}{3}\sqrt{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/3*sqrt(3)*tan(x))
```

Fricas [B] time = 2.11589, size = 76, normalized size = 8.44

$$-\arctan\left(\frac{\sqrt{\frac{4\cos(x)^2-1}{\cos(x)^2}}\cos(x)}{\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt((4*cos(x)^2 - 1)/cos(x)^2)*cos(x)/sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{\sqrt{-(\sec(x)-2)(\sec(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(4-sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt(-(sec(x) - 2)*(sec(x) + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)^2}{\sqrt{-\sec(x)^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(x)^2/sqrt(-sec(x)^2 + 4), x)
```

3.710 $\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin^{-1}(2\tan(x))$$

```
[Out] ArcSin[2*Tan[x]]/2
```

Rubi [A] time = 0.0468873, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3675, 216}

$$\frac{1}{2}\sin^{-1}(2\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]

[Out] ArcSin[2*Tan[x]]/2
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{1-4x^2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\sin^{-1}(2\tan(x))$$

Mathematica [B] time = 0.0622612, size = 47, normalized size = 5.22

$$\frac{\sqrt{5\cos(2x)-3}\sec(x)\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{1-5\sin^2(x)}}\right)}{2\sqrt{2-8\tan^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]
```

```
[Out] (ArcTan[(2*Sin[x])/Sqrt[1 - 5*Sin[x]^2]]*Sqrt[-3 + 5*Cos[2*x]]*Sec[x])/(2*S
qrt[2 - 8*Tan[x]^2])
```

Maple [C] time = 0.361, size = 172, normalized size = 19.1

$$\frac{\sqrt{2}\,(\sin(x))^2}{\sqrt{9+4\sqrt{5}}\cos(x)(-1+\cos(x))}\sqrt{\frac{2\cos(x)\sqrt{5}-2\sqrt{5}+5\cos(x)-4}{1+\cos(x)}}\sqrt{-2\frac{2\cos(x)\sqrt{5}-2\sqrt{5}-5\cos(x)+4}{1+\cos(x)}}\Bigg(2\,\text{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1-4*tan(x)^2)^(1/2),x)
```

```
[Out] 1/(9+4*5^(1/2))^(1/2)*2^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)+5*cos(x)-4)/(1+c
os(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)-5*cos(x)+4)/(1+cos(x)))^(1/2)
*(2*EllipticPi((9+4*5^(1/2))^(1/2)*(-1+cos(x))/sin(x),1/(9+4*5^(1/2)),(9-4*
5^(1/2))^(1/2)/(9+4*5^(1/2))^(1/2))-EllipticF((-1+cos(x))*(5^(1/2)+2)/sin(x
),9-4*5^(1/2)))*sin(x)^2/((5*cos(x)^2-4)/cos(x)^2)^(1/2)/cos(x)/(-1+cos(x))
```

Maxima [A] time = 1.48008, size = 9, normalized size = 1.

$$\frac{1}{2} \arcsin\left(2 \tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2*tan(x))
```

Fricas [B] time = 2.24892, size = 135, normalized size = 15.

$$-\frac{1}{4} \arctan\left(\frac{\left(9 \cos\left(x\right)^{3} - 8 \cos\left(x\right)\right)\sqrt{\frac{5 \cos(x)^{2}-4}{\cos(x)^{2}}}}{4\left(5 \cos\left(x\right)^{2} - 4\right)\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(1/4*(9*cos(x)^3 - 8*cos(x))*sqrt((5*cos(x)^2 - 4)/cos(x)^2)/((5
*cos(x)^2 - 4)*sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(x\right)}{\sqrt{-\left(2\tan\left(x\right) - 1\right)\left(2\tan\left(x\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1-4*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt(-(2*tan(x) - 1)*(2*tan(x) + 1)), x)
```

Giac [A] time = 1.15862, size = 9, normalized size = 1.

$$\frac{1}{2}\arcsin\left(2\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(2*tan(x))
```

3.711 $\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

```
[Out] ArcTanh[Tan[x]/Sqrt[-4 + Tan[x]^2]]
```

Rubi [A] time = 0.0444797, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3675, 217, 206}

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
```

```
[Out] ArcTanh[Tan[x]/Sqrt[-4 + Tan[x]^2]]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-4+x^2}}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$

$$= \tanh^{-1}\left(\frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$

Mathematica [B] time = 0.0518447, size = 46, normalized size = 3.29

$$\frac{\sqrt{5\cos(2x)+3}\sec(x)\tan^{-1}\left(\frac{\sin(x)}{\sqrt{4-5\sin^2(x)}}\right)}{\sqrt{2}\sqrt{\tan^2(x)-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
```

```
[Out] (ArcTan[Sin[x]/Sqrt[4 - 5*Sin[x]^2]]*Sqrt[3 + 5*Cos[2*x]]*Sec[x])/(Sqrt[2]*
Sqrt[-4 + Tan[x]^2])
```

Maple [C] time = 0.394, size = 173, normalized size = 12.4

$$\frac{\sqrt{2}\,(\sin(x))^2}{4\sqrt{3/2-1/2\sqrt{5}}\cos(x)\,(-1+\cos(x))}\sqrt{-2\,\frac{\cos(x)\sqrt{5}-\sqrt{5}-5\,\cos(x)+1}{1+\cos(x)}}\sqrt{\frac{\cos(x)\sqrt{5}-\sqrt{5}+5\,\cos(x)-1}{1+\cos(x)}}\left(2\,\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(-4+tan(x)^2)^(1/2),x)
```

```
[Out] 1/4/(3/2-1/2*5^(1/2))^(1/2)*(-2*(cos(x)*5^(1/2)-5^(1/2)-5*cos(x)+1)/(1+cos(
x)))^(1/2)*2^(1/2)*((cos(x)*5^(1/2)-5^(1/2)+5*cos(x)-1)/(1+cos(x)))^(1/2)*(
2*EllipticPi((3/2-1/2*5^(1/2))^(1/2)*(-1+cos(x))/sin(x),-2/(5^(1/2)-3),(3/2
+1/2*5^(1/2))^(1/2)/(3/2-1/2*5^(1/2))^(1/2))-EllipticF(1/2*(-1+cos(x))*(5^(
1/2)-1)/sin(x),3/2+1/2*5^(1/2)))*sin(x)^2/(-(5*cos(x)^2-1)/cos(x)^2)^(1/2)/
cos(x)/(-1+cos(x))
```

Maxima [A] time = 0.967429, size = 22, normalized size = 1.57

$$\log\left(2\sqrt{\tan(x)^2-4}+2\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(tan(x)^2 - 4) + 2*tan(x))
```

Fricas [B] time = 2.1551, size = 224, normalized size = 16.

$$\frac{1}{4}\log\left(\frac{1}{2}\sqrt{-\frac{5\cos(x)^2-1}{\cos(x)^2}}\cos(x)\sin(x)-\frac{3}{2}\cos(x)^2+\frac{1}{2}\right)-\frac{1}{4}\log\left(-\frac{1}{2}\sqrt{-\frac{5\cos(x)^2-1}{\cos(x)^2}}\cos(x)\sin(x)-\frac{3}{2}\cos(x)^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*cos(x)^2 +
 1/2) - 1/4*log(-1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*c
os(x)^2 + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{\sqrt{(\tan(x)-2)(\tan(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(-4+tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt((tan(x) - 2)*(tan(x) + 2)), x)
```

Giac [A] time = 1.17512, size = 23, normalized size = 1.64

$$-\log\left(\left|\sqrt{\tan(x)^2-4}-\tan(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(sqrt(tan(x)^2 - 4) - tan(x)))
```

3.712 $\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx$

Optimal. Leaf size=19

$$\tan(x)\sqrt{1-\cot^2(x)}+\sin^{-1}(\cot(x))$$

```
[Out] ArcSin[Cot[x]] + Sqrt[1 - Cot[x]^2]*Tan[x]
```

Rubi [A] time = 0.0494782, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3663, 277, 216}

$$\tan(x)\sqrt{1-\cot^2(x)}+\sin^{-1}(\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]
```

```
[Out] ArcSin[Cot[x]] + Sqrt[1 - Cot[x]^2]*Tan[x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx = -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \cot(x)\right)$$
$$= \sqrt{1-\cot^2(x)}\tan(x) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cot(x)\right)$$
$$= \sin^{-1}(\cot(x)) + \sqrt{1-\cot^2(x)}\tan(x)$$

Mathematica [B] time = 0.528552, size = 52, normalized size = 2.74

$$\tan(x)\sqrt{1-\cot^2(x)}\sec(2x)\left(\cos(2x)-\cos(x)\sqrt{-\cos(2x)}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]
```

```
[Out] (-(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Cos[x]*Sqrt[-Cos[2*x]]) + Cos[2*x])*Sqrt[
1 - Cot[x]^2]*Sec[2*x]*Tan[x]
```

Maple [C] time = 0.318, size = 223, normalized size = 11.7

$$-\frac{-1+\cos(x)}{2\cos(x)\sin(x)}\left(4\,i\cos(x)\ln\left(4\frac{-1+\cos(x)}{(\sin(x))^2}\left(2\,i\cos(x)-\cos(x)\sqrt{-\frac{2(\cos(x))^2-1}{(1+\cos(x))^2}}+i-\sqrt{-\frac{2(\cos(x))^2-1}{(1+\cos(x))^2}}\right)\right)-c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1-cot(x)^2)^(1/2),x)
```

```
[Out] -1/2*(-1+cos(x))*(4*I*cos(x)*ln(4*(-1+cos(x))*(2*I*cos(x)-cos(x)*(-(2*cos(x
)^2-1)/(1+cos(x))^2)^(1/2)+I-(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))/sin(x)^2
)-cos(x)*arctan((2*cos(x)^2-3*cos(x)+1)/(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2
)/sin(x)^2)-3*cos(x)*arcsin(1/2*2^(1/2)*(1+2*cos(x))/(1+cos(x)))+2*cos(x)*(
-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+2*(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))
*((2*cos(x)^2-1)/(-1+cos(x)^2))^(1/2)/cos(x)/sin(x)/(-(2*cos(x)^2-1)/(1+cos
```

```
(x))^2)^(1/2)
```

Maxima [A] time = 1.45095, size = 41, normalized size = 2.16

$$\sqrt{-\frac{1}{\tan(x)^2}+1}\tan(x)-\arctan\left(\sqrt{-\frac{1}{\tan(x)^2}+1}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-1/tan(x)^2 + 1)*tan(x) - arctan(sqrt(-1/tan(x)^2 + 1)*tan(x))
```

Fricas [B] time = 2.26284, size = 225, normalized size = 11.84

$$-\frac{\arctan\left(\frac{(3\cos(x)^2-1)\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\,(2\cos(x)^3-\cos(x))}\right)\cos(x)-2\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(arctan(1/2*(3*cos(x)^2 - 1)*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1))*sin
(x)/(2*cos(x)^3 - cos(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1))
*sin(x))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-(\cot(x)-1)(\cot(x)+1)}\sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(cot(x) - 1)*(cot(x) + 1))*sec(x)**2, x)
```

Giac [C] time = 1.20651, size = 192, normalized size = 10.11

$$-\frac{1}{2}\left(\pi+2\arctan\left(-i\right)+2i\right)\mathrm{sgn}\left(\sin\left(x\right)\right)+\frac{1}{4}\left(2\,\pi\mathrm{sgn}\left(\cos\left(x\right)\right)+\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}}{\cos\left(x\right)}-\frac{4\,\cos\left(x\right)}{\sqrt{2}\sqrt{-2\,\cos\left(x\right)^{2}+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(pi + 2*arctan(-I) + 2*I)*sgn(sin(x)) + 1/4*(2*pi*sgn(cos(x)) + sqrt(2
)*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqr
t(-2*cos(x)^2 + 1) - sqrt(2))) + 4*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2*co
s(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1
) - sqrt(2))))*sgn(sin(x))
```

3.713 $\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}+\frac{1}{2}\sin^{-1}(\tan(x))$$

```
[Out] ArcSin[Tan[x]]/2 + (Tan[x]*Sqrt[1 - Tan[x]^2])/2
```

Rubi [A] time = 0.0458562, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3675, 195, 216}

$$\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}+\frac{1}{2}\sin^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

```
[Out] ArcSin[Tan[x]]/2 + (Tan[x]*Sqrt[1 - Tan[x]^2])/2
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx &= \text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\sin^{-1}(\tan(x)) + \frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}\end{aligned}$$

Mathematica [B] time = 0.112636, size = 63, normalized size = 2.42

$$\frac{\cos(2x)\tan(x) + \sqrt{\cos^2(x)}\cos(x)\sqrt{1-\tan^2(x)}\sin^{-1}\left(\frac{\sin(x)}{\sqrt{\cos^2(x)}}\right)}{2\sqrt{\cos^2(x)}\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

```
[Out] (Cos[2*x]*Tan[x] + ArcSin[Sin[x]/Sqrt[Cos[x]^2]]*Cos[x]*Sqrt[Cos[x]^2]*Sqrt
[1 - Tan[x]^2])/(2*Sqrt[Cos[x]^2]*Sqrt[Cos[2*x]])
```

Maple [C] time = 0.21, size = 492, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1-tan(x)^2)^(1/2),x)
```

```
[Out] 1/2/(1+2^(1/2))/(3+2*2^(1/2))^(1/2)*sin(x)*(-2^(1/2)*((cos(x)*2^(1/2)-2^(1/
2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1
+cos(x)))^(1/2)*EllipticF((1+2^(1/2))*(-1+cos(x))/sin(x),3-2*2^(1/2))*cos(x
)^2*sin(x)+2*2^(1/2)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)
*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticPi((3+2*
```

```
2^(1/2))^(1/2)*(-1+cos(x))/sin(x),1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1/2)/(3+2*
2^(1/2))^(1/2))*cos(x)^2*sin(x)-2*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+c
os(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*Ell
ipticF((1+2^(1/2))*(-1+cos(x))/sin(x),3-2*2^(1/2))*cos(x)^2*sin(x)+4*((cos(
x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2
)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticPi((3+2*2^(1/2))^(1/2)*(-1+cos(x))/
sin(x),1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2))*cos(x)^2*si
n(x)+4*cos(x)^3*2^(1/2)-4*cos(x)^2*2^(1/2)+6*cos(x)^3-2*cos(x)*2^(1/2)-6*co
s(x)^2+2*2^(1/2)-3*cos(x)+3)*((2*cos(x)^2-1)/cos(x)^2)^(1/2)/(-1+cos(x))/(2
*cos(x)^2-1)/cos(x)
```

Maxima [A] time = 1.46187, size = 27, normalized size = 1.04

$$\frac{1}{2}\sqrt{-\tan(x)^2+1}\tan(x)+\frac{1}{2}\arcsin(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

Fricas [B] time = 2.34886, size = 215, normalized size = 8.27

$$-\frac{\arctan\left(\frac{\left(3\cos(x)^3-2\cos(x)\right)\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2}}}{2\left(2\cos(x)^2-1\right)\sin(x)}\right)\cos(x)-2\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2}}\sin(x)}{4\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(arctan(1/2*(3*cos(x)^3 - 2*cos(x))*sqrt((2*cos(x)^2 - 1)/cos(x)^2)/((
2*cos(x)^2 - 1)*sin(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/cos(x)^2)*sin(x))
/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(\tan\left(x\right)-1\right)\left(\tan\left(x\right)+1\right)}\sec^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(tan(x) - 1)*(tan(x) + 1))*sec(x)**2, x)
```

Giac [A] time = 1.10653, size = 27, normalized size = 1.04

$$\frac{1}{2}\sqrt{-\tan\left(x\right)^{2}+1}\tan\left(x\right)+\frac{1}{2}\arcsin\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

3.714 $\int e^{\tan(x)} \sec^2(x)\, dx$

Optimal. Leaf size=4

$$e^{\tan(x)}$$

[Out] E^Tan[x]

Rubi [A] time = 0.0123224, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4342, 2194}

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Tan[x]*Sec[x]^2,x]

[Out] E^Tan[x]

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\tan(x)} \sec^2(x)\, dx &= \text{Subst}\left(\int e^x\, dx, x, \tan(x)\right)\\ &= e^{\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.0600031, size = 4, normalized size = 1.

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Tan[x]*Sec[x]^2,x]
```

```
[Out] E^Tan[x]
```

Maple [A] time = 0.012, size = 4, normalized size = 1.

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(tan(x))*sec(x)^2,x)
```

```
[Out] exp(tan(x))
```

Maxima [A] time = 0.961285, size = 4, normalized size = 1.

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] e^tan(x)
```

Fricas [B] time = 1.98432, size = 26, normalized size = 6.5

$$e^{\frac{\sin(x)}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] e^(sin(x)/cos(x))
```

Sympy [A] time = 1.72, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)**2,x)
```

```
[Out] exp(tan(x))
```

Giac [A] time = 1.1086, size = 4, normalized size = 1.

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="giac")
```

```
[Out] e^tan(x)
```

3.715 $\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^8(x)}{8}+\frac{\tan^6(x)}{6}$$

```
[Out] Tan[x]^6/6 + Tan[x]^8/8
```

Rubi [A] time = 0.0666117, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 2607, 14}

$$\frac{\tan^8(x)}{8}+\frac{\tan^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

```
[Out] Tan[x]^6/6 + Tan[x]^8/8
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx = \int \sec^4(x)\tan^5(x)\,dx$$
$$= \text{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, \tan(x)\right)$$
$$= \frac{\tan^6(x)}{6}+\frac{\tan^8(x)}{8}$$

Mathematica [A] time = 0.0175653, size = 25, normalized size = 1.47

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{3}+\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

```
[Out] Sec[x]^4/4 - Sec[x]^6/3 + Sec[x]^8/8
```

Maple [A] time = 0.013, size = 20, normalized size = 1.2

$$\frac{(\sec(x))^8}{8}-\frac{(\sec(x))^6}{3}+\frac{(\sec(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x)
```

```
[Out] 1/8*sec(x)^8-1/3*sec(x)^6+1/4*sec(x)^4
```

Maxima [B] time = 0.955177, size = 57, normalized size = 3.35

$$\frac{6\sin(x)^4-4\sin(x)^2+1}{24\left(\sin(x)^8-4\sin(x)^6+6\sin(x)^4-4\sin(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/24*(6*sin(x)^4 - 4*sin(x)^2 + 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*
sin(x)^2 + 1)
```

Fricas [A] time = 2.09194, size = 61, normalized size = 3.59

$$\frac{6 \cos(x)^4 - 8 \cos(x)^2 + 3}{24 \cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

Sympy [A] time = 9.18103, size = 19, normalized size = 1.12

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{3} + \frac{\sec^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*(-1+sec(x)**2)**2*tan(x),x)
```

```
[Out] sec(x)**8/8 - sec(x)**6/3 + sec(x)**4/4
```

Giac [A] time = 1.10274, size = 27, normalized size = 1.59

$$\frac{6 \cos(x)^4 - 8 \cos(x)^2 + 3}{24 \cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

3.716 $\int \frac{\csc^2(x)}{a+b\cot(x)} \, dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\cot(x))}{b}$$

```
[Out] -(Log[a + b*Cot[x]]/b)
```

Rubi [A] time = 0.0411302, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$-\frac{\log(a+b\cot(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cot[x]),x]
```

```
[Out] -(Log[a + b*Cot[x]]/b)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+b\cot(x)} \, dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x} \, dx, x, b\cot(x)\right)}{b}\\ &= -\frac{\log(a+b\cot(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0586226, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x)) - \log(a\sin(x) + b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cot[x]),x]
```

```
[Out] (Log[Sin[x]] - Log[b*Cos[x] + a*Sin[x]])/b
```

Maple [A] time = 0.025, size = 13, normalized size = 1.1

$$-\frac{\ln(a + b\cot(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cot(x)),x)
```

```
[Out] -ln(a+b*cot(x))/b
```

Maxima [A] time = 0.960791, size = 16, normalized size = 1.33

$$-\frac{\log(b\cot(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="maxima")
```

```
[Out] -log(b*cot(x) + a)/b
```

Fricas [B] time = 2.267, size = 123, normalized size = 10.25

$$-\frac{\log\left(2\,ab\cos(x)\sin(x) - \left(a^2 - b^2\right)\cos(x)^2 + a^2\right) - \log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(log(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 + a^2) - log(-1/4*cos(
x)^2 + 1/4))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{a + b\cot(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cot(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cot(x)), x)
```

Giac [A] time = 1.12864, size = 30, normalized size = 2.5

$$-\frac{\log(|a\tan(x) + b|)}{b} + \frac{\log(|\tan(x)|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="giac")
```

```
[Out] -log(abs(a*tan(x) + b))/b + log(abs(tan(x)))/b
```

3.717 $\int (a+b\cot(x))^n \csc^2(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$$

```
[Out] -((a + b*Cot[x])^(1 + n)/(b*(1 + n)))
```

Rubi [A] time = 0.0412348, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x])^n*Csc[x]^2,x]
```

```
[Out] -((a + b*Cot[x])^(1 + n)/(b*(1 + n)))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\cot(x))^n \csc^2(x)\,dx &= -\frac{\operatorname{Subst}\left(\int (a+x)^n\,dx, x, b\cot(x)\right)}{b}\\
&= -\frac{(a+b\cot(x))^{1+n}}{b(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.183713, size = 19, normalized size = 0.95

$$-\frac{(a+b\cot(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x])^n*Csc[x]^2,x]
```

```
[Out] -((a + b*Cot[x])^(1 + n)/(b + b*n))
```

Maple [A] time = 0.023, size = 21, normalized size = 1.1

$$-\frac{(a+b\cot(x))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^n*csc(x)^2,x)
```

```
[Out] -(a+b*cot(x))^(1+n)/b/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.41513, size = 103, normalized size = 5.15

$$-\frac{(b\cos(x)+a\sin(x))\left(\frac{b\cos(x)+a\sin(x)}{\sin(x)}\right)^{n}}{(bn+b)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(x) + a*sin(x))*((b*cos(x) + a*sin(x))/sin(x))^n/((b*n + b)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**n*csc(x)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cot(x) + a)^n \csc(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(x) + a)^n*csc(x)^2, x)
```

3.718 $\int \csc^2(x) \left(1+\sin^2(x)\right) \, dx$

Optimal. Leaf size=6

$$x-\cot(x)$$

```
[Out] x - Cot[x]
```

Rubi [A] time = 0.0159719, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*(1 + Sin[x]^2),x]
```

```
[Out] x - Cot[x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \csc^2(x) \left(1+\sin^2(x)\right) \, dx &= -\cot(x)+\int 1 \, dx \\ &= x-\cot(x) \end{aligned}$$

Mathematica [A] time = 0.0028742, size = 6, normalized size = 1.

$$x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*(1 + Sin[x]^2),x]
```

```
[Out] x - Cot[x]
```

Maple [A] time = 0.017, size = 7, normalized size = 1.2

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*(1+sin(x)^2),x)
```

```
[Out] x-cot(x)
```

Maxima [A] time = 1.47256, size = 11, normalized size = 1.83

$$x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] x - 1/tan(x)
```

Fricas [B] time = 2.41083, size = 38, normalized size = 6.33

$$\frac{x\sin(x) - \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] (x*sin(x) - cos(x))/sin(x)
```

Sympy [A] time = 13.1619, size = 3, normalized size = 0.5

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*(1+sin(x)**2),x)
```

```
[Out] x - cot(x)
```

Giac [B] time = 1.10361, size = 22, normalized size = 3.67

$$x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.719 $\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx$

Optimal. Leaf size=6

$$x-\cot(x)$$

```
[Out] x - Cot[x]
```

Rubi [A] time = 0.0470889, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {14, 203}

$$x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]
```

```
[Out] x - Cot[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx = \text{Subst}\left(\int \frac{1+\frac{1}{1+\frac{1}{x^2}}}{x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{1+x^2}\right)dx, x, \tan(x)\right)$$
$$= -\cot(x) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= x - \cot(x)$$

Mathematica [A] time = 0.0053662, size = 6, normalized size = 1.

$$x - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]
```

```
[Out] x - Cot[x]
```

Maple [A] time = 0.025, size = 7, normalized size = 1.2

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/(1+cot(x)^2))*csc(x)^2,x)
```

```
[Out] x-cot(x)
```

Maxima [A] time = 1.4416, size = 11, normalized size = 1.83

$$x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="maxima")
```

```
[Out] x - 1/tan(x)
```

Fricas [B] time = 2.31265, size = 38, normalized size = 6.33

$$\frac{x\sin(x) - \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="fricas")
```

```
[Out] (x*sin(x) - cos(x))/sin(x)
```

Sympy [B] time = 0.938944, size = 27, normalized size = 4.5

$$\frac{x\csc^2(x)}{\cot^2(x)+1} - \frac{\cot(x)\csc^2(x)}{\cot^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)**2))*csc(x)**2,x)
```

```
[Out] x*csc(x)**2/(cot(x)**2 + 1) - cot(x)*csc(x)**2/(cot(x)**2 + 1)
```

Giac [B] time = 1.11885, size = 22, normalized size = 3.67

$$x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="giac")
```

```
[Out] x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.720 $\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=28

$$\frac{(bc-ad)\log(c+d\cot(x))}{d^2}-\frac{b\cot(x)}{d}$$

```
[Out] -((b*Cot[x])/d) + ((b*c - a*d)*Log[c + d*Cot[x]])/d^2
```

Rubi [A] time = 0.0818833, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4344, 43}

$$\frac{(bc-ad)\log(c+d\cot(x))}{d^2}-\frac{b\cot(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] -((b*Cot[x])/d) + ((b*c - a*d)*Log[c + d*Cot[x]])/d^2
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx = -\operatorname{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \cot(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \cot(x)\right)$$
$$= -\frac{b\cot(x)}{d}+\frac{(bc-ad)\log(c+d\cot(x))}{d^2}$$

Mathematica [A] time = 0.329919, size = 56, normalized size = 2.

$$\frac{\sin(x)(a+b\cot(x))(-(bc-ad)(\log(\sin(x))-\log(c\sin(x)+d\cos(x)))-bd\cot(x))}{d^2(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] ((a + b*Cot[x])*(-(b*d*Cot[x]) - (b*c - a*d)*(Log[Sin[x]] - Log[d*Cos[x] +
c*Sin[x]]))*Sin[x])/(d^2*(b*Cos[x] + a*Sin[x]))
```

Maple [A] time = 0.046, size = 56, normalized size = 2.

$$-\frac{b}{d\tan(x)}+\frac{\ln(\tan(x))a}{d}-\frac{\ln(\tan(x))cb}{d^2}-\frac{\ln(c\tan(x)+d)a}{d}+\frac{\ln(c\tan(x)+d)cb}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x)
```

```
[Out] -b/d/tan(x)+1/d*ln(tan(x))*a-1/d^2*ln(tan(x))*c*b-1/d*ln(c*tan(x)+d)*a+1/d^
2*ln(c*tan(x)+d)*c*b
```

Maxima [A] time = 0.979703, size = 62, normalized size = 2.21

$$\frac{(bc-ad)\log(c\tan(x)+d)}{d^2}-\frac{(bc-ad)\log(\tan(x))}{d^2}-\frac{b}{d\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*log(c*tan(x) + d)/d^2 - (b*c - a*d)*log(tan(x))/d^2 - b/(d*tan(
x))
```

Fricas [B] time = 2.95262, size = 209, normalized size = 7.46

$$-\frac{2\,bd\cos\left(x\right)-\left(bc-ad\right)\log\left(2\,cd\cos\left(x\right)\sin\left(x\right)-\left(c^2-d^2\right)\cos\left(x\right)^2+c^2\right)\sin\left(x\right)+\left(bc-ad\right)\log\left(-\frac{1}{4}\,\cos\left(x\right)^2+\frac{1}{4}\right)\sin\left(x\right)}{2\,d^2\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*cos(x) - (b*c - a*d)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)*cos(
x)^2 + c^2)*sin(x) + (b*c - a*d)*log(-1/4*cos(x)^2 + 1/4)*sin(x))/(d^2*sin(
x))
```

Sympy [A] time = 11.5975, size = 31, normalized size = 1.11

$$-\frac{b\cot\left(x\right)}{d}-\frac{\left(ad-bc\right)\left(\begin{cases}\frac{\cot\left(x\right)}{c} & \text{for}\: d=0\\\frac{\log\left(c+d\cot\left(x\right)\right)}{d} & \text{otherwise}\end{cases}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b*cot(x)/d - (a*d - b*c)*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x)
)/d, True))/d
```

Giac [B] time = 1.15447, size = 92, normalized size = 3.29

$$-\frac{\left(bc-ad\right)\log\left(\left|\tan\left(x\right)\right|\right)}{d^2}+\frac{\left(bc^2-acd\right)\log\left(\left|c\tan\left(x\right)+d\right|\right)}{cd^2}+\frac{bc\tan\left(x\right)-ad\tan\left(x\right)-bd}{d^2\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*log(abs(tan(x)))/d^2 + (b*c^2 - a*c*d)*log(abs(c*tan(x) + d))/
(c*d^2) + (b*c*tan(x) - a*d*tan(x) - b*d)/(d^2*tan(x))
```

3.721 $\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=53

$$\frac{b\cot(x)(bc-ad)}{d^2}-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}-\frac{(a+b\cot(x))^2}{2d}$$

```
[Out] (b*(b*c - a*d)*Cot[x])/d^2 - (a + b*Cot[x])^2/(2*d) - ((b*c - a*d)^2*Log[c
+ d*Cot[x]])/d^3
```

Rubi [A] time = 0.136435, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$\frac{b\cot(x)(bc-ad)}{d^2}-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}-\frac{(a+b\cot(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] (b*(b*c - a*d)*Cot[x])/d^2 - (a + b*Cot[x])^2/(2*d) - ((b*c - a*d)^2*Log[c
+ d*Cot[x]])/d^3
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = -\operatorname{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \cot(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \cot(x)\right)$$
$$= \frac{b(bc-ad)\cot(x)}{d^2}-\frac{(a+b\cot(x))^2}{2d}-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}$$

Mathematica [A] time = 0.505196, size = 62, normalized size = 1.17

$$\frac{2bd\cot(x)(bc-2ad)+2(bc-ad)^2(\log(\sin(x))-\log(c\sin(x)+d\cos(x)))-b^2d^2\csc^2(x)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] (2*b*d*(b*c - 2*a*d)*Cot[x] - b^2*d^2*Csc[x]^2 + 2*(b*c - a*d)^2*(Log[Sin[x
]] - Log[d*Cos[x] + c*Sin[x]]))/(2*d^3)
```

Maple [B] time = 0.065, size = 119, normalized size = 2.3

$$-\frac{b^2}{2d(\tan(x))^2}+\frac{\ln(\tan(x))a^2}{d}-2\frac{\ln(\tan(x))bac}{d^2}+\frac{\ln(\tan(x))b^2c^2}{d^3}-2\frac{ab}{d\tan(x)}+\frac{cb^2}{d^2\tan(x)}-\frac{\ln(c\tan(x)+d)a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x)
```

```
[Out] -1/2*b^2/d/tan(x)^2+1/d*ln(tan(x))*a^2-2/d^2*ln(tan(x))*b*a*c+1/d^3*ln(tan(
x))*b^2*c^2-2*b/d/tan(x)*a+b^2/d^2/tan(x)*c-1/d*ln(c*tan(x)+d)*a^2+2/d^2*ln
(c*tan(x)+d)*b*a*c-1/d^3*ln(c*tan(x)+d)*b^2*c^2
```

Maxima [A] time = 0.973524, size = 124, normalized size = 2.34

$$-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log(c\tan(x)+d)}{d^3}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log(\tan(x))}{d^3}-\frac{b^2d-2\left(b^2c-2\,abd\right)\tan(x)}{2\,d^2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] -(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(c*tan(x) + d)/d^3 + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*log(tan(x))/d^3 - 1/2*(b^2*d - 2*(b^2*c - 2*a*b*d)*tan(x))/(d
^2*tan(x)^2)
```

Fricas [B] time = 3.29222, size = 417, normalized size = 7.87

$$\frac{b^2d^2-2\left(b^2cd-2\,abd^2\right)\cos\left(x\right)\sin\left(x\right)+\left(b^2c^2-2\,abcd+a^2d^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\cos\left(x\right)^2\right)\log\left(2\,cd\cos\left(x\right)\sin\left(x\right.\right.}{2\left(d^3\cos\left(x\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2 - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(2*c*d*cos(x)*sin
(x) - (c^2 - d^2)*cos(x)^2 + c^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(-1/4*cos(x)^2 + 1/4))/(d^3*cos(x)^2
 - d^3)
```

Sympy [A] time = 28.6935, size = 58, normalized size = 1.09

$$-\frac{b^2\cot^2\left(x\right)}{2d}-\frac{\left(ad-bc\right)^2\left(\begin{cases}\frac{\cot\left(x\right)}{c} & \text{for } d=0\\ \frac{\log\left(c+d\cot\left(x\right)\right)}{d} & \text{otherwise}\end{cases}\right)}{d^2}-\frac{\left(2abd-b^2c\right)\cot\left(x\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**2*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b**2*cot(x)**2/(2*d) - (a*d - b*c)**2*Piecewise((cot(x)/c, Eq(d, 0)), (log
(c + d*cot(x))/d, True))/d**2 - (2*a*b*d - b**2*c)*cot(x)/d**2
```

Giac [B] time = 1.19983, size = 188, normalized size = 3.55

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|\tan\left(x\right)|\right)}{d^3} - \frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\log\left(|c\tan\left(x\right) + d|\right)}{cd^3} - \frac{3\,b^2c^2\tan\left(x\right)^2 - 6\,abcd\tan\left(x\right)^2 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(tan(x)))/d^3 - (b^2*c^3 - 2*a*b*c^2
*d + a^2*c*d^2)*log(abs(c*tan(x) + d))/(c*d^3) - 1/2*(3*b^2*c^2*tan(x)^2 -
6*a*b*c*d*tan(x)^2 + 3*a^2*d^2*tan(x)^2 - 2*b^2*c*d*tan(x) + 4*a*b*d^2*tan(
x) + b^2*d^2)/(d^3*tan(x)^2)
```

3.722 $\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{b\cot(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}+\frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}-\frac{(a+b\cot(x))^3}{3d}$$

```
[Out] -((b*(b*c - a*d)^2*Cot[x])/d^3) + ((b*c - a*d)*(a + b*Cot[x])^2)/(2*d^2) -
(a + b*Cot[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Cot[x]])/d^4
```

Rubi [A] time = 0.138954, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$-\frac{b\cot(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}+\frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}-\frac{(a+b\cot(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] -((b*(b*c - a*d)^2*Cot[x])/d^3) + ((b*c - a*d)*(a + b*Cot[x])^2)/(2*d^2) -
(a + b*Cot[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Cot[x]])/d^4
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx = -\operatorname{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \cot(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right) dx, x, \cot(x)$$
$$= -\frac{b(bc-ad)^2\cot(x)}{d^3} + \frac{(bc-ad)(a+b\cot(x))^2}{2d^2} - \frac{(a+b\cot(x))^3}{3d} + \frac{(bc-ad)^3\log(c+d\,\mathrm{co}}{d^4}$$

Mathematica [A] time = 1.26379, size = 135, normalized size = 1.73

$$\frac{(a+b\cot(x))^3(c\sin(x)+d\cos(x))\left(bd\left(\sin(2x)\left(-9a^2d^2+9abcd+b^2\left(d^2-3c^2\right)\right)+3bd(bc-3ad)\right)-6\sin^2(x)(bc-ad)^3(}{6d^4(c+d\cot(x))(a\sin(x)+b\cos(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] ((a + b*Cot[x])^3*(d*Cos[x] + c*Sin[x])*(-2*b^3*d^3*Cot[x] - 6*(b*c - a*d)^
3*(Log[Sin[x]] - Log[d*Cos[x] + c*Sin[x]])*Sin[x]^2 + b*d*(3*b*d*(b*c - 3*a
*d) + (9*a*b*c*d - 9*a^2*d^2 + b^2*(-3*c^2 + d^2))*Sin[2*x])))/(6*d^4*(c +
d*Cot[x])*(b*Cos[x] + a*Sin[x])^3)
```

Maple [B] time = 0.078, size = 202, normalized size = 2.6

$$-\frac{b^3}{3\,d\,(\tan(x))^3} + \frac{\ln(\tan(x))\,a^3}{d} - 3\,\frac{\ln(\tan(x))\,a^2bc}{d^2} + 3\,\frac{\ln(\tan(x))\,ab^2c^2}{d^3} - \frac{\ln(\tan(x))\,b^3c^3}{d^4} - 3\,\frac{a^2b}{d\tan(x)} + 3\,\frac{ab^2c}{d^2\tan(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x)
```

```
[Out] -1/3*b^3/d/tan(x)^3+1/d*ln(tan(x))*a^3-3/d^2*ln(tan(x))*a^2*b*c+3/d^3*ln(ta
n(x))*a*b^2*c^2-1/d^4*ln(tan(x))*b^3*c^3-3*b/d/tan(x)*a^2+3*b^2/d^2/tan(x)*
a*c-b^3/d^3/tan(x)*c^2-3/2*b^2/d/tan(x)^2*a+1/2*b^3/d^2/tan(x)^2*c-1/d*ln(c
*tan(x)+d)*a^3+3/d^2*ln(c*tan(x)+d)*a^2*b*c-3/d^3*ln(c*tan(x)+d)*a*b^2*c^2+
1/d^4*ln(c*tan(x)+d)*b^3*c^3
```

Maxima [B] time = 0.990708, size = 217, normalized size = 2.78

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(c\tan\left(x\right)+d\right)}{d^4}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\tan\left(x\right)\right)}{d^4}-\frac{2\,b^3d^2+6\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(c*tan(x) + d)/d^4 -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(tan(x))/d^4 - 1/6*
(2*b^3*d^2 + 6*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x)^2 - 3*(b^3*c*d
- 3*a*b^2*d^2)*tan(x))/(d^3*tan(x)^3)
```

Fricas [B] time = 3.42416, size = 705, normalized size = 9.04

$$-\frac{2\left(3\,b^3c^2d-9\,ab^2cd^2+\left(9\,a^2b-b^3\right)d^3\right)\cos\left(x\right)^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3-\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)*d^3)*cos(x)^3 + 3*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)
*cos(x)^2 + c^2)*sin(x) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(-1/
4*cos(x)^2 + 1/4)*sin(x) - 6*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*cos(
x) + 3*(b^3*c*d^2 - 3*a*b^2*d^3)*sin(x))/((d^4*cos(x)^2 - d^4)*sin(x))
```

Sympy [A] time = 75.3299, size = 97, normalized size = 1.24

$$-\frac{b^3\cot^3\left(x\right)}{3d}-\frac{\left(3ab^2d-b^3c\right)\cot^2\left(x\right)}{2d^2}-\frac{\left(ad-bc\right)^3\left(\begin{cases}\frac{\cot\left(x\right)}{c} & \text{for}\ d=0\\ \frac{\log\left(c+d\cot\left(x\right)\right)}{d} & \text{otherwise}\end{cases}\right)}{d^3}-\frac{\left(3a^2bd^2-3ab^2cd+b^3c^2\right)\cot\left(x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**3*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b**3*cot(x)**3/(3*d) - (3*a*b**2*d - b**3*c)*cot(x)**2/(2*d**2) - (a*d - b
*c)**3*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x))/d, True))/d**3 -
(3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*cot(x)/d**3
```

Giac [B] time = 1.17287, size = 313, normalized size = 4.01

$$-\frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(|\tan\left(x\right)|\right)}{d^4} + \frac{\left(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3\right)\log\left(|c\tan\left(x\right) + d|\right)}{cd^4} + \frac{11\,b^3c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(tan(x)))/d^4 +
 (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*log(abs(c*tan(x) +
 d))/(c*d^4) + 1/6*(11*b^3*c^3*tan(x)^3 - 33*a*b^2*c^2*d*tan(x)^3 + 33*a^2*
b*c*d^2*tan(x)^3 - 11*a^3*d^3*tan(x)^3 - 6*b^3*c^2*d*tan(x)^2 + 18*a*b^2*c*
d^2*tan(x)^2 - 18*a^2*b*d^3*tan(x)^2 + 3*b^3*c*d^2*tan(x) - 9*a*b^2*d^3*tan
(x) - 2*b^3*d^3)/(d^4*tan(x)^3)
```

3.723 $\int e^{-\cot(x)} \csc^2(x)\,dx$

Optimal. Leaf size=6

$$e^{-\cot(x)}$$

```
[Out] E^(-Cot[x])
```

Rubi [A] time = 0.0147822, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4344, 2194}

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/E^Cot[x],x]
```

```
[Out] E^(-Cot[x])
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-\cot(x)} \csc^2(x)\,dx &= -\text{Subst}\left(\int e^{-x}\,dx, x, \cot(x)\right)\\ &= e^{-\cot(x)}\end{aligned}$$

Mathematica [A] time = 0.0762972, size = 6, normalized size = 1.

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/E^Cot[x],x]
```

```
[Out] E^(-Cot[x])
```

Maple [A] time = 0.011, size = 6, normalized size = 1.

$$\left(e^{\cot(x)}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/exp(cot(x)),x)
```

```
[Out] 1/exp(cot(x))
```

Maxima [A] time = 0.978347, size = 7, normalized size = 1.17

$$e^{(-\cot(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="maxima")
```

```
[Out] e^(-cot(x))
```

Fricas [A] time = 2.23891, size = 27, normalized size = 4.5

$$e^{\left(-\frac{\cos(x)}{\sin(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="fricas")
```

```
[Out] e^(-cos(x)/sin(x))
```

Sympy [A] time = 115.646, size = 5, normalized size = 0.83

$$e^{-\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/exp(cot(x)),x)
```

```
[Out] exp(-cot(x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(x)^2 e^{(-\cot(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="giac")
```

```
[Out] integrate(csc(x)^2*e^(-cot(x)), x)
```

3.724 $\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sec(x))}{b}$$

```
[Out] Log[a + b*Sec[x]]/b
```

Rubi [A] time = 0.0456383, antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4339, 36, 29, 31}

$$\frac{\log(a\cos(x)+b)}{b}-\frac{\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]
```

```
[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x(b+ax)}\,dx, x, \cos(x)\right)\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cos(x)\right)}{b} + \frac{a\operatorname{Subst}\left(\int \frac{1}{b+ax}\,dx, x, \cos(x)\right)}{b}\\ &= -\frac{\log(\cos(x))}{b} + \frac{\log(b+a\cos(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0176336, size = 20, normalized size = 1.82

$$\frac{\log(a\cos(x)+b)}{b} - \frac{\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]
```

```
[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b
```

Maple [A] time = 0.01, size = 12, normalized size = 1.1

$$\frac{\ln(a+b\sec(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(a+b*sec(x)),x)
```

```
[Out] ln(a+b*sec(x))/b
```

Maxima [A] time = 0.960017, size = 15, normalized size = 1.36

$$\frac{\log(b\sec(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="maxima")
```

```
[Out] log(b*sec(x) + a)/b
```

Fricas [A] time = 2.48189, size = 51, normalized size = 4.64

$$\frac{\log\left(a\cos\left(x\right)+b\right)-\log\left(-\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="fricas")
```

```
[Out] (log(a*cos(x) + b) - log(-cos(x)))/b
```

Sympy [A] time = 0.572914, size = 14, normalized size = 1.27

$$\begin{cases}\frac{\log\left(\frac{a}{b}+\sec\left(x\right)\right)}{b} & \text{for}\: b\neq 0\\ \frac{\sec\left(x\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x)
```

```
[Out] Piecewise((log(a/b + sec(x))/b, Ne(b, 0)), (sec(x)/a, True))
```

Giac [A] time = 1.10937, size = 30, normalized size = 2.73

$$\frac{\log\left(|a\cos\left(x\right)+b|\right)}{b}-\frac{\log\left(|\cos\left(x\right)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="giac")
```

```
[Out] log(abs(a*cos(x) + b))/b - log(abs(cos(x)))/b
```

3.725 $\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx$

Optimal. Leaf size=5

$$-\tan^{-1}(\cos(x))$$

```
[Out] -ArcTan[Cos[x]]
```

Rubi [A] time = 0.0332322, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4339, 203}

$$-\tan^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]

[Out] -ArcTan[Cos[x]]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cos(x)\right)\\
&= -\tan^{-1}(\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.0222478, size = 5, normalized size = 1.

$$-\tan^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]
```

```
[Out] -ArcTan[Cos[x]]
```

Maple [A] time = 0.025, size = 4, normalized size = 0.8

$$\arctan(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(1+sec(x)^2),x)
```

```
[Out] arctan(sec(x))
```

Maxima [A] time = 1.45471, size = 7, normalized size = 1.4

$$-\arctan(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="maxima")
```

```
[Out] -arctan(cos(x))
```

Fricas [A] time = 2.49107, size = 23, normalized size = 4.6

$$-\arctan(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="fricas")
```

```
[Out] -arctan(cos(x))
```

Sympy [A] time = 0.269607, size = 3, normalized size = 0.6

$$\operatorname{atan}(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)**2),x)
```

```
[Out] atan(sec(x))
```

Giac [A] time = 1.08875, size = 7, normalized size = 1.4

$$-\arctan(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(cos(x))
```

3.726 $\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)$$

```
[Out] -ArcTan[(3*Cos[x])/2]/6
```

Rubi [A] time = 0.0347168, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4339, 203}

$$-\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]
```

```
[Out] -ArcTan[(3*Cos[x])/2]/6
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{4+9x^2}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)$$

Mathematica [A] time = 0.0268567, size = 11, normalized size = 1.

$$-\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]
```

```
[Out] -ArcTan[(3*Cos[x])/2]/6
```

Maple [A] time = 0.019, size = 8, normalized size = 0.7

$$\frac{1}{6}\arctan\left(\frac{2\sec(x)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(9+4*sec(x)^2),x)
```

```
[Out] 1/6*arctan(2/3*sec(x))
```

Maxima [A] time = 1.45431, size = 9, normalized size = 0.82

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*arctan(3/2*cos(x))
```

Fricas [A] time = 2.41866, size = 34, normalized size = 3.09

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*arctan(3/2*cos(x))
```

Sympy [A] time = 0.308155, size = 8, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{2\sec(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)**2),x)
```

```
[Out] atan(2*sec(x)/3)/6
```

Giac [A] time = 1.10702, size = 9, normalized size = 0.82

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="giac")
```

```
[Out] -1/6*arctan(3/2*cos(x))
```

3.727 $\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx$

Optimal. Leaf size=7

$$-\log(\cos(x)+1)$$

```
[Out] -Log[1 + Cos[x]]
```

Rubi [A] time = 0.031622, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4339, 31}

$$-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]

[Out] -Log[1 + Cos[x]]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\\ &= -\log(1+\cos(x))\end{aligned}$$

Mathematica [A] time = 0.0055143, size = 9, normalized size = 1.29

$$-2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]
```

```
[Out] -2*Log[Cos[x/2]]
```

Maple [A] time = 0.028, size = 12, normalized size = 1.7

$$\ln(\sec(x)) - \ln(1 + \sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(sec(x)+sec(x)^2),x)
```

```
[Out] ln(sec(x))-ln(1+sec(x))
```

Maxima [A] time = 0.954864, size = 9, normalized size = 1.29

$$-\log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="maxima")
```

```
[Out] -log(cos(x) + 1)
```

Fricas [A] time = 2.41407, size = 32, normalized size = 4.57

$$-\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="fricas")
```

```
[Out] -log(1/2*cos(x) + 1/2)
```

Sympy [B] time = 0.256751, size = 15, normalized size = 2.14

$$\frac{\log\left(\tan^2(x)+1\right)}{2}-\log(\sec(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)**2),x)
```

```
[Out] log(tan(x)**2 + 1)/2 - log(sec(x) + 1)
```

Giac [A] time = 1.0896, size = 9, normalized size = 1.29

$$-\log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="giac")
```

```
[Out] -log(cos(x) + 1)
```

3.728 $\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx$

Optimal. Leaf size=5

$$\operatorname{csch}^{-1}(2\cos(x))$$

```
[Out] ArcCsch[2*Cos[x]]
```

Rubi [A] time = 0.0448581, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4339, 335, 215}

$$\operatorname{csch}^{-1}(2\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]
```

```
[Out] ArcCsch[2*Cos[x]]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{\sqrt{4+\frac{1}{x^2}}x^2}\,dx, x, \cos(x)\right)$$
$$= \operatorname{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sec(x)\right)$$
$$= \sinh^{-1}\left(\frac{\sec(x)}{2}\right)$$

Mathematica [B] time = 0.0303004, size = 38, normalized size = 7.6

$$\frac{\sqrt{2\cos(2x)+3}\sec(x)\tanh^{-1}\left(\sqrt{4\cos^2(x)+1}\right)}{\sqrt{\sec^2(x)+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]
```

```
[Out] (ArcTanh[Sqrt[1 + 4*Cos[x]^2]]*Sqrt[3 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 + Sec[x]
^2]
```

Maple [A] time = 0.032, size = 6, normalized size = 1.2

$$\operatorname{Arcsinh}\left(\frac{\sec(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x)
```

```
[Out] arcsinh(1/2*sec(x))
```

Maxima [B] time = 0.953622, size = 45, normalized size = 9.

$$\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(sqrt(1/cos(x)^2 + 4)*cos(x) + 1) - 1/2*log(sqrt(1/cos(x)^2 + 4)*cos
(x) - 1)
```

Fricas [B] time = 2.71256, size = 80, normalized size = 16.

$$\log\left(-\frac{\sqrt{\frac{4\cos(x)^2+1}{\cos(x)^2}}\cos(x)+1}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(sqrt((4*cos(x)^2 + 1)/cos(x)^2)*cos(x) + 1)/cos(x))
```

Sympy [A] time = 0.950007, size = 5, normalized size = 1.

$$\operatorname{asinh}\left(\frac{\sec(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)**2)**(1/2),x)
```

```
[Out] asinh(sec(x)/2)
```

Giac [B] time = 1.12927, size = 55, normalized size = 11.

$$\frac{\log\left(\sqrt{4\cos(x)^2+1}+1\right)}{2\operatorname{sgn}(\cos(x))}-\frac{\log\left(\sqrt{4\cos(x)^2+1}-1\right)}{2\operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(sqrt(4*cos(x)^2 + 1) + 1)/sgn(cos(x)) - 1/2*log(sqrt(4*cos(x)^2 + 1
) - 1)/sgn(cos(x))
```

3.729 $\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=13

$$\sqrt{\cos^2(x)+1}\,\sec(x)$$

```
[Out] Sqrt[1 + Cos[x]^2]*Sec[x]
```

Rubi [A] time = 0.0792028, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\sqrt{\cos^2(x)+1}\,\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] Sqrt[1 + Cos[x]^2]*Sec[x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \cos(x)\right)\\ &= \sqrt{1+\cos^2(x)}\,\sec(x)\end{aligned}$$

Mathematica [A] time = 0.017403, size = 13, normalized size = 1.

$$\sqrt{\cos^2(x)+1}\,\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] Sqrt[1 + Cos[x]^2]*Sec[x]
```

Maple [B] time = 0.033, size = 25, normalized size = 1.9

$$\frac{1+(\sec(x))^2}{\sec(x)}\frac{1}{\sqrt{\frac{1+(\sec(x))^2}{(\sec(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x)
```

```
[Out] 1/((1+sec(x)^2)/sec(x)^2)^(1/2)/sec(x)*(1+sec(x)^2)
```

Maxima [A] time = 1.45455, size = 18, normalized size = 1.38

$$\frac{\sqrt{\cos(x)^2+1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(cos(x)^2 + 1)/cos(x)
```

Fricas [A] time = 2.41079, size = 51, normalized size = 3.92

$$\frac{\sqrt{\cos(x)^2+1}+\cos(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(cos(x)^2 + 1) + cos(x))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)\sec(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)*sec(x)/sqrt(cos(x)**2 + 1), x)
```

Giac [A] time = 1.11641, size = 28, normalized size = 2.15

$$-\frac{2}{\left(\sqrt{\cos(x)^2+1}-\cos(x)\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/((sqrt(cos(x)^2 + 1) - cos(x))^2 - 1)
```

3.730 $\int e^{\sec(x)} \sec(x) \tan(x) \, dx$

Optimal. Leaf size=4

$$e^{\sec(x)}$$

[Out] E^Sec[x]

Rubi [A] time = 0.0217493, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4339, 2209}

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Sec[x]*Sec[x]*Tan[x],x]

[Out] E^Sec[x]

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{\sec(x)}\sec(x)\tan(x)\,dx = -\text{Subst}\left(\int \frac{e^{\frac{1}{x}}}{x^2}\,dx, x, \cos(x)\right)$$
$$= e^{\sec(x)}$$

Mathematica [A] time = 0.0073632, size = 4, normalized size = 1.

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sec[x]*Sec[x]*Tan[x],x]
```

```
[Out] E^Sec[x]
```

Maple [A] time = 0.006, size = 4, normalized size = 1.

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sec(x))*sec(x)*tan(x),x)
```

```
[Out] exp(sec(x))
```

Maxima [A] time = 0.955632, size = 4, normalized size = 1.

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] e^sec(x)
```

Fricas [A] time = 2.29366, size = 19, normalized size = 4.75

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] e^(1/cos(x))
```

Sympy [A] time = 0.828806, size = 3, normalized size = 0.75

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x)
```

```
[Out] exp(sec(x))
```

Giac [A] time = 1.11965, size = 7, normalized size = 1.75

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] e^(1/cos(x))
```

3.731 $\int 2^{\sec(x)} \sec(x) \tan(x) \, dx$

Optimal. Leaf size=9

$$\frac{2^{\sec(x)}}{\log(2)}$$

[Out] 2^Sec[x]/Log[2]

Rubi [A] time = 0.0216664, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4339, 2209}

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sec[x]*Sec[x]*Tan[x],x]

[Out] 2^Sec[x]/Log[2]

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int 2^{\sec(x)} \sec(x) \tan(x) \, dx = -\text{Subst}\left(\int \frac{2^{\frac{1}{x}}}{x^2} \, dx, x, \cos(x)\right)$$
$$= \frac{2^{\sec(x)}}{\log(2)}$$

Mathematica [A] time = 0.0081172, size = 9, normalized size = 1.

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sec[x]*Sec[x]*Tan[x],x]
```

```
[Out] 2^Sec[x]/Log[2]
```

Maple [A] time = 0.006, size = 10, normalized size = 1.1

$$\frac{2^{\sec(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sec(x)*sec(x)*tan(x),x)
```

```
[Out] 2^sec(x)/ln(2)
```

Maxima [A] time = 0.956561, size = 12, normalized size = 1.33

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 2^sec(x)/log(2)
```

Fricas [A] time = 2.31062, size = 28, normalized size = 3.11

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 2^(1/cos(x))/log(2)
```

Sympy [A] time = 0.927133, size = 7, normalized size = 0.78

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**sec(x)*sec(x)*tan(x),x)
```

```
[Out] 2**sec(x)/log(2)
```

Giac [A] time = 1.0824, size = 15, normalized size = 1.67

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 2^(1/cos(x))/log(2)
```

3.732 $\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

```
[Out] -(1/Sqrt[1 + Sec[2*x]])
```

Rubi [A] time = 0.0545603, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4339, 261}

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
```

```
[Out] -(1/Sqrt[1 + Sec[2*x]])
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2}x^2}\,dx, x, \cos(2x)\right)\right)$$
$$= -\frac{1}{\sqrt{1+\sec(2x)}}$$

Mathematica [A] time = 0.0767221, size = 20, normalized size = 1.67

$$-\frac{2\cos^2(x)\sec(2x)}{(\sec(2x)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
```

```
[Out] (-2*Cos[x]^2*Sec[2*x])/(1 + Sec[2*x])^(3/2)
```

Maple [A] time = 0.023, size = 11, normalized size = 0.9

$$-\frac{1}{\sqrt{1+\sec(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x)
```

```
[Out] -1/(1+sec(2*x))^(1/2)
```

Maxima [A] time = 0.974531, size = 14, normalized size = 1.17

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/sqrt(sec(2*x) + 1)
```

Fricas [B] time = 2.39472, size = 76, normalized size = 6.33

$$-\frac{\sqrt{\frac{\cos(2x)+1}{\cos(2x)}}\cos(2x)}{\cos(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt((cos(2*x) + 1)/cos(2*x))*cos(2*x)/(cos(2*x) + 1)
```

Sympy [A] time = 1.28341, size = 12, normalized size = 1.

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))**(3/2),x)
```

```
[Out] -1/sqrt(sec(2*x) + 1)
```

Giac [B] time = 1.46652, size = 32, normalized size = 2.67

$$\frac{\sqrt{2}\sqrt{-\tan(x)^2+1}}{2\operatorname{sgn}\left(\tan(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-tan(x)^2 + 1)/sgn(tan(x)^2 - 1)
```

3.733 $\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx$

Optimal. Leaf size=43

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\sec(3x)-\frac{1}{3}\sqrt{5}\sinh^{-1}\left(\sqrt{5}\cos(3x)\right)$$

```
[Out] -(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]])/3 + (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/
3
```

Rubi [A] time = 0.0949236, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {277, 215}

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\sec(3x)-\frac{1}{3}\sqrt{5}\sinh^{-1}\left(\sqrt{5}\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

```
[Out] -(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]])/3 + (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/
3
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{1+5x^2}}{x^2}\,dx, x, \cos(3x)\right)\right)$$
$$= \frac{1}{3}\sqrt{1+5\cos^2(3x)}\sec(3x) - \frac{5}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+5x^2}}\,dx, x, \cos(3x)\right)$$
$$= -\frac{1}{3}\sqrt{5}\sinh^{-1}\left(\sqrt{5}\cos(3x)\right) + \frac{1}{3}\sqrt{1+5\cos^2(3x)}\sec(3x)$$

Mathematica [A] time = 0.0573059, size = 43, normalized size = 1.

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\sec(3x) - \frac{1}{3}\sqrt{5}\sinh^{-1}\left(\sqrt{5}\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

```
[Out] -(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]])/3 + (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/
3
```

Maple [A] time = 0.043, size = 66, normalized size = 1.5

$$-\frac{\sec(3x)}{3}\sqrt{\frac{(\sec(3x))^2+5}{(\sec(3x))^2}}\left(\sqrt{5}\text{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{(\sec(3x))^2+5}}\right) - \sqrt{(\sec(3x))^2+5}\right)\frac{1}{\sqrt{(\sec(3x))^2+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x)
```

```
[Out] -1/3*((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)*sec(3*x)*(5^(1/2)*arctanh(5^(1/2)/(s
ec(3*x)^2+5)^(1/2))-(sec(3*x)^2+5)^(1/2))/(sec(3*x)^2+5)^(1/2)
```

Maxima [A] time = 1.46382, size = 47, normalized size = 1.09

$$-\frac{1}{3}\sqrt{5}\,\text{arsinh}\left(\sqrt{5}\cos(3x)\right) + \frac{\sqrt{5\cos(3x)^2+1}}{3\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(5)*arcsinh(sqrt(5)*cos(3*x)) + 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x
)
```

Fricas [B] time = 2.76936, size = 354, normalized size = 8.23

$$\frac{\sqrt{5}\cos(3x)\log\left(80000\cos(3x)^{8}+32000\cos(3x)^{6}+4000\cos(3x)^{4}+160\cos(3x)^{2}-8\left(2000\sqrt{5}\cos(3x)^{7}+600\sqrt{}\right.\right.}{24\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(5)*cos(3*x)*log(80000*cos(3*x)^8 + 32000*cos(3*x)^6 + 4000*cos(3
*x)^4 + 160*cos(3*x)^2 - 8*(2000*sqrt(5)*cos(3*x)^7 + 600*sqrt(5)*cos(3*x)^
5 + 50*sqrt(5)*cos(3*x)^3 + sqrt(5)*cos(3*x))*sqrt(5*cos(3*x)^2 + 1) + 1) +
 8*sqrt(5*cos(3*x)^2 + 1))/cos(3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{5\cos^{2}(3x)+1}\tan(3x)\sec(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)**2)**(1/2)*tan(3*x),x)
```

```
[Out] Integral(sqrt(5*cos(3*x)**2 + 1)*tan(3*x)*sec(3*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{5\cos(3x)^{2}+1}\sec(3x)\tan(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(5*cos(3*x)^2 + 1)*sec(3*x)*tan(3*x), x)
```

3.734 $\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx$

Optimal. Leaf size=22

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\sec(3x)$$

```
[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

Rubi [A] time = 0.0909895, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {264}

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\sec(3x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

```
[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+5x^2}}\,dx, x, \cos(3x)\right)\right)\\ &= \frac{1}{3}\sqrt{1+5\cos^2(3x)}\sec(3x)\end{aligned}$$

Mathematica [A] time = 0.0330922, size = 22, normalized size = 1.

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\sec(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

```
[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

Maple [A] time = 0.032, size = 34, normalized size = 1.6

$$\frac{(\sec(3x))^2+5}{3\sec(3x)}\frac{1}{\sqrt{\frac{(\sec(3x))^2+5}{(\sec(3x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x)
```

```
[Out] 1/3/((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)/sec(3*x)*(sec(3*x)^2+5)
```

Maxima [A] time = 1.43293, size = 27, normalized size = 1.23

$$\frac{\sqrt{5\cos(3x)^2+1}}{3\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

Fricas [A] time = 2.44364, size = 50, normalized size = 2.27

$$\frac{\sqrt{5\cos(3x)^2+1}}{3\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(3x)\sec(3x)}{\sqrt{5\cos^2(3x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)**2)**(1/2),x)
```

```
[Out] Integral(tan(3*x)*sec(3*x)/sqrt(5*cos(3*x)**2 + 1), x)
```

Giac [B] time = 1.24747, size = 159, normalized size = 7.23

$$\frac{2\sqrt{2}\left(\sqrt{3}\tan\left(\frac{3}{2}x\right)^2+\sqrt{3}-\sqrt{3\tan\left(\frac{3}{2}x\right)^4-4\tan\left(\frac{3}{2}x\right)^2+3}\right)}{3\left(\left(\sqrt{3}\tan\left(\frac{3}{2}x\right)^2-\sqrt{3\tan\left(\frac{3}{2}x\right)^4-4\tan\left(\frac{3}{2}x\right)^2+3}\right)^2-2\sqrt{3}\left(\sqrt{3}\tan\left(\frac{3}{2}x\right)^2-\sqrt{3\tan\left(\frac{3}{2}x\right)^4-4\tan\left(\frac{3}{2}x\right)^2+3}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(sqrt(3)*tan(3/2*x)^2 + sqrt(3) - sqrt(3*tan(3/2*x)^4 - 4*tan(3
/2*x)^2 + 3))/((sqrt(3)*tan(3/2*x)^2 - sqrt(3*tan(3/2*x)^4 - 4*tan(3/2*x)^2
 + 3))^2 - 2*sqrt(3)*(sqrt(3)*tan(3/2*x)^2 - sqrt(3*tan(3/2*x)^4 - 4*tan(3/
2*x)^2 + 3)) + 1)
```

3.735 $\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\csc(x))}{b}$$

```
[Out] -(Log[a + b*Csc[x]]/b)
```

Rubi [A] time = 0.0423383, antiderivative size = 20, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4338, 36, 29, 31}

$$\frac{\log(\sin(x))}{b}-\frac{\log(a\sin(x)+b)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]
```

```
[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx, x, \sin(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{b} - \frac{a\,\text{Subst}\left(\int \frac{1}{b+ax}\,dx, x, \sin(x)\right)}{b}\\ &= \frac{\log(\sin(x))}{b} - \frac{\log(b+a\sin(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0185075, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x))}{b} - \frac{\log(a\sin(x)+b)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]
```

```
[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b
```

Maple [A] time = 0.017, size = 13, normalized size = 1.1

$$-\frac{\ln(a+b\csc(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(a+b*csc(x)),x)
```

```
[Out] -ln(a+b*csc(x))/b
```

Maxima [A] time = 0.967199, size = 16, normalized size = 1.33

$$-\frac{\log(b\csc(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] -log(b*csc(x) + a)/b
```

Fricas [A] time = 2.49678, size = 58, normalized size = 4.83

$$-\frac{\log\left(a\sin\left(x\right)+b\right)-\log\left(-\frac{1}{2}\sin\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] -(log(a*sin(x) + b) - log(-1/2*sin(x)))/b
```

Sympy [A] time = 0.482344, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b}+\csc\left(x\right)\right)}{b} & \text{for}\ b\neq 0\\ -\frac{\csc\left(x\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x)
```

```
[Out] Piecewise((-log(a/b + csc(x))/b, Ne(b, 0)), (-csc(x)/a, True))
```

Giac [A] time = 1.09115, size = 30, normalized size = 2.5

$$-\frac{\log\left(|a\sin\left(x\right)+b|\right)}{b}+\frac{\log\left(|\sin\left(x\right)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] -log(abs(a*sin(x) + b))/b + log(abs(sin(x)))/b
```

3.736 $\int 5^{\csc(3x)} \cot(3x)\csc(3x)\,dx$

Optimal. Leaf size=14

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

```
[Out] -5^Csc[3*x]/(3*Log[5])
```

Rubi [A] time = 0.0247173, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4338, 2209}

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Antiderivative was successfully verified.

```
[In] Int[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
```

```
[Out] -5^Csc[3*x]/(3*Log[5])
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int 5^{\csc(3x)}\cot(3x)\csc(3x)\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{5^{\frac{1}{x}}}{x^2}\,dx, x, \sin(3x)\right)$$
$$= -\frac{5^{\csc(3x)}}{3\log(5)}$$

Mathematica [A] time = 0.0313623, size = 14, normalized size = 1.

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
```

```
[Out] -5^Csc[3*x]/(3*Log[5])
```

Maple [A] time = 0.01, size = 13, normalized size = 0.9

$$-\frac{5^{\csc(3x)}}{3\ln(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5^csc(3*x)*cot(3*x)*csc(3*x),x)
```

```
[Out] -1/3*5^csc(3*x)/ln(5)
```

Maxima [A] time = 0.963998, size = 16, normalized size = 1.14

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*5^csc(3*x)/log(5)
```

Fricas [A] time = 2.34667, size = 38, normalized size = 2.71

$$-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3\,\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*5^(1/sin(3*x))/log(5)
```

Sympy [A] time = 0.821964, size = 12, normalized size = 0.86

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5**csc(3*x)*cot(3*x)*csc(3*x),x)
```

```
[Out] -5**csc(3*x)/(3*log(5))
```

Giac [A] time = 1.11444, size = 19, normalized size = 1.36

$$-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3\,\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="giac")
```

```
[Out] -1/3*5^(1/sin(3*x))/log(5)
```

3.737 $\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

```
[Out] ArcTan[Sin[x]]
```

Rubi [A] time = 0.0321606, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4338, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right)\\ &= \tan^{-1}(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0153911, size = 3, normalized size = 1.

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Maple [A] time = 0.015, size = 6, normalized size = 2.

$$-\arctan(\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(1+csc(x)^2),x)
```

```
[Out] -arctan(csc(x))
```

Maxima [A] time = 1.45583, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(sin(x))
```

Fricas [A] time = 2.37124, size = 22, normalized size = 7.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

Sympy [A] time = 0.231967, size = 5, normalized size = 1.67

$$-\operatorname{atan}(\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)**2),x)
```

```
[Out] -atan(csc(x))
```

Giac [A] time = 1.10593, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="giac")
```

```
[Out] arctan(sin(x))
```

3.738 $\int \frac{\cot(6x)\csc(6x)}{\left(5-11\csc^2(6x)\right)^2}\,dx$

Optimal. Leaf size=43

$$\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

```
[Out] -ArcTanh[Sqrt[5/11]*Sin[6*x]]/(60*Sqrt[55]) + Sin[6*x]/(60*(11 - 5*Sin[6*x]
^2))
```

Rubi [A] time = 0.0570532, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4338, 288, 206}

$$\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
```

```
[Out] -ArcTanh[Sqrt[5/11]*Sin[6*x]]/(60*Sqrt[55]) + Sin[6*x]/(60*(11 - 5*Sin[6*x]
^2))
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(6x)\csc(6x)}{\left(5-11\csc^2(6x)\right)^2}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{\left(11-5x^2\right)^2}\,dx, x, \sin(6x)\right)\\ &= \frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{1}{60}\,\text{Subst}\left(\int \frac{1}{11-5x^2}\,dx, x, \sin(6x)\right)\\ &= -\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}+\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}\end{aligned}$$

Mathematica [A] time = 0.680499, size = 41, normalized size = 0.95

$$\frac{\sin(6x)}{30(5\cos(12x)+17)}-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
```

```
[Out] -ArcTanh[Sqrt[5/11]*Sin[6*x]]/(60*Sqrt[55]) + Sin[6*x]/(30*(17 + 5*Cos[12*x
]))
```

Maple [A] time = 0.033, size = 35, normalized size = 0.8

$$\frac{\csc(6\,x)}{660\,(\csc(6\,x))^2-300}-\frac{\sqrt{55}}{3300}\text{Artanh}\left(\frac{\csc(6\,x)\sqrt{55}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x)
```

```
[Out] 1/60*csc(6*x)/(11*csc(6*x)^2-5)-1/3300*55^(1/2)*arctanh(1/5*csc(6*x)*55^(1/
2))
```

Maxima [A] time = 1.47708, size = 66, normalized size = 1.53

$$\frac{1}{6600}\sqrt{55}\log\left(-\frac{\sqrt{55}-5\sin(6x)}{\sqrt{55}+5\sin(6x)}\right)-\frac{\sin(6x)}{60\left(5\sin(6x)^2-11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6600*sqrt(55)*log(-(sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/6
0*sin(6*x)/(5*sin(6*x)^2 - 11)
```

Fricas [B] time = 2.16254, size = 200, normalized size = 4.65

$$\frac{\left(5\sqrt{55}\cos(6x)^2+6\sqrt{55}\right)\log\left(-\frac{5\cos(6x)^2+2\sqrt{55}\sin(6x)-16}{5\cos(6x)^2+6}\right)+110\sin(6x)}{6600\left(5\cos(6x)^2+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6600*((5*sqrt(55)*cos(6*x)^2 + 6*sqrt(55))*log(-(5*cos(6*x)^2 + 2*sqrt(55
)*sin(6*x) - 16)/(5*cos(6*x)^2 + 6)) + 110*sin(6*x))/(5*cos(6*x)^2 + 6)
```

Sympy [B] time = 2.60155, size = 151, normalized size = 3.51

$$\frac{11\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000}-\frac{5\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)}{72600\csc^2(6x)-33000}-\frac{11\sqrt{55}\log\left(\csc(6x)+\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000}+\frac{5\sqrt{55}\log}{72600\,\mathrm{cs}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)**2)**2,x)
```

```
[Out] 11*sqrt(55)*log(csc(6*x) - sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 33
000) - 5*sqrt(55)*log(csc(6*x) - sqrt(55)/11)/(72600*csc(6*x)**2 - 33000) -
 11*sqrt(55)*log(csc(6*x) + sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 3
3000) + 5*sqrt(55)*log(csc(6*x) + sqrt(55)/11)/(72600*csc(6*x)**2 - 33000)
+ 110*csc(6*x)/(72600*csc(6*x)**2 - 33000)
```

Giac [A] time = 1.16217, size = 65, normalized size = 1.51

$$\frac{1}{6600}\sqrt{55}\log\left(\frac{\sqrt{55}-5\sin\left(6x\right)}{\sqrt{55}+5\sin\left(6x\right)}\right)-\frac{\sin\left(6x\right)}{60\left(5\sin\left(6x\right)^{2}-11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6600*sqrt(55)*log((sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/60
*sin(6*x)/(5*sin(6*x)^2 - 11)
```

3.739 $\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx$

Optimal. Leaf size=14

$$\sqrt{\sin^2(x)+1}(-\csc(x))$$

```
[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])
```

Rubi [A] time = 0.0765542, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\sqrt{\sin^2(x)+1}(-\csc(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \sin(x)\right)\\ &= -\csc(x)\sqrt{1+\sin^2(x)}\end{aligned}$$

Mathematica [A] time = 0.01771, size = 14, normalized size = 1.

$$\sqrt{\sin^2(x)+1}(-\csc(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])
```

Maple [A] time = 0.463, size = 15, normalized size = 1.1

$$-\frac{1}{\sin(x)}\sqrt{1+(\sin(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x)
```

```
[Out] -1/sin(x)*(1+sin(x)^2)^(1/2)
```

Maxima [A] time = 1.44459, size = 19, normalized size = 1.36

$$-\frac{\sqrt{\sin(x)^2+1}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(sin(x)^2 + 1)/sin(x)
```

Fricas [A] time = 2.12135, size = 54, normalized size = 3.86

$$-\frac{\sqrt{-\cos(x)^2+2}-\sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-cos(x)^2 + 2) - sin(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)*csc(x)/sqrt(sin(x)**2 + 1), x)
```

Giac [A] time = 1.09895, size = 28, normalized size = 2.

$$\frac{2}{\left(\sqrt{\sin(x)^2+1}-\sin(x)\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(sin(x)^2 + 1) - sin(x))^2 - 1)
```

3.740 $\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx$

Optimal. Leaf size=43

$$\frac{2}{15}\sqrt{\sin^2(5x)+1}\csc(5x)-\frac{1}{15}\sqrt{\sin^2(5x)+1}\csc^3(5x)$$

```
[Out] (2*Csc[5*x]*Sqrt[1 + Sin[5*x]^2])/15 - (Csc[5*x]^3*Sqrt[1 + Sin[5*x]^2])/15
```

Rubi [A] time = 0.10563, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {271, 264}

$$\frac{2}{15}\sqrt{\sin^2(5x)+1}\csc(5x)-\frac{1}{15}\sqrt{\sin^2(5x)+1}\csc^3(5x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
```

```
[Out] (2*Csc[5*x]*Sqrt[1 + Sin[5*x]^2])/15 - (Csc[5*x]^3*Sqrt[1 + Sin[5*x]^2])/15
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}}\,dx, x, \sin(5x)\right)$$
$$= -\frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)} - \frac{2}{15}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \sin(5x)\right)$$
$$= \frac{2}{15}\csc(5x)\sqrt{1+\sin^2(5x)} - \frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)}$$

Mathematica [A] time = 0.0571662, size = 28, normalized size = 0.65

$$-\frac{1}{15}\sqrt{\sin^2(5x)+1}\,\csc(5x)\left(\csc^2(5x)-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
```

```
[Out] -(Csc[5*x]*(-2 + Csc[5*x]^2)*Sqrt[1 + Sin[5*x]^2])/15
```

Maple [A] time = 0.973, size = 38, normalized size = 0.9

$$-\frac{1}{15\,(\sin(5\,x))^3}\sqrt{1+(\sin(5\,x))^2}+\frac{2}{15\,\sin(5\,x)}\sqrt{1+(\sin(5\,x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x)
```

```
[Out] -1/15/sin(5*x)^3*(1+sin(5*x)^2)^(1/2)+2/15/sin(5*x)*(1+sin(5*x)^2)^(1/2)
```

Maxima [A] time = 1.48258, size = 50, normalized size = 1.16

$$\frac{2\sqrt{\sin(5\,x)^2+1}}{15\,\sin(5\,x)}-\frac{\sqrt{\sin(5\,x)^2+1}}{15\,\sin(5\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*sqrt(sin(5*x)^2 + 1)/sin(5*x) - 1/15*sqrt(sin(5*x)^2 + 1)/sin(5*x)^3
```

Fricas [A] time = 2.18866, size = 146, normalized size = 3.4

$$-\frac{2\left(\cos(5x)^2-1\right)\sin(5x)-\left(2\cos(5x)^2-1\right)\sqrt{-\cos(5x)^2+2}}{15\left(\cos(5x)^2-1\right)\sin(5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(cos(5*x)^2 - 1)*sin(5*x) - (2*cos(5*x)^2 - 1)*sqrt(-cos(5*x)^2 +
2))/((cos(5*x)^2 - 1)*sin(5*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot(5x)\csc^3(5x)}{\sqrt{\sin^2(5x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)**3/(1+sin(5*x)**2)**(1/2),x)
```

```
[Out] Integral(cot(5*x)*csc(5*x)**3/sqrt(sin(5*x)**2 + 1), x)
```

Giac [A] time = 1.18201, size = 65, normalized size = 1.51

$$\frac{4\left(3\left(\sqrt{\sin(5x)^2+1}-\sin(5x)\right)^2-1\right)}{15\left(\left(\sqrt{\sin(5x)^2+1}-\sin(5x)\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(3*(sqrt(sin(5*x)^2 + 1) - sin(5*x))^2 - 1)/((sqrt(sin(5*x)^2 + 1) - s
in(5*x))^2 - 1)^3
```

3.741 $\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

```
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rubi [A] time = 0.0366599, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {12, 2176, 2194}

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx = \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \sin(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\sin(a+bx)}}{bn^2} + \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

Mathematica [A] time = 0.0618847, size = 28, normalized size = 0.65

$$\frac{2e^{n\sin(a+bx)}(n\sin(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

Maple [C] time = 0.079, size = 104, normalized size = 2.4

$$\frac{ie^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)}e^{-ibx}e^{-ia}}{nb} - \frac{ie^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)}e^{ibx}e^{ia}}{nb} - 2\,\frac{e^{n(\sin(bx)\cos(a)+\cos(bx)\sin(a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-I/n/b*
exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)-2/n^2/b*exp(n*
(sin(b*x)*cos(a)+cos(b*x)*sin(a)))
```

Maxima [A] time = 1.07819, size = 50, normalized size = 1.16

$$\frac{2\left(ne^{(n\sin(bx+a))}\sin(bx+a) - e^{(n\sin(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

Fricas [A] time = 2.14265, size = 69, normalized size = 1.6

$$\frac{2\left(n\sin\left(bx+a\right)-1\right)e^{\left(n\sin\left(bx+a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sin\left(a+bx\right)}\sin\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\sin\left(bx+a\right)\right)}\sin\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

3.742 $\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

```
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rubi [A] time = 0.0347942, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2176, 2194}

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx = \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \sin(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\sin(a+bx)}}{bn^2} + \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

Mathematica [A] time = 0.0297166, size = 28, normalized size = 0.65

$$\frac{2e^{n\sin(a+bx)}(n\sin(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

Maple [C] time = 0., size = 104, normalized size = 2.4

$$\frac{i e^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)}e^{-ibx}e^{-ia}}{nb} - \frac{i e^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)}e^{ibx}e^{ia}}{nb} - 2\,\frac{e^{n(\sin(bx)\cos(a)+\cos(bx)\sin(a))}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-I/n/b*
exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)-2/n^2/b*exp(n*
(sin(b*x)*cos(a)+cos(b*x)*sin(a)))
```

Maxima [A] time = 1.06917, size = 50, normalized size = 1.16

$$\frac{2\left(ne^{(n\sin(bx+a))}\sin(bx+a) - e^{(n\sin(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

Fricas [A] time = 2.15162, size = 69, normalized size = 1.6

$$\frac{2\left(n\sin\left(bx+a\right)-1\right)e^{\left(n\sin\left(bx+a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sin\left(a+bx\right)}\sin\left(2a+2bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\sin\left(bx+a\right)\right)}\sin\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

3.743 $\int e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4 \sin\left(\frac{a}{2}+\frac{bx}{2}\right) e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4 e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rubi [A] time = 0.0362031, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4 \sin\left(\frac{a}{2}+\frac{bx}{2}\right) e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4 e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx &= \frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\
&= -\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.067914, size = 36, normalized size = 0.56

$$\frac{4e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\left(n\sin\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

Maple [C] time = 0.081, size = 122, normalized size = 1.9

$$\frac{2\,i e^{-\frac{i}{2}bx}e^{-\frac{i}{2}a}}{nb}e^{n\sin\left(\frac{a}{2}\right)\cos\left(\frac{bx}{2}\right)+n\cos\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)} - \frac{2\,i e^{\frac{i}{2}bx}e^{\frac{i}{2}a}}{nb}e^{n\sin\left(\frac{a}{2}\right)\cos\left(\frac{bx}{2}\right)+n\cos\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)} - 4\,\frac{e^{n(\sin(a/2)\cos(1/2\,bx)+\cos(a/2)\sin(1/2\,b}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] 2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*b*x)+n*cos(1/2*a)*sin(1/2*b*x))*exp(-1/2*I
*b*x)*exp(-1/2*I*a)-2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*b*x)+n*cos(1/2*a)*sin(
1/2*b*x))*exp(1/2*I*b*x)*exp(1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*a)*cos(1/2*b*x
)+cos(1/2*a)*sin(1/2*b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A] time = 2.13023, size = 90, normalized size = 1.41

$$\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)
```

Giac [B] time = 1.24357, size = 186, normalized size = 2.91

$$\frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+1}\right)}\right)}{bn^{2}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*
x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 +
1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.744 $\int e^{n \sin\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rubi [A] time = 0.0384814, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx &= \frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\
&= -\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.0302343, size = 36, normalized size = 0.56

$$\frac{4e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\left(n\sin\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

Maple [C] time = 0., size = 122, normalized size = 1.9

$$\frac{2\,ie^{-\frac{i}{2}bx}e^{-\frac{i}{2}a}}{nb}e^{n\sin\left(\frac{a}{2}\right)\cos\left(\frac{bx}{2}\right)+n\cos\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)} - \frac{2\,ie^{\frac{i}{2}bx}e^{\frac{i}{2}a}}{nb}e^{n\sin\left(\frac{a}{2}\right)\cos\left(\frac{bx}{2}\right)+n\cos\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)} - 4\,\frac{e^{n(\sin(a/2)\cos(1/2\,bx)+\cos(a/2)\sin(1/2\,b}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] 2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*b*x)+n*cos(1/2*a)*sin(1/2*b*x))*exp(-1/2*I
*b*x)*exp(-1/2*I*a)-2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*b*x)+n*cos(1/2*a)*sin(
1/2*b*x))*exp(1/2*I*b*x)*exp(1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*a)*cos(1/2*b*x
)+cos(1/2*a)*sin(1/2*b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A] time = 2.16646, size = 90, normalized size = 1.41

$$\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)

Giac [B] time = 1.26535, size = 186, normalized size = 2.91

$$\frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*
x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 +
1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.745 $\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rubi [A] time = 0.0405534, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {12, 2176, 2194}

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx = -\frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn} + \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \cos(a+bx)\right)}{bn}$$
$$= \frac{2e^{n\cos(a+bx)}}{bn^2} - \frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

Mathematica [A] time = 0.148723, size = 28, normalized size = 0.65

$$-\frac{2e^{n\cos(a+bx)}(n\cos(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

Maple [C] time = 0.07, size = 106, normalized size = 2.5

$$-\frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{-ibx}e^{-ia}}{bn} - \frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{ibx}e^{ia}}{bn} + 2\,\frac{e^{-n(\sin(bx)\sin(a)-\cos(bx)\cos(a))}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] -1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-1/b/n
*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+2/b/n^2*exp(-
n*(sin(b*x)*sin(a)-cos(b*x)*cos(a)))
```

Maxima [A] time = 1.0477, size = 50, normalized size = 1.16

$$-\frac{2\left(n\cos\left(bx+a\right)e^{(n\cos(bx+a))} - e^{(n\cos(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

Fricas [A] time = 2.08946, size = 70, normalized size = 1.63

$$-\frac{2\left(n\cos\left(bx+a\right)-1\right)e^{\left(n\cos\left(bx+a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cos\left(a+bx\right)}\sin\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\cos\left(bx+a\right)\right)}\sin\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

3.746 $\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rubi [A] time = 0.0349637, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2176, 2194}

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = -\frac{\text{Subst}\left(\int 2e^{nx}x\,dx,x,\cos(a+bx)\right)}{b}$$
$$= -\frac{2\,\text{Subst}\left(\int e^{nx}x\,dx,x,\cos(a+bx)\right)}{b}$$
$$= -\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn} + \frac{2\,\text{Subst}\left(\int e^{nx}\,dx,x,\cos(a+bx)\right)}{bn}$$
$$= \frac{2e^{n\cos(a+bx)}}{bn^2} - \frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

Mathematica [A] time = 0.0346088, size = 28, normalized size = 0.65

$$-\frac{2e^{n\cos(a+bx)}(n\cos(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

Maple [C] time = 0., size = 106, normalized size = 2.5

$$-\frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{-ibx}e^{-ia}}{bn} - \frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{ibx}e^{ia}}{bn} + 2\,\frac{e^{-n(\sin(bx)\sin(a)-\cos(bx)\cos(a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] -1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-1/b/n
*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+2/b/n^2*exp(-
n*(sin(b*x)*sin(a)-cos(b*x)*cos(a)))
```

Maxima [A] time = 1.05626, size = 50, normalized size = 1.16

$$-\frac{2\left(n\cos\left(bx+a\right)e^{\left(n\cos\left(bx+a\right)\right)} - e^{\left(n\cos\left(bx+a\right)\right)}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

Fricas [A] time = 2.03885, size = 70, normalized size = 1.63

$$-\frac{2\,(n\cos{(bx+a)}-1)e^{(n\cos(bx+a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cos{(a+bx)}}\sin{(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(n\cos(bx+a))}\sin{(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

3.747 $\int e^{n \cos\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx)\, dx$

Optimal. Leaf size=64

$$\frac{4e^{n \cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} - \frac{4 \cos\left(\frac{a}{2}+\frac{bx}{2}\right) e^{n \cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rubi [A] time = 0.0374232, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4e^{n \cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} - \frac{4 \cos\left(\frac{a}{2}+\frac{bx}{2}\right) e^{n \cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx &= -\frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= -\frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= -\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}+\frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\
&= \frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}
$$

Mathematica [A] time = 0.176593, size = 36, normalized size = 0.56

$$
-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(n\cos\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

Maple [C] time = 0.072, size = 124, normalized size = 1.9

$$
-2\,\frac{\mathrm{e}^{n\cos(a/2)\cos(1/2\,bx)-n\sin(a/2)\sin(1/2\,bx)}\mathrm{e}^{i/2bx}\mathrm{e}^{i/2a}}{bn}-2\,\frac{\mathrm{e}^{n\cos(a/2)\cos(1/2\,bx)-n\sin(a/2)\sin(1/2\,bx)}\mathrm{e}^{-i/2bx}\mathrm{e}^{-i/2a}}{bn}+4\,\frac{\mathrm{e}^{-n(\sin(a/2)\sin(1/2\,bx}}{n^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] -2/b/n*exp(n*cos(1/2*a)*cos(1/2*b*x)-n*sin(1/2*a)*sin(1/2*b*x))*exp(1/2*I*b
*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*a)*cos(1/2*b*x)-n*sin(1/2*a)*sin(1/2*b
*x))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(-n*(sin(1/2*a)*sin(1/2*b*x)-
cos(1/2*a)*cos(1/2*b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\cos\left(\frac{1}{2}bx+\frac{1}{2}a\right)\right)}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A] time = 2.09486, size = 92, normalized size = 1.44

$$-\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

Giac [B] time = 1.2496, size = 263, normalized size = 4.11

$$\frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}+e^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+}\right.}\right.}{bn^2\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1
/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a
)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(ta
n(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*
x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.748 $\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rubi [A] time = 0.0374376, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = -\frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= -\frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= -\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}+\frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= \frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.0340926, size = 36, normalized size = 0.56

$$-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(n\cos\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

Maple [C] time = 0., size = 124, normalized size = 1.9

$$-2\,\frac{e^{n\cos(a/2)\cos(1/2\,bx)-n\sin(a/2)\sin(1/2\,bx)}e^{i/2bx}e^{i/2a}}{bn}-2\,\frac{e^{n\cos(a/2)\cos(1/2\,bx)-n\sin(a/2)\sin(1/2\,bx)}e^{-i/2bx}e^{-i/2a}}{bn}+4\,\frac{e^{-n(\sin(a/2)\sin(1/2\,bx}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] -2/b/n*exp(n*cos(1/2*a)*cos(1/2*b*x)-n*sin(1/2*a)*sin(1/2*b*x))*exp(1/2*I*b
*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*a)*cos(1/2*b*x)-n*sin(1/2*a)*sin(1/2*b
*x))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(-n*(sin(1/2*a)*sin(1/2*b*x)-
cos(1/2*a)*cos(1/2*b*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A] time = 2.17702, size = 92, normalized size = 1.44

$$-\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

Giac [B] time = 1.25338, size = 263, normalized size = 4.11

$$\frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}+e^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+}\right.}\right.}{bn^2\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1
/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a
)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(ta
n(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*
x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.749 $\int \csc(x)\log(\tan(x))\sec(x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\tan(x))$$

```
[Out] Log[Tan[x]]^2/2
```

Rubi [A] time = 0.0219136, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2620, 29, 6686}

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

Mathematica [A] time = 0.0056025, size = 9, normalized size = 1.

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Maple [A] time = 0.017, size = 8, normalized size = 0.9

$$\frac{(\ln(\tan(x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*ln(tan(x))*sec(x),x)
```

```
[Out] 1/2*ln(tan(x))^2
```

Maxima [A] time = 0.96757, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x))^2
```

Fricas [A] time = 1.92328, size = 35, normalized size = 3.89

$$\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x)/cos(x))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(\tan(x))\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*ln(tan(x))*sec(x),x)
```

```
[Out] Integral(log(tan(x))*csc(x)*sec(x), x)
```

Giac [A] time = 1.07955, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="giac")
```

```
[Out] 1/2*log(tan(x))^2
```

3.750 $\int \csc(2x) \log(\tan(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{4}\log^2(\tan(x))$$

```
[Out] Log[Tan[x]]^2/4
```

Rubi [A] time = 0.0195538, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3770, 6686}

$$\frac{1}{4}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*Log[Tan[x]],x]
```

```
[Out] Log[Tan[x]]^2/4
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(2x)\log(\tan(x))\,dx = \frac{1}{4}\log^2(\tan(x))$$

Mathematica [A] time = 0.0109094, size = 9, normalized size = 1.

$$\frac{1}{4}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*Log[Tan[x]],x]
```

```
[Out] Log[Tan[x]]^2/4
```

Maple [A] time = 0.016, size = 8, normalized size = 0.9

$$\frac{(\ln(\tan(x)))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*ln(tan(x)),x)
```

```
[Out] 1/4*ln(tan(x))^2
```

Maxima [B] time = 1.54833, size = 358, normalized size = 39.78

$$\frac{1}{4}(\pi - 2\arctan(\sin(x), \cos(x) + 1) - 2\arctan(\sin(x), \cos(x) - 1))\arctan(\sin(2x), \cos(2x) + 1) + \frac{1}{4}\arctan(\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="maxima")
```

```
[Out] 1/4*(pi - 2*arctan2(sin(x), cos(x) + 1) - 2*arctan2(sin(x), cos(x) - 1))*ar
ctan2(sin(2*x), cos(2*x) + 1) + 1/4*arctan2(sin(2*x), cos(2*x) + 1)^2 - 1/4
*(pi - 2*arctan2(sin(x), cos(x) - 1))*arctan2(sin(x), cos(x) + 1) + 1/4*arc
tan2(sin(x), cos(x) + 1)^2 - 1/4*pi*arctan2(sin(x), cos(x) - 1) + 1/4*arcta
n2(sin(x), cos(x) - 1)^2 + 1/8*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + l
og(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos
(2*x) + 1) - 1/16*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^2 - 1/16*lo
g(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^2 - 1/8*log(cos(x)^2 + sin(x)^2 + 2*c
os(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/16*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1)^2 - 1/2*log(cot(2*x) + csc(2*x))*log(tan(x))
```

Fricas [A] time = 2.01379, size = 26, normalized size = 2.89

$$\frac{1}{4}\log\left(\tan\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*log(tan(x))^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*ln(tan(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc\left(2\,x\right)\log\left(\tan\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="giac")
```

```
[Out] integrate(csc(2*x)*log(tan(x)), x)
```

3.751 $\int e^{\cos^2(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=3

$$ex$$

```
[Out] E*x
```

Rubi [A] time = 0.0087911, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 203}

$$ex$$

Antiderivative was successfully verified.

```
[In] Int[E^(Cos[x]^2 + Sin[x]^2),x]
```

```
[Out] E*x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\cos^2(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{e}{1+x^2}\,dx,x,\tan(x)\right)\\ &= e\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\ &= ex\end{aligned}$$

Mathematica [A] time = 0.0004321, size = 3, normalized size = 1.

$$ex$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x]^2 + Sin[x]^2),x]
```

```
[Out] E*x
```

Maple [C] time = 0.012, size = 5, normalized size = 1.7

$$ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x)^2+sin(x)^2),x)
```

```
[Out] exp(1)*x
```

Maxima [C] time = 1.45693, size = 5, normalized size = 1.67

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

```
[Out] x*e
```

Fricas [C] time = 1.9375, size = 7, normalized size = 2.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

```
[Out] x*e
```

Sympy [B] time = 0.14168, size = 14, normalized size = 4.67

$$xe^{\sin^2(x)}e^{\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)**2+sin(x)**2),x)
```

```
[Out] x*exp(sin(x)**2)*exp(cos(x)**2)
```

Giac [C] time = 1.09118, size = 5, normalized size = 1.67

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="giac")
```

```
[Out] x*e
```

3.752 $\int x \sec^2(x) \, dx$

Optimal. Leaf size=8

$$x \tan(x) + \log(\cos(x))$$

```
[Out] Log[Cos[x]] + x*Tan[x]
```

Rubi [A] time = 0.0177246, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]^2,x]
```

```
[Out] Log[Cos[x]] + x*Tan[x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \sec^2(x) \, dx &= x \tan(x) - \int \tan(x) \, dx \\ &= \log(\cos(x)) + x \tan(x) \end{aligned}$$

Mathematica [A] time = 0.0053494, size = 8, normalized size = 1.

$$x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]^2,x]
```

```
[Out] Log[Cos[x]] + x*Tan[x]
```

Maple [A] time = 0.005, size = 9, normalized size = 1.1

$$\ln\left(\cos\left(x\right)\right)+x\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)^2,x)
```

```
[Out] ln(cos(x))+x*tan(x)
```

Maxima [B] time = 1.46013, size = 100, normalized size = 12.5

$$\frac{\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)\log\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)+4\,x\sin\left(2\,x\right)}{2\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2
 + 2*cos(2*x) + 1) + 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) +
1)
```

Fricas [B] time = 2.14376, size = 55, normalized size = 6.88

$$\frac{\cos\left(x\right)\log\left(-\cos\left(x\right)\right)+x\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (cos(x)*log(-cos(x)) + x*sin(x))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)**2,x)
```

```
[Out] Integral(x*sec(x)**2, x)
```

Giac [B] time = 1.13281, size = 139, normalized size = 17.38

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-4x\tan\left(\frac{1}{2}x\right)-\log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)
^2 + 1))*tan(1/2*x)^2 - 4*x*tan(1/2*x) - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)
^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(tan(1/2*x)^2 - 1)
```

3.753 $\int x\cos^4\left(x^2\right)\,dx$

Optimal. Leaf size=34

$$\frac{3x^2}{16}+\frac{1}{8}\sin\left(x^2\right)\cos^3\left(x^2\right)+\frac{3}{16}\sin\left(x^2\right)\cos\left(x^2\right)$$

```
[Out] (3*x^2)/16 + (3*Cos[x^2]*Sin[x^2])/16 + (Cos[x^2]^3*Sin[x^2])/8
```

Rubi [A] time = 0.0224351, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 2635, 8}

$$\frac{3x^2}{16}+\frac{1}{8}\sin\left(x^2\right)\cos^3\left(x^2\right)+\frac{3}{16}\sin\left(x^2\right)\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x^2]^4,x]
```

```
[Out] (3*x^2)/16 + (3*Cos[x^2]*Sin[x^2])/16 + (Cos[x^2]^3*Sin[x^2])/8
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int x\cos^4\left(x^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \cos^4(x)\,dx, x, x^2\right)\\
&= \frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)+\frac{3}{8}\,\text{Subst}\left(\int \cos^2(x)\,dx, x, x^2\right)\\
&= \frac{3}{16}\cos\left(x^2\right)\sin\left(x^2\right)+\frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)+\frac{3}{16}\,\text{Subst}\left(\int 1\,dx, x, x^2\right)\\
&= \frac{3x^2}{16}+\frac{3}{16}\cos\left(x^2\right)\sin\left(x^2\right)+\frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0167307, size = 28, normalized size = 0.82

$$
\frac{3x^2}{16}+\frac{1}{8}\sin\left(2x^2\right)+\frac{1}{64}\sin\left(4x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x^2]^4,x]
```

```
[Out] (3*x^2)/16 + Sin[2*x^2]/8 + Sin[4*x^2]/64
```

Maple [A] time = 0.01, size = 26, normalized size = 0.8

$$
\frac{\sin\left(x^2\right)}{8}\left(\left(\cos\left(x^2\right)\right)^3+\frac{3\cos\left(x^2\right)}{2}\right)+\frac{3\,x^2}{16}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)^4,x)
```

```
[Out] 1/8*(cos(x^2)^3+3/2*cos(x^2))*sin(x^2)+3/16*x^2
```

Maxima [A] time = 0.957417, size = 30, normalized size = 0.88

$$
\frac{3}{16}\,x^2+\frac{1}{64}\,\sin\left(4\,x^2\right)+\frac{1}{8}\,\sin\left(2\,x^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="maxima")
```

```
[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

Fricas [A] time = 2.02743, size = 73, normalized size = 2.15

$$\frac{3}{16}\,x^{2}+\frac{1}{16}\left(2\,\cos\left(x^{2}\right)^{3}+3\,\cos\left(x^{2}\right)\right)\sin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="fricas")
```

```
[Out] 3/16*x^2 + 1/16*(2*cos(x^2)^3 + 3*cos(x^2))*sin(x^2)
```

Sympy [B] time = 1.20831, size = 76, normalized size = 2.24

$$\frac{3x^{2}\sin^{4}\left(x^{2}\right)}{16}+\frac{3x^{2}\sin^{2}\left(x^{2}\right)\cos^{2}\left(x^{2}\right)}{8}+\frac{3x^{2}\cos^{4}\left(x^{2}\right)}{16}+\frac{3\sin^{3}\left(x^{2}\right)\cos\left(x^{2}\right)}{16}+\frac{5\sin\left(x^{2}\right)\cos^{3}\left(x^{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)**4,x)
```

```
[Out] 3*x**2*sin(x**2)**4/16 + 3*x**2*sin(x**2)**2*cos(x**2)**2/8 + 3*x**2*cos(x*
*2)**4/16 + 3*sin(x**2)**3*cos(x**2)/16 + 5*sin(x**2)*cos(x**2)**3/16
```

Giac [A] time = 1.08616, size = 30, normalized size = 0.88

$$\frac{3}{16}\,x^{2}+\frac{1}{64}\,\sin\left(4\,x^{2}\right)+\frac{1}{8}\,\sin\left(2\,x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

3.754 $\int \sqrt{\cos(x)}\sin(x)\,dx$

Optimal. Leaf size=10

$$-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

```
[Out] (-2*Cos[x]^(3/2))/3
```

Rubi [A] time = 0.0121782, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 30}

$$-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]]*Sin[x],x]
```

```
[Out] (-2*Cos[x]^(3/2))/3
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(x)}\sin(x)\,dx &= -\text{Subst}\left(\int \sqrt{x}\,dx, x, \cos(x)\right)\\ &= -\frac{2}{3}\cos^{\frac{3}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.002313, size = 10, normalized size = 1.

$$-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]]*Sin[x],x]
```

```
[Out] (-2*Cos[x]^(3/2))/3
```

Maple [A] time = 0.003, size = 7, normalized size = 0.7

$$-\frac{2}{3}(\cos(x))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*cos(x)^(1/2),x)
```

```
[Out] -2/3*cos(x)^(3/2)
```

Maxima [A] time = 0.961796, size = 8, normalized size = 0.8

$$-\frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*cos(x)^(3/2)
```

Fricas [A] time = 2.07166, size = 26, normalized size = 2.6

$$-\frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*cos(x)^(3/2)
```

Sympy [A] time = 0.283816, size = 10, normalized size = 1.

$$-\frac{2\cos^{\frac{3}{2}}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)**(1/2),x)
```

```
[Out] -2*cos(x)**(3/2)/3
```

Giac [A] time = 1.10316, size = 8, normalized size = 0.8

$$-\frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*cos(x)^(3/2)
```

3.755 $\int e^{-2x} \tan\left(e^{-2x}\right) dx$

Optimal. Leaf size=11

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

```
[Out] Log[Cos[E^(-2*x)]]/2
```

Rubi [A] time = 0.0117125, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 3475}

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[E^(-2*x)]/E^(2*x),x]

[Out] Log[Cos[E^(-2*x)]]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2x} \tan\left(e^{-2x}\right) dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \tan(x)\,dx, x, e^{-2x}\right)\right)\\ &= \frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0080568, size = 11, normalized size = 1.

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[E^(-2*x)]/E^(2*x),x]
```

```
[Out] Log[Cos[E^(-2*x)]]/2
```

Maple [A] time = 0.012, size = 9, normalized size = 0.8

$$\frac{\ln\left(\cos\left(e^{-2x}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(exp(-2*x))/exp(2*x),x)
```

```
[Out] 1/2*ln(cos(exp(-2*x)))
```

Maxima [A] time = 0.958688, size = 11, normalized size = 1.

$$-\frac{1}{2}\log\left(\sec\left(e^{(-2x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(sec(e^(-2*x)))
```

Fricas [A] time = 2.06475, size = 46, normalized size = 4.18

$$\frac{1}{4}\log\left(\frac{1}{\tan\left(e^{(-2x)}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(1/(tan(e^(-2*x))^2 + 1))
```

Sympy [A] time = 0.37184, size = 15, normalized size = 1.36

$$-\frac{\log\left(\tan^{2}\left(e^{-2x}\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x)
```

```
[Out] -log(tan(exp(-2*x))**2 + 1)/4
```

Giac [A] time = 1.07857, size = 12, normalized size = 1.09

$$\frac{1}{2}\,\log\left(\left|\cos\left(e^{(-2\,x)}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(cos(e^(-2*x))))
```

3.756 $\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx$

Optimal. Leaf size=7

$$-2\log(\cos(x)+1)$$

```
[Out] -2*Log[1 + Cos[x]]
```

Rubi [A] time = 0.0443849, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {12, 31}

$$-2\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
```

```
[Out] -2*Log[1 + Cos[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{2}{1+x}\,dx, x, \cos(x)\right)\\
&= -\left(2\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\right)\\
&= -2\log(1+\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.0055193, size = 9, normalized size = 1.29

$$-4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
```

```
[Out] -4*Log[Cos[x/2]]
```

Maple [A] time = 0.033, size = 8, normalized size = 1.1

$$-2\ln\left(1+\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*sin(2*x)/(1+cos(x)),x)
```

```
[Out] -2*ln(1+cos(x))
```

Maxima [A] time = 0.979069, size = 9, normalized size = 1.29

$$-2\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] -2*log(cos(x) + 1)
```

Fricas [A] time = 2.08175, size = 35, normalized size = 5.

$$-2\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] -2*log(1/2*cos(x) + 1/2)
```

Sympy [A] time = 7.66422, size = 8, normalized size = 1.14

$$-2\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x)
```

```
[Out] -2*log(cos(x) + 1)
```

Giac [B] time = 1.10496, size = 23, normalized size = 3.29

$$2\,\log\left(-\frac{\cos\left(x\right)-1}{\cos\left(x\right)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="giac")
```

```
[Out] 2*log(-(cos(x) - 1)/(cos(x) + 1) + 1)
```

3.757 $\int x \sec^2(3x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}x\tan(3x)+\frac{1}{9}\log(\cos(3x))$$

```
[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

Rubi [A] time = 0.0185204, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4184, 3475}

$$\frac{1}{3}x\tan(3x)+\frac{1}{9}\log(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[3*x]^2,x]
```

```
[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sec^2(3x)\,dx &= \frac{1}{3}x\tan(3x)-\frac{1}{3}\int \tan(3x)\,dx\\ &= \frac{1}{9}\log(\cos(3x))+\frac{1}{3}x\tan(3x)\end{aligned}$$

Mathematica [A] time = 0.0084289, size = 19, normalized size = 1.

$$\frac{1}{3}x\tan(3x)+\frac{1}{9}\log(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[3*x]^2,x]
```

```
[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

Maple [A] time = 0.007, size = 16, normalized size = 0.8

$$\frac{\ln(\cos(3x))}{9}+\frac{x\tan(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(3*x)^2,x)
```

```
[Out] 1/9*ln(cos(3*x))+1/3*x*tan(3*x)
```

Maxima [B] time = 1.46553, size = 100, normalized size = 5.26

$$\frac{\left(\cos(6x)^2+\sin(6x)^2+2\cos(6x)+1\right)\log\left(\cos(6x)^2+\sin(6x)^2+2\cos(6x)+1\right)+12x\sin(6x)}{18\left(\cos(6x)^2+\sin(6x)^2+2\cos(6x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*((cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x) + 1)*log(cos(6*x)^2 + sin(6*x)^
2 + 2*cos(6*x) + 1) + 12*x*sin(6*x))/(cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x)
+ 1)
```

Fricas [A] time = 2.00315, size = 74, normalized size = 3.89

$$\frac{\cos(3x)\log(-\cos(3x))+3x\sin(3x)}{9\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(cos(3*x)*log(-cos(3*x)) + 3*x*sin(3*x))/cos(3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sec^2(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)**2,x)
```

```
[Out] Integral(x*sec(3*x)**2, x)
```

Giac [B] time = 1.14042, size = 139, normalized size = 7.32

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4-2\tan\left(\frac{3}{2}x\right)^2+1\right)}{\tan\left(\frac{3}{2}x\right)^4+2\tan\left(\frac{3}{2}x\right)^2+1}\right)\tan\left(\frac{3}{2}x\right)^2-12x\tan\left(\frac{3}{2}x\right)-\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4-2\tan\left(\frac{3}{2}x\right)^2+1\right)}{\tan\left(\frac{3}{2}x\right)^4+2\tan\left(\frac{3}{2}x\right)^2+1}\right)}{18\left(\tan\left(\frac{3}{2}x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/18*(log(4*(tan(3/2*x)^4 - 2*tan(3/2*x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x
)^2 + 1))*tan(3/2*x)^2 - 12*x*tan(3/2*x) - log(4*(tan(3/2*x)^4 - 2*tan(3/2*
x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x)^2 + 1)))/(tan(3/2*x)^2 - 1)
```

3.758 $\int e^{-2\pi x} \cos(2\pi x) \, dx$

Optimal. Leaf size=37

$$\frac{e^{-2\pi x} \sin(2\pi x)}{4\pi} - \frac{e^{-2\pi x} \cos(2\pi x)}{4\pi}$$

```
[Out] -Cos[2*Pi*x]/(4*E^(2*Pi*x)*Pi) + Sin[2*Pi*x]/(4*E^(2*Pi*x)*Pi)
```

Rubi [A] time = 0.0135448, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4433}

$$\frac{e^{-2\pi x} \sin(2\pi x)}{4\pi} - \frac{e^{-2\pi x} \cos(2\pi x)}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*Pi*x]/E^(2*Pi*x),x]
```

```
[Out] -Cos[2*Pi*x]/(4*E^(2*Pi*x)*Pi) + Sin[2*Pi*x]/(4*E^(2*Pi*x)*Pi)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-2\pi x} \cos(2\pi x) \, dx = -\frac{e^{-2\pi x} \cos(2\pi x)}{4\pi} + \frac{e^{-2\pi x} \sin(2\pi x)}{4\pi}$$

Mathematica [A] time = 0.0281835, size = 26, normalized size = 0.7

$$\frac{e^{-2\pi x}(\sin(2\pi x) - \cos(2\pi x))}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*Pi*x]/E^(2*Pi*x),x]
```

```
[Out] (-Cos[2*Pi*x] + Sin[2*Pi*x])/(4*E^(2*Pi*x)*Pi)
```

Maple [A] time = 0.01, size = 31, normalized size = 0.8

$$\frac{1}{2\pi}\left(-\frac{e^{-2\pi x}\cos(2\pi x)}{2}+\frac{e^{-2\pi x}\sin(2\pi x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*Pi*x)/exp(2*Pi*x),x)
```

```
[Out] 1/2/Pi*(-1/2*exp(-2*Pi*x)*cos(2*Pi*x)+1/2*exp(-2*Pi*x)*sin(2*Pi*x))
```

Maxima [A] time = 0.967941, size = 35, normalized size = 0.95

$$-\frac{(\pi\cos(2\pi x)-\pi\sin(2\pi x))e^{(-2\pi x)}}{4\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="maxima")
```

```
[Out] -1/4*(pi*cos(2*pi*x) - pi*sin(2*pi*x))*e^(-2*pi*x)/pi^2
```

Fricas [A] time = 1.82783, size = 82, normalized size = 2.22

$$-\frac{\cos(2\pi x)e^{(-2\pi x)}-e^{(-2\pi x)}\sin(2\pi x)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(2*pi*x)*e^(-2*pi*x) - e^(-2*pi*x)*sin(2*pi*x))/pi
```

Sympy [A] time = 0.486224, size = 32, normalized size = 0.86

$$\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x)
```

```
[Out] exp(-2*pi*x)*sin(2*pi*x)/(4*pi) - exp(-2*pi*x)*cos(2*pi*x)/(4*pi)
```

Giac [A] time = 1.12773, size = 36, normalized size = 0.97

$$-\frac{1}{4}\left(\frac{\cos(2\pi x)}{\pi}-\frac{\sin(2\pi x)}{\pi}\right)e^{(-2\pi x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="giac")
```

```
[Out] -1/4*(cos(2*pi*x)/pi - sin(2*pi*x)/pi)*e^(-2*pi*x)
```

3.759 $\int \left(\cos^{12}(x)\sin^{10}(x) - \cos^{10}(x)\sin^{12}(x)\right) dx$

Optimal. Leaf size=12

$$\frac{1}{11}\sin^{11}(x)\cos^{11}(x)$$

```
[Out] (Cos[x]^11*Sin[x]^11)/11
```

Rubi [B] time = 0.3238, antiderivative size = 129, normalized size of antiderivative = 10.75, number of steps used = 25, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2568, 2635, 8}

$$-\frac{1}{22}\sin^9(x)\cos^{13}(x) - \frac{9}{440}\sin^7(x)\cos^{13}(x) - \frac{7}{880}\sin^5(x)\cos^{13}(x) - \frac{7\sin^3(x)\cos^{13}(x)}{2816} + \frac{1}{22}\sin^{11}(x)\cos^{11}(x) + \frac{1}{40}\sin^9$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

```
[Out] (3*Cos[x]^11*Sin[x])/5632 - (3*Cos[x]^13*Sin[x])/5632 + (Cos[x]^11*Sin[x]^3
)/512 - (7*Cos[x]^13*Sin[x]^3)/2816 + (7*Cos[x]^11*Sin[x]^5)/1280 - (7*Cos[
x]^13*Sin[x]^5)/880 + (Cos[x]^11*Sin[x]^7)/80 - (9*Cos[x]^13*Sin[x]^7)/440
+ (Cos[x]^11*Sin[x]^9)/40 - (Cos[x]^13*Sin[x]^9)/22 + (Cos[x]^11*Sin[x]^11)
/22
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x)\right)dx = \int \cos^{12}(x)\sin^{10}(x)\,dx - \int \cos^{10}(x)\sin^{12}(x)\,dx$$
$$= -\frac{1}{22}\cos^{13}(x)\sin^{9}(x) + \frac{1}{22}\cos^{11}(x)\sin^{11}(x) + \frac{9}{22}\int \cos^{12}(x)\sin^{8}(x)\,dx$$
$$= -\frac{9}{440}\cos^{13}(x)\sin^{7}(x) + \frac{1}{40}\cos^{11}(x)\sin^{9}(x) - \frac{1}{22}\cos^{13}(x)\sin^{9}(x) + \frac{1}{22}$$
$$= -\frac{7}{880}\cos^{13}(x)\sin^{5}(x) + \frac{1}{80}\cos^{11}(x)\sin^{7}(x) - \frac{9}{440}\cos^{13}(x)\sin^{7}(x) + \frac{1}{4}$$
$$= -\frac{7\cos^{13}(x)\sin^{3}(x)}{2816} + \frac{7\cos^{11}(x)\sin^{5}(x)}{1280} - \frac{7}{880}\cos^{13}(x)\sin^{5}(x) + \frac{1}{80}\cos$$
$$= -\frac{3\cos^{13}(x)\sin(x)}{5632} + \frac{1}{512}\cos^{11}(x)\sin^{3}(x) - \frac{7\cos^{13}(x)\sin^{3}(x)}{2816} + \frac{7\cos^{1}}{}$$
$$= \frac{3\cos^{11}(x)\sin(x)}{5632} - \frac{3\cos^{13}(x)\sin(x)}{5632} + \frac{1}{512}\cos^{11}(x)\sin^{3}(x) - \frac{7\cos^{13}(x}{28}$$

Mathematica [B] time = 0.0262099, size = 49, normalized size = 4.08

$$\frac{21\sin(2x)}{1048576} - \frac{15\sin(6x)}{1048576} + \frac{15\sin(10x)}{2097152} - \frac{5\sin(14x)}{2097152} + \frac{\sin(18x)}{2097152} - \frac{\sin(22x)}{23068672}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

```
[Out] (21*Sin[2*x])/1048576 - (15*Sin[6*x])/1048576 + (15*Sin[10*x])/2097152 - (5
*Sin[14*x])/2097152 + Sin[18*x]/2097152 - Sin[22*x]/23068672
```

Maple [B] time = 0.081, size = 176, normalized size = 14.7

$$-\frac{(\cos(x))^{13}(\sin(x))^{9}}{22} - \frac{9\,(\sin(x))^{7}(\cos(x))^{13}}{440} - \frac{7\,(\sin(x))^{5}(\cos(x))^{13}}{880} - \frac{7\,(\sin(x))^{3}(\cos(x))^{13}}{2816} - \frac{3\,\sin(x)(\cos(x))^{1}}{5632}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x)
```

```
[Out] -1/22*cos(x)^13*sin(x)^9-9/440*sin(x)^7*cos(x)^13-7/880*sin(x)^5*cos(x)^13-
7/2816*sin(x)^3*cos(x)^13-3/5632*sin(x)*cos(x)^13+1/22528*(cos(x)^11+11/10*
cos(x)^9+99/80*cos(x)^7+231/160*cos(x)^5+231/128*cos(x)^3+693/256*cos(x))*s
in(x)+1/22*cos(x)^11*sin(x)^11+1/40*sin(x)^9*cos(x)^11+1/80*sin(x)^7*cos(x)
^11+7/1280*sin(x)^5*cos(x)^11+1/512*sin(x)^3*cos(x)^11+1/2048*sin(x)*cos(x)
^11-1/20480*(cos(x)^9+9/8*cos(x)^7+21/16*cos(x)^5+105/64*cos(x)^3+315/128*c
os(x))*sin(x)
```

Maxima [A] time = 0.983177, size = 11, normalized size = 0.92

$$\frac{1}{22528}\sin(2x)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="maxima")
```

```
[Out] 1/22528*sin(2*x)^11
```

Fricas [B] time = 2.45203, size = 130, normalized size = 10.83

$$-\frac{1}{11}\left(\cos(x)^{21} - 5\cos(x)^{19} + 10\cos(x)^{17} - 10\cos(x)^{15} + 5\cos(x)^{13} - \cos(x)^{11}\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(cos(x)^21 - 5*cos(x)^19 + 10*cos(x)^17 - 10*cos(x)^15 + 5*cos(x)^13
- cos(x)^11)*sin(x)
```

Sympy [B] time = 0.089651, size = 236, normalized size = 19.67

$$-\frac{\sin^{21}(x)\cos(x)}{22} + \frac{89\sin^{19}(x)\cos(x)}{440} - \frac{301\sin^{17}(x)\cos(x)}{880} + \frac{3683\sin^{15}(x)\cos(x)}{14080} - \frac{433\sin^{13}(x)\cos(x)}{5632} + \frac{\sin^{11}(x}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**12*sin(x)**10-cos(x)**10*sin(x)**12,x)
```

```
[Out] -sin(x)**21*cos(x)/22 + 89*sin(x)**19*cos(x)/440 - 301*sin(x)**17*cos(x)/88
0 + 3683*sin(x)**15*cos(x)/14080 - 433*sin(x)**13*cos(x)/5632 + sin(x)**11*
cos(x)/22528 + sin(x)**9*cos(x)/20480 + 9*sin(x)**7*cos(x)/163840 + 21*sin(
x)**5*cos(x)/327680 + 21*sin(x)**3*cos(x)/262144 - sin(x)*cos(x)**21/22 + 8
9*sin(x)*cos(x)**19/440 - 301*sin(x)*cos(x)**17/880 + 3683*sin(x)*cos(x)**1
5/14080 - 433*sin(x)*cos(x)**13/5632 + sin(x)*cos(x)**11/22528 + sin(x)*cos
(x)**9/20480 + 9*sin(x)*cos(x)**7/163840 + 21*sin(x)*cos(x)**5/327680 + 21*
sin(x)*cos(x)**3/262144 + 63*sin(x)*cos(x)/262144
```

Giac [B] time = 1.11688, size = 50, normalized size = 4.17

$$-\frac{1}{23068672}\sin\left(22\,x\right)+\frac{1}{2097152}\sin\left(18\,x\right)-\frac{5}{2097152}\sin\left(14\,x\right)+\frac{15}{2097152}\sin\left(10\,x\right)-\frac{15}{1048576}\sin\left(6\,x\right)+\frac{21}{104857\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="giac")
```

```
[Out] -1/23068672*sin(22*x) + 1/2097152*sin(18*x) - 5/2097152*sin(14*x) + 15/2097
152*sin(10*x) - 15/1048576*sin(6*x) + 21/1048576*sin(2*x)
```

3.760 $\int x \cot(x^2) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

```
[Out] Log[Sin[x^2]]/2
```

Rubi [A] time = 0.0066126, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3748, 3475}

$$\frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[x^2],x]
```

```
[Out] Log[Sin[x^2]]/2
```

Rule 3748

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \cot\left(x^2\right) \, dx &= \frac{1}{2}\text{Subst}\left(\int \cot(x) \, dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(\sin\left(x^2\right)\right) \end{aligned}$$

Mathematica [B] time = 0.0086499, size = 19, normalized size = 2.11

$$\frac{1}{2}\log\left(\tan\left(x^2\right)\right)+\frac{1}{2}\log\left(\cos\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[x^2],x]
```

```
[Out] Log[Cos[x^2]]/2 + Log[Tan[x^2]]/2
```

Maple [A] time = 0.002, size = 8, normalized size = 0.9

$$\frac{\ln\left(\sin\left(x^2\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(x^2),x)
```

```
[Out] 1/2*ln(sin(x^2))
```

Maxima [A] time = 0.969597, size = 9, normalized size = 1.

$$\frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x^2))
```

Fricas [A] time = 2.08113, size = 43, normalized size = 4.78

$$\frac{1}{4}\log\left(-\frac{1}{2}\cos\left(2x^2\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="fricas")
```

```
[Out] 1/4*log(-1/2*cos(2*x^2) + 1/2)
```

Sympy [B] time = 0.151614, size = 19, normalized size = 2.11

$$-\frac{\log\left(\tan^{2}\left(x^{2}\right)+1\right)}{4}+\frac{\log\left(\tan\left(x^{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x**2),x)
```

```
[Out] -log(tan(x**2)**2 + 1)/4 + log(tan(x**2))/2
```

Giac [A] time = 1.11234, size = 16, normalized size = 1.78

$$\frac{1}{4}\log\left(\left|\cos\left(x^{2}\right)^{2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(cos(x^2)^2 - 1))
```

3.761 $\int x \sec^2\left(x^2\right)\,dx$

Optimal. Leaf size=8

$$\frac{\tan\left(x^2\right)}{2}$$

```
[Out] Tan[x^2]/2
```

Rubi [A] time = 0.0130906, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4204, 3767, 8}

$$\frac{\tan\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x^2]^2,x]
```

```
[Out] Tan[x^2]/2
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int x\sec^2\left(x^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sec^2(x)\,dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int 1\,dx, x, -\tan\left(x^2\right)\right)\right)\\
&= \frac{\tan\left(x^2\right)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.0163709, size = 8, normalized size = 1.

$$\frac{\tan\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x^2]^2,x]
```

```
[Out] Tan[x^2]/2
```

Maple [A] time = 0.005, size = 7, normalized size = 0.9

$$\frac{\tan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)^2,x)
```

```
[Out] 1/2*tan(x^2)
```

Maxima [B] time = 0.964929, size = 47, normalized size = 5.88

$$\frac{\sin\left(2\,x^2\right)}{\cos\left(2\,x^2\right)^2 + \sin\left(2\,x^2\right)^2 + 2\,\cos\left(2\,x^2\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="maxima")
```

```
[Out] sin(2*x^2)/(cos(2*x^2)^2 + sin(2*x^2)^2 + 2*cos(2*x^2) + 1)
```

Fricas [A] time = 1.97484, size = 31, normalized size = 3.88

$$\frac{\sin\left(x^2\right)}{2\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2)/cos(x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sec^2\left(x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)**2,x)
```

```
[Out] Integral(x*sec(x**2)**2, x)
```

Giac [A] time = 1.07706, size = 8, normalized size = 1.

$$\frac{1}{2}\tan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*tan(x^2)
```

3.762 $\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)$$

```
[Out] ArcTan[Sin[4*x]^2/3]/12
```

Rubi [A] time = 0.0290622, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {12, 275, 203}

$$\frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[8*x]/(9 + Sin[4*x]^4),x]
```

```
[Out] ArcTan[Sin[4*x]^2/3]/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{2x}{9+x^4}\,dx, x, \sin(4x)\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{9+x^4}\,dx, x, \sin(4x)\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \sin^2(4x)\right)\\ &= \frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)\end{aligned}$$

Mathematica [A] time = 0.0162902, size = 15, normalized size = 1.

$$\frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[8*x]/(9 + Sin[4*x]^4),x]
```

```
[Out] ArcTan[Sin[4*x]^2/3]/12
```

Maple [A] time = 0.05, size = 12, normalized size = 0.8

$$\frac{1}{12}\arctan\left(\frac{(\sin(4x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(8*x)/(9+sin(4*x)^4),x)
```

```
[Out] 1/12*arctan(1/3*sin(4*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(8x)}{\sin(4x)^4+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="maxima")
```

```
[Out] integrate(sin(8*x)/(sin(4*x)^4 + 9), x)
```

Fricas [A] time = 2.14511, size = 49, normalized size = 3.27

$$-\frac{1}{12}\arctan\left(\frac{1}{3}\cos\left(4x\right)^{2}-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="fricas")
```

```
[Out] -1/12*arctan(1/3*cos(4*x)^2 - 1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11829, size = 20, normalized size = 1.33

$$\frac{1}{12}\arctan\left(\frac{3}{\cos\left(4x\right)^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="giac")
```

```
[Out] 1/12*arctan(3/(cos(4*x)^2 - 1))
```

3.763 $\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] ArcTan[Sin[2*x]/(2*Sqrt[2])]/(4*Sqrt[2])
```

Rubi [A] time = 0.0218966, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3190, 203}

$$\frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]/(8 + Sin[2*x]^2),x]
```

```
[Out] ArcTan[Sin[2*x]/(2*Sqrt[2])]/(4*Sqrt[2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{8+x^2}\,dx, x, \sin(2x)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.0127909, size = 20, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]/(8 + Sin[2*x]^2),x]
```

```
[Out] ArcTan[(Cos[x]*Sin[x])/Sqrt[2]]/(4*Sqrt[2])
```

Maple [A] time = 0.013, size = 16, normalized size = 0.7

$$\frac{\sqrt{2}}{8}\arctan\left(\frac{\sin(2x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/(8+sin(2*x)^2),x)
```

```
[Out] 1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.45835, size = 20, normalized size = 0.87

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin(2x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

Fricas [A] time = 2.00101, size = 57, normalized size = 2.48

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

Sympy [A] time = 0.318515, size = 19, normalized size = 0.83

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sin\left(2x\right)}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)**2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*sin(2*x)/4)/8
```

Giac [A] time = 1.08792, size = 20, normalized size = 0.87

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

3.764 $\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx$

Optimal. Leaf size=37

$$-\frac{1}{6}\sin^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\cos^3\left(x^2\right)+\frac{\cos\left(x^2\right)}{2}$$

```
[Out] Cos[x^2]/2 - Cos[x^2]^3/6 + Sin[x^2]/2 - Sin[x^2]^3/6
```

Rubi [A] time = 0.0335579, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {14, 3380, 2633, 3379}

$$-\frac{1}{6}\sin^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\cos^3\left(x^2\right)+\frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
```

```
[Out] Cos[x^2]/2 - Cos[x^2]^3/6 + Sin[x^2]/2 - Sin[x^2]^3/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx &= \int\left(x\cos^3\left(x^2\right)-x\sin^3\left(x^2\right)\right)\,dx\\
&= \int x\cos^3\left(x^2\right)\,dx-\int x\sin^3\left(x^2\right)\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\cos^3(x)\,dx,x,x^2\right)-\frac{1}{2}\operatorname{Subst}\left(\int\sin^3(x)\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos\left(x^2\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin\left(x^2\right)\right)\\
&= \frac{\cos\left(x^2\right)}{2}-\frac{1}{6}\cos^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\sin^3\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0230532, size = 37, normalized size = 1.

$$-\frac{1}{6}\sin^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}+\frac{3\cos\left(x^2\right)}{8}-\frac{1}{24}\cos\left(3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
```

```
[Out] (3*Cos[x^2])/8 - Cos[3*x^2]/24 + Sin[x^2]/2 - Sin[x^2]^3/6
```

Maple [A] time = 0.024, size = 30, normalized size = 0.8

$$\frac{\left(2+\left(\cos\left(x^2\right)\right)^2\right)\sin\left(x^2\right)}{6}+\frac{\left(2+\left(\sin\left(x^2\right)\right)^2\right)\cos\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x^2)^3-sin(x^2)^3),x)
```

```
[Out] 1/6*(2+cos(x^2)^2)*sin(x^2)+1/6*(2+sin(x^2)^2)*cos(x^2)
```

Maxima [A] time = 0.968764, size = 39, normalized size = 1.05

$$-\frac{1}{24}\cos\left(3x^2\right)+\frac{3}{8}\cos\left(x^2\right)+\frac{1}{24}\sin\left(3x^2\right)+\frac{3}{8}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="maxima")
```

```
[Out] -1/24*cos(3*x^2) + 3/8*cos(x^2) + 1/24*sin(3*x^2) + 3/8*sin(x^2)
```

Fricas [A] time = 2.08791, size = 86, normalized size = 2.32

$$-\frac{1}{6}\cos\left(x^2\right)^3+\frac{1}{6}\left(\cos\left(x^2\right)^2+2\right)\sin\left(x^2\right)+\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="fricas")
```

```
[Out] -1/6*cos(x^2)^3 + 1/6*(cos(x^2)^2 + 2)*sin(x^2) + 1/2*cos(x^2)
```

Sympy [A] time = 0.609055, size = 42, normalized size = 1.14

$$\frac{\sin^3\left(x^2\right)}{3}+\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)\cos^2\left(x^2\right)}{2}+\frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x**2)**3-sin(x**2)**3),x)
```

```
[Out] sin(x**2)**3/3 + sin(x**2)**2*cos(x**2)/2 + sin(x**2)*cos(x**2)**2/2 + cos(
x**2)**3/3
```

Giac [A] time = 1.07506, size = 39, normalized size = 1.05

$$-\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{6}\sin\left(x^2\right)^3+\frac{1}{2}\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="giac")
```

```
[Out] -1/6*cos(x^2)^3 - 1/6*sin(x^2)^3 + 1/2*cos(x^2) + 1/2*sin(x^2)
```

3.765 $\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx$

Optimal. Leaf size=10

$$\cos(x) + \log(1 - \cos(x))$$

```
[Out] Cos[x] + Log[1 - Cos[x]]
```

Rubi [A] time = 0.0330559, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2833, 43}

$$\cos(x) + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(1 - Cos[x]),x]
```

```
[Out] Cos[x] + Log[1 - Cos[x]]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = -\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, -\cos(x)\right)$$
$$= \cos(x) + \log(1-\cos(x))$$

Mathematica [A] time = 0.0174454, size = 12, normalized size = 1.2

$$\cos(x) + 2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(1 - Cos[x]),x]
```

```
[Out] Cos[x] + 2*Log[Sin[x/2]]
```

Maple [A] time = 0.012, size = 9, normalized size = 0.9

$$\cos(x) + \ln(-1+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1-cos(x)),x)
```

```
[Out] cos(x)+ln(-1+cos(x))
```

Maxima [A] time = 0.9537, size = 11, normalized size = 1.1

$$\cos(x) + \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] cos(x) + log(cos(x) - 1)
```

Fricas [A] time = 1.92776, size = 45, normalized size = 4.5

$$\cos(x) + \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] cos(x) + log(-1/2*cos(x) + 1/2)
```

Sympy [A] time = 0.19146, size = 8, normalized size = 0.8

$$\log(\cos(x) - 1) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x)
```

```
[Out] log(cos(x) - 1) + cos(x)
```

Giac [A] time = 1.109, size = 14, normalized size = 1.4

$$\cos(x) + \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="giac")
```

```
[Out] cos(x) + log(-cos(x) + 1)
```

3.766 $\int x \cos\left(x^2\right) dx$

Optimal. Leaf size=8

$$\frac{\sin\left(x^2\right)}{2}$$

```
[Out] Sin[x^2]/2
```

Rubi [A] time = 0.0069327, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3380, 2637}

$$\frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x^2],x]
```

```
[Out] Sin[x^2]/2
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cos\left(x^2\right)\,dx = \frac{1}{2}\,\text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)$$
$$= \frac{\sin\left(x^2\right)}{2}$$

Mathematica [A] time = 0.0013442, size = 8, normalized size = 1.

$$\frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x^2],x]
```

```
[Out] Sin[x^2]/2
```

Maple [A] time = 0.005, size = 7, normalized size = 0.9

$$\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2),x)
```

```
[Out] 1/2*sin(x^2)
```

Maxima [A] time = 0.962249, size = 8, normalized size = 1.

$$\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sin(x^2)
```

Fricas [A] time = 1.89449, size = 19, normalized size = 2.38

$$\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2)
```

Sympy [A] time = 0.164242, size = 5, normalized size = 0.62

$$\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2),x)
```

```
[Out] sin(x**2)/2
```

Giac [A] time = 1.11061, size = 8, normalized size = 1.

$$\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="giac")
```

```
[Out] 1/2*sin(x^2)
```

3.767 $\int x^2 \cos\left(4x^3\right) dx$

Optimal. Leaf size=10

$$\frac{1}{12}\sin\left(4x^3\right)$$

[Out] Sin[4*x^3]/12

Rubi [A] time = 0.0121073, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3380, 2637}

$$\frac{1}{12}\sin\left(4x^3\right)$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[4*x^3],x]

[Out] Sin[4*x^3]/12

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 \cos\left(4x^3\right) dx &= \frac{1}{3}\text{Subst}\left(\int \cos(4x)\,dx, x, x^3\right)\\ &= \frac{1}{12}\sin\left(4x^3\right)\end{aligned}$$

Mathematica [A] time = 0.003212, size = 10, normalized size = 1.

$$\frac{1}{12}\sin\left(4x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[4*x^3],x]
```

```
[Out] Sin[4*x^3]/12
```

Maple [A] time = 0.005, size = 9, normalized size = 0.9

$$\frac{\sin\left(4x^3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3),x)
```

```
[Out] 1/12*sin(4*x^3)
```

Maxima [A] time = 0.967451, size = 11, normalized size = 1.1

$$\frac{1}{12}\sin\left(4x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="maxima")
```

```
[Out] 1/12*sin(4*x^3)
```

Fricas [A] time = 1.99023, size = 23, normalized size = 2.3

$$\frac{1}{12}\sin\left(4x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="fricas")
```

```
[Out] 1/12*sin(4*x^3)
```

Sympy [A] time = 0.293403, size = 7, normalized size = 0.7

$$\frac{\sin\left(4x^3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(4*x**3),x)
```

```
[Out] sin(4*x**3)/12
```

Giac [A] time = 1.1051, size = 11, normalized size = 1.1

$$\frac{1}{12}\sin\left(4\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="giac")
```

```
[Out] 1/12*sin(4*x^3)
```

3.768 $\int x^3 \cos\left(x^4\right) dx$

Optimal. Leaf size=8

$$\frac{\sin\left(x^4\right)}{4}$$

```
[Out] Sin[x^4]/4
```

Rubi [A] time = 0.0094394, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3380, 2637}

$$\frac{\sin\left(x^4\right)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x^4],x]
```

```
[Out] Sin[x^4]/4
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^3 \cos\left(x^4\right) dx = \frac{1}{4} \operatorname{Subst}\left(\int \cos(x) \, dx, x, x^4\right)$$
$$= \frac{\sin\left(x^4\right)}{4}$$

Mathematica [A] time = 0.001593, size = 8, normalized size = 1.

$$\frac{\sin\left(x^4\right)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x^4],x]
```

```
[Out] Sin[x^4]/4
```

Maple [A] time = 0.005, size = 7, normalized size = 0.9

$$\frac{\sin\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^4),x)
```

```
[Out] 1/4*sin(x^4)
```

Maxima [A] time = 0.982466, size = 8, normalized size = 1.

$$\frac{1}{4} \sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="maxima")
```

```
[Out] 1/4*sin(x^4)
```

Fricas [A] time = 1.94804, size = 19, normalized size = 2.38

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="fricas")
```

```
[Out] 1/4*sin(x^4)
```

Sympy [A] time = 0.538404, size = 5, normalized size = 0.62

$$\frac{\sin\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x**4),x)
```

```
[Out] sin(x**4)/4
```

Giac [A] time = 1.0835, size = 8, normalized size = 1.

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="giac")
```

```
[Out] 1/4*sin(x^4)
```

3.769 $\int x \sin\left(\frac{x^2}{2}\right) dx$

Optimal. Leaf size=10

$$-\cos\left(\frac{x^2}{2}\right)$$

[Out] -Cos[x^2/2]

Rubi [A] time = 0.0077483, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3379, 2638}

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[x^2/2],x]

[Out] -Cos[x^2/2]

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x\sin\left(\frac{x^2}{2}\right)dx = \frac{1}{2}\,\text{Subst}\left(\int \sin\left(\frac{x}{2}\right)dx, x, x^2\right)$$
$$= -\cos\left(\frac{x^2}{2}\right)$$

Mathematica [A] time = 0.0104019, size = 10, normalized size = 1.

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2/2],x]
```

```
[Out] -Cos[x^2/2]
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(1/2*x^2),x)
```

```
[Out] -cos(1/2*x^2)
```

Maxima [A] time = 0.956508, size = 11, normalized size = 1.1

$$-\cos\left(\frac{1}{2}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="maxima")
```

```
[Out] -cos(1/2*x^2)
```

Fricas [A] time = 1.8549, size = 20, normalized size = 2.

$$-\cos\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="fricas")
```

```
[Out] -cos(1/2*x^2)
```

Sympy [A] time = 0.163217, size = 7, normalized size = 0.7

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x**2),x)
```

```
[Out] -cos(x**2/2)
```

Giac [A] time = 1.08726, size = 11, normalized size = 1.1

$$-\cos\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="giac")
```

```
[Out] -cos(1/2*x^2)
```

3.770 $\int x \sec\left(x^2\right) \tan\left(x^2\right) dx$

Optimal. Leaf size=8

$$\frac{\sec\left(x^2\right)}{2}$$

```
[Out] Sec[x^2]/2
```

Rubi [A] time = 0.063054, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6715, 2606, 8}

$$\frac{\sec\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x^2]*Tan[x^2],x]
```

```
[Out] Sec[x^2]/2
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x \sec\left(x^2\right) \tan\left(x^2\right) \, dx = \frac{1}{2} \text{Subst}\left(\int \sec(x) \tan(x) \, dx, x, x^2\right)$$
$$= \frac{1}{2} \text{Subst}\left(\int 1 \, dx, x, \sec\left(x^2\right)\right)$$
$$= \frac{\sec\left(x^2\right)}{2}$$

Mathematica [A] time = 0.0056369, size = 8, normalized size = 1.

$$\frac{\sec\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x^2]*Tan[x^2],x]
```

```
[Out] Sec[x^2]/2
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$\frac{\sec\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)*tan(x^2),x)
```

```
[Out] 1/2*sec(x^2)
```

Maxima [B] time = 0.972905, size = 76, normalized size = 9.5

$$\frac{\cos\left(2\,x^2\right)\cos\left(x^2\right) + \sin\left(2\,x^2\right)\sin\left(x^2\right) + \cos\left(x^2\right)}{\cos\left(2\,x^2\right)^2 + \sin\left(2\,x^2\right)^2 + 2\,\cos\left(2\,x^2\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="maxima")
```

```
[Out] (cos(2*x^2)*cos(x^2) + sin(2*x^2)*sin(x^2) + cos(x^2))/(cos(2*x^2)^2 + sin(
2*x^2)^2 + 2*cos(2*x^2) + 1)
```

Fricas [A] time = 1.95153, size = 19, normalized size = 2.38

$$\frac{1}{2\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x^2)
```

Sympy [A] time = 0.331556, size = 5, normalized size = 0.62

$$\frac{\sec\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)*tan(x**2),x)
```

```
[Out] sec(x**2)/2
```

Giac [A] time = 1.08242, size = 11, normalized size = 1.38

$$\frac{1}{2\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="giac")
```

```
[Out] 1/2/cos(x^2)
```

3.771 $\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} dx$

Optimal. Leaf size=10

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

```
[Out] x^(-1) - Tan[x^(-1)]
```

Rubi [A] time = 0.0178687, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3747, 3473, 8}

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x^(-1)]^2/x^2,x]
```

```
[Out] x^(-1) - Tan[x^(-1)]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\operatorname{Subst}\left(\int \tan^2(x)\,dx, x, \frac{1}{x}\right)$$
$$= -\tan\left(\frac{1}{x}\right) + \operatorname{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Mathematica [A] time = 0.0233596, size = 12, normalized size = 1.2

$$\tan^{-1}\left(\tan\left(\frac{1}{x}\right)\right) - \tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x^(-1)]^2/x^2,x]
```

```
[Out] ArcTan[Tan[x^(-1)]] - Tan[x^(-1)]
```

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$x^{-1} - \tan\left(x^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(1/x)^2/x^2,x)
```

```
[Out] 1/x-tan(1/x)
```

Maxima [B] time = 0.961496, size = 90, normalized size = 9.

$$\frac{\cos\left(\frac{2}{x}\right)^2 - 2\,x\sin\left(\frac{2}{x}\right) + \sin\left(\frac{2}{x}\right)^2 + 2\,\cos\left(\frac{2}{x}\right) + 1}{\left(\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2\,\cos\left(\frac{2}{x}\right) + 1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] (cos(2/x)^2 - 2*x*sin(2/x) + sin(2/x)^2 + 2*cos(2/x) + 1)/((cos(2/x)^2 + si
n(2/x)^2 + 2*cos(2/x) + 1)*x)
```

Fricas [A] time = 2.03941, size = 28, normalized size = 2.8

$$-\frac{x\tan\left(\frac{1}{x}\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(x*tan(1/x) - 1)/x
```

Sympy [A] time = 0.382807, size = 7, normalized size = 0.7

$$-\tan\left(\frac{1}{x}\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)**2/x**2,x)
```

```
[Out] -tan(1/x) + 1/x
```

Giac [A] time = 1.09455, size = 14, normalized size = 1.4

$$\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/x - tan(1/x)
```

3.772 $\int x \tan\left(1+x^2\right) dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\cos\left(x^2+1\right)\right)$$

```
[Out] -Log[Cos[1 + x^2]]/2
```

Rubi [A] time = 0.010176, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3747, 3475}

$$-\frac{1}{2}\log\left(\cos\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[1 + x^2],x]
```

```
[Out] -Log[Cos[1 + x^2]]/2
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\tan\left(1+x^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \tan(1+x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}\log\left(\cos\left(1+x^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0175364, size = 11, normalized size = 1.

$$-\frac{1}{2}\log\left(\cos\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[1 + x^2],x]
```

```
[Out] -Log[Cos[1 + x^2]]/2
```

Maple [A] time = 0.003, size = 10, normalized size = 0.9

$$-\frac{\ln\left(\cos\left(x^2+1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x^2+1),x)
```

```
[Out] -1/2*ln(cos(x^2+1))
```

Maxima [A] time = 0.963871, size = 12, normalized size = 1.09

$$\frac{1}{2}\log\left(\sec\left(x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(sec(x^2 + 1))
```

Fricas [A] time = 1.8903, size = 46, normalized size = 4.18

$$-\frac{1}{4}\log\left(\frac{1}{\tan\left(x^2+1\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*log(1/(tan(x^2 + 1)^2 + 1))
```

Sympy [A] time = 0.167439, size = 12, normalized size = 1.09

$$\frac{\log\left(\tan^2\left(x^2+1\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x**2+1),x)
```

```
[Out] log(tan(x**2 + 1)**2 + 1)/4
```

Giac [A] time = 1.11254, size = 14, normalized size = 1.27

$$-\frac{1}{2}\log\left(\left|\cos\left(x^2+1\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(cos(x^2 + 1)))
```

3.773 $\int \sin(\pi(1+2x))\,dx$

Optimal. Leaf size=12

$$\frac{\cos(2\pi x)}{2\pi}$$

```
[Out] Cos[2*Pi*x]/(2*Pi)
```

Rubi [A] time = 0.0045919, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2638}

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Pi*(1 + 2*x)],x]

[Out] Cos[2*Pi*x]/(2*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(\pi(1+2x))\,dx=\frac{\cos(2\pi x)}{2\pi}$$

Mathematica [A] time = 0.0050726, size = 12, normalized size = 1.

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Pi*(1 + 2*x)],x]
```

```
[Out] Cos[2*Pi*x]/(2*Pi)
```

Maple [A] time = 0.005, size = 11, normalized size = 0.9

$$\frac{\cos\left(2\,\pi\,x\right)}{2\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*(1+2*x)),x)
```

```
[Out] 1/2*cos(2*Pi*x)/Pi
```

Maxima [A] time = 0.951001, size = 14, normalized size = 1.17

$$\frac{\cos\left(2\,\pi x\right)}{2\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*cos(2*pi*x)/pi
```

Fricas [A] time = 1.92326, size = 35, normalized size = 2.92

$$-\frac{\cos\left(\pi + 2\,\pi x\right)}{2\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*cos(pi + 2*pi*x)/pi
```

Sympy [A] time = 1.04472, size = 12, normalized size = 1.

$$-\frac{\cos(\pi(2x+1))}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x)
```

```
[Out] -cos(pi*(2*x + 1))/(2*pi)
```

Giac [A] time = 1.08439, size = 14, normalized size = 1.17

$$\frac{\cos(2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="giac")
```

```
[Out] 1/2*cos(2*pi*x)/pi
```

3.774 $\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{3}\cot^3(x)-\frac{\cot^2(x)}{2}-\cot(x)$$

```
[Out] -Cot[x] - Cot[x]^2/2 - Cot[x]^3/3
```

Rubi [A] time = 0.0595257, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{1}{3}\cot^3(x)-\frac{\cot^2(x)}{2}-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -Cot[x] - Cot[x]^2/2 - Cot[x]^3/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x+x^2}{x^4}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^4}+\frac{1}{x^3}+\frac{1}{x^2}\right)dx,x,\tan(x)\right)\\ &= -\cot(x)-\frac{\cot^2(x)}{2}-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0182162, size = 25, normalized size = 1.19

$$-\frac{2\cot(x)}{3}-\frac{\csc^2(x)}{2}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] (-2*Cot[x])/3 - Csc[x]^2/2 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0.053, size = 20, normalized size = 1.

$$-(\tan(x))^{-1}-\frac{1}{3(\tan(x))^3}-\frac{1}{2(\tan(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x)^2)/(1-cos(x)^2),x)
```

```
[Out] -1/tan(x)-1/3/tan(x)^3-1/2/tan(x)^2
```

Maxima [A] time = 0.956223, size = 24, normalized size = 1.14

$$-\frac{6\tan(x)^2+3\tan(x)+2}{6\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

Fricas [A] time = 2.01622, size = 88, normalized size = 4.19

$$-\frac{4\cos(x)^3-6\cos(x)-3\sin(x)}{6\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cos(x)^3 - 6*cos(x) - 3*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cot(x)}{\cos^2(x)-1}\,dx-\int \frac{\csc^2(x)}{\cos^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)**2)/(1-cos(x)**2),x)
```

```
[Out] -Integral(cot(x)/(cos(x)**2 - 1), x) - Integral(csc(x)**2/(cos(x)**2 - 1),
x)
```

Giac [A] time = 1.10748, size = 24, normalized size = 1.14

$$-\frac{6\,\tan(x)^2+3\,\tan(x)+2}{6\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

3.775 $\int x^2 \cos(4x^3) \cos(5x^3) \, dx$

Optimal. Leaf size=19

$$\frac{\sin(x^3)}{6} + \frac{1}{54} \sin(9x^3)$$

```
[Out] Sin[x^3]/6 + Sin[9*x^3]/54
```

Rubi [A] time = 0.0372446, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4572, 3380, 2637}

$$\frac{\sin(x^3)}{6} + \frac{1}{54} \sin(9x^3)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[4*x^3]*Cos[5*x^3],x]
```

```
[Out] Sin[x^3]/6 + Sin[9*x^3]/54
```

Rule 4572

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandTrigRedu
ce[x^m, Cos[v]^p*Cos[w]^q, x], x] /; IGtQ[m, 0] && IGtQ[p, 0] && IGtQ[q, 0]
 && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && I
ndependentQ[Cancel[v/w], x]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cos\left(4x^3\right)\cos\left(5x^3\right)\,dx &= \int \left(\frac{1}{2}x^2\cos\left(x^3\right)+\frac{1}{2}x^2\cos\left(9x^3\right)\right)dx\\
&= \frac{1}{2}\int x^2\cos\left(x^3\right)\,dx+\frac{1}{2}\int x^2\cos\left(9x^3\right)\,dx\\
&= \frac{1}{6}\,\mathrm{Subst}\left(\int \cos(x)\,dx,x,x^3\right)+\frac{1}{6}\,\mathrm{Subst}\left(\int \cos(9x)\,dx,x,x^3\right)\\
&= \frac{\sin\left(x^3\right)}{6}+\frac{1}{54}\sin\left(9x^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.0086818, size = 19, normalized size = 1.

$$\frac{\sin\left(x^3\right)}{6}+\frac{1}{54}\sin\left(9x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[4*x^3]*Cos[5*x^3],x]
```

```
[Out] Sin[x^3]/6 + Sin[9*x^3]/54
```

Maple [A] time = 0.045, size = 16, normalized size = 0.8

$$\frac{\sin\left(x^3\right)}{6}+\frac{\sin\left(9\,x^3\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3)*cos(5*x^3),x)
```

```
[Out] 1/6*sin(x^3)+1/54*sin(9*x^3)
```

Maxima [A] time = 0.964867, size = 20, normalized size = 1.05

$$\frac{1}{54}\sin\left(9\,x^3\right)+\frac{1}{6}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="maxima")
```

```
[Out] 1/54*sin(9*x^3) + 1/6*sin(x^3)
```

Fricas [B] time = 2.14163, size = 116, normalized size = 6.11

$$\frac{1}{27}\left(128\cos\left(x^3\right)^8-224\cos\left(x^3\right)^6+120\cos\left(x^3\right)^4-20\cos\left(x^3\right)^2+5\right)\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="fricas")
```

```
[Out] 1/27*(128*cos(x^3)^8 - 224*cos(x^3)^6 + 120*cos(x^3)^4 - 20*cos(x^3)^2 + 5)
*sin(x^3)
```

Sympy [B] time = 7.69553, size = 32, normalized size = 1.68

$$-\frac{4\sin\left(4x^3\right)\cos\left(5x^3\right)}{27}+\frac{5\sin\left(5x^3\right)\cos\left(4x^3\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(4*x**3)*cos(5*x**3),x)
```

```
[Out] -4*sin(4*x**3)*cos(5*x**3)/27 + 5*sin(5*x**3)*cos(4*x**3)/27
```

Giac [B] time = 1.08441, size = 53, normalized size = 2.79

$$\frac{128}{27}\sin\left(x^3\right)^9-\frac{32}{3}\sin\left(x^3\right)^7+8\sin\left(x^3\right)^5-\frac{20}{9}\sin\left(x^3\right)^3+\frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="giac")
```

```
[Out] 128/27*sin(x^3)^9 - 32/3*sin(x^3)^7 + 8*sin(x^3)^5 - 20/9*sin(x^3)^3 + 1/3*
sin(x^3)
```

3.776 $\int x^{14} \sin\left(x^3\right) dx$

Optimal. Leaf size=47

$$\frac{4}{3}x^9 \sin\left(x^3\right) - 8x^3 \sin\left(x^3\right) - \frac{1}{3}x^{12} \cos\left(x^3\right) + 4x^6 \cos\left(x^3\right) - 8 \cos\left(x^3\right)$$

```
[Out] -8*Cos[x^3] + 4*x^6*Cos[x^3] - (x^12*Cos[x^3])/3 - 8*x^3*Sin[x^3] + (4*x^9*
Sin[x^3])/3
```

Rubi [A] time = 0.0641899, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2638}

$$\frac{4}{3}x^9 \sin\left(x^3\right) - 8x^3 \sin\left(x^3\right) - \frac{1}{3}x^{12} \cos\left(x^3\right) + 4x^6 \cos\left(x^3\right) - 8 \cos\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sin[x^3],x]
```



```
[Out] -8*Cos[x^3] + 4*x^6*Cos[x^3] - (x^12*Cos[x^3])/3 - 8*x^3*Sin[x^3] + (4*x^9*
Sin[x^3])/3
```


Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^{14}\sin\left(x^3\right)\,dx = \frac{1}{3}\operatorname{Subst}\left(\int x^4\sin(x)\,dx,x,x^3\right)$$
$$= -\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}\operatorname{Subst}\left(\int x^3\cos(x)\,dx,x,x^3\right)$$
$$= -\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)-4\operatorname{Subst}\left(\int x^2\sin(x)\,dx,x,x^3\right)$$
$$= 4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)-8\operatorname{Subst}\left(\int x\cos(x)\,dx,x,x^3\right)$$
$$= 4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)-8x^3\sin\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)+8\operatorname{Subst}\left(\int \sin(x)\,dx,x,x^3\right)$$
$$= -8\cos\left(x^3\right)+4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)-8x^3\sin\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)$$

Mathematica [A] time = 0.0315941, size = 35, normalized size = 0.74

$$\frac{4}{3}x^3\left(x^6-6\right)\sin\left(x^3\right)-\frac{1}{3}\left(x^{12}-12x^6+24\right)\cos\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sin[x^3],x]
```

```
[Out] -((24 - 12*x^6 + x^12)*Cos[x^3])/3 + (4*x^3*(-6 + x^6)*Sin[x^3])/3
```

Maple [C] time = 0.023, size = 64, normalized size = 1.4

$$-\frac{\left(x^{12}+4\,ix^9-12\,x^6-24\,ix^3+24\right)e^{ix^3}}{6}-\frac{\left(x^{12}-4\,ix^9-12\,x^6+24\,ix^3+24\right)e^{-ix^3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*sin(x^3),x)
```

```
[Out] -1/6*(x^12+4*I*x^9-12*x^6-24*I*x^3+24)*exp(I*x^3)-1/6*(x^12-4*I*x^9-12*x^6+
24*I*x^3+24)*exp(-I*x^3)
```

Maxima [A] time = 0.963759, size = 43, normalized size = 0.91

$$-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="maxima")
```

```
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

Fricas [A] time = 2.06486, size = 88, normalized size = 1.87

$$-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="fricas")
```

```
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

Sympy [A] time = 123.675, size = 48, normalized size = 1.02

$$-\frac{x^{12}\cos\left(x^{3}\right)}{3}+\frac{4x^{9}\sin\left(x^{3}\right)}{3}+4x^{6}\cos\left(x^{3}\right)-8x^{3}\sin\left(x^{3}\right)-8\cos\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*sin(x**3),x)
```

```
[Out] -x**12*cos(x**3)/3 + 4*x**9*sin(x**3)/3 + 4*x**6*cos(x**3) - 8*x**3*sin(x**
3) - 8*cos(x**3)
```

Giac [A] time = 1.08185, size = 43, normalized size = 0.91

$$-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="giac")
```

```
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

3.777 $\int e^{-3x^3} x^2 \sin\left(2x^3\right) dx$

Optimal. Leaf size=35

$$-\frac{1}{13}e^{-3x^3}\sin\left(2x^3\right)-\frac{2}{39}e^{-3x^3}\cos\left(2x^3\right)$$

```
[Out] (-2*Cos[2*x^3])/(39*E^(3*x^3)) - Sin[2*x^3]/(13*E^(3*x^3))
```

Rubi [A] time = 0.157812, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6715, 4432}

$$-\frac{1}{13}e^{-3x^3}\sin\left(2x^3\right)-\frac{2}{39}e^{-3x^3}\cos\left(2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[2*x^3])/E^(3*x^3),x]
```

```
[Out] (-2*Cos[2*x^3])/(39*E^(3*x^3)) - Sin[2*x^3]/(13*E^(3*x^3))
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-3x^3}x^2\sin\left(2x^3\right)dx &= \frac{1}{3}\operatorname{Subst}\left(\int e^{-3x}\sin(2x)\,dx, x, x^3\right)\\ &= -\frac{2}{39}e^{-3x^3}\cos\left(2x^3\right)-\frac{1}{13}e^{-3x^3}\sin\left(2x^3\right)\end{aligned}$$

Mathematica [A] time = 0.0474003, size = 28, normalized size = 0.8

$$-\frac{1}{39}e^{-3x^3}\left(3\sin\left(2x^3\right)+2\cos\left(2x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[2*x^3])/E^(3*x^3),x]
```

```
[Out] -(2*Cos[2*x^3] + 3*Sin[2*x^3])/(39*E^(3*x^3))
```

Maple [A] time = 0.018, size = 36, normalized size = 1.

$$\frac{1}{\left(1+\left(\tan\left(x^3\right)\right)^2\right)e^{3x^3}}\left(-\frac{2}{39}+\frac{2\left(\tan\left(x^3\right)\right)^2}{39}-\frac{2\tan\left(x^3\right)}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(2*x^3)/exp(3*x^3),x)
```

```
[Out] (-2/39+2/39*tan(x^3)^2-2/13*tan(x^3))/(1+tan(x^3)^2)/exp(3*x^3)
```

Maxima [A] time = 0.961542, size = 34, normalized size = 0.97

$$-\frac{1}{39}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)e^{\left(-3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="maxima")
```

```
[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

Fricas [A] time = 2.03139, size = 78, normalized size = 2.23

$$-\frac{2}{39}\cos\left(2x^3\right)e^{\left(-3x^3\right)}-\frac{1}{13}e^{\left(-3x^3\right)}\sin\left(2x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="fricas")
```

```
[Out] -2/39*cos(2*x^3)*e^(-3*x^3) - 1/13*e^(-3*x^3)*sin(2*x^3)
```

Sympy [A] time = 2.31183, size = 32, normalized size = 0.91

$$-\frac{e^{-3x^3}\sin\left(2x^3\right)}{13}-\frac{2e^{-3x^3}\cos\left(2x^3\right)}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(2*x**3)/exp(3*x**3),x)
```

```
[Out] -exp(-3*x**3)*sin(2*x**3)/13 - 2*exp(-3*x**3)*cos(2*x**3)/39
```

Giac [A] time = 1.09278, size = 34, normalized size = 0.97

$$-\frac{1}{39}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)e^{\left(-3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="giac")
```

```
[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

3.778 $\int 2x\cos(x^2)\,dx$

Optimal. Leaf size=4

$$\sin(x^2)$$

```
[Out] Sin[x^2]
```

Rubi [A] time = 0.0065855, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 3380, 2637}

$$\sin(x^2)$$

Antiderivative was successfully verified.

```
[In] Int[2*x*Cos[x^2],x]

[Out] Sin[x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int 2x\cos\left(x^2\right)\,dx = 2\int x\cos\left(x^2\right)\,dx$$
$$= \text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)$$
$$= \sin\left(x^2\right)$$

Mathematica [A] time = 0.0014906, size = 4, normalized size = 1.

$$\sin\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x*Cos[x^2],x]
```

```
[Out] Sin[x^2]
```

Maple [A] time = 0.001, size = 5, normalized size = 1.3

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*cos(x^2),x)
```

```
[Out] sin(x^2)
```

Maxima [A] time = 0.960729, size = 5, normalized size = 1.25

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="maxima")
```

```
[Out] sin(x^2)
```

Fricas [A] time = 2.01673, size = 14, normalized size = 3.5

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="fricas")
```

```
[Out] sin(x^2)
```

Sympy [A] time = 0.163815, size = 3, normalized size = 0.75

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x**2),x)
```

```
[Out] sin(x**2)
```

Giac [A] time = 1.09887, size = 5, normalized size = 1.25

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="giac")
```

```
[Out] sin(x^2)
```

3.779 $\int 3x^2 \cos\left(7 + x^3\right) dx$

Optimal. Leaf size=6

$$\sin\left(x^3+7\right)$$

```
[Out] Sin[7 + x^3]
```

Rubi [A] time = 0.0128978, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {12, 3380, 2637}

$$\sin\left(x^3+7\right)$$

Antiderivative was successfully verified.

```
[In] Int[3*x^2*Cos[7 + x^3],x]
```

```
[Out] Sin[7 + x^3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int 3x^2 \cos\left(7+x^3\right) dx = 3\int x^2 \cos\left(7+x^3\right) dx$$
$$= \text{Subst}\left(\int \cos(7+x)\,dx, x, x^3\right)$$
$$= \sin\left(7+x^3\right)$$

Mathematica [A] time = 0.0035533, size = 6, normalized size = 1.

$$\sin\left(x^3+7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*x^2*Cos[7 + x^3],x]
```

```
[Out] Sin[7 + x^3]
```

Maple [A] time = 0.005, size = 7, normalized size = 1.2

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3+7),x)
```

```
[Out] sin(x^3+7)
```

Maxima [A] time = 0.963454, size = 8, normalized size = 1.33

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="maxima")
```

```
[Out] sin(x^3 + 7)
```

Fricas [A] time = 2.02274, size = 19, normalized size = 3.17

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="fricas")
```

```
[Out] sin(x^3 + 7)
```

Sympy [A] time = 0.302243, size = 5, normalized size = 0.83

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x**2*cos(x**3+7),x)
```

```
[Out] sin(x**3 + 7)
```

Giac [A] time = 1.09105, size = 8, normalized size = 1.33

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="giac")
```

```
[Out] sin(x^3 + 7)
```

3.780 $\int \left(\frac{1}{1+x^2}+\sin(x)\right) dx$

Optimal. Leaf size=7

$$\tan^{-1}(x)-\cos(x)$$

```
[Out] ArcTan[x] - Cos[x]
```

Rubi [A] time = 0.0040017, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {203, 2638}

$$\tan^{-1}(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-1) + Sin[x],x]
```

```
[Out] ArcTan[x] - Cos[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{1+x^2}+\sin(x)\right) dx &= \int \frac{1}{1+x^2}\, dx+\int \sin(x)\, dx\\ &= \tan^{-1}(x)-\cos(x)\end{aligned}$$

Mathematica [A] time = 0.007671, size = 7, normalized size = 1.

$$\tan^{-1}(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-1) + Sin[x],x]
```

```
[Out] ArcTan[x] - Cos[x]
```

Maple [A] time = 0.002, size = 8, normalized size = 1.1

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)+sin(x),x)
```

```
[Out] arctan(x)-cos(x)
```

Maxima [A] time = 1.46228, size = 9, normalized size = 1.29

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="maxima")
```

```
[Out] arctan(x) - cos(x)
```

Fricas [A] time = 2.16774, size = 27, normalized size = 3.86

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="fricas")
```

```
[Out] arctan(x) - cos(x)
```

Sympy [A] time = 0.086205, size = 5, normalized size = 0.71

$$-\cos(x) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)+sin(x),x)
```

```
[Out] -cos(x) + atan(x)
```

Giac [A] time = 1.11073, size = 9, normalized size = 1.29

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="giac")
```

```
[Out] arctan(x) - cos(x)
```

3.781 $\int x \sin \left(1+x^2\right) dx$

Optimal. Leaf size=10

$$-\frac{1}{2} \cos \left(x^2+1\right)$$

```
[Out] -Cos[1 + x^2]/2
```

Rubi [A] time = 0.0093647, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 2638}

$$-\frac{1}{2} \cos \left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[1 + x^2],x]
```

```
[Out] -Cos[1 + x^2]/2
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \sin \left(1+x^2\right) dx &= \frac{1}{2} \text{Subst}\left(\int \sin (1+x) \, dx, x, x^2\right) \\ &= -\frac{1}{2} \cos \left(1+x^2\right) \end{aligned}$$

Mathematica [B] time = 0.013242, size = 21, normalized size = 2.1

$$\frac{1}{2}\sin(1)\sin\left(x^2\right)-\frac{1}{2}\cos(1)\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[1 + x^2],x]
```

```
[Out] -(Cos[1]*Cos[x^2])/2 + (Sin[1]*Sin[x^2])/2
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$-\frac{\cos\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2+1),x)
```

```
[Out] -1/2*cos(x^2+1)
```

Maxima [A] time = 0.961171, size = 11, normalized size = 1.1

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x^2 + 1)
```

Fricas [A] time = 2.01179, size = 26, normalized size = 2.6

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x^2 + 1)
```

Sympy [A] time = 0.168666, size = 8, normalized size = 0.8

$$-\frac{\cos\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2+1),x)
```

```
[Out] -cos(x**2 + 1)/2
```

Giac [A] time = 1.072, size = 11, normalized size = 1.1

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*cos(x^2 + 1)
```

3.782 $\int x \cos(1+x^2)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin\left(x^2+1\right)$$

```
[Out] Sin[1 + x^2]/2
```

Rubi [A] time = 0.0087427, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3380, 2637}

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[1 + x^2],x]
```

```
[Out] Sin[1 + x^2]/2
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos\left(1+x^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \cos(1+x)\,dx, x, x^2\right)\\ &= \frac{1}{2}\sin\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0022885, size = 10, normalized size = 1.

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[1 + x^2],x]
```

```
[Out] Sin[1 + x^2]/2
```

Maple [A] time = 0.005, size = 9, normalized size = 0.9

$$\frac{\sin\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2+1),x)
```

```
[Out] 1/2*sin(x^2+1)
```

Maxima [A] time = 0.952117, size = 11, normalized size = 1.1

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sin(x^2 + 1)
```

Fricas [A] time = 2.04218, size = 24, normalized size = 2.4

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2 + 1)
```

Sympy [A] time = 0.170368, size = 7, normalized size = 0.7

$$\frac{\sin\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2+1),x)
```

```
[Out] sin(x**2 + 1)/2
```

Giac [A] time = 1.06857, size = 11, normalized size = 1.1

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sin(x^2 + 1)
```

3.783 $\int \left(1+x^2\cos\left(x^3\right)\right)\,dx$

Optimal. Leaf size=10

$$\frac{\sin\left(x^3\right)}{3}+x$$

```
[Out] x + Sin[x^3]/3
```

Rubi [A] time = 0.0098915, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3380, 2637}

$$\frac{\sin\left(x^3\right)}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 + x^2*Cos[x^3],x]
```

```
[Out] x + Sin[x^3]/3
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(1+x^2\cos\left(x^3\right)\right)\,dx &= x+\int x^2\cos\left(x^3\right)\,dx\\
&= x+\frac{1}{3}\,\text{Subst}\left(\int \cos(x)\,dx,x,x^3\right)\\
&= x+\frac{\sin\left(x^3\right)}{3}
\end{aligned}
$$

Mathematica [A] time = 0.0027596, size = 10, normalized size = 1.

$$\frac{\sin\left(x^3\right)}{3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^2*Cos[x^3],x]
```

```
[Out] x + Sin[x^3]/3
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$x+\frac{\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+x^2*cos(x^3),x)
```

```
[Out] x+1/3*sin(x^3)
```

Maxima [A] time = 0.949289, size = 11, normalized size = 1.1

$$x+\frac{1}{3}\,\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] x + 1/3*sin(x^3)
```

Fricas [A] time = 2.05855, size = 24, normalized size = 2.4

$$x + \frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="fricas")
```

```
[Out] x + 1/3*sin(x^3)
```

Sympy [A] time = 0.297757, size = 7, normalized size = 0.7

$$x + \frac{\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x**2*cos(x**3),x)
```

```
[Out] x + sin(x**3)/3
```

Giac [A] time = 1.0812, size = 11, normalized size = 1.1

$$x + \frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="giac")
```

```
[Out] x + 1/3*sin(x^3)
```

3.784 $\int x^2 \sin\left(1+x^3\right)\,dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

```
[Out] -Cos[1 + x^3]/3
```

Rubi [A] time = 0.0119009, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3379, 2638}

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[1 + x^3],x]
```

```
[Out] -Cos[1 + x^3]/3
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 \sin\left(1+x^3\right)\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \sin(1+x)\,dx, x, x^3\right)\\ &= -\frac{1}{3}\cos\left(1+x^3\right)\end{aligned}$$

Mathematica [B] time = 0.0137952, size = 21, normalized size = 2.1

$$\frac{1}{3}\sin(1)\sin\left(x^3\right)-\frac{1}{3}\cos(1)\cos\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[1 + x^3],x]
```

```
[Out] -(Cos[1]*Cos[x^3])/3 + (Sin[1]*Sin[x^3])/3
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$-\frac{\cos\left(x^3+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^3+1),x)
```

```
[Out] -1/3*cos(x^3+1)
```

Maxima [A] time = 0.967999, size = 11, normalized size = 1.1

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*cos(x^3 + 1)
```

Fricas [A] time = 2.1322, size = 26, normalized size = 2.6

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*cos(x^3 + 1)
```

Sympy [A] time = 0.299217, size = 8, normalized size = 0.8

$$-\frac{\cos\left(x^3+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x**3+1),x)
```

```
[Out] -cos(x**3 + 1)/3
```

Giac [A] time = 1.0784, size = 11, normalized size = 1.1

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*cos(x^3 + 1)
```

3.785 $\int 12x^2 \cos(x^3) \, dx$

Optimal. Leaf size=6

$$4\sin(x^3)$$

```
[Out] 4*Sin[x^3]
```

Rubi [A] time = 0.0091064, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 3380, 2637}

$$4\sin(x^3)$$

Antiderivative was successfully verified.

```
[In] Int[12*x^2*Cos[x^3],x]
```

```
[Out] 4*Sin[x^3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int 12x^2\cos\left(x^3\right)\,dx &= 12\int x^2\cos\left(x^3\right)\,dx\\ &= 4\,\text{Subst}\left(\int \cos(x)\,dx, x, x^3\right)\\ &= 4\sin\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.0016396, size = 6, normalized size = 1.

$$4\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[12*x^2*Cos[x^3],x]
```

```
[Out] 4*Sin[x^3]
```

Maple [A] time = 0.001, size = 7, normalized size = 1.2

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(12*x^2*cos(x^3),x)
```

```
[Out] 4*sin(x^3)
```

Maxima [A] time = 0.962417, size = 8, normalized size = 1.33

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] 4*sin(x^3)
```

Fricas [A] time = 2.03581, size = 16, normalized size = 2.67

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="fricas")
```

```
[Out] 4*sin(x^3)
```

Sympy [A] time = 0.296115, size = 5, normalized size = 0.83

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x**2*cos(x**3),x)
```

```
[Out] 4*sin(x**3)
```

Giac [A] time = 1.06525, size = 8, normalized size = 1.33

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="giac")
```

```
[Out] 4*sin(x^3)
```

3.786 $\int (1+x)\sin(1+x)\,dx$

Optimal. Leaf size=14

$$\sin(x+1)-(x+1)\cos(x+1)$$

```
[Out] -((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

Rubi [A] time = 0.0111327, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3296, 2637}

$$\sin(x+1)-(x+1)\cos(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)*Sin[1 + x],x]
```

```
[Out] -((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+x)\sin(1+x)\,dx &= -(1+x)\cos(1+x)+\int \cos(1+x)\,dx\\ &= -(1+x)\cos(1+x)+\sin(1+x)\end{aligned}$$

Mathematica [A] time = 0.0300698, size = 14, normalized size = 1.

$$\sin(x+1)-(x+1)\cos(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)*Sin[1 + x],x]
```

```
[Out] -((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

Maple [A] time = 0.006, size = 15, normalized size = 1.1

$$-(1+x)\cos(1+x)+\sin(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*sin(1+x),x)
```

```
[Out] -(1+x)*cos(1+x)+sin(1+x)
```

Maxima [A] time = 0.965903, size = 19, normalized size = 1.36

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

Fricas [A] time = 1.95472, size = 46, normalized size = 3.29

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="fricas")
```

```
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

Sympy [A] time = 0.170035, size = 15, normalized size = 1.07

$$-x\cos(x+1)+\sin(x+1)-\cos(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x)
```

```
[Out] -x*cos(x + 1) + sin(x + 1) - cos(x + 1)
```

Giac [A] time = 1.08431, size = 19, normalized size = 1.36

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="giac")
```

```
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

3.787 $\int x^5 \cos \left(x^3\right) dx$

Optimal. Leaf size=20

$$\frac{1}{3}x^3 \sin \left(x^3\right) + \frac{\cos \left(x^3\right)}{3}$$

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rubi [A] time = 0.0174761, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$\frac{1}{3}x^3 \sin \left(x^3\right) + \frac{\cos \left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^5\cos\left(x^3\right)\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x\cos(x)\,dx,x,x^3\right)\\ &= \frac{1}{3}x^3\sin\left(x^3\right)-\frac{1}{3}\,\text{Subst}\left(\int \sin(x)\,dx,x,x^3\right)\\ &= \frac{\cos\left(x^3\right)}{3}+\frac{1}{3}x^3\sin\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.0096345, size = 20, normalized size = 1.

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$\frac{\cos\left(x^3\right)}{3}+\frac{x^3\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x)
```

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Maxima [A] time = 0.960973, size = 22, normalized size = 1.1

$$\frac{1}{3}\,x^3\sin\left(x^3\right)+\frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Fricas [A] time = 2.02844, size = 45, normalized size = 2.25

$$\frac{1}{3}x^3 \sin\left(x^3\right) + \frac{1}{3}\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Sympy [A] time = 1.99397, size = 15, normalized size = 0.75

$$\frac{x^3 \sin\left(x^3\right)}{3} + \frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(x**3),x)
```

```
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

Giac [A] time = 1.09325, size = 22, normalized size = 1.1

$$\frac{1}{3}x^3 \sin\left(x^3\right) + \frac{1}{3}\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="giac")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

3.788 $\int e^{-3x} \cos(x)\,dx$

Optimal. Leaf size=23

$$\frac{1}{10}e^{-3x}\sin(x) - \frac{3}{10}e^{-3x}\cos(x)$$

```
[Out] (-3*Cos[x])/(10*E^(3*x)) + Sin[x]/(10*E^(3*x))
```

Rubi [A] time = 0.0089591, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4433}

$$\frac{1}{10}e^{-3x}\sin(x) - \frac{3}{10}e^{-3x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/E^(3*x),x]
```

```
[Out] (-3*Cos[x])/(10*E^(3*x)) + Sin[x]/(10*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(x)\,dx = -\frac{3}{10}e^{-3x}\cos(x) + \frac{1}{10}e^{-3x}\sin(x)$$

Mathematica [A] time = 0.0131711, size = 16, normalized size = 0.7

$$\frac{1}{10}e^{-3x}(\sin(x) - 3\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/E^(3*x),x]
```

```
[Out] (-3*Cos[x] + Sin[x])/(10*E^(3*x))
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{-3\,x}\cos\left(x\right)}{10}+\frac{\mathrm{e}^{-3\,x}\sin\left(x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/exp(3*x),x)
```

```
[Out] -3/10*exp(-3*x)*cos(x)+1/10*exp(-3*x)*sin(x)
```

Maxima [A] time = 0.960083, size = 20, normalized size = 0.87

$$-\frac{1}{10}\,(3\,\cos\left(x\right)-\sin\left(x\right))e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

Fricas [A] time = 2.17241, size = 62, normalized size = 2.7

$$-\frac{3}{10}\,\cos\left(x\right)e^{(-3\,x)}+\frac{1}{10}\,e^{(-3\,x)}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -3/10*cos(x)*e^(-3*x) + 1/10*e^(-3*x)*sin(x)
```

Sympy [A] time = 0.4711, size = 20, normalized size = 0.87

$$\frac{e^{-3x}\sin(x)}{10} - \frac{3e^{-3x}\cos(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x)
```

```
[Out] exp(-3*x)*sin(x)/10 - 3*exp(-3*x)*cos(x)/10
```

Giac [A] time = 1.0824, size = 20, normalized size = 0.87

$$-\frac{1}{10}\left(3\cos(x) - \sin(x)\right)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="giac")
```

```
[Out] -1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

3.789 $\int x^3 \sin(x^2)\, dx$

Optimal. Leaf size=20

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rubi [A] time = 0.0166995, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2637}

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^3 \sin\left(x^2\right) dx &= \frac{1}{2}\text{Subst}\left(\int x\sin(x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.0020512, size = 20, normalized size = 1.

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$-\frac{x^2\cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A] time = 0.961134, size = 22, normalized size = 1.1

$$-\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A] time = 2.00763, size = 46, normalized size = 2.3

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A] time = 0.545752, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Giac [A] time = 1.08967, size = 22, normalized size = 1.1

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.790 $\int x^3 \cos(x^2)\, dx$

Optimal. Leaf size=20

$$\frac{1}{2}x^2\sin\left(x^2\right)+\frac{\cos\left(x^2\right)}{2}$$

```
[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

Rubi [A] time = 0.0168783, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$\frac{1}{2}x^2\sin\left(x^2\right)+\frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x^2],x]
```

```
[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3 \cos\left(x^2\right) dx = \frac{1}{2}\operatorname{Subst}\left(\int x\cos(x)\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^2\sin\left(x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \sin(x)\,dx, x, x^2\right)$$
$$= \frac{\cos\left(x^2\right)}{2} + \frac{1}{2}x^2\sin\left(x^2\right)$$

Mathematica [A] time = 0.0075433, size = 20, normalized size = 1.

$$\frac{1}{2}x^2\sin\left(x^2\right) + \frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x^2],x]
```

```
[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$\frac{\cos\left(x^2\right)}{2} + \frac{x^2\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^2),x)
```

```
[Out] 1/2*cos(x^2)+1/2*x^2*sin(x^2)
```

Maxima [A] time = 0.974607, size = 22, normalized size = 1.1

$$\frac{1}{2}\,x^2\sin\left(x^2\right) + \frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

Fricas [A] time = 1.96105, size = 45, normalized size = 2.25

$$\frac{1}{2}\,x^2\sin\left(x^2\right)+\frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

Sympy [A] time = 0.549555, size = 15, normalized size = 0.75

$$\frac{x^2\sin\left(x^2\right)}{2}+\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x**2),x)
```

```
[Out] x**2*sin(x**2)/2 + cos(x**2)/2
```

Giac [A] time = 1.09492, size = 22, normalized size = 1.1

$$\frac{1}{2}\,x^2\sin\left(x^2\right)+\frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

3.791 $\int \cos(x)\cos(2\sin(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin(2\sin(x))$$

```
[Out] Sin[2*Sin[x]]/2
```

Rubi [A] time = 0.0104997, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4334, 2637}

$$\frac{1}{2}\sin(2\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*Sin[x]],x]
```

```
[Out] Sin[2*Sin[x]]/2
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2\sin(x))\,dx &= \text{Subst}\left(\int \cos(2x)\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\sin(2\sin(x))\end{aligned}$$

Mathematica [A] time = 1.37057, size = 9, normalized size = 1.

$$\frac{1}{2}\sin(2\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*Sin[x]],x]
```

```
[Out] Sin[2*Sin[x]]/2
```

Maple [A] time = 0.013, size = 8, normalized size = 0.9

$$\frac{\sin(2\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*sin(x)),x)
```

```
[Out] 1/2*sin(2*sin(x))
```

Maxima [A] time = 0.964211, size = 9, normalized size = 1.

$$\frac{1}{2}\sin(2\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*sin(x))
```

Fricas [B] time = 2.03069, size = 57, normalized size = 6.33

$$\frac{1}{2}\sin\left(\frac{4\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*sin(4*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

Sympy [A] time = 0.515596, size = 7, normalized size = 0.78

$$\frac{\sin(2\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x)
```

```
[Out] sin(2*sin(x))/2
```

Giac [A] time = 1.09994, size = 9, normalized size = 1.

$$\frac{1}{2}\sin(2\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*sin(2*sin(x))
```

3.792 $\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\cos^2(x)+1\right)$$

```
[Out] -Log[1 + Cos[x]^2]/2
```

Rubi [A] time = 0.0317343, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4335, 260}

$$-\frac{1}{2}\log\left(\cos^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
```

```
[Out] -Log[1 + Cos[x]^2]/2
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\log\left(1+\cos^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.028184, size = 11, normalized size = 1.

$$-\frac{1}{2}\log(\cos(2x)+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
```

```
[Out] -Log[3 + Cos[2*x]]/2
```

Maple [A] time = 0.01, size = 10, normalized size = 0.9

$$-\frac{\ln\left(1+(\cos(x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1+cos(x)^2),x)
```

```
[Out] -1/2*ln(1+cos(x)^2)
```

Maxima [A] time = 0.947922, size = 12, normalized size = 1.09

$$-\frac{1}{2}\log\left(\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x)^2 + 1)
```

Fricas [A] time = 2.05358, size = 41, normalized size = 3.73

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)^2+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x)^2 + 1/2)
```

Sympy [A] time = 0.194233, size = 10, normalized size = 0.91

$$-\frac{\log\left(\cos^2(x)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)**2),x)
```

```
[Out] -log(cos(x)**2 + 1)/2
```

Giac [A] time = 1.07364, size = 12, normalized size = 1.09

$$-\frac{1}{2}\log\left(\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x)^2 + 1)
```

3.793 $\int (1+\cos(x))(x+\sin(x))^3 \, dx$

Optimal. Leaf size=10

$$\frac{1}{4}(x+\sin(x))^4$$

```
[Out] (x + Sin[x])^4/4
```

Rubi [A] time = 0.0378183, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6686}

$$\frac{1}{4}(x+\sin(x))^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*(x + Sin[x])^3,x]
```

```
[Out] (x + Sin[x])^4/4
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int (1+\cos(x))(x+\sin(x))^3 \, dx=\frac{1}{4}(x+\sin(x))^4$$

Mathematica [A] time = 0.0184468, size = 10, normalized size = 1.

$$\frac{1}{4}(x+\sin(x))^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*(x + Sin[x])^3,x]
```

```
[Out] (x + Sin[x])^4/4
```

Maple [B] time = 0.036, size = 65, normalized size = 6.5

$$\sin(x)x^3 - \frac{3(\cos(x))^2 x^2}{2} + 3x(1/2\cos(x)\sin(x) + x/2) - \frac{3x^2}{2} + x(\sin(x))^3 + \frac{(\sin(x))^4}{4} + \frac{x^4}{4} + 3x(-1/2\cos(x)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*(x+sin(x))^3,x)
```

```
[Out] sin(x)*x^3-3/2*cos(x)^2*x^2+3*x*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2+x*sin(x)^
3+1/4*sin(x)^4+1/4*x^4+3*x*(-1/2*cos(x)*sin(x)+1/2*x)
```

Maxima [A] time = 0.954098, size = 11, normalized size = 1.1

$$\frac{1}{4}(x + \sin(x))^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(x + sin(x))^4
```

Fricas [B] time = 2.03644, size = 126, normalized size = 12.6

$$\frac{1}{4}x^4 + \frac{1}{4}\cos(x)^4 - \frac{1}{2}\left(3x^2 + 1\right)\cos(x)^2 + \frac{3}{2}x^2 + \left(x^3 - x\cos(x)^2 + x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/4*cos(x)^4 - 1/2*(3*x^2 + 1)*cos(x)^2 + 3/2*x^2 + (x^3 - x*cos(
x)^2 + x)*sin(x)
```

Sympy [B] time = 0.61257, size = 36, normalized size = 3.6

$$\frac{x^4}{4} + x^3 \sin(x) + \frac{3x^2 \sin^2(x)}{2} + x \sin^3(x) + \frac{\sin^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))**3,x)
```

```
[Out] x**4/4 + x**3*sin(x) + 3*x**2*sin(x)**2/2 + x*sin(x)**3 + sin(x)**4/4
```

Giac [B] time = 1.08073, size = 82, normalized size = 8.2

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{1}{4}\left(3x^2 - 1\right)\cos(2x) - \frac{1}{4}x\sin(3x) + \frac{1}{4}\left(4x^3 - 21x\right)\sin(x) + 6x\sin(x) + \frac{1}{32}\cos(4x) - \frac{3}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 - 1/4*(3*x^2 - 1)*cos(2*x) - 1/4*x*sin(3*x) + 1/4*(4*x^3
- 21*x)*sin(x) + 6*x*sin(x) + 1/32*cos(4*x) - 3/8*cos(2*x)
```

3.794 $\int (1+\cos(x))\csc^2(x)\,dx$

Optimal. Leaf size=9

$$-\cot(x)-\csc(x)$$

```
[Out] -Cot[x] - Csc[x]
```

Rubi [A] time = 0.0286004, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2669, 3767, 8}

$$-\cot(x)-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*Csc[x]^2,x]
```

```
[Out] -Cot[x] - Csc[x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (1+\cos(x))\csc^2(x)\,dx = -\csc(x) + \int \csc^2(x)\,dx$$
$$= -\csc(x) - \text{Subst}(\int 1\,dx, x, \cot(x))$$
$$= -\cot(x) - \csc(x)$$

Mathematica [A] time = 0.003734, size = 9, normalized size = 1.

$$-\cot(x) - \csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x]^2,x]
```

```
[Out] -Cot[x] - Csc[x]
```

Maple [A] time = 0.017, size = 12, normalized size = 1.3

$$-(\sin(x))^{-1} - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*csc(x)^2,x)
```

```
[Out] -1/sin(x)-cot(x)
```

Maxima [A] time = 0.968491, size = 18, normalized size = 2.

$$-\frac{1}{\sin(x)} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="maxima")
```

```
[Out] -1/sin(x) - 1/tan(x)
```

Fricas [A] time = 1.90474, size = 30, normalized size = 3.33

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 3.76437, size = 8, normalized size = 0.89

$$-\cot(x) - \frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)**2,x)
```

```
[Out] -cot(x) - 1/sin(x)
```

Giac [A] time = 1.09225, size = 11, normalized size = 1.22

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.795 $\int \sin(x)\tan^2(x)\,dx$

Optimal. Leaf size=5

$$\cos(x)+\sec(x)$$

```
[Out] Cos[x] + Sec[x]
```

Rubi [A] time = 0.0162149, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\cos(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[x]^2,x]
```

```
[Out] Cos[x] + Sec[x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan^2(x)\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos(x)\right)\\ &= \cos(x)+\sec(x)\end{aligned}$$

Mathematica [A] time = 0.0106083, size = 5, normalized size = 1.

$$\cos(x) + \sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[x]^2,x]
```

```
[Out] Cos[x] + Sec[x]
```

Maple [B] time = 0.008, size = 20, normalized size = 4.

$$\frac{(\sin(x))^4}{\cos(x)} + \left(2 + (\sin(x))^2\right)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^2,x)
```

```
[Out] sin(x)^4/cos(x)+(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 0.95688, size = 9, normalized size = 1.8

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/cos(x) + cos(x)
```

Fricas [B] time = 2.18105, size = 31, normalized size = 6.2

$$\frac{\cos(x)^2 + 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] (cos(x)^2 + 1)/cos(x)
```

Sympy [A] time = 0.076895, size = 7, normalized size = 1.4

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)**2,x)
```

```
[Out] cos(x) + 1/cos(x)
```

Giac [A] time = 1.09293, size = 9, normalized size = 1.8

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/cos(x) + cos(x)
```

3.796 $\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) dx$

Optimal. Leaf size=13

$$e^{\sin(x)}(x\cos(x) - 1)\sec(x)$$

```
[Out] E^Sin[x]*(-1 + x*Cos[x])*Sec[x]
```

Rubi [F] time = 0.640077, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
```

```
[Out] Defer[Int][E^Sin[x]*x*Cos[x], x] - Defer[Int][E^Sin[x]*Sec[x]*Tan[x], x]
```

Rubi steps

$$\begin{aligned}\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) dx &= \int \left(e^{\sin(x)} x \cos(x) - e^{\sin(x)} \sec(x) \tan(x)\right) dx\\ &= \int e^{\sin(x)} x \cos(x)\, dx - \int e^{\sin(x)} \sec(x) \tan(x)\, dx\end{aligned}$$

Mathematica [A] time = 0.276428, size = 13, normalized size = 1.

$$e^{\sin(x)}(x\cos(x) - 1)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
```

```
[Out] E^Sin[x]*(-1 + x*Cos[x])*Sec[x]
```

Maple [C] time = 0.13, size = 30, normalized size = 2.3

$$\frac{\left(x\mathrm{e}^{2 i x}+x-2\,\mathrm{e}^{i x}\right)\mathrm{e}^{\sin(x)}}{1+\mathrm{e}^{2 i x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x)
```

```
[Out] (x*exp(2*I*x)+x-2*exp(I*x))/(1+exp(2*I*x))*exp(sin(x))
```

Maxima [B] time = 2.6413, size = 119, normalized size = 9.15

$$\frac{x\cos\left(2\,x\right)^{2}e^{\sin(x)}+xe^{\sin(x)}\sin\left(2\,x\right)^{2}-2\,e^{\sin(x)}\sin\left(2\,x\right)\sin\left(x\right)+2\left(xe^{\sin(x)}-\cos\left(x\right)e^{\sin(x)}\right)\cos\left(2\,x\right)+xe^{\sin(x)}-2\,\cos\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="maxima")
```

```
[Out] (x*cos(2*x)^2*e^sin(x) + x*e^sin(x)*sin(2*x)^2 - 2*e^sin(x)*sin(2*x)*sin(x)
 + 2*(x*e^sin(x) - cos(x)*e^sin(x))*cos(2*x) + x*e^sin(x) - 2*cos(x)*e^sin(
x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 2.01477, size = 43, normalized size = 3.31

$$\frac{(x\cos\left(x\right)-1)e^{\sin(x)}}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="fricas")
```

```
[Out] (x*cos(x) - 1)*e^sin(x)/cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)**2*(x*cos(x)**3-sin(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16169, size = 1072, normalized size = 82.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="giac")
```

```
[Out] (x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 + e^(2*tan
(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 - 16*x*e^(2*tan(1/2*x
)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^6 + 12*x*e^(2*tan(1/2*x)/(tan
(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7 - x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^8 - 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan
(1/2*x)^6 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7
- e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 30*x*e^(2*tan(1/2*x)/(
tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^4 - 52*x*e^(2*tan(1/2*x)/(tan(1/
2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^5 + 16*x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^6 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1
/2*x)^5 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 16*x*e^(2*t
an(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^2 + 52*x*e^(2*tan(1/2
*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^3 - 30*x*e^(2*tan(1/2*x)/(tan
(1/2*x)^2 + 1))*tan(1/2*x)^4 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3
/2*x)^2*tan(1/2*x)^2 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*ta
n(1/2*x)^3 + x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2 - 12*x*e^(2
*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x) + 16*x*e^(2*tan(1/2*x
)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*ta
n(3/2*x)^2 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x) -
 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - x*e^(2*tan(1/2*x)/(t
an(1/2*x)^2 + 1)) + e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)))/(tan(3/2*x)^2*tan(
1/2*x)^8 + 2*tan(3/2*x)^2*tan(1/2*x)^6 + tan(1/2*x)^8 + 2*tan(1/2*x)^6 - 2*
tan(3/2*x)^2*tan(1/2*x)^2 - tan(3/2*x)^2 - 2*tan(1/2*x)^2 - 1)
```

3.797 $\int x \csc^2(x)\, dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - x\cot(x)$$

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rubi [A] time = 0.0164081, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x\csc^2(x)\, dx &= -x\cot(x) + \int \cot(x)\, dx \\ &= -x\cot(x) + \log(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.016742, size = 9, normalized size = 1.

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Maple [A] time = 0.006, size = 10, normalized size = 1.1

$$-x\cot(x)+\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)^2,x)
```

```
[Out] -x*cot(x)+ln(sin(x))
```

Maxima [B] time = 0.969676, size = 140, normalized size = 15.56

$$\frac{\big(\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1\big)\log\big(\cos(x)^2+\sin(x)^2+2\cos(x)+1\big)+\big(\cos(2x)^2+\sin(2x)^2-2\cos(2x)}{2\big(\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1\big)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x
) + 1)
```

Fricas [B] time = 2.05214, size = 61, normalized size = 6.78

$$-\frac{x\cos(x)-\log\left(\frac{1}{2}\sin(x)\right)\sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\csc^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2,x)
```

```
[Out] Integral(x*csc(x)**2, x)
```

Giac [B] time = 1.10768, size = 70, normalized size = 7.78

$$\frac{x\tan\left(\frac{1}{2}x\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}x\right)^2}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)-x}{2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 +
1))*tan(1/2*x) - x)/tan(1/2*x)
```

3.798 $\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx$

Optimal. Leaf size=20

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rubi [A] time = 0.0167845, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4574, 2638}

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx &= \int\left(\frac{1}{4}+\frac{1}{2}\sin\left(\frac{\pi}{6}+2x\right)\right)dx\\ &= \frac{x}{4}+\frac{1}{2}\int\sin\left(\frac{\pi}{6}+2x\right)dx\\ &= \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.0119861, size = 20, normalized size = 1.

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Maple [A] time = 0.031, size = 15, normalized size = 0.8

$$\frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(1/6*Pi+x),x)
```

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Maxima [A] time = 0.956136, size = 19, normalized size = 0.95

$$\frac{1}{4}x-\frac{1}{4}\cos\left(\frac{1}{6}\pi+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

Fricas [B] time = 2.0857, size = 105, normalized size = 5.25

$$-\frac{1}{4}\sqrt{3}\cos\left(\frac{1}{6}\pi+x\right)^{2}-\frac{1}{4}\cos\left(\frac{1}{6}\pi+x\right)\sin\left(\frac{1}{6}\pi+x\right)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*cos(1/6*pi + x)^2 - 1/4*cos(1/6*pi + x)*sin(1/6*pi + x) + 1/4*
x
```

Sympy [B] time = 0.534328, size = 37, normalized size = 1.85

$$-\frac{x\sin\left(x\right)\cos\left(x+\frac{\pi}{6}\right)}{2}+\frac{x\sin\left(x+\frac{\pi}{6}\right)\cos\left(x\right)}{2}+\frac{\sin\left(x\right)\sin\left(x+\frac{\pi}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x)
```

```
[Out] -x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 + sin(x)*sin(x + pi/6)
/2
```

Giac [A] time = 1.09626, size = 19, normalized size = 0.95

$$\frac{1}{4}\,x-\frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

3.799 $\int x \sin^3(x^2) \, dx$

Optimal. Leaf size=19

$$\frac{1}{6}\cos^3(x^2) - \frac{\cos(x^2)}{2}$$

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rubi [A] time = 0.0149719, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 2633}

$$\frac{1}{6}\cos^3(x^2) - \frac{\cos(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2]^3,x]
```

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x\sin^3\left(x^2\right)dx = \frac{1}{2}\,\text{Subst}\left(\int \sin^3(x)\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(1-x^2\right)dx, x, \cos\left(x^2\right)\right)\right)$$
$$= -\frac{1}{2}\cos\left(x^2\right) + \frac{1}{6}\cos^3\left(x^2\right)$$

Mathematica [A] time = 0.0124647, size = 19, normalized size = 1.

$$\frac{1}{24}\cos\left(3x^2\right) - \frac{3\cos\left(x^2\right)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2]^3,x]
```

```
[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

Maple [A] time = 0.005, size = 15, normalized size = 0.8

$$-\frac{\left(2+\left(\sin\left(x^2\right)\right)^2\right)\cos\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)
```

Maxima [A] time = 0.960243, size = 20, normalized size = 1.05

$$\frac{1}{24}\cos\left(3\,x^2\right) - \frac{3}{8}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)
```

Fricas [A] time = 2.05156, size = 42, normalized size = 2.21

$$\frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

Sympy [A] time = 0.555839, size = 22, normalized size = 1.16

$$-\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2} - \frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2)**3,x)
```

```
[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

Giac [A] time = 1.07936, size = 20, normalized size = 1.05

$$\frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

3.800 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rubi [A] time = 0.0146159, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sin^2(x)\tan(x)\,dx = -\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, \cos(x)\right)$$
$$= \frac{\cos^2(x)}{2} - \log(\cos(x))$$

Mathematica [A] time = 0.0056344, size = 14, normalized size = 1.

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Maple [A] time = 0.011, size = 13, normalized size = 0.9

$$-\frac{(\sin(x))^2}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x)
```

```
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Maxima [A] time = 0.953101, size = 22, normalized size = 1.57

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 2.07138, size = 39, normalized size = 2.79

$$\frac{1}{2}\cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A] time = 0.081264, size = 10, normalized size = 0.71

$$-\log(\cos(x)) + \frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*tan(x),x)
```

```
[Out] -log(cos(x)) + cos(x)**2/2
```

Giac [A] time = 1.0966, size = 24, normalized size = 1.71

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(-\sin(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)
```

3.801 $\int \cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rubi [A] time = 0.0329977, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2590, 266, 43}

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^2(x)\cot^3(x)\,dx = \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx, x, -\sin(x)\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \sin^2(x)\right)$$
$$= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}$$

Mathematica [A] time = 0.0247997, size = 20, normalized size = 0.91

$$\frac{1}{2}\left(\sin^2(x) - \csc^2(x) - 4\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

Maple [A] time = 0.015, size = 29, normalized size = 1.3

$$-\frac{(\cos(x))^6}{2\,(\sin(x))^2} - \frac{(\cos(x))^4}{2} - (\cos(x))^2 - 2\,\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x)
```

```
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Maxima [A] time = 0.961277, size = 27, normalized size = 1.23

$$\frac{1}{2}\sin(x)^2 - \frac{1}{2\sin(x)^2} - \log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Fricas [B] time = 2.17134, size = 116, normalized size = 5.27

$$-\frac{2\cos(x)^4 - 3\cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

Sympy [A] time = 0.095966, size = 20, normalized size = 0.91

$$-2\log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*cot(x)**3,x)
```

```
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

Giac [B] time = 1.07869, size = 49, normalized size = 2.23

$$-\frac{1}{2}\cos(x)^2 + \frac{2\cos(x)^2 - 1}{2\left(\cos(x)^2 - 1\right)} - \log\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")

[Out] -1/2*cos(x)^2 + 1/2*(2*cos(x)^2 - 1)/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

3.802 $\int \sec(x)(1-\sin(x))\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

[Out] Log[1 + Sin[x]]

Rubi [A] time = 0.0147563, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*(1 - Sin[x]),x]

[Out] Log[1 + Sin[x]]

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)(1-\sin(x))\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, -\sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0072989, size = 36, normalized size = 7.2

$$\log(\cos(x)) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.019, size = 6, normalized size = 1.2

$$\ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(1-sin(x)),x)
```

```
[Out] ln(1+sin(x))
```

Maxima [A] time = 0.955349, size = 7, normalized size = 1.4

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x) + 1)
```

Fricas [A] time = 1.9975, size = 23, normalized size = 4.6

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [B] time = 2.3926, size = 12, normalized size = 2.4

$$\log(\tan(x) + \sec(x)) + \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x)
```

```
[Out] log(tan(x) + sec(x)) + log(cos(x))
```

Giac [A] time = 1.09274, size = 7, normalized size = 1.4

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="giac")
```

```
[Out] log(sin(x) + 1)
```

3.803 $\int (1+\cos(x))\csc(x)\,dx$

Optimal. Leaf size=7

$$\log(1-\cos(x))$$

```
[Out] Log[1 - Cos[x]]
```

Rubi [A] time = 0.0157383, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2667, 31}

$$\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*Csc[x],x]
```

```
[Out] Log[1 - Cos[x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\cos(x))\csc(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos(x)\right)\\ &= \log(1-\cos(x))\end{aligned}$$

Mathematica [B] time = 0.0060792, size = 20, normalized size = 2.86

$$\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

Maple [A] time = 0.013, size = 6, normalized size = 0.9

$$\ln\left(-1+\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*csc(x),x)
```

```
[Out] ln(-1+cos(x))
```

Maxima [A] time = 0.95, size = 7, normalized size = 1.

$$\log\left(\cos\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="maxima")
```

```
[Out] log(cos(x) - 1)
```

Fricas [A] time = 1.99447, size = 32, normalized size = 4.57

$$\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 1.98499, size = 12, normalized size = 1.71

$$-\log(\cot(x)+\csc(x))+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x)
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

Giac [A] time = 1.07245, size = 9, normalized size = 1.29

$$\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="giac")
```

```
[Out] log(-cos(x) + 1)
```

3.804 $\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx$

Optimal. Leaf size=5

$$\sin(x)\cos(x)$$

```
[Out] Cos[x]*Sin[x]
```

Rubi [A] time = 0.0218715, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 383}

$$\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*(1 - Tan[x]^2),x]

[Out] Cos[x]*Sin[x]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx = \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= \cos(x)\sin(x)$$

Mathematica [A] time = 0.001838, size = 8, normalized size = 1.6

$$\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Sin[2*x]/2
```

Maple [A] time = 0.016, size = 6, normalized size = 1.2

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*(1-tan(x)^2),x)
```

```
[Out] cos(x)*sin(x)
```

Maxima [B] time = 0.974787, size = 15, normalized size = 3.

$$\frac{\tan(x)}{\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="maxima")
```

```
[Out] tan(x)/(tan(x)^2 + 1)
```

Fricas [A] time = 1.96089, size = 20, normalized size = 4.

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="fricas")
```

```
[Out] cos(x)*sin(x)
```

Sympy [B] time = 5.9755, size = 14, normalized size = 2.8

$$\frac{\sin(x)\cos(x)}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*(1-tan(x)**2),x)
```

```
[Out] sin(x)*cos(x)/2 + sin(2*x)/4
```

Giac [A] time = 1.08616, size = 12, normalized size = 2.4

$$\frac{1}{\frac{1}{\tan(x)} + \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/(1/tan(x) + tan(x))
```

3.805 $\int \csc(2x)(\cos(x) + \sin(x))\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rubi [A] time = 0.0453018, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4401, 4287, 3770, 4288}

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)(\cos(x)+\sin(x))\,dx &= \int (\cos(x)\csc(2x)+\csc(2x)\sin(x))\,dx\\ &= \int \cos(x)\csc(2x)\,dx+\int \csc(2x)\sin(x)\,dx\\ &= \frac{1}{2}\int \csc(x)\,dx+\frac{1}{2}\int \sec(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0091776, size = 61, normalized size = 4.07

$$\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x/2]]/2 - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Sin[x/2]]/2 + Log[Cos[x
/2] + Sin[x/2]]/2
```

Maple [A] time = 0.077, size = 20, normalized size = 1.3

$$\frac{\ln(\sec(x)+\tan(x))}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*(cos(x)+sin(x)),x)
```

```
[Out] 1/2*ln(sec(x)+tan(x))+1/2*ln(csc(x)-cot(x))
```

Maxima [B] time = 1.47665, size = 93, normalized size = 6.2

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos
(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 2.17789, size = 149, normalized size = 9.93

$$-\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)+1)\sin(x)+\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)-1)\sin(x)-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-1/2*(cos(x
) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

Sympy [B] time = 1.86656, size = 32, normalized size = 2.13

$$-\frac{\log(\sin(x)-1)}{4}+\frac{\log(\sin(x)+1)}{4}+\frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(cos(x) + 1
)/4
```

Giac [B] time = 1.13731, size = 39, normalized size = 2.6

$$\frac{1}{2}\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)-\frac{1}{2}\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)+\frac{1}{2}\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*x) + 1)) - 1/2*log(abs(tan(1/2*x) - 1)) + 1/2*log(abs(t
an(1/2*x)))
```

3.806 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rubi [A] time = 0.0463967, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4334, 628}

$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{-3+2x}{2-3x+x^2}\,dx, x, \sin(x)\right)\\ &= \log\left(2-3\sin(x)+\sin^2(x)\right)\end{aligned}$$

Mathematica [B] time = 0.0935853, size = 26, normalized size = 2.36

$$\log(2-\sin(x))+2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] 2*Log[Cos[x/2] - Sin[x/2]] + Log[2 - Sin[x]]
```

Maple [A] time = 0.027, size = 12, normalized size = 1.1

$$\ln\left(2-3\sin(x)+(\sin(x))^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x)
```

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Maxima [A] time = 0.950533, size = 15, normalized size = 1.36

$$\log\left(\sin(x)^{2}-3\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")
```

```
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

Fricas [A] time = 2.22075, size = 55, normalized size = 5.

$$\log\left(-\frac{1}{2}\sin(x)+1\right)+\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

Sympy [A] time = 0.216341, size = 12, normalized size = 1.09

$$\log(\sin(x) - 2) + \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
```

```
[Out] log(sin(x) - 2) + log(sin(x) - 1)
```

Giac [A] time = 1.08891, size = 20, normalized size = 1.82

$$\log(-\sin(x) + 2) + \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")
```

```
[Out] log(-sin(x) + 2) + log(-sin(x) + 1)
```

3.807 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rubi [A] time = 0.0527477, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4335, 321, 203}

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^2}{5+x^2}\,dx, x, \cos(x)\right)\\ &= -\cos(x) + 5\,\text{Subst}\left(\int \frac{1}{5+x^2}\,dx, x, \cos(x)\right)\\ &= \sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right) - \cos(x)\end{aligned}$$

Mathematica [B] time = 0.17334, size = 82, normalized size = 4.1

$$\frac{1}{20}\left(-20\cos(x) + 21\sqrt{5}\tan^{-1}\left(\frac{1}{\sqrt{5}} - \sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right) + 21\sqrt{5}\tan^{-1}\left(\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right) + \frac{1}{\sqrt{5}}\right) - \sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] (-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5] - Sqrt[6/5
]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*Tan[x/2]] - 20*Cos[x]
)/20
```

Maple [A] time = 0.015, size = 18, normalized size = 0.9

$$-\cos(x) + \arctan\left(\frac{\cos(x)\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(5+cos(x)^2),x)
```

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.46218, size = 23, normalized size = 1.15

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos\left(x\right)\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Fricas [A] time = 2.28918, size = 61, normalized size = 3.05

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos\left(x\right)\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Sympy [A] time = 0.5756, size = 19, normalized size = 0.95

$$-\cos\left(x\right)+\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\cos\left(x\right)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
```

```
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

Giac [A] time = 1.07772, size = 23, normalized size = 1.15

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos\left(x\right)\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

3.808 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\, dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x)+1)$$

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rubi [A] time = 0.0214209, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3258, 615}

$$\log(\sin(x)) - \log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 3258

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\, dx &= \text{Subst}\left(\int \frac{1}{x+x^2}\, dx, x, \sin(x)\right)\\
&= \log(\sin(x)) - \log(1+\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.0080147, size = 11, normalized size = 1.

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Maple [A] time = 0.029, size = 12, normalized size = 1.1

$$\ln(\sin(x)) - \ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^2),x)
```

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Maxima [A] time = 0.997682, size = 15, normalized size = 1.36

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Fricas [A] time = 2.21227, size = 47, normalized size = 4.27

$$\log\left(\frac{1}{2}\sin(x)\right) - \log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

Sympy [A] time = 0.194317, size = 10, normalized size = 0.91

$$-\log(\sin(x)+1)+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**2),x)
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Giac [A] time = 1.07757, size = 16, normalized size = 1.45

$$-\log(\sin(x)+1)+\log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")
```

```
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

3.809 $\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)} \, dx$

Optimal. Leaf size=26

$$\log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(\sin^{\sqrt{2}-1}(x)+1\right)$$

```
[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

Rubi [A] time = 0.0490014, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4334, 266, 36, 29, 31}

$$\log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(\sin^{\sqrt{2}-1}(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
```

```
[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)}\,dx,x,\sin(x)\right)\\ &= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,\sin^{-1+\sqrt{2}}(x)\right)\\ &= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sin^{-1+\sqrt{2}}(x)\right)+\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,\sin^{-1+\sqrt{2}}(x)\right)\\ &= \log(\sin(x))-\left(1+\sqrt{2}\right)\log\left(1+\sin^{-1+\sqrt{2}}(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0416734, size = 26, normalized size = 1.

$$\log(\sin(x))-\left(1+\sqrt{2}\right)\log\left(\sin^{\sqrt{2}-1}(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
```

```
[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

Maple [C] time = 0.566, size = 1856, normalized size = 71.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x)
```

[Out] -I*Pi-1/2*I*2^(1/2)*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))*csgn(I*(exp(I*x)-1
))*csgn(I*(exp(I*x)+1))+1/2*I*2^(1/2)*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))*
csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn(I*exp(-I*x))-I*Pi*csgn(I*(exp(I*x)
-1))*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))*csgn(I*(exp(I*x)+1))-1/2*I*2^(1/2)*P
i-2*ln(2)-2*ln(exp(I*x))+I*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^3+I*Pi*cs
gn((exp(I*x)+1)*(-1+exp(-I*x)))^3+I*Pi*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^2+
2*ln(exp(I*x)+1)+2*ln(exp(I*x)-1)-1/2*I*2^(1/2)*Pi*csgn(I*(exp(I*x)-1)*(exp
(I*x)+1))^3+1/2*I*2^(1/2)*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^3+1/2*I*2^
(1/2)*Pi*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^3+1/2*I*2^(1/2)*Pi*csgn((exp(I*x
)+1)*(-1+exp(-I*x)))^2+I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))^2*csgn(I*(exp
(I*x)-1))+I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))^2*csgn(I*(exp(I*x)+1))+I*P
i*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^2+I
*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))-ln(exp(-1/2*2^(
1/2)*(-I*Pi*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^3+I*Pi*csgn(I*(exp(I*x)+1)*(-
1+exp(-I*x)))*csgn((exp(I*x)+1)*(-1+exp(-I*x)))-I*Pi*csgn(I*(exp(I*x)+1)*(-
1+exp(-I*x)))^3+I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))^3+I*Pi*csgn(I*(exp(I
x)-1)(exp(I*x)+1))*csgn(I*(exp(I*x)-1))*csgn(I*(exp(I*x)+1))+I*Pi-I*Pi*cs
gn(I*(exp(I*x)-1)*(exp(I*x)+1))^2*csgn(I*(exp(I*x)+1))-I*Pi*csgn(I*(exp(I*x
)-1)*(exp(I*x)+1))^2*csgn(I*(exp(I*x)-1))+I*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(
-I*x)))*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^2-I*Pi*csgn(I*(exp(I*x)-1)*(exp(I
*x)+1))*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn(I*exp(-I*x))-I*Pi*csgn(I*(
exp(I*x)+1)*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))-I*Pi*csgn((exp(I*x)+1)*(-1+
exp(-I*x)))^2-I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))*csgn(I*(exp(I*x)+1)*(-
1+exp(-I*x)))^2+2*ln(2)+2*ln(exp(I*x))-2*ln(exp(I*x)-1)-2*ln(exp(I*x)+1)))+
sin(x))-I*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn((exp(I*x)+1)*(-1+exp(
-I*x)))^2-I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))^3-I*Pi*csgn(I*(exp(I*x)+1)
*(-1+exp(-I*x)))*csgn((exp(I*x)+1)*(-1+exp(-I*x)))+1/2*I*2^(1/2)*Pi*csgn(I*
(exp(I*x)-1)*(exp(I*x)+1))^2*csgn(I*(exp(I*x)-1))+1/2*I*2^(1/2)*Pi*csgn(I*(
exp(I*x)-1)*(exp(I*x)+1))^2*csgn(I*(exp(I*x)+1))+1/2*I*2^(1/2)*Pi*csgn(I*(e
xp(I*x)-1)*(exp(I*x)+1))*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^2+1/2*I*2^(1/2
)*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))-1/2*I*2^(1/2)*
Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^2-
1/2*I*2^(1/2)*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn((exp(I*x)+1)*(-1+
exp(-I*x)))-ln(exp(-1/2*2^(1/2)*(-I*Pi*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^3+
I*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn((exp(I*x)+1)*(-1+exp(-I*x)))-
I*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^3+I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*
x)+1))^3+I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))*csgn(I*(exp(I*x)-1))*csgn(I
*(exp(I*x)+1))+I*Pi-I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))^2*csgn(I*(exp(I*
x)+1))-I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)+1))^2*csgn(I*(exp(I*x)-1))+I*Pi*c
sgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^2-I*Pi
csgn(I(exp(I*x)-1)*(exp(I*x)+1))*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn
(I*exp(-I*x))-I*Pi*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))-
I*Pi*csgn((exp(I*x)+1)*(-1+exp(-I*x)))^2-I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*x)
+1))*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))^2+2*ln(2)+2*ln(exp(I*x))-2*ln(exp(
I*x)-1)-2*ln(exp(I*x)+1)))+sin(x))*2^(1/2)+I*Pi*csgn(I*(exp(I*x)-1)*(exp(I*

```
x)+1))*csgn(I*(exp(I*x)+1)*(-1+exp(-I*x)))*csgn(I*exp(-I*x))-2^(1/2)*ln(2)-
2^(1/2)*ln(exp(I*x))+2^(1/2)*ln(exp(I*x)-1)+2^(1/2)*ln(exp(I*x)+1)
```

Maxima [A] time = 1.44062, size = 46, normalized size = 1.77

$$\frac{\sqrt{2}\log\left(\sin\left(x\right)\right)}{\sqrt{2}-1}-\frac{\log\left(\sin\left(x\right)^{\left(\sqrt{2}\right)}+\sin\left(x\right)\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(sin(x))/(sqrt(2) - 1) - log(sin(x)^sqrt(2) + sin(x))/(sqrt(2) -
 1)
```

Fricas [A] time = 2.42289, size = 99, normalized size = 3.81

$$-\left(\sqrt{2}+1\right)\log\left(\sin\left(x\right)^{\left(\sqrt{2}\right)}+\sin\left(x\right)\right)+\left(\sqrt{2}+2\right)\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="fricas")
```

```
[Out] -(sqrt(2) + 1)*log(sin(x)^sqrt(2) + sin(x)) + (sqrt(2) + 2)*log(sin(x))
```

Sympy [B] time = 1.26123, size = 82, normalized size = 3.15

$$\frac{\sqrt{2}\log\left(\sin\left(x\right)+\sin^{\sqrt{2}}\left(x\right)\right)}{-3+2\sqrt{2}}-\frac{\log\left(\sin\left(x\right)+\sin^{\sqrt{2}}\left(x\right)\right)}{-3+2\sqrt{2}}+\frac{\sqrt{2}\log\left(\sin\left(x\right)\right)}{-3+2\sqrt{2}}-\frac{2\log\left(\sin\left(x\right)\right)}{-3+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**(2**(1/2))),x)
```

```
[Out] sqrt(2)*log(sin(x) + sin(x)**(sqrt(2)))/(-3 + 2*sqrt(2)) - log(sin(x) + sin
(x)**(sqrt(2)))/(-3 + 2*sqrt(2)) + sqrt(2)*log(sin(x))/(-3 + 2*sqrt(2)) - 2
```

```
*log(sin(x))/(-3 + 2*sqrt(2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\sin(x)^{(\sqrt{2})} + \sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/(sin(x)^sqrt(2) + sin(x)), x)
```

3.810 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rubi [A] time = 0.0282516, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 14}

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = 2\,\text{Subst}\left(\int \frac{1+x^2}{8x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.0322641, size = 39, normalized size = 1.62

$$\frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))
```

Maple [A] time = 0.054, size = 24, normalized size = 1.

$$\frac{1}{4+4\cos(x)}-\frac{\ln(1+\cos(x))}{8}+\frac{\ln(-1+\cos(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] 1/4/(1+cos(x))-1/8*ln(1+cos(x))+1/8*ln(-1+cos(x))
```

Maxima [B] time = 0.972077, size = 297, normalized size = 12.38

$$4\cos(2x)\cos(x)+8\cos(x)^2-\left(2(2\cos(x)+1)\cos(2x)+\cos(2x)^2+4\cos(x)^2+\sin(2x)^2+4\sin(2x)\sin(x)+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)
^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) +
1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(x) + 1)*cos(2*x) + c
os(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*co
s(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 8*s
in(x)^2 + 4*cos(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 +
sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

Fricas [B] time = 2.31742, size = 132, normalized size = 5.5

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{8(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

Giac [A] time = 1.07389, size = 38, normalized size = 1.58

$$-\frac{\cos(x)-1}{8(\cos(x)+1)}+\frac{1}{8}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.811 $\int (-3+4x+x^2)\sin(2x)\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rubi [A] time = 0.0650418, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 2638, 3296, 2637}

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(-3+4x+x^2\right)\sin(2x)\,dx &= \int \left(-3\sin(2x)+4x\sin(2x)+x^2\sin(2x)\right)\,dx\\
&= -(3\int \sin(2x)\,dx)+4\int x\sin(2x)\,dx+\int x^2\sin(2x)\,dx\\
&= \frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+2\int \cos(2x)\,dx+\int x\cos(2x)\,dx\\
&= \frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)-\frac{1}{2}\int \sin(2x)\,dx\\
&= \frac{7}{4}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)
\end{aligned}$$

Mathematica [A] time = 0.0457935, size = 29, normalized size = 0.72

$$\frac{1}{4}\left(\left(-2x^2-8x+7\right)\cos(2x)+2(x+2)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]

[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$\frac{7\cos(2x)}{4}-2x\cos(2x)-\frac{x^2\cos(2x)}{2}+\sin(2x)+\frac{x\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-3)*sin(2*x),x)

[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

Maxima [A] time = 0.990136, size = 51, normalized size = 1.27

$$-\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)-2\,x\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{3}{2}\,\cos\left(2\,x\right)+\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) +
sin(2*x)
```

Fricas [A] time = 2.28436, size = 76, normalized size = 1.9

$$-\frac{1}{4}\left(2\,x^2+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

Sympy [A] time = 0.323327, size = 39, normalized size = 0.98

$$-\frac{x^2\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}-2x\cos\left(2x\right)+\sin\left(2x\right)+\frac{7\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

Giac [A] time = 1.08716, size = 35, normalized size = 0.88

$$-\frac{1}{4}\left(2\,x^2+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

3.812 $\int e^{-3x} \cos(4x)\, dx$

Optimal. Leaf size=27

$$\frac{4}{25}e^{-3x}\sin(4x) - \frac{3}{25}e^{-3x}\cos(4x)$$

```
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rubi [A] time = 0.0101598, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{4}{25}e^{-3x}\sin(4x) - \frac{3}{25}e^{-3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(4x)\,dx = -\frac{3}{25}e^{-3x}\cos(4x) + \frac{4}{25}e^{-3x}\sin(4x)$$

Mathematica [A] time = 0.0298575, size = 22, normalized size = 0.81

$$\frac{1}{25}e^{-3x}(4\sin(4x) - 3\cos(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{-3\,x}\cos\left(4\,x\right)}{25}+\frac{4\,\mathrm{e}^{-3\,x}\sin\left(4\,x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/exp(3*x),x)
```

```
[Out] -3/25*exp(-3*x)*cos(4*x)+4/25*exp(-3*x)*sin(4*x)
```

Maxima [A] time = 0.947286, size = 26, normalized size = 0.96

$$-\frac{1}{25}\,\left(3\,\cos\left(4\,x\right)-4\,\sin\left(4\,x\right)\right)e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

Fricas [A] time = 2.22763, size = 68, normalized size = 2.52

$$-\frac{3}{25}\,\cos\left(4\,x\right)e^{\left(-3\,x\right)}+\frac{4}{25}\,e^{\left(-3\,x\right)}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

Sympy [A] time = 0.466063, size = 26, normalized size = 0.96

$$\frac{4e^{-3x}\sin(4x)}{25} - \frac{3e^{-3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x)
```

```
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

Giac [A] time = 1.07245, size = 26, normalized size = 0.96

$$-\frac{1}{25}(3\cos(4x) - 4\sin(4x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="giac")
```

```
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

3.813 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rubi [A] time = 0.0397623, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2833, 43}

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)dx, x, \sin(x)\right)$$
$$= -2\sqrt{1+\sin(x)}+\frac{2}{3}(1+\sin(x))^{3/2}$$

Mathematica [A] time = 0.0244546, size = 31, normalized size = 1.35

$$\frac{2(\sin(x)-2)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}{3\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

Maple [A] time = 0.009, size = 18, normalized size = 0.8

$$\frac{2}{3}(1+\sin(x))^{\frac{3}{2}}-2\sqrt{1+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1+sin(x))^(1/2),x)
```

```
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

Maxima [A] time = 0.963013, size = 23, normalized size = 1.

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}}-2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

Fricas [A] time = 2.23375, size = 47, normalized size = 2.04

$$\frac{2}{3}\sqrt{\sin(x)+1}(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

Sympy [A] time = 0.315203, size = 26, normalized size = 1.13

$$\frac{2\sqrt{\sin(x)+1}\sin(x)}{3}-\frac{4\sqrt{\sin(x)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
```

```
[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

Giac [A] time = 1.08573, size = 23, normalized size = 1.

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}}-2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

3.814 $\int \left(x+60 \cos^5(x) \sin^4(x)\right) \, dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+\frac{20 \sin^9(x)}{3}-\frac{120 \sin^7(x)}{7}+12 \sin^5(x)$$

```
[Out] x^2/2 + 12*Sin[x]^5 - (120*Sin[x]^7)/7 + (20*Sin[x]^9)/3
```

Rubi [A] time = 0.0303572, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2564, 270}

$$\frac{x^2}{2}+\frac{20 \sin^9(x)}{3}-\frac{120 \sin^7(x)}{7}+12 \sin^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[x + 60*Cos[x]^5*Sin[x]^4,x]
```

```
[Out] x^2/2 + 12*Sin[x]^5 - (120*Sin[x]^7)/7 + (20*Sin[x]^9)/3
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int\left(x+60\cos^5(x)\sin^4(x)\right)dx &= \frac{x^2}{2}+60\int\cos^5(x)\sin^4(x)\,dx\\ &= \frac{x^2}{2}+60\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2dx,x,\sin(x)\right)\\ &= \frac{x^2}{2}+60\,\text{Subst}\left(\int\left(x^4-2x^6+x^8\right)dx,x,\sin(x)\right)\\ &= \frac{x^2}{2}+12\sin^5(x)-\frac{120\sin^7(x)}{7}+\frac{20\sin^9(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0175568, size = 46, normalized size = 1.53

$$\frac{x^2}{2}+\frac{45\sin(x)}{32}-\frac{5}{16}\sin(3x)-\frac{3}{16}\sin(5x)+\frac{15}{448}\sin(7x)+\frac{5}{192}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x + 60*Cos[x]^5*Sin[x]^4,x]
```

```
[Out] x^2/2 + (45*Sin[x])/32 - (5*Sin[3*x])/16 - (3*Sin[5*x])/16 + (15*Sin[7*x])/
448 + (5*Sin[9*x])/192
```

Maple [A] time = 0.007, size = 41, normalized size = 1.4

$$\frac{x^2}{2}-\frac{20\,(\cos(x))^6(\sin(x))^3}{3}-\frac{20\,\sin(x)(\cos(x))^6}{7}+\frac{4\,\sin(x)}{7}\left(\frac{8}{3}+(\cos(x))^4+\frac{4\,(\cos(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+60*cos(x)^5*sin(x)^4,x)
```

```
[Out] 1/2*x^2-20/3*cos(x)^6*sin(x)^3-20/7*sin(x)*cos(x)^6+4/7*(8/3+cos(x)^4+4/3*c
os(x)^2)*sin(x)
```

Maxima [A] time = 0.962846, size = 32, normalized size = 1.07

$$\frac{20}{3}\sin(x)^9-\frac{120}{7}\sin(x)^7+12\sin(x)^5+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

Fricas [A] time = 2.33896, size = 109, normalized size = 3.63

$$\frac{1}{2}x^2 + \frac{4}{21}\left(35\cos(x)^8 - 50\cos(x)^6 + 3\cos(x)^4 + 4\cos(x)^2 + 8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 4/21*(35*cos(x)^8 - 50*cos(x)^6 + 3*cos(x)^4 + 4*cos(x)^2 + 8)*si
n(x)
```

Sympy [A] time = 0.068034, size = 27, normalized size = 0.9

$$\frac{x^2}{2} + \frac{20\sin^9(x)}{3} - \frac{120\sin^7(x)}{7} + 12\sin^5(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)**5*sin(x)**4,x)
```

```
[Out] x**2/2 + 20*sin(x)**9/3 - 120*sin(x)**7/7 + 12*sin(x)**5
```

Giac [A] time = 1.10109, size = 32, normalized size = 1.07

$$\frac{20}{3}\sin(x)^9 - \frac{120}{7}\sin(x)^7 + 12\sin(x)^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="giac")
```

```
[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

3.815 $\int \cos(x)(\sec(x) + \tan(x))\,dx$

Optimal. Leaf size=6

$$x - \cos(x)$$

```
[Out] x - Cos[x]
```

Rubi [A] time = 0.0106472, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3161, 2638}

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(Sec[x] + Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Rule 3161

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cos[d + e*x]
+ c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)(\sec(x) + \tan(x))\,dx &= \int (1 + \sin(x))\,dx \\
&= x + \int \sin(x)\,dx \\
&= x - \cos(x)
\end{aligned}$$

Mathematica [A] time = 0.0015746, size = 6, normalized size = 1.

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(Sec[x] + Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Maple [A] time = 0.031, size = 7, normalized size = 1.2

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sec(x)+tan(x)),x)
```

```
[Out] x-cos(x)
```

Maxima [A] time = 0.952995, size = 8, normalized size = 1.33

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] x - cos(x)
```

Fricas [A] time = 2.31247, size = 16, normalized size = 2.67

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] x - cos(x)
```

Sympy [A] time = 1.80056, size = 3, normalized size = 0.5

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x)
```

```
[Out] x - cos(x)
```

Giac [B] time = 1.11234, size = 19, normalized size = 3.17

$$x - \frac{2}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] x - 2/(tan(1/2*x)^2 + 1)
```

3.816 $\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx$

Optimal. Leaf size=7

$$\tan(x)-\cos(x)$$

```
[Out] -Cos[x] + Tan[x]
```

Rubi [A] time = 0.0376473, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4401, 3767, 8, 2638}

$$\tan(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(Sec[x]^3 + Tan[x]),x]
```

```
[Out] -Cos[x] + Tan[x]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx &= \int \left(\sec^2(x)+\sin(x)\right)\,dx\\ &= \int \sec^2(x)\,dx+\int \sin(x)\,dx\\ &= -\cos(x)-\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= -\cos(x)+\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0036795, size = 7, normalized size = 1.

$$\tan(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(Sec[x]^3 + Tan[x]),x]
```

```
[Out] -Cos[x] + Tan[x]
```

Maple [A] time = 0.031, size = 8, normalized size = 1.1

$$-\cos(x)+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sec(x)^3+tan(x)),x)
```

```
[Out] -cos(x)+tan(x)
```

Maxima [A] time = 0.959994, size = 9, normalized size = 1.29

$$-\cos(x)+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="maxima")
```

```
[Out] -cos(x) + tan(x)
```

Fricas [B] time = 2.36697, size = 39, normalized size = 5.57

$$-\frac{\cos(x)^2-\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x)^2 - sin(x))/cos(x)
```

Sympy [A] time = 23.3179, size = 8, normalized size = 1.14

$$\frac{\sin(x)}{\cos(x)}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)**3+tan(x)),x)
```

```
[Out] sin(x)/cos(x) - cos(x)
```

Giac [B] time = 1.0972, size = 41, normalized size = 5.86

$$-\frac{2\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)-1\right)}{\tan\left(\frac{1}{2}x\right)^4-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) - 1)/(tan(1/2*x)^4 - 1)
```

3.817 $\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right) dx$

Optimal. Leaf size=13

$$\frac{\csc(x)}{2}-\frac{\cot(x)}{2}$$

```
[Out] -Cot[x]/2 + Csc[x]/2
```

Rubi [A] time = 0.0135493, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {12, 2606, 8, 3767}

$$\frac{\csc(x)}{2}-\frac{\cot(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
```

```
[Out] -Cot[x]/2 + Csc[x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)dx &= \frac{1}{2}\int \left(-\cot(x)\csc(x)+\csc^2(x)\right)dx\\ &= -\left(\frac{1}{2}\int \cot(x)\csc(x)\,dx\right)+\frac{1}{2}\int \csc^2(x)\,dx\\ &= -\left(\frac{1}{2}\,\text{Subst}(\int 1\,dx, x, \cot(x))\right)+\frac{1}{2}\,\text{Subst}(\int 1\,dx, x, \csc(x))\\ &= -\frac{\cot(x)}{2}+\frac{\csc(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0041819, size = 10, normalized size = 0.77

$$\frac{1}{2}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
```

```
[Out] Tan[x/2]/2
```

Maple [A] time = 0.007, size = 10, normalized size = 0.8

$$-\frac{\cot(x)}{2}+\frac{\csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x)
```

```
[Out] -1/2*cot(x)+1/2*csc(x)
```

Maxima [A] time = 0.958026, size = 18, normalized size = 1.38

$$\frac{1}{2\sin(x)}-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/sin(x) - 1/2/tan(x)
```

Fricas [A] time = 2.20441, size = 34, normalized size = 2.62

$$\frac{\sin(x)}{2(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*sin(x)/(cos(x) + 1)
```

Sympy [A] time = 0.070277, size = 14, normalized size = 1.08

$$-\frac{\cos(x)}{2\sin(x)}+\frac{1}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)**2,x)
```

```
[Out] -cos(x)/(2*sin(x)) + 1/(2*sin(x))
```

Giac [A] time = 1.08474, size = 18, normalized size = 1.38

$$\frac{1}{2\sin(x)}-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="giac")
```

```
[Out] 1/2/sin(x) - 1/2/tan(x)
```

3.818 $\int \left(-\csc^2(x)+\sin(2x)\right) dx$

Optimal. Leaf size=11

$$\cot(x)-\frac{1}{2}\cos(2x)$$

```
[Out] -Cos[2*x]/2 + Cot[x]
```

Rubi [A] time = 0.0082446, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3767, 8, 2638}

$$\cot(x)-\frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[-Csc[x]^2 + Sin[2*x],x]
```

```
[Out] -Cos[2*x]/2 + Cot[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(-\csc^2(x)+\sin(2x)\right)\,dx &= -\int \csc^2(x)\,dx+\int \sin(2x)\,dx \\
&= -\frac{1}{2}\cos(2x)+\text{Subst}(\int 1\,dx, x, \cot(x)) \\
&= -\frac{1}{2}\cos(2x)+\cot(x)
\end{aligned}
$$

Mathematica [A] time = 0.0063004, size = 11, normalized size = 1.

$$\cot(x)-\frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Csc[x]^2 + Sin[2*x],x]
```

```
[Out] -Cos[2*x]/2 + Cot[x]
```

Maple [A] time = 0.005, size = 10, normalized size = 0.9

$$-\frac{\cos(2x)}{2}+\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csc(x)^2+sin(2*x),x)
```

```
[Out] -1/2*cos(2*x)+cot(x)
```

Maxima [A] time = 0.967175, size = 15, normalized size = 1.36

$$\frac{1}{\tan(x)}-\frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="maxima")
```

```
[Out] 1/tan(x) - 1/2*cos(2*x)
```

Fricas [B] time = 2.28629, size = 68, normalized size = 6.18

$$-\frac{\left(2\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)-2\,\cos\left(x\right)}{2\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*((2*cos(x)^2 - 1)*sin(x) - 2*cos(x))/sin(x)
```

Sympy [A] time = 0.065527, size = 12, normalized size = 1.09

$$-\frac{\cos\left(2x\right)}{2}+\frac{\cos\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)**2+sin(2*x),x)
```

```
[Out] -cos(2*x)/2 + cos(x)/sin(x)
```

Giac [A] time = 1.09132, size = 15, normalized size = 1.36

$$\frac{1}{\tan\left(x\right)}-\frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="giac")
```

```
[Out] 1/tan(x) - 1/2*cos(2*x)
```

3.819 $\int (2\cot(2x) - 3\sin(3x))\,dx$

Optimal. Leaf size=10

$$\cos(3x) + \log(\sin(2x))$$

```
[Out] Cos[3*x] + Log[Sin[2*x]]
```

Rubi [A] time = 0.0076479, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3475, 2638}

$$\cos(3x) + \log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[2*Cot[2*x] - 3*Sin[3*x],x]
```

```
[Out] Cos[3*x] + Log[Sin[2*x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2\cot(2x) - 3\sin(3x))\,dx &= 2\int \cot(2x)\,dx - 3\int \sin(3x)\,dx\\ &= \cos(3x) + \log(\sin(2x))\end{aligned}$$

Mathematica [A] time = 0.0102941, size = 10, normalized size = 1.

$$\cos(3x) + \log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[2*Cot[2*x] - 3*Sin[3*x],x]
```

```
[Out] Cos[3*x] + Log[Sin[2*x]]
```

Maple [A] time = 0.005, size = 17, normalized size = 1.7

$$-\frac{\ln\left(\left(\cot\left(2x\right)\right)^{2}+1\right)}{2}+\cos\left(3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*cot(2*x)-3*sin(3*x),x)
```

```
[Out] -1/2*ln(cot(2*x)^2+1)+cos(3*x)
```

Maxima [A] time = 0.977552, size = 14, normalized size = 1.4

$$\cos\left(3x\right)+\log\left(\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="maxima")
```

```
[Out] cos(3*x) + log(sin(2*x))
```

Fricas [A] time = 2.46583, size = 66, normalized size = 6.6

$$4\cos\left(x\right)^{3}-3\cos\left(x\right)+\log\left(-\frac{1}{2}\cos\left(x\right)\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4*cos(x)^3 - 3*cos(x) + log(-1/2*cos(x)*sin(x))
```

Sympy [A] time = 0.065541, size = 10, normalized size = 1.

$$\log(\sin(2x)) + \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x)
```

```
[Out] log(sin(2*x)) + cos(3*x)
```

Giac [A] time = 1.09397, size = 15, normalized size = 1.5

$$\cos(3x) + \log(|\sin(2x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="giac")
```

```
[Out] cos(3*x) + log(abs(sin(2*x)))
```

3.820 $\int x \sin(2x^2) \, dx$

Optimal. Leaf size=10

$$-\frac{1}{4}\cos(2x^2)$$

```
[Out] -Cos[2*x^2]/4
```

Rubi [A] time = 0.0077968, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 2638}

$$-\frac{1}{4}\cos(2x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[2*x^2],x]
```

```
[Out] -Cos[2*x^2]/4
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x \sin(2x^2) \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sin(2x) \, dx, x, x^2\right)\\ &= -\frac{1}{4}\cos(2x^2)\end{aligned}$$

Mathematica [A] time = 0.0090049, size = 10, normalized size = 1.

$$-\frac{1}{4}\cos\left(2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[2*x^2],x]
```

```
[Out] -Cos[2*x^2]/4
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$-\frac{\cos\left(2\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(2*x^2),x)
```

```
[Out] -1/4*cos(2*x^2)
```

Maxima [A] time = 0.952914, size = 11, normalized size = 1.1

$$-\frac{1}{4}\cos\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*cos(2*x^2)
```

Fricas [A] time = 2.14754, size = 23, normalized size = 2.3

$$-\frac{1}{4}\cos\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*cos(2*x^2)
```

Sympy [A] time = 0.166031, size = 8, normalized size = 0.8

$$-\frac{\cos\left(2x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x**2),x)
```

```
[Out] -cos(2*x**2)/4
```

Giac [A] time = 1.07296, size = 11, normalized size = 1.1

$$-\frac{1}{4}\cos\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="giac")
```

```
[Out] -1/4*cos(2*x^2)
```

3.821 $\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

```
[Out] (1 + Sin[1 - x]^2)^(3/2)/3
```

Rubi [A] time = 0.0411609, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3198, 261}

$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
```

```
[Out] (1 + Sin[1 - x]^2)^(3/2)/3
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \text{Subst}\left(\int x\sqrt{1+x^2}\,dx, x, \sin(1-x)\right)$$
$$= \frac{1}{3}\left(1+\sin^2(1-x)\right)^{3/2}$$

Mathematica [A] time = 0.034236, size = 18, normalized size = 1.

$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
```

```
[Out] (1 + Sin[1 - x]^2)^(3/2)/3
```

Maple [A] time = 0.015, size = 13, normalized size = 0.7

$$\frac{1}{3}\left(1+(\sin(-1+x))^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x)
```

```
[Out] 1/3*(1+sin(-1+x)^2)^(3/2)
```

Maxima [A] time = 0.95522, size = 16, normalized size = 0.89

$$\frac{1}{3}\left(\sin(x-1)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)
```

Fricas [A] time = 2.43615, size = 42, normalized size = 2.33

$$\frac{1}{3}\left(-\cos\left(x-1\right)^{2}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-cos(x - 1)^2 + 2)^(3/2)
```

Sympy [B] time = 0.856694, size = 32, normalized size = 1.78

$$\frac{\sqrt{\sin^{2}\left(x-1\right)+1}\sin^{2}\left(x-1\right)}{3}+\frac{\sqrt{\sin^{2}\left(x-1\right)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)**2)**(1/2),x)
```

```
[Out] sqrt(sin(x - 1)**2 + 1)*sin(x - 1)**2/3 + sqrt(sin(x - 1)**2 + 1)/3
```

Giac [A] time = 1.07729, size = 16, normalized size = 0.89

$$\frac{1}{3}\left(\sin\left(x-1\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)
```

3.822 $\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

```
[Out] -Sin[x^(-1)]^2/2
```

Rubi [A] time = 0.0123809, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3441}

$$-\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]

[Out] -Sin[x^(-1)]^2/2
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

Mathematica [A] time = 0.0053399, size = 10, normalized size = 1.

$$\frac{1}{2}\cos^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]
```

```
[Out] Cos[x^(-1)]^2/2
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$\frac{\left(\cos\left(x^{-1}\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)*sin(1/x)/x^2,x)
```

```
[Out] 1/2*cos(1/x)^2
```

Maxima [A] time = 0.949932, size = 11, normalized size = 1.1

$$\frac{1}{2}\cos\left(\frac{1}{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*cos(1/x)^2
```

Fricas [A] time = 2.19054, size = 22, normalized size = 2.2

$$\frac{1}{2}\cos\left(\frac{1}{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(1/x)^2
```

Sympy [B] time = 1.74315, size = 31, normalized size = 3.1

$$-\frac{2\tan^2\left(\frac{1}{2x}\right)}{\tan^4\left(\frac{1}{2x}\right)+2\tan^2\left(\frac{1}{2x}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x**2,x)
```

```
[Out] -2*tan(1/(2*x))**2/(tan(1/(2*x))**4 + 2*tan(1/(2*x))**2 + 1)
```

Giac [A] time = 1.06668, size = 11, normalized size = 1.1

$$\frac{1}{2}\cos\left(\frac{1}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*cos(1/x)^2
```

3.823 $\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{6}\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)$$

```
[Out] Sin[1/2 + (3*x)/2]^4/6
```

Rubi [A] time = 0.0161971, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2564, 30}

$$\frac{1}{6}\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]
```

```
[Out] Sin[1/2 + (3*x)/2]^4/6
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx &= \frac{2}{3}\operatorname{Subst}\left(\int x^3\,dx, x, \sin\left(\frac{1}{2}+\frac{3x}{2}\right)\right)\\ &= \frac{1}{6}\sin^4\left(\frac{1}{2}+\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0210782, size = 25, normalized size = 1.56

$$\frac{1}{2}\left(\frac{1}{24}\cos(6x+2)-\frac{1}{6}\cos(3x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]
```

```
[Out] (-Cos[1 + 3*x]/6 + Cos[2 + 6*x]/24)/2
```

Maple [A] time = 0.006, size = 11, normalized size = 0.7

$$\frac{1}{6}\left(\sin\left(\frac{1}{2}+\frac{3x}{2}\right)\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x)
```

```
[Out] 1/6*sin(1/2+3/2*x)^4
```

Maxima [A] time = 0.9477, size = 14, normalized size = 0.88

$$\frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*sin(3/2*x + 1/2)^4
```

Fricas [A] time = 2.326, size = 66, normalized size = 4.12

$$\frac{1}{6}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^{4}-\frac{1}{3}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(3/2*x + 1/2)^4 - 1/3*cos(3/2*x + 1/2)^2
```

Sympy [B] time = 0.572874, size = 39, normalized size = 2.44

$$-\frac{\sin^{2}\left(\frac{3x}{2}+\frac{1}{2}\right)\cos^{2}\left(\frac{3x}{2}+\frac{1}{2}\right)}{3}-\frac{\cos^{4}\left(\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)**3,x)
```

```
[Out] -sin(3*x/2 + 1/2)**2*cos(3*x/2 + 1/2)**2/3 - cos(3*x/2 + 1/2)**4/6
```

Giac [A] time = 1.10547, size = 14, normalized size = 0.88

$$\frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="giac")
```

```
[Out] 1/6*sin(3/2*x + 1/2)^4
```

3.824 $\int 4x\tan\left(x^2\right)\,dx$

Optimal. Leaf size=7

$$-2\log\left(\cos\left(x^2\right)\right)$$

```
[Out] -2*Log[Cos[x^2]]
```

Rubi [A] time = 0.0072001, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 3747, 3475}

$$-2\log\left(\cos\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[4*x*Tan[x^2],x]

[Out] -2*Log[Cos[x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int 4x\tan\left(x^2\right)\,dx = 4\int x\tan\left(x^2\right)\,dx$$
$$= 2\,\text{Subst}\left(\int \tan(x)\,dx, x, x^2\right)$$
$$= -2\log\left(\cos\left(x^2\right)\right)$$

Mathematica [A] time = 0.0058107, size = 7, normalized size = 1.

$$-2\log\left(\cos\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x*Tan[x^2],x]
```

```
[Out] -2*Log[Cos[x^2]]
```

Maple [A] time = 0.003, size = 8, normalized size = 1.1

$$-2\ln\left(\cos\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*tan(x^2),x)
```

```
[Out] -2*ln(cos(x^2))
```

Maxima [A] time = 0.961662, size = 9, normalized size = 1.29

$$2\log\left(\sec\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="maxima")
```

```
[Out] 2*log(sec(x^2))
```

Fricas [A] time = 2.27923, size = 35, normalized size = 5.

$$-\log\left(\frac{1}{\tan\left(x^2\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="fricas")
```

```
[Out] -log(1/(tan(x^2)^2 + 1))
```

Sympy [A] time = 0.127445, size = 8, normalized size = 1.14

$$\log\left(\tan^2\left(x^2\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x**2),x)
```

```
[Out] log(tan(x**2)**2 + 1)
```

Giac [A] time = 1.06695, size = 12, normalized size = 1.71

$$\log\left(\tan\left(x^2\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="giac")
```

```
[Out] log(tan(x^2)^2 + 1)
```

3.825 $\int x \sec \left(5-x^2\right) \, dx$

Optimal. Leaf size=13

$$-\frac{1}{2} \tanh^{-1}\left(\sin \left(5-x^2\right)\right)$$

```
[Out] -ArcTanh[Sin[5 - x^2]]/2
```

Rubi [A] time = 0.0117344, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4204, 3770}

$$-\frac{1}{2} \tanh^{-1}\left(\sin \left(5-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[5 - x^2],x]
```

```
[Out] -ArcTanh[Sin[5 - x^2]]/2
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \sec \left(5-x^2\right) \, dx &= \frac{1}{2} \text{Subst}\left(\int \sec (5-x) \, dx,x,x^2\right)\\ &= -\frac{1}{2} \tanh^{-1}\left(\sin \left(5-x^2\right)\right) \end{aligned}$$

Mathematica [A] time = 0.0175405, size = 13, normalized size = 1.

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[5 - x^2],x]
```

```
[Out] -ArcTanh[Sin[5 - x^2]]/2
```

Maple [A] time = 0.002, size = 17, normalized size = 1.3

$$\frac{\ln\left(\sec\left(x^2-5\right)+\tan\left(x^2-5\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2-5),x)
```

```
[Out] 1/2*ln(sec(x^2-5)+tan(x^2-5))
```

Maxima [A] time = 0.962089, size = 22, normalized size = 1.69

$$\frac{1}{2}\log\left(\sec\left(x^2-5\right)+\tan\left(x^2-5\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="maxima")
```

```
[Out] 1/2*log(sec(x^2 - 5) + tan(x^2 - 5))
```

Fricas [B] time = 2.32914, size = 76, normalized size = 5.85

$$\frac{1}{4}\log\left(\sin\left(x^2-5\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x^2-5\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="fricas")
```

```
[Out] 1/4*log(sin(x^2 - 5) + 1) - 1/4*log(-sin(x^2 - 5) + 1)
```

Sympy [A] time = 0.97906, size = 15, normalized size = 1.15

$$\frac{\log\left(\tan\left(x^2-5\right)+\sec\left(x^2-5\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2-5),x)
```

```
[Out] log(tan(x**2 - 5) + sec(x**2 - 5))/2
```

Giac [B] time = 1.10437, size = 55, normalized size = 4.23

$$\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)+2\right|\right)-\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(1/sin(x^2 - 5) + sin(x^2 - 5) + 2)) - 1/8*log(abs(1/sin(x^2 - 5
) + sin(x^2 - 5) - 2))
```

3.826 $\int \frac{\csc\left(\frac{1}{x}\right)}{x^2} dx$

Optimal. Leaf size=5

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

```
[Out] ArcTanh[Cos[x^(-1)]]
```

Rubi [A] time = 0.0090749, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4205, 3770}

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x^(-1)]/x^2,x]
```

```
[Out] ArcTanh[Cos[x^(-1)]]
```

Rule 4205

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = -\text{Subst}\left(\int \csc(x)\,dx, x, \frac{1}{x}\right)$$
$$= \tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

Mathematica [B] time = 0.0148051, size = 21, normalized size = 4.2

$$\log\left(\cos\left(\frac{1}{2x}\right)\right) - \log\left(\sin\left(\frac{1}{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x^(-1)]/x^2,x]
```

```
[Out] Log[Cos[1/(2*x)]] - Log[Sin[1/(2*x)]]
```

Maple [A] time = 0.003, size = 11, normalized size = 2.2

$$\ln\left(\csc\left(x^{-1}\right) + \cot\left(x^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(1/x)/x^2,x)
```

```
[Out] ln(csc(1/x)+cot(1/x))
```

Maxima [A] time = 0.96226, size = 14, normalized size = 2.8

$$\log\left(\cot\left(\frac{1}{x}\right) + \csc\left(\frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] log(cot(1/x) + csc(1/x))
```

Fricas [B] time = 2.29585, size = 81, normalized size = 16.2

$$\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)-\frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*log(1/2*cos(1/x) + 1/2) - 1/2*log(-1/2*cos(1/x) + 1/2)
```

Sympy [A] time = 1.35122, size = 10, normalized size = 2.

$$\log\left(\cot\left(\frac{1}{x}\right)+\csc\left(\frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x**2,x)
```

```
[Out] log(cot(1/x) + csc(1/x))
```

Giac [A] time = 1.14418, size = 14, normalized size = 2.8

$$-\log\left(\left|\tan\left(\frac{1}{2\,x}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="giac")
```

```
[Out] -log(abs(tan(1/2/x)))
```

3.827 $\int (\csc(x)-\sec(x))(\cos(x)+\sin(x))\,dx$

Optimal. Leaf size=7

$$\log(\sin(x))+\log(\cos(x))$$

```
[Out] Log[Cos[x]] + Log[Sin[x]]
```

Rubi [A] time = 0.0454076, antiderivative size = 9, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 72}

$$\log(\tan(x))+2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]
```

```
[Out] 2*Log[Cos[x]] + Log[Tan[x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (\csc(x)-\sec(x))(\cos(x)+\sin(x))\,dx &= \text{Subst}\left(\int \frac{1-x^2}{x\left(1+x^2\right)}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1-x}{x(1+x)}\,dx, x, \tan^2(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{1+x}\right)dx, x, \tan^2(x)\right)\\ &= 2\log(\cos(x))+\log(\tan(x))\end{aligned}$$

Mathematica [A] time = 0.0071322, size = 7, normalized size = 1.

$$\log(\sin(x))+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]

[Out] Log[Cos[x]] + Log[Sin[x]]
```

Maple [A] time = 0.04, size = 8, normalized size = 1.1

$$\ln(\cos(x))+\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sec(x))*(cos(x)+sin(x)),x)

[Out] ln(cos(x))+ln(sin(x))
```

Maxima [B] time = 0.965771, size = 20, normalized size = 2.86

$$\frac{1}{2}\log\left(-\sin(x)^2+1\right)+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*log(-sin(x)^2 + 1) + log(sin(x))
```

Fricas [A] time = 2.27852, size = 34, normalized size = 4.86

$$\log\left(-\frac{1}{2}\cos(x)\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x)*sin(x))
```

Sympy [A] time = 6.47874, size = 8, normalized size = 1.14

$$\log(\sin(x)) + \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x)
```

```
[Out] log(sin(x)) + log(cos(x))
```

Giac [B] time = 1.08246, size = 26, normalized size = 3.71

$$\frac{1}{2}\log\left(\cos(x)^2\right) + \frac{1}{2}\log\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(cos(x)^2) + 1/2*log(-cos(x)^2 + 1)
```

3.828 $\int (-\cos(3x)\sin(2x) + \cos(2x)\sin(3x))\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -Cos[x]
```

Rubi [A] time = 0.0173933, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4284}

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]

[Out] -Cos[x]
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\cos(3x)\sin(2x) + \cos(2x)\sin(3x))\,dx &= -\int \cos(3x)\sin(2x)\,dx + \int \cos(2x)\sin(3x)\,dx\\ &= -\cos(x)\end{aligned}$$

Mathematica [A] time = 0.0013774, size = 4, normalized size = 1.

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[x]
```

Maple [A] time = 0.05, size = 5, normalized size = 1.3

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)
```

```
[Out] -cos(x)
```

Maxima [A] time = 0.952717, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -cos(x)
```

Fricas [A] time = 2.22345, size = 12, normalized size = 3.

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)
```

Sympy [B] time = 0.830159, size = 20, normalized size = 5.

$$-\sin(2x)\sin(3x) - \cos(2x)\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)
```

```
[Out] -sin(2*x)*sin(3*x) - cos(2*x)*cos(3*x)
```

Giac [A] time = 1.05694, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -cos(x)
```

3.829 $\int 4x\sec^2(2x)\,dx$

Optimal. Leaf size=13

$$2x\tan(2x)+\log(\cos(2x))$$

```
[Out] Log[Cos[2*x]] + 2*x*Tan[2*x]
```

Rubi [A] time = 0.0200766, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 4184, 3475}

$$2x\tan(2x)+\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[4*x*Sec[2*x]^2,x]
```

```
[Out] Log[Cos[2*x]] + 2*x*Tan[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int 4x\sec^2(2x)\,dx = 4\int x\sec^2(2x)\,dx$$
$$= 2x\tan(2x) - 2\int \tan(2x)\,dx$$
$$= \log(\cos(2x)) + 2x\tan(2x)$$

Mathematica [A] time = 0.0071375, size = 21, normalized size = 1.62

$$4\left(\frac{1}{2}x\tan(2x) + \frac{1}{4}\log(\cos(2x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x*Sec[2*x]^2,x]
```

```
[Out] 4*(Log[Cos[2*x]]/4 + (x*Tan[2*x])/2)
```

Maple [A] time = 0.006, size = 14, normalized size = 1.1

$$\ln(\cos(2x)) + 2x\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*sec(2*x)^2,x)
```

```
[Out] ln(cos(2*x))+2*x*tan(2*x)
```

Maxima [B] time = 1.47006, size = 100, normalized size = 7.69

$$\frac{\left(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1\right)\log\left(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1\right) + 8x\sin(4x)}{2\left(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*log(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1) + 8*x*sin(4*x))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)
```

Fricas [A] time = 2.50494, size = 69, normalized size = 5.31

$$\frac{\cos(2x)\log(-\cos(2x)) + 2x\sin(2x)}{\cos(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="fricas")
```

```
[Out] (cos(2*x)*log(-cos(2*x)) + 2*x*sin(2*x))/cos(2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$4\int x\sec^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)**2,x)
```

```
[Out] 4*Integral(x*sec(2*x)**2, x)
```

Giac [B] time = 1.10374, size = 109, normalized size = 8.38

$$\frac{\log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)\tan(x)^2 - 8x\tan(x) - \log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)}{2\left(\tan(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^
2 - 8*x*tan(x) - log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 +
 1)))/(tan(x)^2 - 1)
```

3.830 $\int 4\sin^2(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$-6x+6\tan(x)-2\sin^2(x)\tan(x)$$

```
[Out] -6*x + 6*Tan[x] - 2*Sin[x]^2*Tan[x]
```

Rubi [A] time = 0.0290192, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {12, 2591, 288, 321, 203}

$$-6x+6\tan(x)-2\sin^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[4*Sin[x]^2*Tan[x]^2,x]
```

```
[Out] -6*x + 6*Tan[x] - 2*Sin[x]^2*Tan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int 4\sin^2(x)\tan^2(x)\,dx &= 4\int \sin^2(x)\tan^2(x)\,dx\\
&= 4\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= -2\sin^2(x)\tan(x) + 6\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(x)\right)\\
&= 6\tan(x) - 2\sin^2(x)\tan(x) - 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -6x + 6\tan(x) - 2\sin^2(x)\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.024689, size = 18, normalized size = 1.12

$$4\left(-\frac{3x}{2} + \frac{1}{4}\sin(2x) + \tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*Sin[x]^2*Tan[x]^2,x]
```

```
[Out] 4*((-3*x)/2 + Sin[2*x]/4 + Tan[x])
```

Maple [A] time = 0.01, size = 28, normalized size = 1.8

$$4\,\frac{(\sin(x))^5}{\cos(x)} + 4\left((\sin(x))^3 + 3/2\,\sin(x)\right)\cos(x) - 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*sin(x)^2*tan(x)^2,x)
```

```
[Out] 4*sin(x)^5/cos(x)+4*(sin(x)^3+3/2*sin(x))*cos(x)-6*x
```

Maxima [A] time = 1.45194, size = 27, normalized size = 1.69

$$-6\,x+\frac{2\,\tan\left(x\right)}{\tan\left(x\right)^{2}+1}+4\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

Fricas [A] time = 2.39315, size = 65, normalized size = 4.06

$$-\frac{2\left(3\,x\cos\left(x\right)-\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

Sympy [A] time = 0.062212, size = 20, normalized size = 1.25

$$-6x+\frac{4\sin^{3}\left(x\right)}{\cos\left(x\right)}+6\sin\left(x\right)\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)**2*tan(x)**2,x)
```

```
[Out] -6*x + 4*sin(x)**3/cos(x) + 6*sin(x)*cos(x)
```

Giac [A] time = 1.07805, size = 27, normalized size = 1.69

$$-6x + \frac{2\tan(x)}{\tan(x)^2 + 1} + 4\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="giac")
```

```
[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

3.831 $\int \cos^4(x)\cot^2(x)\,dx$

Optimal. Leaf size=32

$$-\frac{15x}{8}-\frac{15\cot(x)}{8}+\frac{1}{4}\cos^4(x)\cot(x)+\frac{5}{8}\cos^2(x)\cot(x)$$

```
[Out] (-15*x)/8 - (15*Cot[x])/8 + (5*Cos[x]^2*Cot[x])/8 + (Cos[x]^4*Cot[x])/4
```

Rubi [A] time = 0.0336789, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2591, 288, 321, 203}

$$-\frac{15x}{8}-\frac{15\cot(x)}{8}+\frac{1}{4}\cos^4(x)\cot(x)+\frac{5}{8}\cos^2(x)\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Cot[x]^2,x]
```

```
[Out] (-15*x)/8 - (15*Cot[x])/8 + (5*Cos[x]^2*Cot[x])/8 + (Cos[x]^4*Cot[x])/4
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(x)\cot^2(x)\,dx &= -\operatorname{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx, x, \cot(x)\right)\\
&= \frac{1}{4}\cos^4(x)\cot(x) - \frac{5}{4}\operatorname{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \cot(x)\right)\\
&= \frac{5}{8}\cos^2(x)\cot(x) + \frac{1}{4}\cos^4(x)\cot(x) - \frac{15}{8}\operatorname{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\frac{15\cot(x)}{8} + \frac{5}{8}\cos^2(x)\cot(x) + \frac{1}{4}\cos^4(x)\cot(x) + \frac{15}{8}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\frac{15x}{8} - \frac{15\cot(x)}{8} + \frac{5}{8}\cos^2(x)\cot(x) + \frac{1}{4}\cos^4(x)\cot(x)
\end{aligned}$$

Mathematica [A] time = 0.0219386, size = 26, normalized size = 0.81

$$-\frac{15x}{8} - \frac{1}{2}\sin(2x) - \frac{1}{32}\sin(4x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Cot[x]^2,x]
```

```
[Out] (-15*x)/8 - Cot[x] - Sin[2*x]/2 - Sin[4*x]/32
```

Maple [A] time = 0.01, size = 34, normalized size = 1.1

$$-\frac{(\cos(x))^7}{\sin(x)} - \left((\cos(x))^5 + \frac{5\,(\cos(x))^3}{4} + \frac{15\,\cos(x)}{8}\right)\sin(x) - \frac{15\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cot(x)^2,x)
```

```
[Out] -1/sin(x)*cos(x)^7-(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)-15/8*x
```

Maxima [A] time = 1.44403, size = 47, normalized size = 1.47

$$-\frac{15}{8}\,x-\frac{15\,\tan\left(x\right)^{4}+25\,\tan\left(x\right)^{2}+8}{8\left(\tan\left(x\right)^{5}+2\,\tan\left(x\right)^{3}+\tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="maxima")
```

```
[Out] -15/8*x - 1/8*(15*tan(x)^4 + 25*tan(x)^2 + 8)/(tan(x)^5 + 2*tan(x)^3 + tan(
x))
```

Fricas [A] time = 2.46964, size = 86, normalized size = 2.69

$$\frac{2\,\cos\left(x\right)^{5}+5\,\cos\left(x\right)^{3}-15\,x\sin\left(x\right)-15\,\cos\left(x\right)}{8\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^5 + 5*cos(x)^3 - 15*x*sin(x) - 15*cos(x))/sin(x)
```

Sympy [A] time = 0.066273, size = 36, normalized size = 1.12

$$-\frac{15x}{8}-\frac{5\sin\left(x\right)\cos^{3}\left(x\right)}{4}-\frac{15\sin\left(x\right)\cos\left(x\right)}{8}-\frac{\cos^{5}\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*cot(x)**2,x)
```

```
[Out] -15*x/8 - 5*sin(x)*cos(x)**3/4 - 15*sin(x)*cos(x)/8 - cos(x)**5/sin(x)
```

Giac [A] time = 1.1096, size = 42, normalized size = 1.31

$$-\frac{15}{8}\,x - \frac{7\,\tan\left(x\right)^{3} + 9\,\tan\left(x\right)}{8\left(\tan\left(x\right)^{2} + 1\right)^{2}} - \frac{1}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="giac")
```

```
[Out] -15/8*x - 1/8*(7*tan(x)^3 + 9*tan(x))/(tan(x)^2 + 1)^2 - 1/tan(x)
```

3.832 $\int 16 \cos^2(x) \sin^2(x) \, dx$

Optimal. Leaf size=18

$$2x - 4\sin(x)\cos^3(x) + 2\sin(x)\cos(x)$$

```
[Out] 2*x + 2*Cos[x]*Sin[x] - 4*Cos[x]^3*Sin[x]
```

Rubi [A] time = 0.0282526, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {12, 2568, 2635, 8}

$$2x - 4\sin(x)\cos^3(x) + 2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[16*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] 2*x + 2*Cos[x]*Sin[x] - 4*Cos[x]^3*Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int 16\cos^2(x)\sin^2(x)\,dx &= 16\int \cos^2(x)\sin^2(x)\,dx\\ &= -4\cos^3(x)\sin(x)+4\int \cos^2(x)\,dx\\ &= 2\cos(x)\sin(x)-4\cos^3(x)\sin(x)+2\int 1\,dx\\ &= 2x+2\cos(x)\sin(x)-4\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0079291, size = 16, normalized size = 0.89

$$4\left(\frac{x}{2}-\frac{1}{8}\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[16*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] 4*(x/2 - Sin[4*x]/8)
```

Maple [A] time = 0.005, size = 19, normalized size = 1.1

$$2\,x+2\,\cos(x)\sin(x)-4\,(\cos(x))^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*cos(x)^2*sin(x)^2,x)
```

```
[Out] 2*x+2*cos(x)*sin(x)-4*cos(x)^3*sin(x)
```

Maxima [A] time = 0.959415, size = 14, normalized size = 0.78

$$2\,x-\frac{1}{2}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 2*x - 1/2*sin(4*x)
```

Fricas [A] time = 2.23492, size = 53, normalized size = 2.94

$$-2\left(2\cos(x)^{3}-\cos(x)\right)\sin(x)+2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -2*(2*cos(x)^3 - cos(x))*sin(x) + 2*x
```

Sympy [A] time = 0.066772, size = 12, normalized size = 0.67

$$2x-\sin(2x)\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)**2*sin(x)**2,x)
```

```
[Out] 2*x - sin(2*x)*cos(2*x)
```

Giac [A] time = 1.07518, size = 14, normalized size = 0.78

$$2x-\frac{1}{2}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 2*x - 1/2*sin(4*x)
```

3.833 $\int 8\cos^2(x)\sin^4(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{2}-\frac{4}{3}\sin^3(x)\cos^3(x)-\sin(x)\cos^3(x)+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2 - Cos[x]^3*Sin[x] - (4*Cos[x]^3*Sin[x]^3)/3
```

Rubi [A] time = 0.0502599, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {12, 2568, 2635, 8}

$$\frac{x}{2}-\frac{4}{3}\sin^3(x)\cos^3(x)-\sin(x)\cos^3(x)+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[8*Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2 - Cos[x]^3*Sin[x] - (4*Cos[x]^3*Sin[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int 8\cos^2(x)\sin^4(x)\,dx &= 8\int \cos^2(x)\sin^4(x)\,dx \\
&= -\frac{4}{3}\cos^3(x)\sin^3(x)+4\int \cos^2(x)\sin^2(x)\,dx \\
&= -\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)+\int \cos^2(x)\,dx \\
&= \frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)+\frac{\int 1\,dx}{2} \\
&= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0100087, size = 32, normalized size = 0.94

$$8\left(\frac{x}{16}-\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)+\frac{1}{192}\sin(6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[8*Cos[x]^2*Sin[x]^4,x]
```

```
[Out] 8*(x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192)
```

Maple [A] time = 0.005, size = 29, normalized size = 0.9

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}-(\cos(x))^3\sin(x)-\frac{4\,(\cos(x))^3(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*cos(x)^2*sin(x)^4,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3
```

Maxima [A] time = 0.960433, size = 24, normalized size = 0.71

$$-\frac{1}{6}\sin\left(2x\right)^{3} + \frac{1}{2}x - \frac{1}{8}\sin\left(4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*sin(2*x)^3 + 1/2*x - 1/8*sin(4*x)
```

Fricas [A] time = 2.39833, size = 78, normalized size = 2.29

$$\frac{1}{6}\left(8\cos\left(x\right)^{5} - 14\cos\left(x\right)^{3} + 3\cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/2*x
```

Sympy [A] time = 0.061079, size = 32, normalized size = 0.94

$$\frac{x}{2} + \frac{4\sin^{5}\left(x\right)\cos\left(x\right)}{3} - \frac{\sin^{3}\left(x\right)\cos\left(x\right)}{3} - \frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)**2*sin(x)**4,x)
```

```
[Out] x/2 + 4*sin(x)**5*cos(x)/3 - sin(x)**3*cos(x)/3 - sin(x)*cos(x)/2
```

Giac [A] time = 1.08186, size = 30, normalized size = 0.88

$$\frac{1}{2}x + \frac{1}{24}\sin\left(6x\right) - \frac{1}{8}\sin\left(4x\right) - \frac{1}{8}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/24*sin(6*x) - 1/8*sin(4*x) - 1/8*sin(2*x)
```

3.834 $\int 35\cos^3(x)\sin^4(x)\,dx$

Optimal. Leaf size=13

$$7\sin^5(x) - 5\sin^7(x)$$

```
[Out] 7*Sin[x]^5 - 5*Sin[x]^7
```

Rubi [A] time = 0.0241216, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {12, 2564, 14}

$$7\sin^5(x) - 5\sin^7(x)$$

Antiderivative was successfully verified.

```
[In] Int[35*Cos[x]^3*Sin[x]^4,x]
```

```
[Out] 7*Sin[x]^5 - 5*Sin[x]^7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int 35\cos^3(x)\sin^4(x)\,dx = 35\int \cos^3(x)\sin^4(x)\,dx$$
$$= 35\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \sin(x)\right)$$
$$= 35\,\text{Subst}\left(\int \left(x^4-x^6\right)\,dx, x, \sin(x)\right)$$
$$= 7\sin^5(x) - 5\sin^7(x)$$

Mathematica [B] time = 0.0094767, size = 33, normalized size = 2.54

$$35\left(\frac{3\sin(x)}{64} - \frac{1}{64}\sin(3x) - \frac{1}{320}\sin(5x) + \frac{1}{448}\sin(7x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[35*Cos[x]^3*Sin[x]^4,x]
```

```
[Out] 35*((3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448)
```

Maple [B] time = 0.005, size = 29, normalized size = 2.2

$$-5\,(\cos(x))^4(\sin(x))^3 - 3\,\sin(x)(\cos(x))^4 + \left(2 + (\cos(x))^2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(35*cos(x)^3*sin(x)^4,x)
```

```
[Out] -5*cos(x)^4*sin(x)^3-3*sin(x)*cos(x)^4+(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.979526, size = 18, normalized size = 1.38

$$-5\,\sin(x)^7 + 7\,\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -5*sin(x)^7 + 7*sin(x)^5
```

Fricas [A] time = 2.29107, size = 66, normalized size = 5.08

$$\left(5\,\cos\left(x\right)^{6}-8\,\cos\left(x\right)^{4}+\cos\left(x\right)^{2}+2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="fricas")
```

```
[Out] (5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.061939, size = 12, normalized size = 0.92

$$-5\sin^{7}\left(x\right)+7\sin^{5}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)**3*sin(x)**4,x)
```

```
[Out] -5*sin(x)**7 + 7*sin(x)**5
```

Giac [A] time = 1.07909, size = 18, normalized size = 1.38

$$-5\,\sin\left(x\right)^{7}+7\,\sin\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="giac")
```

```
[Out] -5*sin(x)^7 + 7*sin(x)^5
```

3.835 $\int 4\cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{32}-\frac{1}{2}\sin^3(x)\cos^5(x)-\frac{1}{4}\sin(x)\cos^5(x)+\frac{1}{16}\sin(x)\cos^3(x)+\frac{3}{32}\sin(x)\cos(x)$$

```
[Out] (3*x)/32 + (3*Cos[x]*Sin[x])/32 + (Cos[x]^3*Sin[x])/16 - (Cos[x]^5*Sin[x])/
4 - (Cos[x]^5*Sin[x]^3)/2
```

Rubi [A] time = 0.056109, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {12, 2568, 2635, 8}

$$\frac{3x}{32}-\frac{1}{2}\sin^3(x)\cos^5(x)-\frac{1}{4}\sin(x)\cos^5(x)+\frac{1}{16}\sin(x)\cos^3(x)+\frac{3}{32}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[4*Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/32 + (3*Cos[x]*Sin[x])/32 + (Cos[x]^3*Sin[x])/16 - (Cos[x]^5*Sin[x])/
4 - (Cos[x]^5*Sin[x]^3)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int 4\cos^4(x)\sin^4(x)\,dx &= 4\int \cos^4(x)\sin^4(x)\,dx\\
&= -\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{3}{2}\int \cos^4(x)\sin^2(x)\,dx\\
&= -\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{1}{4}\int \cos^4(x)\,dx\\
&= \frac{1}{16}\cos^3(x)\sin(x)-\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{3}{16}\int \cos^2(x)\,dx\\
&= \frac{3}{32}\cos(x)\sin(x)+\frac{1}{16}\cos^3(x)\sin(x)-\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{3\int 1\,dx}{32}\\
&= \frac{3x}{32}+\frac{3}{32}\cos(x)\sin(x)+\frac{1}{16}\cos^3(x)\sin(x)-\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0071436, size = 24, normalized size = 0.52

$$4\left(\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*Cos[x]^4*Sin[x]^4,x]
```

```
[Out] 4*((3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024)
```

Maple [A] time = 0.007, size = 36, normalized size = 0.8

$$-\frac{(\cos(x))^5(\sin(x))^3}{2}-\frac{(\cos(x))^5\sin(x)}{4}+\frac{\sin(x)}{16}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{3x}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*cos(x)^4*sin(x)^4,x)
```

```
[Out] -1/2*cos(x)^5*sin(x)^3-1/4*cos(x)^5*sin(x)+1/16*(cos(x)^3+3/2*cos(x))*sin(x
)+3/32*x
```

Maxima [A] time = 0.959757, size = 22, normalized size = 0.48

$$\frac{3}{32}\,x + \frac{1}{256}\,\sin\left(8\,x\right) - \frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

Fricas [A] time = 2.29601, size = 100, normalized size = 2.17

$$\frac{1}{32}\,\left(16\,\cos\left(x\right)^{7} - 24\,\cos\left(x\right)^{5} + 2\,\cos\left(x\right)^{3} + 3\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{32}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/32*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/32*x
```

Sympy [A] time = 0.069102, size = 31, normalized size = 0.67

$$\frac{3x}{32} - \frac{\sin^{3}\left(2x\right)\cos\left(2x\right)}{32} - \frac{3\sin\left(2x\right)\cos\left(2x\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/32 - sin(2*x)**3*cos(2*x)/32 - 3*sin(2*x)*cos(2*x)/64
```

Giac [A] time = 1.07825, size = 22, normalized size = 0.48

$$\frac{3}{32}\,x + \frac{1}{256}\,\sin\left(8\,x\right) - \frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

3.836 $\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx$

Optimal. Leaf size=9

$$\log(\cos(x)) - \log(\sin(x))$$

```
[Out] Log[Cos[x]] - Log[Sin[x]]
```

Rubi [A] time = 0.028245, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4334, 266, 36, 31, 29}

$$\log(\cos(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(-Sin[x] + Sin[x]^3),x]
```

```
[Out] Log[Cos[x]] - Log[Sin[x]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)x}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \sin^2(x)\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)\\
&= \log(\cos(x)) - \log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.0049069, size = 9, normalized size = 1.

$$\log(\cos(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(-Sin[x] + Sin[x]^3),x]
```

```
[Out] Log[Cos[x]] - Log[Sin[x]]
```

Maple [B] time = 0.02, size = 21, normalized size = 2.3

$$-\ln(\sin(x)) + \frac{\ln(1+\sin(x))}{2} + \frac{\ln(\sin(x)-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(-sin(x)+sin(x)^3),x)
```

```
[Out] -ln(sin(x))+1/2*ln(1+sin(x))+1/2*ln(sin(x)-1)
```

Maxima [B] time = 0.969199, size = 27, normalized size = 3.

$$\frac{1}{2} \log (\sin (x) + 1) + \frac{1}{2} \log (\sin (x) - 1) - \log (\sin (x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - log(sin(x))
```

Fricas [B] time = 2.12816, size = 68, normalized size = 7.56

$$\frac{1}{2} \log \left(\cos (x)^2\right) - \frac{1}{2} \log \left(-\frac{1}{4} \cos (x)^2 + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*log(cos(x)^2) - 1/2*log(-1/4*cos(x)^2 + 1/4)
```

Sympy [B] time = 0.360291, size = 20, normalized size = 2.22

$$\frac{\log (\sin (x) - 1)}{2} + \frac{\log (\sin (x) + 1)}{2} - \log (\sin (x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)**3),x)
```

```
[Out] log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(sin(x))
```

Giac [B] time = 1.09292, size = 26, normalized size = 2.89

$$-\frac{1}{2} \log \left(\sin (x)^2\right) + \frac{1}{2} \log \left(-\sin (x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="giac")
```

```
[Out] -1/2*log(sin(x)^2) + 1/2*log(-sin(x)^2 + 1)
```

3.837 $\int \left(-1+2 \cos^2(x)+\cos(x) \sin(x)\right) \, dx$

Optimal. Leaf size=14

$$\frac{\sin^2(x)}{2}+\sin(x) \cos(x)$$

```
[Out] Cos[x]*Sin[x] + Sin[x]^2/2
```

Rubi [A] time = 0.0166557, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2635, 8, 2564, 30}

$$\frac{\sin^2(x)}{2}+\sin(x) \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
```

```
[Out] Cos[x]*Sin[x] + Sin[x]^2/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx &= -x+2\int \cos^2(x)\,dx+\int \cos(x)\sin(x)\,dx\\
&= -x+\cos(x)\sin(x)+\int 1\,dx+\text{Subst}(\int x\,dx,x,\sin(x))\\
&= \cos(x)\sin(x)+\frac{\sin^2(x)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.0053653, size = 17, normalized size = 1.21

$$\frac{1}{2}\sin(2x)-\frac{\cos^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
```

```
[Out] -Cos[x]^2/2 + Sin[2*x]/2
```

Maple [A] time = 0.005, size = 13, normalized size = 0.9

$$\cos(x)\sin(x)+\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1+2*cos(x)^2+cos(x)*sin(x),x)
```

```
[Out] cos(x)*sin(x)+1/2*sin(x)^2
```

Maxima [A] time = 0.949812, size = 18, normalized size = 1.29

$$-\frac{1}{2}\cos(x)^2+\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)
```

Fricas [A] time = 1.99195, size = 42, normalized size = 3.

$$-\frac{1}{2}\cos(x)^2+\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2 + cos(x)*sin(x)
```

Sympy [A] time = 0.061004, size = 12, normalized size = 0.86

$$\frac{\sin^2(x)}{2}+\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)**2+cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2 + sin(x)*cos(x)
```

Giac [A] time = 1.08018, size = 18, normalized size = 1.29

$$-\frac{1}{2}\cos(x)^2+\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)
```

3.838 $\int \left(\cos^2(x)+\sin^2(x)\right)\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0103859, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2635, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] x
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(\cos^2(x)+\sin^2(x)\right)\,dx &= \int \cos^2(x)\,dx+\int \sin^2(x)\,dx\\ &= 2\frac{\int 1\,dx}{2}\\ &= x\end{aligned}$$

Mathematica [A] time = 0.000303, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] x
```

Maple [A] time = 0.003, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2+sin(x)^2,x)
```

```
[Out] x
```

Maxima [A] time = 0.963386, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.82605, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A] time = 0.059741, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2+sin(x)**2,x)
```

```
[Out] x
```

Giac [A] time = 1.068, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="giac")
```

```
[Out] x
```

3.839 $\int \left(-\cos^2(x)+\sin^2(x)\right)\,dx$

Optimal. Leaf size=6

$$\sin(x)(-\cos(x))$$

```
[Out] -(Cos[x]*Sin[x])
```

Rubi [A] time = 0.0110818, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2635, 8}

$$\sin(x)(-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[-Cos[x]^2 + Sin[x]^2,x]

[Out] -(Cos[x]*Sin[x])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(-\cos^2(x)+\sin^2(x)\right)\,dx &= -\int \cos^2(x)\,dx+\int \sin^2(x)\,dx\\ &= -\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0023956, size = 8, normalized size = 1.33

$$-\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] -Sin[2*x]/2
```

Maple [A] time = 0.001, size = 7, normalized size = 1.2

$$-\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)^2+sin(x)^2,x)
```

```
[Out] -cos(x)*sin(x)
```

Maxima [A] time = 0.952364, size = 8, normalized size = 1.33

$$-\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(2*x)
```

Fricas [A] time = 2.09111, size = 22, normalized size = 3.67

$$-\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="fricas")
```

```
[Out] -cos(x)*sin(x)
```

Sympy [A] time = 0.058978, size = 7, normalized size = 1.17

$$-\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)**2+sin(x)**2,x)
```

```
[Out] -sin(x)*cos(x)
```

Giac [A] time = 1.05358, size = 8, normalized size = 1.33

$$-\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(2*x)
```

3.840 $\int 2^{\sin(x)} \cos(x)\,dx$

Optimal. Leaf size=9

$$\frac{2^{\sin(x)}}{\log(2)}$$

[Out] 2^Sin[x]/Log[2]

Rubi [A] time = 0.0089618, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4334, 2194}

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sin[x]*Cos[x],x]

[Out] 2^Sin[x]/Log[2]

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int 2^{\sin(x)} \cos(x)\,dx &= \text{Subst}\left(\int 2^x\,dx, x, \sin(x)\right)\\ &= \frac{2^{\sin(x)}}{\log(2)}\end{aligned}$$

Mathematica [A] time = 0.0063061, size = 9, normalized size = 1.

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sin[x]*Cos[x],x]
```

```
[Out] 2^Sin[x]/Log[2]
```

Maple [A] time = 0.006, size = 10, normalized size = 1.1

$$\frac{2^{\sin(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sin(x)*cos(x),x)
```

```
[Out] 2^sin(x)/ln(2)
```

Maxima [A] time = 0.959556, size = 12, normalized size = 1.33

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="maxima")
```

```
[Out] 2^sin(x)/log(2)
```

Fricas [A] time = 1.94472, size = 23, normalized size = 2.56

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="fricas")
```

```
[Out] 2^sin(x)/log(2)
```

Sympy [A] time = 0.289783, size = 7, normalized size = 0.78

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**sin(x)*cos(x),x)
```

```
[Out] 2**sin(x)/log(2)
```

Giac [A] time = 1.06874, size = 12, normalized size = 1.33

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="giac")
```

```
[Out] 2^sin(x)/log(2)
```

3.841 $\int \left(\tan^3(x)+\tan^5(x)\right)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^4(x)}{4}$$

```
[Out] Tan[x]^4/4
```

Rubi [A] time = 0.0167934, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3473, 3475}

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3 + Tan[x]^5,x]
```

```
[Out] Tan[x]^4/4
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\tan^3(x)+\tan^5(x)\right)\,dx &= \int \tan^3(x)\,dx+\int \tan^5(x)\,dx\\ &= \frac{\tan^2(x)}{2}+\frac{\tan^4(x)}{4}-\int \tan(x)\,dx-\int \tan^3(x)\,dx\\ &= \log(\cos(x))+\frac{\tan^4(x)}{4}+\int \tan(x)\,dx\\ &= \frac{\tan^4(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.0048798, size = 8, normalized size = 1.

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3 + Tan[x]^5,x]
```

```
[Out] Tan[x]^4/4
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$\frac{(\tan(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3+tan(x)^5,x)
```

```
[Out] 1/4*tan(x)^4
```

Maxima [B] time = 0.948578, size = 47, normalized size = 5.88

$$\frac{4\sin(x)^2-3}{4\left(\sin(x)^4-2\sin(x)^2+1\right)}-\frac{1}{2\left(\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2/(sin(x)^2 - 1)
```

Fricas [A] time = 1.92514, size = 19, normalized size = 2.38

$$\frac{1}{4}\tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*tan(x)^4
```

Sympy [B] time = 0.125427, size = 22, normalized size = 2.75

$$-\frac{4\cos^2(x)-1}{4\cos^4(x)}+\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3+tan(x)**5,x)
```

```
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) + 1/(2*cos(x)**2)
```

Giac [A] time = 1.07061, size = 8, normalized size = 1.

$$\frac{1}{4}\tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/4*tan(x)^4
```

3.842 $\int x \sec(x)(2 + x \tan(x)) \, dx$

Optimal. Leaf size=6

$$x^2 \sec(x)$$

```
[Out] x^2*Sec[x]
```

Rubi [A] time = 0.178028, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6742, 4181, 2279, 2391, 3757}

$$x^2 \sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]*(2 + x*Tan[x]),x]
```

```
[Out] x^2*Sec[x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int x\sec(x)(2+x\tan(x))\,dx &= \int \left(2x\sec(x)+x^2\sec(x)\tan(x)\right)\,dx\\
&= 2\int x\sec(x)\,dx+\int x^2\sec(x)\tan(x)\,dx\\
&= -4ix\tan^{-1}\left(e^{ix}\right)+x^2\sec(x)-2\int \log\left(1-ie^{ix}\right)\,dx+2\int \log\left(1+ie^{ix}\right)\,dx-2\int x\sec(x)\\
&= x^2\sec(x)+2i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)-2i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{ix}\right)+2\int\\
&= 2i\text{Li}_2\left(-ie^{ix}\right)-2i\text{Li}_2\left(ie^{ix}\right)+x^2\sec(x)-2i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)+2i\,\text{Subst}\left(\int\right.\\
&= x^2\sec(x)
\end{aligned}$$

Mathematica [A] time = 0.024492, size = 6, normalized size = 1.

$$x^2\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]*(2 + x*Tan[x]),x]
```

```
[Out] x^2*Sec[x]
```

Maple [C] time = 0.035, size = 20, normalized size = 3.3

$$2\,\frac{x^2 e^{ix}}{1+e^{2\,ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)*(2+x*tan(x)),x)
```

```
[Out] 2*x^2*exp(I*x)/(1+exp(2*I*x))
```

Maxima [B] time = 1.66423, size = 69, normalized size = 11.5

$$\frac{2\left(x^2\cos\left(2x\right)\cos\left(x\right)+x^2\sin\left(2x\right)\sin\left(x\right)+x^2\cos\left(x\right)\right)}{\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\cos\left(2x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="maxima")
```

```
[Out] 2*(x^2*cos(2*x)*cos(x) + x^2*sin(2*x)*sin(x) + x^2*cos(x))/(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 1.96152, size = 16, normalized size = 2.67

$$\frac{x^2}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="fricas")
```

```
[Out] x^2/cos(x)
```

Sympy [A] time = 0.552568, size = 5, normalized size = 0.83

$$x^2\sec\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x)
```

```
[Out] x**2*sec(x)
```

Giac [B] time = 1.09754, size = 35, normalized size = 5.83

$$-\frac{x^2\tan\left(\frac{1}{2}x\right)^2+x^2}{\tan\left(\frac{1}{2}x\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="giac")
```

```
[Out] -(x^2*tan(1/2*x)^2 + x^2)/(tan(1/2*x)^2 - 1)
```

3.843 $\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\csc\left(\sqrt{x}\right)$$

```
[Out] -2*Csc[Sqrt[x]]
```

Rubi [A] time = 0.196644, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$-2\csc\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csc[Sqrt[x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \cot(x)\csc(x)\,dx, x, \sqrt{x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int 1\,dx, x, \csc\left(\sqrt{x}\right)\right)\right)$$
$$= -2\csc\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0179433, size = 8, normalized size = 1.

$$-2\csc\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csc[Sqrt[x]]
```

Maple [A] time = 0.016, size = 7, normalized size = 0.9

$$-2\csc\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*csc(x^(1/2))
```

Maxima [A] time = 0.952835, size = 11, normalized size = 1.38

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/sin(sqrt(x))
```

Fricas [A] time = 2.0916, size = 23, normalized size = 2.88

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/sin(sqrt(x))
```

Sympy [A] time = 0.327789, size = 8, normalized size = 1.

$$-2\csc\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x**(1/2))*csc(x**(1/2))/x**(1/2),x)
```

```
[Out] -2*csc(sqrt(x))
```

Giac [A] time = 1.06237, size = 11, normalized size = 1.38

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -2/sin(sqrt(x))
```

3.844 $\int \frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$\sin^2\left(\sqrt{x}\right)$$

```
[Out] Sin[Sqrt[x]]^2
```

Rubi [A] time = 0.0110285, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3441}

$$\sin^2\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]

[Out] Sin[Sqrt[x]]^2
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sin^2\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0123849, size = 12, normalized size = 1.5

$$-\frac{1}{2}\cos\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -Cos[2*Sqrt[x]]/2
```

Maple [A] time = 0.007, size = 9, normalized size = 1.1

$$-\left(\cos\left(\sqrt{x}\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x)
```

```
[Out] -cos(x^(1/2))^2
```

Maxima [A] time = 0.957622, size = 11, normalized size = 1.38

$$-\cos\left(\sqrt{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -cos(sqrt(x))^2
```

Fricas [A] time = 1.89804, size = 23, normalized size = 2.88

$$-\cos\left(\sqrt{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -cos(sqrt(x))^2
```

Sympy [A] time = 0.305078, size = 8, normalized size = 1.

$$-\cos^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))*sin(x**(1/2))/x**(1/2),x)
```

```
[Out] -cos(sqrt(x))**2
```

Giac [A] time = 1.06722, size = 11, normalized size = 1.38

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -cos(sqrt(x))^2
```

3.845 $\int \frac{\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\sec\left(\sqrt{x}\right)$$

[Out] 2*Sec[Sqrt[x]]

Rubi [A] time = 0.183876, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$2\sec\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]

[Out] 2*Sec[Sqrt[x]]

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \sec(x)\tan(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int 1\,dx, x, \sec\left(\sqrt{x}\right)\right)$$
$$= 2\sec\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0179848, size = 8, normalized size = 1.

$$2\sec\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*Sec[Sqrt[x]]
```

Maple [A] time = 0.01, size = 7, normalized size = 0.9

$$2\,\sec\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*sec(x^(1/2))
```

Maxima [A] time = 0.963457, size = 11, normalized size = 1.38

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/cos(sqrt(x))
```

Fricas [A] time = 2.00175, size = 22, normalized size = 2.75

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/cos(sqrt(x))
```

Sympy [A] time = 0.347173, size = 7, normalized size = 0.88

$$2\sec\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x**(1/2))*tan(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sec(sqrt(x))
```

Giac [A] time = 1.06791, size = 11, normalized size = 1.38

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/cos(sqrt(x))
```

3.846 $\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(a+b\sin(2x))}{4b}$$

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[a + b*Sin[
2*x]]/(4*b)
```

Rubi [A] time = 0.169904, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1075, 12, 634, 618, 204, 628, 260}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{4b}-\frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[Cos[x]]/(2
*b) - Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+2bx+ax^2)}\,dx,x,\tan(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{2bx}{1+x^2}\,dx,x,\tan(x)\right)}{4b^2}+\frac{\text{Subst}\left(\int -\frac{2abx}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b^2}\\
&= \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,\tan(x)\right)}{2b}-\frac{a\,\text{Subst}\left(\int \frac{x}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{2b}\\
&= -\frac{\log(\cos(x))}{2b}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan(x)\right)-\frac{\text{Subst}\left(\int \frac{2b+2ax}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b}\\
&= -\frac{\log(\cos(x))}{2b}-\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}-\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan(x\right.\\
&= \frac{\tan^{-1}\left(\frac{2b+2a\tan(x)}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(\cos(x))}{2b}-\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0805036, size = 55, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(a+b\sin(2x))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[a + b*Sin[
2*x]]/(4*b)
```

Maple [A] time = 0.072, size = 69, normalized size = 1.3

$$\frac{\ln\left(1+(\tan(x))^2\right)}{4b}-\frac{\ln\left(a+2b\tan(x)+a(\tan(x))^2\right)}{4b}+\frac{1}{2}\arctan\left(\frac{2a\tan(x)+2b}{2}\frac{1}{\sqrt{a^2-b^2}}\right)\frac{1}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(2*x)),x)
```

```
[Out] 1/4/b*ln(1+tan(x)^2)-1/4*ln(a+2*b*tan(x)+a*tan(x)^2)/b+1/2/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47573, size = 767, normalized size = 13.95

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{4\left(2a^2-b^2\right)\cos(x)^4-4ab\cos(x)\sin(x)-4\left(2a^2-b^2\right)\cos(x)^2+a^2-2b^2+2\left(2b\cos(x)^2+2\left(2a\cos(x)^3-a\cos(x)\right)\sin(x)-b\right)\sqrt{-a^2+b^2}}{4b^2\cos(x)^4-4b^2\cos(x)^2-4ab\cos(x)\sin(x)-a^2}\right)}{8\left(a^2b-b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin
(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos
(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*cos
(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b
^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(a^2 -
 b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2)*cos(
x)^2 - a^2 + b^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^2 + 4*a
*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

Sympy [B] time = 9.90773, size = 136, normalized size = 2.47

$$-\frac{\begin{cases}\frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{2b} & \text{for } b\neq 0\\ \frac{\sin(2x)}{2a} & \text{otherwise}\end{cases}}{2}+\frac{\begin{cases}\frac{\log(\tan(x))}{2b} & \text{for } a=0\\ -\frac{1}{b-\sqrt{b^2}\tan(x)} & \text{for } a=-\sqrt{b^2}\\ -\frac{1}{b+\sqrt{b^2}\tan(x)} & \text{for } a=\sqrt{b^2}\\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{2\sqrt{-a^2+b^2}}-\frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{2\sqrt{-a^2+b^2}} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(2*x)),x)
```

```
[Out] -Piecewise((log(a/b + sin(2*x))/(2*b), Ne(b, 0)), (sin(2*x)/(2*a), True))/2
 + Piecewise((log(tan(x))/(2*b), Eq(a, 0)), (-1/(b - sqrt(b**2)*tan(x)), Eq
(a, -sqrt(b**2))), (-1/(b + sqrt(b**2)*tan(x)), Eq(a, sqrt(b**2))), (log(ta
n(x) + b/a - sqrt(-a**2 + b**2)/a)/(2*sqrt(-a**2 + b**2)) - log(tan(x) + b/
a + sqrt(-a**2 + b**2)/a)/(2*sqrt(-a**2 + b**2)), True))/2
```

Giac [A] time = 1.09334, size = 104, normalized size = 1.89

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log\left(a\tan(x)^2+2\,b\tan(x)+a\right)}{4\,b}+\frac{\log\left(\tan(x)^2+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/
sqrt(a^2 - b^2) - 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b + 1/4*log(tan(x)^2
 + 1)/b
```

3.847 $\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{4b}$$

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) + Log[a + b*Sin[
2*x]]/(4*b)
```

Rubi [A] time = 0.132885, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {981, 634, 618, 204, 628, 12, 260}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{4b}+\frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) + Log[Cos[x]]/(2
*b) + Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)
```

Rule 981

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol]
 :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*
d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d
*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c,
d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\,(a+2bx+ax^2)}\,dx,x,\tan(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{2bx}{1+x^2}\,dx,x,\tan(x)\right)}{4b^2}+\frac{\text{Subst}\left(\int \frac{4b^2+2abx}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b^2}\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan(x)\right)+\frac{\text{Subst}\left(\int \frac{2b+2ax}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b}-\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,d\right.}{2b}\\
&= \frac{\log(\cos(x))}{2b}+\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}-\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan(\right.\\
&= \frac{\tan^{-1}\left(\frac{2b+2a\tan(x)}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log(\cos(x))}{2b}+\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.062303, size = 54, normalized size = 0.98

$$
\frac{1}{4}\left(\frac{2\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + Log[a + b*Sin
[2*x]]/b)/4
```

Maple [A] time = 0.061, size = 69, normalized size = 1.3

$$
-\frac{\ln\left(1+(\tan(x))^2\right)}{4b}+\frac{\ln\left(a+2b\tan(x)+a(\tan(x))^2\right)}{4b}+\frac{1}{2}\arctan\left(\frac{2a\tan(x)+2b}{2}\frac{1}{\sqrt{a^2-b^2}}\right)\frac{1}{\sqrt{a^2-b^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(2*x)),x)
```

```
[Out] -1/4/b*ln(1+tan(x)^2)+1/4*ln(a+2*b*tan(x)+a*tan(x)^2)/b+1/2/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4104, size = 767, normalized size = 13.95

$$\left[-\frac{\sqrt{-a^2+b^2}b\log\left(-\frac{4\left(2a^2-b^2\right)\cos(x)^4-4ab\cos(x)\sin(x)-4\left(2a^2-b^2\right)\cos(x)^2+a^2-2b^2+2\left(2b\cos(x)^2+2\left(2a\cos(x)^3-a\cos(x)\right)\sin(x)-b\right)\sqrt{-a^2+b^2}}{4b^2\cos(x)^4-4b^2\cos(x)^2-4ab\cos(x)\sin(x)-a^2}\right)-}{8\left(a^2b-b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin
(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos
(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*cos
(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b
^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(a^2 -
 b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2)*cos(
x)^2 - a^2 + b^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^2 + 4*a
*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

Sympy [A] time = 10.0253, size = 136, normalized size = 2.47

$$\frac{\begin{cases}\frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{2b} & \text{for } b\neq 0\\ \frac{\sin(2x)}{2a} & \text{otherwise}\end{cases}}{2}+\frac{\begin{cases}\frac{\log(\tan(x))}{2b} & \text{for } a=0\\ -\frac{1}{b-\sqrt{b^2}\tan(x)} & \text{for } a=-\sqrt{b^2}\\ -\frac{1}{b+\sqrt{b^2}\tan(x)} & \text{for } a=\sqrt{b^2}\\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{2\sqrt{-a^2+b^2}}-\frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{2\sqrt{-a^2+b^2}} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(2*x)),x)
```

```
[Out] Piecewise((log(a/b + sin(2*x))/(2*b), Ne(b, 0)), (sin(2*x)/(2*a), True))/2
+ Piecewise((log(tan(x))/(2*b), Eq(a, 0)), (-1/(b - sqrt(b**2)*tan(x)), Eq(
a, -sqrt(b**2))), (-1/(b + sqrt(b**2)*tan(x)), Eq(a, sqrt(b**2))), (log(tan
(x) + b/a - sqrt(-a**2 + b**2)/a)/(2*sqrt(-a**2 + b**2)) - log(tan(x) + b/a
 + sqrt(-a**2 + b**2)/a)/(2*sqrt(-a**2 + b**2)), True))/2
```

Giac [A] time = 1.08591, size = 104, normalized size = 1.89

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log\left(a\tan(x)^2+2\,b\tan(x)+a\right)}{4\,b}-\frac{\log\left(\tan(x)^2+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/
sqrt(a^2 - b^2) + 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b - 1/4*log(tan(x)^2
 + 1)/b
```

3.848 $\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}-\frac{x}{2b}$$

```
[Out] -x/(2*b) + (Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*Sqrt[a
 - b]*b)
```

Rubi [A] time = 0.124984, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1130, 205}

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] -x/(2*b) + (Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*Sqrt[a
 - b]*b)
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx = \text{Subst}\left(\int \frac{x^2}{a+b+2ax^2+(a-b)x^4}\,dx, x, \tan(x)\right)$$
$$= -\left(\frac{1}{2}\left(-1+\frac{a}{b}\right)\text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2}\,dx, x, \tan(x)\right)\right) + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \text{ta}\right.}{2b}$$
$$= -\frac{x}{2b} + \frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}b}$$

Mathematica [A] time = 0.0895307, size = 48, normalized size = 0.92

$$-\frac{\frac{(a+b)\tanh^{-1}\left(\frac{(a-b)\tan(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] -(x + ((a + b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
)/(2*b)
```

Maple [A] time = 0.045, size = 80, normalized size = 1.5

$$-\frac{\arctan(\tan(x))}{2b} + \frac{a}{2b}\arctan\left(\tan(x)(a-b)\frac{1}{\sqrt{(a-b)(a+b)}}\right)\frac{1}{\sqrt{(a-b)(a+b)}} + \frac{1}{2}\arctan\left(\tan(x)(a-b)\frac{1}{\sqrt{(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*cos(2*x)),x)
```

```
[Out] -1/2/b*arctan(tan(x))+1/2/b/((a-b)*(a+b))^(1/2)*arctan(tan(x)*(a-b)/((a-b)*
(a+b))^(1/2))*a+1/2/((a-b)*(a+b))^(1/2)*arctan(tan(x)*(a-b)/((a-b)*(a+b))^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30778, size = 535, normalized size = 10.29

$$\left[\frac{\sqrt{-\frac{a+b}{a-b}}\log\left(\frac{4\left(2a^2-b^2\right)\cos(x)^4-4\left(2a^2-ab-b^2\right)\cos(x)^2-4\left(2\left(a^2-ab\right)\cos(x)^3-\left(a^2-2ab+b^2\right)\cos(x)\right)\sqrt{-\frac{a+b}{a-b}}\sin(x)+a^2-2ab+b^2}{4b^2\cos(x)^4+4\left(ab-b^2\right)\cos(x)^2+a^2-2ab+b^2}\right)-4x}{8b}, -\frac{\sqrt{\frac{a+b}{a-b}}\operatorname{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-(a + b)/(a - b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*b
 - b^2)*cos(x)^2 - 4*(2*(a^2 - a*b)*cos(x)^3 - (a^2 - 2*a*b + b^2)*cos(x))*
sqrt(-(a + b)/(a - b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(a*b
 - b^2)*cos(x)^2 + a^2 - 2*a*b + b^2)) - 4*x)/b, -1/4*(sqrt((a + b)/(a - b)
)*arctan(1/2*(2*a*cos(x)^2 - a + b)*sqrt((a + b)/(a - b))/((a + b)*cos(x)*s
in(x))) + 2*x)/b]
```

Sympy [B] time = 63.917, size = 432, normalized size = 8.31

$$\frac{\begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0 \wedge b=0 \\ \frac{\tan(x)}{2b} & \text{for } a=b \\ \frac{1}{2b\tan(x)} & \text{for } a=-b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{2a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{2a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}}{2} - \begin{cases} \tilde{\infty}x \\ \frac{x}{b}-\frac{\tan(x)}{2b} \\ \frac{x}{b}+\frac{1}{2b\tan(x)} \\ \frac{\sin(2x)}{2a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{2ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\cdots}\right.}{2ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2b\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*cos(2*x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0
)), (tan(x)/(2*b), Eq(a, b)), (1/(2*b*tan(x)), Eq(a, -b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x))/(2*a*sqrt(-a/(a - b) - b/(a - b)) - 2*b*sqrt(-
a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(2*a*s
qrt(-a/(a - b) - b/(a - b)) - 2*b*sqrt(-a/(a - b) - b/(a - b))), True))/2 -
 Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b - tan(x)/(2*b), Eq(a, b)), (x
/b + 1/(2*b*tan(x)), Eq(a, -b)), (sin(2*x)/(2*a), Eq(b, 0)), (2*a*x*sqrt(-a
/(a - b) - b/(a - b))/(2*a*b*sqrt(-a/(a - b) - b/(a - b)) - 2*b**2*sqrt(-a/
(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(2*a*
b*sqrt(-a/(a - b) - b/(a - b)) - 2*b**2*sqrt(-a/(a - b) - b/(a - b))) + a*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(2*a*b*sqrt(-a/(a - b) - b/(a - b
)) - 2*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b/(a -
b))/(2*a*b*sqrt(-a/(a - b) - b/(a - b)) - 2*b**2*sqrt(-a/(a - b) - b/(a - b
))), True))/2
```

Giac [A] time = 1.08859, size = 93, normalized size = 1.79

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(x\right)-b\tan\left(x\right)}{\sqrt{a^2-b^2}}\right)\right)(a+b)}{2\,\sqrt{a^2-b^2}b}-\frac{x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*(pi*floor(x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(x) - b*tan(x))/
sqrt(a^2 - b^2)))*(a + b)/(sqrt(a^2 - b^2)*b) - 1/2*x/b
```

3.849 $\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{2b}-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*b*Sqrt[
a + b])
```

Rubi [A] time = 0.0936759, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1093, 205}

$$\frac{x}{2b}-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*b*Sqrt[
a + b])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx = \text{Subst}\left(\int \frac{1}{a+b+2ax^2+(a-b)x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2}\,dx, x, \tan(x)\right)}{2b} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan(x)\right)}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Mathematica [A] time = 0.0557233, size = 50, normalized size = 0.96

$$\frac{\frac{(a-b)\tanh^{-1}\left(\frac{(a-b)\tan(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] (x + ((a - b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

Maple [A] time = 0.03, size = 80, normalized size = 1.5

$$\frac{\arctan(\tan(x))}{2b} - \frac{a}{2b}\arctan\left(\tan(x)(a-b)\frac{1}{\sqrt{(a-b)(a+b)}}\right)\frac{1}{\sqrt{(a-b)(a+b)}} + \frac{1}{2}\arctan\left(\tan(x)(a-b)\frac{1}{\sqrt{(a-b)(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*cos(2*x)),x)
```

```
[Out] 1/2/b*arctan(tan(x))-1/2/b/((a-b)*(a+b))^(1/2)*arctan(tan(x)*(a-b)/((a-b)*(
a+b))^(1/2))*a+1/2/((a-b)*(a+b))^(1/2)*arctan(tan(x)*(a-b)/((a-b)*(a+b))^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22565, size = 525, normalized size = 10.1

$$\left[\frac{\sqrt{-\frac{a-b}{a+b}}\log\left(\frac{4\left(2a^2-b^2\right)\cos(x)^4-4\left(2a^2-ab-b^2\right)\cos(x)^2+4\left(2\left(a^2+ab\right)\cos(x)^3-\left(a^2-b^2\right)\cos(x)\right)\sqrt{-\frac{a-b}{a+b}}\sin(x)+a^2-2ab+b^2}{4b^2\cos(x)^4+4\left(ab-b^2\right)\cos(x)^2+a^2-2ab+b^2}\right)+4x}{8b}, -\frac{\sqrt{\frac{a-b}{a+b}}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-(a - b)/(a + b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*b
 - b^2)*cos(x)^2 + 4*(2*(a^2 + a*b)*cos(x)^3 - (a^2 - b^2)*cos(x))*sqrt(-(a
 - b)/(a + b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(a*b - b^2)*
cos(x)^2 + a^2 - 2*a*b + b^2)) + 4*x)/b, -1/4*(sqrt((a - b)/(a + b))*arctan
(-1/2*(2*a*cos(x)^2 - a + b)*sqrt((a - b)/(a + b))/((a - b)*cos(x)*sin(x)))
 - 2*x)/b]
```

Sympy [B] time = 63.9309, size = 432, normalized size = 8.31

$$\frac{\begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0\wedge b=0 \\ \frac{\tan(x)}{2b} & \text{for } a=b \\ \frac{1}{2b\tan(x)} & \text{for } a=-b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{2a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{2a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}}{2}+\begin{cases} \tilde{\infty}x \\ \frac{x}{b}-\frac{\tan(x)}{2b} \\ \frac{x}{b}+\frac{1}{2b\tan(x)} \\ \frac{\sin(2x)}{2a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{2ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\right.}{2ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-2\right.} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*cos(2*x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0
)), (tan(x)/(2*b), Eq(a, b)), (1/(2*b*tan(x)), Eq(a, -b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x))/(2*a*sqrt(-a/(a - b) - b/(a - b)) - 2*b*sqrt(-
a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(2*a*s
qrt(-a/(a - b) - b/(a - b)) - 2*b*sqrt(-a/(a - b) - b/(a - b))), True))/2 +
 Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b - tan(x)/(2*b), Eq(a, b)), (x
/b + 1/(2*b*tan(x)), Eq(a, -b)), (sin(2*x)/(2*a), Eq(b, 0)), (2*a*x*sqrt(-a
/(a - b) - b/(a - b))/(2*a*b*sqrt(-a/(a - b) - b/(a - b)) - 2*b**2*sqrt(-a/
(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(2*a*
b*sqrt(-a/(a - b) - b/(a - b)) - 2*b**2*sqrt(-a/(a - b) - b/(a - b))) + a*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(2*a*b*sqrt(-a/(a - b) - b/(a - b
)) - 2*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b/(a -
b))/(2*a*b*sqrt(-a/(a - b) - b/(a - b)) - 2*b**2*sqrt(-a/(a - b) - b/(a - b
))), True))/2
```

Giac [A] time = 1.08719, size = 96, normalized size = 1.85

$$\frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}\left(-2\,a + 2\,b\right) + \arctan\left(-\frac{a\tan\left(x\right) - b\tan\left(x\right)}{\sqrt{a^2 - b^2}}\right)\right)(a - b)}{2\,\sqrt{a^2 - b^2}b} + \frac{x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(x) - b*tan(x))/s
qrt(a^2 - b^2)))*(a - b)/(sqrt(a^2 - b^2)*b) + 1/2*x/b
```

3.850 $\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] ArcTanh[Sqrt[a*Sin[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rubi [A] time = 0.0351015, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a*Sin[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\sin^2(c+dx)}\right)}{ad}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0385442, size = 31, normalized size = 1.03

$$\frac{\sin(c+dx)\tanh^{-1}(\sin(c+dx))}{d\sqrt{a\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a*Sin[c + d*x]^2])
```

Maple [A] time = 0.043, size = 30, normalized size = 1.

$$\frac{\sin(dx+c)\,\text{Artanh}\,(\sin(dx+c))}{d}\frac{1}{\sqrt{a(\sin(dx+c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x)
```

```
[Out] 1/(a*sin(d*x+c)^2)^(1/2)*sin(d*x+c)*arctanh(sin(d*x+c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32307, size = 221, normalized size = 7.37

$$\left[\frac{\sqrt{-a\cos\left(dx+c\right)^{2}+a}\log\left(-\frac{\sin(dx+c)+1}{\sin(dx+c)-1}\right)}{2\,ad\sin\left(dx+c\right)},-\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a\cos(dx+c)^{2}+a}\sqrt{-a}}{a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a*cos(d*x + c)^2 + a)*log(-(sin(d*x + c) + 1)/(sin(d*x + c) - 1)
)/(a*d*sin(d*x + c)), -sqrt(-a)*arctan(sqrt(-a*cos(d*x + c)^2 + a)*sqrt(-a)
/a)/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(c+dx\right)}{\sqrt{a\sin^{2}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a*sin(c + d*x)**2), x)
```

Giac [B] time = 1.23317, size = 82, normalized size = 2.73

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/sgn(tan(1/2*d*x + 1/2*c)) - log(abs(tan
(1/2*d*x + 1/2*c) - 1))/sgn(tan(1/2*d*x + 1/2*c)))/(sqrt(a)*d)
```

3.851 $\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcTanh[Sqrt[a*Cos[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Rubi [A] time = 0.0319034, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cos[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(c+dx)\right)}{2d}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(c+dx)}\right)}{ad}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 0.0618504, size = 49, normalized size = 1.58

$$\frac{\cos(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

```
[Out] (Cos[c + d*x]*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))/(d*Sqrt[a*C
os[c + d*x]^2])
```

Maple [A] time = 0.043, size = 31, normalized size = 1.

$$-\frac{\cos(dx+c)\,\text{Artanh}(\cos(dx+c))}{d}\frac{1}{\sqrt{a(\cos(dx+c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/(a*cos(d*x+c)^2)^(1/2)*cos(d*x+c)*arctanh(cos(d*x+c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05116, size = 208, normalized size = 6.71

$$\left[-\frac{\sqrt{a\cos\left(dx+c\right)^{2}}\log\left(-\frac{\cos(dx+c)+1}{\cos(dx+c)-1}\right)}{2\,ad\cos\left(dx+c\right)},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(dx+c)^{2}}\sqrt{-a}}{a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(d*x + c)^2)*log(-(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/(a
*d*cos(d*x + c)), sqrt(-a)*arctan(sqrt(a*cos(d*x + c)^2)*sqrt(-a)/a)/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(c+dx\right)}{\sqrt{a\cos^{2}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a*cos(c + d*x)**2), x)
```

Giac [A] time = 1.13357, size = 42, normalized size = 1.35

$$\frac{\arctan\left(\frac{\sqrt{-a\sin(dx+c)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a*sin(d*x + c)^2 + a)/sqrt(-a))/(sqrt(-a)*d)
```

3.852 $\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx$

Optimal. Leaf size=8

$$\sqrt{\sin(x^2)}$$

```
[Out] Sqrt[Sin[x^2]]
```

Rubi [A] time = 0.0127035, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3441}

$$\sqrt{\sin(x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]

[Out] Sqrt[Sin[x^2]]
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx = \sqrt{\sin(x^2)}$$

Mathematica [A] time = 0.0027532, size = 8, normalized size = 1.

$$\sqrt{\sin(x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]
```

```
[Out] Sqrt[Sin[x^2]]
```

Maple [A] time = 0.007, size = 7, normalized size = 0.9

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)/sin(x^2)^(1/2),x)
```

```
[Out] sin(x^2)^(1/2)
```

Maxima [A] time = 0.955825, size = 8, normalized size = 1.

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(sin(x^2))
```

Fricas [A] time = 1.9853, size = 22, normalized size = 2.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(sin(x^2))
```

Sympy [A] time = 0.309172, size = 7, normalized size = 0.88

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)/sin(x**2)**(1/2),x)
```

```
[Out] sqrt(sin(x**2))
```

Giac [A] time = 1.07353, size = 8, normalized size = 1.

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(sin(x^2))
```

3.853 $\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx$

Optimal. Leaf size=19

$$\frac{\sin(x)\log(\sin(x))}{\sqrt{2}\sqrt{\sin^2(x)}}$$

```
[Out] (Log[Sin[x]]*Sin[x])/(Sqrt[2]*Sqrt[Sin[x]^2])
```

Rubi [A] time = 0.0303875, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4356, 12, 15, 29}

$$\frac{\sin(x)\log(\sin(x))}{\sqrt{2}\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] (Log[Sin[x]]*Sin[x])/(Sqrt[2]*Sqrt[Sin[x]^2])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}\,dx, x, \sin(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x^2}}\,dx, x, \sin(x)\right)}{\sqrt{2}}\\
&= \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{\sqrt{2}\sqrt{\sin^2(x)}}\\
&= \frac{\log(\sin(x))\sin(x)}{\sqrt{2}\sqrt{\sin^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0137804, size = 18, normalized size = 0.95

$$\frac{\sin(x)\log(\sin(x))}{\sqrt{1-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] (Log[Sin[x]]*Sin[x])/Sqrt[1 - Cos[2*x]]
```

Maple [A] time = 1.99, size = 25, normalized size = 1.3

$$\frac{\sin(x)\left(\ln(-1+\cos(x))+\ln(1+\cos(x))\right)\sqrt{2}}{4}\frac{1}{\sqrt{(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1-cos(2*x))^(1/2),x)
```

```
[Out] 1/4*sin(x)*(ln(-1+cos(x))+ln(1+cos(x)))*2^(1/2)/(sin(x)^2)^(1/2)
```

Maxima [B] time = 1.48877, size = 55, normalized size = 2.89

$$\frac{1}{4}\sqrt{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)+\frac{1}{4}\sqrt{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*sqrt(2)*log(cos(x
)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 1.99703, size = 68, normalized size = 3.58

$$\frac{\sqrt{-2\cos\left(x\right)^{2}+2}\log\left(\frac{1}{2}\sin\left(x\right)\right)}{2\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-2*cos(x)^2 + 2)*log(1/2*sin(x))/sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0917, size = 19, normalized size = 1.

$$\frac{\sqrt{2}\log\left(|\sin\left(x\right)|\right)}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(sin(x)))/sgn(sin(x))
```

3.854 $\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx$

Optimal. Leaf size=29

$$\frac{\log(x)}{8}-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))+\frac{1}{8}\sin(\log(x))\cos(\log(x))$$

```
[Out] Log[x]/8 + (Cos[Log[x]]*Sin[Log[x]])/8 - (Cos[Log[x]]^3*Sin[Log[x]])/4
```

Rubi [A] time = 0.0563327, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2568, 2635, 8}

$$\frac{\log(x)}{8}-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))+\frac{1}{8}\sin(\log(x))\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]
```

```
[Out] Log[x]/8 + (Cos[Log[x]]*Sin[Log[x]])/8 - (Cos[Log[x]]^3*Sin[Log[x]])/4
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx &= \text{Subst}\left(\int \cos^2(x)\sin^2(x)\,dx, x, \log(x)\right)\\ &= -\frac{1}{4}\cos^3(\log(x))\sin(\log(x)) + \frac{1}{4}\text{Subst}\left(\int \cos^2(x)\,dx, x, \log(x)\right)\\ &= \frac{1}{8}\cos(\log(x))\sin(\log(x)) - \frac{1}{4}\cos^3(\log(x))\sin(\log(x)) + \frac{1}{8}\text{Subst}\left(\int 1\,dx, x, \log(x)\right)\\ &= \frac{\log(x)}{8} + \frac{1}{8}\cos(\log(x))\sin(\log(x)) - \frac{1}{4}\cos^3(\log(x))\sin(\log(x))\end{aligned}$$

Mathematica [A] time = 0.0164266, size = 16, normalized size = 0.55

$$\frac{\log(x)}{8} - \frac{1}{32}\sin(4\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]
```

```
[Out] Log[x]/8 - Sin[4*Log[x]]/32
```

Maple [A] time = 0.009, size = 24, normalized size = 0.8

$$\frac{\ln(x)}{8} + \frac{\cos(\ln(x))\sin(\ln(x))}{8} - \frac{(\cos(\ln(x)))^3\sin(\ln(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x))^2*sin(ln(x))^2/x,x)
```

```
[Out] 1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))
```

Maxima [A] time = 0.962125, size = 16, normalized size = 0.55

$$\frac{1}{8}\log(x) - \frac{1}{32}\sin(4\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*log(x) - 1/32*sin(4*log(x))
```

Fricas [A] time = 2.1343, size = 85, normalized size = 2.93

$$-\frac{1}{8}\left(2\cos\left(\log\left(x\right)\right)^{3}-\cos\left(\log\left(x\right)\right)\right)\sin\left(\log\left(x\right)\right)+\frac{1}{8}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(log(x))^3 - cos(log(x)))*sin(log(x)) + 1/8*log(x)
```

Sympy [B] time = 66.3406, size = 476, normalized size = 16.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x))**2*sin(ln(x))**2/x,x)
```

```
[Out] log(x)*tan(log(x)/2)**8/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(
log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(log(x)/2)**6/(8*tan(
log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)
**2 + 8) + 6*log(x)*tan(log(x)/2)**4/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)
**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(log(x)/
2)**2/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*
tan(log(x)/2)**2 + 8) + log(x)/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 +
48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 2*tan(log(x)/2)**7/(8*tan(
log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)
**2 + 8) - 14*tan(log(x)/2)**5/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 +
48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 14*tan(log(x)/2)**3/(8*tan
(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2
)**2 + 8) - 2*tan(log(x)/2)/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*
tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8)
```

Giac [A] time = 1.07292, size = 16, normalized size = 0.55

$$\frac{1}{8}\,\log\left(x\right)-\frac{1}{32}\,\sin\left(4\,\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="giac")
```

```
[Out] 1/8*log(x) - 1/32*sin(4*log(x))
```

3.855 $\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{2}+\frac{1}{3}\log(2-\sin(2x))-\frac{1}{6}\log(\sin(x)+\cos(x))$$

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/6 + Log[2 - Sin[2*x]]/3
```

Rubi [A] time = 0.132845, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2074, 635, 203, 260, 628}

$$\frac{x}{2}+\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)-\frac{1}{6}\log(\tan(x)+1)+\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 + Log[Cos[x]]/2 - Log[1 + Tan[x]]/6 + Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{x^3}{1+x^2+x^3+x^5}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{6(1+x)}+\frac{1-x}{2\left(1+x^2\right)}+\frac{-1+2x}{3\left(1-x+x^2\right)}\right)dx, x, \tan(x)\right)\\
&= -\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan(x)\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \tan(x\right.\\
&= -\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)-\frac{1}{2}\,\text{Su}\\
&= \frac{x}{2}+\frac{1}{2}\log(\cos(x))-\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.102501, size = 29, normalized size = 1.

$$\frac{x}{2}+\frac{1}{3}\log(2-\sin(2x))-\frac{1}{6}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/6 + Log[2 - Sin[2*x]]/3
```

Maple [A] time = 0.058, size = 34, normalized size = 1.2

$$\frac{\ln\left(1-\tan(x)+(\tan(x))^2\right)}{3}-\frac{\ln(1+\tan(x))}{6}-\frac{\ln\left(1+(\tan(x))^2\right)}{4}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x)^3+sin(x)^3),x)
```

```
[Out] 1/3*ln(1-tan(x)+tan(x)^2)-1/6*ln(1+tan(x))-1/4*ln(1+tan(x)^2)+1/2*x
```

Maxima [B] time = 1.47655, size = 139, normalized size = 4.79

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1}+\frac{2\sin(x)^2}{(\cos(x)+1)^2}+\frac{2\sin(x)^3}{(\cos(x)+1)^3}+\frac{\sin(x)^4}{(\cos(x)+1)^4}+1\right)-\frac{1}{6}\log\left(-\frac{2\sin(x}{\cos(x)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) + 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(
cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) -
1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) - 1/2*log(sin
(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 2.17078, size = 93, normalized size = 3.21

$$\frac{1}{2}x-\frac{1}{12}\log(2\cos(x)\sin(x)+1)+\frac{1}{3}\log(-\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/12*log(2*cos(x)*sin(x) + 1) + 1/3*log(-cos(x)*sin(x) + 1)
```

Sympy [A] time = 0.405274, size = 32, normalized size = 1.1

$$\frac{x}{2}-\frac{\log(\sin(x)+\cos(x))}{6}+\frac{\log\left(\sin^2(x)-\sin(x)\cos(x)+\cos^2(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(cos(x)**3+sin(x)**3),x)
```

```
[Out] x/2 - log(sin(x) + cos(x))/6 + log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3
```

Giac [A] time = 1.09594, size = 46, normalized size = 1.59

$$\frac{1}{2}x + \frac{1}{3}\log\left(\tan(x)^2 - \tan(x) + 1\right) - \frac{1}{4}\log\left(\tan(x)^2 + 1\right) - \frac{1}{6}\log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(ab
s(tan(x) + 1))
```

3.856 $\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)} dx$

Optimal. Leaf size=29

$$\frac{x}{2} - \frac{1}{3}\log(2 - \sin(2x)) + \frac{1}{6}\log(\sin(x) + \cos(x))$$

```
[Out] x/2 + Log[Cos[x] + Sin[x]]/6 - Log[2 - Sin[2*x]]/3
```

Rubi [A] time = 0.0906246, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2058, 635, 203, 260, 628}

$$\frac{x}{2} - \frac{1}{3}\log\left(\tan^2(x) - \tan(x) + 1\right) + \frac{1}{6}\log(\tan(x) + 1) - \frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2+x^3+x^5}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx, x, \tan(x)\right)\\
&= \frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\text{Subst}\left(\int \frac{1-2x}{1-x+x^2}\,dx, x, \tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)+\frac{1}{2}\text{Sul}\\
&= \frac{x}{2}-\frac{1}{2}\log(\cos(x))+\frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0798164, size = 29, normalized size = 1.

$$\frac{x}{2}-\frac{1}{3}\log(2-\sin(2x))+\frac{1}{6}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 + Log[Cos[x] + Sin[x]]/6 - Log[2 - Sin[2*x]]/3
```

Maple [A] time = 0.061, size = 34, normalized size = 1.2

$$-\frac{\ln\left(1-\tan(x)+(\tan(x))^2\right)}{3}+\frac{\ln(1+\tan(x))}{6}+\frac{\ln\left(1+(\tan(x))^2\right)}{4}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(cos(x)^3+sin(x)^3),x)
```

```
[Out] -1/3*ln(1-tan(x)+tan(x)^2)+1/6*ln(1+tan(x))+1/4*ln(1+tan(x)^2)+1/2*x
```

Maxima [B] time = 1.46563, size = 139, normalized size = 4.79

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right) + \frac{1}{6}\log\left(-\frac{2\sin(x}{\cos(x)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(
cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) +
1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) + 1/2*log(sin
(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 2.16915, size = 93, normalized size = 3.21

$$\frac{1}{2}x + \frac{1}{12}\log\left(2\cos(x)\sin(x)+1\right) - \frac{1}{3}\log\left(-\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*x + 1/12*log(2*cos(x)*sin(x) + 1) - 1/3*log(-cos(x)*sin(x) + 1)
```

Sympy [A] time = 0.42424, size = 32, normalized size = 1.1

$$\frac{x}{2} + \frac{\log(\sin(x)+\cos(x))}{6} - \frac{\log\left(\sin^2(x) - \sin(x)\cos(x) + \cos^2(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(cos(x)**3+sin(x)**3),x)
```

```
[Out] x/2 + log(sin(x) + cos(x))/6 - log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3
```

Giac [A] time = 1.12169, size = 46, normalized size = 1.59

$$\frac{1}{2}x - \frac{1}{3}\log\left(\tan(x)^2 - \tan(x) + 1\right) + \frac{1}{4}\log\left(\tan(x)^2 + 1\right) + \frac{1}{6}\log\left(|\tan(x) + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ab
s(tan(x) + 1))
```

3.857 $\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx$

Optimal. Leaf size=44

$$\frac{1}{3(2-\sin(x))}+\frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(\sin(x)+1)$$

```
[Out] Log[1 - Sin[x]]/2 - (4*Log[2 - Sin[x]])/9 - Log[1 + Sin[x]]/18 + 1/(3*(2 - Sin[x]))
```

Rubi [A] time = 0.0556506, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {710, 801}

$$\frac{1}{3(2-\sin(x))}+\frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
```

```
[Out] Log[1 - Sin[x]]/2 - (4*Log[2 - Sin[x]])/9 - Log[1 + Sin[x]]/18 + 1/(3*(2 - Sin[x]))
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = \text{Subst}\left(\int \frac{1}{(2-x)^2\left(-1+x^2\right)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3(2-\sin(x))} + \frac{1}{3}\text{Subst}\left(\int \frac{2+x}{(2-x)\left(-1+x^2\right)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3(2-\sin(x))} + \frac{1}{3}\text{Subst}\left(\int \left(-\frac{4}{3(-2+x)} + \frac{3}{2(-1+x)} - \frac{1}{6(1+x)}\right)dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\log(1-\sin(x)) - \frac{4}{9}\log(2-\sin(x)) - \frac{1}{18}\log(1+\sin(x)) + \frac{1}{3(2-\sin(x))}$$

Mathematica [A] time = 0.0769394, size = 38, normalized size = 0.86

$$\frac{1}{18}\left(-\frac{6}{\sin(x)-2} + 9\log(1-\sin(x)) - 8\log(2-\sin(x)) - \log(\sin(x)+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
```

```
[Out] (9*Log[1 - Sin[x]] - 8*Log[2 - Sin[x]] - Log[1 + Sin[x]] - 6/(-2 + Sin[x]))
/18
```

Maple [A] time = 0.071, size = 31, normalized size = 0.7

$$-\frac{1}{3\sin(x)-6} - \frac{4\ln(\sin(x)-2)}{9} - \frac{\ln(1+\sin(x))}{18} + \frac{\ln(\sin(x)-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(-5+cos(x)^2+4*sin(x)),x)
```

```
[Out] -1/3/(sin(x)-2)-4/9*ln(sin(x)-2)-1/18*ln(1+sin(x))+1/2*ln(sin(x)-1)
```

Maxima [A] time = 0.962276, size = 41, normalized size = 0.93

$$-\frac{1}{3(\sin(x)-2)} - \frac{1}{18}\log(\sin(x)+1) + \frac{1}{2}\log(\sin(x)-1) - \frac{4}{9}\log(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - 4/9*log(si
n(x) - 2)
```

Fricas [A] time = 1.95965, size = 171, normalized size = 3.89

$$-\frac{(\sin(x)-2)\log(\sin(x)+1)+8(\sin(x)-2)\log\left(-\frac{1}{2}\sin(x)+1\right)-9(\sin(x)-2)\log(-\sin(x)+1)+6}{18(\sin(x)-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/18*((sin(x) - 2)*log(sin(x) + 1) + 8*(sin(x) - 2)*log(-1/2*sin(x) + 1) -
 9*(sin(x) - 2)*log(-sin(x) + 1) + 6)/(sin(x) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)}{4\sin(x)+\cos^2(x)-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)**2+4*sin(x)),x)
```

```
[Out] Integral(sec(x)/(4*sin(x) + cos(x)**2 - 5), x)
```

Giac [A] time = 1.06596, size = 46, normalized size = 1.05

$$-\frac{1}{3(\sin(x)-2)}-\frac{1}{18}\log(\sin(x)+1)-\frac{4}{9}\log(-\sin(x)+2)+\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="giac")

[Out] -1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) - 4/9*log(-sin(x) + 2) + 1/2*log(-
sin(x) + 1)
```

3.858 $\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{\sin(x)+3\cos(x)}}{\sqrt{\cos(x)}}$$

```
[Out] (2*Sqrt[3*Cos[x] + Sin[x]])/Sqrt[Cos[x]]
```

Rubi [B] time = 2.24263, antiderivative size = 88, normalized size of antiderivative = 4.63, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6719, 1063, 8}

$$\frac{2\cos^2\left(\frac{x}{2}\right)\left(-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
```

```
[Out] (2*Cos[x/2]^2*(3 + 2*Tan[x/2] - 3*Tan[x/2]^2))/(Sqrt[Cos[x/2]^2*(3 + 2*Tan[
x/2] - 3*Tan[x/2]^2)]*Sqrt[Cos[x/2]^2*(1 - Tan[x/2]^2)])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1063

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2
)^(q + 1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*
d - a*f)))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1)
 + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
```

```
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
 + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4))))*x
 - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]
) &&  !IGtQ[q, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{\frac{3+2x-3x^2}{1+x^2}}\sqrt{\frac{1-x^2}{1+x^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{\left(2\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{3+2x-3x^2}(1-x^2)\sqrt{\frac{1-x^2}{1+x^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{\sqrt{\sec^2\left(\frac{x}{2}\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}}$$

$$= \frac{\left(2\cos^2\left(\frac{x}{2}\right)\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\right)\mathrm{Subst}\left(\int \frac{1+x^2}{\sqrt{3+2x-3x^2}(1-x^2)^{3/2}}\,d\right.}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}}$$

$$= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\sqrt{3+2\right.}{4\sqrt{\cos^2\left(\right.}}$$

$$= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}}$$

Mathematica [A] time = 0.0668912, size = 19, normalized size = 1.

$$\frac{2\sqrt{\sin(x)+3\cos(x)}}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
```

```
[Out] (2*Sqrt[3*Cos[x] + Sin[x]])/Sqrt[Cos[x]]
```

Maple [A] time = 0.26, size = 16, normalized size = 0.8

$$2\,\frac{\sqrt{3\,\cos\left(x\right)+\sin\left(x\right)}}{\sqrt{\cos\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x)
```

```
[Out] 2*(3*cos(x)+sin(x))^(1/2)/cos(x)^(1/2)
```

Maxima [B] time = 1.63236, size = 196, normalized size = 10.32

$$\frac{2\left(\frac{2\,\sin(x)}{\cos(x)+1}-\frac{6\,\sin(x)^2}{(\cos(x)+1)^2}-\frac{2\,\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\,\sin(x)^4}{(\cos(x)+1)^4}+3\right)\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^2}{\sqrt{\frac{2\,\sin(x)}{\cos(x)+1}-\frac{3\,\sin(x)^2}{(\cos(x)+1)^2}+3}\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{2\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^4}{(\cos(x)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(2*sin(x)/(cos(x) + 1) - 6*sin(x)^2/(cos(x) + 1)^2 - 2*sin(x)^3/(cos(x) +
 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 3)*(sin(x)^2/(cos(x) + 1)^2 + 1)^2/(sqr
t(2*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + 3)*(sin(x)/(cos(x) +
1) + 1)^(3/2)*(-sin(x)/(cos(x) + 1) + 1)^(3/2)*(2*sin(x)^2/(cos(x) + 1)^2 +
 sin(x)^4/(cos(x) + 1)^4 + 1))
```

Fricas [A] time = 2.19315, size = 54, normalized size = 2.84

$$\frac{2\,\sqrt{3\,\cos\left(x\right)+\sin\left(x\right)}}{\sqrt{\cos\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3*cos(x) + sin(x))/sqrt(cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(x) + 3\cos(x)}\cos^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**(3/2)/(3*cos(x)+sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sin(x) + 3*cos(x))*cos(x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos(x) + \sin(x)}\cos(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*cos(x) + sin(x))*cos(x)^(3/2)), x)
```

3.859 $\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$

Optimal. Leaf size=44

$$-\log(\sin(x)) + \frac{2\sqrt{\sin(x)+\cos(x)}}{\sqrt{\cos(x)}} + 2\log\left(\sqrt{\sin(x)+\cos(x)} - \sqrt{\cos(x)}\right)$$

```
[Out] -Log[Sin[x]] + 2*Log[-Sqrt[Cos[x]] + Sqrt[Cos[x] + Sin[x]]] + (2*Sqrt[Cos[x
] + Sin[x]])/Sqrt[Cos[x]]
```

Rubi [F] time = 2.56892, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

```
[Out] Defer[Int][(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2), x]
```

Rubi steps

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx = \int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$$

Mathematica [A] time = 0.39238, size = 68, normalized size = 1.55

$$\frac{2\left(\sin(x)+\cos(x)-\sqrt{\cos(x)}\sqrt{\sqrt{\sin^2(x)}+\cos(x)}\coth^{-1}\left(\frac{\sqrt{\sqrt{\sin^2(x)}+\cos(x)}}{\sqrt{\cos(x)}}\right)\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)+\cos(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

```
[Out] (2*(Cos[x] + Sin[x] - ArcCoth[Sqrt[Cos[x] + Sqrt[Sin[x]^2]]/Sqrt[Cos[x]]]*S
qrt[Cos[x]]*Sqrt[Cos[x] + Sqrt[Sin[x]^2]]))/(Sqrt[Cos[x]]*Sqrt[Cos[x] + Sin
[x]])
```

Maple [C] time = 0.435, size = 917, normalized size = 20.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x)
```

```
[Out] 1/(2+2^(1/2))*(-1+cos(x))^2*(1+cos(x))^2*(EllipticPi(1/2*2^(1/2)*((2+2^(1/2
))*2^(1/2)*(sin(x)-1)/cos(x))^(1/2),-2^(1/2)/(2+2^(1/2)),I/(2+2^(1/2))*((2-
2^(1/2))*(2+2^(1/2)))^(1/2))*(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(
x)+2^(1/2)-2)/cos(x))^(1/2)*(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x
)+2^(1/2)+2)/cos(x))^(1/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/cos(x))^(1/2)*si
n(x)-EllipticF(1/2*2^(1/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/cos(x))^(1/2),I/
(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*(2^(1/2)*(cos(x)*2^(1/2)-sin(x
)*2^(1/2)+2*sin(x)+2^(1/2)-2)/cos(x))^(1/2)*(2^(1/2)*(cos(x)*2^(1/2)-sin(x)
*2^(1/2)-2*sin(x)+2^(1/2)+2)/cos(x))^(1/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/
cos(x))^(1/2)*sin(x)+EllipticPi(1/2*2^(1/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)
/cos(x))^(1/2),2^(1/2)/(2+2^(1/2)),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^
(1/2))*(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(1/2)-2)/cos(x))^
(1/2)*(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1/2)+2)/cos(x))^(
1/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/cos(x))^(1/2)*sin(x)+(2^(1/2)*(cos(x)*
2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(1/2)-2)/cos(x))^(1/2)*(2^(1/2)*(cos(x)*2
^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1/2)+2)/cos(x))^(1/2)*EllipticPi(1/2*2^(1
/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/cos(x))^(1/2),-2^(1/2)/(2+2^(1/2)),I/(2
+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/
cos(x))^(1/2)-(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(1/2)-2)/c
os(x))^(1/2)*(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1/2)+2)/co
s(x))^(1/2)*EllipticF(1/2*2^(1/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/cos(x))^(
1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*((2+2^(1/2))*2^(1/2)*(s
in(x)-1)/cos(x))^(1/2)+(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(
1/2)-2)/cos(x))^(1/2)*(2^(1/2)*(cos(x)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1
/2)+2)/cos(x))^(1/2)*EllipticPi(1/2*2^(1/2)*((2+2^(1/2))*2^(1/2)*(sin(x)-1)
/cos(x))^(1/2),2^(1/2)/(2+2^(1/2)),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^
(1/2))*((2+2^(1/2))*2^(1/2)*(sin(x)-1)/cos(x))^(1/2)+2*cos(x)*2^(1/2)+2*sin
```

```
(x)*2^(1/2)+4*cos(x)+4*sin(x))/cos(x)^(1/2)/sin(x)^4/(cos(x)+sin(x))^(1/2)
```

Maxima [B] time = 2.44539, size = 699, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*((2*cos(2*x) + sin(2*x))*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x
) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^3 + (2*cos(2*x) + sin(2*x))*c
os(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x)
 + sin(4*x) + 1))*sin(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 - (cos(2*x) - 2*sin(2*x) + 1)*sin(1/
2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + si
n(4*x) + 1))^3 - (cos(2*x) - sin(2*x) - 1)*cos(1/2*arctan2(-cos(4*x) + sin(
4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1)) - ((cos(2*x)
- 2*sin(2*x) + 1)*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + cos(2*x) + sin(2*x) - 1)*sin(1/2*a
rctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4
*x) + 1)))/((4*(cos(2*x) - sin(2*x))*cos(4*x) + 2*cos(4*x)^2 + 4*cos(2*x)^2
 + 4*(cos(2*x) + sin(2*x) + 1)*sin(4*x) + 2*sin(4*x)^2 + 4*sin(2*x)^2 + 4*c
os(2*x) + 4*sin(2*x) + 2)^(1/4)*(cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*s
in(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + sin(1/2*arctan2(-co
s(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^
2))
```

Fricas [B] time = 2.15771, size = 363, normalized size = 8.25

$$-\frac{\cos\left(x\right)\log\left(\left(2\,\cos\left(x\right)+\sin\left(x\right)\right)\sqrt{\cos\left(x\right)+\sin\left(x\right)}\sqrt{\cos\left(x\right)}+\frac{7}{4}\,\cos\left(x\right)^{2}+2\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\right)-\cos\left(x\right)\log\left(-(2\,\cos\right.}{4\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)*log((2*cos(x) + sin(x))*sqrt(cos(x) + sin(x))*sqrt(cos(x)) + 7
/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) - cos(x)*log(-(2*cos(x) + sin(x))*sqrt
```

```
(cos(x) + sin(x))*sqrt(cos(x)) + 7/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) - 8*
sqrt(cos(x) + sin(x))*sqrt(cos(x)))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(x) + \cos(x)}\csc(x)}{\cos^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))**(1/2)/cos(x)**(3/2),x)
```

```
[Out] Integral(sqrt(sin(x) + cos(x))*csc(x)/cos(x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(x) + \sin(x)}\csc(x)}{\cos(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(x) + sin(x))*csc(x)/cos(x)^(3/2), x)
```

3.860 $\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$

Optimal. Leaf size=19

$$\frac{x\sqrt{\sin(2x)+1}}{\sin(x)+\cos(x)}$$

```
[Out] (x*Sqrt[1 + Sin[2*x]])/(Cos[x] + Sin[x])
```

Rubi [B] time = 1.70716, antiderivative size = 72, normalized size of antiderivative = 3.79, number of steps used = 17, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4401, 6719, 1075, 628, 635, 203, 260, 12, 1023}

$$\frac{2\cos^2\left(\frac{x}{2}\right)\tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
```

```
[Out] (2*ArcTan[Tan[x/2]]*Cos[x/2]^2*(1 + 2*Tan[x/2] - Tan[x/2]^2))/Sqrt[Cos[x/2]
^4*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
```

```
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx = \int \left(\frac{\cos(x)}{\sqrt{1+\sin(2x)}} + \frac{\sin(x)}{\sqrt{1+\sin(2x)}}\right) dx$$

$$= \int \frac{\cos(x)}{\sqrt{1+\sin(2x)}}\,dx + \int \frac{\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{2x}{\left(1+x^2\right)^2\sqrt{\frac{\left(-1-2x+x^2\right)^2}{\left(1+x^2\right)^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2\sqrt{\frac{\left(-1-2x+x^2\right)^2}{\left(1+x^2\right)^2}}}\,dx, x$$

$$= 4\,\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^2\sqrt{\frac{\left(-1-2x+x^2\right)^2}{\left(1+x^2\right)^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + \frac{\left(2\cos^2\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)\right)\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-1-2\,\text{t}\right.}}$$

$$= \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)\right)\right)\text{Subst}\left(\int \frac{-4+4x}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{4\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-1-2\tan\right.\right.}{4\sqrt{\text{co}}}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right)\log\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{2\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right.\right.\right.}{2\sqrt{\cos^4\left(\frac{x}{2}\right.}}$$

$$= \frac{x\cos^2\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{2\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\cos^2\left(\frac{x}{2}\right)\log\left(\cos\left(\frac{x}{2}\right)\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}} -$$

$$= \frac{x\cos^2\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}}$$

Mathematica [A] time = 0.0131581, size = 17, normalized size = 0.89

$$\frac{x(\sin(x)+\cos(x))}{\sqrt{\sin(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
```

```
[Out] (x*(Cos[x] + Sin[x]))/Sqrt[1 + Sin[2*x]]
```

Maple [C] time = 0.287, size = 12372, normalized size = 651.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.66231, size = 444, normalized size = 23.37

$$\frac{1}{16}\sqrt{2}\left(2\sqrt{2}\arctan\left(\sin\left(2x\right)+1,\cos\left(2x\right)\right)+\sqrt{2}\log\left(\cos\left(2x\right)^{2}+\sin\left(2x\right)^{2}+2\sin\left(2x\right)+1\right)+4\left(\cos\left(4x\right)^{2}+4\cos\left(2x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) + sqrt(2)*log(cos(2
*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) + 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos
(2*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(co
s(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))*sin(2*x) + cos
(2*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))))) + 1/
16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) - sqrt(2)*log(cos(2*x
)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) - 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos(2
*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(cos(
2*x)*cos(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))) - sin(2
*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))))
```

Fricas [A] time = 1.96431, size = 5, normalized size = 0.26

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x) + \cos(x)}{\sqrt{\sin(2x) + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))**(1/2),x)
```

```
[Out] Integral((sin(x) + cos(x))/sqrt(sin(2*x) + 1), x)
```

Giac [B] time = 1.11637, size = 57, normalized size = 3.

$$\frac{2\pi\left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor - x}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^3 - 2\tan\left(\frac{1}{2}x\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] (2*pi*floor(1/2*x/pi + 1/2) - x)/sgn(tan(1/2*x)^4 - 2*tan(1/2*x)^3 - 2*tan(
1/2*x) - 1)
```

3.861 $\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=13

$$2\sqrt{(\sin(x)+1)\sec(x)}$$

```
[Out] 2*Sqrt[Sec[x]*(1 + Sin[x])]
```

Rubi [A] time = 0.145602, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 4400, 2705, 2671}

$$2\sqrt{(\sin(x)+1)\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
```

```
[Out] 2*Sqrt[Sec[x]*(1 + Sin[x])]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2705

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]
*(g*Sec[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Cos[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx &= \int \sec(x)\sqrt{\sec(x)(1+\sin(x))}\,dx\\ &= \frac{\sqrt{\sec(x)(1+\sin(x))}\int \sec^{\frac{3}{2}}(x)\sqrt{1+\sin(x)}\,dx}{\sqrt{\sec(x)}\sqrt{1+\sin(x)}}\\ &= \frac{\left(\sqrt{\cos(x)}\sqrt{\sec(x)(1+\sin(x))}\right)\int \frac{\sqrt{1+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{1+\sin(x)}}\\ &= 2\sqrt{\sec(x)(1+\sin(x))}\end{aligned}$$

Mathematica [B] time = 0.0446157, size = 37, normalized size = 2.85

$$2\sqrt{\frac{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
```

```
[Out] 2*Sqrt[(Cos[x/2] + Sin[x/2])/(Cos[x/2] - Sin[x/2])]
```

Maple [A] time = 0.058, size = 10, normalized size = 0.8

$$2\sqrt{\sec(x)+\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(sec(x)+tan(x))^(1/2),x)
```

```
[Out] 2*(sec(x)+tan(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(x) + \tan(x)}\sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(x) + tan(x))*sec(x), x)
```

Fricas [A] time = 2.03483, size = 72, normalized size = 5.54

$$2\sqrt{\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tan(x) + \sec(x)}\sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x) + sec(x))*sec(x), x)
```

Giac [B] time = 1.29174, size = 74, normalized size = 5.69

$$-\frac{4 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^{3}-\tan\left(\frac{1}{2}x\right)^{2}-\tan\left(\frac{1}{2}x\right)-1\right)\operatorname{sgn}\left(\cos\left(x\right)\right)}{\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^{2}+1}-1}{\tan\left(\frac{1}{2}x\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="giac")
```

```
[Out] -4*sgn(-tan(1/2*x)^3 - tan(1/2*x)^2 - tan(1/2*x) - 1)*sgn(cos(x))/((sqrt(-t
an(1/2*x)^2 + 1) - 1)/tan(1/2*x) + 1)
```

3.862 $\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

```
[Out] (2*(4 + 3*Sec[x])^(3/2))/9
```

Rubi [A] time = 0.0438361, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4339, 261}

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]
```

```
[Out] (2*(4 + 3*Sec[x])^(3/2))/9
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{4+\frac{3}{x}}}{x^2}\,dx, x, \cos(x)\right)$$
$$= \frac{2}{9}(4+3\sec(x))^{3/2}$$

Mathematica [A] time = 0.0540786, size = 14, normalized size = 1.

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]
```

```
[Out] (2*(4 + 3*Sec[x])^(3/2))/9
```

Maple [A] time = 0.018, size = 11, normalized size = 0.8

$$\frac{2}{9}(4+3\sec(x))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x)
```

```
[Out] 2/9*(4+3*sec(x))^(3/2)
```

Maxima [A] time = 0.95489, size = 14, normalized size = 1.

$$\frac{2}{9}(3\sec(x)+4)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*sec(x) + 4)^(3/2)
```

Fricas [B] time = 2.04178, size = 74, normalized size = 5.29

$$\frac{2\sqrt{\frac{4\cos(x)+3}{\cos(x)}}(4\cos(x)+3)}{9\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt((4*cos(x) + 3)/cos(x))*(4*cos(x) + 3)/cos(x)
```

Sympy [B] time = 0.910715, size = 29, normalized size = 2.07

$$\frac{2\sqrt{3\sec(x)+4}\sec(x)}{3}+\frac{8\sqrt{3\sec(x)+4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))**(1/2)*tan(x),x)
```

```
[Out] 2*sqrt(3*sec(x) + 4)*sec(x)/3 + 8*sqrt(3*sec(x) + 4)/9
```

Giac [B] time = 1.09666, size = 92, normalized size = 6.57

$$\frac{2\left(4\left(\sqrt{4\cos(x)^2+3\cos(x)}-2\cos(x)\right)^2-6\sqrt{4\cos(x)^2+3\cos(x)}+12\cos(x)+3\right)\mathrm{sgn}(\cos(x))}{\left(\sqrt{4\cos(x)^2+3\cos(x)}-2\cos(x)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] 2*(4*(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^2 - 6*sqrt(4*cos(x)^2 + 3*cos
(x)) + 12*cos(x) + 3)*sgn(cos(x))/(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^
3
```

3.863 $\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx$

Optimal. Leaf size=25

$$\frac{2}{7}(\sec(x)+1)^{7/2}-\frac{4}{5}(\sec(x)+1)^{5/2}$$

```
[Out] (-4*(1 + Sec[x])^(5/2))/5 + (2*(1 + Sec[x])^(7/2))/7
```

Rubi [A] time = 0.0853067, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4373, 1570, 1469, 627, 43}

$$\frac{2}{7}(\sec(x)+1)^{7/2}-\frac{4}{5}(\sec(x)+1)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
```

```
[Out] (-4*(1 + Sec[x])^(5/2))/5 + (2*(1 + Sec[x])^(7/2))/7
```

Rule 4373

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\left(1-x^2\right)}{x^4}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \cos(x)\right)\\
&= \text{Subst}\left(\int \sqrt{1+x}\left(-1+x^2\right)\,dx, x, \sec(x)\right)\\
&= \text{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \sec(x)\right)\\
&= \text{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx, x, \sec(x)\right)\\
&= -\frac{4}{5}(1+\sec(x))^{5/2}+\frac{2}{7}(1+\sec(x))^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.189612, size = 30, normalized size = 1.2

$$-\frac{8}{35}\cos^4\left(\frac{x}{2}\right)(9\cos(x)-5)\sec^3(x)\sqrt{\sec(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
```

```
[Out] (-8*Cos[x/2]^4*(-5 + 9*Cos[x])*Sec[x]^3*Sqrt[1 + Sec[x]])/35
```

Maple [A] time = 0.067, size = 34, normalized size = 1.4

$$-\frac{(18\cos(x)-10)(\sin(x))^4}{35(-1+\cos(x))^2(\cos(x))^3}\sqrt{\frac{1+\cos(x)}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x)
```

```
[Out] -2/35*(9*cos(x)-5)*((1+cos(x))/cos(x))^(1/2)*sin(x)^4/(-1+cos(x))^2/cos(x)^
3
```

Maxima [A] time = 0.957432, size = 28, normalized size = 1.12

$$\frac{2}{7}\left(\frac{1}{\cos(x)}+1\right)^{\frac{7}{2}}-\frac{4}{5}\left(\frac{1}{\cos(x)}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] 2/7*(1/cos(x) + 1)^(7/2) - 4/5*(1/cos(x) + 1)^(5/2)
```

Fricas [B] time = 2.03862, size = 111, normalized size = 4.44

$$-\frac{2\left(9\cos(x)^3+13\cos(x)^2-\cos(x)-5\right)\sqrt{\frac{\cos(x)+1}{\cos(x)}}}{35\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -2/35*(9*cos(x)^3 + 13*cos(x)^2 - cos(x) - 5)*sqrt((cos(x) + 1)/cos(x))/cos
(x)^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(x)+1}\tan^3(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(sec(x) + 1)*tan(x)**3*sec(x), x)
```

Giac [B] time = 1.092, size = 173, normalized size = 6.92

$$-\frac{2\left(35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^6-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^5-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^4+1\cdots\right.}{35\left(\sqrt{\cos(x\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^6 - 35*(sqrt(cos(x)^2 + cos(x)
) - cos(x))^5 - 35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^4 + 105*(sqrt(cos(x)^
2 + cos(x)) - cos(x))^3 - 91*(sqrt(cos(x)^2 + cos(x)) - cos(x))^2 + 35*sqrt
(cos(x)^2 + cos(x)) - 35*cos(x) - 5)*sgn(cos(x))/(sqrt(cos(x)^2 + cos(x)) -
 cos(x))^7
```

3.864 $\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx$

Optimal. Leaf size=25

$$\frac{4}{5}(\csc(x)+1)^{5/2}-\frac{2}{7}(\csc(x)+1)^{7/2}$$

```
[Out] (4*(1 + Csc[x])^(5/2))/5 - (2*(1 + Csc[x])^(7/2))/7
```

Rubi [A] time = 0.0810866, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4372, 1570, 1469, 627, 43}

$$\frac{4}{5}(\csc(x)+1)^{5/2}-\frac{2}{7}(\csc(x)+1)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] (4*(1 + Csc[x])^(5/2))/5 - (2*(1 + Csc[x])^(7/2))/7
```

Rule 4372

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c*d^(n - 1)), Subst[Int[SubstFor[(
1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\left(1-x^2\right)}{x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \sin(x)\right)\\
&= -\text{Subst}\left(\int \sqrt{1+x}\left(-1+x^2\right)\,dx, x, \csc(x)\right)\\
&= -\text{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \csc(x)\right)\\
&= -\text{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx, x, \csc(x)\right)\\
&= \frac{4}{5}(1+\csc(x))^{5/2}-\frac{2}{7}(1+\csc(x))^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.0388715, size = 18, normalized size = 0.72

$$-\frac{2}{35}(\csc(x)+1)^{5/2}(5\csc(x)-9)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] (-2*(1 + Csc[x])^(5/2)*(-9 + 5*Csc[x]))/35
```

Maple [B] time = 0.084, size = 38, normalized size = 1.5

$$-\frac{18\left(\cos\left(x\right)\right)^{2}\sin\left(x\right)+26\left(\cos\left(x\right)\right)^{2}-16\sin\left(x\right)-16}{35\left(\sin\left(x\right)\right)^{3}}\sqrt{\frac{1+\sin\left(x\right)}{\sin\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x)
```

```
[Out] -2/35*((1+sin(x))/sin(x))^(1/2)*(9*cos(x)^2*sin(x)+13*cos(x)^2-8*sin(x)-8)/
sin(x)^3
```

Maxima [A] time = 0.96642, size = 28, normalized size = 1.12

$$-\frac{2}{7}\left(\frac{1}{\sin\left(x\right)}+1\right)^{\frac{7}{2}}+\frac{4}{5}\left(\frac{1}{\sin\left(x\right)}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/7*(1/sin(x) + 1)^(7/2) + 4/5*(1/sin(x) + 1)^(5/2)
```

Fricas [B] time = 2.07529, size = 135, normalized size = 5.4

$$\frac{2\left(13\cos\left(x\right)^{2}+\left(9\cos\left(x\right)^{2}-8\right)\sin\left(x\right)-8\right)\sqrt{\frac{\sin(x)+1}{\sin(x)}}}{35\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(13*cos(x)^2 + (9*cos(x)^2 - 8)*sin(x) - 8)*sqrt((sin(x) + 1)/sin(x))/
((cos(x)^2 - 1)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(x)+1}\cot^3(x)\csc(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*csc(x)*(1+csc(x))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(x) + 1)*cot(x)**3*csc(x), x)
```

Giac [B] time = 1.09556, size = 173, normalized size = 6.92

$$\frac{2\left(35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^6-35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^5-35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^4+105\left(\right.\right.}{35\left(\sqrt{\sin(x)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^6 - 35*(sqrt(sin(x)^2 + sin(x))
 - sin(x))^5 - 35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^4 + 105*(sqrt(sin(x)^2
 + sin(x)) - sin(x))^3 - 91*(sqrt(sin(x)^2 + sin(x)) - sin(x))^2 + 35*sqrt(
sin(x)^2 + sin(x)) - 35*sin(x) - 5)*sgn(sin(x))/(sqrt(sin(x)^2 + sin(x)) -
sin(x))^7
```

3.865 $\int \sqrt{\csc(x)}(x\cos(x) - 4\sec(x)\tan(x))\,dx$

Optimal. Leaf size=20

$$\frac{2x}{\sqrt{\csc(x)}} - \frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

```
[Out] (2*x)/Sqrt[Csc[x]] - (4*Sec[x])/Csc[x]^(3/2)
```

Rubi [A] time = 0.151274, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6742, 4213, 3771, 2639, 2626}

$$\frac{2x}{\sqrt{\csc(x)}} - \frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]
```

```
[Out] (2*x)/Sqrt[Csc[x]] - (4*Sec[x])/Csc[x]^(3/2)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(x)}(x\cos(x)-4\sec(x)\tan(x))\,dx &= \int \left(x\cos(x)\sqrt{\csc(x)}-\frac{4\sec^2(x)}{\sqrt{\csc(x)}}\right)dx\\ &= -\left(4\int \frac{\sec^2(x)}{\sqrt{\csc(x)}}\,dx\right)+\int x\cos(x)\sqrt{\csc(x)}\,dx\\ &= \frac{2x}{\sqrt{\csc(x)}}-\frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] time = 0.447148, size = 17, normalized size = 0.85

$$\frac{2(x\csc(x)-2\sec(x))}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]
```

```
[Out] (2*(x*Csc[x] - 2*Sec[x]))/Csc[x]^(3/2)
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \sqrt{\csc(x)}\,(x\cos(x)-4\sec(x)\tan(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

```
[Out] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x\cos(x) - 4\sec(x)\tan(x))\sqrt{\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="giac")
```

```
[Out] integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)
```

3.866 $\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=76

$$-\frac{35}{16}\sqrt{\cot^2(x)}+\frac{1}{6}\cos^6(x)\sqrt{\cot^2(x)}+\frac{7}{24}\cos^4(x)\sqrt{\cot^2(x)}+\frac{35}{48}\cos^2(x)\sqrt{\cot^2(x)}-\frac{35}{16}x\tan(x)\sqrt{\cot^2(x)}$$

```
[Out] (-35*Sqrt[Cot[x]^2])/16 + (35*Cos[x]^2*Sqrt[Cot[x]^2])/48 + (7*Cos[x]^4*Sqr
t[Cot[x]^2])/24 + (Cos[x]^6*Sqrt[Cot[x]^2])/6 - (35*x*Sqrt[Cot[x]^2]*Tan[x]
)/16
```

Rubi [A] time = 0.162233, antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3175, 4360, 25, 266, 47, 50, 63, 203}

$$-\frac{35}{16}\sqrt{\csc^2(x)-1}+\frac{1}{6}\sin^6(x)\left(\csc^2(x)-1\right)^{7/2}+\frac{7}{24}\sin^4(x)\left(\csc^2(x)-1\right)^{5/2}+\frac{35}{48}\sin^2(x)\left(\csc^2(x)-1\right)^{3/2}+\frac{35}{16}\tan^{-1}\left(\sqrt{cs\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (35*ArcTan[Sqrt[-1 + Csc[x]^2]])/16 - (35*Sqrt[-1 + Csc[x]^2])/16 + (35*(-1
 + Csc[x]^2)^(3/2)*Sin[x]^2)/48 + (7*(-1 + Csc[x]^2)^(5/2)*Sin[x]^4)/24 + (
(-1 + Csc[x]^2)^(7/2)*Sin[x]^6)/6
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4360

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = \int \cos^6(x)\cot(x)\sqrt{-1+\csc^2(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(1-x^2\right)^3}{x}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\,dx, x, \sin(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \csc^2(x)\right)\right)$$
$$= \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) - \frac{7}{12}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \csc^2(x)\right)$$
$$= \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) - \frac{35}{48}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x\right.$$
$$= \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) - \frac{35}{32}\text{Sul}$$
$$= -\frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}$$
$$= -\frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}$$
$$= \frac{35}{16}\tan^{-1}\left(\sqrt{\cot^2(x)}\right) - \frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}$$

Mathematica [A] time = 0.0889674, size = 40, normalized size = 0.53

$$\frac{1}{384}\sqrt{\cot^2(x)}\sec(x)(-840x\sin(x) - 525\cos(x) + 126\cos(3x) + 14\cos(5x) + \cos(7x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (Sqrt[Cot[x]^2]*Sec[x]*(-525*Cos[x] + 126*Cos[3*x] + 14*Cos[5*x] + Cos[7*x]
 - 840*x*Sin[x]))/384
```

Maple [A] time = 0.194, size = 54, normalized size = 0.7

$$-\frac{\sqrt{4}\left(-8\,(\cos(x))^7 - 14\,(\cos(x))^5 - 35\,(\cos(x))^3 + 105\,x\sin(x) + 105\,\cos(x)\right)}{96\,\cos(x)}\sqrt{-\frac{(\cos(x))^2}{-1+(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x)
```

```
[Out] -1/96*4^(1/2)*(-8*cos(x)^7-14*cos(x)^5-35*cos(x)^3+105*x*sin(x)+105*cos(x))
*(-cos(x)^2/(-1+cos(x)^2))^(1/2)/cos(x)
```

Maxima [B] time = 1.47936, size = 184, normalized size = 2.42

$$-\frac{3}{2}\sqrt{\frac{1}{\sin(x)^2}-1}\sin(x)^2-\sqrt{\frac{1}{\sin(x)^2}-1}+\frac{3\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{\sin(x)^2}-1}}{48\left(\left(\frac{1}{\sin(x)^2}-1\right)^3+3\left(\frac{1}{\sin(x)^2}-1\right)^2+\frac{3}{\sin(x)^2}-2\right)}-\frac{3\left(\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}}-\sqrt{\cdots}\right.}{8\left(\left(\frac{1}{\sin(x)^2}-1\right)^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(1/sin(x)^2 - 1)*sin(x)^2 - sqrt(1/sin(x)^2 - 1) + 1/48*(3*(1/sin(
x)^2 - 1)^(5/2) + 8*(1/sin(x)^2 - 1)^(3/2) - 3*sqrt(1/sin(x)^2 - 1))/((1/si
n(x)^2 - 1)^3 + 3*(1/sin(x)^2 - 1)^2 + 3/sin(x)^2 - 2) - 3/8*((1/sin(x)^2 -
 1)^(3/2) - sqrt(1/sin(x)^2 - 1))/((1/sin(x)^2 - 1)^2 + 2/sin(x)^2 - 1) + 3
5/16*arctan(sqrt(1/sin(x)^2 - 1))
```

Fricas [A] time = 2.11007, size = 112, normalized size = 1.47

$$-\frac{8\cos(x)^7+14\cos(x)^5+35\cos(x)^3-105\,x\sin(x)-105\cos(x)}{48\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^7 + 14*cos(x)^5 + 35*cos(x)^3 - 105*x*sin(x) - 105*cos(x))/
sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09798, size = 131, normalized size = 1.72

$$-\frac{1}{48}\left(\left(2\left(4\sin(x)^2-19\right)\sin(x)^2+87\right)\sqrt{-\sin(x)^2+1}\sin(x)-105\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-x\right)(-1)^{\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor}+\frac{24\left(\sqrt{-\sin(x)^2+1}-\right.}{\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*((2*(4*sin(x)^2 - 19)*sin(x)^2 + 87)*sqrt(-sin(x)^2 + 1)*sin(x) - 105
*(pi*floor(x/pi + 1/2) - x)*(-1)^floor(x/pi + 1/2) + 24*(sqrt(-sin(x)^2 + 1
) - 1)/sin(x) - 24*sin(x)/(sqrt(-sin(x)^2 + 1) - 1))*sgn(sin(x))
```

3.867 $\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=81

$$\sin(x)\sqrt{\cot^2(x)}+\frac{1}{7}\sin(x)\cos^6(x)\sqrt{\cot^2(x)}+\frac{1}{5}\sin(x)\cos^4(x)\sqrt{\cot^2(x)}+\frac{1}{3}\sin(x)\cos^2(x)\sqrt{\cot^2(x)}-\tan(x)\sqrt{\cot^2(x}$$

```
[Out] Sqrt[Cot[x]^2]*Sin[x] + (Cos[x]^2*Sqrt[Cot[x]^2]*Sin[x])/3 + (Cos[x]^4*Sqrt
[Cot[x]^2]*Sin[x])/5 + (Cos[x]^6*Sqrt[Cot[x]^2]*Sin[x])/7 - ArcTanh[Cos[x]]
*Sqrt[Cot[x]^2]*Tan[x]
```

Rubi [A] time = 0.158987, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3175, 4121, 3658, 2592, 302, 206}

$$\sin(x)\sqrt{\cot^2(x)}+\frac{1}{7}\sin(x)\cos^6(x)\sqrt{\cot^2(x)}+\frac{1}{5}\sin(x)\cos^4(x)\sqrt{\cot^2(x)}+\frac{1}{3}\sin(x)\cos^2(x)\sqrt{\cot^2(x)}-\tan(x)\sqrt{\cot^2(x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] Sqrt[Cot[x]^2]*Sin[x] + (Cos[x]^2*Sqrt[Cot[x]^2]*Sin[x])/3 + (Cos[x]^4*Sqrt
[Cot[x]^2]*Sin[x])/5 + (Cos[x]^6*Sqrt[Cot[x]^2]*Sin[x])/7 - ArcTanh[Cos[x]]
*Sqrt[Cot[x]^2]*Tan[x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
```

```
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx &= \int \cos^7(x)\sqrt{-1+\csc^2(x)}\,dx \\
&= \int \cos^7(x)\sqrt{\cot^2(x)}\,dx \\
&= \left(\sqrt{\cot^2(x)}\tan(x)\right)\int \cos^7(x)\cot(x)\,dx \\
&= -\left(\left(\sqrt{\cot^2(x)}\tan(x)\right)\text{Subst}\left(\int \frac{x^8}{1-x^2}\,dx, x, \cos(x)\right)\right) \\
&= -\left(\left(\sqrt{\cot^2(x)}\tan(x)\right)\text{Subst}\left(\int \left(-1-x^2-x^4-x^6+\frac{1}{1-x^2}\right)dx, x, \cos(x)\right)\right) \\
&= \sqrt{\cot^2(x)}\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{\cot^2(x)}\sin(x)+ \\
&= \sqrt{\cot^2(x)}\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{\cot^2(x)}\sin(x)+
\end{aligned}
$$

Mathematica [A] time = 0.0635501, size = 55, normalized size = 0.68

$$\frac{\tan(x)\sqrt{\cot^2(x)}\left(9765\cos(x)+1295\cos(3x)+189\cos(5x)+15\cos(7x)+6720\log\left(\sin\left(\frac{x}{2}\right)\right)-6720\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{6720}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (Sqrt[Cot[x]^2]*(9765*Cos[x] + 1295*Cos[3*x] + 189*Cos[5*x] + 15*Cos[7*x] -
 6720*Log[Cos[x/2]] + 6720*Log[Sin[x/2]])*Tan[x])/6720
```

Maple [A] time = 0.157, size = 65, normalized size = 0.8

$$\frac{\sqrt{4}\sin(x)}{210\cos(x)}\left(15(\cos(x))^7+21(\cos(x))^5+35(\cos(x))^3+105\cos(x)+105\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+176\right)\sqrt{-\frac{(\cos(x)}{-1+(\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x)
```

```
[Out] 1/210*4^(1/2)*(15*cos(x)^7+21*cos(x)^5+35*cos(x)^3+105*cos(x)+105*ln(-(-1+c
os(x))/sin(x))+176)*sin(x)*(-cos(x)^2/(-1+cos(x)^2))^(1/2)/cos(x)
```

Maxima [A] time = 0.985823, size = 116, normalized size = 1.43

$$\frac{1}{7}\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{7}{2}}\sin(x)^7+\frac{1}{5}\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{5}{2}}\sin(x)^5+\frac{1}{3}\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}}\sin(x)^3+\sqrt{\frac{1}{\sin(x)^2}-1}\sin(x)-\frac{1}{2}\log\left(\sqrt{\frac{}{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(1/sin(x)^2 - 1)^(7/2)*sin(x)^7 + 1/5*(1/sin(x)^2 - 1)^(5/2)*sin(x)^5 +
 1/3*(1/sin(x)^2 - 1)^(3/2)*sin(x)^3 + sqrt(1/sin(x)^2 - 1)*sin(x) - 1/2*lo
g(sqrt(1/sin(x)^2 - 1)*sin(x) + 1) + 1/2*log(sqrt(1/sin(x)^2 - 1)*sin(x) -
1)
```

Fricas [A] time = 2.18922, size = 150, normalized size = 1.85

$$-\frac{1}{7}\cos(x)^7-\frac{1}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3-\cos(x)+\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/7*cos(x)^7 - 1/5*cos(x)^5 - 1/3*cos(x)^3 - cos(x) + 1/2*log(1/2*cos(x) +
 1/2) - 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09033, size = 138, normalized size = 1.7

$$-\frac{1}{210}\left(30\left(\sin(x)^2-1\right)^3\sqrt{-\sin(x)^2+1}-42\left(\sin(x)^2-1\right)^2\sqrt{-\sin(x)^2+1}-70\left(-\sin(x)^2+1\right)^{\frac{3}{2}}-210\sqrt{-\sin(x)^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/210*(30*(sin(x)^2 - 1)^3*sqrt(-sin(x)^2 + 1) - 42*(sin(x)^2 - 1)^2*sqrt(
-sin(x)^2 + 1) - 70*(-sin(x)^2 + 1)^(3/2) - 210*sqrt(-sin(x)^2 + 1) + 105*l
og(sqrt(-sin(x)^2 + 1) + 1) - 105*log(-sqrt(-sin(x)^2 + 1) + 1))*sgn(sin(x)
)
```

3.868 $\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$

<u>**Optimal**</u>. Leaf size=76

$$\frac{i\sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{i\sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2x\sec(x)\tanh^{-1}\left(e^{ix}\right)}{\sqrt{a\sec^2(x)}}$$

```
[Out] (-2*x*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (I*PolyLog[2, -E^(I*x)]*S
ec[x])/Sqrt[a*Sec[x]^2] - (I*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```

<u>**Rubi [A]**</u> time = 0.534855, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 4183, 2279, 2391}

$$\frac{i\sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{i\sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2x\sec(x)\tanh^{-1}\left(e^{ix}\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (I*PolyLog[2, -E^(I*x)]*S
ec[x])/Sqrt[a*Sec[x]^2] - (I*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```

<u>Rule 6720</u>

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

<u>Rule 4183</u>

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

<u>Rule 2279</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx &= \frac{\sec(x)\int x\csc(x)\,dx}{\sqrt{a\sec^2(x)}}\\
&= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\sec(x)\int\log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}}+\frac{\sec(x)\int\log\left(1+e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}}\\
&= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}+\frac{(i\sec(x))\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{(i\sec(x))\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,\right.}{\sqrt{a\sec^2(x)}}\\
&= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}+\frac{i\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{i\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0614066, size = 69, normalized size = 0.91

$$\frac{\sec(x)\left(i\text{PolyLog}\left(2,-e^{ix}\right)-i\text{PolyLog}\left(2,e^{ix}\right)+x\left(\log\left(1-e^{ix}\right)-\log\left(1+e^{ix}\right)\right)\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((x*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + I*PolyLog[2, -E^(I*x)] - I*Poly
Log[2, E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.078, size = 98, normalized size = 1.3

$$\frac{-2\,i}{1+e^{2\,ix}}\left(-\frac{i}{2}e^{ix}x\ln\left(e^{ix}+1\right)-\frac{e^{ix}\text{polylog}\left(2,-e^{ix}\right)}{2}+\frac{i}{2}e^{ix}x\ln\left(1-e^{ix}\right)+\frac{e^{ix}\text{polylog}\left(2,e^{ix}\right)}{2}\right)\frac{1}{\sqrt{\frac{ae^{2\,ix}}{\left(1+e^{2\,ix}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)
```

```
[Out] -2*I/(a*exp(2*I*x)/(1+exp(2*I*x))^2)^(1/2)/(1+exp(2*I*x))*(-1/2*I*exp(I*x)*
x*ln(exp(I*x)+1)-1/2*exp(I*x)*polylog(2,-exp(I*x))+1/2*I*exp(I*x)*x*ln(1-ex
p(I*x))+1/2*exp(I*x)*polylog(2,exp(I*x)))
```

Maxima [A] time = 1.52844, size = 107, normalized size = 1.41

$$-\frac{2ix\arctan(\sin(x),\cos(x)+1)+2ix\arctan(\sin(x),-\cos(x)+1)+x\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)-x\log}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) + 1
) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log(cos(x)^2 + sin(x)^2 -
 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I*x)))/sqrt(a)
```

Fricas [B] time = 2.45447, size = 439, normalized size = 5.78

$$-\frac{(x\cos(x)\log(\cos(x)+i\sin(x)+1)+x\cos(x)\log(\cos(x)-i\sin(x)+1)-x\cos(x)\log(-\cos(x)+i\sin(x)+1)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
+ 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(
x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin
(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(
x)))*sqrt(a/cos(x)^2)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

3.869 $\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$

Optimal. Leaf size=128

$$\frac{2ix\sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2ix\sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2\sec(x)\text{PolyLog}\left(3,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}+\frac{2\sec(x)\text{PolyLog}\left(3,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2\cdots}{\cdots}$$

```
[Out] (-2*x^2*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((2*I)*x*PolyLog[2, -E^
(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((2*I)*x*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt
[a*Sec[x]^2] - (2*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (2*PolyLo
g[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.591856, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 4183, 2531, 2282, 6589}

$$\frac{2ix\sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2ix\sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2\sec(x)\text{PolyLog}\left(3,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}+\frac{2\sec(x)\text{PolyLog}\left(3,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```


```
[Out] (-2*x^2*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((2*I)*x*PolyLog[2, -E^
(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((2*I)*x*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt
[a*Sec[x]^2] - (2*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (2*PolyLo
g[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = \frac{\sec(x)\int x^2\csc(x)\,dx}{\sqrt{a\sec^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2\sec(x))\int x\log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}} + \frac{(2\sec(x))\int x\log\left(1+e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2i\sec(x))\int \text{Li}_2\left(-e^{ix}\right)\ldots}{\sqrt{a\sec^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2\sec(x))\,\text{Subst}\left(\int \frac{\text{Li}_2(\ldots}{x}\ldots\right.}{\sqrt{a\sec^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2\text{Li}_3\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2\text{L}\ldots}{}$$

Mathematica [A] time = 0.0687644, size = 99, normalized size = 0.77

$$\frac{\sec(x)\left(2ix\text{PolyLog}\left(2,-e^{ix}\right)-2ix\text{PolyLog}\left(2,e^{ix}\right)-2\text{PolyLog}\left(3,-e^{ix}\right)+2\text{PolyLog}\left(3,e^{ix}\right)+x^2\log\left(1-e^{ix}\right)-x^2\log\right.}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((x^2*Log[1 - E^(I*x)] - x^2*Log[1 + E^(I*x)] + (2*I)*x*PolyLog[2, -E^(I*x)
] - (2*I)*x*PolyLog[2, E^(I*x)] - 2*PolyLog[3, -E^(I*x)] + 2*PolyLog[3, E^(
I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.066, size = 132, normalized size = 1.

$$-2\,\frac{1/2\,e^{ix}x^2\ln\left(e^{ix}+1\right)-ie^{ix}x\text{polylog}\left(2,-e^{ix}\right)+e^{ix}\text{polylog}\left(3,-e^{ix}\right)-1/2\,e^{ix}x^2\ln\left(1-e^{ix}\right)+ie^{ix}x\text{polylog}\left(2,e^{ix}\right)-e}{1+e^{2\,ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)
```

```
[Out] -2/(a*exp(2*I*x)/(1+exp(2*I*x))^2)^(1/2)/(1+exp(2*I*x))*(1/2*exp(I*x)*x^2*l
n(exp(I*x)+1)-I*exp(I*x)*x*polylog(2,-exp(I*x))+exp(I*x)*polylog(3,-exp(I*x
))-1/2*exp(I*x)*x^2*ln(1-exp(I*x))+I*exp(I*x)*x*polylog(2,exp(I*x))-exp(I*x
)*polylog(3,exp(I*x)))
```

Maxima [A] time = 1.52118, size = 144, normalized size = 1.12

$$-\frac{2i\,x^2\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)+2i\,x^2\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x^2\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)-}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x^2*arctan2(sin(x), cos(x) + 1) + 2*I*x^2*arctan2(sin(x), -cos(x)
 + 1) + x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^2*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*I*x*dilog(-e^(I*x)) + 4*I*x*dilog(e^(I*x)) + 4*p
olylog(3, -e^(I*x)) - 4*polylog(3, e^(I*x)))/sqrt(a)
```

Fricas [C] time = 2.4141, size = 788, normalized size = 6.16

$$\frac{2\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,\cos(x)+i\sin(x))+2\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,\cos(x)-i\sin(x))-2\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{po}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/cos
(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*pol
ylog(3, -cos(x) + I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x)
- I*sin(x)) - (x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x
) - I*sin(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log
(-cos(x) - I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*co
s(x)*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*
I*x*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^2))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

3.870 $\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$

Optimal. Leaf size=186

$$\frac{3ix^2 \sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a \sec^2(x)}}-\frac{3ix^2 \sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a \sec^2(x)}}-\frac{6x \sec(x)\text{PolyLog}\left(3,-e^{ix}\right)}{\sqrt{a \sec^2(x)}}+\frac{6x \sec(x)\text{PolyLog}\left(3,e^{ix}\right)}{\sqrt{a \sec^2(x)}}-$$

```
[Out] (-2*x^3*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((3*I)*x^2*PolyLog[2, -
E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((3*I)*x^2*PolyLog[2, E^(I*x)]*Sec[x])/
Sqrt[a*Sec[x]^2] - (6*x*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (6*
x*PolyLog[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((6*I)*PolyLog[4, -E^(I*x)
]*Sec[x])/Sqrt[a*Sec[x]^2] + ((6*I)*PolyLog[4, E^(I*x)]*Sec[x])/Sqrt[a*Sec[
x]^2]
```

Rubi [A] time = 0.569598, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 4183, 2531, 6609, 2282, 6589}

$$\frac{3ix^2 \sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a \sec^2(x)}}-\frac{3ix^2 \sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a \sec^2(x)}}-\frac{6x \sec(x)\text{PolyLog}\left(3,-e^{ix}\right)}{\sqrt{a \sec^2(x)}}+\frac{6x \sec(x)\text{PolyLog}\left(3,e^{ix}\right)}{\sqrt{a \sec^2(x)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((3*I)*x^2*PolyLog[2, -
E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((3*I)*x^2*PolyLog[2, E^(I*x)]*Sec[x])/
Sqrt[a*Sec[x]^2] - (6*x*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (6*
x*PolyLog[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((6*I)*PolyLog[4, -E^(I*x)
]*Sec[x])/Sqrt[a*Sec[x]^2] + ((6*I)*PolyLog[4, E^(I*x)]*Sec[x])/Sqrt[a*Sec[
x]^2]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = \frac{\sec(x) \int x^3 \csc(x) \, dx}{\sqrt{a \sec^2(x)}}$$
$$= -\frac{2x^3 \tanh^{-1}\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{(3 \sec(x)) \int x^2 \log\left(1 - e^{ix}\right) \, dx}{\sqrt{a \sec^2(x)}} + \frac{(3 \sec(x)) \int x^2 \log\left(1 + e^{ix}\right) \, dx}{\sqrt{a \sec^2(x)}}$$
$$= -\frac{2x^3 \tanh^{-1}\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{Li}_2\left(-e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{(6i \sec(x)) \int x \text{Li}_2\left(-e^{\right.}}{\sqrt{a \sec^2(x)}}$$
$$= -\frac{2x^3 \tanh^{-1}\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{Li}_2\left(-e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{Li}_3\left(-e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} +$$
$$= -\frac{2x^3 \tanh^{-1}\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{Li}_2\left(-e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{Li}_3\left(-e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} +$$
$$= -\frac{2x^3 \tanh^{-1}\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{Li}_2\left(-e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{Li}_3\left(-e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}} +$$

Mathematica [A] time = 0.0986147, size = 147, normalized size = 0.79

$$-\frac{i \sec(x) \left(-24x^2 \text{PolyLog}\left(2, e^{-ix}\right) - 24x^2 \text{PolyLog}\left(2, -e^{ix}\right) + 48ix \text{PolyLog}\left(3, e^{-ix}\right) - 48ix \text{PolyLog}\left(3, -e^{ix}\right) + 48\text{PolyL}\right.}{8\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((-I/8)*(Pi^4 - 2*x^4 + (8*I)*x^3*Log[1 - E^((-I)*x)] - (8*I)*x^3*Log[1 + E
^(I*x)] - 24*x^2*PolyLog[2, E^((-I)*x)] - 24*x^2*PolyLog[2, -E^(I*x)] + (48
*I)*x*PolyLog[3, E^((-I)*x)] - (48*I)*x*PolyLog[3, -E^(I*x)] + 48*PolyLog[4
, E^((-I)*x)] + 48*PolyLog[4, -E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.068, size = 172, normalized size = 0.9

$$\frac{2i}{1 + e^{2ix}} \left(\frac{i}{2} e^{ix} x^3 \ln\left(e^{ix} + 1\right) + \frac{3 e^{ix} x^2 \text{polylog}\left(2, -e^{ix}\right)}{2} + 3 i e^{ix} x \text{polylog}\left(3, -e^{ix}\right) - 3 e^{ix} \text{polylog}\left(4, -e^{ix}\right) - \frac{i}{2} e^{ix} x^3 \ln\left(1 \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*I/(a*exp(2*I*x)/(1+exp(2*I*x))^2)^(1/2)/(1+exp(2*I*x))*(1/2*I*exp(I*x)*x^
3*ln(exp(I*x)+1)+3/2*exp(I*x)*x^2*polylog(2,-exp(I*x))+3*I*exp(I*x)*x*polyl
og(3,-exp(I*x))-3*exp(I*x)*polylog(4,-exp(I*x))-1/2*I*exp(I*x)*x^3*ln(1-exp
(I*x))-3/2*exp(I*x)*x^2*polylog(2,exp(I*x))-3*I*exp(I*x)*x*polylog(3,exp(I*
x))+3*exp(I*x)*polylog(4,exp(I*x)))
```

Maxima [A] time = 1.56115, size = 177, normalized size = 0.95

$$-\frac{2ix^3 \arctan\left(\sin\left(x\right), \cos\left(x\right)+1\right) + 2ix^3 \arctan\left(\sin\left(x\right), -\cos\left(x\right)+1\right) + x^3 \log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\cos\left(x\right)+1\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x^3*arctan2(sin(x), cos(x) + 1) + 2*I*x^3*arctan2(sin(x), -cos(x)
 + 1) + x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^3*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 6*I*x^2*dilog(-e^(I*x)) + 6*I*x^2*dilog(e^(I*x)) +
 12*x*polylog(3, -e^(I*x)) - 12*x*polylog(3, e^(I*x)) + 12*I*polylog(4, -e^
(I*x)) - 12*I*polylog(4, e^(I*x)))/sqrt(a)
```

Fricas [C] time = 2.70255, size = 1137, normalized size = 6.11

$$\frac{6x\sqrt{\frac{a}{\cos(x)^2}}\cos\left(x\right)\mathrm{polylog}\left(3, \cos\left(x\right)+i\sin\left(x\right)\right) + 6x\sqrt{\frac{a}{\cos(x)^2}}\cos\left(x\right)\mathrm{polylog}\left(3, \cos\left(x\right)-i\sin\left(x\right)\right) - 6x\sqrt{\frac{a}{\cos(x)^2}}\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt(a
/cos(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(
x)*polylog(3, -cos(x) + I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3,
-cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin
(x)) - 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 6*I*sqrt
(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^2)*c
os(x)*polylog(4, -cos(x) - I*sin(x)) - (x^3*cos(x)*log(cos(x) + I*sin(x) +
```

```
1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) + I*sin
(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*dilog(co
s(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(x)
*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt
(a/cos(x)^2))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

3.871 $\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$

Optimal. Leaf size=81

$$-\frac{i \sec^2(x) \text{PolyLog}\left(2, e^{2ix}\right)}{2\sqrt{a \sec^4(x)}} - \frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log\left(1 - e^{2ix}\right) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

```
[Out] ((-I/2)*x^2*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x*Log[1 - E^((2*I)*x)]*Sec[x]^2)/
Sqrt[a*Sec[x]^4] - ((I/2)*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4
]
```

Rubi [A] time = 0.487143, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6720, 3717, 2190, 2279, 2391}

$$-\frac{i \sec^2(x) \text{PolyLog}\left(2, e^{2ix}\right)}{2\sqrt{a \sec^4(x)}} - \frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log\left(1 - e^{2ix}\right) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-I/2)*x^2*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x*Log[1 - E^((2*I)*x)]*Sec[x]^2)/
Sqrt[a*Sec[x]^4] - ((I/2)*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4
]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx &= \frac{\sec^2(x)\int x\cot(x)\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} - \frac{\left(2i\sec^2(x)\right)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{\sec^2(x)\int \log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\left(i\sec^2(x)\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{i\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0381124, size = 50, normalized size = 0.62

$$-\frac{i\sec^2(x)\left(\text{PolyLog}\left(2,e^{2ix}\right)+x\left(x+2i\log\left(1-e^{2ix}\right)\right)\right)}{2\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-I/2)*(x*(x + (2*I)*Log[1 - E^((2*I)*x)]) + PolyLog[2, E^((2*I)*x)])*Sec[
x]^2)/Sqrt[a*Sec[x]^4]
```

Maple [B] time = 0.077, size = 147, normalized size = 1.8

$$\frac{\frac{i}{2}e^{2ix}x^2}{\left(1+e^{2ix}\right)^2}\frac{1}{\sqrt{\frac{ae^{4ix}}{\left(1+e^{2ix}\right)^4}}}-\frac{2i}{\left(1+e^{2ix}\right)^2}\left(\frac{e^{2ix}x^2}{2}+\frac{i}{2}e^{2ix}x\ln\left(e^{ix}+1\right)+\frac{e^{2ix}\mathrm{polylog}\left(2,-e^{ix}\right)}{2}+\frac{i}{2}e^{2ix}x\ln\left(1-e^{ix}\right)+\frac{e^{2ix}\mathrm{p}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)
```

```
[Out] 1/2*I/(a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)/(1+exp(2*I*x))^2*exp(2*I*x)*x^2
-2*I/(a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)/(1+exp(2*I*x))^2*(1/2*exp(2*I*x)
*x^2+1/2*I*exp(2*I*x)*x*ln(exp(I*x)+1)+1/2*exp(2*I*x)*polylog(2,-exp(I*x))+
1/2*I*exp(2*I*x)*x*ln(1-exp(I*x))+1/2*exp(2*I*x)*polylog(2,exp(I*x)))
```

Maxima [A] time = 1.51372, size = 112, normalized size = 1.38

$$\frac{-ix^2+2ix\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2ix\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right.}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -co
s(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))/sqrt(a)
```

Fricas [B] time = 3.07491, size = 459, normalized size = 5.67

$$\frac{\left(x\cos\left(x\right)^2\log\left(\cos\left(x\right)+i\sin\left(x\right)+1\right)+x\cos\left(x\right)^2\log\left(\cos\left(x\right)-i\sin\left(x\right)+1\right)+x\cos\left(x\right)^2\log\left(-\cos\left(x\right)+i\sin\left(x\right)+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(
x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(-cos(x) -
 I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)^2*dilog(cos
(x) - I*sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*cos(x)^2*dilog(-
cos(x) - I*sin(x)))*sqrt(a/cos(x)^4)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec(x)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

3.872 $\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$

Optimal. Leaf size=109

$$-\frac{ix\sec^2(x)\text{PolyLog}\left(2,e^{2ix}\right)}{\sqrt{a\sec^4(x)}}+\frac{\sec^2(x)\text{PolyLog}\left(3,e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}-\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}}+\frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

```
[Out] ((-I/3)*x^3*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^2*Log[1 - E^((2*I)*x)]*Sec[x]^2
)/Sqrt[a*Sec[x]^4] - (I*x*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4
] + (PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4])
```

Rubi [A] time = 0.57421, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 3717, 2190, 2531, 2282, 6589}

$$-\frac{ix\sec^2(x)\text{PolyLog}\left(2,e^{2ix}\right)}{\sqrt{a\sec^4(x)}}+\frac{\sec^2(x)\text{PolyLog}\left(3,e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}-\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}}+\frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-I/3)*x^3*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^2*Log[1 - E^((2*I)*x)]*Sec[x]^2
)/Sqrt[a*Sec[x]^4] - (I*x*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4
] + (PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \frac{\sec^2(x)\int x^2\cot(x)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} - \frac{\left(2i\sec^2(x)\right)\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{\left(2\sec^2(x)\right)\int x\log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\mathrm{Li}_2\left(e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\left(i\sec^2(x)\right)\int \mathrm{Li}_2\left(e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\mathrm{Li}_2\left(e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\sec^2(x)\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(x)}{x}\,dx, x,\right.}{2\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\mathrm{Li}_2\left(e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\mathrm{Li}_3\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}$$

Mathematica [A] time = 0.0595842, size = 75, normalized size = 0.69

$$\frac{\sec^2(x)\left(24ix\mathrm{PolyLog}\left(2,e^{-2ix}\right)+12\mathrm{PolyLog}\left(3,e^{-2ix}\right)+8ix^3+24x^2\log\left(1-e^{-2ix}\right)-i\pi^3\right)}{24\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] (((-I)*Pi^3 + (8*I)*x^3 + 24*x^2*Log[1 - E^((-2*I)*x)] + (24*I)*x*PolyLog[2
, E^((-2*I)*x)] + 12*PolyLog[3, E^((-2*I)*x)])*Sec[x]^2)/(24*Sqrt[a*Sec[x]^
4])
```

Maple [B] time = 0.067, size = 183, normalized size = 1.7

$$\frac{\frac{i}{3}\mathrm{e}^{2\,ix}x^3}{\left(1+\mathrm{e}^{2\,ix}\right)^2}\frac{1}{\sqrt{\frac{a\mathrm{e}^{4\,ix}}{\left(1+\mathrm{e}^{2\,ix}\right)^4}}} - 2\,\frac{i/3\mathrm{e}^{2\,ix}x^3 - 1/2\,\mathrm{e}^{2\,ix}x^2\ln\left(\mathrm{e}^{ix}+1\right) + i\mathrm{e}^{2\,ix}x\mathrm{polylog}\left(2,-\mathrm{e}^{ix}\right) - \mathrm{e}^{2\,ix}\mathrm{polylog}\left(3,-\mathrm{e}^{ix}\right) - 1/2\,\mathrm{e}^{2\,ix}x}{\left(1+\mathrm{e}^{2\,ix}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)
```

```
[Out] 1/3*I/(a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)/(1+exp(2*I*x))^2*exp(2*I*x)*x^3
-2/(a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)/(1+exp(2*I*x))^2*(1/3*I*exp(2*I*x)
*x^3-1/2*exp(2*I*x)*x^2*ln(exp(I*x)+1)+I*exp(2*I*x)*x*polylog(2,-exp(I*x))-
exp(2*I*x)*polylog(3,-exp(I*x))-1/2*exp(2*I*x)*x^2*ln(1-exp(I*x))+I*exp(2*I
*x)*x*polylog(2,exp(I*x))-exp(2*I*x)*polylog(3,exp(I*x)))
```

Maxima [A] time = 1.53651, size = 153, normalized size = 1.4

$$\frac{-2i\,x^3 + 6i\,x^2 \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) - 6i\,x^2 \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + 3\,x^2 \log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\,\cos\left(x\right.\right.}{6\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-2*I*x^3 + 6*I*x^2*arctan2(sin(x), cos(x) + 1) - 6*I*x^2*arctan2(sin(x
), -cos(x) + 1) + 3*x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 3*x^2*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x*dilog(-e^(I*x)) - 12*I*x*dilo
g(e^(I*x)) + 12*polylog(3, -e^(I*x)) + 12*polylog(3, e^(I*x)))/sqrt(a)
```

Fricas [C] time = 2.68834, size = 821, normalized size = 7.53

$$\frac{2\,\sqrt{\frac{a}{\cos\left(x\right)^4}} \cos\left(x\right)^2 \operatorname{polylog}\left(3, \cos\left(x\right) + i\,\sin\left(x\right)\right) + 2\,\sqrt{\frac{a}{\cos\left(x\right)^4}} \cos\left(x\right)^2 \operatorname{polylog}\left(3, \cos\left(x\right) - i\,\sin\left(x\right)\right) + 2\,\sqrt{\frac{a}{\cos\left(x\right)^4}} \cos\left(x\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/c
os(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)
^2*polylog(3, -cos(x) + I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3,
-cos(x) - I*sin(x)) + (x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)
^2*log(cos(x) - I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) + I*sin(x) + 1) +
x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)^2*dilog(cos(x) + I*
sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)^2*dilog(-c
os(x) + I*sin(x)) - 2*I*x*cos(x)^2*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)
```

```
^4))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec(x)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

3.873 $\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$

Optimal. Leaf size=143

$$-\frac{3ix^2 \sec^2(x)\text{PolyLog}\left(2, e^{2ix}\right)}{2\sqrt{a \sec^4(x)}} + \frac{3x \sec^2(x)\text{PolyLog}\left(3, e^{2ix}\right)}{2\sqrt{a \sec^4(x)}} + \frac{3i \sec^2(x)\text{PolyLog}\left(4, e^{2ix}\right)}{4\sqrt{a \sec^4(x)}} - \frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log\left(1 - \right.}{\sqrt{a \,\right.}}$$

```
[Out] ((-I/4)*x^4*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^3*Log[1 - E^((2*I)*x)]*Sec[x]^2
)/Sqrt[a*Sec[x]^4] - (((3*I)/2)*x^2*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[
a*Sec[x]^4] + (3*x*PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4]) +
 (((3*I)/4)*PolyLog[4, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```

Rubi [A] time = 0.610126, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6720, 3717, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3ix^2 \sec^2(x)\text{PolyLog}\left(2, e^{2ix}\right)}{2\sqrt{a \sec^4(x)}} + \frac{3x \sec^2(x)\text{PolyLog}\left(3, e^{2ix}\right)}{2\sqrt{a \sec^4(x)}} + \frac{3i \sec^2(x)\text{PolyLog}\left(4, e^{2ix}\right)}{4\sqrt{a \sec^4(x)}} - \frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log\left(1 - \right.}{\sqrt{a \,\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```



```
[Out] ((-I/4)*x^4*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^3*Log[1 - E^((2*I)*x)]*Sec[x]^2
)/Sqrt[a*Sec[x]^4] - (((3*I)/2)*x^2*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[
a*Sec[x]^4] + (3*x*PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4]) +
 (((3*I)/4)*PolyLog[4, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \frac{\sec^2(x)\int x^3\cot(x)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} - \frac{\left(2i\sec^2(x)\right)\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{\left(3\sec^2(x)\right)\int x^2\log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{\left(3i\sec^2(x)\right)\int x\text{Li}_2\left(e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} - \frac{(3\sec}{}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{(3i\sec}{}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3i\text{Li}_4}{4}$$

Mathematica [A] time = 0.0693036, size = 87, normalized size = 0.61

$$-\frac{i\sec^2(x)\left(-96x^2\text{PolyLog}\left(2,e^{-2ix}\right)+96ix\text{PolyLog}\left(3,e^{-2ix}\right)+48\text{PolyLog}\left(4,e^{-2ix}\right)-16x^4+64ix^3\log\left(1-e^{-2ix}\right)+\pi^4\right)}{64\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-I/64)*(Pi^4 - 16*x^4 + (64*I)*x^3*Log[1 - E^((-2*I)*x)] - 96*x^2*PolyLog
[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^((-
2*I)*x)])*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```

Maple [A] time = 0.069, size = 221, normalized size = 1.6

$$\frac{\frac{i}{4}e^{2ix}x^4}{\left(1+e^{2ix}\right)^2}\frac{1}{\sqrt{\frac{ae^{4ix}}{\left(1+e^{2ix}\right)^4}}} + \frac{2i}{\left(1+e^{2ix}\right)^2}\left(-\frac{e^{2ix}x^4}{4} - \frac{i}{2}e^{2ix}x^3\ln\left(e^{ix}+1\right) - \frac{3e^{2ix}x^2\text{polylog}\left(2,-e^{ix}\right)}{2} - 3ie^{2ix}x\text{polylog}\left(3,-e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)
```

```
[Out] 1/4*I/(a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)/(1+exp(2*I*x))^2*exp(2*I*x)*x^4
+2*I/(a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)/(1+exp(2*I*x))^2*(-1/4*exp(2*I*x
)*x^4-1/2*I*exp(2*I*x)*x^3*ln(exp(I*x)+1)-3/2*exp(2*I*x)*x^2*polylog(2,-exp
(I*x))-3*I*exp(2*I*x)*x*polylog(3,-exp(I*x))+3*exp(2*I*x)*polylog(4,-exp(I*
x))-1/2*I*exp(2*I*x)*x^3*ln(1-exp(I*x))-3/2*exp(2*I*x)*x^2*polylog(2,exp(I*
x))-3*I*exp(2*I*x)*x*polylog(3,exp(I*x))+3*exp(2*I*x)*polylog(4,exp(I*x)))
```

Maxima [A] time = 1.55314, size = 185, normalized size = 1.29

$$\frac{-i x^4 + 4i x^3 \arctan(\sin(x), \cos(x) + 1) - 4i x^3 \arctan(\sin(x), -\cos(x) + 1) + 2 x^3 \log\left(\cos(x)^2 + \sin(x)^2 + 2 \cos(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-I*x^4 + 4*I*x^3*arctan2(sin(x), cos(x) + 1) - 4*I*x^3*arctan2(sin(x),
 -cos(x) + 1) + 2*x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*x^3*log(c
os(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x^2*dilog(-e^(I*x)) - 12*I*x^2*di
log(e^(I*x)) + 24*x*polylog(3, -e^(I*x)) + 24*x*polylog(3, e^(I*x)) + 24*I*
polylog(4, -e^(I*x)) + 24*I*polylog(4, e^(I*x)))/sqrt(a)
```

Fricas [C] time = 2.69629, size = 1180, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt
(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 6*x*sqrt(a/cos(x)^4)*
cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 6*x*sqrt(a/cos(x)^4)*cos(x)^2*pol
ylog(3, -cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(
x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(x) - I*sin(x)
```

```
) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) + I*sin(x)) + 6*I*sqrt
(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) - I*sin(x)) + (x^3*cos(x)^2*log(co
s(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)
^2*log(-cos(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) -
 3*I*x^2*cos(x)^2*dilog(cos(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x)
- I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2
*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^4))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

3.874 $\int x \csc(x) \sec(x) \sqrt{a \sec^2(x)} \, dx$

Optimal. Leaf size=105

$$i \cos(x) \text{PolyLog}\left(2, -e^{ix}\right) \sqrt{a \sec^2(x)} - i \cos(x) \text{PolyLog}\left(2, e^{ix}\right) \sqrt{a \sec^2(x)} + x\sqrt{a \sec^2(x)} - 2x \cos(x) \tanh^{-1}\left(e^{ix}\right) \sqrt{a \text{se}$$

```
[Out] x*Sqrt[a*Sec[x]^2] - 2*x*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - ArcTanh
[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] + I*Cos[x]*PolyLog[2, -E^(I*x)]*Sqrt[a*Sec
[x]^2] - I*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.34337, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6720, 2622, 321, 207, 4420, 6271, 4183, 2279, 2391, 3770}

$$i \cos(x) \text{PolyLog}\left(2, -e^{ix}\right) \sqrt{a \sec^2(x)} - i \cos(x) \text{PolyLog}\left(2, e^{ix}\right) \sqrt{a \sec^2(x)} + x\sqrt{a \sec^2(x)} - 2x \cos(x) \tanh^{-1}\left(e^{ix}\right) \sqrt{a \text{se}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x*Sqrt[a*Sec[x]^2] - 2*x*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - ArcTanh
[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] + I*Cos[x]*PolyLog[2, -E^(I*x)]*Sqrt[a*Sec
[x]^2] - I*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x\csc(x)\sec^2(x)\,dx$$
$$= x\sqrt{a\sec^2(x)} - x\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int\left(-\tanh^{-1}\right.$$
$$= x\sqrt{a\sec^2(x)} - x\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int\tanh^{-1}(\mathrm{co}$$
$$= x\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x\csc(x)\,dx$$
$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} -$$
$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} +$$
$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} +$$

Mathematica [A] time = 0.0792283, size = 108, normalized size = 1.03

$$\sqrt{a\sec^2(x)}\left(i\cos(x)\left(\text{PolyLog}\left(2,-e^{ix}\right)-\text{PolyLog}\left(2,e^{ix}\right)\right)+x+x\left(\log\left(1-e^{ix}\right)-\log\left(1+e^{ix}\right)\right)\cos(x)+\cos(x)\log\left(\mathrm{c}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] (x + x*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + Cos[x]*Log[Cos[x/2] -
 Sin[x/2]] - Cos[x]*Log[Cos[x/2] + Sin[x/2]] + I*Cos[x]*(PolyLog[2, -E^(I*x
)] - PolyLog[2, E^(I*x)]))*Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.107, size = 86, normalized size = 0.8

$$2\sqrt{\frac{a\mathrm{e}^{2\,ix}}{\left(1+\mathrm{e}^{2\,ix}\right)^2}}x+4\sqrt{\frac{a\mathrm{e}^{2\,ix}}{\left(1+\mathrm{e}^{2\,ix}\right)^2}}\left(i\arctan\left(\mathrm{e}^{ix}\right)+i/2\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)-1/2\,x\ln\left(\mathrm{e}^{ix}+1\right)+i/2\mathrm{dilog}\left(\mathrm{e}^{ix}\right)\right)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*I*x)/(1+exp(2*I*x))^2)^(1/2)*x+4*(a*exp(2*I*x)/(1+exp(2*I*x))^2)
^(1/2)*(I*arctan(exp(I*x))+1/2*I*dilog(exp(I*x)+1)-1/2*x*ln(exp(I*x)+1)+1/2
*I*dilog(exp(I*x)))*cos(x)
```

Maxima [B] time = 1.57098, size = 405, normalized size = 3.86

$$\frac{\left(2 \left(\cos \left(2 x\right) + i \sin \left(2 x\right) + 1\right) \arctan \left(\cos \left(x\right), \sin \left(x\right) + 1\right) + 2 \left(\cos \left(2 x\right) + i \sin \left(2 x\right) + 1\right) \arctan \left(\cos \left(x\right), - \sin \left(x\right) + 1\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (2*(cos(2*x) + I*sin(2*x) + 1)*arctan2(cos(x), sin(x) + 1) + 2*(cos(2*x) +
I*sin(2*x) + 1)*arctan2(cos(x), -sin(x) + 1) - (2*x*cos(2*x) + 2*I*x*sin(2*
x) + 2*x)*arctan2(sin(x), cos(x) + 1) - (2*x*cos(2*x) + 2*I*x*sin(2*x) + 2*
x)*arctan2(sin(x), -cos(x) + 1) - 4*I*x*cos(x) + 2*(cos(2*x) + I*sin(2*x) +
 1)*dilog(-e^(I*x)) - 2*(cos(2*x) + I*sin(2*x) + 1)*dilog(e^(I*x)) - (-I*x*
cos(2*x) + x*sin(2*x) - I*x)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - (I*x
*cos(2*x) - x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (-I
*cos(2*x) + sin(2*x) - I)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (I*cos(
2*x) - sin(2*x) + I)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*x*sin(x))*
sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

Fricas [A] time = 2.41178, size = 500, normalized size = 4.76

$$-\frac{1}{2} \left(x \cos \left(x\right) \log \left(\cos \left(x\right) + i \sin \left(x\right) + 1\right) + x \cos \left(x\right) \log \left(\cos \left(x\right) - i \sin \left(x\right) + 1\right) - x \cos \left(x\right) \log \left(- \cos \left(x\right) + i \sin \left(x\right) + 1\right) -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
+ 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(
x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin
(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(
x)) + cos(x)*log(-(sin(x) + 1)/(sin(x) - 1)) - 2*x)*sqrt(a/cos(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\sec^2(x)}\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(x)^2}x\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x*csc(x)*sec(x), x)
```

3.875 $\int x^2 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx$

Optimal. Leaf size=225

$$2ix\cos(x)\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)}-2ix\cos(x)\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)}-2i\cos(x)\text{PolyLog}\left(2,-ie^{ix}\right)\sqrt{a\sec^2(x)}$$

```
[Out] x^2*Sqrt[a*Sec[x]^2] + (4*I)*x*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - 2*
x^2*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (2*I)*x*Cos[x]*PolyLog[2, -E
^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt[a*Sec
[x]^2] + (2*I)*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*x*Cos[
x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 2*Cos[x]*PolyLog[3, -E^(I*x)]*Sqr
t[a*Sec[x]^2] + 2*Cos[x]*PolyLog[3, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.531386, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6720, 2622, 321, 207, 4420, 14, 6273, 4183, 2531, 2282, 6589, 4181, 2279, 2391}

$$2ix\cos(x)\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)}-2ix\cos(x)\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)}-2i\cos(x)\text{PolyLog}\left(2,-ie^{ix}\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```



```
[Out] x^2*Sqrt[a*Sec[x]^2] + (4*I)*x*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - 2*
x^2*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (2*I)*x*Cos[x]*PolyLog[2, -E
^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt[a*Sec
[x]^2] + (2*I)*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*x*Cos[
x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 2*Cos[x]*PolyLog[3, -E^(I*x)]*Sqr
t[a*Sec[x]^2] + 2*Cos[x]*PolyLog[3, E^(I*x)]*Sqrt[a*Sec[x]^2]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
```

[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^2 \csc(x)\sec^2(x)\,dx$$
$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(2\cos(x)\sqrt{a\sec^2(x)}\right)\int x\left(-\mathrm{t}\right.$$
$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(2\cos(x)\sqrt{a\sec^2(x)}\right)\int\left(-x\,\mathrm{ta}\right.$$
$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \left(2\cos(x)\sqrt{a\sec^2(x)}\right)\int x\tan$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^2\csc(x)\,d$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}$$

Mathematica [A] time = 0.127748, size = 174, normalized size = 0.77

$$\sqrt{a\sec^2(x)}\left(2ix\cos(x)\left(\text{PolyLog}\left(2,-e^{ix}\right)-\text{PolyLog}\left(2,e^{ix}\right)\right)+2\cos(x)\left(\text{PolyLog}\left(3,e^{ix}\right)-\text{PolyLog}\left(3,-e^{ix}\right)\right)-2\cos$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] (x^2 + x^2*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) - 2*Cos[x]*(x*(Log[
1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) + I*(PolyLog[2, (-I)*E^(I*x)] - PolyLo
g[2, I*E^(I*x)])) + (2*I)*x*Cos[x]*(PolyLog[2, -E^(I*x)] - PolyLog[2, E^(I*
x)]) + 2*Cos[x]*(-PolyLog[3, -E^(I*x)] + PolyLog[3, E^(I*x)]))*Sqrt[a*Sec[x
]^2]
```

Maple [A] time = 0.157, size = 200, normalized size = 0.9

$$2\sqrt{\frac{a\mathrm{e}^{2\,ix}}{\left(1+\mathrm{e}^{2\,ix}\right)^2}}x^2-4\,i\sqrt{\frac{a\mathrm{e}^{2\,ix}}{\left(1+\mathrm{e}^{2\,ix}\right)^2}}\left(2\,i\left(\frac{x\ln\left(1+i\mathrm{e}^{ix}\right)}{2}-\frac{x\ln\left(1-i\mathrm{e}^{ix}\right)}{2}-\frac{i}{2}\mathrm{dilog}\left(1+i\mathrm{e}^{ix}\right)+\frac{i}{2}\mathrm{dilog}\left(1-i\mathrm{e}^{ix}\right)\right)-\frac{i}{2}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*I*x)/(1+exp(2*I*x))^2)^(1/2)*x^2-4*I*(a*exp(2*I*x)/(1+exp(2*I*x)
)^2)^(1/2)*(2*I*(1/2*x*ln(1+I*exp(I*x))-1/2*x*ln(1-I*exp(I*x))-1/2*I*dilog(
1+I*exp(I*x))+1/2*I*dilog(1-I*exp(I*x)))-1/2*I*(-1/3*I*x^3+x^2*ln(exp(I*x)+
1)-2*I*x*polylog(2,-exp(I*x))+2*polylog(3,-exp(I*x)))-1/2*I*(1/3*I*x^3-x^2*
ln(1-exp(I*x))+2*I*x*polylog(2,exp(I*x))-2*polylog(3,exp(I*x))))*cos(x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.67575, size = 1210, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^2)*co
s(x)*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -co
s(x) + I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) - I*sin(x)) -
 1/2*(x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x) - I*sin
(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log(-cos(x)
- I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*cos(x)*dilo
g(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x
)*dilog(-cos(x) - I*sin(x)) + 2*x*cos(x)*log(I*cos(x) + sin(x) + 1) - 2*x*c
os(x)*log(I*cos(x) - sin(x) + 1) + 2*x*cos(x)*log(-I*cos(x) + sin(x) + 1) -
 2*x*cos(x)*log(-I*cos(x) - sin(x) + 1) - 2*x^2 - 2*I*cos(x)*dilog(I*cos(x)
 + sin(x)) - 2*I*cos(x)*dilog(I*cos(x) - sin(x)) + 2*I*cos(x)*dilog(-I*cos(
x) + sin(x)) + 2*I*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(x)^2}x^2\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x^2*csc(x)*sec(x), x)
```

3.876 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)} \, dx$

Optimal. Leaf size=341

$$3ix^2 \cos(x)\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)} - 3ix^2\cos(x)\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)} - 6ix\cos(x)\text{PolyLog}\left(2,-ie^{ix}\right)\sqrt{a\sec^2}$$

```
[Out] x^3*Sqrt[a*Sec[x]^2] + (6*I)*x^2*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] -
2*x^3*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (3*I)*x^2*Cos[x]*PolyLog[2
, -E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*x*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt
[a*Sec[x]^2] + (6*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (3*I
)*x^2*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 6*x*Cos[x]*PolyLog[3, -
E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*Cos[x]*PolyLog[3, (-I)*E^(I*x)]*Sqrt[a*Sec[x]
^2] - 6*Cos[x]*PolyLog[3, I*E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*x*Cos[x]*PolyLog[
3, E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*Cos[x]*PolyLog[4, -E^(I*x)]*Sqrt[a*Sec
[x]^2] + (6*I)*Cos[x]*PolyLog[4, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.62853, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2622, 321, 207, 4420, 14, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$3ix^2 \cos(x)\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)} - 3ix^2\cos(x)\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)} - 6ix\cos(x)\text{PolyLog}\left(2,-ie^{ix}\right)\sqrt{a\sec^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```


```
[Out] x^3*Sqrt[a*Sec[x]^2] + (6*I)*x^2*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] -
2*x^3*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (3*I)*x^2*Cos[x]*PolyLog[2
, -E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*x*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt
[a*Sec[x]^2] + (6*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (3*I
)*x^2*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 6*x*Cos[x]*PolyLog[3, -
E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*Cos[x]*PolyLog[3, (-I)*E^(I*x)]*Sqrt[a*Sec[x]
^2] - 6*Cos[x]*PolyLog[3, I*E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*x*Cos[x]*PolyLog[
3, E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*Cos[x]*PolyLog[4, -E^(I*x)]*Sqrt[a*Sec
[x]^2] + (6*I)*Cos[x]*PolyLog[4, E^(I*x)]*Sqrt[a*Sec[x]^2]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
```

```
+ 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx = \left(\cos(x)\sqrt{a \sec^2(x)}\right) \int x^3 \csc(x) \sec^2(x)\, dx$$
$$= x^3\sqrt{a \sec^2(x)} - x^3 \tanh^{-1}(\cos(x)) \cos(x)\sqrt{a \sec^2(x)} - \left(3 \cos(x)\sqrt{a \sec^2(x)}\right) \int x^2 \big(-$$
$$= x^3\sqrt{a \sec^2(x)} - x^3 \tanh^{-1}(\cos(x)) \cos(x)\sqrt{a \sec^2(x)} - \left(3 \cos(x)\sqrt{a \sec^2(x)}\right) \int \big(-x^2$$
$$= x^3\sqrt{a \sec^2(x)} - x^3 \tanh^{-1}(\cos(x)) \cos(x)\sqrt{a \sec^2(x)} + \left(3 \cos(x)\sqrt{a \sec^2(x)}\right) \int x^2 \tan$$
$$= x^3\sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x)} + \left(\cos(x)\sqrt{a \sec^2(x)}\right) \int x^3 \csc(x)$$
$$= x^3\sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x}$$
$$= x^3\sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x}$$
$$= x^3\sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x}$$
$$= x^3\sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x}$$
$$= x^3\sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}\left(e^{ix}\right) \cos(x)\sqrt{a \sec^2(x}$$

Mathematica [A] time = 0.432337, size = 290, normalized size = 0.85

$$\frac{1}{8}\sqrt{a \sec^2(x)} \left(24ix^2 \cos(x)\text{PolyLog}\left(2, e^{-ix}\right) + 24ix^2 \cos(x)\text{PolyLog}\left(2, -e^{ix}\right) - 48ix \cos(x)\text{PolyLog}\left(2, -ie^{ix}\right) + 48ix \text{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((8*x^3 - I*Pi^4*Cos[x] + (2*I)*x^4*Cos[x] + 8*x^3*Cos[x]*Log[1 - E^((-I)*x
)] - 24*x^2*Cos[x]*Log[1 - I*E^(I*x)] + 24*x^2*Cos[x]*Log[1 + I*E^(I*x)] -
8*x^3*Cos[x]*Log[1 + E^(I*x)] + (24*I)*x^2*Cos[x]*PolyLog[2, E^((-I)*x)] +
(24*I)*x^2*Cos[x]*PolyLog[2, -E^(I*x)] - (48*I)*x*Cos[x]*PolyLog[2, (-I)*E^
(I*x)] + (48*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)] + 48*x*Cos[x]*PolyLog[3, E^(
(-I)*x)] - 48*x*Cos[x]*PolyLog[3, -E^(I*x)] + 48*Cos[x]*PolyLog[3, (-I)*E^(
I*x)] - 48*Cos[x]*PolyLog[3, I*E^(I*x)] - (48*I)*Cos[x]*PolyLog[4, E^((-I)*
x)] - (48*I)*Cos[x]*PolyLog[4, -E^(I*x)])*Sqrt[a*Sec[x]^2])/8
```

Maple [A] time = 0.226, size = 250, normalized size = 0.7

$$2\sqrt{\frac{a e^{2ix}}{\left(1+e^{2ix}\right)^2}}x^3+4\sqrt{\frac{a e^{2ix}}{\left(1+e^{2ix}\right)^2}}\left(-3/2\,x^2\ln\left(1-ie^{ix}\right)+3\,ix\mathrm{polylog}\left(2,ie^{ix}\right)-3\,\mathrm{polylog}\left(3,ie^{ix}\right)+3/2\,x^2\ln\left(1+ie^{ix}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*I*x)/(1+exp(2*I*x))^2)^(1/2)*x^3+4*(a*exp(2*I*x)/(1+exp(2*I*x))^
2)^(1/2)*(-3/2*x^2*ln(1-I*exp(I*x))+3*I*x*polylog(2,I*exp(I*x))-3*polylog(3
,I*exp(I*x))+3/2*x^2*ln(1+I*exp(I*x))-3*I*x*polylog(2,-I*exp(I*x))+3*polylo
g(3,-I*exp(I*x))+1/2*I*(1/4*x^4+I*x^3*ln(exp(I*x)+1)+3*x^2*polylog(2,-exp(I
*x))+6*I*x*polylog(3,-exp(I*x))-6*polylog(4,-exp(I*x)))+1/2*I*(-1/4*x^4-I*x
^3*ln(1-exp(I*x))-3*x^2*polylog(2,exp(I*x))-6*I*x*polylog(3,exp(I*x))+6*pol
ylog(4,exp(I*x))))*cos(x)
```

Maxima [B] time = 1.65314, size = 790, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*I*x^3*cos(x) - 4*x^3*sin(x) - (6*x^2*cos(2*x) + 6*I*x^2*sin(2*x) + 6*x^
2)*arctan2(cos(x), sin(x) + 1) - (6*x^2*cos(2*x) + 6*I*x^2*sin(2*x) + 6*x^2
)*arctan2(cos(x), -sin(x) + 1) + (2*x^3*cos(2*x) + 2*I*x^3*sin(2*x) + 2*x^3
)*arctan2(sin(x), cos(x) + 1) + (2*x^3*cos(2*x) + 2*I*x^3*sin(2*x) + 2*x^3)
*arctan2(sin(x), -cos(x) + 1) - (12*x*cos(2*x) + 12*I*x*sin(2*x) + 12*x)*di
log(I*e^(I*x)) + (12*x*cos(2*x) + 12*I*x*sin(2*x) + 12*x)*dilog(-I*e^(I*x))
 - (6*x^2*cos(2*x) + 6*I*x^2*sin(2*x) + 6*x^2)*dilog(-e^(I*x)) + (6*x^2*cos
(2*x) + 6*I*x^2*sin(2*x) + 6*x^2)*dilog(e^(I*x)) + (-I*x^3*cos(2*x) + x^3*s
in(2*x) - I*x^3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^3*cos(2*x)
- x^3*sin(2*x) + I*x^3)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + (-3*I*x^2
*cos(2*x) + 3*x^2*sin(2*x) - 3*I*x^2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) +
1) + (3*I*x^2*cos(2*x) - 3*x^2*sin(2*x) + 3*I*x^2)*log(cos(x)^2 + sin(x)^2
- 2*sin(x) + 1) + 12*(cos(2*x) + I*sin(2*x) + 1)*polylog(4, -e^(I*x)) - 12*
(cos(2*x) + I*sin(2*x) + 1)*polylog(4, e^(I*x)) + (-12*I*cos(2*x) + 12*sin(
```

```
2*x) - 12*I)*polylog(3, I*e^(I*x)) + (12*I*cos(2*x) - 12*sin(2*x) + 12*I)*p
olylog(3, -I*e^(I*x)) + (-12*I*x*cos(2*x) + 12*x*sin(2*x) - 12*I*x)*polylog
(3, -e^(I*x)) + (12*I*x*cos(2*x) - 12*x*sin(2*x) + 12*I*x)*polylog(3, e^(I*
x)))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

Fricas [C] time = 3.03825, size = 1906, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/cos(
x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*po
lylog(3, -cos(x) + I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(
x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin(x))
- 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 3*I*sqrt(a/co
s(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^2)*cos(x)
*polylog(4, -cos(x) - I*sin(x)) + 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*co
s(x) + sin(x)) - 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*cos(x) - sin(x)) +
3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -I*cos(x) + sin(x)) - 3*sqrt(a/cos(x)^
2)*cos(x)*polylog(3, -I*cos(x) - sin(x)) - 1/2*(x^3*cos(x)*log(cos(x) + I*s
in(x) + 1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x)
 + I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*
dilog(cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^
2*cos(x)*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x
)) + 3*x^2*cos(x)*log(I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(I*cos(x) -
sin(x) + 1) + 3*x^2*cos(x)*log(-I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(-
I*cos(x) - sin(x) + 1) - 2*x^3 - 6*I*x*cos(x)*dilog(I*cos(x) + sin(x)) - 6*
I*x*cos(x)*dilog(I*cos(x) - sin(x)) + 6*I*x*cos(x)*dilog(-I*cos(x) + sin(x)
) + 6*I*x*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec(x)^2} x^3 \csc(x) \sec(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x^3*csc(x)*sec(x), x)
```

3.877 $\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx$

Optimal. Leaf size=142

$$\frac{1}{2}i\cos^2(x)\text{PolyLog}\left(2,-e^{2ix}\right)\sqrt{a\sec^4(x)}-\frac{1}{2}i\cos^2(x)\text{PolyLog}\left(2,e^{2ix}\right)\sqrt{a\sec^4(x)}+\frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)}+\frac{1}{2}x\sin^2(x$$

```
[Out] (x*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*
Sec[x]^4] + (I/2)*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (I/2
)*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (Cos[x]*Sqrt[a*Sec[x]
^4]*Sin[x])/2 + (x*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rubi [A] time = 0.399437, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {6720, 2620, 14, 4420, 2548, 4419, 4183, 2279, 2391, 3473, 8}

$$\frac{1}{2}i\cos^2(x)\text{PolyLog}\left(2,-e^{2ix}\right)\sqrt{a\sec^4(x)}-\frac{1}{2}i\cos^2(x)\text{PolyLog}\left(2,e^{2ix}\right)\sqrt{a\sec^4(x)}+\frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)}+\frac{1}{2}x\sin^2(x$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```


```
[Out] (x*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*
Sec[x]^4] + (I/2)*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (I/2
)*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (Cos[x]*Sqrt[a*Sec[x]
^4]*Sin[x])/2 + (x*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x\csc(x)\sec^3(x)\,dx$$

$$= x\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x\sqrt{a\sec^4(x)}\sin^2(x) - \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \Big(\mathrm{lo}$$

$$= x\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x\sqrt{a\sec^4(x)}\sin^2(x) - \frac{1}{2}\left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int$$

$$= -\frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}x\sqrt{a\sec^4(x)}\sin^2(x) + \frac{1}{2}\left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int 1\,dx$$

$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}x\sqrt{a\sec^4(x)}\sin^2(x) + \Big(2\cos^2($$

$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\sin($$

$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\sin($$

$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}i\cos^2(x)\mathrm{Li}_2\left(-e^{2ix}\right)\sqrt{}$$

Mathematica [A] time = 0.236901, size = 85, normalized size = 0.6

$$\frac{1}{2}\cos^2(x)\sqrt{a\sec^4(x)}\left(i\mathrm{PolyLog}\left(2,-e^{2ix}\right) - i\mathrm{PolyLog}\left(2,e^{2ix}\right) + 2x\log\left(1-e^{2ix}\right) - 2x\log\left(1+e^{2ix}\right) - \tan(x) + x\sec^2(x$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*(2*x*Log[1 - E^((2*I)*x)] - 2*x*Log[1 + E^((2*I)
*x)] + I*PolyLog[2, -E^((2*I)*x)] - I*PolyLog[2, E^((2*I)*x)] + x*Sec[x]^2
- Tan[x]))/2
```

Maple [A] time = 0.077, size = 165, normalized size = 1.2

$$\sqrt{\frac{a e^{4 i x}}{\left(1+e^{2 i x}\right)^4}}\left(-i+2 x-i e^{-2 i x}\right)-4 i \sqrt{\frac{a e^{4 i x}}{\left(1+e^{2 i x}\right)^4}}\left(1+e^{2 i x}\right)^2\left(-\frac{i}{4} e^{-2 i x} x \ln\left(1+e^{2 i x}\right)-\frac{e^{-2 i x} \operatorname{polylog}\left(2,-e^{2 i x}\right)}{8}+\frac{i}{4} e^{-2 i x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)*(-I+2*x-I*exp(-2*I*x))-4*I*(a*exp(4*I
*x)/(1+exp(2*I*x))^4)^(1/2)*(1+exp(2*I*x))^2*(-1/4*I*exp(-2*I*x)*x*ln(1+exp
(2*I*x))-1/8*exp(-2*I*x)*polylog(2,-exp(2*I*x))+1/4*I*exp(-2*I*x)*x*ln(exp(
I*x)+1)+1/4*exp(-2*I*x)*polylog(2,-exp(I*x))+1/4*I*exp(-2*I*x)*x*ln(1-exp(I
*x))+1/4*exp(-2*I*x)*polylog(2,exp(I*x)))
```

Maxima [B] time = 1.7041, size = 583, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arc
tan2(sin(2*x), cos(2*x) + 1) - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*sin(4*x
) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(x), cos(x) + 1) + (2*x*cos(4*x) + 4*x
*cos(2*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(x), -cos(x)
+ 1) - 2*(-2*I*x - 1)*cos(2*x) - (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*
sin(2*x) + 1)*dilog(-e^(2*I*x)) + (2*cos(4*x) + 4*cos(2*x) + 2*I*sin(4*x) +
 4*I*sin(2*x) + 2)*dilog(-e^(I*x)) + (2*cos(4*x) + 4*cos(2*x) + 2*I*sin(4*x
) + 4*I*sin(2*x) + 2)*dilog(e^(I*x)) + (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*
sin(4*x) + 2*x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin(4*x) - 2*x*sin(2*x) + I*x)*log(c
os(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin
(4*x) - 2*x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (4*x
- 2*I)*sin(2*x) + 2)*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4
*sin(2*x) - 2*I)
```

Fricas [B] time = 3.12056, size = 910, normalized size = 6.41

$$\frac{1}{2}\left(x\cos(x)^2\log(\cos(x)+i\sin(x)+1)+x\cos(x)^2\log(\cos(x)-i\sin(x)+1)-x\cos(x)^2\log(i\cos(x)+\sin(x)+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(
x) + 1) - x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(I*cos(x) -
 sin(x) + 1) - x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(-I*c
os(x) - sin(x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*l
og(-cos(x) - I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)
^2*dilog(cos(x) - I*sin(x)) - I*cos(x)^2*dilog(I*cos(x) + sin(x)) + I*cos(x
)^2*dilog(I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-I*cos(x) + sin(x)) - I*cos
(x)^2*dilog(-I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*
cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)*sin(x) + x)*sqrt(a/cos(x)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\sec^4(x)}\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(x)^4}x\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x*csc(x)*sec(x), x)
```

3.878 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$

Optimal. Leaf size=220

$$ix \cos^2(x) \text{PolyLog}\left(2, -e^{2ix}\right) \sqrt{a \sec^4(x)} - ix \cos^2(x) \text{PolyLog}\left(2, e^{2ix}\right) \sqrt{a \sec^4(x)} - \frac{1}{2} \cos^2(x) \text{PolyLog}\left(3, -e^{2ix}\right) \sqrt{a \sec^4(}$$

```
[Out] (x^2*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x^2*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqr
t[a*Sec[x]^4] - Cos[x]^2*Log[Cos[x]]*Sqrt[a*Sec[x]^4] + I*x*Cos[x]^2*PolyLo
g[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - I*x*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*
Sqrt[a*Sec[x]^4] - (Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 +
 (Cos[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - x*Cos[x]*Sqrt[a*Se
c[x]^4]*Sin[x] + (x^2*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rubi [A] time = 0.537266, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2620, 14, 4420, 2551, 4419, 4183, 2531, 2282, 6589, 3720, 3475, 30}

$$ix \cos^2(x) \text{PolyLog}\left(2, -e^{2ix}\right) \sqrt{a \sec^4(x)} - ix \cos^2(x) \text{PolyLog}\left(2, e^{2ix}\right) \sqrt{a \sec^4(x)} - \frac{1}{2} \cos^2(x) \text{PolyLog}\left(3, -e^{2ix}\right) \sqrt{a \sec^4(}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```


```
[Out] (x^2*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x^2*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqr
t[a*Sec[x]^4] - Cos[x]^2*Log[Cos[x]]*Sqrt[a*Sec[x]^4] + I*x*Cos[x]^2*PolyLo
g[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - I*x*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*
Sqrt[a*Sec[x]^4] - (Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 +
 (Cos[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - x*Cos[x]*Sqrt[a*Se
c[x]^4]*Sin[x] + (x^2*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x^2\csc(x)\sec^3(x)\,dx$$
$$= x^2\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\sin^2(x) - \left(2\cos^2(x)\sqrt{a\sec^4(x)}\right)$$
$$= x^2\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\sin^2(x) - \left(2\cos^2(x)\sqrt{a\sec^4(x)}\right)$$
$$= x^2\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\sin^2(x) - \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int$$
$$= -x\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\sin^2(x) + \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x\,dx$$
$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(\cos(x))\sqrt{a\sec^4(x)} - x\cos(x)\sqrt{a\sec^4(x)}\sin(x)$$
$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(\cos(x))$$
$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(\cos(x))$$
$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(\cos(x))$$
$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(\cos(x))$$

Mathematica [A] time = 0.64321, size = 138, normalized size = 0.63

$$\frac{1}{24}\cos^2(x)\sqrt{a\sec^4(x)}\left(24ix\text{PolyLog}\left(2,e^{-2ix}\right) + 24ix\text{PolyLog}\left(2,-e^{2ix}\right) + 12\text{PolyLog}\left(3,e^{-2ix}\right) - 12\text{PolyLog}\left(3,-e^{2ix}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^3 + (16*I)*x^3 + 24*x^2*Log[1 - E^((-2*
I)*x)] - 24*x^2*Log[1 + E^((2*I)*x)] - 24*Log[Cos[x]] + (24*I)*x*PolyLog[2,
 E^((-2*I)*x)] + (24*I)*x*PolyLog[2, -E^((2*I)*x)] + 12*PolyLog[3, E^((-2*I
)*x)] - 12*PolyLog[3, -E^((2*I)*x)] + 12*x^2*Sec[x]^2 - 24*x*Tan[x]))/24
```

Maple [C] time = 0.098, size = 254, normalized size = 1.2

$$2\sqrt{\frac{a e^{4ix}}{\left(1+e^{2ix}\right)^4}}x\left(x-i-ie^{-2ix}\right) + 2\sqrt{\frac{a e^{4ix}}{\left(1+e^{2ix}\right)^4}}\left(1+e^{2ix}\right)^2\left(-1/2\,e^{-2ix}\ln\left(1+e^{2ix}\right) - e^{-2ix}\Im(x) + e^{-2ix}\ln\left(e^{i\Re(x)}\right) - 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)
```

```
[Out] 2*(a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)*x*(x-I-I*exp(-2*I*x))+2*(a*exp(4*I*
x)/(1+exp(2*I*x))^4)^(1/2)*(1+exp(2*I*x))^2*(-1/2*exp(-2*I*x)*ln(1+exp(2*I*
x))-exp(-2*I*x)*Im(x)+exp(-2*I*x)*ln(exp(I*Re(x)))-1/2*exp(-2*I*x)*x^2*ln(1
+exp(2*I*x))+1/2*I*exp(-2*I*x)*x*polylog(2,-exp(2*I*x))-1/4*exp(-2*I*x)*pol
ylog(3,-exp(2*I*x))+1/2*exp(-2*I*x)*x^2*ln(exp(I*x)+1)-I*exp(-2*I*x)*x*poly
log(2,-exp(I*x))+exp(-2*I*x)*polylog(3,-exp(I*x))+1/2*exp(-2*I*x)*x^2*ln(1-
exp(I*x))-I*exp(-2*I*x)*x*polylog(2,exp(I*x))+exp(-2*I*x)*polylog(3,exp(I*x
)))
```

Maxima [B] time = 1.78614, size = 882, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((2*x^2 + 2*(x^2 + 1)*cos(4*x) + 4*(x^2 + 1)*cos(2*x) + (2*I*x^2 + 2*I)*si
n(4*x) + (4*I*x^2 + 4*I)*sin(2*x) + 2)*arctan2(sin(2*x), cos(2*x) + 1) - (2
*x^2*cos(4*x) + 4*x^2*cos(2*x) + 2*I*x^2*sin(4*x) + 4*I*x^2*sin(2*x) + 2*x^
2)*arctan2(sin(x), cos(x) + 1) + (2*x^2*cos(4*x) + 4*x^2*cos(2*x) + 2*I*x^2
*sin(4*x) + 4*I*x^2*sin(2*x) + 2*x^2)*arctan2(sin(x), -cos(x) + 1) - 4*x*co
s(4*x) + (4*I*x^2 - 4*x)*cos(2*x) - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*si
n(4*x) + 4*I*x*sin(2*x) + 2*x)*dilog(-e^(2*I*x)) + (4*x*cos(4*x) + 8*x*cos(
2*x) + 4*I*x*sin(4*x) + 8*I*x*sin(2*x) + 4*x)*dilog(-e^(I*x)) + (4*x*cos(4*
x) + 8*x*cos(2*x) + 4*I*x*sin(4*x) + 8*I*x*sin(2*x) + 4*x)*dilog(e^(I*x)) +
 (-I*x^2 + (-I*x^2 - I)*cos(4*x) + (-2*I*x^2 - 2*I)*cos(2*x) + (x^2 + 1)*si
n(4*x) + 2*(x^2 + 1)*sin(2*x) - I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x)
 + 1) + (I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x)
+ I*x^2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(4*x) + 2*I*x^
2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1) + (-I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)
*polylog(3, -e^(2*I*x)) + (4*I*cos(4*x) + 8*I*cos(2*x) - 4*sin(4*x) - 8*sin
(2*x) + 4*I)*polylog(3, -e^(I*x)) + (4*I*cos(4*x) + 8*I*cos(2*x) - 4*sin(4*
x) - 8*sin(2*x) + 4*I)*polylog(3, e^(I*x)) - 4*I*x*sin(4*x) - 4*(x^2 + I*x)
*sin(2*x))*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2*x)
- 2*I)
```

Fricas [C] time = 3.26037, size = 1823, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*
cos(x)^2*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(
3, I*cos(x) + sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) - sin
(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) + sin(x)) - sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2
*polylog(3, -cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos
(x) - I*sin(x)) + 1/2*(x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)
^2*log(cos(x) - I*sin(x) + 1) - x^2*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x
^2*cos(x)^2*log(I*cos(x) - sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x) + sin(x
) + 1) - x^2*cos(x)^2*log(-I*cos(x) - sin(x) + 1) + x^2*cos(x)^2*log(-cos(x
) + I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)
^2*dilog(cos(x) + I*sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) - 2*I
*x*cos(x)^2*dilog(I*cos(x) + sin(x)) + 2*I*x*cos(x)^2*dilog(I*cos(x) - sin(
x)) + 2*I*x*cos(x)^2*dilog(-I*cos(x) + sin(x)) - 2*I*x*cos(x)^2*dilog(-I*co
s(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x)^2*
dilog(-cos(x) - I*sin(x)) - cos(x)^2*log(cos(x) + I*sin(x) + I) - cos(x)^2*
log(cos(x) - I*sin(x) + I) - cos(x)^2*log(-cos(x) + I*sin(x) + I) - cos(x)^
2*log(-cos(x) - I*sin(x) + I) - 2*x*cos(x)*sin(x) + x^2)*sqrt(a/cos(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(x)^4}x^2\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x^2*csc(x)*sec(x), x)
```

3.879 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$

Optimal. Leaf size=356

$$\frac{3}{2} i x^2 \cos^2(x) \text{PolyLog}\left(2,-e^{2ix}\right) \sqrt{a \sec^4(x)} - \frac{3}{2} i x^2 \cos^2(x) \text{PolyLog}\left(2,e^{2ix}\right) \sqrt{a \sec^4(x)} - \frac{3}{2} x \cos^2(x) \text{PolyLog}\left(3,-e^{2ix}\right)$$

```
[Out] ((3*I)/2)*x^2*Cos[x]^2*Sqrt[a*Sec[x]^4] + (x^3*Cos[x]^2*Sqrt[a*Sec[x]^4])/2
 - 2*x^3*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*Sec[x]^4] - 3*x*Cos[x]^2*Log[
1 + E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*Cos[x]^2*PolyLog[2, -E^((2*I)
*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqr
t[a*Sec[x]^4] - ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x
]^4] - (3*x*Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 + (3*x*Co
s[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - ((3*I)/4)*Cos[x]^2*Pol
yLog[4, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/4)*Cos[x]^2*PolyLog[4, E^((
2*I)*x)]*Sqrt[a*Sec[x]^4] - (3*x^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x])/2 + (x^3
*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rubi [A] time = 0.636957, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 17, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.944, Rules used = {6720, 2620, 14, 4420, 2551, 4419, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 2279, 2391, 30}

$$\frac{3}{2} i x^2 \cos^2(x) \text{PolyLog}\left(2,-e^{2ix}\right) \sqrt{a \sec^4(x)} - \frac{3}{2} i x^2 \cos^2(x) \text{PolyLog}\left(2,e^{2ix}\right) \sqrt{a \sec^4(x)} - \frac{3}{2} x \cos^2(x) \text{PolyLog}\left(3,-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```


```
[Out] ((3*I)/2)*x^2*Cos[x]^2*Sqrt[a*Sec[x]^4] + (x^3*Cos[x]^2*Sqrt[a*Sec[x]^4])/2
 - 2*x^3*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*Sec[x]^4] - 3*x*Cos[x]^2*Log[
1 + E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*Cos[x]^2*PolyLog[2, -E^((2*I)
*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqr
t[a*Sec[x]^4] - ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x
]^4] - (3*x*Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 + (3*x*Co
s[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - ((3*I)/4)*Cos[x]^2*Pol
yLog[4, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/4)*Cos[x]^2*PolyLog[4, E^((
2*I)*x)]*Sqrt[a*Sec[x]^4] - (3*x^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x])/2 + (x^3
*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x^3 \csc(x)\sec^3(x)\,dx$$
$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) - \left(3\cos^2(x)\sqrt{a\sec^4(x)}\right)$$
$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) - \left(3\cos^2(x)\sqrt{a\sec^4(x)}\right)$$
$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) - \frac{1}{2}\left(3\cos^2(x)\sqrt{a\sec^4(x)}\right.$$
$$= -\frac{3}{2}x^2\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) + \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x^3$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - \frac{3}{2}x^2\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4}$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4}$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4}$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4}$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4}$$

Mathematica [A] time = 1.08983, size = 191, normalized size = 0.54

$$\frac{1}{64}\cos^2(x)\sqrt{a\sec^4(x)}\left(96ix^2\text{PolyLog}\left(2,e^{-2ix}\right) + 96i\left(x^2+1\right)\text{PolyLog}\left(2,-e^{2ix}\right) + 96x\text{PolyLog}\left(3,e^{-2ix}\right) - 96x\text{PolyLo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^4 + (96*I)*x^2 + (32*I)*x^4 + 64*x^3*Lo
g[1 - E^((-2*I)*x)] - 192*x*Log[1 + E^((2*I)*x)] - 64*x^3*Log[1 + E^((2*I)*
x)] + (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] + (96*I)*(1 + x^2)*PolyLog[2, -E^
((2*I)*x)] + 96*x*PolyLog[3, E^((-2*I)*x)] - 96*x*PolyLog[3, -E^((2*I)*x)]
- (48*I)*PolyLog[4, E^((-2*I)*x)] - (48*I)*PolyLog[4, -E^((2*I)*x)] + 32*x^
3*Sec[x]^2 - 96*x^2*Tan[x]))/64
```

Maple [A] time = 0.09, size = 324, normalized size = 0.9

$$\sqrt{\frac{a\mathrm{e}^{4ix}}{\left(1+\mathrm{e}^{2ix}\right)^{4}}}\,x^{2}\left(2x-3i-3i\mathrm{e}^{-2ix}\right)-2i\sqrt{\frac{a\mathrm{e}^{4ix}}{\left(1+\mathrm{e}^{2ix}\right)^{4}}}\left(1+\mathrm{e}^{2ix}\right)^{2}\left(-\frac{3\,\mathrm{e}^{-2ix}x^{2}}{2}-\frac{3i}{2}\mathrm{e}^{-2ix}x\ln\left(1+\mathrm{e}^{2ix}\right)-\frac{3\,\mathrm{e}^{-2ix}\mathrm{polylog}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)*x^2*(2*x-3*I-3*I*exp(-2*I*x))-2*I*(a*
exp(4*I*x)/(1+exp(2*I*x))^4)^(1/2)*(1+exp(2*I*x))^2*(-3/2*exp(-2*I*x)*x^2-3
/2*I*exp(-2*I*x)*x*ln(1+exp(2*I*x))-3/4*exp(-2*I*x)*polylog(2,-exp(2*I*x))-
1/2*I*exp(-2*I*x)*x^3*ln(1+exp(2*I*x))-3/4*exp(-2*I*x)*x^2*polylog(2,-exp(2
*I*x))-3/4*I*exp(-2*I*x)*x*polylog(3,-exp(2*I*x))+3/8*exp(-2*I*x)*polylog(4
,-exp(2*I*x))+1/2*I*exp(-2*I*x)*x^3*ln(exp(I*x)+1)+3/2*exp(-2*I*x)*x^2*poly
log(2,-exp(I*x))+3*I*exp(-2*I*x)*x*polylog(3,-exp(I*x))-3*exp(-2*I*x)*polyl
og(4,-exp(I*x))+1/2*I*exp(-2*I*x)*x^3*ln(1-exp(I*x))+3/2*exp(-2*I*x)*x^2*po
lylog(2,exp(I*x))+3*I*exp(-2*I*x)*x*polylog(3,exp(I*x))-3*exp(-2*I*x)*polyl
og(4,exp(I*x)))
```

Maxima [B] time = 1.9119, size = 1175, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (18*x^2*cos(4*x) + 18*I*x^2*sin(4*x) - (8*x^3 + 2*(4*x^3 + 9*x)*cos(4*x) +
4*(4*x^3 + 9*x)*cos(2*x) + (8*I*x^3 + 18*I*x)*sin(4*x) + (16*I*x^3 + 36*I*x
)*sin(2*x) + 18*x)*arctan2(sin(2*x), cos(2*x) + 1) + (6*x^3*cos(4*x) + 12*x
^3*cos(2*x) + 6*I*x^3*sin(4*x) + 12*I*x^3*sin(2*x) + 6*x^3)*arctan2(sin(x),
 cos(x) + 1) - (6*x^3*cos(4*x) + 12*x^3*cos(2*x) + 6*I*x^3*sin(4*x) + 12*I*
x^3*sin(2*x) + 6*x^3)*arctan2(sin(x), -cos(x) + 1) - (12*I*x^3 - 18*x^2)*co
s(2*x) + (12*x^2 + 3*(4*x^2 + 3)*cos(4*x) + 6*(4*x^2 + 3)*cos(2*x) - (-12*I
*x^2 - 9*I)*sin(4*x) - (-24*I*x^2 - 18*I)*sin(2*x) + 9)*dilog(-e^(2*I*x)) -
 (18*x^2*cos(4*x) + 36*x^2*cos(2*x) + 18*I*x^2*sin(4*x) + 36*I*x^2*sin(2*x)
 + 18*x^2)*dilog(-e^(I*x)) - (18*x^2*cos(4*x) + 36*x^2*cos(2*x) + 18*I*x^2*
sin(4*x) + 36*I*x^2*sin(2*x) + 18*x^2)*dilog(e^(I*x)) - (-4*I*x^3 + (-4*I*x
^3 - 9*I*x)*cos(4*x) + (-8*I*x^3 - 18*I*x)*cos(2*x) + (4*x^3 + 9*x)*sin(4*x
) + 2*(4*x^3 + 9*x)*sin(2*x) - 9*I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2
```

```
*x) + 1) - (3*I*x^3*cos(4*x) + 6*I*x^3*cos(2*x) - 3*x^3*sin(4*x) - 6*x^3*si
n(2*x) + 3*I*x^3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - (3*I*x^3*cos(4*
x) + 6*I*x^3*cos(2*x) - 3*x^3*sin(4*x) - 6*x^3*sin(2*x) + 3*I*x^3)*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1) - (6*cos(4*x) + 12*cos(2*x) + 6*I*sin(4*x)
+ 12*I*sin(2*x) + 6)*polylog(4, -e^(2*I*x)) + (36*cos(4*x) + 72*cos(2*x) +
36*I*sin(4*x) + 72*I*sin(2*x) + 36)*polylog(4, -e^(I*x)) + (36*cos(4*x) + 7
2*cos(2*x) + 36*I*sin(4*x) + 72*I*sin(2*x) + 36)*polylog(4, e^(I*x)) - (-12
*I*x*cos(4*x) - 24*I*x*cos(2*x) + 12*x*sin(4*x) + 24*x*sin(2*x) - 12*I*x)*p
olylog(3, -e^(2*I*x)) - (36*I*x*cos(4*x) + 72*I*x*cos(2*x) - 36*x*sin(4*x)
- 72*x*sin(2*x) + 36*I*x)*polylog(3, -e^(I*x)) - (36*I*x*cos(4*x) + 72*I*x*
cos(2*x) - 36*x*sin(4*x) - 72*x*sin(2*x) + 36*I*x)*polylog(3, e^(I*x)) + 6*
(2*x^3 + 3*I*x^2)*sin(2*x))*sqrt(a)/(-6*I*cos(4*x) - 12*I*cos(2*x) + 6*sin(
4*x) + 12*sin(2*x) - 6*I)
```

Fricas [C] time = 4.53984, size = 2446, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x
)^2*polylog(3, I*cos(x) + sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3
, I*cos(x) - sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) +
 sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + 3
*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 3*x*sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(
x)^2*polylog(4, cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
4, cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) +
 sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) - sin(x)) - 3*
I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -I*cos(x) + sin(x)) + 3*I*sqrt(a/cos
(x)^4)*cos(x)^2*polylog(4, -I*cos(x) - sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x
)^2*polylog(4, -cos(x) + I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
4, -cos(x) - I*sin(x)) + 1/2*(x^3*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^3
*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) + I*sin(x)
+ 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 3*I*x^2*cos(x)^2*dilog(co
s(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(
x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2*dilog(-cos(x) - I*sin(x))
 + (-3*I*x^2 - 3*I)*cos(x)^2*dilog(I*cos(x) + sin(x)) + (3*I*x^2 + 3*I)*cos
(x)^2*dilog(I*cos(x) - sin(x)) + (3*I*x^2 + 3*I)*cos(x)^2*dilog(-I*cos(x) +
```

```
 sin(x)) + (-3*I*x^2 - 3*I)*cos(x)^2*dilog(-I*cos(x) - sin(x)) - (x^3 + 3*x
)*cos(x)^2*log(I*cos(x) + sin(x) + 1) - (x^3 + 3*x)*cos(x)^2*log(I*cos(x) -
 sin(x) + 1) - (x^3 + 3*x)*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - (x^3 + 3*
x)*cos(x)^2*log(-I*cos(x) - sin(x) + 1) - 3*x^2*cos(x)*sin(x) + x^3)*sqrt(a
/cos(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec(x)^4} x^3 \csc(x) \sec(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x^3*csc(x)*sec(x), x)
```

3.880 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi [A] time = 0.031031, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx\\ &= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx\\ &= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A] time = 0.0101799, size = 25, normalized size = 1.

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A] time = 0.048, size = 20, normalized size = 0.8

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A] time = 0.946402, size = 26, normalized size = 1.04

$$\frac{1}{24}\cos(6x)-\frac{1}{16}\cos(4x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A] time = 2.33245, size = 54, normalized size = 2.16

$$\frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] time = 14.387, size = 116, normalized size = 4.64

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{5\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(3*x)*cos(2*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/3 - 3*cos(x)*cos(2*x)*cos(3*x)/8
```

Giac [A] time = 1.06461, size = 18, normalized size = 0.72

$$-\frac{4}{3}\sin(x)^6 + \frac{3}{2}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

3.881 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi [A] time = 0.0326408, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)dx \\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx \\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A] time = 0.0098102, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A] time = 0.041, size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A] time = 0.966645, size = 30, normalized size = 1.

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A] time = 2.34773, size = 81, normalized size = 2.7

$$\frac{1}{12}\left(16\cos(x)^5-10\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] time = 14.5589, size = 114, normalized size = 3.8

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4}+\frac{x\sin(x)\sin(3x)\cos(2x)}{4}+\frac{x\sin(2x)\sin(3x)\cos(x)}{4}+\frac{x\cos(x)\cos(2x)\cos(3x)}{4}+\frac{\sin(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
6 + sin(x)*cos(2*x)*cos(3*x)/8 + 5*sin(3*x)*cos(x)*cos(2*x)/24
```

Giac [A] time = 1.09322, size = 30, normalized size = 1.

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.882 $\int \cos(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

```
[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

Rubi [A] time = 0.0334716, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)-\frac{1}{4}\cos(4x)-\frac{1}{4}\cos(6x)\right)dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx-\frac{1}{4}\int \cos(4x)\,dx-\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A] time = 0.0080781, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

Maple [A] time = 0.036, size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sin(2x)}{8}-\frac{\sin(4x)}{16}-\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)
```

Maxima [A] time = 0.940329, size = 30, normalized size = 1.

$$\frac{1}{4}x-\frac{1}{24}\sin(6x)-\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A] time = 2.42285, size = 82, normalized size = 2.73

$$-\frac{1}{12}\left(16\cos\left(x\right)^{5}-10\cos\left(x\right)^{3}-3\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -1/12*(16*cos(x)^5 - 10*cos(x)^3 - 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] time = 14.2556, size = 116, normalized size = 3.87

$$-\frac{x\sin\left(x\right)\sin\left(2x\right)\cos\left(3x\right)}{4}+\frac{x\sin\left(x\right)\sin\left(3x\right)\cos\left(2x\right)}{4}+\frac{x\sin\left(2x\right)\sin\left(3x\right)\cos\left(x\right)}{4}+\frac{x\cos\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{4}+\frac{\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
3 + 3*sin(x)*cos(2*x)*cos(3*x)/8 - 5*sin(3*x)*cos(x)*cos(2*x)/24
```

Giac [A] time = 1.07059, size = 30, normalized size = 1.

$$\frac{1}{4}x-\frac{1}{24}\sin\left(6x\right)-\frac{1}{16}\sin\left(4x\right)+\frac{1}{8}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.883 $\int \cos(2x)\cos(3x)\sin(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

```
[Out] -Cos[2*x]/8 + Cos[4*x]/16 - Cos[6*x]/24
```

Rubi [A] time = 0.0310978, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Cos[3*x]*Sin[x],x]
```

```
[Out] -Cos[2*x]/8 + Cos[4*x]/16 - Cos[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\cos(3x)\sin(x)\,dx &= \int \left(\frac{1}{4}\sin(2x) - \frac{1}{4}\sin(4x) + \frac{1}{4}\sin(6x)\right) dx \\ &= \frac{1}{4}\int \sin(2x)\,dx - \frac{1}{4}\int \sin(4x)\,dx + \frac{1}{4}\int \sin(6x)\,dx \\ &= -\frac{1}{8}\cos(2x) + \frac{1}{16}\cos(4x) - \frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A] time = 0.0087282, size = 25, normalized size = 1.

$$-\frac{1}{8}\cos(2x) + \frac{1}{16}\cos(4x) - \frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Cos[3*x]*Sin[x],x]
```

```
[Out] -Cos[2*x]/8 + Cos[4*x]/16 - Cos[6*x]/24
```

Maple [A] time = 0.035, size = 20, normalized size = 0.8

$$-\frac{\cos(2x)}{8} + \frac{\cos(4x)}{16} - \frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*sin(x),x)
```

```
[Out] -1/8*cos(2*x)+1/16*cos(4*x)-1/24*cos(6*x)
```

Maxima [A] time = 0.948219, size = 26, normalized size = 1.04

$$-\frac{1}{24}\cos(6x) + \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*cos(6*x) + 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A] time = 2.3858, size = 61, normalized size = 2.44

$$-\frac{4}{3}\cos(x)^6 + \frac{5}{2}\cos(x)^4 - \frac{3}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -4/3*cos(x)^6 + 5/2*cos(x)^4 - 3/2*cos(x)^2
```

Sympy [B] time = 14.3147, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} + \frac{5\sin(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 + 5*sin(x)*sin(3*x)*cos(2*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/6 - cos(x)*cos(2*x)*cos(3*x)/8
```

Giac [A] time = 1.06464, size = 26, normalized size = 1.04

$$\frac{4}{3}\sin(x)^6 - \frac{3}{2}\sin(x)^4 + \frac{1}{2}\sin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] 4/3*sin(x)^6 - 3/2*sin(x)^4 + 1/2*sin(x)^2
```

3.884 $\int x \sin\left(x^2\right) dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\cos\left(x^2\right)$$

```
[Out] -Cos[x^2]/2
```

Rubi [A] time = 0.0062987, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3379, 2638}

$$-\frac{1}{2}\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2],x]

[Out] -Cos[x^2]/2
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x \sin\left(x^2\right) dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sin(x)\, dx, x, x^2\right)\\ &= -\frac{1}{2}\cos\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0060461, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2],x]
```

```
[Out] -Cos[x^2]/2
```

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$-\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2),x)
```

```
[Out] -1/2*cos(x^2)
```

Maxima [A] time = 0.946036, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x^2)
```

Fricas [A] time = 2.35517, size = 20, normalized size = 2.5

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x^2)
```

Sympy [A] time = 0.158996, size = 7, normalized size = 0.88

$$-\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2),x)
```

```
[Out] -cos(x**2)/2
```

Giac [A] time = 1.07883, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*cos(x^2)
```

3.885 $\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx$

Optimal. Leaf size=11

$$-\frac{1}{6}(\sin(x)+\cos(x))^6$$

```
[Out] -(Cos[x] + Sin[x])^6/6
```

Rubi [A] time = 0.0207024, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3145}

$$-\frac{1}{6}(\sin(x)+\cos(x))^6$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

```
[Out] -(Cos[x] + Sin[x])^6/6
```

Rule 3145

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[((c*B - b*C)*(b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(
b^2 + c^2)), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

Rubi steps

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx=-\frac{1}{6}(\cos(x)+\sin(x))^6$$

Mathematica [B] time = 0.0777261, size = 25, normalized size = 2.27

$$-\frac{5}{8}\sin(2x)+\frac{1}{24}\sin(6x)+\frac{1}{4}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

```
[Out] Cos[4*x]/4 - (5*Sin[2*x])/8 + Sin[6*x]/24
```

Maple [B] time = 0.032, size = 97, normalized size = 8.8

$$-\frac{\cos(x)}{6}\left((\sin(x))^5+\frac{5\,(\sin(x))^3}{4}+\frac{15\,\sin(x)}{8}\right)+\frac{2\,(\sin(x))^6}{3}-\frac{5\,(\cos(x))^3\,(\sin(x))^3}{6}-\frac{5\,(\cos(x))^3\sin(x)}{8}+\frac{5\,\cos(x}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+2/3*sin(x)^6-5/6*cos(x)^3*s
in(x)^3-5/8*cos(x)^3*sin(x)+5/16*cos(x)*sin(x)+5/6*cos(x)^5*sin(x)-5/24*(co
s(x)^3+3/2*cos(x))*sin(x)+2/3*cos(x)^6-1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(
x))*sin(x)
```

Maxima [A] time = 0.943586, size = 12, normalized size = 1.09

$$-\frac{1}{6}\,(\cos(x)+\sin(x))^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(cos(x) + sin(x))^6
```

Fricas [B] time = 2.39975, size = 101, normalized size = 9.18

$$2\,\cos(x)^4-2\,\cos(x)^2+\frac{1}{3}\left(4\,\cos(x)^5-4\,\cos(x)^3-3\,\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="fricas")
```

```
[Out] 2*cos(x)^4 - 2*cos(x)^2 + 1/3*(4*cos(x)^5 - 4*cos(x)^3 - 3*cos(x))*sin(x)
```

Sympy [B] time = 2.08743, size = 54, normalized size = 4.91

$$-\sin^5(x)\cos(x) - 2\sin^4(x)\cos^2(x) - \frac{10\sin^3(x)\cos^3(x)}{3} - 2\sin^2(x)\cos^4(x) - \sin(x)\cos^5(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))**5,x)
```

```
[Out] -sin(x)**5*cos(x) - 2*sin(x)**4*cos(x)**2 - 10*sin(x)**3*cos(x)**3/3 - 2*si
n(x)**2*cos(x)**4 - sin(x)*cos(x)**5
```

Giac [B] time = 1.07106, size = 26, normalized size = 2.36

$$\frac{1}{4}\cos(4x) + \frac{1}{24}\sin(6x) - \frac{5}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="giac")
```

```
[Out] 1/4*cos(4*x) + 1/24*sin(6*x) - 5/8*sin(2*x)
```

3.886 $\int 2x\sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=11

$$x\sec^2(x) - \tan(x)$$

```
[Out] x*Sec[x]^2 - Tan[x]
```

Rubi [A] time = 0.0187379, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {12, 3757, 3767, 8}

$$x\sec^2(x) - \tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[2*x*Sec[x]^2*Tan[x],x]
```

```
[Out] x*Sec[x]^2 - Tan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int 2x\sec^2(x)\tan(x)\,dx &= 2\int x\sec^2(x)\tan(x)\,dx \\
&= x\sec^2(x) - \int \sec^2(x)\,dx \\
&= x\sec^2(x) + \text{Subst}\left(\int 1\,dx, x, -\tan(x)\right) \\
&= x\sec^2(x) - \tan(x)
\end{aligned}
$$

Mathematica [A] time = 0.011037, size = 18, normalized size = 1.64

$$2\left(\frac{1}{2}x\sec^2(x) - \frac{\tan(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x*Sec[x]^2*Tan[x],x]
```

```
[Out] 2*((x*Sec[x]^2)/2 - Tan[x]/2)
```

Maple [A] time = 0.01, size = 12, normalized size = 1.1

$$\frac{x}{(\cos(x))^2} - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*sec(x)^2*tan(x),x)
```

```
[Out] x/cos(x)^2-tan(x)
```

Maxima [B] time = 0.954077, size = 180, normalized size = 16.36

$$\frac{2\left(4\,x\cos(2\,x)^2 + 4\,x\sin(2\,x)^2 + (2\,x\cos(2\,x) + \sin(2\,x))\cos(4\,x) + 2\,x\cos(2\,x) + (2\,x\sin(2\,x) - \cos(2\,x) - 1)\sin(4\,x)\right.}{2\,(2\,\cos(2\,x) + 1)\cos(4\,x) + \cos(4\,x)^2 + 4\,\cos(2\,x)^2 + \sin(4\,x)^2 + 4\,\sin(4\,x)\sin(2\,x) + 4\,\sin(2\,x)^2 + 4\,\cos(2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 2*(4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2
*x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Fricas [A] time = 2.28016, size = 42, normalized size = 3.82

$$-\frac{\cos(x)\sin(x) - x}{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] -(cos(x)*sin(x) - x)/cos(x)^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int x\tan(x)\sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)**2*tan(x),x)
```

```
[Out] 2*Integral(x*tan(x)*sec(x)**2, x)
```

Giac [B] time = 1.08604, size = 70, normalized size = 6.36

$$\frac{x\tan\left(\frac{1}{2}x\right)^4 + 2x\tan\left(\frac{1}{2}x\right)^2 + 2\tan\left(\frac{1}{2}x\right)^3 + x - 2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] (x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))/(ta
n(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

3.887 $\int \frac{1+\cos^2(x)}{1+\cos(2x)} \, dx$

Optimal. Leaf size=12

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

```
[Out] x/2 + Tan[x]/2
```

Rubi [A] time = 0.0468899, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 203}

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

```
[Out] x/2 + Tan[x]/2
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx = \text{Subst}\left(\int \frac{2+x^2}{2+2x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{2} + \text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{x}{2} + \frac{\tan(x)}{2}$$

Mathematica [A] time = 0.0157699, size = 12, normalized size = 1.

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

```
[Out] x/2 + Tan[x]/2
```

Maple [A] time = 0.047, size = 9, normalized size = 0.8

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)/(1+cos(2*x)),x)
```

```
[Out] 1/2*x+1/2*tan(x)
```

Maxima [B] time = 0.945971, size = 24, normalized size = 2.

$$\frac{1}{2}\,x + \frac{\sin(2\,x)}{2\,(\cos(2\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*sin(2*x)/(cos(2*x) + 1)
```

Fricas [A] time = 2.30898, size = 43, normalized size = 3.58

$$\frac{x\cos(x) + \sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x) + sin(x))/cos(x)
```

Sympy [A] time = 1.68321, size = 7, normalized size = 0.58

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(1+cos(2*x)),x)
```

```
[Out] x/2 + tan(x)/2
```

Giac [A] time = 1.0669, size = 11, normalized size = 0.92

$$\frac{1}{2}x + \frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)
```

3.888 $\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2} + \log(\tan(x))$$

```
[Out] Log[Tan[x]] + Tan[x]^2/2
```

Rubi [A] time = 0.0390701, antiderivative size = 17, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4335, 266, 44}

$$\frac{\sec^2(x)}{2} + \log(\sin(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]
```

```
[Out] -Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{x^3\left(1-x^2\right)}\,dx, x, \cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx, x, \cos^2(x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)dx, x, \cos^2(x)\right)\right)\\ &= -\log(\cos(x))+\log(\sin(x))+\frac{\sec^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0150483, size = 17, normalized size = 1.42

$$\frac{\sec^2(x)}{2}+\log(\sin(x))-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]
```

```
[Out] -Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2
```

Maple [B] time = 0.024, size = 27, normalized size = 2.3

$$\frac{1}{2\left(\cos(x)\right)^2}-\ln(\cos(x))+\frac{\ln(1+\cos(x))}{2}+\frac{\ln(-1+\cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3-cos(x)^5),x)
```

```
[Out] 1/2/cos(x)^2-ln(cos(x))+1/2*ln(1+cos(x))+1/2*ln(-1+cos(x))
```

Maxima [B] time = 0.950405, size = 35, normalized size = 2.92

$$\frac{1}{2\cos(x)^2}+\frac{1}{2}\log(\cos(x)+1)+\frac{1}{2}\log(\cos(x)-1)-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="maxima")
```

```
[Out] 1/2/cos(x)^2 + 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1) - log(cos(x))
```

Fricas [B] time = 2.31086, size = 108, normalized size = 9.

$$-\frac{\cos\left(x\right)^{2}\log\left(\cos\left(x\right)^{2}\right)-\cos\left(x\right)^{2}\log\left(-\frac{1}{4}\,\cos\left(x\right)^{2}+\frac{1}{4}\right)-1}{2\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)^2*log(cos(x)^2) - cos(x)^2*log(-1/4*cos(x)^2 + 1/4) - 1)/cos(x
)^2
```

Sympy [B] time = 1.64249, size = 29, normalized size = 2.42

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2}+\frac{\log\left(\cos\left(x\right)+1\right)}{2}-\log\left(\cos\left(x\right)\right)+\frac{1}{2\cos^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)**3-cos(x)**5),x)
```

```
[Out] log(cos(x) - 1)/2 + log(cos(x) + 1)/2 - log(cos(x)) + 1/(2*cos(x)**2)
```

Giac [B] time = 1.0692, size = 42, normalized size = 3.5

$$\frac{\cos\left(x\right)^{2}+1}{2\,\cos\left(x\right)^{2}}-\frac{1}{2}\,\log\left(\cos\left(x\right)^{2}\right)+\frac{1}{2}\,\log\left(-\cos\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="giac")
```

```
[Out] 1/2*(cos(x)^2 + 1)/cos(x)^2 - 1/2*log(cos(x)^2) + 1/2*log(-cos(x)^2 + 1)
```

3.889 $\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx$

Optimal. Leaf size=19

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

```
[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

Rubi [A] time = 0.0396507, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4339, 270}

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]
```

```
[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx = -\text{Subst}\left(\int \frac{\left(11-5x^5\right)^2}{x^{12}}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{121}{x^{12}}-\frac{110}{x^7}+\frac{25}{x^2}\right)dx, x, \cos(x)\right)$$
$$= 25\sec(x)-\frac{55\sec^6(x)}{3}+11\sec^{11}(x)$$

Mathematica [A] time = 0.013386, size = 19, normalized size = 1.

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]
```

```
[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

Maple [A] time = 0.017, size = 18, normalized size = 1.

$$25\sec(x)-\frac{55\left(\sec(x)\right)^6}{3}+11\left(\sec(x)\right)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(5-11*sec(x)^5)^2*tan(x),x)
```

```
[Out] 25*sec(x)-55/3*sec(x)^6+11*sec(x)^11
```

Maxima [A] time = 0.957646, size = 27, normalized size = 1.42

$$\frac{75\cos(x)^{10}-55\cos(x)^5+33}{3\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

Fricas [A] time = 2.52757, size = 66, normalized size = 3.47

$$\frac{75 \cos(x)^{10} - 55 \cos(x)^{5} + 33}{3 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

Sympy [A] time = 33.4383, size = 19, normalized size = 1.

$$11 \sec^{11}(x) - \frac{55 \sec^{6}(x)}{3} + 25 \sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)**5)**2*tan(x),x)
```

```
[Out] 11*sec(x)**11 - 55*sec(x)**6/3 + 25*sec(x)
```

Giac [A] time = 1.07273, size = 27, normalized size = 1.42

$$\frac{75 \cos(x)^{10} - 55 \cos(x)^{5} + 33}{3 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

3.890 $\int \sin^3(5x)\tan^3(5x)\,dx$

Optimal. Leaf size=44

$$\frac{1}{6}\sin^3(5x)+\frac{1}{2}\sin(5x)+\frac{1}{10}\sin^3(5x)\tan^2(5x)-\frac{1}{2}\tanh^{-1}(\sin(5x))$$

```
[Out] -ArcTanh[Sin[5*x]]/2 + Sin[5*x]/2 + Sin[5*x]^3/6 + (Sin[5*x]^3*Tan[5*x]^2)/
10
```

Rubi [A] time = 0.0380145, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2592, 288, 302, 206}

$$\frac{1}{6}\sin^3(5x)+\frac{1}{2}\sin(5x)+\frac{1}{10}\sin^3(5x)\tan^2(5x)-\frac{1}{2}\tanh^{-1}(\sin(5x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]^3*Tan[5*x]^3,x]
```

```
[Out] -ArcTanh[Sin[5*x]]/2 + Sin[5*x]/2 + Sin[5*x]^3/6 + (Sin[5*x]^3*Tan[5*x]^2)/
10
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sin^3(5x)\tan^3(5x)\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \sin(5x)\right)\\ &= \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(5x)\right)\\ &= \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\operatorname{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(5x)\right)\\ &= \frac{1}{2}\sin(5x) + \frac{1}{6}\sin^3(5x) + \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(5x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\sin(5x)) + \frac{1}{2}\sin(5x) + \frac{1}{6}\sin^3(5x) + \frac{1}{10}\sin^3(5x)\tan^2(5x)\end{aligned}$$

Mathematica [A] time = 0.0448652, size = 52, normalized size = 1.18

$$-\frac{1}{15}\sin^3(5x)\tan^2(5x) - \frac{1}{3}\sin(5x)\tan^2(5x) - \frac{1}{2}\tanh^{-1}(\sin(5x)) + \frac{1}{2}\tan(5x)\sec(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]^3*Tan[5*x]^3,x]
```

```
[Out] -ArcTanh[Sin[5*x]]/2 + (Sec[5*x]*Tan[5*x])/2 - (Sin[5*x]*Tan[5*x]^2)/3 - (S
in[5*x]^3*Tan[5*x]^2)/15
```

Maple [A] time = 0.025, size = 50, normalized size = 1.1

$$\frac{(\sin(5x))^7}{10\,(\cos(5x))^2} + \frac{(\sin(5x))^5}{10} + \frac{(\sin(5x))^3}{6} + \frac{\sin(5x)}{2} - \frac{\ln(\sec(5x)+\tan(5x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^3,x)
```

```
[Out] 1/10*sin(5*x)^7/cos(5*x)^2+1/10*sin(5*x)^5+1/6*sin(5*x)^3+1/2*sin(5*x)-1/2*
ln(sec(5*x)+tan(5*x))
```

Maxima [A] time = 0.943373, size = 66, normalized size = 1.5

$$\frac{1}{15}\sin(5x)^3 - \frac{\sin(5x)}{10\left(\sin(5x)^2 - 1\right)} - \frac{1}{4}\log(\sin(5x)+1) + \frac{1}{4}\log(\sin(5x)-1) + \frac{2}{5}\sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="maxima")
```

```
[Out] 1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) +
1/4*log(sin(5*x) - 1) + 2/5*sin(5*x)
```

Fricas [A] time = 2.51175, size = 182, normalized size = 4.14

$$-\frac{15\cos(5x)^2\log(\sin(5x)+1) - 15\cos(5x)^2\log(-\sin(5x)+1) + 2\left(2\cos(5x)^4 - 14\cos(5x)^2 - 3\right)\sin(5x)}{60\cos(5x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(15*cos(5*x)^2*log(sin(5*x) + 1) - 15*cos(5*x)^2*log(-sin(5*x) + 1) +
 2*(2*cos(5*x)^4 - 14*cos(5*x)^2 - 3)*sin(5*x))/cos(5*x)^2
```

Sympy [A] time = 0.115596, size = 51, normalized size = 1.16

$$\frac{\log(\sin(5x)-1)}{4} - \frac{\log(\sin(5x)+1)}{4} + \frac{\sin^3(5x)}{15} + \frac{2\sin(5x)}{5} - \frac{\sin(5x)}{5\left(2\sin^2(5x) - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)**3*tan(5*x)**3,x)
```

```
[Out] log(sin(5*x) - 1)/4 - log(sin(5*x) + 1)/4 + sin(5*x)**3/15 + 2*sin(5*x)/5 -
 sin(5*x)/(5*(2*sin(5*x)**2 - 2))
```

Giac [B] time = 3.06945, size = 657, normalized size = 14.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="giac")
```

```
[Out] -1/60*(15*log(2*(tan(5/2*x)^2 + 2*tan(5/2*x) + 1)/(tan(5/2*x)^2 + 1))*tan(5
/2*x)^10 - 15*log(2*(tan(5/2*x)^2 - 2*tan(5/2*x) + 1)/(tan(5/2*x)^2 + 1))*t
an(5/2*x)^10 + 15*log(2*(tan(5/2*x)^2 + 2*tan(5/2*x) + 1)/(tan(5/2*x)^2 + 1
))*tan(5/2*x)^8 - 15*log(2*(tan(5/2*x)^2 - 2*tan(5/2*x) + 1)/(tan(5/2*x)^2
+ 1))*tan(5/2*x)^8 - 60*tan(5/2*x)^9 - 30*log(2*(tan(5/2*x)^2 + 2*tan(5/2*x
) + 1)/(tan(5/2*x)^2 + 1))*tan(5/2*x)^6 + 30*log(2*(tan(5/2*x)^2 - 2*tan(5/
2*x) + 1)/(tan(5/2*x)^2 + 1))*tan(5/2*x)^6 - 80*tan(5/2*x)^7 - 30*log(2*(ta
n(5/2*x)^2 + 2*tan(5/2*x) + 1)/(tan(5/2*x)^2 + 1))*tan(5/2*x)^4 + 30*log(2*
(tan(5/2*x)^2 - 2*tan(5/2*x) + 1)/(tan(5/2*x)^2 + 1))*tan(5/2*x)^4 + 88*tan
(5/2*x)^5 + 15*log(2*(tan(5/2*x)^2 + 2*tan(5/2*x) + 1)/(tan(5/2*x)^2 + 1))*
tan(5/2*x)^2 - 15*log(2*(tan(5/2*x)^2 - 2*tan(5/2*x) + 1)/(tan(5/2*x)^2 + 1
))*tan(5/2*x)^2 - 80*tan(5/2*x)^3 + 15*log(2*(tan(5/2*x)^2 + 2*tan(5/2*x) +
 1)/(tan(5/2*x)^2 + 1)) - 15*log(2*(tan(5/2*x)^2 - 2*tan(5/2*x) + 1)/(tan(5
/2*x)^2 + 1)) - 60*tan(5/2*x))/(tan(5/2*x)^10 + tan(5/2*x)^8 - 2*tan(5/2*x)
^6 - 2*tan(5/2*x)^4 + tan(5/2*x)^2 + 1)
```

3.891 $\int \sin^3(5x)\tan^4(5x)\,dx$

Optimal. Leaf size=37

$$\frac{1}{15}\cos^3(5x)-\frac{3}{5}\cos(5x)+\frac{1}{15}\sec^3(5x)-\frac{3}{5}\sec(5x)$$

```
[Out] (-3*Cos[5*x])/5 + Cos[5*x]^3/15 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

Rubi [A] time = 0.0328969, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2590, 270}

$$\frac{1}{15}\cos^3(5x)-\frac{3}{5}\cos(5x)+\frac{1}{15}\sec^3(5x)-\frac{3}{5}\sec(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]^3*Tan[5*x]^4,x]
```

```
[Out] (-3*Cos[5*x])/5 + Cos[5*x]^3/15 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sin^3(5x)\tan^4(5x)\,dx = -\left(\frac{1}{5}\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4}\,dx, x, \cos(5x)\right)\right)$$
$$= -\left(\frac{1}{5}\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)dx, x, \cos(5x)\right)\right)$$
$$= -\frac{3}{5}\cos(5x)+\frac{1}{15}\cos^3(5x)-\frac{3}{5}\sec(5x)+\frac{1}{15}\sec^3(5x)$$

Mathematica [A] time = 0.030634, size = 35, normalized size = 0.95

$$-\frac{11}{20}\cos(5x)+\frac{1}{60}\cos(15x)+\frac{1}{15}\sec^3(5x)-\frac{3}{5}\sec(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]^3*Tan[5*x]^4,x]
```

```
[Out] (-11*Cos[5*x])/20 + Cos[15*x]/60 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

Maple [B] time = 0.029, size = 60, normalized size = 1.6

$$\frac{(\sin(5x))^8}{15(\cos(5x))^3}-\frac{(\sin(5x))^8}{3\cos(5x)}-\frac{\cos(5x)}{3}\left(\frac{16}{5}+(\sin(5x))^6+\frac{6(\sin(5x))^4}{5}+\frac{8(\sin(5x))^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^4,x)
```

```
[Out] 1/15*sin(5*x)^8/cos(5*x)^3-1/3*sin(5*x)^8/cos(5*x)-1/3*(16/5+sin(5*x)^6+6/5
*sin(5*x)^4+8/5*sin(5*x)^2)*cos(5*x)
```

Maxima [A] time = 0.957101, size = 45, normalized size = 1.22

$$\frac{1}{15}\cos(5x)^3-\frac{9\cos(5x)^2-1}{15\cos(5x)^3}-\frac{3}{5}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="maxima")
```

```
[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

Fricas [A] time = 2.40515, size = 86, normalized size = 2.32

$$\frac{\cos(5x)^6 - 9\cos(5x)^4 - 9\cos(5x)^2 + 1}{15\cos(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(cos(5*x)^6 - 9*cos(5*x)^4 - 9*cos(5*x)^2 + 1)/cos(5*x)^3
```

Sympy [A] time = 0.098131, size = 34, normalized size = 0.92

$$-\frac{9\cos^2(5x) - 1}{15\cos^3(5x)} + \frac{\cos^3(5x)}{15} - \frac{3\cos(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)**3*tan(5*x)**4,x)
```

```
[Out] -(9*cos(5*x)**2 - 1)/(15*cos(5*x)**3) + cos(5*x)**3/15 - 3*cos(5*x)/5
```

Giac [A] time = 1.27442, size = 45, normalized size = 1.22

$$\frac{1}{15}\cos(5x)^3 - \frac{9\cos(5x)^2 - 1}{15\cos(5x)^3} - \frac{3}{5}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="giac")
```

```
[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

3.892 $\int \sin^5(6x)\tan^3(6x)\,dx$

Optimal. Leaf size=54

$$\frac{7}{60}\sin^5(6x)+\frac{7}{36}\sin^3(6x)+\frac{7}{12}\sin(6x)+\frac{1}{12}\sin^5(6x)\tan^2(6x)-\frac{7}{12}\tanh^{-1}(\sin(6x))$$

```
[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sin[6*x])/12 + (7*Sin[6*x]^3)/36 + (7*Sin[6*
x]^5)/60 + (Sin[6*x]^5*Tan[6*x]^2)/12
```

Rubi [A] time = 0.0411704, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2592, 288, 302, 206}

$$\frac{7}{60}\sin^5(6x)+\frac{7}{36}\sin^3(6x)+\frac{7}{12}\sin(6x)+\frac{1}{12}\sin^5(6x)\tan^2(6x)-\frac{7}{12}\tanh^{-1}(\sin(6x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[6*x]^5*Tan[6*x]^3,x]
```

```
[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sin[6*x])/12 + (7*Sin[6*x]^3)/36 + (7*Sin[6*
x]^5)/60 + (Sin[6*x]^5*Tan[6*x]^2)/12
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sin^5(6x)\tan^3(6x)\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x^8}{\left(1-x^2\right)^2}\,dx, x, \sin(6x)\right)\\ &= \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\,\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(6x)\right)\\ &= \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\,\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)dx, x, \sin(6x)\right)\\ &= \frac{7}{12}\sin(6x) + \frac{7}{36}\sin^3(6x) + \frac{7}{60}\sin^5(6x) + \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \right.\\ &= -\frac{7}{12}\tanh^{-1}(\sin(6x)) + \frac{7}{12}\sin(6x) + \frac{7}{36}\sin^3(6x) + \frac{7}{60}\sin^5(6x) + \frac{1}{12}\sin^5(6x)\tan^2(6x)\end{aligned}$$

Mathematica [A] time = 0.0917743, size = 68, normalized size = 1.26

$$-\frac{1}{30}\sin^5(6x)\tan^2(6x) - \frac{7}{90}\sin^3(6x)\tan^2(6x) - \frac{7}{18}\sin(6x)\tan^2(6x) - \frac{7}{12}\tanh^{-1}(\sin(6x)) + \frac{7}{12}\tan(6x)\sec(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[6*x]^5*Tan[6*x]^3,x]
```

```
[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sec[6*x]*Tan[6*x])/12 - (7*Sin[6*x]*Tan[6*x]
^2)/18 - (7*Sin[6*x]^3*Tan[6*x]^2)/90 - (Sin[6*x]^5*Tan[6*x]^2)/30
```

Maple [A] time = 0.038, size = 58, normalized size = 1.1

$$\frac{(\sin(6x))^9}{12(\cos(6x))^2} + \frac{(\sin(6x))^7}{12} + \frac{7(\sin(6x))^5}{60} + \frac{7(\sin(6x))^3}{36} + \frac{7\sin(6x)}{12} - \frac{7\ln(\sec(6x)+\tan(6x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^5*tan(6*x)^3,x)
```

```
[Out] 1/12*sin(6*x)^9/cos(6*x)^2+1/12*sin(6*x)^7+7/60*sin(6*x)^5+7/36*sin(6*x)^3+
7/12*sin(6*x)-7/12*ln(sec(6*x)+tan(6*x))
```

Maxima [A] time = 0.941967, size = 77, normalized size = 1.43

$$\frac{1}{30}\sin\left(6x\right)^{5}+\frac{1}{9}\sin\left(6x\right)^{3}-\frac{\sin\left(6x\right)}{12\left(\sin\left(6x\right)^{2}-1\right)}-\frac{7}{24}\log\left(\sin\left(6x\right)+1\right)+\frac{7}{24}\log\left(\sin\left(6x\right)-1\right)+\frac{1}{2}\sin\left(6x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*lo
g(sin(6*x) + 1) + 7/24*log(sin(6*x) - 1) + 1/2*sin(6*x)
```

Fricas [A] time = 2.55227, size = 211, normalized size = 3.91

$$-\frac{105\cos\left(6x\right)^{2}\log\left(\sin\left(6x\right)+1\right)-105\cos\left(6x\right)^{2}\log\left(-\sin\left(6x\right)+1\right)-2\left(6\cos\left(6x\right)^{6}-32\cos\left(6x\right)^{4}+116\cos\left(6x\right)^{2}+\right.}{360\cos\left(6x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/360*(105*cos(6*x)^2*log(sin(6*x) + 1) - 105*cos(6*x)^2*log(-sin(6*x) + 1
) - 2*(6*cos(6*x)^6 - 32*cos(6*x)^4 + 116*cos(6*x)^2 + 15)*sin(6*x))/cos(6*
x)^2
```

Sympy [A] time = 0.1208, size = 61, normalized size = 1.13

$$\frac{7\log\left(\sin\left(6x\right)-1\right)}{24}-\frac{7\log\left(\sin\left(6x\right)+1\right)}{24}+\frac{\sin^{5}\left(6x\right)}{30}+\frac{\sin^{3}\left(6x\right)}{9}+\frac{\sin\left(6x\right)}{2}-\frac{\sin\left(6x\right)}{6\left(2\sin^{2}\left(6x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)**5*tan(6*x)**3,x)
```

```
[Out] 7*log(sin(6*x) - 1)/24 - 7*log(sin(6*x) + 1)/24 + sin(6*x)**5/30 + sin(6*x)
**3/9 + sin(6*x)/2 - sin(6*x)/(6*(2*sin(6*x)**2 - 2))
```

Giac [B] time = 2.93253, size = 890, normalized size = 16.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="giac")
```

```
[Out] -1/360*(105*log(2*(tan(3*x)^2 + 2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^
14 - 105*log(2*(tan(3*x)^2 - 2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^14
+ 315*log(2*(tan(3*x)^2 + 2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^12 - 3
15*log(2*(tan(3*x)^2 - 2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^12 - 420*
tan(3*x)^13 + 105*log(2*(tan(3*x)^2 + 2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan
(3*x)^10 - 105*log(2*(tan(3*x)^2 - 2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*
x)^10 - 1400*tan(3*x)^11 - 525*log(2*(tan(3*x)^2 + 2*tan(3*x) + 1)/(tan(3*x
)^2 + 1))*tan(3*x)^8 + 525*log(2*(tan(3*x)^2 - 2*tan(3*x) + 1)/(tan(3*x)^2
+ 1))*tan(3*x)^8 - 924*tan(3*x)^9 - 525*log(2*(tan(3*x)^2 + 2*tan(3*x) + 1)
/(tan(3*x)^2 + 1))*tan(3*x)^6 + 525*log(2*(tan(3*x)^2 - 2*tan(3*x) + 1)/(ta
n(3*x)^2 + 1))*tan(3*x)^6 + 1648*tan(3*x)^7 + 105*log(2*(tan(3*x)^2 + 2*tan
(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^4 - 105*log(2*(tan(3*x)^2 - 2*tan(3*x
) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^4 - 924*tan(3*x)^5 + 315*log(2*(tan(3*x)^
2 + 2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^2 - 315*log(2*(tan(3*x)^2 -
2*tan(3*x) + 1)/(tan(3*x)^2 + 1))*tan(3*x)^2 - 1400*tan(3*x)^3 + 105*log(2*
(tan(3*x)^2 + 2*tan(3*x) + 1)/(tan(3*x)^2 + 1)) - 105*log(2*(tan(3*x)^2 - 2
*tan(3*x) + 1)/(tan(3*x)^2 + 1)) - 420*tan(3*x))/(tan(3*x)^14 + 3*tan(3*x)^
12 + tan(3*x)^10 - 5*tan(3*x)^8 - 5*tan(3*x)^6 + tan(3*x)^4 + 3*tan(3*x)^2
+ 1)
```

3.893 $\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\cos(2x)+\frac{1}{10}\sec^5(2x)-\frac{1}{2}\sec^3(2x)+\frac{3}{2}\sec(2x)$$

```
[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

Rubi [A] time = 0.0391507, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 2590, 270}

$$\frac{1}{2}\cos(2x)+\frac{1}{10}\sec^5(2x)-\frac{1}{2}\sec^3(2x)+\frac{3}{2}\sec(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
```

```
[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx &= \int \sin(2x)\tan^6(2x)\,dx\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^6}\,dx, x, \cos(2x)\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)dx, x, \cos(2x)\right)\right)\\ &= \frac{1}{2}\cos(2x)+\frac{3}{2}\sec(2x)-\frac{1}{2}\sec^3(2x)+\frac{1}{10}\sec^5(2x)\end{aligned}$$

Mathematica [A] time = 0.0292605, size = 37, normalized size = 1.

$$\frac{1}{2}\cos(2x)+\frac{1}{10}\sec^5(2x)-\frac{1}{2}\sec^3(2x)+\frac{3}{2}\sec(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
```

```
[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

Maple [A] time = 0.031, size = 32, normalized size = 0.9

$$\frac{1}{10\,(\cos(2\,x))^5}-\frac{1}{2\,(\cos(2\,x))^3}+\frac{3}{2\,\cos(2\,x)}+\frac{\cos(2\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sec(2*x)^2)^3*sin(2*x),x)
```

```
[Out] 1/10/cos(2*x)^5-1/2/cos(2*x)^3+3/2/cos(2*x)+1/2*cos(2*x)
```

Maxima [A] time = 0.947299, size = 42, normalized size = 1.14

$$\frac{3}{2\,\cos(2\,x)}-\frac{1}{2\,\cos(2\,x)^3}+\frac{1}{10\,\cos(2\,x)^5}+\frac{1}{2}\,\cos(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="maxima")
```

```
[Out] 3/2/cos(2*x) - 1/2/cos(2*x)^3 + 1/10/cos(2*x)^5 + 1/2*cos(2*x)
```

Fricas [A] time = 2.35321, size = 90, normalized size = 2.43

$$\frac{5\cos(2x)^6 + 15\cos(2x)^4 - 5\cos(2x)^2 + 1}{10\cos(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/10*(5*cos(2*x)^6 + 15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)**2)**3*sin(2*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07006, size = 45, normalized size = 1.22

$$\frac{15\cos(2x)^4 - 5\cos(2x)^2 + 1}{10\cos(2x)^5} + \frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="giac")
```

```
[Out] 1/10*(15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5 + 1/2*cos(2*x)
```

3.894 $\int \sin(x)\tan^5(x)\,dx$

Optimal. Leaf size=34

$$-\frac{15\sin(x)}{8}+\frac{1}{4}\sin(x)\tan^4(x)-\frac{5}{8}\sin(x)\tan^2(x)+\frac{15}{8}\tanh^{-1}(\sin(x))$$

```
[Out] (15*ArcTanh[Sin[x]])/8 - (15*Sin[x])/8 - (5*Sin[x]*Tan[x]^2)/8 + (Sin[x]*Ta
n[x]^4)/4
```

Rubi [A] time = 0.0249877, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2592, 288, 321, 206}

$$-\frac{15\sin(x)}{8}+\frac{1}{4}\sin(x)\tan^4(x)-\frac{5}{8}\sin(x)\tan^2(x)+\frac{15}{8}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[x]])/8 - (15*Sin[x])/8 - (5*Sin[x]*Tan[x]^2)/8 + (Sin[x]*Ta
n[x]^4)/4
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan^5(x)\,dx &= \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx,x,\sin(x)\right)\\ &= \frac{1}{4}\sin(x)\tan^4(x)-\frac{5}{4}\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x,\sin(x)\right)\\ &= -\frac{5}{8}\sin(x)\tan^2(x)+\frac{1}{4}\sin(x)\tan^4(x)+\frac{15}{8}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(x)\right)\\ &= -\frac{15\sin(x)}{8}-\frac{5}{8}\sin(x)\tan^2(x)+\frac{1}{4}\sin(x)\tan^4(x)+\frac{15}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(x)\right)\\ &= \frac{15}{8}\tanh^{-1}(\sin(x))-\frac{15\sin(x)}{8}-\frac{5}{8}\sin(x)\tan^2(x)+\frac{1}{4}\sin(x)\tan^4(x)\end{aligned}$$

Mathematica [A] time = 0.0095064, size = 42, normalized size = 1.24

$$-\sin(x)\tan^4(x)+\frac{15}{8}\tanh^{-1}(\sin(x))-\frac{15}{4}\tan(x)\sec^3(x)+5\tan^3(x)\sec(x)+\frac{15}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[x]])/8 + (15*Sec[x]*Tan[x])/8 - (15*Sec[x]^3*Tan[x])/4 + 5*
Sec[x]*Tan[x]^3 - Sin[x]*Tan[x]^4
```

Maple [A] time = 0.013, size = 46, normalized size = 1.4

$$\frac{(\sin(x))^7}{4(\cos(x))^4} - \frac{3(\sin(x))^7}{8(\cos(x))^2} - \frac{3(\sin(x))^5}{8} - \frac{5(\sin(x))^3}{8} - \frac{15\sin(x)}{8} + \frac{15\ln(\sec(x)+\tan(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^5,x)
```

```
[Out] 1/4*sin(x)^7/cos(x)^4-3/8*sin(x)^7/cos(x)^2-3/8*sin(x)^5-5/8*sin(x)^3-15/8*
sin(x)+15/8*ln(sec(x)+tan(x))
```

Maxima [A] time = 0.945148, size = 62, normalized size = 1.82

$$\frac{9\sin(x)^3 - 7\sin(x)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{15}{16}\log(\sin(x)+1) - \frac{15}{16}\log(\sin(x)-1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 15/16*log(sin(x)
+ 1) - 15/16*log(sin(x) - 1) - sin(x)
```

Fricas [A] time = 2.59876, size = 158, normalized size = 4.65

$$\frac{15\cos(x)^4\log(\sin(x)+1) - 15\cos(x)^4\log(-\sin(x)+1) - 2\left(8\cos(x)^4 + 9\cos(x)^2 - 2\right)\sin(x)}{16\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(15*cos(x)^4*log(sin(x) + 1) - 15*cos(x)^4*log(-sin(x) + 1) - 2*(8*cos
(x)^4 + 9*cos(x)^2 - 2)*sin(x))/cos(x)^4
```

Sympy [A] time = 0.149226, size = 49, normalized size = 1.44

$$\frac{9\sin^3(x) - 7\sin(x)}{8\sin^4(x) - 16\sin^2(x) + 8} - \frac{15\log(\sin(x) - 1)}{16} + \frac{15\log(\sin(x) + 1)}{16} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)**5,x)
```

```
[Out] (9*sin(x)**3 - 7*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*log(sin(x) -
 1)/16 + 15*log(sin(x) + 1)/16 - sin(x)
```

Giac [A] time = 1.06898, size = 57, normalized size = 1.68

$$\frac{9\sin(x)^3 - 7\sin(x)}{8\left(\sin(x)^2 - 1\right)^2} + \frac{15}{16}\log(\sin(x) + 1) - \frac{15}{16}\log(-\sin(x) + 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^2 - 1)^2 + 15/16*log(sin(x) + 1) - 15/1
6*log(-sin(x) + 1) - sin(x)
```

3.895 $\int \cos^5(2x) \cot^4(2x) \, dx$

Optimal. Leaf size=43

$$\frac{1}{10}\sin^5(2x) - \frac{2}{3}\sin^3(2x) + 3\sin(2x) - \frac{1}{6}\csc^3(2x) + 2\csc(2x)$$

```
[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

Rubi [A] time = 0.0360787, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2590, 270}

$$\frac{1}{10}\sin^5(2x) - \frac{2}{3}\sin^3(2x) + 3\sin(2x) - \frac{1}{6}\csc^3(2x) + 2\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^5*Cot[2*x]^4,x]
```

```
[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(2x)\cot^4(2x)\,dx = -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^4}\,dx, x, -\sin(2x)\right)\right)$$
$$= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)dx, x, -\sin(2x)\right)\right)$$
$$= 2\csc(2x)-\frac{1}{6}\csc^3(2x)+3\sin(2x)-\frac{2}{3}\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

Mathematica [A] time = 0.0259009, size = 43, normalized size = 1.

$$\frac{1}{10}\sin^5(2x)-\frac{2}{3}\sin^3(2x)+3\sin(2x)-\frac{1}{6}\csc^3(2x)+2\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^5*Cot[2*x]^4,x]
```

```
[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

Maple [A] time = 0.054, size = 68, normalized size = 1.6

$$-\frac{(\cos(2x))^{10}}{6(\sin(2x))^3}+\frac{7(\cos(2x))^{10}}{6\sin(2x)}+\frac{7\sin(2x)}{6}\left(\frac{128}{35}+(\cos(2x))^8+\frac{8(\cos(2x))^6}{7}+\frac{48(\cos(2x))^4}{35}+\frac{64(\cos(2x))^2}{35}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^5*cot(2*x)^4,x)
```

```
[Out] -1/6/sin(2*x)^3*cos(2*x)^10+7/6/sin(2*x)*cos(2*x)^10+7/6*(128/35+cos(2*x)^8
+8/7*cos(2*x)^6+48/35*cos(2*x)^4+64/35*cos(2*x)^2)*sin(2*x)
```

Maxima [A] time = 0.939942, size = 55, normalized size = 1.28

$$\frac{1}{10}\sin(2x)^5-\frac{2}{3}\sin(2x)^3+\frac{12\sin(2x)^2-1}{6\sin(2x)^3}+3\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="maxima")
```

```
[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*s
in(2*x)
```

Fricas [A] time = 2.41084, size = 140, normalized size = 3.26

$$-\frac{3\cos(2x)^8 + 8\cos(2x)^6 + 48\cos(2x)^4 - 192\cos(2x)^2 + 128}{30\left(\cos(2x)^2 - 1\right)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="fricas")
```

```
[Out] -1/30*(3*cos(2*x)^8 + 8*cos(2*x)^6 + 48*cos(2*x)^4 - 192*cos(2*x)^2 + 128)/
((cos(2*x)^2 - 1)*sin(2*x))
```

Sympy [A] time = 0.098547, size = 42, normalized size = 0.98

$$\frac{12\sin^2(2x) - 1}{6\sin^3(2x)} + \frac{\sin^5(2x)}{10} - \frac{2\sin^3(2x)}{3} + 3\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**5*cot(2*x)**4,x)
```

```
[Out] (12*sin(2*x)**2 - 1)/(6*sin(2*x)**3) + sin(2*x)**5/10 - 2*sin(2*x)**3/3 + 3
*sin(2*x)
```

Giac [A] time = 1.08401, size = 55, normalized size = 1.28

$$\frac{1}{10}\sin(2x)^5 - \frac{2}{3}\sin(2x)^3 + \frac{12\sin(2x)^2 - 1}{6\sin(2x)^3} + 3\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="giac")
```

```
[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*s
in(2*x)
```

3.896 $\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx$

Optimal. Leaf size=87

$$\frac{1}{33}\sin^{11}(3x)-\frac{8}{27}\sin^9(3x)+\frac{4}{3}\sin^7(3x)-\frac{56}{15}\sin^5(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{3}\sin(3x)-\frac{1}{15}\csc^5(3x)+\frac{8}{9}\csc^3(3x)-\frac{28}{3}\text{cs}$$

```
[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70
*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27
+ Sin[3*x]^11/33
```

Rubi [A] time = 0.130178, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4120, 2590, 270}

$$\frac{1}{33}\sin^{11}(3x)-\frac{8}{27}\sin^9(3x)+\frac{4}{3}\sin^7(3x)-\frac{56}{15}\sin^5(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{3}\sin(3x)-\frac{1}{15}\csc^5(3x)+\frac{8}{9}\csc^3(3x)-\frac{28}{3}\text{cs}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

```
[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70
*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27
+ Sin[3*x]^11/33
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx &= \int \cos^{11}(3x)\left(-1+\csc^2(3x)\right)^3\,dx\\ &= \int \cos^{11}(3x)\cot^6(3x)\,dx\\ &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^8}{x^6}\,dx, x, -\sin(3x)\right)\right)\\ &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \left(-56+\frac{1}{x^6}-\frac{8}{x^4}+\frac{28}{x^2}+70x^2-56x^4+28x^6-8x^8+x^1\right.\right.\right.\\ &= -\frac{28}{3}\csc(3x)+\frac{8}{9}\csc^3(3x)-\frac{1}{15}\csc^5(3x)-\frac{56}{3}\sin(3x)+\frac{70}{9}\sin^3(3x)-\end{aligned}$$

Mathematica [A] time = 0.0604997, size = 87, normalized size = 1.

$$\frac{1}{33}\sin^{11}(3x)-\frac{8}{27}\sin^9(3x)+\frac{4}{3}\sin^7(3x)-\frac{56}{15}\sin^5(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{3}\sin(3x)-\frac{1}{15}\csc^5(3x)+\frac{8}{9}\csc^3(3x)-\frac{28}{3}\csc($$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

```
[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70
*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27
+ Sin[3*x]^11/33
```

Maple [A] time = 0.056, size = 72, normalized size = 0.8

$$\frac{(\sin(3x))^{11}}{33}-\frac{8(\sin(3x))^9}{27}+\frac{4(\sin(3x))^7}{3}-\frac{56(\sin(3x))^5}{15}+\frac{70(\sin(3x))^3}{9}-\frac{56\sin(3x)}{3}-\frac{28}{3\sin(3x)}+\frac{8}{9(\sin(3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x)
```

```
[Out] 1/33*sin(3*x)^11-8/27*sin(3*x)^9+4/3*sin(3*x)^7-56/15*sin(3*x)^5+70/9*sin(3
*x)^3-56/3*sin(3*x)-28/3/sin(3*x)+8/9/sin(3*x)^3-1/15/sin(3*x)^5
```

Maxima [A] time = 0.943058, size = 99, normalized size = 1.14

$$\frac{1}{33}\sin(3x)^{11}-\frac{8}{27}\sin(3x)^{9}+\frac{4}{3}\sin(3x)^{7}-\frac{56}{15}\sin(3x)^{5}+\frac{70}{9}\sin(3x)^{3}-\frac{420\sin(3x)^{4}-40\sin(3x)^{2}+3}{45\sin(3x)^{5}}-\frac{56}{3}\,\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="maxima"
)
```

```
[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70
/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3
*sin(3*x)
```

Fricas [A] time = 2.70848, size = 275, normalized size = 3.16

$$\frac{45\cos(3x)^{16}+80\cos(3x)^{14}+160\cos(3x)^{12}+384\cos(3x)^{10}+1280\cos(3x)^{8}+10240\cos(3x)^{6}-61440\cos(3}{1485\left(\cos(3x)^{4}-2\cos(3x)^{2}+1\right)\sin(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="fricas"
)
```

```
[Out] 1/1485*(45*cos(3*x)^16 + 80*cos(3*x)^14 + 160*cos(3*x)^12 + 384*cos(3*x)^10
 + 1280*cos(3*x)^8 + 10240*cos(3*x)^6 - 61440*cos(3*x)^4 + 81920*cos(3*x)^2
 - 32768)/((cos(3*x)^4 - 2*cos(3*x)^2 + 1)*sin(3*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18115, size = 99, normalized size = 1.14

$$\frac{1}{33}\sin(3x)^{11} - \frac{8}{27}\sin(3x)^{9} + \frac{4}{3}\sin(3x)^{7} - \frac{56}{15}\sin(3x)^{5} + \frac{70}{9}\sin(3x)^{3} - \frac{420\sin(3x)^{4} - 40\sin(3x)^{2} + 3}{45\sin(3x)^{5}} - \frac{56}{3}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="giac")
```

```
[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70
/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3
*sin(3*x)
```

3.897 $\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx$

Optimal. Leaf size=42

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Rubi [A] time = 0.118309, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4360, 266, 43}

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4360

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx &= \int \cos^4(2x)\cot(2x)\left(-1+\csc^2(2x)\right)^2\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^5}\,dx, x, \sin(2x)\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(2x)\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, \sin^2(2x)\right)\\ &= \csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)\end{aligned}$$

Mathematica [A] time = 0.0341055, size = 42, normalized size = 1.

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Maple [A] time = 0.05, size = 37, normalized size = 0.9

$$\frac{(\sin(2x))^4}{8}+(\cos(2x))^2+3\ln(\sin(2x))+(\sin(2x))^{-2}-\frac{1}{8(\sin(2x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x)
```

```
[Out] 1/8*sin(2*x)^4+cos(2*x)^2+3*ln(sin(2*x))+1/sin(2*x)^2-1/8/sin(2*x)^4
```

Maxima [A] time = 0.939247, size = 59, normalized size = 1.4

$$\frac{1}{8}\sin(2x)^4 - \sin(2x)^2 + \frac{8\sin(2x)^2 - 1}{8\sin(2x)^4} + \frac{3}{2}\log\left(\sin(2x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(s
in(2*x)^2)
```

Fricas [B] time = 2.51972, size = 220, normalized size = 5.24

$$\frac{8\cos(2x)^8 + 32\cos(2x)^6 - 115\cos(2x)^4 + 38\cos(2x)^2 + 192\left(\cos(2x)^4 - 2\cos(2x)^2 + 1\right)\log\left(\frac{1}{2}\sin(2x)\right) + 29}{64\left(\cos(2x)^4 - 2\cos(2x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(
cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(
2*x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)**2)**2*(1-sin(2*x)**2)**2,x)
```

[Out] Timed out

Giac [A] time = 1.09308, size = 70, normalized size = 1.67

$$\frac{1}{8}\sin\left(2x\right)^{4}-\sin\left(2x\right)^{2}-\frac{18\sin\left(2x\right)^{4}-8\sin\left(2x\right)^{2}+1}{8\sin\left(2x\right)^{4}}+\frac{3}{2}\log\left(\sin\left(2x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 - 1/8*(18*sin(2*x)^4 - 8*sin(2*x)^2 + 1)/sin(2*
x)^4 + 3/2*log(sin(2*x)^2)
```

3.898 $\int \cos(2x) \left(-1+\csc^2(2x)\right)^4 \left(1-\sin^2(2x)\right)^2 \, dx$

Optimal. Leaf size=63

$$\frac{1}{10}\sin^5(2x)-\sin^3(2x)+\frac{15}{2}\sin(2x)-\frac{1}{14}\csc^7(2x)+\frac{3}{5}\csc^5(2x)-\frac{5}{2}\csc^3(2x)+10\csc(2x)$$

```
[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin
[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

Rubi [A] time = 0.123567, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4120, 2590, 270}

$$\frac{1}{10}\sin^5(2x)-\sin^3(2x)+\frac{15}{2}\sin(2x)-\frac{1}{14}\csc^7(2x)+\frac{3}{5}\csc^5(2x)-\frac{5}{2}\csc^3(2x)+10\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin
[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx &= \int \cos^5(2x)\left(-1+\csc^2(2x)\right)^4\,dx\\ &= \int \cos^5(2x)\cot^8(2x)\,dx\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^6}{x^8}\,dx, x, -\sin(2x)\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(15+\frac{1}{x^8}-\frac{6}{x^6}+\frac{15}{x^4}-\frac{20}{x^2}-6x^2+x^4\right)dx, x, -\sin(2x)\right)\right.\\ &= 10\csc(2x)-\frac{5}{2}\csc^3(2x)+\frac{3}{5}\csc^5(2x)-\frac{1}{14}\csc^7(2x)+\frac{15}{2}\sin(2x)-\sin\end{aligned}$$

Mathematica [A] time = 0.0306683, size = 63, normalized size = 1.

$$\frac{1}{10}\sin^5(2x)-\sin^3(2x)+\frac{15}{2}\sin(2x)-\frac{1}{14}\csc^7(2x)+\frac{3}{5}\csc^5(2x)-\frac{5}{2}\csc^3(2x)+10\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin
[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

Maple [A] time = 0.051, size = 56, normalized size = 0.9

$$\frac{(\sin(2x))^5}{10}-(\sin(2x))^3+\frac{15\sin(2x)}{2}+10(\sin(2x))^{-1}-\frac{5}{2(\sin(2x))^3}+\frac{3}{5(\sin(2x))^5}-\frac{1}{14(\sin(2x))^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x)
```

```
[Out] 1/10*sin(2*x)^5-sin(2*x)^3+15/2*sin(2*x)+10/sin(2*x)-5/2/sin(2*x)^3+3/5/sin
(2*x)^5-1/14/sin(2*x)^7
```

Maxima [A] time = 0.955094, size = 77, normalized size = 1.22

$$\frac{1}{10}\sin(2x)^5-\sin(2x)^3+\frac{700\sin(2x)^6-175\sin(2x)^4+42\sin(2x)^2-5}{70\sin(2x)^7}+\frac{15}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] 1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*s
in(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

Fricas [A] time = 2.42875, size = 238, normalized size = 3.78

$$-\frac{7\cos(2x)^{12}+28\cos(2x)^{10}+280\cos(2x)^8-2240\cos(2x)^6+4480\cos(2x)^4-3584\cos(2x)^2+1024}{70\left(\cos(2x)^6-3\cos(2x)^4+3\cos(2x)^2-1\right)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] -1/70*(7*cos(2*x)^12 + 28*cos(2*x)^10 + 280*cos(2*x)^8 - 2240*cos(2*x)^6 +
4480*cos(2*x)^4 - 3584*cos(2*x)^2 + 1024)/((cos(2*x)^6 - 3*cos(2*x)^4 + 3*c
os(2*x)^2 - 1)*sin(2*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)**2)**4*(1-sin(2*x)**2)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.09792, size = 77, normalized size = 1.22

$$\frac{1}{10} \sin(2x)^5 - \sin(2x)^3 + \frac{700 \sin(2x)^6 - 175 \sin(2x)^4 + 42 \sin(2x)^2 - 5}{70 \sin(2x)^7} + \frac{15}{2} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*s
in(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

3.899 $\int \cot(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^2 dx$

Optimal. Leaf size=60

$$-\frac{1}{12}\sin^4(3x)+\frac{5}{6}\sin^2(3x)-\frac{1}{18}\csc^6(3x)+\frac{5}{12}\csc^4(3x)-\frac{5}{3}\csc^2(3x)-\frac{10}{3}\log(\sin(3x))$$

```
[Out] (-5*Csc[3*x]^2)/3 + (5*Csc[3*x]^4)/12 - Csc[3*x]^6/18 - (10*Log[Sin[3*x]])/
3 + (5*Sin[3*x]^2)/6 - Sin[3*x]^4/12
```

Rubi [A] time = 0.126059, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4360, 266, 43}

$$-\frac{1}{12}\sin^4(3x)+\frac{5}{6}\sin^2(3x)-\frac{1}{18}\csc^6(3x)+\frac{5}{12}\csc^4(3x)-\frac{5}{3}\csc^2(3x)-\frac{10}{3}\log(\sin(3x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
```

```
[Out] (-5*Csc[3*x]^2)/3 + (5*Csc[3*x]^4)/12 - Csc[3*x]^6/18 - (10*Log[Sin[3*x]])/
3 + (5*Sin[3*x]^2)/6 - Sin[3*x]^4/12
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4360

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx &= \int \cos^4(3x)\cot(3x)\left(-1+\csc^2(3x)\right)^3\,dx\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^5}{x^7}\,dx, x, \sin(3x)\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{(1-x)^5}{x^4}\,dx, x, \sin^2(3x)\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \left(5+\frac{1}{x^4}-\frac{5}{x^3}+\frac{10}{x^2}-\frac{10}{x}-x\right)dx, x, \sin^2(3x)\right)\\ &= -\frac{5}{3}\csc^2(3x)+\frac{5}{12}\csc^4(3x)-\frac{1}{18}\csc^6(3x)-\frac{10}{3}\log(\sin(3x))+\frac{5}{6}\sin^2(3\end{aligned}$$

Mathematica [A] time = 0.123967, size = 52, normalized size = 0.87

$$\frac{1}{36}\left(-3\sin^4(3x)+30\sin^2(3x)-2\csc^6(3x)+15\csc^4(3x)-60\csc^2(3x)-120\log(\sin(3x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
```

```
[Out] (-60*Csc[3*x]^2 + 15*Csc[3*x]^4 - 2*Csc[3*x]^6 - 120*Log[Sin[3*x]] + 30*Sin
[3*x]^2 - 3*Sin[3*x]^4)/36
```

Maple [A] time = 0.056, size = 49, normalized size = 0.8

$$-\frac{(\sin(3x))^4}{12}-\frac{5(\cos(3x))^2}{6}-\frac{10\ln(\sin(3x))}{3}-\frac{5}{3(\sin(3x))^2}+\frac{5}{12(\sin(3x))^4}-\frac{1}{18(\sin(3x))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x)
```

```
[Out] -1/12*sin(3*x)^4-5/6*cos(3*x)^2-10/3*ln(sin(3*x))-5/3/sin(3*x)^2+5/12/sin(3
*x)^4-1/18/sin(3*x)^6
```

Maxima [A] time = 0.944821, size = 70, normalized size = 1.17

$$-\frac{1}{12}\sin(3x)^4+\frac{5}{6}\sin(3x)^2-\frac{60\sin(3x)^4-15\sin(3x)^2+2}{36\sin(3x)^6}-\frac{5}{3}\log\left(\sin(3x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] -1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 - 1/36*(60*sin(3*x)^4 - 15*sin(3*x)^2 + 2
)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

Fricas [B] time = 2.28558, size = 293, normalized size = 4.88

$$-\frac{24\cos(3x)^{10}+120\cos(3x)^8-609\cos(3x)^6+387\cos(3x)^4+333\cos(3x)^2+960\left(\cos(3x)^6-3\cos(3x)^4+3\right.}{288\left(\cos(3x)^6-3\cos(3x)^4+3\cos(3x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] -1/288*(24*cos(3*x)^10 + 120*cos(3*x)^8 - 609*cos(3*x)^6 + 387*cos(3*x)^4 +
 333*cos(3*x)^2 + 960*(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)*log(1/
2*sin(3*x)) - 271)/(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12859, size = 81, normalized size = 1.35

$$-\frac{1}{12}\sin(3x)^4+\frac{5}{6}\sin(3x)^2+\frac{110\sin(3x)^6-60\sin(3x)^4+15\sin(3x)^2-2}{36\sin(3x)^6}-\frac{5}{3}\log\left(\sin(3x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 + 1/36*(110*sin(3*x)^6 - 60*sin(3*x)^4 +
15*sin(3*x)^2 - 2)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

3.900 $\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 \, dx$

Optimal. Leaf size=47

$$\frac{1}{45}\tan^5(9x)+\frac{4}{27}\tan^3(9x)+\frac{2}{3}\tan(9x)-\frac{1}{27}\cot^3(9x)-\frac{4}{9}\cot(9x)$$

```
[Out] (-4*Cot[9*x])/9 - Cot[9*x]^3/27 + (2*Tan[9*x])/3 + (4*Tan[9*x]^3)/27 + Tan[
9*x]^5/45
```

Rubi [A] time = 0.102327, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3657, 2620, 270}

$$\frac{1}{45}\tan^5(9x)+\frac{4}{27}\tan^3(9x)+\frac{2}{3}\tan(9x)-\frac{1}{27}\cot^3(9x)-\frac{4}{9}\cot(9x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

```
[Out] (-4*Cot[9*x])/9 - Cot[9*x]^3/27 + (2*Tan[9*x])/3 + (4*Tan[9*x]^3)/27 + Tan[
9*x]^5/45
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3\,dx &= \int \left(1+\cot^2(9x)\right)^2 \sec^6(9x)\,dx\\ &= \int \csc^4(9x)\sec^6(9x)\,dx\\ &= \frac{1}{9}\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^4}\,dx, x, \tan(9x)\right)\\ &= \frac{1}{9}\operatorname{Subst}\left(\int \left(6+\frac{1}{x^4}+\frac{4}{x^2}+4x^2+x^4\right)dx, x, \tan(9x)\right)\\ &= -\frac{4}{9}\cot(9x)-\frac{1}{27}\cot^3(9x)+\frac{2}{3}\tan(9x)+\frac{4}{27}\tan^3(9x)+\frac{1}{45}\tan^5(9x)\end{aligned}$$

Mathematica [A] time = 0.0500226, size = 59, normalized size = 1.26

$$\frac{73}{135}\tan(9x)-\frac{11}{27}\cot(9x)-\frac{1}{27}\cot(9x)\csc^2(9x)+\frac{1}{45}\tan(9x)\sec^4(9x)+\frac{14}{135}\tan(9x)\sec^2(9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

```
[Out] (-11*Cot[9*x])/27 - (Cot[9*x]*Csc[9*x]^2)/27 + (73*Tan[9*x])/135 + (14*Sec[
9*x]^2*Tan[9*x])/135 + (Sec[9*x]^4*Tan[9*x])/45
```

Maple [A] time = 0.044, size = 38, normalized size = 0.8

$$-\frac{4\cot(9x)}{9}-\frac{(\cot(9x))^3}{27}+\frac{2\tan(9x)}{3}+\frac{4(\tan(9x))^3}{27}+\frac{(\tan(9x))^5}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x)
```

```
[Out] -4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5
```

Maxima [A] time = 0.949618, size = 55, normalized size = 1.17

$$\frac{1}{45}\tan(9x)^5+\frac{4}{27}\tan(9x)^3-\frac{12\tan(9x)^2+1}{27\tan(9x)^3}+\frac{2}{3}\tan(9x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/45*tan(9*x)^5 + 4/27*tan(9*x)^3 - 1/27*(12*tan(9*x)^2 + 1)/tan(9*x)^3 + 2
/3*tan(9*x)
```

Fricas [A] time = 2.06821, size = 115, normalized size = 2.45

$$\frac{3\tan(9x)^8+20\tan(9x)^6+90\tan(9x)^4-60\tan(9x)^2-5}{135\tan(9x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/135*(3*tan(9*x)^8 + 20*tan(9*x)^6 + 90*tan(9*x)^4 - 60*tan(9*x)^2 - 5)/ta
n(9*x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)**2)**2*(1+tan(9*x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.901 $\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}+63\sin^2(x)-49\sin(x)-2\log(1-\sin(x))+128\log(\sin(x)+1)$$

```
[Out] -2*Log[1 - Sin[x]] + 128*Log[1 + Sin[x]] - 49*Sin[x] + 63*Sin[x]^2 - (49*Si
n[x]^3)/3 - (49*Sin[x]^5)/5
```

Rubi [A] time = 0.115741, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3175, 3223, 1810, 633, 31}

$$-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}+63\sin^2(x)-49\sin(x)-2\log(1-\sin(x))+128\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

```
[Out] -2*Log[1 - Sin[x]] + 128*Log[1 + Sin[x]] - 49*Sin[x] + 63*Sin[x]^2 - (49*Si
n[x]^3)/3 - (49*Sin[x]^5)/5
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx &= \int \sec(x)\left(9-7\sin^3(x)\right)^2\,dx\\
&= \text{Subst}\left(\int \frac{\left(9-7x^3\right)^2}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(-49+126x-49x^2-49x^4+\frac{2(65-63x)}{1-x^2}\right)dx, x, \sin(x)\right)\\
&= -49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}+2\,\text{Subst}\left(\int \frac{65-63x}{1-x^2}\,dx, x, \sin(x)\right)\\
&= -49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}+2\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sin(x)\right)-12\\
&= -2\log(1-\sin(x))+128\log(1+\sin(x))-49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.0220506, size = 71, normalized size = 1.65

$$-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}-49\sin(x)-63\cos^2(x)+49\tanh^{-1}(\sin(x))+126\log(\cos(x))-81\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

```
[Out] 49*ArcTanh[Sin[x]] - 63*Cos[x]^2 + 126*Log[Cos[x]] - 81*Log[Cos[x/2] - Sin[
x/2]] + 81*Log[Cos[x/2] + Sin[x/2]] - 49*Sin[x] - (49*Sin[x]^3)/3 - (49*Sin
[x]^5)/5
```

Maple [A] time = 0.024, size = 38, normalized size = 0.9

$$-\frac{49\,(\sin(x))^5}{5}-\frac{49\,(\sin(x))^3}{3}+63\,(\sin(x))^2-49\,\sin(x)-2\,\ln(\sin(x)-1)+128\,\ln(1+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x)
```

```
[Out] -49/5*sin(x)^5-49/3*sin(x)^3+63*sin(x)^2-49*sin(x)-2*ln(sin(x)-1)+128*ln(1+
sin(x))
```

Maxima [A] time = 0.94411, size = 50, normalized size = 1.16

$$-\frac{49}{5}\,\sin(x)^5-\frac{49}{3}\,\sin(x)^3+63\,\sin(x)^2+128\,\log(\sin(x)+1)-2\,\log(\sin(x)-1)-49\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="maxima")
```

```
[Out] -49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log(
sin(x) - 1) - 49*sin(x)
```

Fricas [A] time = 2.23806, size = 140, normalized size = 3.26

$$-63\,\cos(x)^2-\frac{49}{15}\left(3\,\cos(x)^4-11\,\cos(x)^2+23\right)\sin(x)+128\,\log(\sin(x)+1)-2\,\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="fricas")
```

```
[Out] -63*cos(x)^2 - 49/15*(3*cos(x)^4 - 11*cos(x)^2 + 23)*sin(x) + 128*log(sin(x
) + 1) - 2*log(-sin(x) + 1)
```

Sympy [A] time = 4.25094, size = 44, normalized size = 1.02

$$-2\log(\sin(x)-1)+128\log(\sin(x)+1)-\frac{49\sin^5(x)}{5}-\frac{49\sin^3(x)}{3}-49\sin(x)-63\cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)**3)**2/(1-sin(x)**2),x)
```

```
[Out] -2*log(sin(x) - 1) + 128*log(sin(x) + 1) - 49*sin(x)**5/5 - 49*sin(x)**3/3
- 49*sin(x) - 63*cos(x)**2
```

Giac [A] time = 1.11644, size = 53, normalized size = 1.23

$$-\frac{49}{5}\sin(x)^5-\frac{49}{3}\sin(x)^3+63\sin(x)^2+128\log(\sin(x)+1)-2\log(-\sin(x)+1)-49\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="giac")
```

```
[Out] -49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log(
-sin(x) + 1) - 49*sin(x)
```

3.902 $\int \cos^4(2x)\cot^5(2x)\,dx$

Optimal. Leaf size=42

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Rubi [A] time = 0.0406121, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2590, 266, 43}

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^4*Cot[2*x]^5,x]
```


```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```


Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^4(2x)\cot^5(2x)\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^5}\,dx, x, -\sin(2x)\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(2x)\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, \sin^2(2x)\right)$$
$$= \csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)$$

Mathematica [A] time = 0.0279163, size = 42, normalized size = 1.

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^4*Cot[2*x]^5,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Maple [A] time = 0.027, size = 69, normalized size = 1.6

$$-\frac{(\cos(2x))^{10}}{8(\sin(2x))^4}+\frac{3(\cos(2x))^{10}}{8(\sin(2x))^2}+\frac{3(\cos(2x))^8}{8}+\frac{(\cos(2x))^6}{2}+\frac{3(\cos(2x))^4}{4}+\frac{3(\cos(2x))^2}{2}+3\ln(\sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*cot(2*x)^5,x)
```

```
[Out] -1/8/sin(2*x)^4*cos(2*x)^10+3/8/sin(2*x)^2*cos(2*x)^10+3/8*cos(2*x)^8+1/2*c
os(2*x)^6+3/4*cos(2*x)^4+3/2*cos(2*x)^2+3*ln(sin(2*x))
```

Maxima [A] time = 0.95173, size = 59, normalized size = 1.4

$$\frac{1}{8}\sin(2x)^4-\sin(2x)^2+\frac{8\sin(2x)^2-1}{8\sin(2x)^4}+\frac{3}{2}\log\left(\sin(2x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(s
in(2*x)^2)
```

Fricas [B] time = 2.23845, size = 220, normalized size = 5.24

$$\frac{8\cos(2x)^8 + 32\cos(2x)^6 - 115\cos(2x)^4 + 38\cos(2x)^2 + 192\left(\cos(2x)^4 - 2\cos(2x)^2 + 1\right)\log\left(\frac{1}{2}\sin(2x)\right) + 29}{64\left(\cos(2x)^4 - 2\cos(2x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="fricas")
```

```
[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(
cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(
2*x)^2 + 1)
```

Sympy [A] time = 0.108527, size = 41, normalized size = 0.98

$$\frac{8\sin^2(2x) - 1}{8\sin^4(2x)} + 3\log(\sin(2x)) + \frac{\sin^4(2x)}{8} - \sin^2(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**4*cot(2*x)**5,x)
```

```
[Out] (8*sin(2*x)**2 - 1)/(8*sin(2*x)**4) + 3*log(sin(2*x)) + sin(2*x)**4/8 - sin
(2*x)**2
```

Giac [B] time = 1.14504, size = 300, normalized size = 7.14

$$-\frac{\left(\frac{28(\cos(2x)-1)}{\cos(2x)+1} + \frac{288(\cos(2x)-1)^2}{(\cos(2x)+1)^2} + 1\right)(\cos(2x)+1)^2}{128(\cos(2x)-1)^2} - \frac{7(\cos(2x)-1)}{32(\cos(2x)+1)} - \frac{(\cos(2x)-1)^2}{128(\cos(2x)+1)^2} - \frac{\frac{84(\cos(2x)-1)}{\cos(2x)+1} - \frac{126(\cos(2}{(\cos(2x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="giac")
```

```
[Out] -1/128*(28*(cos(2*x) - 1)/(cos(2*x) + 1) + 288*(cos(2*x) - 1)^2/(cos(2*x) +
 1)^2 + 1)*(cos(2*x) + 1)^2/(cos(2*x) - 1)^2 - 7/32*(cos(2*x) - 1)/(cos(2*x
) + 1) - 1/128*(cos(2*x) - 1)^2/(cos(2*x) + 1)^2 - 1/4*(84*(cos(2*x) - 1)/(
cos(2*x) + 1) - 126*(cos(2*x) - 1)^2/(cos(2*x) + 1)^2 + 84*(cos(2*x) - 1)^3
/(cos(2*x) + 1)^3 - 25*(cos(2*x) - 1)^4/(cos(2*x) + 1)^4 - 25)/((cos(2*x) -
 1)/(cos(2*x) + 1) - 1)^4 - 3*log(-(cos(2*x) - 1)/(cos(2*x) + 1) + 1) + 3/2
*log(-(cos(2*x) - 1)/(cos(2*x) + 1))
```

3.903 $\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx$

Optimal. Leaf size=74

$$\frac{\tan(x)}{3}-\frac{4}{9}\tanh^{-1}(\sin(x))-\frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{9}\sqrt{7}\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{7}\cos\left(\frac{x}{2}\right)\right)$$

```
[Out] (-4*ArcTanh[Sin[x]])/9 - (Sqrt[7]*Log[Sqrt[7]*Cos[x/2] - Sin[x/2]])/9 + (Sq
rt[7]*Log[Sqrt[7]*Cos[x/2] + Sin[x/2]])/9 + Tan[x]/3
```

Rubi [A] time = 0.246008, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4397, 2723, 3056, 3001, 3770, 2659, 206}

$$\frac{\tan(x)}{3}-\frac{4}{9}\tanh^{-1}(\sin(x))-\frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{9}\sqrt{7}\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{7}\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
```



```
[Out] (-4*ArcTanh[Sin[x]])/9 - (Sqrt[7]*Log[Sqrt[7]*Cos[x/2] - Sin[x/2]])/9 + (Sq
rt[7]*Log[Sqrt[7]*Cos[x/2] + Sin[x/2]])/9 + Tan[x]/3
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
```

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = \int \frac{\tan^2(x)}{3+4\cos(x)}\,dx$$
$$= \int \frac{\left(1-\cos^2(x)\right)\sec^2(x)}{3+4\cos(x)}\,dx$$
$$= \frac{\tan(x)}{3} + \frac{1}{3}\int \frac{(-4-3\cos(x))\sec(x)}{3+4\cos(x)}\,dx$$
$$= \frac{\tan(x)}{3} - \frac{4}{9}\int \sec(x)\,dx + \frac{7}{9}\int \frac{1}{3+4\cos(x)}\,dx$$
$$= -\frac{4}{9}\tanh^{-1}(\sin(x)) + \frac{\tan(x)}{3} + \frac{14}{9}\,\mathrm{Subst}\left(\int \frac{1}{7-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\frac{4}{9}\tanh^{-1}(\sin(x)) - \frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) + \frac{\mathrm{ta}}{}$$

Mathematica [A] time = 0.0782563, size = 63, normalized size = 0.85

$$\frac{1}{9}\left(3\tan(x) + 2\sqrt{7}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right) + 4\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) - 4\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
```

```
[Out] (2*Sqrt[7]*ArcTanh[Tan[x/2]/Sqrt[7]] + 4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[C
os[x/2] + Sin[x/2]] + 3*Tan[x])/9
```

Maple [A] time = 0.021, size = 55, normalized size = 0.7

$$-\frac{1}{3}\left(1+\tan\left(\frac{x}{2}\right)\right)^{-1} - \frac{4}{9}\ln\left(1+\tan\left(\frac{x}{2}\right)\right) - \frac{1}{3}\left(\tan\left(\frac{x}{2}\right)-1\right)^{-1} + \frac{4}{9}\ln\left(\tan\left(\frac{x}{2}\right)-1\right) + \frac{2\sqrt{7}}{9}\mathrm{Artanh}\left(\frac{\sqrt{7}}{7}\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^2/(4+3*sec(x)),x)
```

```
[Out] -1/3/(1+tan(1/2*x))-4/9*ln(1+tan(1/2*x))-1/3/(tan(1/2*x)-1)+4/9*ln(tan(1/2*
x)-1)+2/9*7^(1/2)*arctanh(1/7*tan(1/2*x)*7^(1/2))
```

Maxima [A] time = 1.43588, size = 123, normalized size = 1.66

$$-\frac{1}{9}\sqrt{7}\log\left(-\frac{\sqrt{7}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{7}+\frac{\sin(x)}{\cos(x)+1}}\right)-\frac{2\sin(x)}{3\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)}-\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(7)*log(-(sqrt(7) - sin(x)/(cos(x) + 1))/(sqrt(7) + sin(x)/(cos(x)
 + 1))) - 2/3*sin(x)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 4/9*log
(sin(x)/(cos(x) + 1) + 1) + 4/9*log(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A] time = 2.45333, size = 274, normalized size = 3.7

$$\frac{\sqrt{7}\cos(x)\log\left(\frac{2\cos(x)^2+2\left(3\sqrt{7}\cos(x)+4\sqrt{7}\right)\sin(x)+24\cos(x)+23}{16\cos(x)^2+24\cos(x)+9}\right)-4\cos(x)\log(\sin(x)+1)+4\cos(x)\log(-\sin(x)+1)+6}{18\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(7)*cos(x)*log((2*cos(x)^2 + 2*(3*sqrt(7)*cos(x) + 4*sqrt(7))*sin
(x) + 24*cos(x) + 23)/(16*cos(x)^2 + 24*cos(x) + 9)) - 4*cos(x)*log(sin(x)
+ 1) + 4*cos(x)*log(-sin(x) + 1) + 6*sin(x))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(x)\sec(x)}{3\sec(x)+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**2/(4+3*sec(x)),x)
```

```
[Out] Integral(tan(x)**2*sec(x)/(3*sec(x) + 4), x)
```

Giac [A] time = 1.22474, size = 97, normalized size = 1.31

$$-\frac{1}{9}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}\right)-\frac{2\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^{2}-1\right)}-\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)+\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(7)*log(abs(-2*sqrt(7) + 2*tan(1/2*x))/abs(2*sqrt(7) + 2*tan(1/2*x
))) - 2/3*tan(1/2*x)/(tan(1/2*x)^2 - 1) - 4/9*log(abs(tan(1/2*x) + 1)) + 4/
9*log(abs(tan(1/2*x) - 1))
```

3.904 $\int x \sec(1+x) \tan(1+x) \, dx$

Optimal. Leaf size=14

$$x \sec(x+1) - \tanh^{-1}(\sin(x+1))$$

```
[Out] -ArcTanh[Sin[1 + x]] + x*Sec[1 + x]
```

Rubi [A] time = 0.0106727, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3757, 3770}

$$x \sec(x+1) - \tanh^{-1}(\sin(x+1))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[1 + x]*Tan[1 + x],x]

[Out] -ArcTanh[Sin[1 + x]] + x*Sec[1 + x]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \sec(1+x) \tan(1+x) \, dx &= x \sec(1+x) - \int \sec(1+x) \, dx \\ &= -\tanh^{-1}(\sin(1+x)) + x \sec(1+x) \end{aligned}$$

Mathematica [B] time = 0.0400587, size = 47, normalized size = 3.36

$$x\sec(x+1)+\log\left(\cos\left(\frac{x+1}{2}\right)-\sin\left(\frac{x+1}{2}\right)\right)-\log\left(\sin\left(\frac{x+1}{2}\right)+\cos\left(\frac{x+1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[1 + x]*Tan[1 + x],x]
```

```
[Out] Log[Cos[(1 + x)/2] - Sin[(1 + x)/2]] - Log[Cos[(1 + x)/2] + Sin[(1 + x)/2]]
 + x*Sec[1 + x]
```

Maple [B] time = 0.011, size = 32, normalized size = 2.3

$$\frac{1+x}{\cos(1+x)}-\ln(\sec(1+x)+\tan(1+x))-(\cos(1+x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(1+x)*tan(1+x),x)
```

```
[Out] (1+x)/cos(1+x)-ln(sec(1+x)+tan(1+x))-1/cos(1+x)
```

Maxima [B] time = 1.46521, size = 238, normalized size = 17.

$$\frac{4(x+1)\cos(2x+2)\cos(x+1)+4(x+1)\sin(2x+2)\sin(x+1)+4(x+1)\cos(x+1)-\left(\cos(2x+2)^2+\sin(2x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(x + 1)*cos(2*x + 2)*cos(x + 1) + 4*(x + 1)*sin(2*x + 2)*sin(x + 1)
+ 4*(x + 1)*cos(x + 1) - (cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2)
+ 1)*log(cos(x + 1)^2 + sin(x + 1)^2 + 2*sin(x + 1) + 1) + (cos(2*x + 2)^2
+ sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1)*log(cos(x + 1)^2 + sin(x + 1)^2 - 2*
sin(x + 1) + 1))/(cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1) - 1
/cos(x + 1)
```

Fricas [B] time = 2.05271, size = 122, normalized size = 8.71

$$-\frac{\cos(x+1)\log(\sin(x+1)+1) - \cos(x+1)\log(-\sin(x+1)+1) - 2x}{2\cos(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x + 1)*log(sin(x + 1) + 1) - cos(x + 1)*log(-sin(x + 1) + 1) - 2*
x)/cos(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\tan(x+1)\sec(x+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x)
```

```
[Out] Integral(x*tan(x + 1)*sec(x + 1), x)
```

Giac [B] time = 1.44736, size = 1592, normalized size = 113.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="giac")
```

```
[Out] 1/2*(2*x*tan(1/2)^2*tan(1/2*x)^2 + log(2*(tan(1/2)^2 + 1)/(tan(1/2)^2*tan(1
/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^
2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 +
tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1))*tan(1/2)^2*ta
n(1/2*x)^2 - log(2*(tan(1/2)^2 + 1)/(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2
*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1
/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2
```

```
*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1))*tan(1/2)^2*tan(1/2*x)^2 + 2*x*tan(1
/2)^2 - log(2*(tan(1/2)^2 + 1)/(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(
1/2*x)^3 + 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)
^4 + 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2
 - 2*tan(1/2) - 2*tan(1/2*x) + 1))*tan(1/2)^2 + log(2*(tan(1/2)^2 + 1)/(tan
(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4
+ 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*ta
n(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1))*
tan(1/2)^2 - 4*log(2*(tan(1/2)^2 + 1)/(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)
^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan
(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1
/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1))*tan(1/2)*tan(1/2*x) + 4*log(2*(ta
n(1/2)^2 + 1)/(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(
1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2
*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2
*tan(1/2*x) + 1))*tan(1/2)*tan(1/2*x) + 2*x*tan(1/2*x)^2 - log(2*(tan(1/2)^
2 + 1)/(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*ta
n(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/
2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/
2*x) + 1))*tan(1/2*x)^2 + log(2*(tan(1/2)^2 + 1)/(tan(1/2)^2*tan(1/2*x)^4 -
 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2
*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^
2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1))*tan(1/2*x)^2 + 2*x + l
og(2*(tan(1/2)^2 + 1)/(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3
+ 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*ta
n(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(
1/2) - 2*tan(1/2*x) + 1)) - log(2*(tan(1/2)^2 + 1)/(tan(1/2)^2*tan(1/2*x)^4
 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1
/2*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2
)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)))/(tan(1/2)^2*tan(1/2
*x)^2 - tan(1/2)^2 - 4*tan(1/2)*tan(1/2*x) - tan(1/2*x)^2 + 1)
```

3.905 $\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}} dx$

Optimal. Leaf size=14

$$-2\sqrt{9-\sin^2(x)}$$

```
[Out] -2*Sqrt[9 - Sin[x]^2]
```

Rubi [A] time = 0.0379825, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {12, 261}

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
```

```
[Out] -2*Sqrt[9 - Sin[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = \text{Subst}\left(\int \frac{2x}{\sqrt{9-x^2}}\,dx, x, \sin(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{9-x^2}}\,dx, x, \sin(x)\right)$$
$$= -2\sqrt{9-\sin^2(x)}$$

Mathematica [A] time = 0.0151059, size = 14, normalized size = 1.

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
```

```
[Out] -2*Sqrt[9 - Sin[x]^2]
```

Maple [A] time = 0.018, size = 13, normalized size = 0.9

$$-2\sqrt{9-(\sin(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-sin(x)^2)^(1/2),x)
```

```
[Out] -2*(9-sin(x)^2)^(1/2)
```

Maxima [A] time = 0.943657, size = 16, normalized size = 1.14

$$-2\sqrt{-\sin(x)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-sin(x)^2 + 9)
```

Fricas [A] time = 2.13119, size = 31, normalized size = 2.21

$$-2\sqrt{\cos(x)^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(cos(x)^2 + 8)
```

Sympy [A] time = 1.26045, size = 12, normalized size = 0.86

$$-2\sqrt{9 - \sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)**2)**(1/2),x)
```

```
[Out] -2*sqrt(9 - sin(x)**2)
```

Giac [B] time = 1.16975, size = 136, normalized size = 9.71

$$-\frac{8\left(3\tan\left(\frac{1}{2}x\right)^2 - \sqrt{9\tan\left(\frac{1}{2}x\right)^4 + 14\tan\left(\frac{1}{2}x\right)^2 + 9} - 3\right)}{\left(3\tan\left(\frac{1}{2}x\right)^2 - \sqrt{9\tan\left(\frac{1}{2}x\right)^4 + 14\tan\left(\frac{1}{2}x\right)^2 + 9}\right)^2 + 18\tan\left(\frac{1}{2}x\right)^2 - 6\sqrt{9\tan\left(\frac{1}{2}x\right)^4 + 14\tan\left(\frac{1}{2}x\right)^2 + 9} + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8*(3*tan(1/2*x)^2 - sqrt(9*tan(1/2*x)^4 + 14*tan(1/2*x)^2 + 9) - 3)/((3*ta
n(1/2*x)^2 - sqrt(9*tan(1/2*x)^4 + 14*tan(1/2*x)^2 + 9))^2 + 18*tan(1/2*x)^
2 - 6*sqrt(9*tan(1/2*x)^4 + 14*tan(1/2*x)^2 + 9) + 5)
```

3.906 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

```
[Out] -ArcSin[Cos[x]^2/3]
```

Rubi [A] time = 0.0538037, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {12, 1107, 619, 216}

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
```

```
[Out] -ArcSin[Cos[x]^2/3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{8+2x-x^2}}\,dx, x, \sin^2(x)\right)\\
&= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 2\cos^2(x)\right)\right)\\
&= -\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.0158111, size = 11, normalized size = 1.

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
```

```
[Out] -ArcSin[Cos[x]^2/3]
```

Maple [A] time = 0.033, size = 10, normalized size = 0.9

$$-\arcsin\left(\frac{(\cos(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-cos(x)^4)^(1/2),x)
```

```
[Out] -arcsin(1/3*cos(x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)}{\sqrt{-\cos(x)^4+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

Fricas [B] time = 2.65232, size = 72, normalized size = 6.55

$$\arctan\left(\frac{\sqrt{-\cos(x)^4+9}\cos(x)^2}{\cos(x)^4-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)}{\sqrt{-\cos(x)^4+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

3.907 $\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx$

Optimal. Leaf size=34

$$-\frac{\sin\left(\frac{1}{x}\right)}{x^3}-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}+\frac{6\sin\left(\frac{1}{x}\right)}{x}+6\cos\left(\frac{1}{x}\right)$$

```
[Out] 6*Cos[x^(-1)] - (3*Cos[x^(-1)])/x^2 - Sin[x^(-1)]/x^3 + (6*Sin[x^(-1)])/x
```

Rubi [A] time = 0.0485441, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$-\frac{\sin\left(\frac{1}{x}\right)}{x^3}-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}+\frac{6\sin\left(\frac{1}{x}\right)}{x}+6\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x^(-1)]/x^5,x]
```

```
[Out] 6*Cos[x^(-1)] - (3*Cos[x^(-1)])/x^2 - Sin[x^(-1)]/x^3 + (6*Sin[x^(-1)])/x
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx &= -\operatorname{Subst}\left(\int x^3\cos(x)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sin\left(\frac{1}{x}\right)}{x^3} + 3\operatorname{Subst}\left(\int x^2\sin(x)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + 6\operatorname{Subst}\left(\int x\cos(x)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x} - 6\operatorname{Subst}\left(\int \sin(x)\,dx, x, \frac{1}{x}\right)\\
&= 6\cos\left(\frac{1}{x}\right) - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x}
\end{aligned}
$$

Mathematica [A] time = 0.0252488, size = 32, normalized size = 0.94

$$\frac{(6x^2-1)\sin\left(\frac{1}{x}\right)}{x^3} + \frac{3(2x^2-1)\cos\left(\frac{1}{x}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x^(-1)]/x^5,x]
```

```
[Out] (3*(-1 + 2*x^2)*Cos[x^(-1)])/x^2 + ((-1 + 6*x^2)*Sin[x^(-1)])/x^3
```

Maple [A] time = 0.008, size = 35, normalized size = 1.

$$6\cos\left(x^{-1}\right) - 3\,\frac{\cos\left(x^{-1}\right)}{x^2} - \frac{\sin\left(x^{-1}\right)}{x^3} + 6\,\frac{\sin\left(x^{-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)/x^5,x)
```

```
[Out] 6*cos(1/x)-3*cos(1/x)/x^2-sin(1/x)/x^3+6*sin(1/x)/x
```

Maxima [C] time = 1.07376, size = 26, normalized size = 0.76

$$\frac{1}{2}\,\Gamma\left(4,\frac{i}{x}\right)+\frac{1}{2}\,\Gamma\left(4,-\frac{i}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(4, I/x) + 1/2*gamma(4, -I/x)
```

Fricas [A] time = 2.02326, size = 72, normalized size = 2.12

$$\frac{3\left(2\,x^3-x\right)\cos\left(\frac{1}{x}\right)+\left(6\,x^2-1\right)\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="fricas")
```

```
[Out] (3*(2*x^3 - x)*cos(1/x) + (6*x^2 - 1)*sin(1/x))/x^3
```

Sympy [A] time = 4.17184, size = 32, normalized size = 0.94

$$6\cos\left(\frac{1}{x}\right)+\frac{6\sin\left(\frac{1}{x}\right)}{x}-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x**5,x)
```

```
[Out] 6*cos(1/x) + 6*sin(1/x)/x - 3*cos(1/x)/x**2 - sin(1/x)/x**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(cos(1/x)/x^5, x)
```

3.908 $\int \cos^3(1+x)\sin^3(1+x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\sin^4(x+1)-\frac{1}{6}\sin^6(x+1)$$

```
[Out] Sin[1 + x]^4/4 - Sin[1 + x]^6/6
```

Rubi [A] time = 0.0298582, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 14}

$$\frac{1}{4}\sin^4(x+1)-\frac{1}{6}\sin^6(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[1 + x]^3*Sin[1 + x]^3,x]
```

```
[Out] Sin[1 + x]^4/4 - Sin[1 + x]^6/6
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(1+x)\sin^3(1+x)\,dx &= \text{Subst}\left(\int x^3\left(1-x^2\right)\,dx, x, \sin(1+x)\right)\\ &= \text{Subst}\left(\int \left(x^3-x^5\right)\,dx, x, \sin(1+x)\right)\\ &= \frac{1}{4}\sin^4(1+x) - \frac{1}{6}\sin^6(1+x)\end{aligned}$$

Mathematica [A] time = 0.0127767, size = 25, normalized size = 1.19

$$\frac{1}{8}\left(\frac{1}{24}\cos(6(x+1)) - \frac{3}{8}\cos(2(x+1))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[1 + x]^3*Sin[1 + x]^3,x]
```

```
[Out] ((-3*Cos[2*(1 + x)])/8 + Cos[6*(1 + x)]/24)/8
```

Maple [A] time = 0.013, size = 24, normalized size = 1.1

$$-\frac{(\cos(1+x))^4(\sin(1+x))^2}{6} - \frac{(\cos(1+x))^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1+x)^3*sin(1+x)^3,x)
```

```
[Out] -1/6*cos(1+x)^4*sin(1+x)^2-1/12*cos(1+x)^4
```

Maxima [A] time = 0.9565, size = 23, normalized size = 1.1

$$-\frac{1}{6}\sin(x+1)^6 + \frac{1}{4}\sin(x+1)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

Fricas [A] time = 1.97505, size = 50, normalized size = 2.38

$$\frac{1}{6}\cos\left(x+1\right)^{6}-\frac{1}{4}\cos\left(x+1\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x + 1)^6 - 1/4*cos(x + 1)^4
```

Sympy [A] time = 2.07261, size = 22, normalized size = 1.05

$$\frac{\sin^{6}\left(x+1\right)}{12}+\frac{\sin^{4}\left(x+1\right)\cos^{2}\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)**3*sin(1+x)**3,x)
```

```
[Out] sin(x + 1)**6/12 + sin(x + 1)**4*cos(x + 1)**2/4
```

Giac [A] time = 1.08373, size = 23, normalized size = 1.1

$$-\frac{1}{6}\sin\left(x+1\right)^{6}+\frac{1}{4}\sin\left(x+1\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

3.909 $\int (1+2x)^3 \sin^2(1+2x)\,dx$

Optimal. Leaf size=99

$$-\frac{3x^2}{4}+\frac{1}{16}(2x+1)^4-\frac{3x}{4}+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{3}{16}\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)+\frac{3}{8}(2x+1)\mathrm{s}$$

```
[Out] (-3*x)/4 - (3*x^2)/4 + (1 + 2*x)^4/16 + (3*(1 + 2*x)*Cos[1 + 2*x]*Sin[1 + 2
*x])/8 - ((1 + 2*x)^3*Cos[1 + 2*x]*Sin[1 + 2*x])/4 - (3*Sin[1 + 2*x]^2)/16
+ (3*(1 + 2*x)^2*Sin[1 + 2*x]^2)/8
```

Rubi [A] time = 0.0595834, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3311, 32, 3310}

$$-\frac{3x^2}{4}+\frac{1}{16}(2x+1)^4-\frac{3x}{4}+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{3}{16}\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)+\frac{3}{8}(2x+1)\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

```
[Out] (-3*x)/4 - (3*x^2)/4 + (1 + 2*x)^4/16 + (3*(1 + 2*x)*Cos[1 + 2*x]*Sin[1 + 2
*x])/8 - ((1 + 2*x)^3*Cos[1 + 2*x]*Sin[1 + 2*x])/4 - (3*Sin[1 + 2*x]^2)/16
+ (3*(1 + 2*x)^2*Sin[1 + 2*x]^2)/8
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (1+2x)^3 \sin^2(1+2x)\,dx = -\frac{1}{4}(1+2x)^3\cos(1+2x)\sin(1+2x) + \frac{3}{8}(1+2x)^2\sin^2(1+2x) + \frac{1}{2}\int (1+2x)^3\,dx - \frac{3}{2}\int$$
$$= \frac{1}{16}(1+2x)^4 + \frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x) - \frac{1}{4}(1+2x)^3\cos(1+2x)\sin(1+2x) -$$
$$= -\frac{3x}{4} - \frac{3x^2}{4} + \frac{1}{16}(1+2x)^4 + \frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x) - \frac{1}{4}(1+2x)^3\cos(1+2x)\,\mathrm{s}$$

Mathematica [A] time = 0.229936, size = 55, normalized size = 0.56

$$\frac{1}{32}\left(2(2x+1)\left(\left(-8x^2-8x+1\right)\sin(4x+2)+(2x+1)^3\right)-3\left(8x^2+8x+1\right)\cos(4x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

```
[Out] (-3*(1 + 8*x + 8*x^2)*Cos[2 + 4*x] + 2*(1 + 2*x)*((1 + 2*x)^3 + (1 - 8*x -
8*x^2)*Sin[2 + 4*x]))/32
```

Maple [A] time = 0.019, size = 97, normalized size = 1.

$$\frac{(1+2x)^3}{2}\left(-\frac{\sin(1+2x)\cos(1+2x)}{2}+x+\frac{1}{2}\right)-\frac{3(\cos(1+2x))^2(1+2x)^2}{8}+\frac{3+6x}{4}\left(\frac{\sin(1+2x)\cos(1+2x)}{2}+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*sin(1+2*x)^2,x)
```

```
[Out] 1/2*(1+2*x)^3*(-1/2*sin(1+2*x)*cos(1+2*x)+x+1/2)-3/8*cos(1+2*x)^2*(1+2*x)^2
+3/4*(1+2*x)*(1/2*sin(1+2*x)*cos(1+2*x)+x+1/2)-3/16*(1+2*x)^2-3/16*sin(1+2*
```

```
x)^2-3/16*(1+2*x)^4
```

Maxima [A] time = 0.983696, size = 69, normalized size = 0.7

$$\frac{1}{16}(2x+1)^4-\frac{3}{32}\left(2(2x+1)^2-1\right)\cos(4x+2)-\frac{1}{16}\left(2(2x+1)^3-6x-3\right)\sin(4x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*x + 1)^4 - 3/32*(2*(2*x + 1)^2 - 1)*cos(4*x + 2) - 1/16*(2*(2*x + 1
)^3 - 6*x - 3)*sin(4*x + 2)
```

Fricas [A] time = 2.15695, size = 177, normalized size = 1.79

$$x^4+2x^3-\frac{3}{16}\left(8x^2+8x+1\right)\cos(2x+1)^2-\frac{1}{8}\left(16x^3+24x^2+6x-1\right)\cos(2x+1)\sin(2x+1)+\frac{9}{4}x^2+\frac{5}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] x^4 + 2*x^3 - 3/16*(8*x^2 + 8*x + 1)*cos(2*x + 1)^2 - 1/8*(16*x^3 + 24*x^2
+ 6*x - 1)*cos(2*x + 1)*sin(2*x + 1) + 9/4*x^2 + 5/4*x
```

Sympy [B] time = 1.31698, size = 189, normalized size = 1.91

$$x^4\sin^2(2x+1)+x^4\cos^2(2x+1)+2x^3\sin^2(2x+1)-2x^3\sin(2x+1)\cos(2x+1)+2x^3\cos^2(2x+1)+\frac{9x^2\sin^2(2x+}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*sin(1+2*x)**2,x)
```

```
[Out] x**4*sin(2*x + 1)**2 + x**4*cos(2*x + 1)**2 + 2*x**3*sin(2*x + 1)**2 - 2*x*
*3*sin(2*x + 1)*cos(2*x + 1) + 2*x**3*cos(2*x + 1)**2 + 9*x**2*sin(2*x + 1)
**2/4 - 3*x**2*sin(2*x + 1)*cos(2*x + 1) + 3*x**2*cos(2*x + 1)**2/4 + 5*x*s
```

```
in(2*x + 1)**2/4 - 3*x*sin(2*x + 1)*cos(2*x + 1)/4 - x*cos(2*x + 1)**2/4 +
sin(2*x + 1)*cos(2*x + 1)/8 - 3*cos(2*x + 1)**2/16
```

Giac [A] time = 1.0762, size = 78, normalized size = 0.79

$$x^4 + 2\,x^3 + \frac{3}{2}\,x^2 - \frac{3}{32}\left(8\,x^2 + 8\,x + 1\right)\cos\left(4\,x + 2\right) - \frac{1}{16}\left(16\,x^3 + 24\,x^2 + 6\,x - 1\right)\sin\left(4\,x + 2\right) + \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="giac")
```

```
[Out] x^4 + 2*x^3 + 3/2*x^2 - 3/32*(8*x^2 + 8*x + 1)*cos(4*x + 2) - 1/16*(16*x^3
+ 24*x^2 + 6*x - 1)*sin(4*x + 2) + 1/2*x
```

3.910 $\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{x}{2}+\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{\tanh^{-1}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -x/2 + ArcTanh[(Cos[x]*(1 + Tan[x]))/Sqrt[2]]/Sqrt[2] + Log[Cos[x] - Sin[x]
]/2
```

Rubi [A] time = 0.0895854, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4401, 3484, 3530, 3509, 206}

$$-\frac{x}{2}+\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{\tanh^{-1}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sec[x])/(1 - Tan[x]),x]
```


```
[Out] -x/2 + ArcTanh[(Cos[x]*(1 + Tan[x]))/Sqrt[2]]/Sqrt[2] + Log[Cos[x] - Sin[x]
]/2
```


Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx &= \int \left(\frac{1}{-1+\tan(x)}-\frac{\sec(x)}{-1+\tan(x)}\right)dx\\ &= \int \frac{1}{-1+\tan(x)}\,dx-\int \frac{\sec(x)}{-1+\tan(x)}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}\int \frac{1+\tan(x)}{-1+\tan(x)}\,dx+\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\cos(x)(1+\tan(x))\right)\\ &= -\frac{x}{2}+\frac{\tanh^{-1}\left(\frac{\cos(x)(1+\tan(x))}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log(\cos(x)-\sin(x))\end{aligned}$$

Mathematica [C] time = 0.0602444, size = 40, normalized size = 1.08

$$\frac{1}{2}\left(-x+(2-2i)\sqrt[4]{-1}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sec[x])/(1 - Tan[x]),x]
```

```
[Out] (-x + (2 - 2*I)*(-1)^(1/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + Log[Cos[x] - S
in[x]])/2
```

Maple [A] time = 0.045, size = 51, normalized size = 1.4

$$\frac{1}{2}\ln\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+2\tan\left(x/2\right)-1\right)+\sqrt{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{4}\left(2+2\tan\left(x/2\right)\right)\right)-\frac{1}{2}\ln\left(1+\left(\tan\left(\frac{x}{2}\right)\right)^{2}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sec(x))/(1-tan(x)),x)
```

```
[Out] 1/2*ln(tan(1/2*x)^2+2*tan(1/2*x)-1)+2^(1/2)*arctanh(1/4*(2+2*tan(1/2*x))*2^
(1/2))-1/2*ln(1+tan(1/2*x)^2)-1/2*x
```

Maxima [A] time = 1.45566, size = 80, normalized size = 2.16

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}-1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}+1}\right)-\frac{1}{2}x-\frac{1}{4}\log\left(\tan\left(x\right)^{2}+1\right)+\frac{1}{2}\log\left(\tan\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) - 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) + 1)) - 1/2*x - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) - 1)
```

Fricas [A] time = 2.20353, size = 180, normalized size = 4.86

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos\left(x\right)\right)\sin\left(x\right)+2\sqrt{2}\cos\left(x\right)+3}{2\cos\left(x\right)\sin\left(x\right)-1}\right)-\frac{1}{2}x+\frac{1}{4}\log\left(-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) + 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) - 1)) - 1/2*x + 1/4*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sec(x)}{\tan(x) - 1}\, dx - \int -\frac{1}{\tan(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x)
```

```
[Out] -Integral(sec(x)/(tan(x) - 1), x) - Integral(-1/(tan(x) - 1), x)
```

Giac [B] time = 1.16572, size = 95, normalized size = 2.57

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 2\tan\left(\frac{1}{2}x\right) + 2\right|}{\left|2\sqrt{2} + 2\tan\left(\frac{1}{2}x\right) + 2\right|}\right) - \frac{1}{2}x - \frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + \frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)^2 + 2\tan\left(\frac{1}{2}x\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) + 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) + 2)) - 1/2*x - 1/2*log(tan(1/2*x)^2 + 1) + 1/2*log(abs(tan(1/2*x)^2
+ 2*tan(1/2*x) - 1))
```

3.911 $\int x^2 \cos(3x) \cos(5x) \, dx$

Optimal. Leaf size=57

$$\frac{1}{4}x^2 \sin(2x) + \frac{1}{16}x^2 \sin(8x) - \frac{1}{8}\sin(2x) - \frac{1}{512}\sin(8x) + \frac{1}{4}x\cos(2x) + \frac{1}{64}x\cos(8x)$$

```
[Out] (x*Cos[2*x])/4 + (x*Cos[8*x])/64 - Sin[2*x]/8 + (x^2*Sin[2*x])/4 - Sin[8*x]
/512 + (x^2*Sin[8*x])/16
```

Rubi [A] time = 0.0734621, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4429, 3296, 2637}

$$\frac{1}{4}x^2 \sin(2x) + \frac{1}{16}x^2 \sin(8x) - \frac{1}{8}\sin(2x) - \frac{1}{512}\sin(8x) + \frac{1}{4}x\cos(2x) + \frac{1}{64}x\cos(8x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[3*x]*Cos[5*x],x]
```

```
[Out] (x*Cos[2*x])/4 + (x*Cos[8*x])/64 - Sin[2*x]/8 + (x^2*Sin[2*x])/4 - Sin[8*x]
/512 + (x^2*Sin[8*x])/16
```

Rule 4429

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f
_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cos[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\cos(3x)\cos(5x)\,dx &= \int\left(\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x^2\cos(8x)\right)dx\\
&= \frac{1}{2}\int x^2\cos(2x)\,dx+\frac{1}{2}\int x^2\cos(8x)\,dx\\
&= \frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{8}\int x\sin(8x)\,dx-\frac{1}{2}\int x\sin(2x)\,dx\\
&= \frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)+\frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{64}\int\cos(8x)\,dx-\frac{1}{4}\int\cos(2x)\\
&= \frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)-\frac{1}{8}\sin(2x)+\frac{1}{4}x^2\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{16}x^2\sin(8x)
\end{aligned}
$$

Mathematica [A] time = 0.0914518, size = 49, normalized size = 0.86

$$\frac{1}{512}\left(128x^2\sin(2x)+32x^2\sin(8x)-64\sin(2x)-\sin(8x)+128x\cos(2x)+8x\cos(8x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[3*x]*Cos[5*x],x]
```

```
[Out] (128*x*Cos[2*x] + 8*x*Cos[8*x] - 64*Sin[2*x] + 128*x^2*Sin[2*x] - Sin[8*x]
+ 32*x^2*Sin[8*x])/512
```

Maple [A] time = 0.046, size = 46, normalized size = 0.8

$$\frac{x\cos(2x)}{4}+\frac{x\cos(8x)}{64}-\frac{\sin(2x)}{8}+\frac{x^2\sin(2x)}{4}-\frac{\sin(8x)}{512}+\frac{x^2\sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x)*cos(5*x),x)
```

```
[Out] 1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x)
+1/16*x^2*sin(8*x)
```

Maxima [A] time = 0.985726, size = 55, normalized size = 0.96

$$\frac{1}{64}\,x\cos\left(8\,x\right)+\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{512}\left(32\,x^{2}-1\right)\sin\left(8\,x\right)+\frac{1}{8}\left(2\,x^{2}-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="maxima")
```

```
[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2
 - 1)*sin(2*x)
```

Fricas [A] time = 2.25457, size = 220, normalized size = 3.86

$$2\,x\cos\left(x\right)^{8}-4\,x\cos\left(x\right)^{6}+\frac{5}{2}\,x\cos\left(x\right)^{4}+\frac{1}{64}\left(16\left(32\,x^{2}-1\right)\cos\left(x\right)^{7}-24\left(32\,x^{2}-1\right)\cos\left(x\right)^{5}+10\left(32\,x^{2}-1\right)\cos\left(x\right)^{3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="fricas")
```

```
[Out] 2*x*cos(x)^8 - 4*x*cos(x)^6 + 5/2*x*cos(x)^4 + 1/64*(16*(32*x^2 - 1)*cos(x)
^7 - 24*(32*x^2 - 1)*cos(x)^5 + 10*(32*x^2 - 1)*cos(x)^3 - 15*cos(x))*sin(x
) - 15/64*x
```

Sympy [A] time = 7.72247, size = 90, normalized size = 1.58

$$-\frac{3x^{2}\sin\left(3x\right)\cos\left(5x\right)}{16}+\frac{5x^{2}\sin\left(5x\right)\cos\left(3x\right)}{16}+\frac{15x\sin\left(3x\right)\sin\left(5x\right)}{64}+\frac{17x\cos\left(3x\right)\cos\left(5x\right)}{64}+\frac{63\sin\left(3x\right)\cos\left(5x\right)}{512}-\frac{65}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(3*x)*cos(5*x),x)
```

```
[Out] -3*x**2*sin(3*x)*cos(5*x)/16 + 5*x**2*sin(5*x)*cos(3*x)/16 + 15*x*sin(3*x)*
sin(5*x)/64 + 17*x*cos(3*x)*cos(5*x)/64 + 63*sin(3*x)*cos(5*x)/512 - 65*sin
(5*x)*cos(3*x)/512
```

Giac [A] time = 1.09981, size = 55, normalized size = 0.96

$$\frac{1}{64}\,x\cos\left(8\,x\right)+\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{512}\,\left(32\,x^{2}-1\right)\sin\left(8\,x\right)+\frac{1}{8}\,\left(2\,x^{2}-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="giac")
```

```
[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2
 - 1)*sin(2*x)
```

3.912 $\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=57

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]) + Sqrt[2]*ArcTan
[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]
```

Rubi [B] time = 0.211369, antiderivative size = 243, normalized size of antiderivative = 4.26, number of steps used = 22, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3107, 2575, 297, 1162, 617, 204, 1165, 628, 2574}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 + (Sqrt[
2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/
Sqrt[Cos[x]]]/Sqrt[2] + ArcTan[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqr
t[2] - Log[1 + Cot[x] - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) +
Log[1 + Cot[x] + (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) + Log[1 -
 (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[
2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
```

```
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx = \int \left(\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)dx$$

$$= \int \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\,dx + \int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$

$$= -\left(2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)\right) + 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

$$= \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right) - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x\right.$$

$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) -$$

$$= -\frac{\log\left(1+\cot(x)-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{2\sqrt{2}} + \frac{\log\left(1+\cot(x)+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{2\sqrt{2}} + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} -$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.0567925, size = 68, normalized size = 1.19

$$\frac{2\sqrt{\sin(x)}\sqrt[4]{\cos^2(x)}\left(\sin(x)\sqrt{\cos^2(x)}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\sin^2(x)\right)+3\cos(x)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin\right.\right.}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] (2*(Cos[x]^2)^(1/4)*Sqrt[Sin[x]]*(3*Cos[x]*Hypergeometric2F1[1/4, 1/4, 5/4,
 Sin[x]^2] + Sqrt[Cos[x]^2]*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[
```

```
x]))/(3*Cos[x]^(3/2))
```

Maple [C] time = 0.157, size = 137, normalized size = 2.4

$$-\frac{\sqrt{2}}{-1+\cos(x)}\sqrt{\frac{\sin(x)+1-\cos(x)}{\sin(x)}}\sqrt{\frac{\cos(x)-1+\sin(x)}{\sin(x)}}\sqrt{\frac{-1+\cos(x)}{\sin(x)}}\,(\sin(x))^{\frac{3}{2}}\left(i\mathrm{EllipticPi}\left(\sqrt{\frac{\sin(x)+1-\cos(x)}{\sin(x)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x)
```

```
[Out] -((sin(x)+1-cos(x))/sin(x))^(1/2)*2^(1/2)*((cos(x)-1+sin(x))/sin(x))^(1/2)*
((-1+cos(x))/sin(x))^(1/2)*sin(x)^(3/2)*(I*EllipticPi(((sin(x)+1-cos(x))/si
n(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((sin(x)+1-cos(x))/sin(x))^
(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticF(((sin(x)+1-cos(x))/sin(x))^(1/2),1/2
*2^(1/2)))/cos(x)^(1/2)/(-1+cos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

Fricas [B] time = 2.39579, size = 275, normalized size = 4.82

$$-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(32\sqrt{2}\cos(x)^4-32\sqrt{2}\cos(x)^2+16\sqrt{2}\cos(x)\sin(x)-\sqrt{2}\right)\sqrt{\cos(x)}\sqrt{\sin(x)}}{8\left(4\cos(x)^5-3\cos(x)^3-\left(4\cos(x)^4-5\cos(x)^2\right)\sin(x)-\cos(x)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-1/8*(32*sqrt(2)*cos(x)^4 - 32*sqrt(2)*cos(x)^2 + 16*sq
rt(2)*cos(x)*sin(x) - sqrt(2))*sqrt(cos(x))*sqrt(sin(x))/(4*cos(x)^5 - 3*co
s(x)^3 - (4*cos(x)^4 - 5*cos(x)^2)*sin(x) - cos(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x) + \cos(x)}{\sqrt{\sin(x)}\sqrt{\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)**(1/2)/sin(x)**(1/2),x)
```

```
[Out] Integral((sin(x) + cos(x))/(sqrt(sin(x))*sqrt(cos(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

3.913 $\int \sec^2(x)(1+\sin(x))\,dx$

Optimal. Leaf size=5

$$\tan(x)+\sec(x)$$

```
[Out] Sec[x] + Tan[x]
```

Rubi [A] time = 0.0231152, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2669, 3767, 8}

$$\tan(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*(1 + Sin[x]),x]
```

```
[Out] Sec[x] + Tan[x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^2(x)(1+\sin(x))\,dx &= \sec(x) + \int \sec^2(x)\,dx \\
&= \sec(x) - \mathrm{Subst}\left(\int 1\,dx, x, -\tan(x)\right) \\
&= \sec(x) + \tan(x)
\end{aligned}
$$

Mathematica [A] time = 0.0037442, size = 5, normalized size = 1.

$$\tan(x) + \sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(1 + Sin[x]),x]
```

```
[Out] Sec[x] + Tan[x]
```

Maple [A] time = 0.018, size = 8, normalized size = 1.6

$$\tan(x) + (\cos(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1+sin(x)),x)
```

```
[Out] tan(x)+1/cos(x)
```

Maxima [A] time = 0.94676, size = 9, normalized size = 1.8

$$\frac{1}{\cos(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="maxima")
```

```
[Out] 1/cos(x) + tan(x)
```

Fricas [B] time = 2.04508, size = 61, normalized size = 12.2

$$\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A] time = 5.50021, size = 7, normalized size = 1.4

$$\tan(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1+sin(x)),x)
```

```
[Out] tan(x) + 1/cos(x)
```

Giac [A] time = 1.08856, size = 14, normalized size = 2.8

$$-\frac{2}{\tan\left(\frac{1}{2}x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

3.914 $\int \left(10x^9 \cos \left(x^5 \log(x)\right) - x^{10} \left(x^4 + 5x^4 \log(x)\right) \sin \left(x^5 \log(x\right.\right.$

Optimal. Leaf size=11

$$x^{10} \cos \left(x^5 \log(x)\right)$$

[Out] x^10*Cos[x^5*Log[x]]

Rubi [F] time = 0.275911, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \left(10x^9 \cos \left(x^5 \log(x)\right) - x^{10} \left(x^4 + 5x^4 \log(x)\right) \sin \left(x^5 \log(x)\right)\right) dx$$

Verification is Not applicable to the result.

[In] Int[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]],x]

[Out] 10*Defer[Int][x^9*Cos[x^5*Log[x]], x] - Defer[Int][x^14*Sin[x^5*Log[x]], x] - 5*Defer[Int][x^14*Log[x]*Sin[x^5*Log[x]], x]

Rubi steps

$$\begin{aligned} \int \left(10x^9 \cos \left(x^5 \log(x)\right) - x^{10} \left(x^4 + 5x^4 \log(x)\right) \sin \left(x^5 \log(x)\right)\right) dx &= 10 \int x^9 \cos \left(x^5 \log(x)\right) dx - \int x^{10} \left(x^4 + 5x^4 \log(x\right. \\ &= 10 \int x^9 \cos \left(x^5 \log(x)\right) dx - \int x^{14}(1 + 5 \log(x)) \sin \\ &= 10 \int x^9 \cos \left(x^5 \log(x)\right) dx - \int \left(x^{14} \sin \left(x^5 \log(x)\right)\right. \\ &= -\left(5 \int x^{14} \log(x) \sin \left(x^5 \log(x)\right) dx\right) + 10 \int x^9 \cos \end{aligned}$$

Mathematica [A] time = 0.319045, size = 11, normalized size = 1.

$$x^{10} \cos \left(x^5 \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]
],x]
```

```
[Out] x^10*Cos[x^5*Log[x]]
```

Maple [C] time = 0.109, size = 30, normalized size = 2.7

$$\frac{x^{10}x^{ix^5}}{2}+\frac{x^{10}}{2\,x^{ix^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10*x^9*cos(x^5*ln(x))-x^10*(x^4+5*x^4*ln(x))*sin(x^5*ln(x)),x)
```

```
[Out] 1/2*x^10*x^(I*x^5)+1/2*x^10/(x^(I*x^5))
```

Maxima [A] time = 1.20728, size = 15, normalized size = 1.36

$$x^{10}\cos\left(x^5\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="maxima")
```

```
[Out] x^10*cos(x^5*log(x))
```

Fricas [A] time = 2.15781, size = 30, normalized size = 2.73

$$x^{10}\cos\left(x^5\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="fricas")
```

```
[Out] x^10*cos(x^5*log(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x**9*cos(x**5*ln(x))-x**10*(x**4+5*x**4*ln(x))*sin(x**5*ln(x))
,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="giac")
```

```
[Out] Timed out
```

3.915 $\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=27

$$\frac{x}{2}-\frac{\cos(x)}{2}-\log\left(\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)$$

```
[Out] x/2 - Cos[x]/2 - Log[Cos[Pi/4 + x/2]]
```

Rubi [F] time = 0.0625198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
```

```
[Out] Defer[Int][Cos[x/2]^2*Tan[Pi/4 + x/2], x]
```

Rubi steps

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)dx=\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

Mathematica [A] time = 0.166814, size = 24, normalized size = 0.89

$$\frac{1}{2}\left(x-\cos(x)-\log(\cos(x))+2\tanh^{-1}\left(\cot\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
```

```
[Out] (x + 2*ArcTanh[Cot[x/2]] - Cos[x] - Log[Cos[x]])/2
```

Maple [A] time = 0.14, size = 22, normalized size = 0.8

$$\frac{x}{2}-\frac{\cos \left(x\right)}{2}+\frac{\ln \left(\sec \left(x\right)+\tan \left(x\right)\right)}{2}-\frac{\ln \left(\cos \left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*x)^2*tan(1/4*Pi+1/2*x),x)
```

```
[Out] 1/2*x-1/2*cos(x)+1/2*ln(sec(x)+tan(x))-1/2*ln(cos(x))
```

Maxima [B] time = 1.44345, size = 100, normalized size = 3.7

$$\frac{2\,x\cos \left(x\right)^{2}+2\,x\sin \left(x\right)^{2}-\cos \left(2\,x\right)\cos \left(x\right)-2\left(\cos \left(x\right)^{2}+\sin \left(x\right)^{2}\right)\log \left(\cos \left(x\right)^{2}+\sin \left(x\right)^{2}-2\,\sin \left(x\right)+1\right)-\sin \left(2\,x\right)\sin}{4\left(\cos \left(x\right)^{2}+\sin \left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x*cos(x)^2 + 2*x*sin(x)^2 - cos(2*x)*cos(x) - 2*(cos(x)^2 + sin(x)^2
)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - sin(2*x)*sin(x) - cos(x))/(cos(
x)^2 + sin(x)^2)
```

Fricas [A] time = 2.06683, size = 85, normalized size = 3.15

$$-\cos \left(\frac{1}{2}\,x\right)^{2}+\frac{1}{2}\,x-\frac{1}{2}\,\log \left(-2\,\cos \left(\frac{1}{2}\,x\right)\sin \left(\frac{1}{2}\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="fricas")
```

```
[Out] -cos(1/2*x)^2 + 1/2*x - 1/2*log(-2*cos(1/2*x)*sin(1/2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)**2*tan(1/4*pi+1/2*x),x)
```

```
[Out] Integral(cos(x/2)**2*tan(x/2 + pi/4), x)
```

Giac [B] time = 1.1435, size = 126, normalized size = 4.67

$$\frac{x\tan\left(\frac{1}{2}x\right)^2-\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2-2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)^2+x-\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2-2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)-1}{2\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2
 + 1))*tan(1/2*x)^2 + tan(1/2*x)^2 + x - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x)
 + 1)/(tan(1/2*x)^2 + 1)) - 1)/(tan(1/2*x)^2 + 1)
```

3.916 $\int (2+3x)^2 \sin^3(x)\, dx$

Optimal. Leaf size=65

$$\frac{2}{3}(3x+2)\sin^3(x)+4(3x+2)\sin(x)-\frac{2}{3}\cos^3(x)-\frac{2}{3}(3x+2)^2\cos(x)+14\cos(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

```
[Out] 14*Cos[x] - (2*(2 + 3*x)^2*Cos[x])/3 - (2*Cos[x]^3)/3 + 4*(2 + 3*x)*Sin[x]
- ((2 + 3*x)^2*Cos[x]*Sin[x]^2)/3 + (2*(2 + 3*x)*Sin[x]^3)/3
```

Rubi [A] time = 0.0683732, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 3296, 2638, 2633}

$$\frac{2}{3}(3x+2)\sin^3(x)+4(3x+2)\sin(x)-\frac{2}{3}\cos^3(x)-\frac{2}{3}(3x+2)^2\cos(x)+14\cos(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2*Sin[x]^3,x]
```

```
[Out] 14*Cos[x] - (2*(2 + 3*x)^2*Cos[x])/3 - (2*Cos[x]^3)/3 + 4*(2 + 3*x)*Sin[x]
- ((2 + 3*x)^2*Cos[x]*Sin[x]^2)/3 + (2*(2 + 3*x)*Sin[x]^3)/3
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (2+3x)^2\sin^3(x)\,dx &= -\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x)+\frac{2}{3}(2+3x)\sin^3(x)+\frac{2}{3}\int (2+3x)^2\sin(x)\,dx-2\int \sin^3(x)\,dx\\ &= -\frac{2}{3}(2+3x)^2\cos(x)-\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x)+\frac{2}{3}(2+3x)\sin^3(x)+2\,\text{Subst}\left(\int \left(1-x^2\right)\,dx\right.\\ &= 2\cos(x)-\frac{2}{3}(2+3x)^2\cos(x)-\frac{2\cos^3(x)}{3}+4(2+3x)\sin(x)-\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x)+\frac{2}{3}(2\\ &= 14\cos(x)-\frac{2}{3}(2+3x)^2\cos(x)-\frac{2\cos^3(x)}{3}+4(2+3x)\sin(x)-\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x)+\frac{2}{3}(\end{aligned}$$

Mathematica [A] time = 0.0874298, size = 50, normalized size = 0.77

$$\frac{1}{12}\left(-9\left(9x^2+12x-14\right)\cos(x)+\left(9x^2+12x+2\right)\cos(3x)-2(3x+2)(\sin(3x)-27\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2*Sin[x]^3,x]
```

```
[Out] (-9*(-14 + 12*x + 9*x^2)*Cos[x] + (2 + 12*x + 9*x^2)*Cos[3*x] - 2*(2 + 3*x)
*(-27*Sin[x] + Sin[3*x]))/12
```

Maple [A] time = 0.026, size = 62, normalized size = 1.

$$-3\,x^2\left(2+(\sin(x))^2\right)\cos(x)+12\cos(x)+12\,x\sin(x)+2\,x(\sin(x))^3-\frac{\left(4+2\,(\sin(x))^2\right)\cos(x)}{3}-4\,x\left(2+(\sin(x))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*sin(x)^3,x)
```

```
[Out] -3*x^2*(2+sin(x)^2)*cos(x)+12*cos(x)+12*x*sin(x)+2*x*sin(x)^3-2/3*(2+sin(x)
^2)*cos(x)-4*x*(2+sin(x)^2)*cos(x)+4/3*sin(x)^3+8*sin(x)
```

Maxima [A] time = 0.970957, size = 89, normalized size = 1.37

$$\frac{4}{3}\cos\left(x\right)^{3}+\frac{1}{12}\left(9x^{2}-2\right)\cos\left(3x\right)+x\cos\left(3x\right)-\frac{27}{4}\left(x^{2}-2\right)\cos\left(x\right)-9x\cos\left(x\right)-\frac{1}{2}x\sin\left(3x\right)+\frac{27}{2}x\sin\left(x\right)-4\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 4/3*cos(x)^3 + 1/12*(9*x^2 - 2)*cos(3*x) + x*cos(3*x) - 27/4*(x^2 - 2)*cos(
x) - 9*x*cos(x) - 1/2*x*sin(3*x) + 27/2*x*sin(x) - 4*cos(x) - 1/3*sin(3*x)
+ 9*sin(x)
```

Fricas [A] time = 2.05968, size = 146, normalized size = 2.25

$$\frac{1}{3}\left(9x^{2}+12x+2\right)\cos\left(x\right)^{3}-\left(9x^{2}+12x-10\right)\cos\left(x\right)-\frac{2}{3}\left(\left(3x+2\right)\cos\left(x\right)^{2}-21x-14\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 12*x + 2)*cos(x)^3 - (9*x^2 + 12*x - 10)*cos(x) - 2/3*((3*x +
2)*cos(x)^2 - 21*x - 14)*sin(x)
```

Sympy [A] time = 1.23187, size = 100, normalized size = 1.54

$$-9x^{2}\sin^{2}\left(x\right)\cos\left(x\right)-6x^{2}\cos^{3}\left(x\right)+14x\sin^{3}\left(x\right)-12x\sin^{2}\left(x\right)\cos\left(x\right)+12x\sin\left(x\right)\cos^{2}\left(x\right)-8x\cos^{3}\left(x\right)+\frac{28\sin^{3}\left(x\right)}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*sin(x)**3,x)
```

```
[Out] -9*x**2*sin(x)**2*cos(x) - 6*x**2*cos(x)**3 + 14*x*sin(x)**3 - 12*x*sin(x)*
*2*cos(x) + 12*x*sin(x)*cos(x)**2 - 8*x*cos(x)**3 + 28*sin(x)**3/3 + 10*sin
(x)**2*cos(x) + 8*sin(x)*cos(x)**2 + 32*cos(x)**3/3
```

Giac [A] time = 1.09012, size = 69, normalized size = 1.06

$$\frac{1}{12}\left(9x^2+12x+2\right)\cos(3x)-\frac{3}{4}\left(9x^2+12x-14\right)\cos(x)-\frac{1}{6}(3x+2)\sin(3x)+\frac{9}{2}(3x+2)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*(9*x^2 + 12*x + 2)*cos(3*x) - 3/4*(9*x^2 + 12*x - 14)*cos(x) - 1/6*(3*
x + 2)*sin(3*x) + 9/2*(3*x + 2)*sin(x)
```

3.917 $\int \sec^{1+m}(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^m(x)}{m}$$

```
[Out] Sec[x]^m/m
```

Rubi [A] time = 0.0232947, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2622, 30}

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^(1 + m)*Sin[x],x]
```

```
[Out] Sec[x]^m/m
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^{1+m}(x)\sin(x)\,dx &= \text{Subst}\left(\int x^{-1+m}\,dx, x, \sec(x)\right)\\ &= \frac{\sec^m(x)}{m}\end{aligned}$$

Mathematica [A] time = 0.016455, size = 8, normalized size = 1.

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^(1 + m)*Sin[x],x]
```

```
[Out] Sec[x]^m/m
```

Maple [A] time = 0.019, size = 11, normalized size = 1.4

$$\frac{\left((\cos(x))^{-1}\right)^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^(1+m)*sin(x),x)
```

```
[Out] 1/m*(1/cos(x))^m
```

Maxima [A] time = 0.946384, size = 14, normalized size = 1.75

$$\frac{\cos(x)^{-m}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="maxima")
```

```
[Out] cos(x)^(-m)/m
```

Fricas [A] time = 2.11288, size = 39, normalized size = 4.88

$$\frac{\frac{1}{\cos(x)}^{m+1}\cos(x)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="fricas")
```

```
[Out] (1/cos(x))^(m + 1)*cos(x)/m
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)\sec^{m+1}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**(1+m)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(x)**(m + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(x)^{m+1}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="giac")
```

```
[Out] integrate(sec(x)^(m + 1)*sin(x), x)
```

3.918 $\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx$

Optimal. Leaf size=32

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] -((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

Rubi [A] time = 0.0400978, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2563}

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx=-\frac{\cos^{1+n}(a+bx)\sin^{-1-n}(a+bx)}{b(1+n)}$$

Mathematica [A] time = 0.081499, size = 32, normalized size = 1.

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int (\cos (bx + a))^{n} (\sin (bx + a))^{-2-n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
```

```
[Out] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
```

Maxima [B] time = 1.48542, size = 169, normalized size = 5.28

$$\frac{2\left(\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}} - 1\right)(\cos (bx + a) + 1)e^{\left(n\log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}+1\right) - n\log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right) + n\log\left(-\frac{\sin(bx+a)}{\cos(bx+a)+1}+1\right)\right)}}{\left(2^{n+2}n + 2^{n+2}\right)b\sin (bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="maxima")
```

```
[Out] 2*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)*e^(n*log(sin
(b*x + a)/(cos(b*x + a) + 1) + 1) - n*log(sin(b*x + a)/(cos(b*x + a) + 1))
+ n*log(-sin(b*x + a)/(cos(b*x + a) + 1) + 1))/((2^(n + 2)*n + 2^(n + 2))*b
*sin(b*x + a))
```

Fricas [A] time = 2.30636, size = 101, normalized size = 3.16

$$-\frac{\cos (bx + a)^{n} \sin (bx + a)^{-n-2} \cos (bx + a) \sin (bx + a)}{bn + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="fricas")
```

```
[Out] -cos(b*x + a)^n*sin(b*x + a)^(-n - 2)*cos(b*x + a)*sin(b*x + a)/(b*n + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**n*sin(b*x+a)**(-2-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^n \sin(bx+a)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^n*sin(b*x + a)^(-n - 2), x)
```

3.919 $\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

```
[Out] ArcTan[Sin[x]]
```

Rubi [A] time = 0.0301497, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4397, 3190, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx = \int \frac{\cos(x)}{1+\sin^2(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right)$$
$$= \tan^{-1}(\sin(x))$$

Mathematica [A] time = 0.0175787, size = 3, normalized size = 1.

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
```

```
[Out] ArcTan[Sin[x]]
```

Maple [A] time = 0.043, size = 4, normalized size = 1.3

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+sin(x)*tan(x)),x)
```

```
[Out] arctan(sin(x))
```

Maxima [B] time = 0.971993, size = 61, normalized size = 20.33

$$\frac{1}{2}\arctan\left(\sin(2x)+2\sin(x),\cos(2x)+2\cos(x)-1\right)-\frac{1}{2}\arctan\left(\sin(2x)-2\sin(x),\cos(2x)-2\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(sin(2*x) + 2*sin(x), cos(2*x) + 2*cos(x) - 1) - 1/2*arctan2(sin
(2*x) - 2*sin(x), cos(2*x) - 2*cos(x) - 1)
```

Fricas [A] time = 2.09466, size = 22, normalized size = 7.33

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin\left(x\right)\tan\left(x\right) + \sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x)
```

```
[Out] Integral(1/(sin(x)*tan(x) + sec(x)), x)
```

Giac [A] time = 1.09693, size = 4, normalized size = 1.33

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="giac")
```

```
[Out] arctan(sin(x))
```

3.920 $\int \left(a+bx+cx^2\right)\sin(x)\,dx$

Optimal. Leaf size=35

$$-a\cos(x)+b\sin(x)-bx\cos(x)-cx^2\cos(x)+2cx\sin(x)+2c\cos(x)$$

```
[Out] -(a*Cos[x]) + 2*c*Cos[x] - b*x*Cos[x] - c*x^2*Cos[x] + b*Sin[x] + 2*c*x*Sin
[x]
```

Rubi [A] time = 0.0654437, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 2638, 3296, 2637}

$$-a\cos(x)+b\sin(x)-bx\cos(x)-cx^2\cos(x)+2cx\sin(x)+2c\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*Sin[x],x]
```

```
[Out] -(a*Cos[x]) + 2*c*Cos[x] - b*x*Cos[x] - c*x^2*Cos[x] + b*Sin[x] + 2*c*x*Sin
[x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx+cx^2\right)\sin(x)\,dx &= \int \left(a\sin(x)+bx\sin(x)+cx^2\sin(x)\right)\,dx \\
&= a\int \sin(x)\,dx+b\int x\sin(x)\,dx+c\int x^2\sin(x)\,dx \\
&= -a\cos(x)-bx\cos(x)-cx^2\cos(x)+b\int \cos(x)\,dx+(2c)\int x\cos(x)\,dx \\
&= -a\cos(x)-bx\cos(x)-cx^2\cos(x)+b\sin(x)+2cx\sin(x)-(2c)\int \sin(x)\,dx \\
&= -a\cos(x)+2c\cos(x)-bx\cos(x)-cx^2\cos(x)+b\sin(x)+2cx\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0398081, size = 32, normalized size = 0.91

$$-a\cos(x)+b\sin(x)-bx\cos(x)-c\left(x^2-2\right)\cos(x)+2cx\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*Sin[x],x]
```

```
[Out] -(a*Cos[x]) - b*x*Cos[x] - c*(-2 + x^2)*Cos[x] + b*Sin[x] + 2*c*x*Sin[x]
```

Maple [A] time = 0.002, size = 36, normalized size = 1.

$$c\left(-x^2\cos(x)+2\cos(x)+2x\sin(x)\right)+b(\sin(x)-x\cos(x))-a\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*sin(x),x)
```

```
[Out] c*(-x^2*cos(x)+2*cos(x)+2*x*sin(x))+b*(sin(x)-x*cos(x))-a*cos(x)
```

Maxima [A] time = 0.95894, size = 47, normalized size = 1.34

$$-(x\cos(x)-\sin(x))b-\left(\left(x^2-2\right)\cos(x)-2x\sin(x)\right)c-a\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="maxima")
```

```
[Out] -(x*cos(x) - sin(x))*b - ((x^2 - 2)*cos(x) - 2*x*sin(x))*c - a*cos(x)
```

Fricas [A] time = 1.8922, size = 73, normalized size = 2.09

$$-\left(cx^2 + bx + a - 2\,c\right)\cos\left(x\right) + \left(2\,cx + b\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="fricas")
```

```
[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

Sympy [A] time = 0.336708, size = 39, normalized size = 1.11

$$-a\cos\left(x\right) - bx\cos\left(x\right) + b\sin\left(x\right) - cx^2\cos\left(x\right) + 2cx\sin\left(x\right) + 2c\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*sin(x),x)
```

```
[Out] -a*cos(x) - b*x*cos(x) + b*sin(x) - c*x**2*cos(x) + 2*c*x*sin(x) + 2*c*cos(
x)
```

Giac [A] time = 1.0951, size = 36, normalized size = 1.03

$$-\left(cx^2 + bx + a - 2\,c\right)\cos\left(x\right) + \left(2\,cx + b\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="giac")
```

```
[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

3.921 $\int \frac{\sin(x^5)}{x} \, dx$

Optimal. Leaf size=8

$$\frac{\text{Si}(x^5)}{5}$$

```
[Out] SinIntegral[x^5]/5
```

Rubi [A] time = 0.0065648, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3375}

$$\frac{\text{Si}(x^5)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^5]/x,x]
```

```
[Out] SinIntegral[x^5]/5
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sin(x^5)}{x} \, dx = \frac{\text{Si}(x^5)}{5}$$

Mathematica [A] time = 0.0019638, size = 8, normalized size = 1.

$$\frac{\text{Si}(x^5)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^5]/x,x]
```

```
[Out] SinIntegral[x^5]/5
```

Maple [A] time = 0.005, size = 7, normalized size = 0.9

$$\frac{\mathrm{Si}\left(x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^5)/x,x)
```

```
[Out] 1/5*Si(x^5)
```

Maxima [C] time = 1.08548, size = 23, normalized size = 2.88

$$-\frac{1}{10} i \,\mathrm{Ei}\left(i x^5\right) + \frac{1}{10} i \,\mathrm{Ei}\left(-i x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/10*I*Ei(I*x^5) + 1/10*I*Ei(-I*x^5)
```

Fricas [A] time = 1.98848, size = 31, normalized size = 3.88

$$\frac{1}{5} \,\mathrm{Si}\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="fricas")
```

```
[Out] 1/5*sin_integral(x^5)
```

Sympy [A] time = 0.571996, size = 5, normalized size = 0.62

$$\frac{\operatorname{Si}\left(x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**5)/x,x)
```

```
[Out] Si(x**5)/5
```

Giac [A] time = 1.10321, size = 8, normalized size = 1.

$$\frac{1}{5}\operatorname{Si}\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="giac")
```

```
[Out] 1/5*sin_integral(x^5)
```

3.922 $\int \frac{\sin(2^x)}{1+2^x} dx$

Optimal. Leaf size=37

$$\frac{\sin(1)\text{CosIntegral}(2^x+1)}{\log(2)} + \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

```
[Out] (CosIntegral[1 + 2^x]*Sin[1])/Log[2] + SinIntegral[2^x]/Log[2] - (Cos[1]*Si
nIntegral[1 + 2^x])/Log[2]
```

Rubi [A] time = 0.173205, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2282, 6742, 3299, 3303, 3302}

$$\frac{\sin(1)\text{CosIntegral}(2^x+1)}{\log(2)} + \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2^x]/(1 + 2^x),x]
```

```
[Out] (CosIntegral[1 + 2^x]*Sin[1])/Log[2] + SinIntegral[2^x]/Log[2] - (Cos[1]*Si
nIntegral[1 + 2^x])/Log[2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2^x)}{1+2^x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x(1+x)}\,dx,x,2^x\right)}{\log(2)}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{x}-\frac{\sin(x)}{1+x}\right)dx,x,2^x\right)}{\log(2)}\\
&= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx,x,2^x\right)}{\log(2)}-\frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx,x,2^x\right)}{\log(2)}\\
&= \frac{\mathbf{Si}(2^x)}{\log(2)}-\frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx,x,2^x\right)}{\log(2)}+\frac{\sin(1)\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx,x,2^x\right)}{\log(2)}\\
&= \frac{\mathbf{Ci}(1+2^x)\sin(1)}{\log(2)}+\frac{\mathbf{Si}(2^x)}{\log(2)}-\frac{\cos(1)\mathbf{Si}(1+2^x)}{\log(2)}
\end{aligned}$$

Mathematica [A] time = 0.0709211, size = 29, normalized size = 0.78

$$\frac{\sin(1)\mathbf{CosIntegral}(2^x+1)+\mathbf{Si}(2^x)-\cos(1)\mathbf{Si}(1+2^x)}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2^x]/(1 + 2^x),x]
```

```
[Out] (CosIntegral[1 + 2^x]*Sin[1] + SinIntegral[2^x] - Cos[1]*SinIntegral[1 + 2^
x])/Log[2]
```

Maple [A] time = 0.01, size = 38, normalized size = 1.

$$\frac{\mathrm{Si}\left(2^{x}\right)}{\ln\left(2\right)}-\frac{\cos\left(1\right)\mathrm{Si}\left(1+2^{x}\right)}{\ln\left(2\right)}+\frac{\mathrm{Ci}\left(1+2^{x}\right)\sin\left(1\right)}{\ln\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2^x)/(1+2^x),x)
```

```
[Out] Si(2^x)/ln(2)-cos(1)*Si(1+2^x)/ln(2)+Ci(1+2^x)*sin(1)/ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(2^{x}\right)}{2^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="maxima")
```

```
[Out] integrate(sin(2^x)/(2^x + 1), x)
```

Fricas [A] time = 2.14152, size = 176, normalized size = 4.76

$$\frac{\mathrm{Ci}\left(2^{x}+1\right)\sin\left(1\right)+\mathrm{Ci}\left(-2^{x}-1\right)\sin\left(1\right)-2\,\cos\left(1\right)\mathrm{Si}\left(2^{x}+1\right)+2\,\mathrm{Si}\left(2^{x}\right)}{2\,\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="fricas")
```

```
[Out] 1/2*(cos_integral(2^x + 1)*sin(1) + cos_integral(-2^x - 1)*sin(1) - 2*cos(1
)*sin_integral(2^x + 1) + 2*sin_integral(2^x))/log(2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(2^{x}\right)}{2^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2**x)/(1+2**x),x)
```

```
[Out] Integral(sin(2**x)/(2**x + 1), x)
```

Giac [A] time = 1.09163, size = 39, normalized size = 1.05

$$\frac{\operatorname{Ci}\left(2^{x}+1\right)\sin\left(1\right)-\cos\left(1\right)\operatorname{Si}\left(2^{x}+1\right)+\operatorname{Si}\left(2^{x}\right)}{\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="giac")
```

```
[Out] (cos_integral(2^x + 1)*sin(1) - cos(1)*sin_integral(2^x + 1) + sin_integral
(2^x))/log(2)
```

3.923 $\int x \cos \left(2 x^2\right) \sin^{\frac{3}{4}}\left(2 x^2\right) \, dx$

Optimal. Leaf size=14

$$\frac{1}{7} \sin^{\frac{7}{4}}\left(2 x^2\right)$$

```
[Out] Sin[2*x^2]^(7/4)/7
```

Rubi [A] time = 0.0131242, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3441}

$$\frac{1}{7} \sin^{\frac{7}{4}}\left(2 x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]

[Out] Sin[2*x^2]^(7/4)/7
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x \cos \left(2 x^2\right) \sin^{\frac{3}{4}}\left(2 x^2\right) \, dx = \frac{1}{7} \sin^{\frac{7}{4}}\left(2 x^2\right)$$

Mathematica [A] time = 0.0065822, size = 14, normalized size = 1.

$$\frac{1}{7} \sin^{\frac{7}{4}}\left(2 x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]
```

```
[Out] Sin[2*x^2]^(7/4)/7
```

Maple [A] time = 0.004, size = 11, normalized size = 0.8

$$\frac{1}{7}\left(\sin\left(2x^{2}\right)\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x)
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Maxima [A] time = 0.953281, size = 14, normalized size = 1.

$$\frac{1}{7}\sin\left(2x^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Fricas [A] time = 2.15446, size = 30, normalized size = 2.14

$$\frac{1}{7}\sin\left(2x^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Sympy [A] time = 120.493, size = 10, normalized size = 0.71

$$\frac{\sin^{\frac{7}{4}}\left(2x^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x**2)*sin(2*x**2)**(3/4),x)
```

```
[Out] sin(2*x**2)**(7/4)/7
```

Giac [A] time = 1.09325, size = 14, normalized size = 1.

$$\frac{1}{7}\,\sin\left(2\,x^2\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="giac")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

3.924 $\int x \sec^2(x^2) \tan^2(x^2) \, dx$

Optimal. Leaf size=10

$$\frac{1}{6}\tan^3(x^2)$$

[Out] Tan[x^2]^3/6

Rubi [A] time = 0.0369431, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6686}

$$\frac{1}{6}\tan^3(x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x^2]^2*Tan[x^2]^2,x]

[Out] Tan[x^2]^3/6

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \sec^2(x^2) \tan^2(x^2) \, dx = \frac{1}{6}\tan^3(x^2)$$

Mathematica [A] time = 0.00331, size = 10, normalized size = 1.

$$\frac{1}{6}\tan^3(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x^2]^2*Tan[x^2]^2,x]

```
[Out] Tan[x^2]^3/6
```

Maple [A] time = 0.022, size = 15, normalized size = 1.5

$$\frac{\left(\sin\left(x^2\right)\right)^3}{6\left(\cos\left(x^2\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)^2*tan(x^2)^2,x)
```

```
[Out] 1/6*sin(x^2)^3/cos(x^2)^3
```

Maxima [A] time = 0.935444, size = 11, normalized size = 1.1

$$\frac{1}{6}\tan\left(x^2\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*tan(x^2)^3
```

Fricas [B] time = 2.05899, size = 58, normalized size = 5.8

$$-\frac{\left(\cos\left(x^2\right)^2-1\right)\sin\left(x^2\right)}{6\cos\left(x^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(cos(x^2)^2 - 1)*sin(x^2)/cos(x^2)^3
```

Sympy [A] time = 1.73092, size = 7, normalized size = 0.7

$$\frac{\tan^3(x^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)**2*tan(x**2)**2,x)
```

```
[Out] tan(x**2)**3/6
```

Giac [A] time = 1.06285, size = 11, normalized size = 1.1

$$\frac{1}{6}\tan\left(x^2\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*tan(x^2)^3
```

3.925 $\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$

Optimal. Leaf size=17

$$-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

```
[Out] -Cos[a + b*x^3]^8/(24*b)
```

Rubi [A] time = 0.0241569, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3442}

$$-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]

[Out] -Cos[a + b*x^3]^8/(24*b)
```

Rule 3442

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[Cos[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ
[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx=-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

Mathematica [A] time = 0.0211926, size = 17, normalized size = 1.

$$-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] -Cos[a + b*x^3]^8/(24*b)
```

Maple [A] time = 0.006, size = 16, normalized size = 0.9

$$-\frac{\left(\cos\left(bx^3+a\right)\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x)
```

```
[Out] -1/24*cos(b*x^3+a)^8/b
```

Maxima [A] time = 0.959524, size = 20, normalized size = 1.18

$$-\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/24*cos(b*x^3 + a)^8/b
```

Fricas [A] time = 2.22122, size = 35, normalized size = 2.06

$$-\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/24*cos(b*x^3 + a)^8/b
```

Sympy [A] time = 30.8132, size = 27, normalized size = 1.59

$$\begin{cases} -\frac{\cos^8\left(a+bx^3\right)}{24b} & \text{for } b \neq 0 \\ \frac{x^3\sin\left(a\right)\cos^7\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

```
[Out] Piecewise((-cos(a + b*x**3)**8/(24*b), Ne(b, 0)), (x**3*sin(a)*cos(a)**7/3,
 True))
```

Giac [A] time = 1.1732, size = 20, normalized size = 1.18

$$-\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/24*cos(b*x^3 + a)^8/b
```

3.926 $\int x^5 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$

Optimal. Leaf size=129

$$\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{192b^2}+\frac{7\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{1152b^2}+\frac{35\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4608b^2}+\frac{35\sin\left(a+bx^3\right)\cos\left(a\right.}{3072b^2}$$

```
[Out] (35*x^3)/(3072*b) - (x^3*Cos[a + b*x^3]^8)/(24*b) + (35*Cos[a + b*x^3]*Sin[
a + b*x^3])/(3072*b^2) + (35*Cos[a + b*x^3]^3*Sin[a + b*x^3])/(4608*b^2) +
(7*Cos[a + b*x^3]^5*Sin[a + b*x^3])/(1152*b^2) + (Cos[a + b*x^3]^7*Sin[a +
b*x^3])/(192*b^2)
```

Rubi [A] time = 0.14444, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3444, 3380, 2635, 8}

$$\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{192b^2}+\frac{7\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{1152b^2}+\frac{35\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4608b^2}+\frac{35\sin\left(a+bx^3\right)\cos\left(a\right.}{3072b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] (35*x^3)/(3072*b) - (x^3*Cos[a + b*x^3]^8)/(24*b) + (35*Cos[a + b*x^3]*Sin[
a + b*x^3])/(3072*b^2) + (35*Cos[a + b*x^3]^3*Sin[a + b*x^3])/(4608*b^2) +
(7*Cos[a + b*x^3]^5*Sin[a + b*x^3])/(1152*b^2) + (Cos[a + b*x^3]^7*Sin[a +
b*x^3])/(192*b^2)
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x^5\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\int x^2\cos^8\left(a+bx^3\right)\,dx}{8b}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\text{Subst}\left(\int\cos^8(a+bx)\,dx,x,x^3\right)}{24b}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)}{192b^2}+\frac{7\,\text{Subst}\left(\int\cos^6(a+bx)\,dx\right.}{192b}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{7\cos^5\left(a+bx^3\right)\sin\left(a+bx^3\right)}{1152b^2}+\frac{\cos^7\left(a+bx^3\right)\sin\left(a+b\right.}{192b^2}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos^3\left(a+bx^3\right)\sin\left(a+bx^3\right)}{4608b^2}+\frac{7\cos^5\left(a+bx^3\right)\sin\left(a+\right.}{1152b^2}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos\left(a+bx^3\right)\sin\left(a+bx^3\right)}{3072b^2}+\frac{35\cos^3\left(a+bx^3\right)\sin\left(a\right.}{4608b^2}\\
&= \frac{35x^3}{3072b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos\left(a+bx^3\right)\sin\left(a+bx^3\right)}{3072b^2}+\frac{35\cos^3\left(a+bx^3\right)}{4608}\end{aligned}$$

Mathematica [A] time = 0.547363, size = 120, normalized size = 0.93

$$\frac{672\sin\left(2\left(a+bx^3\right)\right)+168\sin\left(4\left(a+bx^3\right)\right)+32\sin\left(6\left(a+bx^3\right)\right)+3\sin\left(8\left(a+bx^3\right)\right)-1344bx^3\cos\left(2\left(a+bx^3\right)\right)-}{73728b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] (-1344*b*x^3*Cos[2*(a + b*x^3)] - 672*b*x^3*Cos[4*(a + b*x^3)] - 192*b*x^3*
Cos[6*(a + b*x^3)] - 24*b*x^3*Cos[8*(a + b*x^3)] + 672*Sin[2*(a + b*x^3)] +
 168*Sin[4*(a + b*x^3)] + 32*Sin[6*(a + b*x^3)] + 3*Sin[8*(a + b*x^3)])/(73
728*b^2)
```

Maple [B] time = 0.248, size = 436, normalized size = 3.4

$$\frac{1}{128+128\left(\tan\left(bx^3+a\right)\right)^2}\left(-\frac{4x^3}{3b}+\frac{4\tan\left(bx^3+a\right)}{3b^2}+\frac{4x^3\left(\tan\left(bx^3+a\right)\right)^2}{3b}\right)+\frac{1}{128\left(1+\left(\tan\left(bx^3+a\right)\right)^2\right)^2}\left(\frac{\tan\left(bx^3\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x)
```

```
[Out] 1/128*(-4/3*x^3/b+4/3/b^2*tan(b*x^3+a)+4/3*x^3/b*tan(b*x^3+a)^2)/(1+tan(b*x
^3+a)^2)+1/128*(1/b^2*tan(b*x^3+a)-x^3/b-1/b^2*tan(b*x^3+a)^3+6*x^3/b*tan(b
*x^3+a)^2-x^3/b*tan(b*x^3+a)^4)/(1+tan(b*x^3+a)^2)^2+3/64*(-1/18*x^3/b+1/54
/b^2*tan(3*b*x^3+3*a)+1/18*x^3/b*tan(3*b*x^3+3*a)^2)/(1+tan(3*b*x^3+3*a)^2)
+3/64*(-1/6*x^3/b+1/6/b^2*tan(b*x^3+a)+1/6*x^3/b*tan(b*x^3+a)^2)/(1+tan(b*x
^3+a)^2)+1/128*(-1/6*x^3/b+1/12/b^2*tan(2*b*x^3+2*a)+1/6*x^3/b*tan(2*b*x^3+
2*a)^2)/(1+tan(2*b*x^3+2*a)^2)+1/128*(-1/24*x^3/b+1/48/b^2*tan(2*b*x^3+2*a)
-1/48/b^2*tan(2*b*x^3+2*a)^3+1/4*x^3/b*tan(2*b*x^3+2*a)^2-1/24*x^3/b*tan(2*
b*x^3+2*a)^4)/(1+tan(2*b*x^3+2*a)^2)^2
```

Maxima [A] time = 1.02108, size = 170, normalized size = 1.32

$$-\frac{24\,bx^3\cos\left(8\,bx^3+8\,a\right)+192\,bx^3\cos\left(6\,bx^3+6\,a\right)+672\,bx^3\cos\left(4\,bx^3+4\,a\right)+1344\,bx^3\cos\left(2\,bx^3+2\,a\right)-3\sin\left(8\,bx^3\right.}{73728\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672*
b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 +
8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2
*a))/b^2
```

Fricas [A] time = 2.18843, size = 213, normalized size = 1.65

$$-\frac{384\,bx^3\cos\left(bx^3+a\right)^8-105\,bx^3-\left(48\cos\left(bx^3+a\right)^7+56\cos\left(bx^3+a\right)^5+70\cos\left(bx^3+a\right)^3+105\cos\left(bx^3+a\right)\right)\sin}{9216\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/9216*(384*b*x^3*cos(b*x^3 + a)^8 - 105*b*x^3 - (48*cos(b*x^3 + a)^7 + 56
*cos(b*x^3 + a)^5 + 70*cos(b*x^3 + a)^3 + 105*cos(b*x^3 + a))*sin(b*x^3 + a
))/b^2
```

Sympy [A] time = 115.585, size = 241, normalized size = 1.87

$$\begin{cases}\frac{35x^3\sin^8\left(a+bx^3\right)}{3072b}+\frac{35x^3\sin^6\left(a+bx^3\right)\cos^2\left(a+bx^3\right)}{768b}+\frac{35x^3\sin^4\left(a+bx^3\right)\cos^4\left(a+bx^3\right)}{512b}+\frac{35x^3\sin^2\left(a+bx^3\right)\cos^6\left(a+bx^3\right)}{768b}-\frac{31x^3\cos^8\left(a+bx^3\right)}{1024b}+\frac{35\,\mathrm{s}}{} \\ \frac{x^6\sin\left(a\right)\cos^7\left(a\right)}{6}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

```
[Out] Piecewise((35*x**3*sin(a + b*x**3)**8/(3072*b) + 35*x**3*sin(a + b*x**3)**6
*cos(a + b*x**3)**2/(768*b) + 35*x**3*sin(a + b*x**3)**4*cos(a + b*x**3)**4
/(512*b) + 35*x**3*sin(a + b*x**3)**2*cos(a + b*x**3)**6/(768*b) - 31*x**3*
cos(a + b*x**3)**8/(1024*b) + 35*sin(a + b*x**3)**7*cos(a + b*x**3)/(3072*b
**2) + 385*sin(a + b*x**3)**5*cos(a + b*x**3)**3/(9216*b**2) + 511*sin(a +
b*x**3)**3*cos(a + b*x**3)**5/(9216*b**2) + 31*sin(a + b*x**3)*cos(a + b*x*
*3)**7/(1024*b**2), Ne(b, 0)), (x**6*sin(a)*cos(a)**7/6, True))
```

Giac [A] time = 1.18963, size = 170, normalized size = 1.32

$$-\frac{24\,bx^3\cos\left(8\,bx^3+8\,a\right)+192\,bx^3\cos\left(6\,bx^3+6\,a\right)+672\,bx^3\cos\left(4\,bx^3+4\,a\right)+1344\,bx^3\cos\left(2\,bx^3+2\,a\right)-3\sin\left(8\right.}{73728\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672*
b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 +
8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2
*a))/b^2
```

3.927 $\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx$

Optimal. Leaf size=110

$$-\frac{5\tanh^{-1}\left(\sin\left(a+bx^3\right)\right)}{336b^2}-\frac{\tan\left(a+bx^3\right)\sec^5\left(a+bx^3\right)}{126b^2}-\frac{5\tan\left(a+bx^3\right)\sec^3\left(a+bx^3\right)}{504b^2}-\frac{5\tan\left(a+bx^3\right)\sec\left(a+b\right.}{336b^2}$$

```
[Out] (-5*ArcTanh[Sin[a + b*x^3]])/(336*b^2) + (x^3*Sec[a + b*x^3]^7)/(21*b) - (5
*Sec[a + b*x^3]*Tan[a + b*x^3])/(336*b^2) - (5*Sec[a + b*x^3]^3*Tan[a + b*x
^3])/(504*b^2) - (Sec[a + b*x^3]^5*Tan[a + b*x^3])/(126*b^2)
```

Rubi [A] time = 0.112444, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3757, 4204, 3768, 3770}

$$-\frac{5\tanh^{-1}\left(\sin\left(a+bx^3\right)\right)}{336b^2}-\frac{\tan\left(a+bx^3\right)\sec^5\left(a+bx^3\right)}{126b^2}-\frac{5\tan\left(a+bx^3\right)\sec^3\left(a+bx^3\right)}{504b^2}-\frac{5\tan\left(a+bx^3\right)\sec\left(a+b\right.}{336b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

```
[Out] (-5*ArcTanh[Sin[a + b*x^3]])/(336*b^2) + (x^3*Sec[a + b*x^3]^7)/(21*b) - (5
*Sec[a + b*x^3]*Tan[a + b*x^3])/(336*b^2) - (5*Sec[a + b*x^3]^3*Tan[a + b*x
^3])/(504*b^2) - (Sec[a + b*x^3]^5*Tan[a + b*x^3])/(126*b^2)
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx &= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{\int x^2\sec^7\left(a+bx^3\right)\,dx}{7b} \\
&= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{\text{Subst}\left(\int \sec^7(a+bx)\,dx, x, x^3\right)}{21b} \\
&= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{\sec^5\left(a+bx^3\right)\tan\left(a+bx^3\right)}{126b^2} - \frac{5\,\text{Subst}\left(\int \sec^5(a+bx)\,dx, x\right.}{126b} \\
&= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{5\sec^3\left(a+bx^3\right)\tan\left(a+bx^3\right)}{504b^2} - \frac{\sec^5\left(a+bx^3\right)\tan\left(a+bx^3\right.}{126b^2} \\
&= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{5\sec\left(a+bx^3\right)\tan\left(a+bx^3\right)}{336b^2} - \frac{5\sec^3\left(a+bx^3\right)\tan\left(a+bx^3\right.}{504b^2} \\
&= -\frac{5\tanh^{-1}\left(\sin\left(a+bx^3\right)\right)}{336b^2} + \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{5\sec\left(a+bx^3\right)\tan\left(a+bx^3\right)}{336b^2} -
\end{aligned}$$

Mathematica [B] time = 0.883299, size = 352, normalized size = 3.2

$$\frac{\sec^7\left(a+bx^3\right)\left(-566\sin\left(2\left(a+bx^3\right)\right)-200\sin\left(4\left(a+bx^3\right)\right)-30\sin\left(6\left(a+bx^3\right)\right)+105\cos\left(5\left(a+bx^3\right)\right)\log\left(\cos\left(\frac{1}{2}\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

```
[Out] (Sec[a + b*x^3]^7*(3072*b*x^3 + 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/
2] - Sin[(a + b*x^3)/2]] + 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] - S
in[(a + b*x^3)/2]] + 525*Cos[a + b*x^3]*(Log[Cos[(a + b*x^3)/2] - Sin[(a +
b*x^3)/2]] - Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]]) + 315*Cos[3*(a +
 b*x^3)]*(Log[Cos[(a + b*x^3)/2] - Sin[(a + b*x^3)/2]] - Log[Cos[(a + b*x^3
```

```
)/2] + Sin[(a + b*x^3)/2]]) - 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2]
 + Sin[(a + b*x^3)/2]] - 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] + Sin
[(a + b*x^3)/2]] - 566*Sin[2*(a + b*x^3)] - 200*Sin[4*(a + b*x^3)] - 30*Sin
[6*(a + b*x^3)]))/(64512*b^2)
```

Maple [C] time = 0.148, size = 160, normalized size = 1.5

$$\frac{\frac{i}{504}\left(15\,e^{13\,i\left(b x^3+a\right)}-3072\,i b x^3 e^{7\,i\left(b x^3+a\right)}+100\,e^{11\,i\left(b x^3+a\right)}+283\,e^{9\,i\left(b x^3+a\right)}-283\,e^{5\,i\left(b x^3+a\right)}-100\,e^{3\,i\left(b x^3+a\right)}-15\,e^{i\left(b x^3+a\right)}\right)}{b^2\left(e^{2\,i\left(b x^3+a\right)}+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x)
```

```
[Out] 1/504*I/b^2/(exp(2*I*(b*x^3+a))+1)^7*(15*exp(13*I*(b*x^3+a))-3072*I*b*x^3*e
xp(7*I*(b*x^3+a))+100*exp(11*I*(b*x^3+a))+283*exp(9*I*(b*x^3+a))-283*exp(5*
I*(b*x^3+a))-100*exp(3*I*(b*x^3+a))-15*exp(I*(b*x^3+a)))-5/336/b^2*ln(exp(I
*(b*x^3+a))+I)+5/336/b^2*ln(exp(I*(b*x^3+a))-I)
```

Maxima [B] time = 2.32208, size = 5171, normalized size = 47.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2016*(4*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 15*sin(13*b*x^3 + 13*a) - 100*si
n(11*b*x^3 + 11*a) - 283*sin(9*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*
sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(14*b*x^3 + 14*a) + 420*(sin(12*
b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x
^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(13*b*x^3 + 13*a)
 + 28*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 100*sin(11*b*x^3 + 11*a) - 283*sin(9
*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*
x^3 + a))*cos(12*b*x^3 + 12*a) + 2800*(3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x
^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a
))*cos(11*b*x^3 + 11*a) + 84*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 283*sin(9*b*x
^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*x^3
```

+ a))*cos(10*b*x^3 + 10*a) + 7924*(5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(9*b*x^3 + 9*a) + 140*(3072*b*x^3*cos(7*b*x^3 + 7*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(8*b*x^3 + 8*a) + 12288*(35*b*x^3*cos(6*b*x^3 + 6*a) + 21*b*x^3*cos(4*b*x^3 + 4*a) + 7*b*x^3*cos(2*b*x^3 + 2*a) + b*x^3)*cos(7*b*x^3 + 7*a) + 140*(283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(6*b*x^3 + 6*a) - 7924*(3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(5*b*x^3 + 5*a) + 420*(20*sin(3*b*x^3 + 3*a) + 3*sin(b*x^3 + a))*cos(4*b*x^3 + 4*a) + 15*(2*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(14*b*x^3 + 14*a) + cos(14*b*x^3 + 14*a)^2 + 14*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(12*b*x^3 + 12*a) + 49*cos(12*b*x^3 + 12*a)^2 + 42*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(10*b*x^3 + 10*a) + 441*cos(10*b*x^3 + 10*a)^2 + 70*(35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(8*b*x^3 + 8*a) + 1225*cos(8*b*x^3 + 8*a)^2 + 70*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(6*b*x^3 + 6*a) + 1225*cos(6*b*x^3 + 6*a)^2 + 42*(7*cos(2*b*x^3 + 2*a) + 1)*cos(4*b*x^3 + 4*a) + 441*cos(4*b*x^3 + 4*a)^2 + 49*cos(2*b*x^3 + 2*a)^2 + 14*(sin(12*b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(14*b*x^3 + 14*a) + sin(14*b*x^3 + 14*a)^2 + 98*(3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(12*b*x^3 + 12*a) + 49*sin(12*b*x^3 + 12*a)^2 + 294*(5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(10*b*x^3 + 10*a) + 441*sin(10*b*x^3 + 10*a)^2 + 490*(5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(8*b*x^3 + 8*a) + 1225*sin(8*b*x^3 + 8*a)^2 + 490*(3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(6*b*x^3 + 6*a) + 1225*sin(6*b*x^3 + 6*a)^2 + 441*sin(4*b*x^3 + 4*a)^2 + 294*sin(4*b*x^3 + 4*a)*sin(2*b*x^3 + 2*a) + 49*sin(2*b*x^3 + 2*a)^2 + 14*cos(2*b*x^3 + 2*a) + 1)*log((cos(b*x^3 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x^3 + 2*a) + sin(b*x^3 + 2*a)^2 + 2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)/(cos(b*x^3 + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x^3 + 2*a) + sin(b*x^3 + 2*a)^2 - 2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)) + 4*(3072*b*x^3*sin(7*b*x^3 + 7*a) + 15*cos(13*b*x^3 + 13*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(14*b*x^3 + 14*a) - 60*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(13*b*x^3 + 13*a) + 28*(3072*b*x^3*sin(7*b*x^3 + 7*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(12*b*x^3 + 12*a) - 400*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(11*b*x^3 + 11*a) + 84*(3072*b*x^3*sin(7*b*x^3 + 7*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5

```
*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(10*b*x^3 +
10*a) - 1132*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^
3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(9*b*x^3 + 9*a) + 140*(3072*b*x^3*s
in(7*b*x^3 + 7*a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*co
s(b*x^3 + a))*sin(8*b*x^3 + 8*a) + 86016*(5*b*x^3*sin(6*b*x^3 + 6*a) + 3*b*
x^3*sin(4*b*x^3 + 4*a) + b*x^3*sin(2*b*x^3 + 2*a))*sin(7*b*x^3 + 7*a) - 140
*(283*cos(5*b*x^3 + 5*a) + 100*cos(3*b*x^3 + 3*a) + 15*cos(b*x^3 + a))*sin(
6*b*x^3 + 6*a) + 1132*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*si
n(5*b*x^3 + 5*a) - 420*(20*cos(3*b*x^3 + 3*a) + 3*cos(b*x^3 + a))*sin(4*b*x
^3 + 4*a) + 400*(7*cos(2*b*x^3 + 2*a) + 1)*sin(3*b*x^3 + 3*a) - 2800*cos(3*
b*x^3 + 3*a)*sin(2*b*x^3 + 2*a) - 420*cos(b*x^3 + a)*sin(2*b*x^3 + 2*a) + 4
20*cos(2*b*x^3 + 2*a)*sin(b*x^3 + a) + 60*sin(b*x^3 + a))/(b^2*cos(14*b*x^3
 + 14*a)^2 + 49*b^2*cos(12*b*x^3 + 12*a)^2 + 441*b^2*cos(10*b*x^3 + 10*a)^2
 + 1225*b^2*cos(8*b*x^3 + 8*a)^2 + 1225*b^2*cos(6*b*x^3 + 6*a)^2 + 441*b^2*
cos(4*b*x^3 + 4*a)^2 + 49*b^2*cos(2*b*x^3 + 2*a)^2 + b^2*sin(14*b*x^3 + 14*
a)^2 + 49*b^2*sin(12*b*x^3 + 12*a)^2 + 441*b^2*sin(10*b*x^3 + 10*a)^2 + 122
5*b^2*sin(8*b*x^3 + 8*a)^2 + 1225*b^2*sin(6*b*x^3 + 6*a)^2 + 441*b^2*sin(4*
b*x^3 + 4*a)^2 + 294*b^2*sin(4*b*x^3 + 4*a)*sin(2*b*x^3 + 2*a) + 49*b^2*sin
(2*b*x^3 + 2*a)^2 + 14*b^2*cos(2*b*x^3 + 2*a) + b^2 + 2*(7*b^2*cos(12*b*x^3
 + 12*a) + 21*b^2*cos(10*b*x^3 + 10*a) + 35*b^2*cos(8*b*x^3 + 8*a) + 35*b^2
*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a)
+ b^2)*cos(14*b*x^3 + 14*a) + 14*(21*b^2*cos(10*b*x^3 + 10*a) + 35*b^2*cos(
8*b*x^3 + 8*a) + 35*b^2*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*
b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(12*b*x^3 + 12*a) + 42*(35*b^2*cos(8*b*x^3
 + 8*a) + 35*b^2*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos
(2*b*x^3 + 2*a) + b^2)*cos(10*b*x^3 + 10*a) + 70*(35*b^2*cos(6*b*x^3 + 6*a)
 + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(8*b*x^3
+ 8*a) + 70*(21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a) + b^2)*co
s(6*b*x^3 + 6*a) + 42*(7*b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(4*b*x^3 + 4*a) +
 14*(b^2*sin(12*b*x^3 + 12*a) + 3*b^2*sin(10*b*x^3 + 10*a) + 5*b^2*sin(8*b*
x^3 + 8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a) + b^2*sin(
2*b*x^3 + 2*a))*sin(14*b*x^3 + 14*a) + 98*(3*b^2*sin(10*b*x^3 + 10*a) + 5*b
^2*sin(8*b*x^3 + 8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a)
 + b^2*sin(2*b*x^3 + 2*a))*sin(12*b*x^3 + 12*a) + 294*(5*b^2*sin(8*b*x^3 +
8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a) + b^2*sin(2*b*x^
3 + 2*a))*sin(10*b*x^3 + 10*a) + 490*(5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(
4*b*x^3 + 4*a) + b^2*sin(2*b*x^3 + 2*a))*sin(8*b*x^3 + 8*a) + 490*(3*b^2*si
n(4*b*x^3 + 4*a) + b^2*sin(2*b*x^3 + 2*a))*sin(6*b*x^3 + 6*a))
```

Fricas [A] time = 2.31409, size = 294, normalized size = 2.67

$$-\frac{15 \cos\left(bx^3+a\right)^7 \log\left(\sin\left(bx^3+a\right)+1\right) - 15 \cos\left(bx^3+a\right)^7 \log\left(-\sin\left(bx^3+a\right)+1\right) - 96\,bx^3 + 2\left(15 \cos\left(bx^3+a\right)^5 + \right.}{2016\,b^2 \cos\left(bx^3+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/2016*(15*cos(b*x^3 + a)^7*log(sin(b*x^3 + a) + 1) - 15*cos(b*x^3 + a)^7*
log(-sin(b*x^3 + a) + 1) - 96*b*x^3 + 2*(15*cos(b*x^3 + a)^5 + 10*cos(b*x^3
 + a)^3 + 8*cos(b*x^3 + a))*sin(b*x^3 + a))/(b^2*cos(b*x^3 + a)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \tan\left(a+bx^3\right) \sec^7\left(a+bx^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*sec(b*x**3+a)**7*tan(b*x**3+a),x)
```

```
[Out] Integral(x**5*tan(a + b*x**3)*sec(a + b*x**3)**7, x)
```

Giac [B] time = 2.43347, size = 1964, normalized size = 17.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/2016*(96*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^14 - 96*a*tan(1/2*b*x^3 + 1/
2*a)^14 + 15*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1
)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^14 - 15*log(2*(tan
(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*
a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^14 + 672*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*
a)^12 - 672*a*tan(1/2*b*x^3 + 1/2*a)^12 - 105*log(2*(tan(1/2*b*x^3 + 1/2*a)
```

```
^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*
b*x^3 + 1/2*a)^12 + 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 +
 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^12 + 13
2*tan(1/2*b*x^3 + 1/2*a)^13 + 2016*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^10 -
2016*a*tan(1/2*b*x^3 + 1/2*a)^10 + 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*
tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 +
 1/2*a)^10 - 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a)
 + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^10 - 112*tan(1
/2*b*x^3 + 1/2*a)^11 + 3360*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^8 - 3360*a*t
an(1/2*b*x^3 + 1/2*a)^8 - 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b
*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^8
 + 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan
(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^8 + 340*tan(1/2*b*x^3 +
1/2*a)^9 + 3360*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^6 - 3360*a*tan(1/2*b*x^3
 + 1/2*a)^6 + 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a
) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^6 - 525*log(2
*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 +
 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^6 + 2016*(b*x^3 + a)*tan(1/2*b*x^3 +
 1/2*a)^4 - 2016*a*tan(1/2*b*x^3 + 1/2*a)^4 - 315*log(2*(tan(1/2*b*x^3 + 1/
2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(
1/2*b*x^3 + 1/2*a)^4 + 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^
3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^4 -
340*tan(1/2*b*x^3 + 1/2*a)^5 + 96*b*x^3 + 672*(b*x^3 + a)*tan(1/2*b*x^3 + 1
/2*a)^2 - 672*a*tan(1/2*b*x^3 + 1/2*a)^2 + 105*log(2*(tan(1/2*b*x^3 + 1/2*a
)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2
*b*x^3 + 1/2*a)^2 - 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 +
 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^2 + 112
*tan(1/2*b*x^3 + 1/2*a)^3 - 15*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*
b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1)) + 15*log(2*(tan(1/2*b*x
^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1
)) - 132*tan(1/2*b*x^3 + 1/2*a))/((tan(1/2*b*x^3 + 1/2*a)^14 - 7*tan(1/2*b*
x^3 + 1/2*a)^12 + 21*tan(1/2*b*x^3 + 1/2*a)^10 - 35*tan(1/2*b*x^3 + 1/2*a)^
8 + 35*tan(1/2*b*x^3 + 1/2*a)^6 - 21*tan(1/2*b*x^3 + 1/2*a)^4 + 7*tan(1/2*b
*x^3 + 1/2*a)^2 - 1)*b^2)
```

3.928 $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx$

Optimal. Leaf size=6

$$-\tan\left(\frac{1}{x}\right)$$

```
[Out] -Tan[x^(-1)]
```

Rubi [A] time = 0.0204549, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4204, 3767, 8}

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x^(-1)]^2/x^2,x]
```

```
[Out] -Tan[x^(-1)]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\text{Subst}\left(\int \sec^2(x)\,dx, x, \frac{1}{x}\right)$$
$$= \text{Subst}\left(\int 1\,dx, x, -\tan\left(\frac{1}{x}\right)\right)$$
$$= -\tan\left(\frac{1}{x}\right)$$

Mathematica [A] time = 0.0175814, size = 6, normalized size = 1.

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x^(-1)]^2/x^2,x]
```

```
[Out] -Tan[x^(-1)]
```

Maple [A] time = 0.007, size = 7, normalized size = 1.2

$$-\tan\left(x^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(1/x)^2/x^2,x)
```

```
[Out] -tan(1/x)
```

Maxima [B] time = 0.961808, size = 49, normalized size = 8.17

$$-\frac{2\sin\left(\frac{2}{x}\right)}{\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2\cos\left(\frac{2}{x}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*sin(2/x)/(cos(2/x)^2 + sin(2/x)^2 + 2*cos(2/x) + 1)
```

Fricas [A] time = 2.11173, size = 27, normalized size = 4.5

$$-\frac{\sin\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -sin(1/x)/cos(1/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)**2/x**2,x)
```

```
[Out] Integral(sec(1/x)**2/x**2, x)
```

Giac [A] time = 1.07121, size = 8, normalized size = 1.33

$$-\tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -tan(1/x)
```

3.929 $\int 3x^2 \cos(x^3)\, dx$

Optimal. Leaf size=4

$$\sin(x^3)$$

```
[Out] Sin[x^3]
```

Rubi [A] time = 0.0097691, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 3380, 2637}

$$\sin(x^3)$$

Antiderivative was successfully verified.

```
[In] Int[3*x^2*Cos[x^3],x]
```

```
[Out] Sin[x^3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int 3x^2\cos\left(x^3\right)\,dx &= 3\int x^2\cos\left(x^3\right)\,dx\\ &= \text{Subst}\left(\int \cos(x)\,dx, x, x^3\right)\\ &= \sin\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.0016308, size = 4, normalized size = 1.

$$\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*x^2*Cos[x^3],x]
```

```
[Out] Sin[x^3]
```

Maple [A] time = 0.002, size = 5, normalized size = 1.3

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3),x)
```

```
[Out] sin(x^3)
```

Maxima [A] time = 0.940966, size = 5, normalized size = 1.25

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] sin(x^3)
```

Fricas [A] time = 2.01688, size = 14, normalized size = 3.5

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="fricas")
```

```
[Out] sin(x^3)
```

Sympy [A] time = 0.292507, size = 3, normalized size = 0.75

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x**2*cos(x**3),x)
```

```
[Out] sin(x**3)
```

Giac [A] time = 1.07257, size = 5, normalized size = 1.25

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="giac")
```

```
[Out] sin(x^3)
```

3.930 $\int (1+2x)\sec^2(1+2x)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}(2x+1)\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

```
[Out] Log[Cos[1 + 2*x]]/2 + ((1 + 2*x)*Tan[1 + 2*x])/2
```

Rubi [A] time = 0.0234298, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4184, 3475}

$$\frac{1}{2}(2x+1)\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

```
[Out] Log[Cos[1 + 2*x]]/2 + ((1 + 2*x)*Tan[1 + 2*x])/2
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+2x)\sec^2(1+2x)\,dx &= \frac{1}{2}(1+2x)\tan(1+2x)-\int \tan(1+2x)\,dx\\ &= \frac{1}{2}\log(\cos(1+2x))+\frac{1}{2}(1+2x)\tan(1+2x)\end{aligned}$$

Mathematica [A] time = 0.0145957, size = 30, normalized size = 1.11

$$x\tan(2x+1)+\frac{1}{2}\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

```
[Out] Log[Cos[1 + 2*x]]/2 + Tan[1 + 2*x]/2 + x*Tan[1 + 2*x]
```

Maple [A] time = 0.008, size = 24, normalized size = 0.9

$$\frac{\ln(\cos(1+2x))}{2}+\frac{(1+2x)\tan(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*sec(1+2*x)^2,x)
```

```
[Out] 1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)
```

Maxima [B] time = 1.44956, size = 132, normalized size = 4.89

$$\frac{\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)\log\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)+4(2x}{4\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((cos(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)*log(cos(4*x + 2
)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1) + 4*(2*x + 1)*sin(4*x + 2))/(cos
(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)
```

Fricas [A] time = 2.02397, size = 104, normalized size = 3.85

$$\frac{\cos(2x+1)\log(-\cos(2x+1))+(2x+1)\sin(2x+1)}{2\cos(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(2*x + 1)*log(-cos(2*x + 1)) + (2*x + 1)*sin(2*x + 1))/cos(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x+1)\sec^2(2x+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)**2,x)
```

```
[Out] Integral((2*x + 1)*sec(2*x + 1)**2, x)
```

Giac [B] time = 1.356, size = 1273, normalized size = 47.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(log(4*(tan(1/2)^4 + 2*tan(1/2)^2 + 1)/(tan(1/2)^4*tan(x)^8 - 8*tan(1/2
)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3
*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*tan
(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^
4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*
tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1))*tan(1/2)^2*tan(x)^2 - 8*x
*tan(1/2)^2*tan(x) - 8*x*tan(1/2)*tan(x)^2 - log(4*(tan(1/2)^4 + 2*tan(1/2)
^2 + 1)/(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^
8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6
+ 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*ta
n(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/
2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2
)*tan(x) + 1))*tan(1/2)^2 - 4*log(4*(tan(1/2)^4 + 2*tan(1/2)^2 + 1)/(tan(1/
2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^
4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*ta
```

```
n(x)^7 + tan(x)^8 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(
1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 -
 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1))*
tan(1/2)*tan(x) - 4*tan(1/2)^2*tan(x) - log(4*(tan(1/2)^4 + 2*tan(1/2)^2 +
1)/(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2
*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*t
an(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^
4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*
tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan
(x) + 1))*tan(x)^2 - 4*tan(1/2)*tan(x)^2 + 8*x*tan(1/2) + 8*x*tan(x) + log(
4*(tan(1/2)^4 + 2*tan(1/2)^2 + 1)/(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x
)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5
 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3*t
an(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan
(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4
- 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1)) + 4*tan(1/2) + 4*tan(x))/(tan(1/2)
^2*tan(x)^2 - tan(1/2)^2 - 4*tan(1/2)*tan(x) - tan(x)^2 + 1)
```

3.931 $\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right) dx$

Optimal. Leaf size=26

$$\frac{2x^2\sqrt{3\sin(a+bx)+x^3}}{3b}$$

```
[Out] (2*x^2*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b)
```

Rubi [F] time = 0.810739, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Si
n[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]
```

```
[Out] Defer[Int][x^4/Sqrt[x^3 + 3*Sin[a + b*x]], x]/b + Defer[Int][(x^2*Cos[a + b
*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x] + (4*Defer[Int][x*Sqrt[x^3 + 3*Sin[a +
b*x]], x])/(3*b)
```

Rubi steps

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right) dx=\frac{\int \frac{x^4}{\sqrt{x^3+3\sin(a+bx)}}\,dx}{b}+\frac{4\int x\sqrt{x^3+3\sin}}{3b}$$

Mathematica [A] time = 0.430944, size = 26, normalized size = 1.

$$\frac{2x^2\sqrt{3\sin(a+bx)+x^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3
+ 3*Sin[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]
```

```
[Out] (2*x^2*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b)
```

Maple [A] time = 0.324, size = 28, normalized size = 1.1

$$\frac{\sqrt{2}x^{2}}{3\,b}\sqrt{2\,x^{3}+6\,\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2)+
4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x)
```

```
[Out] 1/3*(2*x^3+6*sin(b*x+a))^(1/2)/b*2^(1/2)*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\sqrt{x^{3}+3\,\sin\left(bx+a\right)}b}+\frac{x^{2}\cos\left(bx+a\right)}{\sqrt{x^{3}+3\,\sin\left(bx+a\right)}}+\frac{4\,\sqrt{x^{3}+3\,\sin\left(bx+a\right)}x}{3\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{7x^4}{\sqrt{x^3+3\sin(a+bx)}}\,dx + \int \frac{12x\sin(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx + \int \frac{3bx^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/b/(x**3+3*sin(b*x+a))**(1/2)+x**2*cos(b*x+a)/(x**3+3*sin(b*x
+a))**(1/2)+4/3*x*(x**3+3*sin(b*x+a))**(1/2)/b,x)
```

```
[Out] (Integral(7*x**4/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(12*x*sin(a + b*
x)/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(3*b*x**2*cos(a + b*x)/sqrt(x*
*3 + 3*sin(a + b*x)), x))/(3*b)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^3+3\sin(bx+a)}\,b} + \frac{x^2\cos(bx+a)}{\sqrt{x^3+3\sin(bx+a)}} + \frac{4\sqrt{x^3+3\sin(bx+a)}\,x}{3\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

3.932 $\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$

Optimal. Leaf size=28

$$\text{CannotIntegrate}\left(\frac{x^2\cos(a+bx)}{\sqrt{3\sin(a+bx)+x^3}},x\right)$$

```
[Out] CannotIntegrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

Rubi [A] time = 0.110789, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]

[Out] Defer[Int][(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

Rubi steps

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx=\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Mathematica [A] time = 7.65393, size = 0, normalized size = 0.

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]
```

```
[Out] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

Maple [A] time = 0.38, size = 0, normalized size = 0.

$$\int x^2 \cos(bx+a) \frac{1}{\sqrt{x^3+3\,\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

```
[Out] int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cos(bx+a)}{\sqrt{x^3+3\,\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)/(x**3+3*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2*cos(a + b*x)/sqrt(x**3 + 3*sin(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cos(bx+a)}{\sqrt{x^3+3\,\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

3.933 $\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx$

Optimal. Leaf size=9

$$\log\left(e^x\sin(x)+1\right)$$

```
[Out] Log[1 + E^x*Sin[x]]
```

Rubi [F] time = 0.380774, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]
```

```
[Out] x + Log[Sin[x]] - Defer[Int][(1 + E^x*Sin[x])^(-1), x] - Defer[Int][Cot[x]/
(1 + E^x*Sin[x]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx &= \int \left(1+\cot(x)-\frac{(1+\cot(x))\csc(x)}{e^x+\csc(x)}\right)dx\\
&= x+\int \cot(x)\,dx-\int \frac{(1+\cot(x))\csc(x)}{e^x+\csc(x)}\,dx\\
&= x+\log(\sin(x))-\int \left(\frac{1}{1+e^x\sin(x)}+\frac{\cot(x)}{1+e^x\sin(x)}\right)dx\\
&= x+\log(\sin(x))-\int \frac{1}{1+e^x\sin(x)}\,dx-\int \frac{\cot(x)}{1+e^x\sin(x)}\,dx
\end{aligned}$$

Mathematica [A] time = 0.123442, size = 9, normalized size = 1.

$$\log\left(e^x\sin(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]
```

```
[Out] Log[1 + E^x*Sin[x]]
```

Maple [B] time = 0.063, size = 57, normalized size = 6.3

$$\left(x+x\left(\tan\left(\frac{x}{2}\right)\right)^{2}\right)\left(1+\left(\tan\left(\frac{x}{2}\right)\right)^{2}\right)^{-1}-\ln\left(1+\left(\tan\left(\frac{x}{2}\right)\right)^{2}\right)+\ln\left(\mathrm{e}^{-x}\left(\tan\left(\frac{x}{2}\right)\right)^{2}+\mathrm{e}^{-x}+2\tan\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(exp(-x)+sin(x)),x)
```

```
[Out] (x+x*tan(1/2*x)^2)/(1+tan(1/2*x)^2)-ln(1+tan(1/2*x)^2)+ln(exp(-x)*tan(1/2*x
)^2+exp(-x)+2*tan(1/2*x))
```

Maxima [B] time = 1.61049, size = 111, normalized size = 12.33

$$x+\frac{1}{2}\log\left(\left(\cos\left(2x\right)^{2}e^{(2x)}+4\cos\left(x\right)e^{x}\sin\left(2x\right)+e^{(2x)}\sin\left(2x\right)^{2}-2\left(2e^{x}\sin\left(x\right)+e^{(2x)}\right)\cos\left(2x\right)+4\cos\left(x\right)^{2}+4e^{x}\text{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="maxima")
```

```
[Out] x + 1/2*log((cos(2*x)^2*e^(2*x) + 4*cos(x)*e^x*sin(2*x) + e^(2*x)*sin(2*x)^
2 - 2*(2*e^x*sin(x) + e^(2*x))*cos(2*x) + 4*cos(x)^2 + 4*e^x*sin(x) + 4*sin
(x)^2 + e^(2*x))*e^(-2*x))
```

Fricas [A] time = 2.1199, size = 35, normalized size = 3.89

$$x+\log\left(e^{(-x)}+\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="fricas")
```

```
[Out] x + log(e^(-x) + sin(x))
```

Sympy [A] time = 0.331089, size = 10, normalized size = 1.11

$$x + \log\left(\sin\left(x\right) + e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x)
```

```
[Out] x + log(sin(x) + exp(-x))
```

Giac [B] time = 1.06841, size = 112, normalized size = 12.44

$$x + \frac{1}{2}\,\log\left(\frac{4\left(e^{(-2\,x)}\tan\left(\frac{1}{2}\,x\right)^{4} + 4\,e^{(-x)}\tan\left(\frac{1}{2}\,x\right)^{3} + 2\,e^{(-2\,x)}\tan\left(\frac{1}{2}\,x\right)^{2} + 4\,e^{(-x)}\tan\left(\frac{1}{2}\,x\right) + 4\,\tan\left(\frac{1}{2}\,x\right)^{2} + e^{(-2\,x)}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4} + 2\,\tan\left(\frac{1}{2}\,x\right)^{2} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="giac")
```

```
[Out] x + 1/2*log(4*(e^(-2*x)*tan(1/2*x)^4 + 4*e^(-x)*tan(1/2*x)^3 + 2*e^(-2*x)*t
an(1/2*x)^2 + 4*e^(-x)*tan(1/2*x) + 4*tan(1/2*x)^2 + e^(-2*x))/(tan(1/2*x)^
4 + 2*tan(1/2*x)^2 + 1))
```

3.934 $\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)dx$

Optimal. Leaf size=77

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] (3*b*x)/8 - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*b*Cos[c + d*
x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.12597, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4393, 2748, 2633, 2635, 8}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*b*x)/8 - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*b*Cos[c + d*
x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 4393

```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)dx &= \int \sin^3(c+dx)(a+b\sin(c+dx))\,dx\\ &= a\int \sin^3(c+dx)\,dx+b\int \sin^4(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}(3b)\int \sin^2(c+dx)\,dx-\frac{a\,\mathrm{Subst}\big(}{}\\ &= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3b\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\,\mathrm{co}}{}\\ &= \frac{3bx}{8}-\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3b\cos(c+dx)\sin(c+dx)}{8d}\end{aligned}$$

Mathematica [A] time = 0.159127, size = 76, normalized size = 0.99

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}+\frac{3b(c+dx)}{8d}-\frac{b\sin(2(c+dx))}{4d}+\frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12
*d) - (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.012, size = 60, normalized size = 0.8

$$\frac{1}{d}\left(b\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x)
```

```
[Out] 1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a*
(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A] time = 0.968368, size = 77, normalized size = 1.

$$\frac{32\left(\cos(dx+c)^3-3\cos(dx+c)\right)a+3\left(12dx+12c+\sin(4dx+4c)-8\sin(2dx+2c)\right)b}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(d*x + c)^3 - 3*cos(d*x + c))*a + 3*(12*d*x + 12*c + sin(4*d*x
 + 4*c) - 8*sin(2*d*x + 2*c))*b)/d
```

Fricas [A] time = 2.09194, size = 157, normalized size = 2.04

$$\frac{8a\cos(dx+c)^3+9bdx-24a\cos(dx+c)+3\left(2b\cos(dx+c)^3-5b\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x - 24*a*cos(d*x + c) + 3*(2*b*cos(d*x + c
)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.0998, size = 150, normalized size = 1.95

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2a\cos^3(c+dx)}{3d} + \frac{3bx\sin^4(c+dx)}{8} + \frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3bx\cos^4(c+dx)}{8} - \frac{5b\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{3b\sin(c\cdot \\ x\left(a\sin^2(c) + b\sin^3(c)\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) +
3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*c
os(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*b*sin(c + d*x
)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*sin(c)**3)*sin(c),
True))
```

Giac [A] time = 1.07828, size = 84, normalized size = 1.09

$$\frac{3}{8}\,bx + \frac{a\cos(3\,dx + 3\,c)}{12\,d} - \frac{3\,a\cos(dx + c)}{4\,d} + \frac{b\sin(4\,dx + 4\,c)}{32\,d} - \frac{b\sin(2\,dx + 2\,c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/12*a*cos(3*d*x + 3*c)/d - 3/4*a*cos(d*x + c)/d + 1/32*b*sin(4*d
*x + 4*c)/d - 1/4*b*sin(2*d*x + 2*c)/d
```

3.935 $\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=161

$$-\frac{\left(a^2+3b^2\right)\cos^5(c+dx)}{5d}+\frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(a^2+b^2\right)\cos(c+dx)}{d}-\frac{ab\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{5ab\sin^3(}{}$$

```
[Out] (5*a*b*x)/8 - ((a^2 + b^2)*Cos[c + d*x])/d + ((2*a^2 + 3*b^2)*Cos[c + d*x]^
3)/(3*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^5)/(5*d) + (b^2*Cos[c + d*x]^7)/(7*d
) - (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (5*a*b*Cos[c + d*x]*Sin[c + d
*x]^3)/(12*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```

Rubi [A] time = 0.269948, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4393, 2789, 2635, 8, 3013, 373}

$$-\frac{\left(a^2+3b^2\right)\cos^5(c+dx)}{5d}+\frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(a^2+b^2\right)\cos(c+dx)}{d}-\frac{ab\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{5ab\sin^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```


```
[Out] (5*a*b*x)/8 - ((a^2 + b^2)*Cos[c + d*x])/d + ((2*a^2 + 3*b^2)*Cos[c + d*x]^
3)/(3*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^5)/(5*d) + (b^2*Cos[c + d*x]^7)/(7*d
) - (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (5*a*b*Cos[c + d*x]*Sin[c + d
*x]^3)/(12*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```


Rule 4393

```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx = \int \sin^5(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$= (2ab)\int \sin^6(c+dx)\,dx + \int \sin^5(c+dx)\left(a^2+b^2\sin^2(c+dx)\right)\,dx$$

$$= -\frac{ab\cos(c+dx)\sin^5(c+dx)}{3d} + \frac{1}{3}(5ab)\int \sin^4(c+dx)\,dx - \frac{\text{Subst}}{}$$

$$= -\frac{5ab\cos(c+dx)\sin^3(c+dx)}{12d} - \frac{ab\cos(c+dx)\sin^5(c+dx)}{3d} + \frac{1}{4}(5$$

$$= -\frac{\left(a^2+b^2\right)\cos(c+dx)}{d} + \frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d} - \frac{\left(a^2+3b^2\right)\cos}{5d}$$

$$= \frac{5abx}{8} - \frac{\left(a^2+b^2\right)\cos(c+dx)}{d} + \frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d} - \frac{\left(a^2+3\right.}{}$$

Mathematica [A] time = 0.208935, size = 134, normalized size = 0.83

$$\frac{-525\left(8a^2+7b^2\right)\cos(c+dx)+35\left(20a^2+21b^2\right)\cos(3(c+dx))-84a^2\cos(5(c+dx))-3150ab\sin(2(c+dx))+630ab\sin}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (4200*a*b*c + 4200*a*b*d*x - 525*(8*a^2 + 7*b^2)*Cos[c + d*x] + 35*(20*a^2
+ 21*b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] - 147*b^2*Cos[5*(c + d
*x)] + 15*b^2*Cos[7*(c + d*x)] - 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin[4*
(c + d*x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.018, size = 125, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{b^2\cos(dx+c)}{7}\left(\frac{16}{5}+(\sin(dx+c))^6+\frac{6\,(\sin(dx+c))^4}{5}+\frac{8\,(\sin(dx+c))^2}{5}\right)+2\,ab\left(-1/6\,\left((\sin(dx+c))^5+5/4\,(\mathrm{s}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-1/7*b^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+2*a*b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c)-1/5*a^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A] time = 0.971191, size = 177, normalized size = 1.1

$$-\frac{224\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)a^2-35\left(4\sin(2\,dx+2\,c)^3+60\,dx+60\,c+9\sin(4\,dx+4\,c\right.}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="maxima
")
```

```
[Out] -1/3360*(224*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a^2 -
 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*a*b - 96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)
^3 - 35*cos(d*x + c))*b^2)/d
```

Fricas [A] time = 2.30429, size = 321, normalized size = 1.99

$$\frac{120\,b^2\cos(dx+c)^7 - 168\left(a^2+3\,b^2\right)\cos(dx+c)^5 + 525\,abdx + 280\left(2\,a^2+3\,b^2\right)\cos(dx+c)^3 - 840\left(a^2+b^2\right)\cos(dx+}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 3*b^2)*cos(d*x + c)^5 + 525*a*b*
d*x + 280*(2*a^2 + 3*b^2)*cos(d*x + c)^3 - 840*(a^2 + b^2)*cos(d*x + c) - 3
5*(8*a*b*cos(d*x + c)^5 - 26*a*b*cos(d*x + c)^3 + 33*a*b*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [A] time = 8.14724, size = 326, normalized size = 2.02

$$\begin{cases} -\frac{a^2\sin^4(c+dx)\cos(c+dx)}{d} - \frac{4a^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{8a^2\cos^5(c+dx)}{15d} + \frac{5abx\sin^6(c+dx)}{8} + \frac{15abx\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{15abx\sin^2(c+dx)\mathrm{co}}{8} \\ x\left(a\sin^2(c)+b\sin^3(c)\right)^2\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*a**2*sin(c + d*x)**2*co
s(c + d*x)**3/(3*d) - 8*a**2*cos(c + d*x)**5/(15*d) + 5*a*b*x*sin(c + d*x)*
*6/8 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a*b*x*sin(c + d*x)**
2*cos(c + d*x)**4/8 + 5*a*b*x*cos(c + d*x)**6/8 - 11*a*b*sin(c + d*x)**5*co
s(c + d*x)/(8*d) - 5*a*b*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*a*b*sin(
c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*b*
*2*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**2*sin(c + d*x)**2*cos(c + d*x)*
*5/(5*d) - 16*b**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*s
in(c)**3)**2*sin(c), True))
```

Giac [A] time = 1.12286, size = 193, normalized size = 1.2

$$\frac{5}{8}\,abx + \frac{b^2\cos(7\,dx+7\,c)}{448\,d} - \frac{ab\sin(6\,dx+6\,c)}{96\,d} + \frac{3\,ab\sin(4\,dx+4\,c)}{32\,d} - \frac{15\,ab\sin(2\,dx+2\,c)}{32\,d} - \frac{\left(4\,a^2+7\,b^2\right)\cos(5\,dx+}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 5/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d + 3/
32*a*b*sin(4*d*x + 4*c)/d - 15/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 + 7
*b^2)*cos(5*d*x + 5*c)/d + 1/192*(20*a^2 + 21*b^2)*cos(3*d*x + 3*c)/d - 5/6
4*(8*a^2 + 7*b^2)*cos(d*x + c)/d
```

3.936 $\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right.$

Optimal. Leaf size=89

$$-\frac{(4a+3c)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3c)+\frac{b\cos^3(c+dx)}{3d}-\frac{b\cos(c+dx)}{d}-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

```
[Out] ((4*a + 3*c)*x)/8 - (b*Cos[c + d*x])/d + (b*Cos[c + d*x]^3)/(3*d) - ((4*a +
 3*c)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (c*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```

Rubi [A] time = 0.108528, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4237, 3023, 2748, 2635, 8, 2633}

$$-\frac{(4a+3c)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3c)+\frac{b\cos^3(c+dx)}{3d}-\frac{b\cos(c+dx)}{d}-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3),x]
```


```
[Out] ((4*a + 3*c)*x)/8 - (b*Cos[c + d*x])/d + (b*Cos[c + d*x]^3)/(3*d) - ((4*a +
 3*c)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (c*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```


Rule 4237

```
Int[(u_)*((A_.)*sin[(a_.) + (b_.)*(x_)]^(n_.) + (B_.)*sin[(a_.) + (b_.)*(x_
)]^(n1_) + (C_.)*sin[(a_.) + (b_.)*(x_)]^(n2_)), x_Symbol] :> Int[ActivateT
rig[u]*Sin[a + b*x]^n*(A + B*Sin[a + b*x] + C*Sin[a + b*x]^2), x] /; FreeQ[
{a, b, A, B, C, n}, x] && EqQ[n1, n + 1] && EqQ[n2, n + 2]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)dx &= \int \sin^2(c+dx)\left(a+b\sin(c+dx)+c\sin^2(c+dx)\right)\\ &= -\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}\int \sin^2(c+dx)(4a\\ &= -\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}+b\int \sin^3(c+dx)\,dx\\ &= -\frac{(4a+3c)\cos(c+dx)\sin(c+dx)}{8d}-\frac{c\cos(c+dx)}{4d}\\ &= \frac{1}{8}(4a+3c)x-\frac{b\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}-\end{aligned}$$

Mathematica [A] time = 0.154986, size = 105, normalized size = 1.18

$$\frac{a(c+dx)}{2d}-\frac{a\sin(2(c+dx))}{4d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d}+\frac{3c(c+dx)}{8d}-\frac{c\sin(2(c+dx))}{4d}+\frac{c\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^
3),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (3*c*(c + d*x))/(8*d) - (3*b*Cos[c + d*x])/(4*d) + (b
*Cos[3*(c + d*x)])/(12*d) - (a*Sin[2*(c + d*x)])/(4*d) - (c*Sin[2*(c + d*x)
])/(4*d) + (c*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.016, size = 84, normalized size = 0.9

$$\frac{1}{d}\left(c\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}+a\left(-\frac{\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x)
```

```
[Out] 1/d*(c*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*b*
(2+sin(d*x+c)^2)*cos(d*x+c)+a*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.970018, size = 107, normalized size = 1.2

$$\frac{24(2dx+2c-\sin(2dx+2c))a+32\left(\cos(dx+c)^3-3\cos(dx+c)\right)b+3(12dx+12c+\sin(4dx+4c)-8\sin(2dx+}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c - sin(2*d*x + 2*c))*a + 32*(cos(d*x + c)^3 - 3*cos(d*
x + c))*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*c)/d
```

Fricas [A] time = 2.09546, size = 181, normalized size = 2.03

$$\frac{8b\cos(dx+c)^3+3(4a+3c)dx-24b\cos(dx+c)+3\left(2c\cos(dx+c)^3-(4a+5c)\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="fricas")
```

```
[Out] 1/24*(8*b*cos(d*x + c)^3 + 3*(4*a + 3*c)*d*x - 24*b*cos(d*x + c) + 3*(2*c*c
os(d*x + c)^3 - (4*a + 5*c)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.20449, size = 201, normalized size = 2.26

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} - \frac{a\sin(c+dx)\cos(c+dx)}{2d} - \frac{b\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b\cos^3(c+dx)}{3d} + \frac{3cx\sin^4(c+dx)}{8} + \frac{3cx\sin^2(c+dx)\cos^2(c+d \\ x\left(a\sin(c) + b\sin^2(c) + c\sin^3(c)\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c + d*x)**3/
(3*d) + 3*c*x*sin(c + d*x)**4/8 + 3*c*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
 3*c*x*cos(c + d*x)**4/8 - 5*c*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*c*sin
(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)**2 + c*
sin(c)**3)*sin(c), True))
```

Giac [A] time = 1.08788, size = 95, normalized size = 1.07

$$\frac{1}{8}(4a + 3c)x + \frac{b\cos(3dx + 3c)}{12d} - \frac{3b\cos(dx + c)}{4d} + \frac{c\sin(4dx + 4c)}{32d} - \frac{(a + c)\sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="giac")
```

```
[Out] 1/8*(4*a + 3*c)*x + 1/12*b*cos(3*d*x + 3*c)/d - 3/4*b*cos(d*x + c)/d + 1/32
*c*sin(4*d*x + 4*c)/d - 1/4*(a + c)*sin(2*d*x + 2*c)/d
```

3.937 $\int \sin(c+dx) \left(a \sin(c+dx) + b \sin^2(c+dx) + c \sin^3(c+dx)\right.$

Optimal. Leaf size=288

$$\frac{a^2 \cos^3(c+dx)}{3d} - \frac{a^2 \cos(c+dx)}{d} - \frac{\left(2ac+b^2\right)\cos^5(c+dx)}{5d} + \frac{2\left(2ac+b^2\right)\cos^3(c+dx)}{3d} - \frac{\left(2ac+b^2\right)\cos(c+dx)}{d} - \frac{ab\sin}{}$$

```
[Out] (3*a*b*x)/4 + (5*b*c*x)/8 - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d - (
(b^2 + 2*a*c)*Cos[c + d*x])/d + (a^2*Cos[c + d*x]^3)/(3*d) + (c^2*Cos[c + d
*x]^3)/d + (2*(b^2 + 2*a*c)*Cos[c + d*x]^3)/(3*d) - (3*c^2*Cos[c + d*x]^5)/
(5*d) - ((b^2 + 2*a*c)*Cos[c + d*x]^5)/(5*d) + (c^2*Cos[c + d*x]^7)/(7*d) -
 (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) - (5*b*c*Cos[c + d*x]*Sin[c + d*x]
)/(8*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^3)/(2*d) - (5*b*c*Cos[c + d*x]*Sin
[c + d*x]^3)/(12*d) - (b*c*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```

Rubi [A] time = 0.400114, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {4394, 3256, 2633, 2635, 8}

$$\frac{a^2 \cos^3(c+dx)}{3d} - \frac{a^2 \cos(c+dx)}{d} - \frac{\left(2ac+b^2\right)\cos^5(c+dx)}{5d} + \frac{2\left(2ac+b^2\right)\cos^3(c+dx)}{3d} - \frac{\left(2ac+b^2\right)\cos(c+dx)}{d} - \frac{ab\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3)^2,x
]
```


```
[Out] (3*a*b*x)/4 + (5*b*c*x)/8 - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d - (
(b^2 + 2*a*c)*Cos[c + d*x])/d + (a^2*Cos[c + d*x]^3)/(3*d) + (c^2*Cos[c + d
*x]^3)/d + (2*(b^2 + 2*a*c)*Cos[c + d*x]^3)/(3*d) - (3*c^2*Cos[c + d*x]^5)/
(5*d) - ((b^2 + 2*a*c)*Cos[c + d*x]^5)/(5*d) + (c^2*Cos[c + d*x]^7)/(7*d) -
 (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) - (5*b*c*Cos[c + d*x]*Sin[c + d*x]
)/(8*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^3)/(2*d) - (5*b*c*Cos[c + d*x]*Sin
[c + d*x]^3)/(12*d) - (b*c*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```


Rule 4394

```
Int[(u_)*((a_)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (b_.)*(F_)[(d_.) + (e_.)*(x
_)]^(q_.) + (c_.)*(F_)[(d_.) + (e_.)*(x_)]^(r_.))^(n_.), x_Symbol] :> Int[A
ctivateTrig[u*F[d + e*x]^(n*p)*(a + b*F[d + e*x]^(q - p) + c*F[d + e*x]^(r
- p))^n], x] /; FreeQ[{a, b, c, d, e, p, q, r}, x] && InertTrigQ[F] && Inte
```

```
gerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx = \int \sin^3(c+dx)\left(a+b\sin(c+dx)+c\sin^2(c+dx)\right)^2$$
$$= \int \left(a^2\sin^3(c+dx)+2ab\sin^4(c+dx)+\left(b^2+2ac\right)$$
$$= a^2\int \sin^3(c+dx)\,dx+(2ab)\int \sin^4(c+dx)\,dx+(2$$
$$= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{2d}-\frac{bc\cos(c+dx)\sin^5(}{3d}$$
$$= -\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}-\frac{\left(b^2+2ac\right)\cos(}{d}$$
$$= \frac{3abx}{4}-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}-\frac{\left(b^2+2ac\right.}{}$$
$$= \frac{3abx}{4}+\frac{5bcx}{8}-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}-$$

Mathematica [A] time = 0.479222, size = 167, normalized size = 0.58

$$\frac{-105\left(48a^2+80ac+40b^2+35c^2\right)\cos(c+dx)+35\left(16a^2+40ac+20b^2+21c^2\right)\cos(3(c+dx))-21\left(c(8a+7c)+4b^2\right)c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^
3)^2,x]
```

```
[Out] (840*b*(6*a + 5*c)*(c + d*x) - 105*(48*a^2 + 40*b^2 + 80*a*c + 35*c^2)*Cos[
c + d*x] + 35*(16*a^2 + 20*b^2 + 40*a*c + 21*c^2)*Cos[3*(c + d*x)] - 21*(4*
b^2 + c*(8*a + 7*c))*Cos[5*(c + d*x)] + 15*c^2*Cos[7*(c + d*x)] - 210*b*(16
*a + 15*c)*Sin[2*(c + d*x)] + 210*b*(2*a + 3*c)*Sin[4*(c + d*x)] - 70*b*c*S
in[6*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.023, size = 213, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{c^2\cos(dx+c)}{7}\left(\frac{16}{5}+(\sin(dx+c))^6+\frac{6\,(\sin(dx+c))^4}{5}+\frac{8\,(\sin(dx+c))^2}{5}\right)+2\,cb\left(-1/6\,\left((\sin(dx+c))^5+5/4\,(\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-1/7*c^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+2*c*b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c)-2/5*a*c*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-1/5
*b^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/4*(sin(d*x+c)
^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a^2*(2+sin(d*x+c)^2)*cos(d
*x+c))
```

Maxima [A] time = 0.977532, size = 294, normalized size = 1.02

$$\frac{1120\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{2}+210\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)ab-224\left(3\,\cos\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algo
rithm="maxima")
```

```
[Out] 1/3360*(1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 + 210*(12*d*x + 12*c + s
in(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a*b - 224*(3*cos(d*x + c)^5 - 10*cos(
d*x + c)^3 + 15*cos(d*x + c))*b^2 - 448*(3*cos(d*x + c)^5 - 10*cos(d*x + c)
^3 + 15*cos(d*x + c))*a*c + 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*si
n(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b*c + 96*(5*cos(d*x + c)^7 - 21*cos(d
*x + c)^5 + 35*cos(d*x + c)^3 - 35*cos(d*x + c))*c^2)/d
```

Fricas [A] time = 2.29698, size = 420, normalized size = 1.46

$$\frac{120\,c^{2}\cos\left(dx+c\right)^{7}-168\left(b^{2}+2\,ac+3\,c^{2}\right)\cos\left(dx+c\right)^{5}+280\left(a^{2}+2\,b^{2}+4\,ac+3\,c^{2}\right)\cos\left(dx+c\right)^{3}+105\left(6\,ab+5\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algo
rithm="fricas")
```

```
[Out] 1/840*(120*c^2*cos(d*x + c)^7 - 168*(b^2 + 2*a*c + 3*c^2)*cos(d*x + c)^5 +
280*(a^2 + 2*b^2 + 4*a*c + 3*c^2)*cos(d*x + c)^3 + 105*(6*a*b + 5*b*c)*d*x
- 840*(a^2 + b^2 + 2*a*c + c^2)*cos(d*x + c) - 35*(8*b*c*cos(d*x + c)^5 - 2
```

```
*(6*a*b + 13*b*c)*cos(d*x + c)^3 + 3*(10*a*b + 11*b*c)*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [A] time = 8.98825, size = 541, normalized size = 1.88

$$\begin{cases} -\frac{a^2\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2a^2\cos^3(c+dx)}{3d} + \frac{3abx\sin^4(c+dx)}{4} + \frac{3abx\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3abx\cos^4(c+dx)}{4} - \frac{5ab\sin^3(c+dx)\cos(c+dx)}{4d} - 3 \\ x\left(a\sin(c) + b\sin^2(c) + c\sin^3(c)\right)^2\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)/d - 2*a**2*cos(c + d*x)**3/(3
*d) + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2
 + 3*a*b*x*cos(c + d*x)**4/4 - 5*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 3
*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*a*c*sin(c + d*x)**4*cos(c + d*x
)/d - 8*a*c*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 16*a*c*cos(c + d*x)**5/
(15*d) - b**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*b**2*sin(c + d*x)**2*cos(c
 + d*x)**3/(3*d) - 8*b**2*cos(c + d*x)**5/(15*d) + 5*b*c*x*sin(c + d*x)**6/
8 + 15*b*c*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*b*c*x*sin(c + d*x)**2*c
os(c + d*x)**4/8 + 5*b*c*x*cos(c + d*x)**6/8 - 11*b*c*sin(c + d*x)**5*cos(c
 + d*x)/(8*d) - 5*b*c*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*b*c*sin(c +
 d*x)*cos(c + d*x)**5/(8*d) - c**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*c**2*
sin(c + d*x)**4*cos(c + d*x)**3/d - 8*c**2*sin(c + d*x)**2*cos(c + d*x)**5/
(5*d) - 16*c**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)*
*2 + c*sin(c)**3)**2*sin(c), True))
```

Giac [A] time = 1.15043, size = 251, normalized size = 0.87

$$\frac{1}{8}(6ab + 5bc)x + \frac{c^2\cos(7dx + 7c)}{448d} - \frac{bc\sin(6dx + 6c)}{96d} - \frac{\left(4b^2 + 8ac + 7c^2\right)\cos(5dx + 5c)}{320d} + \frac{\left(16a^2 + 20b^2 + 40ac\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algo
rithm="giac")
```

```
[Out] 1/8*(6*a*b + 5*b*c)*x + 1/448*c^2*cos(7*d*x + 7*c)/d - 1/96*b*c*sin(6*d*x +
 6*c)/d - 1/320*(4*b^2 + 8*a*c + 7*c^2)*cos(5*d*x + 5*c)/d + 1/192*(16*a^2
```

```
+ 20*b^2 + 40*a*c + 21*c^2)*cos(3*d*x + 3*c)/d - 1/64*(48*a^2 + 40*b^2 + 80
*a*c + 35*c^2)*cos(d*x + c)/d + 1/32*(2*a*b + 3*b*c)*sin(4*d*x + 4*c)/d - 1
/32*(16*a*b + 15*b*c)*sin(2*d*x + 2*c)/d
```

3.938 $\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx$

Optimal. Leaf size=61

$$-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{c\sin(c+dx)\cos(c+dx)}{2d}+\frac{cx}{2}$$

```
[Out] (c*x)/2 - (a*Cos[c + d*x])/d + (2*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d - (
c*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.289499, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4395, 4401, 2639, 2638, 2635, 8}

$$-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{c\sin(c+dx)\cos(c+dx)}{2d}+\frac{cx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

```
[Out] (c*x)/2 - (a*Cos[c + d*x])/d + (2*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d - (
c*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 4395

```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx &= \int \sqrt{\sin(c+dx)}\left(b+a\sqrt{\sin(c+dx)}+c\sin^{\frac{3}{2}}(c+dx)\right)dx\\ &= \int \left(b\sqrt{\sin(c+dx)}+a\sin(c+dx)+c\sin^2(c+dx)\right)dx\\ &= a\int \sin(c+dx)\,dx+b\int \sqrt{\sin(c+dx)}\,dx+c\int \sin^2(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{c\cos(c+dx)\sin(c}{2d}\\ &= \frac{cx}{2}-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{c\cos(c+dx)\,\mathrm{s}}{2d}\end{aligned}$$

Mathematica [A] time = 0.180645, size = 55, normalized size = 0.9

$$\frac{-4a\cos(c+dx)-8bE\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+c(-\sin(2(c+dx))+2c+2dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

```
[Out] (-4*a*Cos[c + d*x] - 8*b*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + c*(2*c + 2*d
*x - Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.924, size = 136, normalized size = 2.2

$$cx - \frac{a\cos(dx+c)}{d} - \frac{c}{d}\left(\frac{\sin(dx+c)\cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) - \frac{b}{d\cos(dx+c)}\sqrt{\sin(dx+c)+1}\sqrt{-2\sin(dx+c)+2}\sqrt{-\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x)
```

```
[Out] c*x-a*cos(d*x+c)/d-c/d*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-b*(sin(d*x
+c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*(2*EllipticE((sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))/c
os(d*x+c)/sin(d*x+c)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(2*c*d*x - 4*a*cos(d*x + c) + 2*d*integrate(-(((b*cos(3/2*d*x + 3/2*c)
- b*cos(1/2*d*x + 1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c))
*cos(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)) - (b*cos(3/2*d*x + 3/2*c
) - b*cos(1/2*d*x + 1/2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1/2*c
))*sin(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c) + 1)) + ((b*cos(3/2*d*x + 3/2*c) - b*cos(1/2*d*x + 1/
2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1/2*c))*cos(1/2*arctan2(sin
(d*x + c), -cos(d*x + c) + 1)) + (b*cos(3/2*d*x + 3/2*c) - b*cos(1/2*d*x +
1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c))*sin(1/2*arctan2(s
in(d*x + c), -cos(d*x + c) + 1)))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
) + 1)))/((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^(1/4)*(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1)^(1/4)), x) - c*sin(2*d*x
 + 2*c))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-c\cos\left(dx+c\right)^{2}+a\sin\left(dx+c\right)+b\sqrt{\sin\left(dx+c\right)}+c,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="fric
as")
```

```
[Out] integral(-c*cos(d*x + c)^2 + a*sin(d*x + c) + b*sqrt(sin(d*x + c)) + c, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sqrt{\sin\left(c+dx\right)}+b+c\sin^{\frac{3}{2}}\left(c+dx\right)\right)\sqrt{\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2)),x)
```

```
[Out] Integral((a*sqrt(sin(c + d*x)) + b + c*sin(c + d*x)**(3/2))*sqrt(sin(c + d*
x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(dx+c\right)+a+\frac{b}{\sqrt{\sin\left(dx+c\right)}}\right)\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="giac
")
```

```
[Out] integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))*sin(d*x + c), x)
```

3.939 $\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2\,dx$

Optimal. Leaf size=148

$$\frac{4bc\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d}-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{ac\sin(c+dx)\cos(c+dx)}{d}+acx+b^2x$$

```
[Out] b^2*x + a*c*x - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d + (c^2*Cos[c +
d*x]^3)/(3*d) + (4*a*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d + (4*b*c*Ellipti
cF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (4*b*c*Cos[c + d*x]*Sqrt[Sin[c + d*x]])/
(3*d) - (a*c*Cos[c + d*x]*Sin[c + d*x])/d
```

Rubi [A] time = 0.239463, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4395, 4401, 2639, 2638, 2635, 2641, 8, 2633}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{ac\sin(c+dx)\cos(c+dx)}{d}+acx+b^2x+\frac{4bcF\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d}-4bc\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
```

```
[Out] b^2*x + a*c*x - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d + (c^2*Cos[c +
d*x]^3)/(3*d) + (4*a*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d + (4*b*c*Ellipti
cF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (4*b*c*Cos[c + d*x]*Sqrt[Sin[c + d*x]])/
(3*d) - (a*c*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 4395

```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx = \int \left(b+a\sqrt{\sin(c+dx)}+c\sin^{\frac{3}{2}}(c+dx)\right)^2 dx$$
$$= \int \left(b^2+2ab\sqrt{\sin(c+dx)}+a^2\sin(c+dx)+2bc\sin^{\frac{3}{2}}(c+dx)+\right.$$
$$= b^2x+a^2\int \sin(c+dx)\,dx+(2ab)\int \sqrt{\sin(c+dx)}\,dx+(2ac)\int$$
$$= b^2x-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{4bc\cos(c+}{}$$
$$= b^2x+acx-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}+\frac{c^2\cos^3(c+dx)}{3d}+$$

Mathematica [A] time = 0.276039, size = 137, normalized size = 0.93

$$\frac{-16bc\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)-12a^2\cos(c+dx)-48abE\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+12ac^2+12acdx-6ac\sin(2(c}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
```

```
[Out] (12*b^2*c + 12*a*c^2 + 12*b^2*d*x + 12*a*c*d*x - 12*a^2*Cos[c + d*x] - 9*c^
2*Cos[c + d*x] + c^2*Cos[3*(c + d*x)] - 48*a*b*EllipticE[(-2*c + Pi - 2*d*x
)/4, 2] - 16*b*c*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 16*b*c*Cos[c + d*x]*
Sqrt[Sin[c + d*x]] - 6*a*c*Sin[2*(c + d*x)])/(12*d)
```

Maple [A] time = 1.197, size = 266, normalized size = 1.8

$$b^2x-\frac{a^2\cos(dx+c)}{d}-\frac{c^2\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3\,d}+2\,\frac{ac\,(-1/2\,\sin(dx+c)\cos(dx+c)+1/2\,dx+c/2)}{d}+\frac{}{3\,d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*x-a^2*cos(d*x+c)/d-1/3*c^2/d*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*c/d*(-1/2*
sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2/3*b*(3*a*(sin(d*x+c)+1)^(1/2)*(-2*si
```

```
n(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))+(sin(d*x+c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*Ell
ipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*c-6*a*(sin(d*x+c)+1)^(1/2)*(-2*sin
(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticE((sin(d*x+c)+1)^(1/2),1/2*2^(
1/2))-2*cos(d*x+c)^2*sin(d*x+c)*c)/cos(d*x+c)/sin(d*x+c)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-2\,ac\cos\left(dx+c\right)^{2}+b^{2}+2\,ac-\left(c^{2}\cos\left(dx+c\right)^{2}-a^{2}-c^{2}\right)\sin\left(dx+c\right)+2\left(bc\sin\left(dx+c\right)+ab\right)\sqrt{\sin\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-2*a*c*cos(d*x + c)^2 + b^2 + 2*a*c - (c^2*cos(d*x + c)^2 - a^2 -
c^2)*sin(d*x + c) + 2*(b*c*sin(d*x + c) + a*b)*sqrt(sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2))**2,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(dx+c\right)+a+\frac{b}{\sqrt{\sin\left(dx+c\right)}}\right)^{2}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))^2*sin(d*x + c), x)
```

3.940 $\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx$

Optimal. Leaf size=34

$$\frac{f^{a+bx}\left(e^{i(c+dx)}\right)^n}{b\log(f)+idn}$$

```
[Out] ((E^(I*(c + d*x)))^n*f^(a + b*x))/(I*d*n + b*Log[f])
```

Rubi [A] time = 0.0970483, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4614, 2281, 2287, 2194}

$$\frac{f^{a+bx}\left(e^{i(c+dx)}\right)^n}{b\log(f)+idn}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
```

```
[Out] ((E^(I*(c + d*x)))^n*f^(a + b*x))/(I*d*n + b*Log[f])
```

Rule 4614

```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx &= \int \left(e^{i(c+dx)}\right)^n f^{a+bx}\,dx\\ &= \left(e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\right)\int e^{in(c+dx)}f^{a+bx}\,dx\\ &= \left(e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\right)\int e^{icn+a\log(f)+x(idn+b\log(f))}\,dx\\ &= \frac{\left(e^{i(c+dx)}\right)^n f^{a+bx}}{idn+b\log(f)}\end{aligned}$$

Mathematica [A] time = 0.092996, size = 43, normalized size = 1.26

$$-\frac{if^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n}{dn-ib\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
```

```
[Out] ((-I)*f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n)/(d*n - I*b*Log[f])
```

Maple [B] time = 0.125, size = 86, normalized size = 2.5

$$\frac{e^{(bx+a)\ln(f)}}{idn+b\ln(f)}e^{n\ln\left(2\,i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-1}+\left(1-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x)
```

```
[Out] 1/(I*d*n+b*ln(f))*exp((b*x+a)*ln(f))*exp(n*ln(2*I*tan(1/2*d*x+1/2*c)/(1+tan
(1/2*d*x+1/2*c)^2)+(1-tan(1/2*d*x+1/2*c)^2)/(1+tan(1/2*d*x+1/2*c)^2)))
```

Maxima [A] time = 0.988954, size = 68, normalized size = 2.

$$\frac{-i f^{bx} f^a \cos(dnx + cn) + f^{bx} f^a \sin(dnx + cn)}{dn - i b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] (-I*f^(b*x)*f^a*cos(d*n*x + c*n) + f^(b*x)*f^a*sin(d*n*x + c*n))/(d*n - I*b
*log(f))
```

Fricas [A] time = 2.3028, size = 70, normalized size = 2.06

$$\frac{f^{bx+a} \left(e^{(i dx + i c)}\right)^n}{i dn + b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] f^(b*x + a)*(e^(I*d*x + I*c))^n/(I*d*n + b*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.82261, size = 42, normalized size = 1.24

$$\frac{f^a e^{\left(i\,dnx+bx\log\left(f\right)+i\,cn\right)}}{i\,dn+b\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] f^a*e^(I*d*n*x + b*x*log(f) + I*c*n)/(I*d*n + b*log(f))
```

3.941 $\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx$

Optimal. Leaf size=36

$$-\frac{f^{a+bx}\left(e^{-i(c+dx)}\right)^n}{-b\log(f)+idn}$$

```
[Out] -(((E^((-I)*(c + d*x)))^n*f^(a + b*x))/(I*d*n - b*Log[f]))
```

Rubi [A] time = 0.0973396, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4614, 2281, 2287, 2194}

$$-\frac{f^{a+bx}\left(e^{-i(c+dx)}\right)^n}{-b\log(f)+idn}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]
```

```
[Out] -(((E^((-I)*(c + d*x)))^n*f^(a + b*x))/(I*d*n - b*Log[f]))
```

Rule 4614

```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx &= \int \left(e^{-i(c+dx)}\right)^n f^{a+bx}\,dx\\ &= \left(e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\right)\int e^{-in(c+dx)}f^{a+bx}\,dx\\ &= \left(e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\right)\int \exp(-icn+a\log(f)-x(idn-b\log(f)))\,dx\\ &= -\frac{\left(e^{-i(c+dx)}\right)^n f^{a+bx}}{idn-b\log(f)}\end{aligned}$$

Mathematica [A] time = 0.075935, size = 43, normalized size = 1.19

$$\frac{if^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n}{dn+ib\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]
```

```
[Out] (I*f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n)/(d*n + I*b*Log[f])
```

Maple [B] time = 0.122, size = 86, normalized size = 2.4

$$\frac{e^{(bx+a)\ln(f)}}{-idn+b\ln\left(f\right)}e^{n\ln\left(\left(1-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-1}-2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x)
```

```
[Out] 1/(-I*d*n+b*ln(f))*exp((b*x+a)*ln(f))*exp(n*ln((1-tan(1/2*d*x+1/2*c)^2)/(1+
tan(1/2*d*x+1/2*c)^2)-2*I*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)))
```

Maxima [A] time = 0.995875, size = 84, normalized size = 2.33

$$\frac{f^{bx}f^{a}\cos\left(dnx\right)-i\,f^{bx}f^{a}\sin\left(dnx\right)}{\left(-i\,dn+b\log\left(f\right)\right)\cos\left(cn\right)+\left(dn+i\,b\log\left(f\right)\right)\sin\left(cn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] (f^(b*x)*f^a*cos(d*n*x) - I*f^(b*x)*f^a*sin(d*n*x))/((-I*d*n + b*log(f))*co
s(c*n) + (d*n + I*b*log(f))*sin(c*n))
```

Fricas [A] time = 2.06092, size = 73, normalized size = 2.03

$$\frac{f^{bx+a}\left(e^{(-i\,dx-i\,c)}\right)^{n}}{-i\,dn+b\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] f^(b*x + a)*(e^(-I*d*x - I*c))^n/(-I*d*n + b*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.13264, size = 42, normalized size = 1.17

$$\frac{f^a e^{\left(-i\,dnx+bx\log\left(f\right)-i\,cn\right)}}{-i\,dn+b\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] f^a*e^(-I*d*n*x + b*x*log(f) - I*c*n)/(-I*d*n + b*log(f))
```

3.942 $\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx$

Optimal. Leaf size=120

$$-\frac{4\log\left(2\tan^2(a+bx)-\left(1-\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1-\sqrt{5}\right)b}-\frac{4\log\left(2\tan^2(a+bx)-\left(1+\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1+\sqrt{5}\right)b}+\frac{\log(\tan(a+}{5b}$$

```
[Out] Log[Cos[a + b*x]]/b + Log[1 + Tan[a + b*x]]/(5*b) - (4*Log[2 - (1 - Sqrt[5]
)*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 - Sqrt[5])*b) - (4*Log[2 - (1 + S
qrt[5])*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 + Sqrt[5])*b)
```

Rubi [A] time = 0.701148, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2074, 260, 2086, 628}

$$-\frac{4\log\left(2\tan^2(a+bx)-\left(1-\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1-\sqrt{5}\right)b}-\frac{4\log\left(2\tan^2(a+bx)-\left(1+\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1+\sqrt{5}\right)b}+\frac{\log(\tan(a+}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^5),x]
```

```
[Out] Log[Cos[a + b*x]]/b + Log[1 + Tan[a + b*x]]/(5*b) - (4*Log[2 - (1 - Sqrt[5]
)*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 - Sqrt[5])*b) - (4*Log[2 - (1 + S
qrt[5])*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 + Sqrt[5])*b)
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^5}{1+x^2+x^5+x^7}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{5(1+x)}-\frac{x}{1+x^2}+\frac{2(2+x-4x^2+2x^3)}{5(1-x+x^2-x^3+x^4)}\right) dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\log(1+\tan(a+bx))}{5b}+\frac{2\,\text{Subst}\left(\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx, x, \tan(a+bx)\right)}{5b}-\frac{\text{Subst}\left(\int \frac{x}{1+x^{\cdots}}\right.}{\cdots}$$

$$= \frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{5b}-\frac{2\,\text{Subst}\left(\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right)x}{2+\left(-1-\sqrt{5}\right)x+2x^2}\,dx, x, \tan(a+\cdots\right.}{5\sqrt{5}b}$$

$$= \frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{5b}-\frac{4\log\left(2-\left(1-\sqrt{5}\right)\tan(a+bx)+2\tan^{\cdots}\right.}{5\left(1-\sqrt{5}\right)b}$$

Mathematica [A] time = 0.594612, size = 73, normalized size = 0.61

$$\frac{-\left(\sqrt{5}-1\right)\log\left(\sin(2(a+bx))-\sqrt{5}+1\right)+\left(1+\sqrt{5}\right)\log\left(\sin(2(a+bx))+\sqrt{5}+1\right)+\log(\sin(a+bx)+\cos(a+bx))}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^
5),x]
```

```
[Out] (Log[Cos[a + b*x] + Sin[a + b*x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + Sin[2*
(a + b*x)]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + Sin[2*(a + b*x)]])/(5*b)
```

Maple [A] time = 0.211, size = 184, normalized size = 1.5

$$\frac{\ln\left(\tan\left(bx+a\right)\sqrt{5}+2\,\left(\tan\left(bx+a\right)\right)^{2}-\tan\left(bx+a\right)+2\right)\sqrt{5}}{5\,b}+\frac{\ln\left(\tan\left(bx+a\right)\sqrt{5}+2\,\left(\tan\left(bx+a\right)\right)^{2}-\tan\left(bx+a\right.}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x)
```

```
[Out] 1/5/b*ln(tan(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)*5^(1/2)+1/5/b*ln(t
an(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)-1/5/b*ln(-tan(b*x+a)*5^(1/2)
+2*tan(b*x+a)^2-tan(b*x+a)+2)*5^(1/2)+1/5/b*ln(-tan(b*x+a)*5^(1/2)+2*tan(b*
x+a)^2-tan(b*x+a)+2)+1/5*ln(1+tan(b*x+a))/b-1/2/b*ln(1+tan(b*x+a)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^{5}-\sin\left(bx+a\right)^{5}}{\cos\left(bx+a\right)^{5}+\sin\left(bx+a\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="maxima")
```

```
[Out] integrate((cos(b*x + a)^5 - sin(b*x + a)^5)/(cos(b*x + a)^5 + sin(b*x + a)^
5), x)
```

Fricas [A] time = 2.41077, size = 405, normalized size = 3.38

$$\frac{2\sqrt{5}\log\left(-\frac{2\cos(bx+a)^{4}-2\left(\sqrt{5}+1\right)\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^{2}-\sqrt{5}-3}{\cos(bx+a)^{4}-\cos(bx+a)^{2}-\cos(bx+a)\sin(bx+a)+1}\right)+2\log\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}-\cos\left(bx+a\right)\mathrm{si}\right.}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="fricas")
```

```
[Out] 1/10*(2*sqrt(5)*log(-(2*cos(b*x + a)^4 - 2*(sqrt(5) + 1)*cos(b*x + a)*sin(b
*x + a) - 2*cos(b*x + a)^2 - sqrt(5) - 3)/(cos(b*x + a)^4 - cos(b*x + a)^2
- cos(b*x + a)*sin(b*x + a) + 1)) + 2*log(cos(b*x + a)^4 - cos(b*x + a)^2 -
 cos(b*x + a)*sin(b*x + a) + 1) + log(2*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**5-sin(b*x+a)**5)/(cos(b*x+a)**5+sin(b*x+a)**5),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25929, size = 173, normalized size = 1.44

$$-\frac{2\sqrt{5}\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)\tan\left(bx+a\right)+\tan\left(bx+a\right)^{2}+1\right)-2\sqrt{5}\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)\tan\left(bx+a\right)+\tan\left(bx+a\right)^{2}+1\right)-2\log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="giac")
```

```
[Out] -1/10*(2*sqrt(5)*log(-1/2*(sqrt(5) + 1)*tan(b*x + a) + tan(b*x + a)^2 + 1)
- 2*sqrt(5)*log(1/2*(sqrt(5) - 1)*tan(b*x + a) + tan(b*x + a)^2 + 1) - 2*lo
g(tan(b*x + a)^4 - tan(b*x + a)^3 + tan(b*x + a)^2 - tan(b*x + a) + 1) + 5*
log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

3.943 $\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx$

Optimal. Leaf size=72

$$\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}-\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}$$

```
[Out] -Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b) + Log[1 + Sqr
t[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rubi [A] time = 0.151379, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1165, 628}

$$\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}-\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b) + Log[1 + Sqr
t[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \tan(a+bx)\right)}{2\sqrt{2}b}$$
$$= -\frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b} + \frac{\log\left(1+\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b}$$

Mathematica [A] time = 0.0345226, size = 25, normalized size = 0.35

$$\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^
4),x]
```

```
[Out] ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

Maple [A] time = 0.088, size = 108, normalized size = 1.5

$$\frac{\sqrt{2}}{8\,b}\ln\left(\frac{1+\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}{1-\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}\right)-\frac{\sqrt{2}}{8\,b}\ln\left(\frac{1-\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}{1+\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x)
```

```
[Out] 1/8/b*2^(1/2)*ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*x+a)+
tan(b*x+a)^2))-1/8/b*2^(1/2)*ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(1
/2)*tan(b*x+a)+tan(b*x+a)^2))
```

Maxima [A] time = 1.43376, size = 78, normalized size = 1.08

$$\frac{\sqrt{2}\log\left(\tan\left(bx+a\right)^{2}+\sqrt{2}\tan\left(bx+a\right)+1\right)-\sqrt{2}\log\left(\tan\left(bx+a\right)^{2}-\sqrt{2}\tan\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="maxima")
```

```
[Out] 1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(t
an(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

Fricas [A] time = 2.12544, size = 193, normalized size = 2.68

$$\frac{\sqrt{2}\log\left(-\frac{2\,\cos\left(bx+a\right)^{4}-2\,\sqrt{2}\cos\left(bx+a\right)\sin\left(bx+a\right)-2\,\cos\left(bx+a\right)^{2}-1}{2\,\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 - 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

Sympy [A] time = 28.038, size = 122, normalized size = 1.69

$$\begin{cases} -\frac{\sqrt{2}\log\left(4\sin^{2}\left(a+bx\right)-4\sqrt{2}\sin\left(a+bx\right)\cos\left(a+bx\right)+4\cos^{2}\left(a+bx\right)\right)}{4b}+\frac{\sqrt{2}\log\left(4\sin^{2}\left(a+bx\right)+4\sqrt{2}\sin\left(a+bx\right)\cos\left(a+bx\right)+4\cos^{2}\left(a+bx\right)\right)}{4b} & \text{for}\ b\neq 0\\ \frac{x\left(-\sin^{4}\left(a\right)+\cos^{4}\left(a\right)\right)}{\sin^{4}\left(a\right)+\cos^{4}\left(a\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**4-sin(b*x+a)**4)/(cos(b*x+a)**4+sin(b*x+a)**4),x)
```

```
[Out] Piecewise((-sqrt(2)*log(4*sin(a + b*x)**2 - 4*sqrt(2)*sin(a + b*x)*cos(a +
b*x) + 4*cos(a + b*x)**2)/(4*b) + sqrt(2)*log(4*sin(a + b*x)**2 + 4*sqrt(2)
```

```
*sin(a + b*x)*cos(a + b*x) + 4*cos(a + b*x)**2)/(4*b), Ne(b, 0)), (x*(-sin(
a)**4 + cos(a)**4)/(sin(a)**4 + cos(a)**4), True))
```

Giac [A] time = 1.2328, size = 65, normalized size = 0.9

$$-\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sin(2bx+2a)\right|}{\left|2\sqrt{2}+2\sin(2bx+2a)\right|}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin
(2*b*x + 2*a)))/b
```

3.944 $\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx$

Optimal. Leaf size=55

$$-\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}+\frac{\log(\tan(a+bx)+1)}{3b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -(Log[Cos[a + b*x]]/b) + Log[1 + Tan[a + b*x]]/(3*b) - (2*Log[1 - Tan[a + b
*x] + Tan[a + b*x]^2])/(3*b)
```

Rubi [A] time = 0.408613, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2074, 260, 628}

$$-\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}+\frac{\log(\tan(a+bx)+1)}{3b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^3),x]
```

```
[Out] -(Log[Cos[a + b*x]]/b) + Log[1 + Tan[a + b*x]]/(3*b) - (2*Log[1 - Tan[a + b
*x] + Tan[a + b*x]^2])/(3*b)
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx=\frac{\text{Subst}\left(\int \frac{1-x^3}{1+x^2+x^3+x^5}\,dx,x,\tan(a+bx)\right)}{b}$$
$$=\frac{\text{Subst}\left(\int \left(\frac{1}{3(1+x)}+\frac{x}{1+x^2}-\frac{2(-1+2x)}{3(1-x+x^2)}\right)dx,x,\tan(a+bx)\right)}{b}$$
$$=\frac{\log(1+\tan(a+bx))}{3b}-\frac{2\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\tan(a+bx)\right)}{3b}+\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x\right.}{b}$$
$$=-\frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{3b}-\frac{2\log\left(1-\tan(a+bx)+\tan^2(a+bx)\right)}{3b}$$

Mathematica [A] time = 0.204279, size = 42, normalized size = 0.76

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{3b}-\frac{2\log(2-\sin(2(a+bx)))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^
3),x]
```

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/(3*b) - (2*Log[2 - Sin[2*(a + b*x)]])/(3*b
)
```

Maple [A] time = 0.147, size = 56, normalized size = 1.

$$\frac{\ln(1+\tan(bx+a))}{3b}+\frac{\ln\left(1+(\tan(bx+a))^2\right)}{2b}-\frac{2\ln\left(1-\tan(bx+a)+(\tan(bx+a))^2\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x)
```

```
[Out] 1/3*ln(1+tan(b*x+a))/b+1/2/b*ln(1+tan(b*x+a)^2)-2/3*ln(1-tan(b*x+a)+tan(b*x
+a)^2)/b
```

Maxima [B] time = 1.47065, size = 208, normalized size = 3.78

$$-\frac{2\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{2\sin(bx+a)^3}{(\cos(bx+a)+1)^3}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1\right)-\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}-1\right)-3\log(}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="maxima")
```

```
[Out] -1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x
+ a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b
*x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)
^2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 +
1))/b
```

Fricas [A] time = 2.23759, size = 116, normalized size = 2.11

$$\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)-4\log\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="fricas")
```

```
[Out] 1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x + a
) + 1))/b
```

Sympy [A] time = 1.74592, size = 76, normalized size = 1.38

$$\begin{cases} \frac{\log\left(\sin\left(a+bx\right)+\cos\left(a+bx\right)\right)}{3b}-\frac{2\log\left(\sin^2\left(a+bx\right)-\sin\left(a+bx\right)\cos\left(a+bx\right)+\cos^2\left(a+bx\right)\right)}{3b} & \text{for } b\neq 0 \\ \frac{x\left(-\sin^3\left(a\right)+\cos^3\left(a\right)\right)}{\sin^3\left(a\right)+\cos^3\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**3-sin(b*x+a)**3)/(cos(b*x+a)**3+sin(b*x+a)**3),x)
```

```
[Out] Piecewise((log(sin(a + b*x) + cos(a + b*x))/(3*b) - 2*log(sin(a + b*x)**2 -
 sin(a + b*x)*cos(a + b*x) + cos(a + b*x)**2)/(3*b), Ne(b, 0)), (x*(-sin(a)
**3 + cos(a)**3)/(sin(a)**3 + cos(a)**3), True))
```

Giac [A] time = 1.17244, size = 70, normalized size = 1.27

$$-\frac{4 \log\left(\tan\left(bx+a\right)^{2}-\tan\left(bx+a\right)+1\right)-3 \log\left(\tan\left(bx+a\right)^{2}+1\right)-2 \log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="giac")
```

```
[Out] -1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1)
- 2*log(abs(tan(b*x + a) + 1)))/b
```

3.945 $\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

```
[Out] (Cos[a + b*x]*Sin[a + b*x])/b
```

Rubi [A] time = 0.0541188, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4380, 2635, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^2),x]
```

```
[Out] (Cos[a + b*x]*Sin[a + b*x])/b
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx = \int \left(\cos^2(a+bx)-\sin^2(a+bx)\right)\,dx$$
$$= \int \cos^2(a+bx)\,dx - \int \sin^2(a+bx)\,dx$$
$$= \frac{\cos(a+bx)\sin(a+bx)}{b}$$

Mathematica [B] time = 0.0118323, size = 33, normalized size = 2.06

$$\frac{\sin(2a)\cos(2bx)}{2b} + \frac{\cos(2a)\sin(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^
2),x]
```

```
[Out] (Cos[2*b*x]*Sin[2*a])/(2*b) + (Cos[2*a]*Sin[2*b*x])/(2*b)
```

Maple [A] time = 0.04, size = 17, normalized size = 1.1

$$\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x)
```

```
[Out] cos(b*x+a)*sin(b*x+a)/b
```

Maxima [A] time = 0.951656, size = 30, normalized size = 1.88

$$\frac{\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="maxima")
```

```
[Out] tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)
```

Fricas [A] time = 1.99053, size = 39, normalized size = 2.44

$$\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="fricas")
```

```
[Out] cos(b*x + a)*sin(b*x + a)/b
```

Sympy [B] time = 0.283215, size = 32, normalized size = 2.

$$\frac{\sin(a+bx)\cos(a+bx)}{b\sin^2(a+bx)+b\cos^2(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**2-sin(b*x+a)**2)/(cos(b*x+a)**2+sin(b*x+a)**2),x)
```

```
[Out] sin(a + b*x)*cos(a + b*x)/(b*sin(a + b*x)**2 + b*cos(a + b*x)**2)
```

Giac [A] time = 1.08693, size = 19, normalized size = 1.19

$$\frac{\sin(2bx+2a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="giac")
```

```
[Out] 1/2*sin(2*b*x + 2*a)/b
```

3.946 $\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b
```

Rubi [A] time = 0.0281176, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {3133}

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]
```

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx=\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

Mathematica [A] time = 0.043197, size = 18, normalized size = 1.

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]
```

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b
```

Maple [A] time = 0.026, size = 19, normalized size = 1.1

$$\frac{\ln\left(\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)
```

```
[Out] ln(cos(b*x+a)+sin(b*x+a))/b
```

Maxima [A] time = 0.936929, size = 24, normalized size = 1.33

$$\frac{\log\left(\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="max
ima")
```

```
[Out] log(cos(b*x + a) + sin(b*x + a))/b
```

Fricas [A] time = 2.05172, size = 59, normalized size = 3.28

$$\frac{\log\left(2\,\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="fri
cas")
```

```
[Out] 1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

Sympy [A] time = 0.484454, size = 31, normalized size = 1.72

$$\begin{cases} \frac{\log(\sin(a+bx)+\cos(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x(-\sin(a)+\cos(a))}{\sin(a)+\cos(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)
```

```
[Out] Piecewise((log(sin(a + b*x) + cos(a + b*x))/b, Ne(b, 0)), (x*(-sin(a) + cos
(a))/(sin(a) + cos(a)), True))
```

Giac [A] time = 1.13265, size = 39, normalized size = 2.17

$$-\frac{\log\left(\tan(bx+a)^2+1\right) - 2\log(|\tan(bx+a)+1|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="gia
c")
```

```
[Out] -1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

3.947 $\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)} \, dx$

Optimal. Leaf size=19

$$-\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

```
[Out] -(Log[Cos[a + b*x] + Sin[a + b*x]]/b)
```

Rubi [A] time = 0.31409, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {801, 260}

$$-\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]
```

```
[Out] -(Log[Cos[a + b*x] + Sin[a + b*x]]/b)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x}{(1+x)(1+x^2)}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{x}{1+x^2}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\log(1+\tan(a+bx))}{b}+\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\log(\cos(a+bx))}{b}-\frac{\log(1+\tan(a+bx))}{b}$$

Mathematica [A] time = 0.0578265, size = 19, normalized size = 1.

$$-\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]
```

```
[Out] -(Log[Cos[a + b*x] + Sin[a + b*x]]/b)
```

Maple [A] time = 0.131, size = 32, normalized size = 1.7

$$-\frac{\ln(1+\tan(bx+a))}{b}+\frac{\ln\left(1+(\tan(bx+a))^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)
```

```
[Out] -ln(1+tan(b*x+a))/b+1/2/b*ln(1+tan(b*x+a)^2)
```

Maxima [B] time = 1.4379, size = 95, normalized size = 5.

$$-\frac{\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}-1\right)-\log\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="ma
xima")
```

```
[Out] -(log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^2/(cos(b*x + a) + 1
)^2 - 1) - log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b
```

Fricas [A] time = 2.12089, size = 61, normalized size = 3.21

$$-\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="fr
icas")
```

```
[Out] -1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\csc\left(a+bx\right)}{\csc\left(a+bx\right)+\sec\left(a+bx\right)}\,dx-\int-\frac{\sec\left(a+bx\right)}{\csc\left(a+bx\right)+\sec\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)
```

```
[Out] -Integral(csc(a + b*x)/(csc(a + b*x) + sec(a + b*x)), x) - Integral(-sec(a
+ b*x)/(csc(a + b*x) + sec(a + b*x)), x)
```

Giac [A] time = 1.15848, size = 39, normalized size = 2.05

$$\frac{\log\left(\tan\left(bx+a\right)^{2}+1\right)-2\,\log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

3.948 $\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)} \, dx$

Optimal. Leaf size=17

$$-\frac{\sin(a+bx)\cos(a+bx)}{b}$$

```
[Out] -((Cos[a + b*x]*Sin[a + b*x])/b)
```

Rubi [A] time = 0.174518, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {383}

$$-\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]^2),x]
```

```
[Out] -((Cos[a + b*x]*Sin[a + b*x])/b)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)} \, dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{\left(1+x^2\right)^2} \, dx, x, \tan(a+bx)\right)}{b} \\
&= -\frac{\cos(a+bx)\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0143717, size = 33, normalized size = 1.94

$$-\frac{\sin(2a)\cos(2bx)}{2b}-\frac{\cos(2a)\sin(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]
^2),x]
```

```
[Out] -(Cos[2*b*x]*Sin[2*a])/(2*b) - (Cos[2*a]*Sin[2*b*x])/(2*b)
```

Maple [A] time = 0.084, size = 18, normalized size = 1.1

$$-\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x)
```

```
[Out] -cos(b*x+a)*sin(b*x+a)/b
```

Maxima [A] time = 0.945537, size = 31, normalized size = 1.82

$$-\frac{\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algor
ithm="maxima")
```

```
[Out] -tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)
```

Fricas [A] time = 2.02152, size = 41, normalized size = 2.41

$$-\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algor
ithm="fricas")
```

```
[Out] -cos(b*x + a)*sin(b*x + a)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\csc^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx-\int -\frac{\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**2+sec(b*x+a)**2)/(csc(b*x+a)**2+sec(b*x+a)**2),x)
```

```
[Out] -Integral(csc(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x) - Integra
l(-sec(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x)
```

Giac [A] time = 1.13875, size = 19, normalized size = 1.12

$$-\frac{\sin(2\,bx+2\,a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*sin(2*b*x + 2*a)/b
```

3.949 $\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx$

Optimal. Leaf size=54

$$\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}-\frac{\log(\tan(a+bx)+1)}{3b}+\frac{\log(\cos(a+bx))}{b}$$

```
[Out] Log[Cos[a + b*x]]/b - Log[1 + Tan[a + b*x]]/(3*b) + (2*Log[1 - Tan[a + b*x]
 + Tan[a + b*x]^2])/(3*b)
```

Rubi [A] time = 0.531603, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6725, 260, 628}

$$\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}-\frac{\log(\tan(a+bx)+1)}{3b}+\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]^3),x]
```

```
[Out] Log[Cos[a + b*x]]/b - Log[1 + Tan[a + b*x]]/(3*b) + (2*Log[1 - Tan[a + b*x]
 + Tan[a + b*x]^2])/(3*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^3}{(1+x^2)(1+x^3)}\,dx,x,\tan(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{3(1+x)}-\frac{x}{1+x^2}+\frac{2(-1+2x)}{3(1-x+x^2)}\right)dx,x,\tan(a+bx)\right)}{b}$$

$$= -\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\tan(a+bx)\right)}{3b}-\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\right.}{}$$

$$= \frac{\log(\cos(a+bx))}{b}-\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\log\left(1-\tan(a+bx)+\tan^2(a+bx\right.}{3b}$$

Mathematica [A] time = 0.236641, size = 42, normalized size = 0.78

$$\frac{2\log(2-\sin(2(a+bx)))}{3b}-\frac{\log(\sin(a+bx)+\cos(a+bx))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]
^3),x]
```

```
[Out] -Log[Cos[a + b*x] + Sin[a + b*x]]/(3*b) + (2*Log[2 - Sin[2*(a + b*x)]])/(3*
b)
```

Maple [A] time = 0.26, size = 56, normalized size = 1.

$$-\frac{\ln(1+\tan(bx+a))}{3b}-\frac{\ln\left(1+(\tan(bx+a))^2\right)}{2b}+\frac{2\ln\left(1-\tan(bx+a)+(\tan(bx+a))^2\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x)
```

```
[Out] -1/3*ln(1+tan(b*x+a))/b-1/2/b*ln(1+tan(b*x+a)^2)+2/3*ln(1-tan(b*x+a)+tan(b*
x+a)^2)/b
```

Maxima [B] time = 1.46416, size = 208, normalized size = 3.85

$$\frac{2\,\log\left(-\frac{2\,\sin(bx+a)}{\cos(bx+a)+1}+\frac{2\,\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{2\,\sin(bx+a)^3}{(\cos(bx+a)+1)^3}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1\right)-\log\left(-\frac{2\,\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}-1\right)-3\,\log\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="maxima")
```

```
[Out] 1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x +
 a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b*
x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^
2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1
))/b
```

Fricas [A] time = 2.23174, size = 117, normalized size = 2.17

$$-\frac{\log\left(2\,\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)-4\,\log\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="fricas")
```

```
[Out] -1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x +
a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**3+sec(b*x+a)**3)/(csc(b*x+a)**3+sec(b*x+a)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2281, size = 70, normalized size = 1.3

$$\frac{4\,\log\left(\tan\left(bx+a\right)^{2}-\tan\left(bx+a\right)+1\right)-3\,\log\left(\tan\left(bx+a\right)^{2}+1\right)-2\,\log\left(|\tan\left(bx+a\right)+1|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="giac")
```

```
[Out] 1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1) -
 2*log(abs(tan(b*x + a) + 1)))/b
```

3.950 $\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx$

Optimal. Leaf size=72

$$\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}-\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}$$

```
[Out] Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b) - Log[1 + Sqrt
[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rubi [A] time = 1.39599, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1165, 628}

$$\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}-\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]^4),x]
```

```
[Out] Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b) - Log[1 + Sqrt
[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{1+x^4}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \tan(a+bx)\right.}{2\sqrt{2}b}$$
$$= \frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\sqrt{2}\tan(a+bx)+\tan^2(a+\right.}{2\sqrt{2}b}$$

Mathematica [A] time = 0.0252557, size = 26, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]
^4),x]
```

```
[Out] -(ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

Maple [A] time = 0.156, size = 108, normalized size = 1.5

$$-\frac{\sqrt{2}}{8\,b}\ln\left(\frac{1+\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}{1-\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}\right)+\frac{\sqrt{2}}{8\,b}\ln\left(\frac{1-\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}{1+\sqrt{2}\tan(bx+a)+(\tan(bx+a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x)
```

```
[Out] -1/8/b*2^(1/2)*ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*x+a)
+tan(b*x+a)^2))+1/8/b*2^(1/2)*ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(
1/2)*tan(b*x+a)+tan(b*x+a)^2))
```

Maxima [A] time = 1.43122, size = 78, normalized size = 1.08

$$-\frac{\sqrt{2}\log\left(\tan\left(bx+a\right)^{2}+\sqrt{2}\tan\left(bx+a\right)+1\right)-\sqrt{2}\log\left(\tan\left(bx+a\right)^{2}-\sqrt{2}\tan\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(
tan(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

Fricas [A] time = 2.24861, size = 193, normalized size = 2.68

$$\frac{\sqrt{2}\log\left(-\frac{2\,\cos\left(bx+a\right)^{4}+2\,\sqrt{2}\cos\left(bx+a\right)\sin\left(bx+a\right)-2\,\cos\left(bx+a\right)^{2}-1}{2\,\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 + 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**4+sec(b*x+a)**4)/(csc(b*x+a)**4+sec(b*x+a)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22519, size = 65, normalized size = 0.9

$$\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sin(2bx+2a)\right|}{\left|2\sqrt{2}+2\sin(2bx+2a)\right|}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin(
2*b*x + 2*a)))/b
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```